Initial Solution Heuristic for Portfolio Optimization of Electricity Markets Participation

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Abstract. Meta-heuristic search methods are used to find near optimal global solutions for difficult optimization problems. These meta-heuristic processes usually require some kind of knowledge to overcome the local optimum locations. One way to achieve diversification is to start the search procedure from a solution already obtained through another method. Since this solution is already validated the algorithm will converge easily to a greater global solution. In this work, several well-known meta-heuristics are used to solve the problem of electricity markets participation portfolio optimization. Their search performance is compared to the performance of a proposed hybrid method (ad-hoc heuristic to generate the initial solution, which is combined with the search method). The addressed problem is the portfolio optimization for energy markets participation, where there are different markets where it is possible to negotiate. In this way the result will be the optimal allocation of electricity in the different markets in order to obtain the maximum return quantified through the objective function.

Keywords: Electricity Markets, Heuristic Search, Meta-heuristic Optimization, Portfolio Optimization

1 Introduction

Metaheuristics can be defined as a set of search methods, such as construction heuristics, local search and more general orientation criteria to solve a specific problem. These have attracted the attention of many users due to their simplicity of implementation. In turn, the metaheuristics do not always reach an optimal solution, even for long computing times, but they manage to arrive at a near-optimal solution in a short time, which a deterministic resolution cannot obtain [1].

The performance measures, such as the value of the target solution and the execution time can be seen as random variables, because with an algorithm of this nature it is never known beforehand what the final result will be. Support for research on

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metaheuristics can be provided by statistics. With statistics, it is possible to construct a systematic framework for the collection and evaluation of data, maximizing the objectivity and the reproducibility of the experiences. It is possible to construct a mathematical foundation that provides a probabilistic measure of events based on inference from the empirical data. In [2], it is possible to analyze the use of statistical tools in the study of algorithms and heuristics.

The work presented in [3] analyzes and compares the use of this type of algorithms, where the analysis is executed considering two models defined by the author:

- The univariate model, which considers the cost of the solution or the execution time;
- The multivariate model, in which both the cost of the solution and the execution time are of interest.

In the first case, the user is concerned with the cost of the solution (e.g., maximization / minimization problem) or the execution time as a measure of algorithm performance. When the interest is the cost of the solution, it is assumed that the computational resources are used in the same way by the different algorithms under study, it is called the principle of fairness. On the other hand, if the execution time is the parameter to be analyzed, it will have to be taken into account the number of times that a solution is obtained with the same characteristics in all the algorithms [4].

In the second case, the performance analysis of the algorithm includes the cost of solution and the execution time. In this case, the analysis falls within the scope of multivariate statistics, the authors [4] distinguish two specific scenarios about the multivariate model that may be of interest to the user, both are based on the cost of the solution and the time of execution, although they are distinguished by: In the first scenario the user only registers the final value (cost function x run time), and in the second scenario the user must save a set of values from the beginning of the search to the end, at each iteration it will save the value (cost function x run time).

In order to be able to return a solution to the problems described above, the case of the final value of the cost function can be a random value, the following methodology is proposed, as expressed in the pseudo-code of Fig. 1.

```
Initialise i = 1
while (Stopping condition is not satisfied)
{
        Setp 1. (Generation)
                Construct solution x_i
        Step 2. (Search)
                Apply a search method to improve x_i
                Let x'_i be the solution obtained
        if (s'_i improves the best)
                Update the best
        i = i + 1
}
```

Fig. 1 Multi-Start Procedure [5]

Fig. 1 shows the pseudo-code of a multi start procedure. The solution x_i is constructed in Step 1 at iteration i. This is a typically performed with an iterative algorithm. Step 2 is devoted to improving this solution, obtaining solution x_i'. A simple improvement method can be applied. However, this second phase has recently become more elaborate and, in some cases, is performed with a complex metaheuristic that may or may not improve the initial solution x_i (in this latter case we set $x_i' = x_i$) [5].

