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A NEW APPROACH TO EXPANDABLE STRUCTURES: CROSSED EXPANDABLE FRAMES (X-FRAMES)

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ABSTRACT

The use of expandable structures in the field of building began in the 1960s, based on the pioneering work of Emilio Pérez Piñero. They underwent significant developments at the end of the 20th century, with typologies based on scissors or bundle modules. Until now, this typology has not been further enhanced, despite some very interesting contributions.

These studies are usually based on straight bar expandable structures, although there are some interesting proposal based on the deployability of parallel arc systems, even in real buildings. However, other possible types of expandable structures have not been explored to date.

In this paper, a new system for expandable structures is proposed, which opens new and interesting design possibilities based on the same folding principle. The system consists of deploying elements such as arches or frames with multiple intersections. Solutions for cylindrical vaults with horizontal axis joints and more complex geometries such as conoids are proposed, as well as domes with vertical and horizontal axis joints. Finally, other structures with special kinematic compatibility difficulties, such as concentric domes or toroids, are also studied.

Keywords: *Deployable/Retractable/Transformable Structural Morphology, Metal Spatial Structures*

1. INTRODUCTION

After World War II, there was a widespread development of spatial mesh structures, formed by a three-dimensional lattice of bars. Their lightness and their ability to cover large spans efficiently, led to the construction of numerous impressive structures of great beauty. However, the most complicated aspect of these structures was their assembly at great height, entailing a major risk of accidents. For this reason, several systems were studied, such as construction at ground level and then hoisting the structure into place, as was used in the roof mesh of the Festival Plaza of Expo'70 in Osaka, or Kawaguchi's Pantadome system, used for the roof of the Palau de Sant Jordi built for the 1992 Olympic Games in Barcelona [1].

In 1960, the Spanish architect Emilio Pérez Piñero devised a series of transportable structures that could be deployed in the chosen location and did not require on-site assembly. These were light structures that performed in the same way as space meshes, which could be transported in a compact package. This first idea was reflected in his project for a Deployable Itinerant Theatre, winner of the UIA award in 1961. This proposal, which achieved major international diffusion, laid the foundations of Pérez Piñero's prestige, and allowed him to build the first deployable structure, which was the Transportable Pavilion for the XXV Years of Peace Exhibition in 1964 [2], consisting of flat deployable 9x12 m modules that were successively installed in Madrid, Barcelona and San Sebastián (figure 1). The chosen system used bundle modules with a square floor plan.

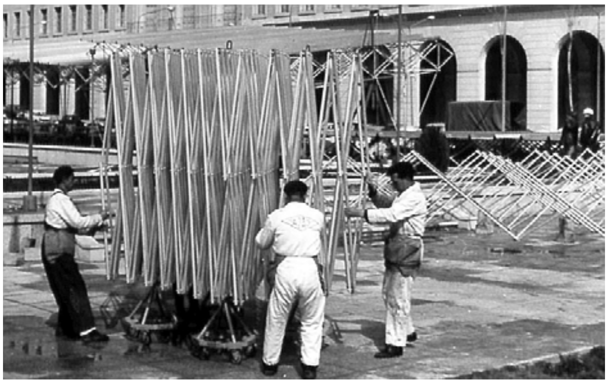

Figure 1: *Deployment of the Transportable Pavilion of E. P. Piñero in Madrid [photo courtesy of E. Pérez Belda]*

From 1964 onwards there was a period of more than thirty years, in which no other structure of the same type was built. In 1995, Escrig, Sánchez and Pérez Valcárcel designed and built the roof of the San Pablo pool in Seville [3]. The cover is composed of two vaulted modules measuring 30x30 m, with quadrangular scissor modules joined to form the roof of an Olympic pool, resulting in a 30x60m enclosure (figure 2). This building is the deployable structure with the largest span that has been built to date.

