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Quantifying the benefit of structural health monitoring: can the value of information be negative?

The benefit of Structural Health Monitoring (SHM) can be properly quantified using the concept of Value of Information (*VoI*), which is, applied to an SHM case, the difference between the utilities of operating the structure with and without the monitoring system. The aim of this contribution is to demonstrate that, in a decision-making process where two different individuals are involved in the decision chain, i.e. the owner and the manager of the structure, the *VoI* can be negative. Indeed, even if the two decision makers are both rational and exposed to the same background information, their optimal actions can diverge after the installation of the monitoring system due to their different appetite for risk: this scenario could generate a negative *VoI*, which corresponds exactly to the amount of money the owner is willing to pay to prevent the manager using the monitoring system. In this paper, starting from a literature review about how to quantify the *VoI,* a mathematical formulation is proposed which allows one to assess when and under which specific conditions, e.g. appropriate combination of prior information and utility functions, the *VoI* becomes negative. Moreover, to illustrate how this framework works, a hypothetical *VoI* is evaluated for the Streicker Bridge, a pedestrian bridge on the Princeton University campus equipped with a fiber optic sensing system: the results show how the predominant factor that determines a negative *VoI* is the different risk appetite of the two decision makers, owner and manager.

Keywords: value of information; Bayesian inference; expected utility theory; decision-making; bridge management; structural health monitoring

1. Introduction

Structural Health Monitoring (SHM) is a powerful tool for bridge management that support decisions concerning maintenance, reconstruction and repairs of assets through reducing uncertainties on the state of the structure. Uncertainty increases the likelihood of unwelcome outcomes such as neglecting necessary repairs while engaging in unnecessary ones. Such decision-making is challenging as it requires the decision maker to trade-off between anticipated risk and benefits to prioritize activities. The prioritization of activities will be determined in part by uncertainty and in part by the appetite for risk of the decision maker, which varies across individuals such that, given the same alternatives with the same state of uncertainty, two rational decision makers may take different courses of action.

Consider a simple car maintenance example as an illustration, where an owner must decide to prioritize changing the battery or tires. This decision should be informed not only by the probability of each failing but also on the consequences of failure. Failure of the battery may be considered more of an inconvenience as it prevents the car from starting but tire failure on a moving vehicle can be a safety critical event. If the probability of tire failure is less than the battery failure then the decision will be influenced by the decision-makers appetite for risk, where the more risk adverse will choose to replace the tires, even with a small probability to avoid the more serious consequences, while more risk neutral decision makers will accept the exposure to the tire failure risk to avoid the more likely and inconvenient battery failure. The extent to which a decision maker is risk adverse can be characterised by the extent to which they avoid exposure to rare but highly consequential events. For a fuller discussion, where this is methodology is explored in the context of programme risk management see Wilson and Quigley (2016).

Through reduction in uncertainty such decision makers become more aligned in their choices. However, monitoring systems might become costly and with limited budgets the anticipated value of the information provided towards the safety of the structure must be considered. Although the utility of SHM has rarely been questioned, very recently a few published papers (Thons & Faber, 2013) (Zonta, et al., 2014) have clarified how to evaluate it. The benefit of information is formally quantified by the so-called Value of Information (*VoI*), a concept anything but new: it was first introduced by

Lindley (1956), as a measure of the information provided by an experiment, and later formalized by Raiffa and Schlaifer (1961) and DeGroot (1984).

Since its introduction, it has been continuously applied in many fields, including statistics, reliability and operational research (Wagner, 1969) (Sahin & Robinson, 2002) (Ketzenberg, et al., 2007) (Goulet, et al., 2015) (Quigley, et al., 2017). Its first appearance in the SHM community was implicitly in the 1980s (Thoft-Christensen & Sorensen, 1987), while explicitly it is much more recent and dates back to a paper published in 2005 by Straub and Faber (2005), followed by Bernal, Zonta and Pozzi (2009), Pozzi, Zonta, Wang and Chen (2010), Pozzi and Der Kiureghian (2011), Thöns and Faber (2013), Zonta, Glisic and Adriaenssens (2014), Limongelli, Omenzetter, Yazgan and Soyoz (2017), Giordano, Prendergast and Limongelli (2020) - a recent state of the art can be found in Straub et al. (2017) and Thöns (2017). In the last few years, quantifying the value of SHM has known a renewed popularity thanks to the activity of the EU-funded COST action TU1402 (Thons, et al., 2017): in particular, Sousa et al. (2019), Diamantidis et al. (2019) and Thöns (2019) are the official guidelines of the COST action TU1402, where efforts have been made to properly present the subject targeting complementary audiences, i.e. decision-makers, practising engineers and scientists.

In summary, the value of a SHM system can be simply defined as the difference between the benefit, or expected utility u_{pp}, of operating the structure *with* the monitoring system and the benefit, or expected utility u_0, of operating the structure *without* the system. Both u_{pp} and u_0 are expected utilities calculated *a priori*, i.e. before actually receiving any information from the monitoring system. While in u_0 it is assumed that the knowledge of the manager is his/her a priori knowledge, u_{pp} is calculated assuming the decision maker has access to the monitoring information and is sometimes referred as to *preposterior utility*. In classical decision theory, one of the main assumptions is that all decisions concerning system installation and operation are taken by the same rational agent. In this case, it is easily proved that the *VoI* can only be positive, consistently with the principle that “information can’t hurt”, as first introduced by Cover and Thomas (2012) and later by Pozzi et al. (2017).

However, there are several cases in the literature where a negative *VoI* is observed: regardless of the field of application, they can be classified in three different classes. The first one (#1) relates to non-cooperative games and decisions against nature: in summary, when agents compete against each other, information can produce a negative value to some of them, precisely because they are in the area of competitive decisions. In the literature, some examples can be found principally in the field of financial markets, see for instance Baiman (1975), Schredelseker (2001), Pfeifer, Schredelseker and Seeber (2009). The second case (#2) is instead about the presence of constraints in the decision process, which can lead the decision maker to take irrational decisions. A clear example is reported in Pozzi et al. (2017) and Pozzi et al. (2020), where a system’s maintenance agent has to blindly follow the prescriptions of codes and regulations, regardless of their inherent rationality: in this case, the decision maker, in order to bypass societal constraints (e.g., legal requirements, etc), may find it convenient to avoid information. As demonstrated in the papers, the constraint affects the *VoI*, which may consequently result in it being negative. Finally, the third case (#3) relates to the presence of multiple rational decision makers that have to take decisions at different levels, which are somehow connected. This case is presented for instance in Bolognani et al. (2018), where two individuals are involved in the decision chain as regards a SHM-based decision process. In the paper it is proved that, because of the different appetite for risk of the two rational agents, the *VoI* may become negative. Table 1 summarizes the main features of the three cases.

Table 1. Main features of the three cases.

Case	Definition	Example of application
#1	When agents compete against each other, information can produce a negative value to some of them, precisely because they are in the area of competitive decisions.	Financial markets (Baiman, 1975) (Schredelseker, 2001) (Pfeifer, et al., 2009)
#2	The presence of constraints (e.g. prescriptions of codes and regulations) in the decision process can lead decision makers to take irrational decisions, e.g. they may find it convenient to avoid information.	Maintenance of engineering systems (Pozzi, et al., 2017) (Pozzi, et al., 2020)
#3	The presence of multiple rational decision makers, that have to take decisions at different levels, may lead to a negative *VoI* because of the different appetite for risk of the agents.	Structural Health Monitoring (Bolognani, et al., 2018)

While in the literature it is easily demonstrated the reason why in the first two cases introduced above it is possible to find a negative *VoI*, respectively because of competitive decisions and because of irrational constrains, it is not so immediate to understand why it may happen in the third case, which is based only on rational behaviours. Consequently, this contribution will focus on this specific third case, which is also the one that mainly affects the field of SHM. A typical situation in SHM is that there are more individuals involved in the decision chain: a first example is the case where there is one agent who decides whether or not to install the monitoring system, and one who decides how to use it (Bolognani, et al., 2018); a second example is the pair decision maker and practising engineer, cited in the COST Action TU1402 (Sousa, et al., 2019) (Diamantidis, et al., 2019).

The case study introduced by Bolognani et al. (2018) is examined herein and the main assumptions are summarised in the following. Two decision makers are involved in the decision chain, and they have to take decisions at two different decision stages. Firstly, a decision is made on whether or not to buy and install the monitoring system on the structure; typically, this decision is carried out by a high-level manager, conventionally referred to as *owner*. The second stage concerns the day-to-day operation of the structure, which includes for example maintenance, repair, retrofit or enforcing traffic limitations, once the monitoring system is installed; if installed these decisions may be informed by the monitoring system. Typically, this decision is carried out by an engineer, referred to as *manager*. The two agents are both rational and with the same background knowledge, they only differ in the weight they apply to the possible economic losses, meaning that they have different utility functions (i.e. different priorities). Therefore, the two decision makers may differ in their choices under uncertainty: for instance, the owner needs to consider the manager's appetite for risk when deciding whether to install a monitoring system, as this will indicate how the system will influence the manager's decision-making and as such the value of this information. As proved in Bolognani et al. (2018), these assumptions can lead to a negative *VoI* because, even if the two agents have the same prior knowledge of the problem, their optimal actions can diverge after the installation of the monitoring system, due to their different attitudes towards risk. While in the paper it is showed that the *VoI* can become negative, the proof under which generic mathematical conditions this is true remains under research.