This type of procedure is called multi-start methods, considering the pseudo-code of the Fig. 1, in the first step the construction of the initial solution can be developed through a simpler search method (Local Search (LS), Tabu Search (TS)), and in step 2, a more elaborate method (Particle Swarm Optimization (PSO), Genetic Algorithm (GA), or Simulated Annealing (SA)), which will require more complexity in implementation. More search time should be given to the second step, because a more powerful algorithm is used to obtain better values in its performance.

The presented pseudo-code can be applied to any type of problem as long as it has a type of objective function to solve. In the case addressed by this paper, the problem in hands is the optimization of portfolios in electricity markets. This paper proposes a heuristic methodology to determine an initial solution in the portfolio optimization problem. The objective of this heuristic is to provide a good initial point for the search process, so that the meta-heuristics can achieve solutions nearer to the global optimum, and in faster execution times (without the need for long search processes). According to the pseudo-code presented above, in the first step a solution is generated through the use of the heuristic created for this purpose. In the second step, the optimization with different algorithms, PSO, GA and SA, is performed. In the end, a comparison is made on the performance of the algorithms with the use of the proposed initial solution heuristic and without its use. This comparison takes into account the multivariate analysis model, where it considers the value of the function cost, execution time and also the number of iterations.

After this introductory section, section 2 presents the mathematical formulation of the electricity markets participation portfolio optimization problem that is addressed. The proposed heuristic for generating the initial solution is also presented in this section. Section 3 presents the description of the case study used to validate the proposed method, and section 4 presents the achieved results. Finally, section 5 presents the most relevant conclusions of this work.

2 Portfolio optimization

The portfolio optimization problem was firstly introduced by Henry Markowitz [6], with application in the field of finance and economics. This problem addressed by Markowitz considers a model which efficiently allocates a number of assets so that the future will bring positive return with a certain level of risk. In energy markets this problem is also relevant, especially concerning the support of market negotiating player's decisions. Given the available market opportunities, players need to decide whether to and how to participate in each market type, in order to obtain as much gain as possible from their negotiations.

The problem of portfolio optimization has been applied in different areas, but more important than that is the techniques that have been applied to try to solve it. In 1956, the author who presented the methodology presented in [7] a discussion about the application of a computational technique to solve the model using the formulation of quadratic problems. But three years later Philip Wolfe in [8], proposes the resolution of the problem using the simplex method.

Years later, with the development of science and technology, the artificial intelligence (AI) was born, and it would bring with it the intelligent research algorithms. Nowadays many types of meta-heuristics have been applied to solve the problem, for example, local search techniques were applied in [9], the SA optimization technique was implemented to the problem in [10], the PSO [11], neural networks (NN) [12] and GA [13] were also already used to solve the problem of portfolio optimization in different fields of application. The application of the portfolio optimization model in the electric sector has, however, been a rather absent subject of research. Among the few exceptions are a model of risk management in the short term, by applying the Markowitz model to optimize the portfolio and minimize risk in energy markets [14].

A methodology for participation in electricity markets for the following day is presented in [15] using portfolio optimization. IA techniques are used in this work, namely the PSO. In this publication, the author's main objective is to provide support to the participants' participation in electricity markets. For this reason, the methodology proposed by the author is part of a decision support system called Multi-Agent Simulator for Competitive Electricity Markets (MASCEM) [16].

In this work, as previously mentioned, a heuristic is presented, which allows the construction of a valid initial solution, which serves as input data for the portfolio optimization problem. This heuristic thus aims at improving the initial search point from different AI algorithms. The final results are used to provide decision support to electricity market participants in order to aid them in taking the most profitable negotiation decisions.

2.1 Mathematical formulation

In this section the presentation of the proposed model for the optimization is made. It should be emphasized that the considered model does not follow mathematically the model proposed by Markowitz, but the basis concept is the same. The proposed model, tries to allocate electricity in the different electricity markets in an optimal way, generating a maximum level of profit, while respecting the imposed rules.

