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Leak Localization in Water Distribution Networks using Fisher Discriminant Analysis

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Abstract: This paper addresses the problem of leak localization in water distribution networks (WDN) using Fisher Discriminant Analysis (FDA). First, the paper introduces how FDA can be used for leak localization using the information of pressure measurements from the sensors available in the WDN. Then, the problem of sensor placement is considered when the proposed leak localization based on FDA is used. The proposed leak localization and sensor placement approaches based on FDA will be used using a well-known WDN case study.

Keywords: Leak localization, Water distribution network, Fisher discriminant analysis, Sensor placement

1. INTRODUCTION

Water leaks in networks can cause significant economic losses in the fluid transportation and an increase on reparation costs, giving as a consequence an extra cost translated to the final consumer. In many water distribution networks (WDN), losses due to leaks are estimated to account up to 30 % of the total amount of extracted water. Such burden cannot be tolerated in a world struggling with satisfying water demands of a growing population.

In the field of leak detection and localization in the WDNs, several techniques are applied and more are in development. In (Fanner et al., 2007; Puust et al., 2010; Mutikanga et al., 2012), recent reviews are presented about the most extended methods and techniques presented so far. Another interesting review is presented in (Li et al., 2015) where the methods are classified as hardware based and software based. In (Wu and Liu, 2017), the data-driven approaches, focused mainly in the leak detection problem, are reviewed.

Several model based approaches have been developed using the hydraulic model. In (Perez et al., 2011), the hydraulic model is simulated under different leak scenarios. Then, applying a sensitivity analysis, a binary leak signature matrix is obtained that is used for matching the residuals obtained from the comparison of the pressure measurements and their estimation using the hydraulic model. In (Quevedo et al., 2012), the matrix is not binarized and the node candidate is the one that has the most correlated signature with the residual, this work was extended by taking into account the demand uncertainty in (Pérez et al., 2016). In (Casillas et al., 2012), the angle between every leak signature in the sensitivity matrix and the residual is calculated, and the closest one (minimum angle) is the node candidate. Additionally, a time horizon is introduced into the analysis. An improved technique (Casillas et al., 2013) to reduce the impact of the non-linearity leak behaviour is proposed. In this approach, one residual is used to normalize the others and then, by means of the Euclidean distance, the leak is located finding the minimum distance between the residual obtained and the different columns of the sensitivity matrix. An alternative to the use of the sensitivity matrix approach, it is based on the application of the structural analysis as explained in (Rosich et al., 2014).

This paper addresses the problem of leak localization in water distribution networks using the Fisher Discriminant Analysis (FDA). FDA is a pattern classification method used to find the optimal linear combination of features which best separate different classes, where in this context each class corresponds to data collected during a specific known leak scenario. It is an empirical method based on observed attributes over the collected examples and has been successfully applied to fault diagnosis (Chiang et al., 2001; Garcia-Alvarez et al., 2009). First, the paper introduces how FDA can be used for leak localization using the information of pressure measurements from the sensors available in the WDN and the sensitivity analysis. Then, the problem of sensor placement is considered when the proposed leak localization based on FDA is used. The proposed leak localization and sensor placement approaches based on FDA have been applied to a well-known WDN case study.

The structure of the paper is the following: In Section 2, after the leak localization problem formulation, the proposed

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approach based on Fisher discriminant analysis is presented. In Section 3, the sensor placement problem is also addressed. Section 4 presents the results of the leak localization and sensor placement approaches in a well-known WDN, the Hanoi case study. Finally, Section 5 summarizes the main conclusions and presents future work.

2. PROPOSED APPROACH

2.1 Problem formulation

The main objective of the proposed approach is to localize leaks in a WDN using pressure measurements from the sensors installed in some of the nodes. A leak will be considered as a water flow loss through a defect of a network element. The paper considers single and continuous leaks once they have appeared.

The proposed leak localization methodology is applied to a WDN once a leak is detected by means of certain data analyses involving the night flows together with the supplied/billed amount of water (Puust et al., 2010).