The aim of this contribution is to demonstrate under which specific conditions, e.g. appropriate combination of prior information (i.e. the knowledge before new data is collected) and utility functions (i.e. functions that reflect the utility of the consequences of an action), it is possible to find a negative *VoI* in this specific case, by developing a mathematical formulation. Section 2 reviews the formulation for the quantification of the *VoI* in a SHM-based decision process, both in the classical case of a singular rational agent and in the case of two different individuals, needed for a better understanding of how it is possible to achieve a negative *VoI* only in the second case. Next, in Section 3, a prototype decision problem is introduced and a mathematical formulation to investigate under which circumstances the *VoI* becomes negative is developed. Finally, to illustrate how this framework works, Section 4 applies it to the same decision problem reported in Zonta et al. (2014), Tonelli et al. (2018) and Bolognani et al. (2018), i.e. the Streicker Bridge case study: it is a pedestrian bridge at Princeton University campus equipped with a continuous monitoring system. Some concluding remarks are presented at the end of the article.

2. Value of information for SHM-based decision

In this section, the concept of *VoI* for SHM-based decision problems is reviewed, following a similar path as in Zonta et al. (2014) and Bolognani et al. (2018). The reader can find recent examples of SHM-based decision problems in Flynn and Todd (2010a), Flynn and Todd (2010b), Flynn et al. (2011), Tonelli et al. (2017), Bolognani et al. (2017), Verzobio et al. (2018), Verzobio et al. (2019).

As observed in Cappello, Zonta and Glisic (2016), SHM-based decision-making, i.e. deciding based on the information from a SHM system, is properly a two-step process, as represented in Figure 1: it includes the judgment of the state S of the structure based on the information $\mathbf{y}$ from the sensors, and subsequently the decision of the optimal action a_{opt} to take based on the knowledge of the state S. As regards the first step, i.e. the judgment, the logical inference process followed by a rational agent is mathematically encoded in Bayes' rule (see for example Lindley (2006), Sivia and Skilling (2006), Bolstad (2010)), which allows one to estimate the posterior distribution on the state of the structure $\mathrm{P}(S|\mathbf{y})$ (i.e. the probability of the state S given the information $\mathbf{y}$ from the sensors). Successively, the decision-making step is about choosing the best action a_{opt}:

Expected Utility Theory (EUT) (Neumann & Morgenstern, 1944) (Raiffa & Schlaifer, 1961) describes the analysis of decision-making under risk and is considered as a normative model of rational choice (Parmigiani & Inoue, 2009).

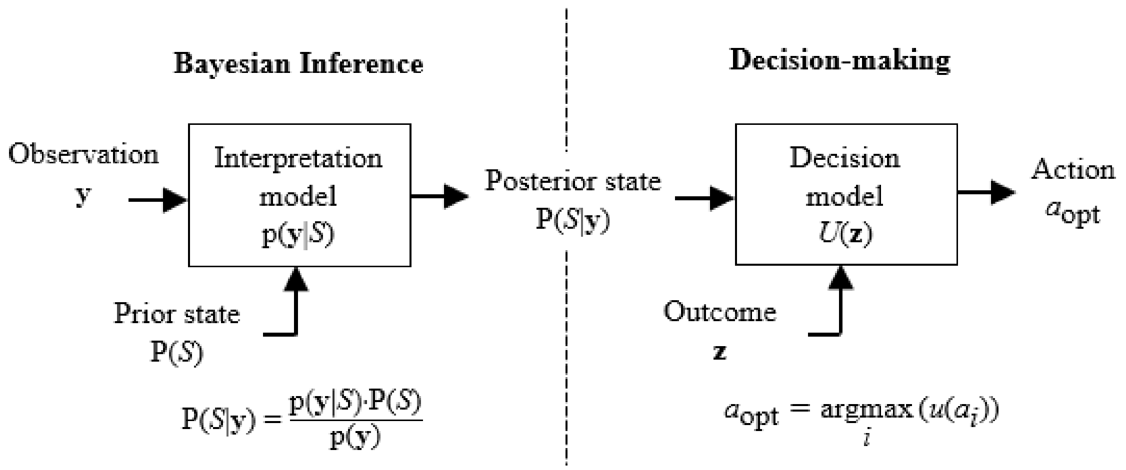

Figure 1. Process of SHM-based decision-making.

Before proceeding with the mathematical formulations of *VoI*, some generic assumptions are introduced:

- The monitoring system provides a dataset that can be represented by a vector **y**, called observation in Figure 1, that is fundamental to evaluate the posterior state through Bayesian inference.
- The structure, for instance a bridge, can be in a one out of N mutually exclusive and exhaustive states S_1, S_2, …, S_N (e.g.: S_1 = ’severely damaged’, S_2 = ’moderately damaged’, S_3 = ’not damaged’, …).
- The state of the structure is generally not deterministically known and can be only described in probabilistic terms.
- The decision maker can choose between a set of M alternative actions a_1, a_2, …, a_M (e.g.: a_1 = ‘do nothing’, a_2 = ‘limit traffic’, a_3 = ‘close the bridge to traffic’, …).

- Taking an action produces measurable consequences (e.g.: a monetary gain or loss, a temporary downtime of the structure, in some cases causalities); the consequences of an action can be mathematically described by several parameters (e.g.: the amount of money lost, the number of days of downtime, the number of casualties), encoded in the outcome vector **z** of Figure 1.
- The outcome **z** of an action depends on the state of the structure, thus it is a function of both action a and state S: $\mathbf{z}(a, S)$; when the state is certain the consequence of an action is also deterministically known; therefore, the only uncertainty in the decision process is the state of the structure S.
- For simplicity and clarity, the case of 'single shot' interrogation is considered, which is the case when the interrogation occurs only following an event which has a single chance to happen during the lifespan; an extension to the case of multiple interrogations can be found in Zonta et al. (2014).

2.1. Unconditional VoI context

In the classical formulation of *VoI* (Zonta, et al., 2014), referred to as *unconditional*, i.e. assuming all decisions concerning system installation and operation taken by the same rational agent, the *VoI* of a monitoring system is simply the difference between the expected utility with the monitoring system u_{pp}, and the corresponding utility without the monitoring system u_0:

$$VoI = u_{pp} - u_0. \tag{1}$$

In the case of a structure not equipped with a monitoring system, the rational manager decides without accessing any SHM data, and they will choose the action a that maximize the expected utility u_0. Consequently, the utility without monitoring, also called *prior utility*, is calculated as follows:

$$u_0 = \max_i u(a_i), \qquad a_{\text{opt}} = \arg\max_i u(a_i), \tag{2a,b}$$

where a_{opt} is the action which carries the maximum expected utility u. Conversely, if a monitoring system is installed and the data are available for the agent, the monitoring observation $\mathbf{y}$ affects the state knowledge, and therefore indirectly their decisions. In this case, the expected utility u_{pp}, also called *preposterior utility*, can be derived from the posterior expected utility $u(\mathbf{y})$ by marginalizing out the variable $\mathbf{y}$ (Zonta, et al., 2014) (Cappello, et al., 2016):

$$u_{pp} = \mathrm{E}_{\mathbf{y}}\left[\max_i u(a_i,\mathbf{y})\right] = \int_{D\mathbf{y}} \max_i u(a_i,\mathbf{y}) \cdot \mathrm{p}(\mathbf{y})\ \mathrm{d}\mathbf{y}, \tag{3}$$

where $\mathrm{E}_{\mathbf{y}}$ is the expected value operator of $\mathbf{y}$**,** while distribution $\mathrm{p}(\mathbf{y})$ is the so-called evidence in classical Bayesian theory (Sivia & Skilling, 2006). In conclusion, the *unconditional VoI* of a monitoring system is calculated as follows:

$$VoI = u_{pp} - u_0 = \int_{D\mathbf{y}} \max_i u(a_i,\mathbf{y}) \cdot \mathrm{p}(\mathbf{y})\ \mathrm{d}\mathbf{y} - \max_i u(a_i)\,. \tag{4}$$

In other words, the *VoI* is the difference between the expected maximum utility and the maximum expected utility. It is mathematically verified that u_{pp} is always greater than or equal to u_0, and therefore the *VoI* as formulated above can only be positive. This is to say that under these assumptions it is never preferred not to have the data if they were available, which is consistent with the principle “information can’t hurt” (Cover & Thomas, 2012).

2.2. Conditional VoI context

Bolognani et al. (2018) have investigated a variant of the decision problem above where two different rational individuals, rather than one, are involved in the decision chain. In particular, there is an *owner* who decides whether or not to install a monitoring system, and a *manager* who decides which is the optimal action once the monitoring system is installed or not. Therefore, all utilities are from the *owner* perspective, but should be evaluated accounting for the action that the *manager*, not the owner, is expected to choose. The prior expected utility of Equation (2), in the case of a structure without the monitoring system, changes to:

$$u_0 = u\left(a_{\text{opt}}^*\right) = u\left\{\arg\max_i u^*(a_i)\right\}, \tag{5}$$

where the star * indicates the optimal action or the utility from the *manager* perspective. Similarly, the expected utility of the owner in the expectation of what the manager would decide if a monitoring system was installed turns into:

$$u_{pp} = \int_{D\mathbf{y}} u\left\{\arg\max_i u^*(a_i,\mathbf{y})\right\} \cdot \mathrm{p}(\mathbf{y})\, \mathrm{d}\mathbf{y}\,. \tag{6}$$

The *VoI* of a monitoring system calculated under these assumptions is labelled *conditional*, to remind us that the utility of the owner is conditional to the action chosen downstream by the manager, and reads (Bolognani, et al., 2018):

$$VoI = u_{pp} - u_0 = \int_{D\mathbf{y}} u\left\{\arg\max_i u^*(a_i,\mathbf{y})\right\} \cdot \mathrm{p}(\mathbf{y})\, \mathrm{d}\mathbf{y} - u\left\{\arg\max_i u^*(a_i)\right\}. \tag{7}$$

Table 2 summarizes the *unconditional* and *conditional* formulations. As observed in Bolognani et al. (2018), in the *conditional* case it is no longer automatically verified that the owner's *preposterior* utility u_{pp} is always greater than or equal to the *prior* utility u_0.