Equation (1) represented the objective function, which models the optimization of players' market participation portfolio. This function considers the expected production of a market player for each period of each day, and the amount of power to be negotiated in each market is optimized to get the maximum income that can be achieved [15].

$$(Spow_{M \dots NumS}, Bpow_{S1 \dots NumS}) = Max \left[\begin{array}{c} \sum_{M=M1}^{NumM} (Spow_{M,d,p} \times ps_{M,d,p} \times Asell_M) - \\ \sum_{S=S1}^{NumS} (Bpow_S \times ps_{S,d,p} \times Abuy_S) \end{array} \right] \quad (1)$$

$$\forall d \in Nday, \forall p \in Nper, Asell_M \in \{0,1\}, Abuy \in \{0,1\}$$

In equation (1) d represents the weekday, $Nday$ represent the number of days, p represents the negotiation period, $Nper$ represent the number of negotiation periods, $Asell_M$ and $Abuy_S$ are boolean variables, indicating if this player can enter negotiations in each market type, M represents the referred market, $NumM$ represents the number of markets, S represents a session of the balancing market, and $NumS$ represents the number of sessions. Variables $ps_{M,d,p}$ and $ps_{S,d,p}$ represent the expected (forecasted) prices of selling and buying electricity in each session of each market type, in each period of each day. The outputs are $Spow_M$ representing the amount of power to sell in market M, and $Bpow_S$ representing the amount of power to buy in session S.

In equation (2) is expressed the way in which the negotiation prices are obtained. As one can see, sale prices $ps_{M,d,p}$ and purchase prices $ps_{S,d,p}$ are considered.

$$\begin{array}{c} ps_{M,d,p} = Value(d, p, Spow_M, M) \\ ps_{S,d,p} = Value(d, p, Bpow_S, S) \end{array} \quad (2)$$

The $Value$ is obtained by equation (3), and is calculated from the application of the clustering and fuzzy approach.

$$\text{Value(day,per,Pow,Market)=Data(fuzzy(pow),day,per,Market)} \quad (3)$$

With the implementation of this model, it is possible to obtain market prices based on the traded amount. In order to achieve this, for the modeling of the prices are considered the expected production of a market player for each period of each day. The results of the application of this methodology can be observed in [17]. Equation (3) defines this condition, where Data refers to the historical data that correlates the amount of transacted power, the day, period of the day and the particular market session.

Equation (4) represents the main constraint of this problem. The constraint imposes that the total power that can be sold in the set of all markets is never higher than the total expect production (TEP) of the player, plus the total of purchased power.

$$\sum_{M=M1}^{NumM} Spow_M \leq TEP + \sum_{S=S1}^{NumS} Bpow_S \quad (4)$$

Equations (5), (6) and (7) represent other constraints that can be applied to the problem. This depends on the nature of the problem itself, e.g. type of each market, negotiation amount, type of supported player (renewable based generation, cogeneration, etc.).

$$TEP = \sum Energy_{prod}, Energy_{prod} \in \{Renew_{prod}, Therm_{prod}\} \quad (5)$$

$$0 \leq Renew_{prod} \leq Max_{prod} \quad (6)$$

$$Min_{prod} \leq Therm_{prod} \leq Max_{prod}, if\ Therm_{prod} > 0 \quad (7)$$

By (5) it can be seen that the energy production may come from renewable sources and thermoelectric sources. If the player is a producer of thermoelectric power, the production must be set at a minimum since it is not feasible to completely turn off the production plant, as can be observed by equation (7). If the producer is based on renewable energy, the only restriction is the maximum production capacity, as in (6).