Figure 2: *Cover of the S. Pablo pool in Seville*

Prior to the construction of this structure, its creators had carried out extensive investigations that made it possible to define the geometric and kinematic conditions of the covering [4, 5]. They also designed the calculation programs necessary to define its resistance conditions [6]. Nevertheless, from this point onwards, construction activity using this type of structure was reduced to a minimum, limited to small experimental constructions. Research activity continued to develop new proposals, such as the works of Hernández Merchán [7], Gantes [8], [9], Hoberman [10], Tibert [11], Pellegrino [12], José Sánchez [13], K. Kawaguchi [14], Sánchez-Cuenca [15], De Temmerman [16], Begiristain [17], or Liew [18].

The main problem with deployable structures is their lack of rigidity. For a structure to be deployable, it needs to be a mechanism, by its own nature. Once deployed, it is necessary to apply enough external constraints in order for it to be able to work as a real structure and therefore be capable of resisting the forces applied. This behaviour will essentially depend on the basic module, which usually consists of straight bars based on the pantograph system (figure 3).

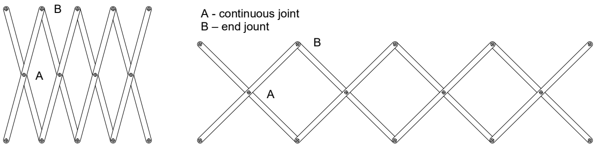

Figure 3: *. Flat X-bars system*

This system can be arranged spatially in several directions to create modular spatial meshes. If the expandable module is based on a triangle, the mesh will be rigid once the supports are fixed. On the other hand, if it is made of square modules it will not be rigid, and it will be necessary to include bracing elements. Despite this disadvantage, meshes made of square modules are lighter, easier to construct, and require much simpler joints. It is no coincidence that the two largest deployable structures built to date use square modules.

Therefore, for the Transportable Pavilion of P. Piñero, auxiliary bars had to be attached after deployment in order to stiffen the mesh, and in the pool of San Pablo de F. Escrig, diagonal bars were placed in the spherical dome for the same purpose (figure 4). Later, Valcárcel and Escrig proposed the use of bars with a lockable internal linkage to solve this problem [19].

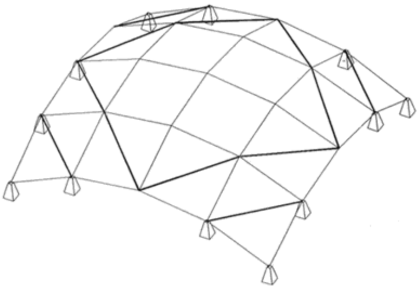

Figure 4: *Bracings in the cover of S. Pablo swimming pool*

This high degree of instability also calls for the use of auxiliary elements throughout the deployment process in order to secure the structure. This requirement, together with the need to have a locking system when finished, means the deployment process is not as fast as would be desirable. A complete analysis of the possibilities of these structures, either in vault format as well as in multiple solutions of domes is described, including their calculation process [6]. Coverage systems other than the usual textile coating are also proposed [20].

As an alternative to the X-bars system, we propose a system based on the folding of linear elements as portal frames, although for aesthetic reasons and ease of construction, it is initially proposed with the figure of the arch. Using a rigid frame means the structure has less freedom of movement, and is therefore more stable in its deployment and in its final state.

The first proposals in this field are the patents of F. Escrig [20] and of S. Toshiaki [21], where the concept of two X-crossed bars is substituted for a pair of arches in the shape of an easel (figure 5). From the side, this looks the same as the previous X-bars design. The joints have to be resosolved using horizontal axis pins.

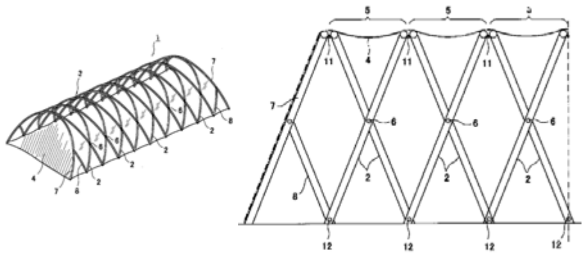

Figure 5: *Patent structure extendable of S.Toshiaki. and side view [drawings by S. Toshiaki patent]*

This system has been developed in practice with great efficiency. Escrig and Sánchez built a roof for a 16x30m pool in Valencina de la Concepción (Seville, Spain). It is a simple cover and so light that the users could open and close it without the need of auxiliary media.