The proposed approach involves several stages. First, data of node pressures are obtained from extensive simulations of normal and leak scenarios. Then, the residuals $r = \begin{bmatrix} r_1 & \cdots & r_{n_s} \end{bmatrix}^T$ are calculated as the difference between the pressure measurements, p_i, and its corresponding estimation, $\hat{p}_i$, obtained from the simulation of the hydraulic model with no leak, i.e, $r_i = p_i - \hat{p}_i$ After that a sensitivity analysis of these residuals is carried out (Casillas et al., 2013)

$$Sens_{ij} = \frac{\hat{p}_i^{f_j} - \hat{p}_i}{f_j} \tag{1}$$

for $i = 1, \cdots, n_s$ and $j = 1, \cdots, n$ (n_s is the number of sensors and n is the number of leaks that corresponds with the number of nodes), and where $\hat{p}_i^{f_j}$ and $\hat{p}_i$ are the pressure estimation obtained from the hydraulic model simulation under the leak f_j scenario and the leak-free scenario, respectively. More precisely, each simulated fault scenario is performed by injecting a leak of a magnitude f_j in the j-th network node in order to compute the sensitivity matrix (1) using a hydraulic simulator (as e.g. EPANET). From these sensitivity data, FDA is applied to characterize the different leak classes. Then, once the different leak classes have been characterized, data from an unknown leak scenario is presented to the FDA algorithm that will try to match the observed scenario to the one of the already characterized leak classes. The class that presents the best matching is proposed as the leak candidate.

In real WDN, the proposed leak localization approach should be applied taking into account that the number of pressure measurements will be reduced ($n_s < n$). This will be discussed in Section 3.

2.2 Proposed Approach

In the following the proposed leak localization approach based on FDA is presented in detail.

2.2.1 Characterization of the different leak classes

The first stage is the construction of the following matrices, D_j, one for each leak scenario *j=1,...,n,* that contains the sensitivity $Sens_{i,k}$ for each node (*i=1,...,n*) and the time instant considered, i.e., the number of observations collected from the network (*k=1,...,m*):

$$D_j = \begin{bmatrix} Sens_{1,1} & \cdots & Sens_{1,n} \\ \vdots & \ddots & \vdots \\ Sens_{m,1} & \cdots & Sens_{m,n} \end{bmatrix} \tag{2}$$

These matrices are the starting point for the application of the FDA methodology. To this aim n classes are defined, one corresponding to each leak scenario considered.

Per each class *j=1,...,n* with $X_j = D_j$, the mean and the within-scatter matrix is defined as follows:

$$\bar{x}_j = \frac{1}{m} \sum_i^m x_i \tag{3}$$

$$S_j = \sum_i^m \left(x_i - \bar{x}_i\right)\left(x_i - \bar{x}_i\right)^T \tag{4}$$

From the dispersion matrix of each class, the within-class-scatter matrix can be evaluated as follows

$$S_w = \sum_j^n S_j \tag{5}$$

The previous analysis is repeated now for all the classes, and the matrix X is defined as follows

$$X = \begin{bmatrix} D_1 \\ \vdots \\ D_n \end{bmatrix} \tag{6}$$

From (6), the mean and total-scatter matrix is given by

$$\bar{x} = \frac{1}{m \cdot n} \sum_i^{m \cdot n} x_i \tag{7}$$

$$S_t = \sum_i^{m \cdot n} \left(x_i - \bar{x}\right)\left(x_i - \bar{x}\right)^T \tag{8}$$

Then, the between-class-scatter matrix, S_b, can be determined taking into account that the total-scatter matrix can be evaluated as follows:

$$S_t = S_b + S_w \tag{9}$$

so, $S_b = S_t - S_w$ that can be calculated using (5) and (8). Now the objective of the first FDA vector, w_1, is to maximize the scatter between classes while minimizing the scatter within classes:

$$\max_{w_1 \neq 0} \frac{w_1^T S_b w_1}{w_1^T S_w w_1} \tag{10}$$

with $w_1 \in R^n$. The second FDA vector, w_2, is computed so as to maximize the scatter between classes while minimizing the scatter within classes on all axes perpendicular to the first FDA vector, and so on for the remaining FDA vectors. These vectors are equal to the eigenvectors w_k of the generalized eigenvalue problem:

$$S_b w_k = \lambda_k S_w w_k \tag{11}$$

where the eigenvalues λ_k indicate the degree of separability of each class and they are sorted in decreasing order and w_k are their eigenvectors associated. From (11), a reduction of dimensionality can be applied by choosing a set of eigenvectors W_a corresponding to the biggest eigenvalues λ_a, i.e., by retaining only those components which eigenvalues satisfying

$$\sum_i^a \lambda_i \geq \alpha \sum_i^n \lambda_i \tag{12}$$

where α is a parameter that establishes the level of approximation (a typical value is $\alpha = 0.95$). Then, the linear transformation of the data matrix (6) from the n-dimensional space to the reduced a-dimensional space generated by the FDA vectors is:

$$z_i = W_a^T x_i \tag{13}$$

FDA computes W_a such as the data matrix (6) for the n classes are optimally separated when projected into the a dimensional space.

2.2.2 Identification of the leak candidate

In the second phase, after being characterized the different leak classes using FDA using data in simulation, given an unknown leak scenario, the leak localization can be done by means of the evaluation of the discriminant function per each class and the current leak scenario.

An observed leak scenario is assigned to the class i when the maximum discriminant function value, g_i, satisfies:

$$g_i(x) > g_j(x) \quad \forall j \neq i \tag{14}$$

where $g_j(x)$ is the discriminant function for class j given a vector $x \in \mathbf{R}^n$. The discriminant function that minimizes the error rate, when the leak f_i occurs is (Chiang et al., 2001):

$$g_i(x) = P(f_i \mid x) \tag{15}$$

where $P(f_i \mid x)$ is the a posteriori probability of x belonging to class i. Using Bayes' rule:

$$P(f_i \mid x) = \frac{P(x \mid f_i) P(f_i)}{P(x)} \tag{16}$$

where $P(f_i)$ is the a priori probability for class f_i, $P(x)$ is the probability density function for x and $P(x \mid f_i)$ is the probability density function for x conditioned to f_i. If the data for each class is normally distributed, $P(x \mid f_i)$ is given by:

$$P(x|f_i) = \frac{1}{(2\pi)^{m/2}\left[\det(\Sigma_i)\right]^{1/2}} \exp\left[-\frac{1}{2}(x-\mu_i)^T \Sigma_i^{-1}(x-\mu_i)\right] \tag{17}$$

where m is the number of measurement variables, and μ_i and Σ_i are the mean vector and covariance matrix for class i, respectively. Following, it can be shown that identical classification occurs when the eq. (15) is replaced by:

$$g_i(x) = ln\, P(x \mid f_i) + ln\, P(f_i) \tag{18}$$

Then, substituting the expression of $P(x \mid f_i)$ into the previous equation leads to:

$$\begin{aligned} g_{j,i} = &-\frac{1}{2}\left(x-\bar{x}_j\right) W_a \left(\frac{1}{n_j - 1} W_a^T S_j W_a\right)^{-1} W_a^T \left(x - \bar{x}_j\right) + \\ &ln\left(p_i\right) - \frac{1}{2} ln\left[det\left(\frac{1}{n_j - 1} W_a^T S_j W_a\right)\right] \end{aligned} \tag{19}$$

where $g_{j,i}$ is the discriminant function of each class $j = 1, \cdots, n$ in each sampling time $i = 1, \cdots, m$, and n_j are the number of samples collected for class j in the training stage, i.e., when the matrix D_j in (2) was built.

Then, the node proposed as candidate to have the leak is the one that provided by

$$g_k = \max\left(\sum_i^m g_{1,i}, \sum_i^m g_{2,i}, \ldots, \sum_i^m g_{n,i}\right) \tag{20}$$

that matches the current scenario with the leak class with a higher level of similarity.

3. SENSOR PLACEMENT

3.1 Problem statement

After being introduced the leak localization method based on FDA, a sensor placement methodology that aims at minimizing the number of sensors to install is presented.