2.2 Proposed initial solution heuristic

The proposed heuristic for initial solutions generation aims at providing an adequate initial point for metaheuristics search process. Thus, the goal is to use ad-hoc knowledge on the problem to create a set of generic rules that allow initial solutions to be generated automatically in order to feed the metaheuristic methods. The proposed heuristic is composed by the following steps:

- 1st - In the Spot market, since it is impossible to buy energy, when the player participates to sell, the value of this variable is automatically zero (8);

$$if\ M = \text{Spot Market}, Bpow_M = 0 \quad (8)$$

- 2nd - The spot market price is compared with the prices of the intraday and balancing market sessions, and the higher price is saved (9);

$$search\ (\max ps_{M,d,p}), save\ ps_{M,d,p} \quad (9)$$

- 3rd - The sale or purchase price is calculated for bilateral contracts and local markets (considered in this model as Smart Grid (SG) level markets), considering the maximum amount of power available for purchase. If the maximum selling price is greater than these, the maximum purchase quantity in the two previous markets is allocated (10). This enables players to purchase power at lower prices in order to sell it in market opportunities with higher expected price;

$$if\ saved\ price \geq\ ps_{M,d,p}(\max Bpow)$$
$$Bpow_M = \max quantity\ , for\ M = Bilateral\ and\ SG \quad (10)$$

where:

  - $\max quantity$ is the maximum purchase quantity

- 4th - Since in the various sessions of the balancing and intraday market it is only possible to perform one of the shares in each negotiation period (buy or sell), the maximum price verified in all market sessions is compared with the purchase price of the maximum quantity in each of the balancing market sessions. If the maximum price is higher, the maximum amount of purchase will be automatically allocated in the balancing or intraday market sessions (11), following the same logic as in step 3;

$$\begin{gathered} if\ ps_{M,d,p}(\max Bpow)\ for\ balancing\ sections \leq price\ saved \\ Bpow_M = \max quantity\ , for\ balancing\ session \end{gathered} \tag{11}$$

- 5th- The sum of the power amount allocated to be bought with the available quantity resulting from own production is obtained, thus obtaining the total quantity available to sell (12);

$$\max quantity\ \ for\ sale = \max quantity\ \ buy + TEP \tag{12}$$

- 6th - Since there are markets where the expected price is highly dependent on the negotiated quantity (e.g. bilateral contracts), the sale price in those markets is calculated for several intervals of quantities (e.g. from 10 in 10 MW) up to the maximum available for sale (13);

$$\begin{gathered} if\ Bpow, Spow\ dependent\ the\ quantity, search\ the\ best\ option \\ best\ option =\ best\ value\ in\ all\ intervals \end{gathered} \tag{13}$$

- 7th - A search is made iteratively to search if there is an amount of electricity in which the price in the market is higher than the maximum found price. This search is done only for markets where the expected price is highly dependent on the negotiated quantity. If any is found, the correspondent quantity is allocated to that market (14).

$$\begin{gathered} if\ M = Bilateral\ and\ SG, \\ search\ quantity\ where\ price\ \ > saved\ price \end{gathered} \tag{14}$$

- 8th - The quantity available for sale is updated based on the amount allocated in the two previous markets (15);

$$\begin{gathered} sale\ quanty = \max quanty\ \ for\ sale -\ Spow_M;\ M \\ = (Bilateral, SG) \end{gathered} \tag{15}$$

- 9th - To allocate the electricity that is lacking, look for the market where the remaining amount can be more profitable and, respecting the impossibility of buying and selling in the same market, this quantity is allocated (16).

$$Bpow = max\ price\ for\ sale\ quantity, for\ Spot\ or\ Balancing \tag{16}$$

After all the steps have been completed, the constraints of the problem must be applied in order to guarantee that the solution is valid and that the research is started without creating a random solution.

3 Case study

This case study considers seven different metaheuristic algorithms to perform the optimization of the presented portfolio optimization formulation, being them: PSO [18], EPSO [19], QPSO [20], NPSO-LRS [21], MPSO-TVAC [22], AG [23] and SA [24]. The proposed heuristic is used to find an initial solution for the algorithms. A comparison is made between the performances of the algorithms when using the proposed initial solution heuristic and when using a random initial solution.