Therefore, unlike the *unconditional* case, it is possible to find a combination of prior probabilities, likelihoods and utility functions which yield a negative *conditional VoI*. The aim of this contribution is then to demonstrate under which mathematical conditions it is possible to find a negative *VoI*.

Table 2. Formulation of *VoI* for SHM in the *unconditional* and *conditional* case.

	Unconditional formulation (one agent profile, i.e. manager profile = owner profile)	*Conditional* formulation (two different agents, i.e. manager profile ≠ owner profile)
Prior utility without monitoring u_0	$\max\limits_i u(a_i)$	$u\left\{\arg\max\limits_i u^*(a_i)\right\}$
Preposterior utility with monitoring u_{pp}	$\int_{D\mathbf{y}} \max\limits_i u(a_i,\mathbf{y}) \cdot \mathrm{p}(\mathbf{y})\ \mathrm{d}\mathbf{y}$	$\int_{D\mathbf{y}} u\left\{\arg\max\limits_i u^*(a_i,\mathbf{y})\right\} \cdot \mathrm{p}(\mathbf{y})\ \mathrm{d}\mathbf{y}$

3. When does the *VoI* become negative?

In the previous section, the concept of conditional value of information has been introduced and it has been noticed that, ‘under certain conditions’, it could become negative. In this section, the goal is to clarify which exactly are the conditions whereby the *conditional VoI* becomes negative. To do so, the analysis will focus on a 2-state 2-action prototype decision problem, graphically illustrated in the decision tree of Figure 2, which is representative of a number of binary decision settings that can be found in the literature (Raiffa & Schlaifer, 1961) (Parmigiani & Inoue, 2009). Particularly, the following assumptions are made:

- The structure can be in one of two mutually exclusive and exhaustive states S_1 and S_2 (e.g.: S_1 = *the bridge is damaged*; S_2 = *the bridge is not damaged*).

- The decision maker can choose between two alternative decisions a_1 and a_2 (e.g.: a_1 = *do nothing*; a_2 = *close the bridge*).
- Both actions may have consequences, depending on the (uncertain) state: $\mathbf{z}(a_i, S_j)$ represents the set of consequences of action a_i on the realization of state S_j. Both manager and owner are equally aware of these consequences.
- The utility function of the *owner* is defined $U(\mathbf{z})$, where the argument $\mathbf{z}$ is a particular set of consequences. To simplify the notation, the utility of the consequences of action a_i on the realization of state S_j is labelled $U_{ij} = U(\mathbf{z}(a_i, S_j))$. To prevent confusion, note that in this paper capital U indicates the utility function, while lowercase u denotes an expected utility.
- Regardless the complexity of the monitoring system, its ultimate output is represented by a single parameter y, defined in the domain $[0, y_{\max}]$. Parameter y could be, for instance, a compensated measurement, or a synthetic damage index calculated using the full dataset recorded to date. The *manager* makes a decision solely based on parameter y.
- The two agents, *owner* and *manager*, have the same prior knowledge of the problem, i.e. their prior probability $\mathrm{P}(S_j)$ of being in one of the two states is identical. They also interpret the data from the SHM system using the same interpretation model, which is encoded in the two likelihood functions $\mathrm{p}(y|S_j)$ (i.e. the probability of observing the data of the monitoring system given the state of the structure). They are both coherent and judge consistently with Bayes' rule (i.e. rationally): therefore, their judgement on the state of the structure, prior or posterior, is always identical.
- Similarly, the two agents decide rationally consistently with EUT. However, their utility functions are generally different (i.e. they weight in a different way

the seriousness of an action), thus their decisions, in the same situation, could differ.

- Parameter y is defined in such a way that the values of y whereby the owner chooses an action rather than the other are separated by a single threshold $\bar{y}$. Without losing generality, it is assumed here that when $y < \bar{y}$ the owner chooses action a_1. The same applies to the manager, except that their threshold, labelled $\bar{y}^*$, could be different.

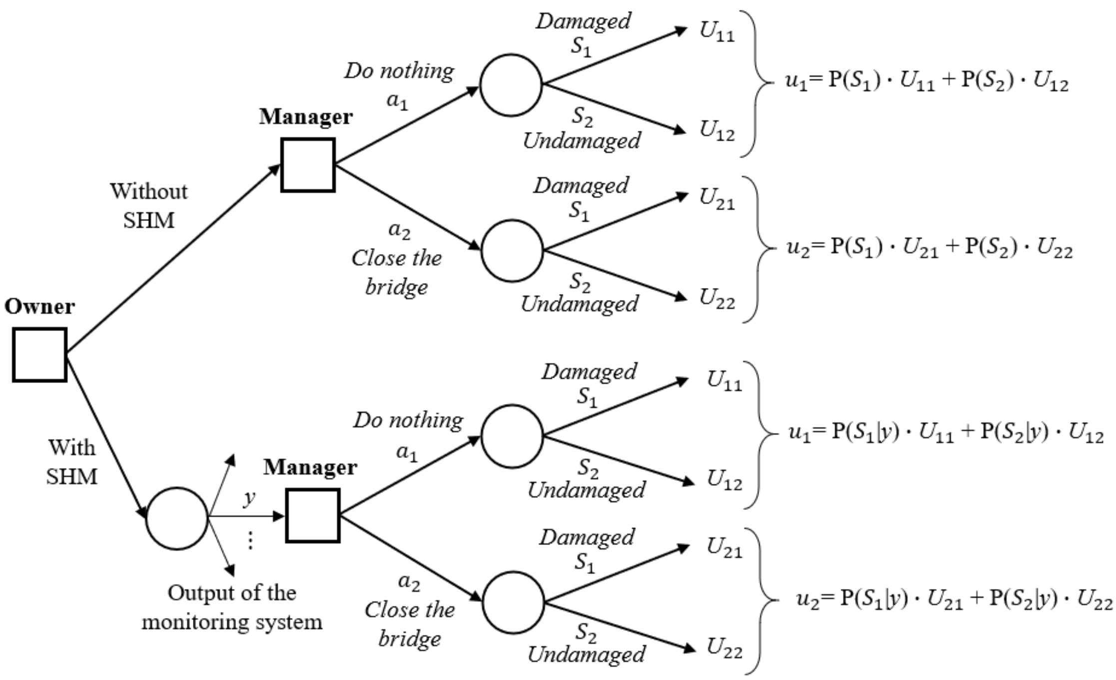

Figure 2. Decision tree of the prototype decision problem.

With the above assumptions, the conditions whereby the *VoI* becomes negative will be established. Before tackling the problem in full, the further assumptions are made: $U_{12} = U_{21} = 0$ and $U_{11} < U_{22} < 0$. This simplifying hypothesis makes the solution much more intuitive and easier to understand and will be released at the end of this section. To further help the reader picturing the problem, imagine you are dealing with a bridge that may be in damaged, i.e. S_1, or undamaged, i.e. S_2, condition. The manager can decide to keep the bridge open or close it. If the bridge is left open and is damaged, the bridge fails (i.e. the bridge collapses) producing a negative utility U_{11}. If the bridge is unnecessarily closed when not damaged, the manager is sanctioned with a penalty U_{22}. The loss for a failure is in absolute value much greater than the penalty for closing the bridge without necessity, i.e. $|U_{11}| > |U_{22}|$ or $U_{11} < U_{22}$, reminding that the utilities are negative. More generally, this is the prototype of any problem where an agent is faced with a binary decision, and each decision can be right or wrong depending on the unknown state. If the agent makes the right choice, nothing happens, otherwise they are sanctioned with a penalty.

In summary, the decision tree of this specific situation is the same as in Figure 2, with the only difference that now both the branch "Without SHM" and "With SHM" have the situation illustrated in Figure 3, i.e. $U_{12} = U_{21} = 0$, and consequently the evaluation of the expected utilities u_1 and u_2 is simplified.

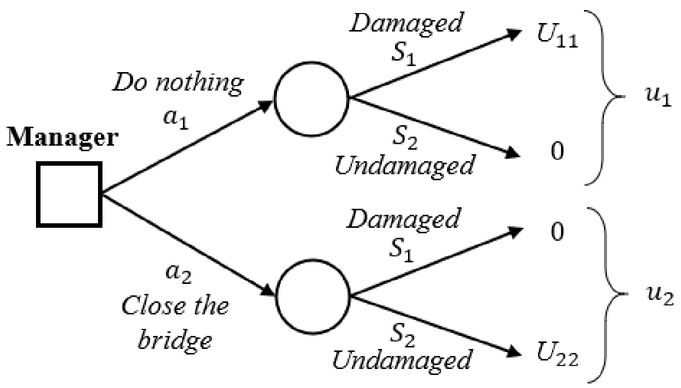

Figure 3. How the decision tree of Figure 2 changes in this specific situation.

3.1. Decision a priori

The problem of decision a priori from the *owner* perspective is analysed. The owner will favour action a_2 (i.e. close the bridge) over a_1 (i.e. do nothing) when the prior expected utility u_2 is greater than the prior expected utility u_1, i.e.:

$$u_1 < u_2. \tag{8}$$

Recall u_1 = P(S_1)U_{11}, u_2 = P(S_2)U_{22}, and both utilities are negative, the inequality of Equation (8) can be rewritten as:

$$R = \frac{\mathrm{P}(S_2)}{\mathrm{P}(S_1)} \frac{U_{22}}{U_{11}} < 1, \tag{9}$$

where R is a discriminant ratio which expresses the optimal action a priori from the owner perspective. It can be observed that, by definition, $R = 1$ corresponds to the indifference in the choice a priori between the two actions a_1 and a_2, i.e. $u_1 = u_2$, while it is preferred to choose action a_1 if $R > 1$, i.e. $u_1 > u_2$, or action a_2 if $R < 1$, i.e. $u_1 < u_2$. It is convenient to express the discriminant R as:

$$R = \frac{r}{q}, \tag{10}$$

where:

$$q = \frac{\mathrm{P}(S_1)}{\mathrm{P}(S_2)}, \qquad r = \frac{U_{22}}{U_{11}}. \tag{11a,b}$$

Index q is the prior odds of state S_1 respect to S_2, while index r is an indicator of the subjective risk appetite of the decision maker: the more the agent is risk seeking, the bigger is index r. The risk seeking index r is subjective and changes with the actor. So, the manager in general may have a different value r^* and therefore a different value of the discriminant ratio R^* a priori. In the following, it is assumed that manager and owner agree that the optimal action a priori is, for example, a_2 (i.e. close the bridge), thus both ratios R and R^* are smaller than one.