Figure 6: *La Alameda Auditorium in Jaén. 1999*

The same system, although of a much larger size and weight, was used by the same authors in the retractable roof of the La Alameda Auditorium in Jaén (Spain), to cover an area of 42x100m (figure 6).

In any of these cases, the operating principle is similar to that of the X-cross bars. They are simple and effective solutions, but they require relatively strong sections, which increases the weight and complexity of the auxiliary mechanisms. On the other hand, they are formally limited to cylindrical shapes.

A variant of these proposals is exposed in reference [22], where C. Morales presents another deployment system based on polygonal arches, with the same limitations.

2. PROPOSAL OF A NEW EXPANSIBLE TYPOLOGY

As an alternative to the cross-bar system and following these last lines, a system based on the foldability of planes is proposed. It is similar to an idea included in the article by F. Escrig [23] of 1999, but which was not developed. This paper aims to present the range of possibilities it offers and how they take shape. The basic idea is explained in figure 7.

Figure 7: *Folding Planes*

Although in reality it is impossible for two planes to intersect and maintain continuity, this helps to better explain this approach. Instead of planes, we can take a part of these planes, for example only the arcs drawn in the previous figure. They need to be of different size in order not to buckle during rotation and have continuity in their movement. Their axis of rotation will coincide with the intersection of the planes, which in this case is horizontal. In the same way as arches were used, we could have used frames or any other figure, with the only condition that through the twirl, the figure of one plane does not collide with that of the other plane.

Figure 8: *Folding planes with multiple intersections*

The next concept we are going to introduce is the multiple intersections of the planes: each plane in one direction can intersect with more than one plane inclined in the opposite direction (figure 8). This is not a basic point for the development of this structural typology, but it does provide more formal versatility. In addition, the system will have better structural performance, as the transmission of stresses between the different elements is distributed in a greater number of points of contact. The stresses transmitted in these points will be minor, and if any of the elements fail, their load will be shared among several points, which helps to guarantee thestructural stability of the whole. This also helps to reduce flexing in the bars, something that has always been a disadvantage of deployable structures.

The basic result of the approach that we have described can be seen in the following model (figure 9), which is the result of applying the structural element arch to the system of planes with multiple intersections. This is the basic development of the type of Crossed Expandable Planes (X-Frames). The result is similar to that of the first patents, except that they have more than one intermediate contact axis, which will provide a better structural behaviour.

Figure 9: *X-Frames structure with vault geometry*

3. NEW POSSIBILITIES OF THE X-FRAME TYPOLOGY.

3.1. Complex surfaces

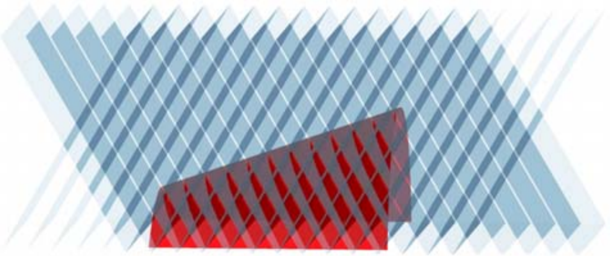

Figure 10: *Planes with the two inclinations and desired conoid surface*

Although the previously shown result is the simplest and most obvious, making small variations the results can be more interesting and different. A first possible line of development in X-Frames would be the definition of the desired final geometry, which opens a wide range of possibilities. The process of defining the structure consists of placing the system of crossed planes deployed in their final position, and intersecting these planes with the desired geometry. A

Figure 11: *Deploying of the conoid vault*

developed example is using a conoid, in which we maintain its base of constant width, while its height varies progressively (figure 10).

The intersections between the planes and the desired surface will define the set of arcs. It consists of two subsets of arcs, one corresponding to the planes inclined in one direction and the other corresponding to the planes inclined in the opposite direction . In order to be able to fold this series of arches, only is necessary that the arches inclined in one direction are a little smaller than those in the other direction, so that they can fit inside each other. Therefore, the external arcs will face outwards, while the internal arcs will face inwards.