In order to define a realistic scenario, five different market types have been considered, thereby enabling the supported market player to sell and buy in all of them. The considered markets are the day-ahead spot market, negotiations by means of bilateral contracts, the balancing (or intra-day) market, and a local market, at the Smart-Grid (SG) level. The balancing market is divided into different sessions. In the day-ahead spot market the player (acting as seller) is only allowed to sell electricity, while in the other market types the player can either buy or sell depending on the expected prices. Limits have also been imposed on the possible amount of negotiation in each market. In this case, it is only possible to buy up to 10 MW in each market in each period of negotiation, which makes a total of 40 MW purchased. It is possible to sell power on any market, and it can be transacted as a whole or in installments. The player has 10 MW of own production for sale.

In this problem, it has also been imposed that in each session of the balancing market, the player can only either sell or buy in each period. In bilateral contracts and in SG negotiation, it is possible to both sell and purchase in the same period (by negotiating with different players). Since the optimization requires real market data, so that it can be used to support players' decisions in a realistic environment, it is necessary that the electricity prices are provided. The real electricity market prices data, concerning the day-ahead spot market, the intraday market, and bilateral contracts have been extracted from website of the Iberian electricity market operator –MIBEL [25]. Local SG market prices are based on the results of previous studies [15].

4 Results

Table 1 presents the results for the first period of the considered simulation day, of the various methods when using a random initial solution, and when using the initial solution generated by the heuristic proposed in section 2.2 (methods with the suffix -ST). When using the proposed heuristic, all the presented algorithms start their search from a solution already defined by the set of rules expressed by the heuristic. It should be noted that the algorithms were all applied in the same conditions (same machine, same input data, etc.) so that there are no variations due to external factors. A total of 1000 executions have been run for each simulation.

As can be seen from Table 1, when the algorithms are executed using the initial solution generated by the proposed heuristic, the observed values for the objective function undergo changes. The algorithms that use the initial solution, as expected, present a minimal solution larger than those that do not use it. In terms of maximum reached value, most of the algorithms can reach very close maxima, with or without initial solution. The algorithms using the initial solution have a higher average than others, as well as a lower standard deviation, which means that the variability of results when using an initial solution is lower. In terms of execution time and required number of iterations, the values generally decrease when using the proposed heuristic.

Figure 2 presents the box plots for all applied methods. The box plots are constructed based on the observed results obtained in the simulations, namely the maximum and minimum, and from three calculated parameters: the median, the value of the first quartile and the third quartile. This representation is very useful because it allows a representation of how the results data is distributed as to the greater or lesser concentration, symmetry or existence of values outside the context of the results. It is also very useful in comparing groups of results.

Table 1. Optimization results for the different methods

Algorithms	Value of Objective Function (€)				Execution Time (s)		Iteration number	
	Min	Max	Mean	STD	Mean	STD	Mean	STD
PSO	571,5	1998,6	1483,8	270	0,184	0,035	64	10,9
PSO - ST	1805,7	2000,6	1981,1	47,4	1,046	0,277	384	101
EPSO	482,8	2000,6	1579,4	307	27,93	54,814	1621	3173,9
EPSO - ST	1875,1	2000,6	1972,7	28,4	13,168	30,568	783	1808,2
QPSO	320,7	1998,9	1232	305	0,292	0,116	61	35,2
QPSO - ST	1730,2	2000,6	1939,2	56,4	0,282	0,09	63	26
NPSO-LRS	1416,4	2000,6	1762,4	144	1,5	0,466	363	112
NPSO-LRS - ST	1889,1	2000,6	1992,1	20,9	1,806	0,499	448	122
MPSO-TVAC	1416,6	2000,6	1947,2	133	6,841	0,871	492	60,2
MPSO-TVAC - ST	1816,7	2000,6	1873,7	85,1	4,059	0,232	298	11,4
AG	1545,5	2000,6	1971,2	76,4	7,478	0,679	2625	218
AG - ST	1730,2	2000,6	1993	40,3	4,728	0,743	1663	255
SA	1781,5	1927,2	1884	55,5	0,551	0,021	1831	26,1
SA - ST	1945	2000,6	1988,3	11,7	0,51	0,02	1730	6,4