The data matrices (2) and (6) consider that measurements in all the nodes of the WDN are available, i.e., assume that a sensor is available in each node. The goal of the proposed approach is to select the nodes where to place the n_s available sensors for installation such that maximize the leak isolability. To this aim a discrete optimization problem is formulated. Defining a binary optimization

$$q = \begin{bmatrix} q_1 & \ldots & q_n \end{bmatrix} \tag{21}$$

that contains as many components as nodes and where $q_i = 1$ means that the sensor has been installed in the i-th node, while $q_i = 0$ otherwise. Then, the discrete optimization problem can be formulated as follows:

$$\begin{aligned} &\min_q \sum_{j=1}^{n} \frac{e_j(q)}{n} \\ &s.t. \\ &\sum_{i=1}^{n} q_i = n_s \end{aligned} \tag{22}$$

where

$$e_j(q) = \begin{cases} 0 & if \;\; j = \arg\max\left(\sum_{i=1}^{m} g_{1,i}(q), \sum_{i=1}^{m} g_{2,i}(q), \ldots, \sum_{i=1}^{m} g_{n,i}(q) \right) \\ 1 & otherwise \end{cases} \tag{23}$$

The discriminant functions

$$g_{j,i}(q) = -\frac{1}{2}\left(x - \bar{x}_j(q)\right) W_a(q) \left(\frac{1}{n_j - 1} W_a^T(q) S_j(q) W_a(q) \right)^{-1} W_a^T(q) \left(x - \bar{x}_j(q)\right) + ln\left(p_i\right) - \frac{1}{2} ln \left[det \left(\frac{1}{n_j - 1} W_a^T(q) S_j(q) W_a(q) \right)^{-1} \right]$$

are parameterized with the optimization variable q in (21). That means that during the optimization process, per each configuration to be assesses the FDA is repeated by considering only the columns of the data matrices (2) and (6) where the sensors are considered to be installed, that is, $q_i = 1$ while the others columns (i.e. those with $q_i = 0$) are removed.

3.2 Problem solution

To solve the discrete optimization problem (22), two heuristic optimization problems are proposed since it cannot be solved with deterministic problems because of the algorithmic nature of the constraints.

3.2.1 Genetic Algorithms

The first heuristic approach that is proposed is based on the use of the Genetic Algorithms (GA), and in particular the implementation available in MATLAB in the global optimization toolbox. GA is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution (Goldberg, 1989). The genetic algorithm repeatedly modifies a population of individual solutions. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them to produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. GA can be applied to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear. The genetic algorithm can address problems of mixed integer programming, where some components are restricted to be integer-valued.

3.2.2 CMA-ES Algorithm

CMA-ES stands for Covariance Matrix Adaptation Evolution Strategy. It belongs to the class of evolutionary algorithms and evolutionary computation. Two main principles for the adaptation of parameters of the search distribution are exploited in the CMA-ES algorithm (Igel, 2006).

First, a maximum-likelihood principle based on the idea to increase the probability of successful candidate solutions and search steps is applied. The mean of the distribution is updated such that the likelihood of previously successful candidate solutions is maximized. The covariance matrix of the distribution is updated (incrementally) such that the likelihood of previously successful search steps is increased. Both updates can be interpreted as a natural gradient descent. Also, in consequence, the CMA conducts an iterated principal components analysis of successful search steps while retaining all principal axes. Estimation of distribution algorithms and the Cross-Entropy Method are based on very similar ideas, but estimate (non-incrementally) the covariance matrix by maximizing the likelihood of successful solution points instead of successful search steps.

Second, two paths of the time evolution of the distribution mean of the strategy are recorded, called search or evolution paths. These paths contain significant information about the correlation between consecutive steps. Specifically, if consecutive steps are taken in a similar direction, the evolution paths become long. The evolution paths are exploited in two ways. One path is used for the covariance matrix adaptation procedure in place of single successful search steps and facilitates a possibly much faster variance increase of favorable directions. The other path is used to conduct an additional step-size control. This step-size control aims to make consecutive movements of the distribution mean orthogonal in expectation. The step-size control effectively prevents premature convergence yet allowing fast convergence to an optimum.