3.2. Limitations

Not all surfaces allow for the design of deployable arch systems using this procedure. In particular, the problem arises when trying to obtain an ellipsoid (more exactly a quarter of the ellipsoid) with this system.

Initially the layout and the way to obtain the arches are identical to the previous case, except that now the surface has double curvature.

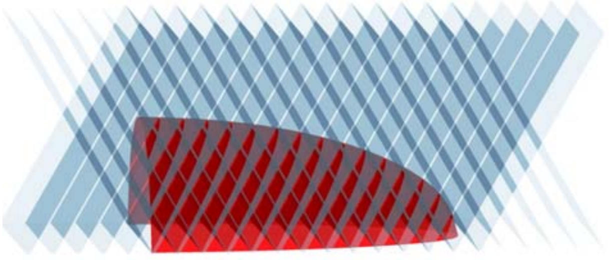

Figure 12: *X-Frames system for obtaining a quarter of an ellipsoid*

Once the guidelines are obtained, which will be the set of exterior and interior arches, we can move on to the manufacture of the model, which results in an attractive appearance.

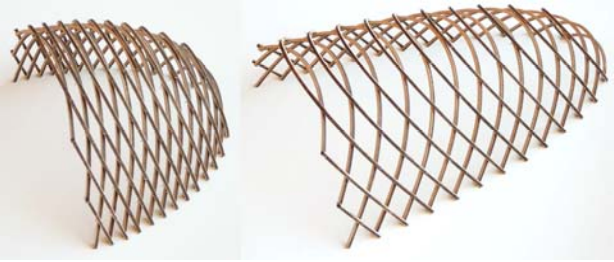

Figure 13: *Deploying the ellipsoid*

The limitation arises in this case when trying to fold the structure, as it certain geometric incompatibilities at its right end (in the major axis). Accordingly, the results obtained show that compact folding is not possible, although this could be an interesting solution in areas where it is necessary to maintain a partially covered area, such as in many sports venues or stages.

3.3. Parallel axes domes

Having presented the possibilities of the translation of planes in the creation of deployable systems, the next step will be to study the rotation. In X-bar systems the bars haven an eccentric crossing point, with the upper sections longer than the lower ones, and deployment takes place in the form of an arch, as can be seen in the following figure.

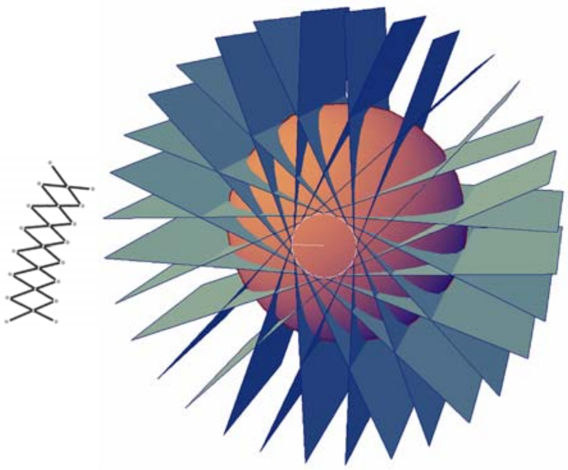

Figure 14: *Deployment with rotation*

By making a comparison with our planes system, we will replace each bar with a plane, and instead of having a single intersection with the ones in the opposite direction, they will have multiple intersections (figure 14). This can also be seen in the illustration the drawing in the final deployed position, as if the planes, previously parallel to each other, were rotating around an axis. In the previous case of parallel planes we could suppose it as a particular case of the latter, in which the centre of rotation is far away, at infinity. This design allows for numerous structural solutions. In the first option, we continue to maintain the intersections between planes as horizontal lines. Then will result in us obtaining dome-shaped surfaces. It is a very interesting possibility, as it makes it possible to cover the enclosure with a textile fixed to the structure, and the tympanum provides two zones of easy access to the interior (figure 15).