3.2 Decision a posteriori

In the previous subsection, how decision is made a priori has been explained, meaning 'before' the decision maker has seen the data acquired by the monitoring system. In this subsection, it is explained instead how judgment and decision change when the information from the monitoring system is available to the decision maker, a situation called a 'posteriori', meaning 'after' the decision maker has seen the data. Recall that the judgment a posteriori is indicated with the notation $\mathrm{P}(S_j|y)$, to separate it from the judgment a priori $\mathrm{P}(S_j)$, where y is the output of the monitoring system.

The analysis a posteriori from the owner's perspective is now analysed. After observing a particular output y from the monitoring system, the owner updates their knowledge of the structural state from prior $\mathrm{P}(S_j)$ to posterior $\mathrm{P}(S_j|y)$. Similar to the prior case, the owner decides a posteriori by comparing the expected utilities of the two actions a posteriori, i.e. $u_1(y) = \mathrm{P}(S_1|y)U_{11}$ and $u_2(y) = \mathrm{P}(S_2|y)U_{22}$, as shown in Figure 4(d) in the example case of Gaussian likelihood distributions (Figure 4(a), 4(b), 4(c)). The threshold $\bar{y}$ is the value of y (i.e. the output of the monitoring system, Figure 2) for which a posteriori the expected utilities are the same, i.e. $u_1(\bar{y}) = u_2(\bar{y})$, which can be expressed in the following way:

$$\bar{y}\colon u_1(\bar{y}) = u_2(\bar{y}). \tag{12}$$

Recall it has been assumed that the owner's choice a priori is action a_2 (i.e. close the bridge), and that y is defined in such a way that the optimal action a posteriori is a_1 (i.e. do nothing) for $y < \bar{y}$. Therefore, a posteriori the owner will change their decision when $y < \bar{y}$ and confirm the prior decision otherwise. Using Bayes' theorem, Equation (12) can be rearranged in the form:

$$\bar{y}:\ \frac{\mathrm{p}(\bar{y}|S_1)}{\mathrm{p}(\bar{y}|S_2)} = \frac{\mathrm{P}(S_2)}{\mathrm{P}(S_1)}\frac{U_{22}}{U_{11}}. \tag{13}$$

It can be immediately recognized that the right-hand term of Equation (13) is the same ratio R a priori introduced in Equation (9). Further, a function $g(y)$ is defined as the ratio between the likelihoods of the two states:

$$g(y) = \frac{\mathrm{p}(y|S_1)}{\mathrm{p}(y|S_2)}. \tag{14}$$

Therefore, the owner threshold $\bar{y}$ is determined by the following simple equation:

$$\bar{y}:\ g(\bar{y}) = R. \tag{15}$$

As such function g, which depends only on the likelihood distributions, equals R when evaluated in the threshold, as shown in Figure 4(e). It is possible to observe that the threshold effectively depends on ratio R, which in turn depends on the risk apatite of the owner.

In a similar manner, the manager threshold $\bar{y}^*$ is such that $g(\bar{y}^*) = R^*$, as illustrated again in Figure 4(e). The manager threshold $\bar{y}^*$ can be bigger or smaller than the owner threshold $\bar{y}$ depending on whether the manager is respectively more or less risk seeking than the owner. Since in general the threshold of the manager $\bar{y}^*$ and the one of the owner $\bar{y}$ do not coincide, it is possible to have three different situations a posteriori (i.e. following a monitoring observation y):

- If observation y is smaller than the two thresholds, both manager and owner agree to change their decision to a_1.
- If observation y is bigger than the two threshold, manager and owner agree to keep the prior decision a_2.

- if observation y is included between the two thresholds, manager and owner disagree on the decision to be made.

3.3 Preposterior analysis

The utility gain resulting from changing a decision a posteriori is defined as $\Delta u(y) = u_1(y) - u_2(y)$. Evidently, changing their mind is convenient to the owner when the monitoring system yield value smaller than their threshold. The *conditional VoI*, introduced in Equation (7), based on the developed assumptions can be calculate as follows:

$$VoI = \int_0^{\bar{y}^*} \Delta u(y) \cdot \mathrm{p}(y)\, \mathrm{d}y, \tag{16}$$

where $\Delta u(y) \cdot \mathrm{p}(y)$ can be seen as an expected utility density function (EUDF), plotted in Figure 4(f). The figure shows that the *VoI* is effectively the area under the expected utility function up to the threshold of the manager $\bar{y}^*$. It is also observed that:

- Because the EUDF is greater than zero under the threshold of the owner $\bar{y}$, evidently the *VoI* is maximum and always positive when the two thresholds coincide; this is the case of the unconditional value of information *uVoI*.
- When the manager is less risk seeking than the owner, i.e. $\bar{y}^* < \bar{y}$, the *conditional VoI* is smaller than the unconditional, but can never be negative – could be at least zero when $\bar{y}^* = 0$.
- When the manager is more risk seeking than the owner, i.e. $\bar{y}^* > \bar{y}$ (this is exactly the situation of Figure 4(f)), the negative integral of the EUDF between the two thresholds can be interpreted as a *Loss for Disagreement (LfD)* of the

two decision makers. If the *LfD* is bigger than the *uVoI*, then the *conditional VoI* results as a negative valued *VoI*.

In order to better clarify the condition whereby the *VoI* is negative, note that in the particular case analysed the EUDF can be written as:

$$\Delta u(y) \cdot \mathrm{p}(y) = (\mathrm{P}(S_1|y)U_{11} - \mathrm{P}(S_2|y)U_{22}) \cdot \mathrm{p}(y) = \mathrm{P}(y|S_1)\mathrm{P}(S_1)U_{11} - \mathrm{P}(y|S_2)\mathrm{P}(S_2)U_{22}. \quad (17)$$

Therefore, the *conditional VoI* becomes:

$$VoI = \int_0^{\bar{y}^*} \mathrm{p}(y|S_1)\mathrm{P}(S_1)U_{11}\,\mathrm{d}y - \int_0^{\bar{y}^*} \mathrm{p}(y|S_2)\mathrm{P}(S_2)U_{22}\,\mathrm{d}y. \quad (18)$$

The *VoI* is equal to zero either if $\bar{y}^* = 0$ or:

$$\bar{y}^*: \quad \frac{\int_0^{\bar{y}^*} \mathrm{p}(y|S_1)\,\mathrm{d}y}{\int_0^{\bar{y}^*} \mathrm{p}(y|S_2)\,\mathrm{d}y} = \frac{\mathrm{P}(S_2)}{\mathrm{P}(S_1)} \frac{U_{22}}{U_{11}}. \quad (19)$$

Notice that the format of Equation (19) is strikingly similar to Equation (13), with the only difference that the left-hand term is the ratio between the cumulative distributions of the two likelihoods, rather than the two mass density functions. Therefore, another function $G(y)$ is defined, as the ratio between the cumulative distributions of the two likelihoods (i.e. $\mathrm{F}(y|S_1)$ and $\mathrm{F}(y|S_2)$):

$$G(\mathrm{y}) = \frac{\mathrm{F}(y|S_1)}{\mathrm{F}(y|S_2)} = \frac{\int_0^{y} \mathrm{p}(y|S_1)\,\mathrm{d}y}{\int_0^{\bar{y}} \mathrm{p}(y|S_2)\,\mathrm{d}y}. \quad (20)$$

Consequently, the minimum manager threshold $\bar{y}^*$ that makes the *VoI* negative is determined by the following simple equation:

$$\bar{y}^*: \quad G(\bar{y}^*) = R. \quad (21)$$

This outcome, together with Equation (15), explicates how the threshold $\bar{y}$ and the index R of the manager, i.e. $\bar{y}^*$ and R^*, must be, in comparison to the ones of the owner, in order to achieve a null *conditional VoI*:

$$\bar{y}^* = G^{-1}\left(g(\bar{y})\right), \qquad R^* = g\left(G^{-1}(R)\right). \tag{22a,b}$$

In other words, in order to have a null *VoI*, the ratio between indexes r and between the thresholds $\bar{y}$ of the two agents are:

$$\frac{r^*}{r} = \frac{R^*}{R} = \frac{g\left(G^{-1}(R)\right)}{R}, \qquad \frac{\bar{y}^*}{\bar{y}} = \frac{G^{-1}(R)}{g^{-1}(R)}. \tag{23a,b}$$

3.4 Generalization and summary

These formulations have been derived under the very stringent assumption that $U_{12} = U_{21} = 0$ (see Figure 3). This assumption is now released: the condition whereby the owner will favour action a_2 (i.e. close the bridge) over a_1 (i.e. do nothing), which was previously encoded into Equation (9), now reads:

$$\frac{\mathrm{P}(S_2)}{\mathrm{P}(S_1)} \frac{U_{22} - U_{12}}{U_{11} - U_{21}} < 1, \tag{24}$$

so, it suffices to redefine the risk seeking factor r (previously encoded into Equation (11b)) as:

$$r = \frac{U_{22} - U_{12}}{U_{11} - U_{21}}, \tag{25}$$

and the rest of the formulation is completely identical. Index r, and consequently also R, is an indicator about the risk appetite of the decision maker based on the definition of the four utilities: even in this general case, the more the agent is risk seeking, the bigger is index r.