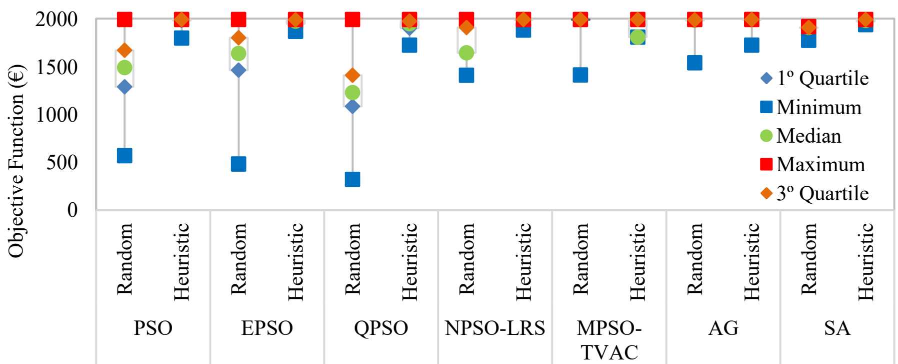

Fig. 2 Box plot for the applied methods

As can be seen from Figure 2, it is visible that the minimum values undergo considerable changes when using the proposed heuristic for generation of the initial solution. It is noteworthy that the QPSO, which was the algorithm that suffered the greatest change of minimum value, went from 1232.7 € to 1956 €. Another of the characteristics to be considered is the value between 1st quartile and 3rd quartile, since this distance represents 50% of the observations; the less the distance the more reliable the method will be. All methods using the proposed heuristic have significantly decreased the distance between these two quartiles. In terms of maximum values, although the algorithms with the initial solution reached the best maximums, the previous versions with random initial solutions also managed to reach very close values (at the cost of higher execution times and variability); except the SA, as it was the method that obtained a greater improvement.

Table 2 presents the results of upper bound, lower bound and error values for the 95% confidence intervals of all the executed simulations (1000 simulations).

Table 2. Error of confidence interval 95%

Type solution		**PSO**	**EPSO**	**QPSO**	**NPSO-LRS**	**MPSO-TVAC**	**AG**	**SA**
Random	Upper bound	1500,58	1598,41	1250,89	1771,31	1955,41	1975,91	1887,48
	Lower bound	1467,09	1560,38	1213,03	1753,42	1938,90	1966,44	1880,61
	Error	16,74	19,01	18,93	8,94	8,25	4,73	3,43
Heuristic	Upper bound	1984,01	1974,49	1942,69	1993,35	1879,00	1995,46	1988,99
	Lower bound	1978,13	1970,97	1935,70	1990,76	1868,46	1990,47	1987,54
	Error	2,90	1,75	3,49	1,29	5,27	2,49	0,72

From Table 2 it can be verified that there is a great difference of values between the results when using the random solution and the heuristic for the majority of the presented methods. The PSO, EPSO and QPSO methods are the ones that benefit most with the proposed method, because as it is possible to observe, both the upper and lower limits have risen considerably to near the maximum value. On the other hand, the error value has decreased considerably. From the analysis of the results of the 95% confidence intervals, the proposed heuristic for the determination of the initial solution shows clear advantages over the random solution.

In Fig. 3 is presented the performance of NPSO-LRS with random solution and heuristic solution. This algorithm is chosen because it showed a great difference in terms of objetive funtion STD when comparing the random and heuristic solution.

From Fig. 3 it can be seen that both algorithms converge to very close maximum solutions. The big difference is in the STD, which decreases considerably when using the proposed initial solution heuristic (from 144 to 20.9 when using the random initial solution). It should also be noted that the results of the heuristic solution have a scale on the y-axis different from the random solution, this is due to the initial solution of the heuristic solution being 1730 €. Fig. 4 shows the energy purchased and sold in each market for the SA whne using the random solution and heuristic solution, this algorithm was chosen because it is possible to observe the large diferences in the two algorithms.