Another novelty in the design is that if we rotate the

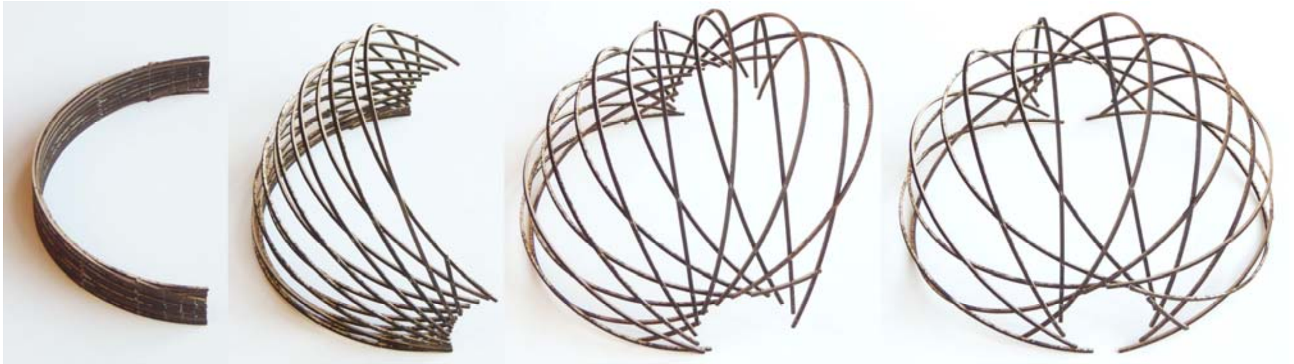

Figure 15: *Deployment process of a horizontal axis dome*

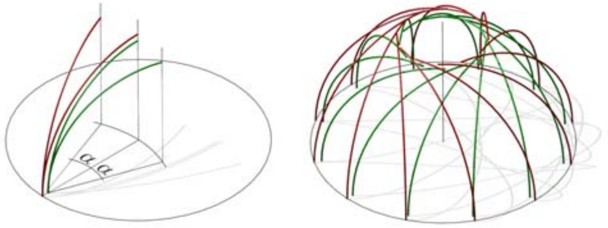

Figure 17: *Generation process of the vertical axes dome*

previous planes 90 degrees, then the axes of intersection of the planes become vertical. If we also take only the upper half of the arcs, over the horizontal plane that would be the ground, we obtain the second model of the dome. This can be seen in the following example (figure 16). The number of arches can vary depending on the frequency and separation between planes. In this case, it may be necessary for the separation between the lower parts of the arches to be wide enough to allow access to the interior of the enclosure.

Creating this last type of domes is very simple. Two maximum circle arcs with different radii are defined. Then, one of the arcs is rotated on the vertical axis in one direction and the other arc is rotated with the same angle in the other direction. Making a polar array with an even number of elements, the dome (Figure 17) is obtained.

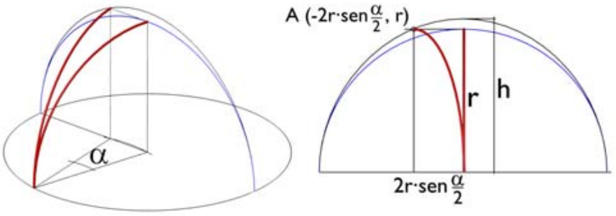

Figure 18: *Elliptical shape of the dome after deployment*

Since the bar is an arc of a circle, the dome when deployed forms an ellipsoid. The height is slightly higher than that of the sphere. The ellipsoid has to pass through point A, which corresponds to the final position of the end of the arch, after the turn. Its coordinates are