In summary, the necessary and sufficient condition to have a negative *VoI* is:

$$R^* > g\left(G^{-1}(R)\right), \qquad \bar{y}^* > G^{-1}\left(g(\bar{y})\right), \tag{26a,b}$$

where the ratio R depends on the prior odds q and on the risk seeking ratio r, defined in Equation (25). The conclusion is that, in order to achieve a negative *VoI*, the manager has to be more risk seeking than the owner, i.e. $r^* > r$, so that their threshold $\bar{y}^*$ is bigger of an amount that only depends on the choice of the likelihood distributions.

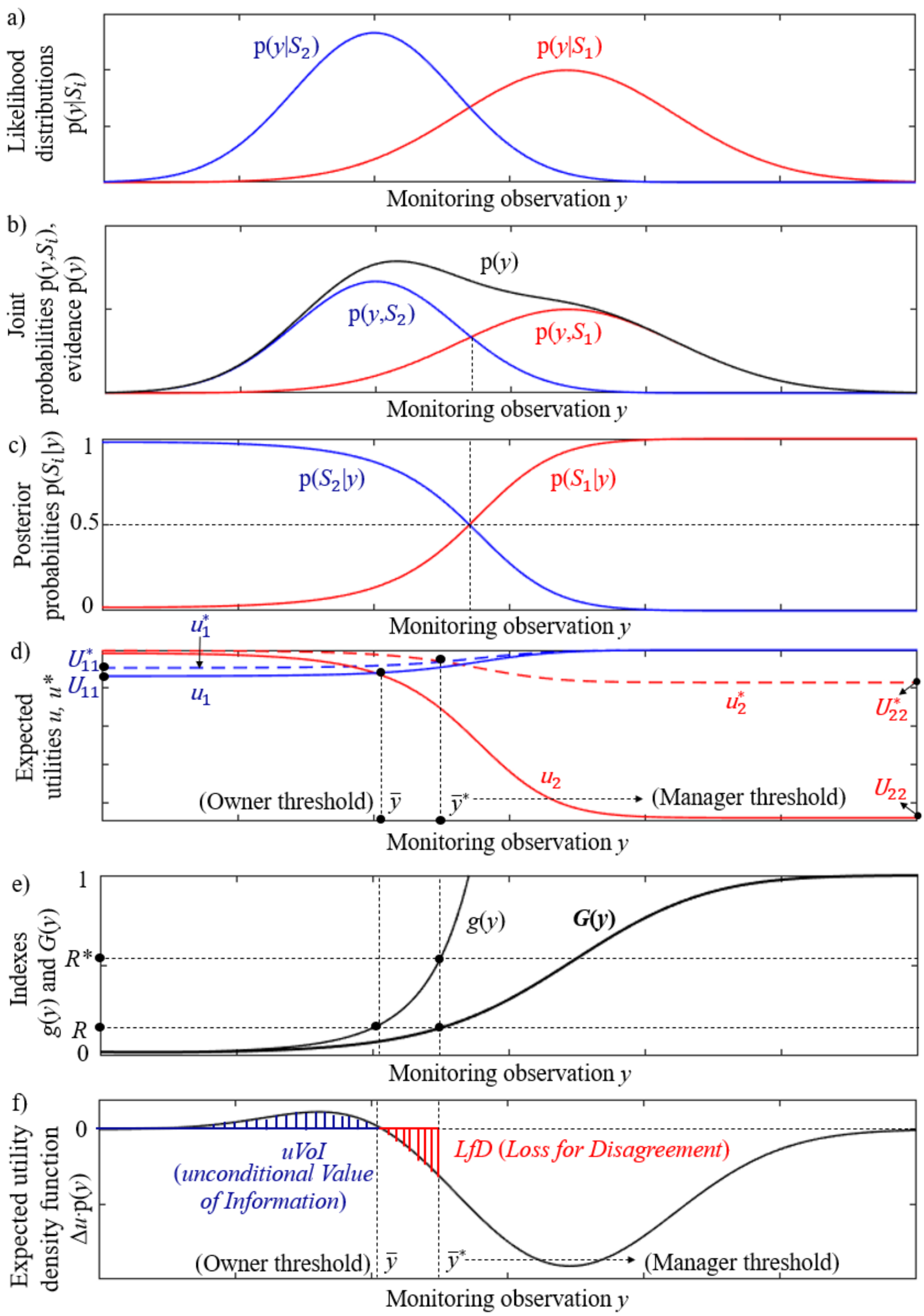

Figure 4. Graphical representation of how the *conditional VoI* may become negative: likelihood distributions (a), joint probabilities and evidence (b), posterior probabilities (c), expected utilities (d), indexes g and G (e), expected utility density functions (f).

3.5. Notable case

Equation (26a) shows that the ratio R^* that produces a negative *VoI* depends only on the choice of the likelihood distributions (illustrated for example in Figure 4(a) as Gaussian) and on the owner ratio *R*. In order to calculate R^*, functions $g(y)$ and $G(y)$ have to be expressed, and their inverse functions $g^{-1}(R)$ and $G^{-1}(R)$ have to be calculated. Unfortunately, in most cases it is not easy, and sometime not even possible, to express the inverse functions in closed form. A notable exception is when the likelihood distributions are described with polynomial functions, as follows:

$$\mathrm{p}(y|S_1) = (\mathrm{n}+1)\, y^{\mathrm{n}}, \quad \mathrm{p}(y|S_2) = \frac{\mathrm{n}+1}{\mathrm{n}}(1 - y^{\mathrm{n}}), \quad \text{with } y \in [0, 1]. \tag{27}$$

These likelihoods are presented in Figure 5, as an example, for the polynomial degree n varying from 1 to 4. For instance, n=1 corresponds to the case of triangular distributions over an interval. In order to make this case more intuitive to the reader, imagine that the monitoring system ultimately yields a damage index that is equal to 0 if there is no damage, and equal to 1 if the structure is fully damaged. In this case, the undamaged likelihood (i.e. $\mathrm{p}(y|S_2)$) could be described with a triangular distribution that has its maximum at 0, at its minimum at 1. Similarly, the damaged likelihood (i.e. $\mathrm{p}(y|S_1)$) could be described with a triangular distribution that has its maximum at 1, at its minimum at 0.

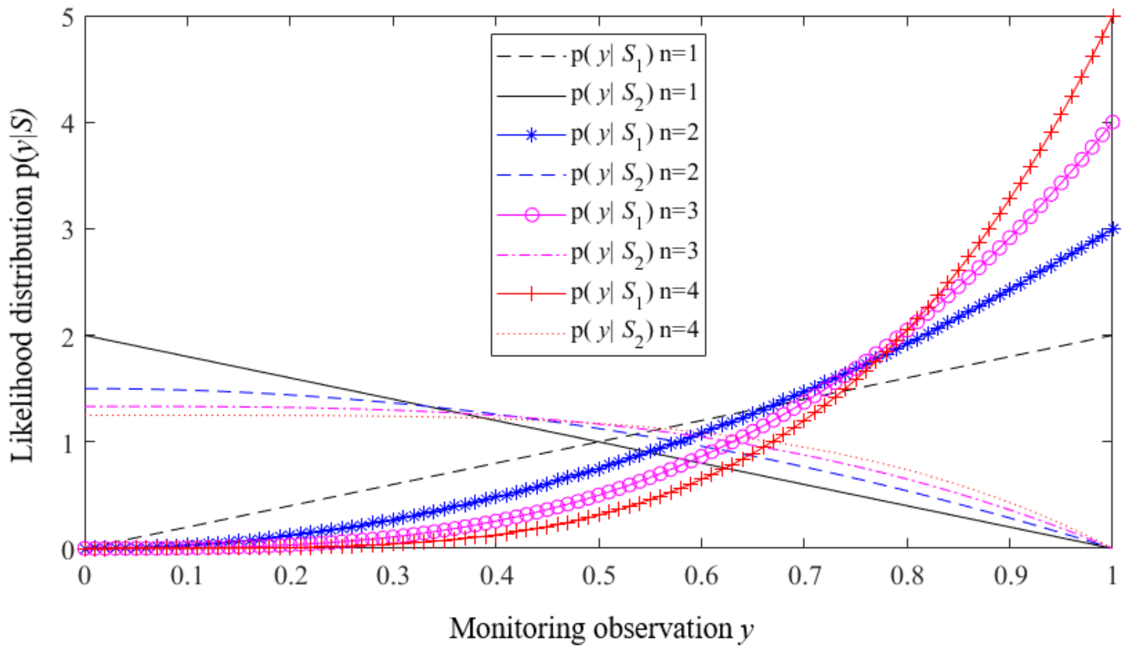

Figure 5. Likelihood distributions according to the polynomial degree n (this is a specific case of likelihood distributions that can be compared to Figure 4(a)).

In this case, functions $g(y)$, $G(y)$ and their inverse are:

$$g(y) = \mathrm{n}\,\frac{y^{\mathrm{n}}}{1 - y^{\mathrm{n}}}, \qquad g^{-1}(R) = \sqrt[\mathrm{n}]{\frac{R}{\mathrm{n}+R}}\,. \tag{28a,b}$$

$$G(y) = \mathrm{n}\,\frac{y^{\mathrm{n}}}{(\mathrm{n}+1) - y^{\mathrm{n}}}, \qquad G^{-1}(R) = \sqrt[\mathrm{n}]{\frac{R(\mathrm{n}+1)}{\mathrm{n}+R}}. \tag{29a,b}$$

An interesting feature of this class of likelihood functions is that the rate between the manager and owner threshold is constant and equal to:

$$\frac{\bar{y}^*}{\bar{y}} = \sqrt[\mathrm{n}]{\mathrm{n}+1}. \tag{30}$$

This means that, to achieve a null *conditional VoI*, the threshold of the manager has to be bigger than the one of the owner of a quantity that depends only on the polynomial degree

n. For instance, in the linear case, i.e. n = 1, it results that $\bar{y}^*$ has to be double of $\bar{y}$. Table 3 reports the results for n from 1 to 4.