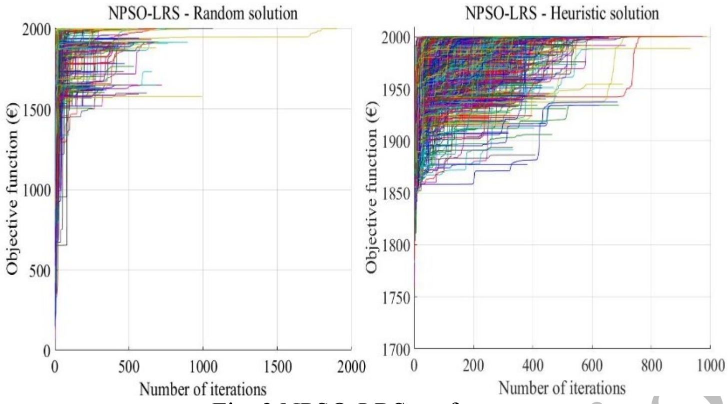

Fig. 3 NPSO-LRS performance

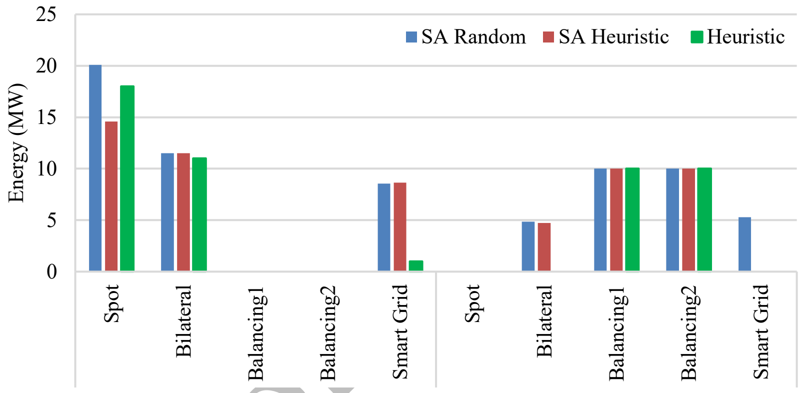

Fig. 4 Purchase and the sale in different markets

Figure 4 shows the results of the SA algorithm when using the random solution, when using the proposed heuristic for initial solution and also the values generated by the proposed heuristic by itself. By analyzing the objective function values, the SA Random registered 1927 €, SA heuristic 2000,6 € and heuristic 1730 €. It is possible to verify that the heuristic presents a solution that is already close to the maximum vale achieved by the meta-heuristic search process, and then it is up to the algorithms to refine that solution to obtain a better result. The SA heuristic was the one that registered the best value of objective function and one can see that it defined to buy the maximum quantity of electricity in the balancing markets (10 MW), and to purchase of 4.7 MW in bilateral contracts. The sale is set to the Smart Grid in 8.6 MW, 13.3 MW in Bilateral Contracts and 13.8 MW in the Spot market.

A curiosity is that SA random has a higher volume of transacted electricity (40 MW), but does not represent a greater profit because it was necessary to buy more electricity. The SA heuristic traded 34.7 MW and made a better profit.

5 Conclusions

This paper presented a heuristic to generate a good initial solution for meta-heuristic methods. The proposed heuristic proved to be advantageous since the results present considerable advantages in relation to the previous results, using random initial solutions. By defining this starting point for the algorithms they can obtain better values in their general search process. This is visible by the analysis of the STD, because the lower value means that the solutions are closer to the average. The average with this modification also gets closer to the maximum value which, coupled with the small STD, makes the set of solutions very strong.

As future work, authors intend create a multiobjective a model of that considers the risk in calculation of the return, as well as to develop a heuristic that allows the same association between profit and risk.

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