$$A\ (-2r \cdot \text{sen}\ \frac{\alpha}{2}, r) \qquad (1)$$

The equation of the ellipse passing through this point will be

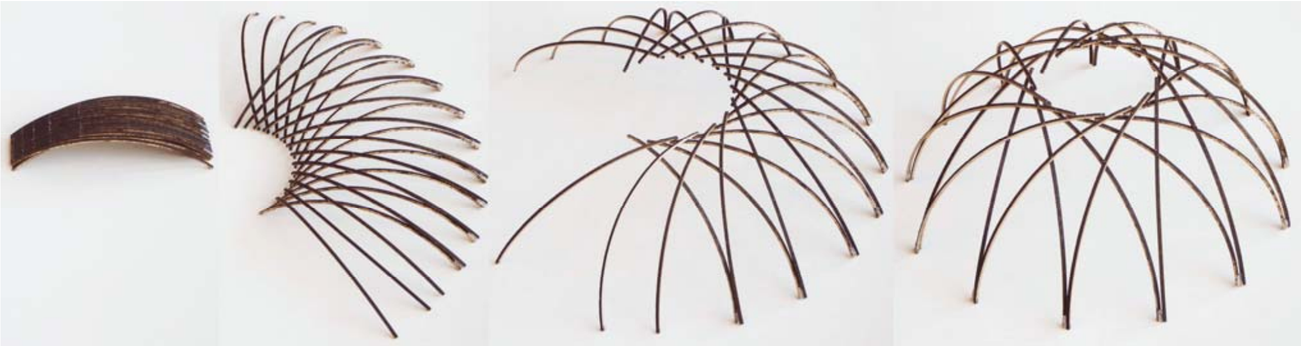

Figure 16: *Deployment process with vertical axes*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{4r^2 \cdot \mathrm{sen}^2\alpha / 2}{r^2} + \frac{r^2}{h^2} = 1 \tag{2}$$

$$h^2 \cdot \left(1 - 4\mathrm{sen}^2\alpha / 2\right) = r^2$$

$$h = \frac{r}{\sqrt{1 - 4\mathrm{sen}^2\alpha / 2}}$$

This allows defining the ellipsoid. The difference is small, but must be taken into account in the design of the cover.

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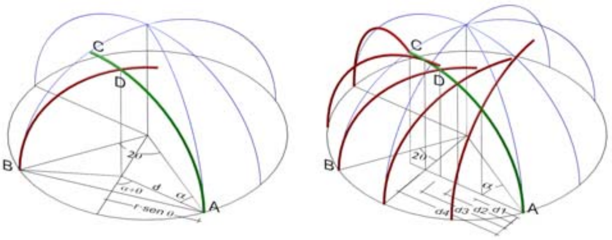

Figure 19: *Bolts position in the dome of vertical axis*

It is important to define the cross points of the bars to determine the placement of the bolts and make the work templates. If we use α to define the angle that the arc rotates on its vertical support axis (point A) and 2θ at the angle between the vertical planes that define two intersecting arcs, the horizontal distance d between point A and the position of the bolt is

$$d = \frac{r \cdot \mathrm{sen}\,\theta}{\mathrm{sen}\left(\alpha + \theta\right)} \tag{3}$$

The angle θ is defined by the frequency of the dome, which must be an even number. In a dome of frequency n the angle would be $\theta = \pi / n$. The intersections would be :

$$d_1 = \frac{r \cdot \mathrm{sen}\,\theta}{\mathrm{sen}\left(\alpha + \theta\right)} \;\ldots\; d_n = \frac{r \cdot \mathrm{sen}\,n\theta}{\mathrm{sen}\left(\alpha + n\theta\right)} \tag{4}$$

It is not necessary or advisable to place bolts at all of the possible intersections. We analysed the behaviour of a model with bolts in all of the intersections, and then removed two rows of connections, d_4 and the final row. The displacements hardly varied.

In order to check the geometrical and mechanical conditions of these domes, a 1/2 scale model of a frequency dome 12 was made, comprised of arcs of 6060-type aluminium tubes (aluminium-magnesium-silicon), T5 quality, and 8 mm 5.6 grade steel bolts, according to ISO 898-1. The tubes had a diameter of 40 mm and were 2 mm thick. Its section could be lower, although it was considered appropriate to use larger sections in order to detect possible misalignments in the folding and unfolding process. To prevent friction between the tubes, Teflon washers were inserted.