Table 3. How the ratio between $\bar{y}^*$ and $\bar{y}$ varies according to n to achieve *VoI* = 0.

n	1	2	3	4
$\frac{\bar{y}^*}{\bar{y}}$	2	$\sqrt{3}$	$\sqrt[4]{3}$	$\sqrt[5]{4}$

It is evident that, as n increases, it decreases how much $\bar{y}^*$ has to be bigger than $\bar{y}$ in order to have $^{(O|M)}VoI$ = 0, and consequently a negative *conditional VoI*. It is possible to understand easily the reason of this outcome by analysing it graphically: in the linear case, presented in Figure 6(a), the threshold of the manager has to be clearly double of the one of the owner, because *uVoI* and *LfD* are two triangles. Conversely, with n > 1, as for example Figure 6(b) shows for n = 2, it is evident that, in order to have the area of *uVoI* and the one of *LfD* equal, $\bar{y}^*$ has to be bigger than $\bar{y}$, but less than double.

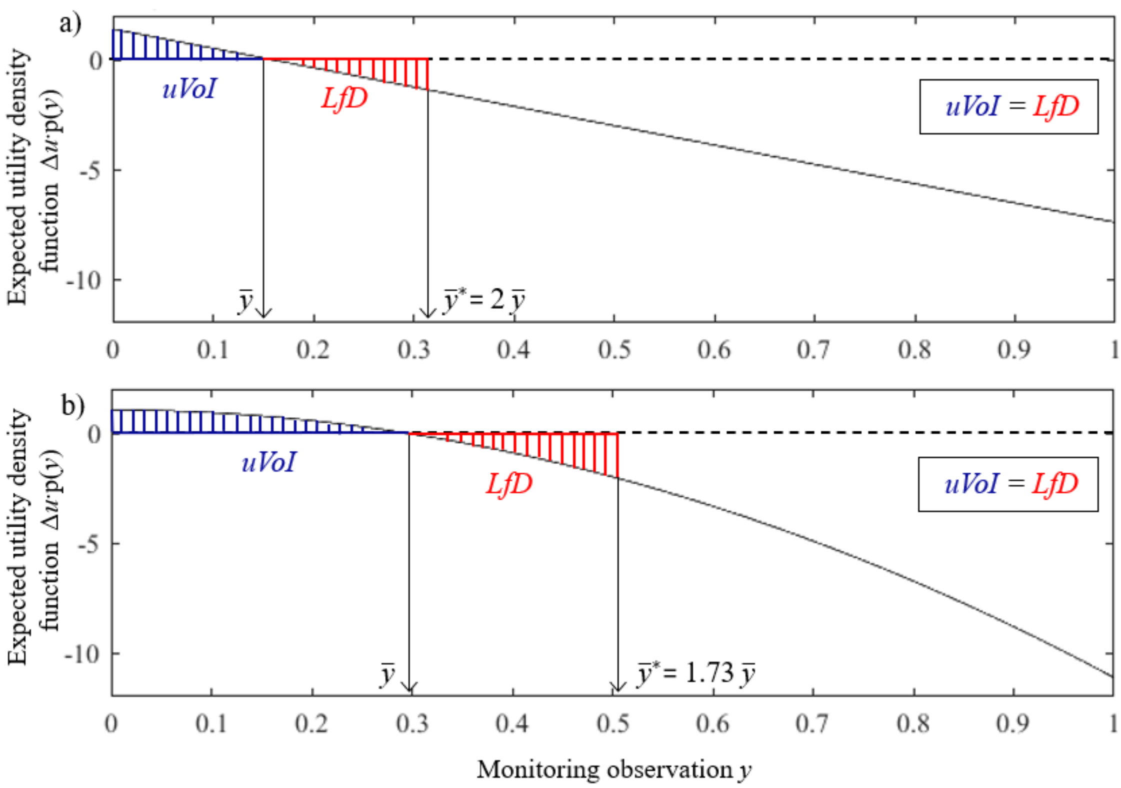

Figure 6. Expected utility density function for n = 1 (a), and n = 2 (b) (*uVoI* = *LfD* based on the values of Table 3).

In addition, it has already been anticipated that a bigger threshold corresponds to a bigger index *R*, meaning that the manager has to be more risk seeking than the owner in order to have a null *conditional VoI*, and consequently a negative one. It is possible to verify this sentence by developing Equation (23a), in this case of polynomial likelihood distributions:

$$\frac{r^*}{r} = \frac{R^*}{R} = \frac{g\left(G^{-1}(R)\right)}{R} = \frac{\mathrm{n}+1}{1-R} = \frac{\mathrm{n}+1}{1-r/q}. \tag{31}$$

This means that, in order to have a null *conditional VoI*, the manager has to be more risk seeking than the owner, i.e. $r^* > r$, by an amount that increases as the polynomial degree n rises, and which depends also on *r* itself. In conclusion, while it is clear that in real-life the likelihood distributions may have various different shapes, e.g. Gaussian as in the case study of Section 4, defining them with polynomial functions allows us to achieve results in closed form, which is useful to understand better the practical meaning of the developed formulation.

4. The Streicker Bridge case study

To illustrate how the developed framework works, it is analysed the same case study as in Zonta et al. (2014) and Bolognani et al. (2018), i.e. the Streicker bridge, since it respects all the assumptions introduced in the previous sections. The bridge is a pedestrian steel-concrete structure located at Princeton University Campus and, from a structural point of view, it consists of a thin post-tensioned supported by a high resistance steel lattice. The main span of the bridge overpasses Washington Road, a busy public road of the campus (see Figure 7(a) and 7(b)). The SHM-*lab* of Princeton University instrumented the bridge with two SHM systems: global structural monitoring using discrete long-gauge strain Fiber Optic Sensors (FOS), based on fiber Bragg-grating (FBG) (Kang, et al., 2007); and integrity monitoring, using truly distributed FOS based on Brillouin Optical Time Domain Analysis (BOTDA) (Nikles, et al., 1996). These two approaches are complementary: discrete sensors monitor an average strain at discrete points, while the distributed sensors monitor one-dimensional strain field. Discrete FOS embedded in the bridge deck have gauge length 60 cm and feature excellent measurement properties with error limits of ±4 με. Thus, they are excellent for assessment of global structural behaviour and for structural identification. Instead, distributed FOS have accuracy an order of magnitude lower than discrete sensors and so cannot be used for accurate structural identification; they are used for damage detection and localization. Figure 7(c) shows the sensors map in the main span. More details about the Streicker bridge and its case study are provided in a number of past publications (Glisic & Adriaenssens, 2010) (Glisic, et al., 2011) (Glisic & Inaudi, 2012).

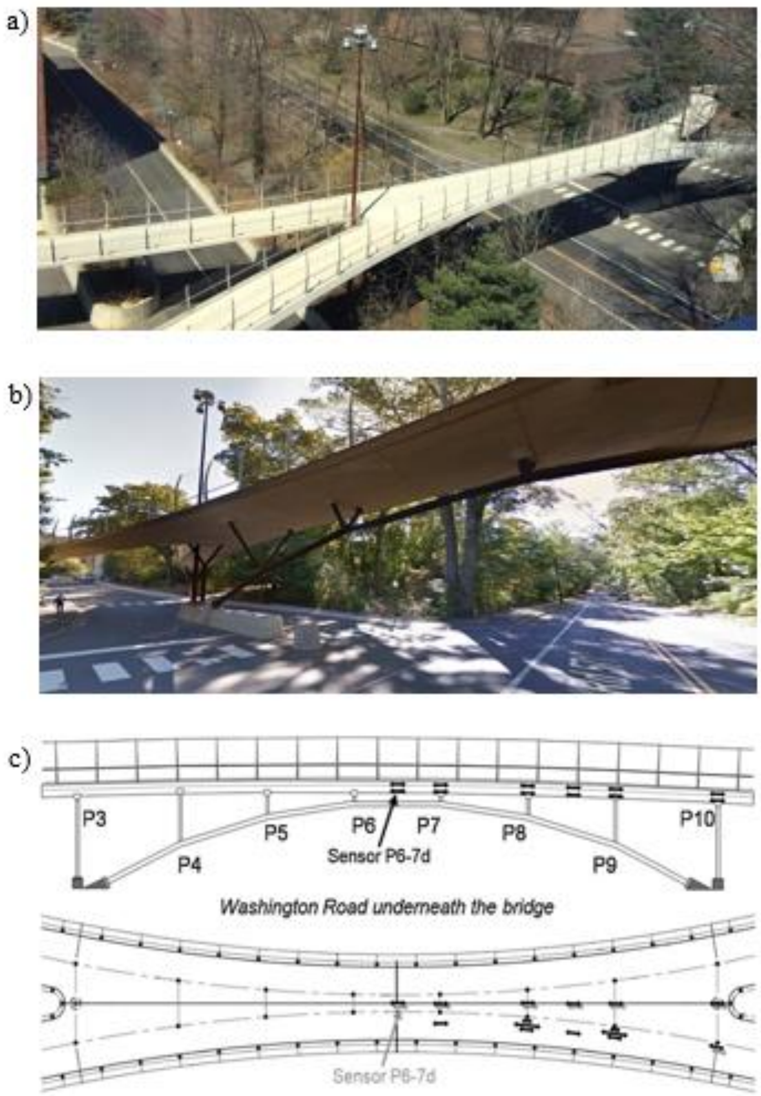

Figure 7. The Streicker bridge: view of the bridge (a)(b), location of the sensor analysed in the case study (c).