4. APPLICATION RESULTS

4.1 Hanoi case study

To test the above methodologies, a case study based on the Hanoi WDN is used (Rodríguez et al 2006). It will allow analyzing the effectiveness of the proposed methods in a network with big flows. The demand pattern is designed according to (Rodríguez et al 2006). Matlab® and Epanet® are used altogether to simulate the leaks and to obtain and analyze the network data using the algorithms proposed in the paper.

A simulation of 24 hours with a sampling time of 15 minutes is carried out. This is because the demand is measured each 15 minutes. This gives a total of m=97 samples.

This network has 31 demand nodes with indexes from 1 to 31. A leak of 50 liters per second magnitude is used to compute the sensitivity matrixes shown in Figure 1, just for training purposes, i.e., to calculate the matrices D_j in (2), after that the method has been tested for different leaks magnitudes.

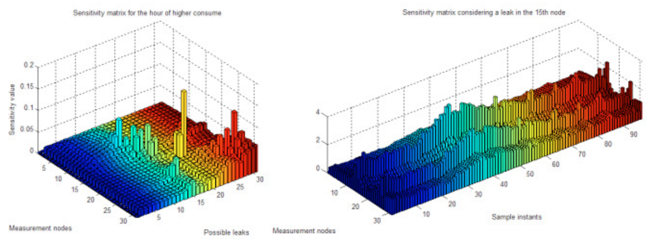

Figure 1. Sensitivity matrices for Hanoi Network

4.2 Sensor placement results

The choice of sensor placement is affected by the leak magnitude taken to build the sensitivities, but in real scenarios this magnitude cannot be determined in advance. To improve the robustness of the sensor placement to this situation, a set of sensitives that are computed from the different leak magnitudes, is introduced in the evaluation function. If there is l possible leak magnitudes each one associated to a sensitivity matrix, it is necessary to calculate the discriminant function (19) as $g_{k,j,i}$, for each class $j = 1,\cdots,n$ in each sampling time $i = 1,\cdots,m$ and for each leak magnitude, $k = 1,\cdots,l$. So the error function that it is necessary to minimize (23) is changed to:

$$e_j(q) = \begin{cases} 0 & \text{if } j = \arg\max\left(\frac{1}{l}\sum_{k=1}^{l}\sum_{i=1}^{m} g_{1,k,i}(q),\ldots,\frac{1}{l}\sum_{k=1}^{l}\sum_{i=1}^{m} g_{n,k,i}(q)\right) \\ 1 & \text{otherwise} \end{cases}$$

(24)

Using the sensor placement approach presented in Section 3, and using the GA and CMA-ES algorithms for solving the optimization problem (22), the results obtained are presented in Table 1. The GA used is the implementation available in MATLAB in the global optimization toolbox. The chromosomes correspond to the possible presence or absence of a sensor in a given node, i.e, the vector q in (21). The fitness function is (22) with the error calculated taking into account the unknown leak magnitude, i.e., the error index is calculated as (24), with l=7, (7 possible magnitudes of leak are considered: 10, 20, 30, 40, 50, 70 and 80 liters per second). Additionally, a Gaussian white noise with mean amplitude corresponding to approximately 0.5% of the expected measurement is added to the measurements. The algorithm performs five main iterations with ten generations in each of them, and 30 vectors in the initial population.

For the CMA-ES algorithm, the implementation used can be downloaded from http://yarpiz.com/235/ypea108-cma-es, in MATLAB language and the initial values for this algorithm are: 100 iterations with an initial population of 50 vectors, and a target of 0 for the fitness function, that as before is (22), calculated with the error index (24) taking into account the unknown leak magnitude (l=7) and the addition of Gaussian white noise to the measurements. In this case the initial vector is not a binary one, is an integer vector. So, the vector q in (21) is changed by a vector $z=[i_{q1},\ldots,i_{qn}]$ that contains only the number of the nodes where the sensors are installed.