Figure 20: *Prototype of a vertical axis dome.*

Figure 21: *Prototype after folding.*

Figure 22: *Detail of joint with Teflon washers*

The intermediate linkages are solved with vertical shaft bolts (figure 22). The lower linkages should allow the vertical rotation of the tubes, with different radii. For this reason, the chosen model uses a simple plastic section that allows each tube to move on its own axis. For real structures this linkage can be easily made of steel.

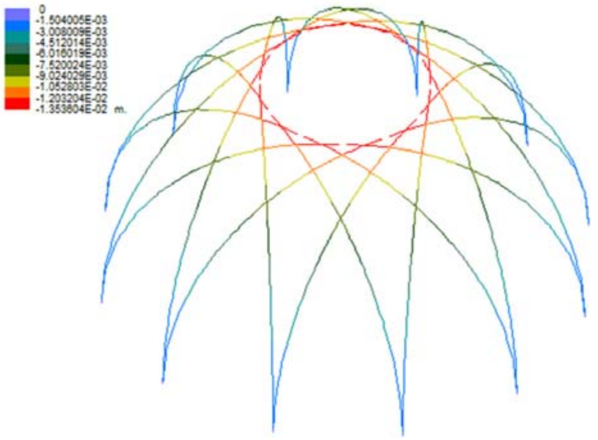

Figure 21: *Analysis of the vertical displacements of the prototype*

The model was tested at the laboratory of the Structures Department of the School of Architecture of the University of A Coruña. The results coincided quite accurately with the theoretical model. Firstly, the dome was tested with all of the joints fixed in place, and then two rows of bolts were removed. The differences were very small, since they worked as a reciprocal support.

The construction of this model made it possible to verify the formal viability of the proposal and the geometric relationships that result between each element. The structural behaviour and the stability were verified by a simple gravitational load test that revealed the strongly rigid nature of this structure, the fundamental object of this study. Also, there was a high degree of approximation between the calculations and the results that were measured. After assuring the geometric condition, this opens a new field of analysis and testing in relation to the singular behaviour of each component in relation to its formal and material condition.

5.4. Concentric axes domes

The structures described in this article are based on arcs that rotate on horizontal or vertical pivots. If the arcs are drawn as maximum circles of a sphere, it is possible to create very interesting domes. In this case, the intersection axes are no longer all horizontal or vertical, as in the previous cases, but instead converge in the centre of the dome. The bars are arcs of maximum circles that rotate around radial bolts. This makes it possible to create pinned joints with bolts perpendicular to the bars, which are the simplest joints possible (figures 24 to 27).

The movements of the bars are always compatible in all folding and unfolding positions. In the same way as the vertical axis dome, it is only necessary, to separate the bars with Teflon washers, to avoid friction between bars.

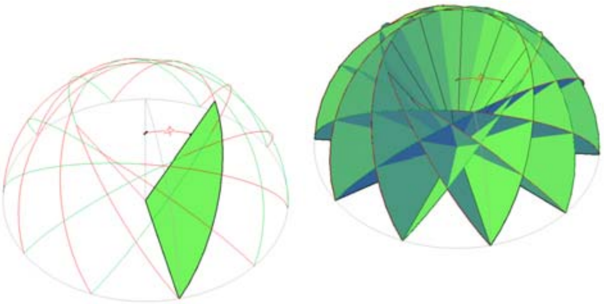

Figure 24: *Generation scheme with concurrent intersections in the centre of the sphere*

Figure 25: *Deployment process of the concentric axes dome*

The generation system is also simple. As in the case of the vertical axis dome, the base arches are drawn first. They must be two maximum circle arcs with different radii, since one order of arcs must rotate above the other. Then one of the arcs is rotated on the vertical axis in one direction and the other with the same angle in the other direction. The dome is generated with a polar array with an even number of elements.