4.1 Introduction of the SHM-decision problem

The SHM-based decision problem, the main assumptions and the individuals involved are the same as in Bolognani et al. (2018). The bridge is managed by two fictitious agents with distinct roles:

- Ophelia (O) is the *owner* responsible for Princeton's estate, who has to decide on whether or not to install the monitoring system; she is Malcolm's supervisor.

- Malcolm (M) is the *manager* responsible for the bridge operation and maintenance, who has to take decisions on the state of the bridge based on monitoring data.

They are both rational individuals and they have the same background knowledge, they only differ in the way how to weight the seriousness of the consequences of a failure. They are concerned by a single specific scenario: a truck, driving along Washington road, could collide with the steel arch of the bridge. After the incident, the bridge will be in one of the following two states:

- S_1 = *damaged* (D), i.e. the bridge is still standing but has suffered major damage, and there is a change of collapse of the entire bridge.
- S_2 = *undamaged* (U), i.e. the structure has either no damage or some minor damage.

According to Malcolm and Ophelia, the two states are mutually exclusive and exhaustive, i.e. P(D) + P(U) = 1. It is assumed that they focus on the sensor installed at the bottom of the middle cross-section between P6 and P7 (called Sensor P6-7d, see Figure 7(c)). The output of the monitoring system is then represented by the strain ε of this specific fiber optic sensor. It is also assumed that the two agents use the same interpretation model, i.e. they interpret identically the data from the monitoring system, as it will be presented in Section 4.2.

After Malcolm the manager estimates the state of the bridge, he may decide between two alternative actions:

- a_1 = *do nothing* (DN), i.e. no special restrictions to traffic under and over the bridge.

- a_2 = *close bridge* (CB), i.e. both Streicker Bridge and Washington Road are closed to traffic for the time needed for a thorough inspection, estimated to be 1 month.

Finally, Ophelia and Malcolm agree that the costs, denoted by z, related to each action, for each state, are the same as estimated in Glisic and Adriaenssens (2010) and reported in Table 4: z_F is the failure cost while z_{DT} is a 1-month downtime cost. The resultant decision tree of this case study is illustrated in Figure 8.

Table 4. Costs per action and state.

	State S_1 = D (*damaged*)	State S_2 = U (*undamaged*)
Action a_1 = DN (*do nothing*)	$z_{11} = z_F$ = 881.60 k$	z_{12} = 0.00 k$
Action a_2 = CB (*close bridge*)	$z_{21} = z_{DT}$ = 139.80 k$	$z_{22} = z_{DT}$ = 139.80 k$

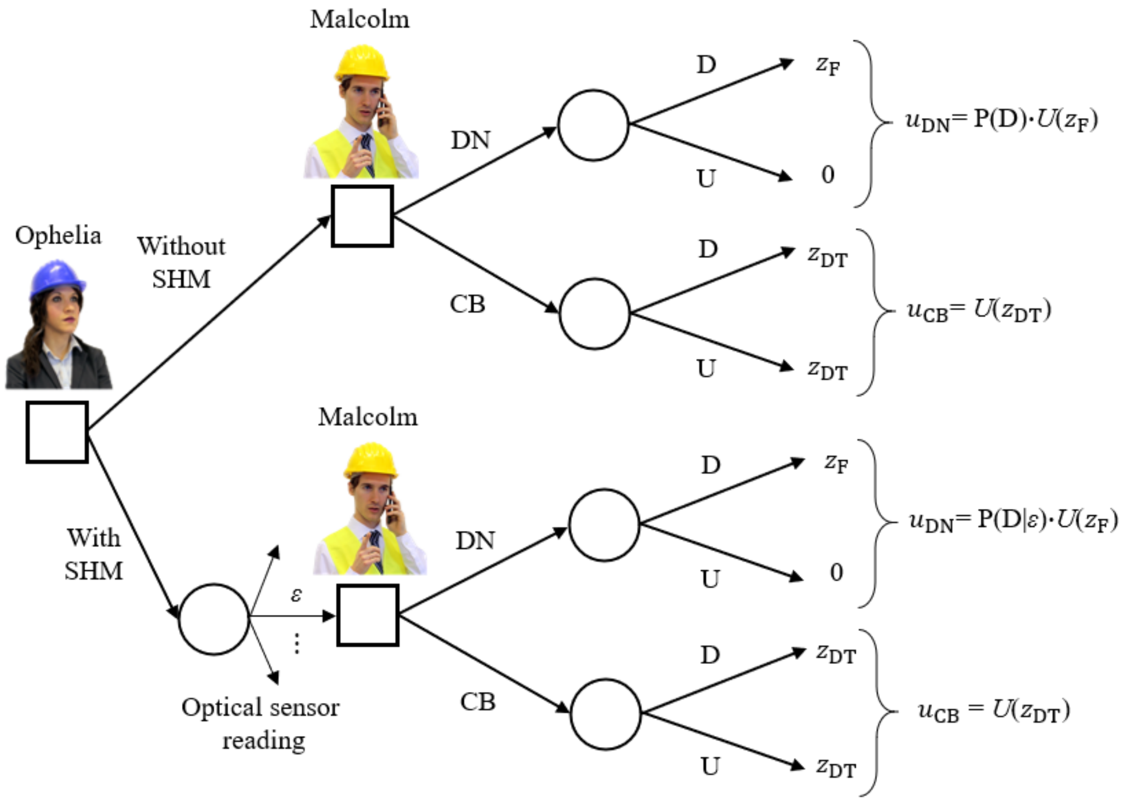

Figure 8. Decision tree for the Streicker bridge case study (application of Figure 2).

In order to apply the formulation about the negative *VoI* introduced in Section 3, the likelihood distributions, i.e. indexes $g(\varepsilon)$ and $G(\varepsilon)$ (see Equation (14) and (20)), the appetite for risk of the decision makers and the choice of prior probabilities, i.e. index R, have to be analysed.

4.2 Analysis of likelihood distributions

In this subsection, firstly the likelihood distributions of the case study are introduced, then the resultant indexes $g(\varepsilon)$ and $G(\varepsilon)$ are evaluated.

Similar to Zonta et al. (2014), the likelihoods of the two states are described by Gaussian distributions: p(ε|U) is the likelihood of no damage, defined with mean value $\mu = 0$ με and standard deviation $\sigma = 300$ με, since Malcolm and Ophelia expect the bridge to be undamaged if the change in strain will be close to zero, along with a natural fluctuation of the strain due to thermal effects and to a certain extent due to creep and shrinkage; p(ε|D) is instead the likelihood of damage, defined with mean value $\mu = 1000$ με and standard deviation $\sigma = 600$ με, since in this case they expect a significant change in strain. Table 5 summarizes the main features of these likelihoods.

Table 5. Main features of the Gaussian likelihood distributions.

	Mean value μ [με]	Standard deviation σ [με]
Likelihood of no damage p(ε|U)	0	300
Likelihood of damage p(ε|D)	1000	600

Before the data are available, Malcolm and Ophelia can predict the distribution of ε, which is practically the so-called evidence in classical Bayesian theory, through the formula:

$$p(\varepsilon) = p(\varepsilon|D) \cdot P(D) + p(\varepsilon|U) \cdot P(U). \tag{32}$$

When the measurement ε is instead available, both the agents update their estimation of the probability of damage consistently with Bayes' theorem:

$$p(D|\varepsilon) = \frac{p(\varepsilon|D) \cdot P(D)}{p(\varepsilon)}, \tag{33}$$

where $p(D|\varepsilon)$ is the posterior probability of damage.

The defined likelihood distributions, illustrated in Figure 9(a), allow us to calculate the resulting indexes $g(\varepsilon)$ (from Equation (14)) and $G(\varepsilon)$ (from Equation (20)), which are presented in Figure 9(b). Note that the plot of $g(\varepsilon)$ has been cut at $g(\varepsilon) = 2$, since for $\varepsilon > 500$ με $g(\varepsilon)$ tends to infinity.

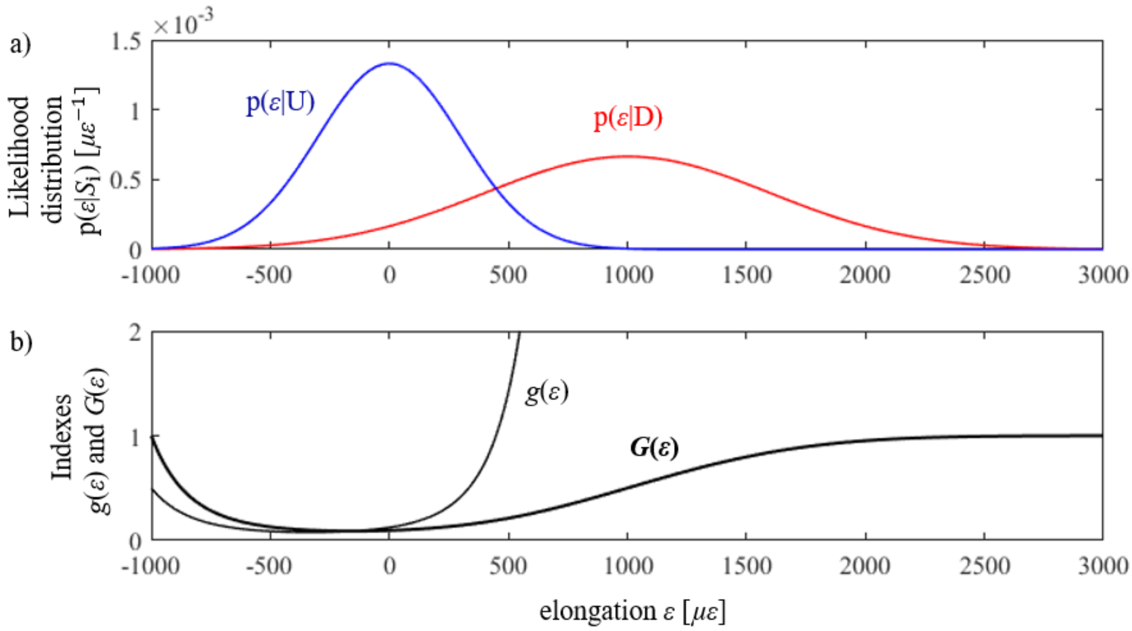

Figure 9. Analysis of the likelihood distributions: likelihoods (a), index $g(\varepsilon)$ and $G(\varepsilon)$ (b)

(this figure corresponds to Figure 4(a) and 4(e)).