From this table, it can be seen that the results obtained with both heuristic optimization algorithms are the same what allows to cross-check them. In these experiments, as have been said before, the uncertainty about the unknown leak magnitude and the addition of Gaussian white noise to the measurements are taken into account. It can be noticed that the error index for the three sensors configuration is 0.09, meaning that 9% of the leaks (i.e., only three leaks from the possible 31 leaks) are not located in the right node. This suggests that three sensors would be a good choice to have reliable leak detection and location for this network.

Figure 2 and 3 presents respectively the exact localization of the sensors in the Hanoi network when two and three sensors are respectively placed.

Table 1. Results of the sensor placement

# Sensors	**GA**		**CMA**	
	Nodes	***Error***	***Nodes***	***Error***
2	[12 29]	0.2459	[12 29]	0.2459
3	[11 15 28]	0.0903	[11 15 28]	0.0903

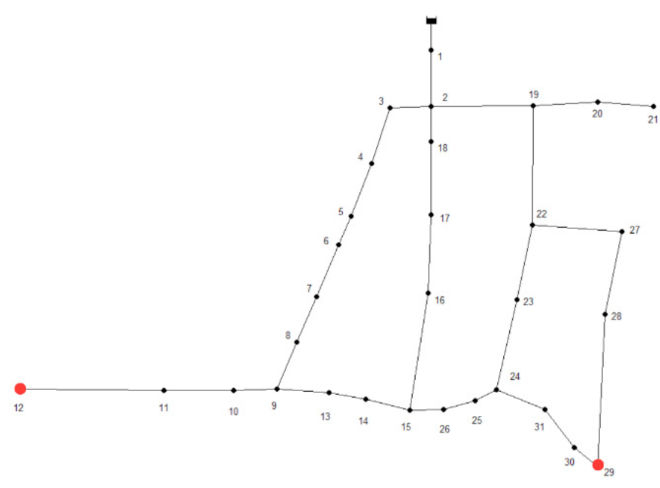

Figure 2. Optimal location of two sensors

4.3 Leak localization results

Using the leak localization method presented in Section 3 based on the FDA approach, the results obtained are assessed using the error index (22) and (24) are presented in Table 2, but in this case without noise in the measurements, for comparison purposes with those obtained using the angle method proposed in (Casillas et al, 2013). From this table it can be seen that in the optimal two sensors configuration the FDA method outperforms the angle method, that in this case the error is zero. This means that the FDA method can located all the leaks (31 leaks) in the right node, just with two pressure sensors in the network. However, for the optimal three sensor configuration the angle method outperforms the FDA method, but with no so big difference.

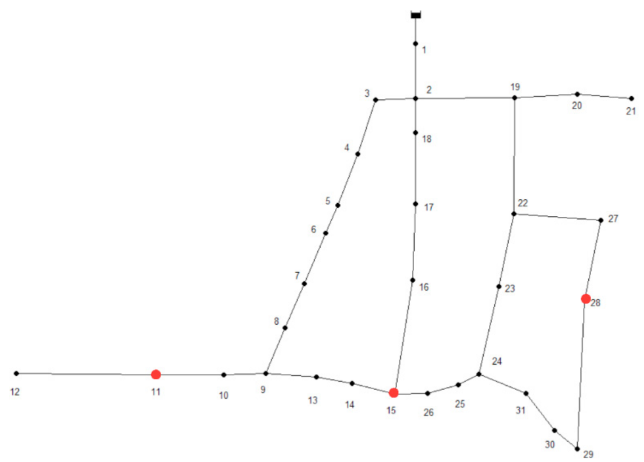

Figure 3. Optimal location of three sensors

Table 2. Leak localization results

# Sensors	Angle method		FDA	
	Configuration	***Error***	***Configuration***	***Error***
2	[12 21]	0,061	[12 29]	0
3	[12 14 21]	0.011	[3 12 29]	0.0359

5. CONCLUSIONS

This paper has proposed a leak localization method for water distribution networks using the Fisher Discriminant Analysis (FDA). This leak localization approach uses the information of pressure measurements from the sensors available in the WDN. A method for the optimal sensor placement is also proposed when the proposed leak localization based on FDA is used. The proposed leak localization and sensor placement approaches based on FDA has been used in a well-known WDN case study obtaining good results.

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