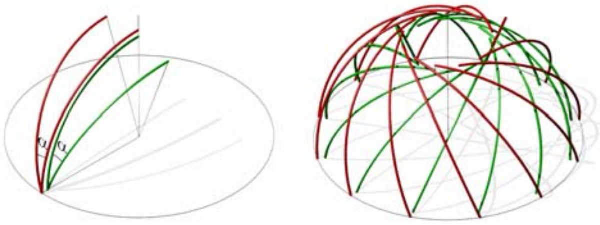

Figure 26: *Generation process of the concentric axis dome*

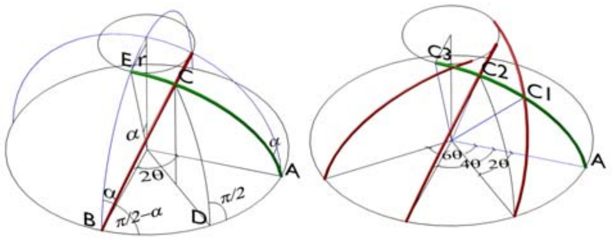

Figure 27: *Bolts position in the dome of concentric axis.*

According to the rotation applied to the arc, the radius of the oculus can be easily determined, being maximum circles and a rectangular spherical triangle. The angle B and the side AC measure 90º. Consequently the side BC measures α

$$r = R.\cos\alpha \tag{5}$$

One necessary datum is the spherical angle c, which makes it possible to fix the position of the turning bolt on the arc, for a predetermined rotation α. By applying the theorems of cosine and sine in spherical trigonometry we obtain:

$$\mathrm{tg}\,AC_1 = \frac{\mathrm{tg}\,\theta}{\mathrm{sen}\,\alpha} \quad .... \quad \mathrm{tg}\,AC_n = \frac{\mathrm{tg}\,n\theta}{\mathrm{sen}\,\alpha} \tag{6}$$

By fixing the horizontal angle between any two tubes, the position of the axis of rotation of the bolt can be obtained. With these data, the working templates can be easily produced.

5.5. Toroids

Now we can move to another subsystem within the overall system. If in the case of X-Frame vertical intersection axis structures (the first case of the domes), we move the centre of rotation outside of the arch, thereby obtaining figures similar to toroids, or semi-toroids to be more precise, as we only make use of the upper half (figure 28).

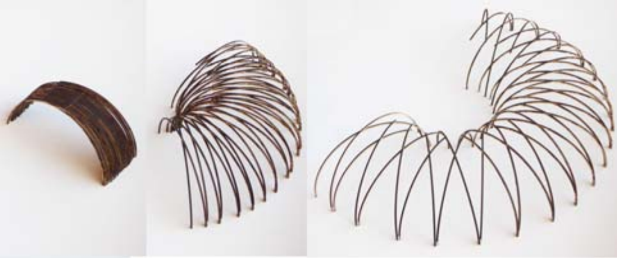

Figure 26: *Deployment of the toroid*

6. COMMENTS AND CONCLUSIONS

A new typology of deployable structures called X-frames is proposed. There are a number of precedents of analogous typologies that only allow for cylindrically-deployed structures. The indicated technique allows for its application in other curved surfaces, such as conoids, spheres or toroids.

After deployment, these structures prove to be more stable than those made using straight bars, due to the intermediate joints. Usually the linkages are the main problem in the design of deployable structures, but here the joints are solved simply and economically in most cases using bolts.

The proposed system permits the design of translation surfaces with horizontal joints, different from cylindrical joints. Fully compatible surfaces are developed in folding, such as conoids, and other partially closing shapes, such as the ellipsoid. Also, spherical surfaces with parallel axes of rotation, either vertical or horizontal, are proposed. The geometric conditions that permit folding and unfolding are analysed. Similarly, a type of deployable structure formed by maximum circle arcs with radial axes of rotation is developed. Finally, the conditions for arcs with toroidal development are presented.

A 1/2 scale model has been built, which made it possible to verify the conditions of geometric compatibility in the folding and unfolding processes. It also made it possible to verify the resistance capacity of this type of structures.

These have been shown to be structures with an attractive appearance. Combined with a textile cover system, they can be a solution for the need to temporarily close medium-sized spaces, with a transportable structure, which are easily and quickly expandable. Their size is generally limited by the size of the maximum arch that can be accommodated in a conventional transport, about 12 meters long by 3 meters high. This makes it possible to build domes with spans of 18 m and cylindrical vaults with spans of 12 m and of the necessary length, which can be used in a wide variety of applications.

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