4.3 Analysis of appetite for risk of decision makers

The index R varies according to the appetite for risk of the decision maker, i.e. index r (see Equation (25)), and to the choice of prior probabilities, i.e. index q (see Equation (11(a))). While the prior probabilities will be analysed later, the different utility functions of the two agents are now presented.

As introduced before, Ophelia and Malcolm differ in their utility functions, which is the weight they apply to the possible economic losses. In general, the risk appetite of an individual depends on multiple factors, which include: their confidence in making decision, funded on their professional experience and maturity; their position and responsibility into the decision chain; and most importantly their personal attitude and scale of value respect to the possible consequences of a collapse. Clearly these quantities can change according to the age and cultural background of the individual. In principle, understanding the risk appetite can be done with an elicitation process (Verzobio, et al., 2020). In the following, indices (M) and (O) will indicate that a quantity is intended respectively from Malcolm the manager's perspective and Ophelia the owner's perspective.

According to Bolognani et al. (2018), Ophelia is defined as risk neutral with respect to the loss compared to the value of a single structure, since she is in charge of a large stock of structures and then the loss corresponding to an individual structure is much smaller than the overall asset value. This means that, according to her behaviour, a negative utility is linear with the incurred loss. Strictly speaking, a utility function is defined except for a multiplicative factor, therefore it should be expressed in an arbitrary unit sometime referred to as *util* (McConnell, 1966). Since Ophelia's utility is linear with loss, for the sake of clarity negative utility will be deliberately confused with loss, and therefore Ophelia's utility will be measured in k$.

Unlike Ophelia, in order to demonstrate the formulation introduced in Section 3 about the negative *VoI*, it is supposed that the behaviour of Malcolm the manager can be risk adverse or risk seeking: in this way, since it is assumed the owner to be always risk neutral, it is possible to analyse both the situations of a manager more risk seeking and more risk adverse than the owner. These behaviours can be described mathematically using various models, for instance the Arrow-Pratt's utility model (Pratt, 1964) (Arrow, 1965), where the different aptitude of an agent is encoded in the coefficient of Absolute Risk Aversion (ARA) θ. Similarly to Bolognani et al. (2018), it is assumed that the manager's utility has constant ARA, and then the utility function takes the form of an exponential:

$$^{(\mathrm{M})}U(z) > \frac{1 - \mathrm{e}^{-z \cdot \theta}}{\theta}, \tag{34}$$

where θ is the constant ARA coefficient. Figure 10 shows the linear utility function of Ophelia's behaviour and both Malcolm's utility functions, which depend on his particular behaviour:

- Risk adverse, i.e. his negative utility increases more than proportionally with the loss. To calibrate θ, it is assumed that for a loss equal to the failure cost, Malcolm's negative utility is twice that of Ophelia's. This results in a constant ARA coefficient θ = -1.423 M^{-1}
- Risk seeking, i.e. his negative utility increases less than proportionally with the loss. In this case, it is assumed that for a loss equal to the failure cost, Malcolm's negative utility has to be less than half that of Ophelia's.

Consequently, constant ARA coefficient $\theta = 3.034$ M\$$^{-1}$ is chosen.

Based on these utility functions, the utilities U of the costs related to each action, for each state can be calculated, and consequently the index r, which in this case turns into:

$$r = \frac{U(z_{DT})}{U(z_F) - U(z_{DT})}. \tag{35}$$

All the outcomes are reported in Table 6. As expected, it is possible to notice that the more the decision maker is risk seeking, the bigger is the index r: ${}^{(O)}r = 0.189 > {}^{(M)}r = 0.096$ if the manager is risk adverse and then he is less risk seeking than the owner, ${}^{(M)}r = 0.590 > {}^{(O)}r = 0.189$ if the manager is risk seeking and then he is more risk seeking than the owner (who is considered always risk neutral).

In summary, index r has been fixed, while R will depend on the choice of the prior probabilities. In the next subsections, to understand how the VoI varies depending on the different appetite for risk of the two agents, and when it may consequently become negative, it will be evaluated both in the case of the manager risk adverse and risk seeking.

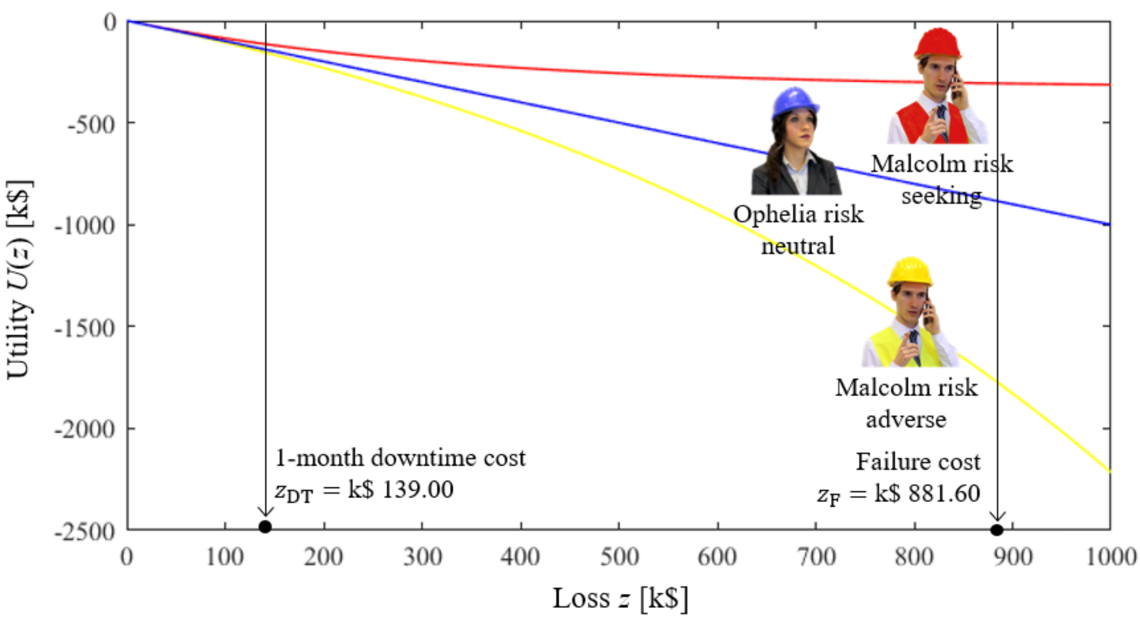

Figure 10. Representation of the utility functions for Malcolm the manager (Equation 34)

and Ophelia the owner.

Table 6. Ophelia's and Malcolm's loss perception.

Ophelia the owner RISK NEUTRAL			
	State D	State U	$^{(O)}r$
Action DN	$^{(O)}U(z_F)$ = -881.60 k$	$^{(O)}U(z)$ = 0.00 k$	0.189
Action CB	$^{(O)}U(z_{DT})$ = -139.80 k$	$^{(O)}U(z_{DT})$ = -139.80 k$	
Malcolm the manager RISK ADVERSE			
	State D	State U	$^{(M)}r$
Action DN	$^{(M)}U(z_F)$ = -1762.94 k$	$^{(M)}U(z)$ = 0.00 k$	0.096
Action CB	$^{(M)}U(z_{DT})$ = -154.94 k$	$^{(M)}U(z_{DT})$ = -154.94 k$	
Malcolm the manager RISK SEEKING			
	State D	State U	$^{(M)}r$
Action DN	$^{(M)}U(z_F)$ = -306.88 k$	$^{(M)}U(z)$ = 0.00 k$	0.590
Action CB	$^{(M)}U(z_{DT})$ = -113.93 k$	$^{(M)}U(z_{DT})$ = -113.93 k$	

4.4 Case 1: Malcolm the manager risk adverse

In this first case Ophelia is risk neutral while Malcolm is risk adverse, meaning that the manager is less risk seeking than the owner. According to the formulation introduced in Section 3, the goal is to verify that in this case it is not possible to find a negative *conditional VoI*. Since all the indexes have been defined about the formulation except q, in the following everything will be analysed in term of P(D), i.e. in term of q.

To start, the expected utilities u_0 a priori are evaluated, i.e. if the monitoring system is not installed. A decision maker would always choose to close the bridge when their utility related to the action CB is less negative than the utility of DN, in formula:

$$u_{CB} \geq u_{DN}, \qquad U(z_{DT}) \geq U(z_F) \cdot P(D). \qquad (36a,b)$$

Consequently, it is obtained that for Ophelia it is always convenient a priori to close the bridge if P(D) > 0.16, while for Malcolm if P(D) > 0.09, that is smaller because of his risk adverse behaviour. The outcomes are presented in Figure 11(a), along with the conditional expected utility $^{(O|M)}u_0$ calculated as in Equation (5), which is what is really needed in order to evaluate the *conditional VoI*. Note that $^{(O|M)}u_0$ has a discontinuity for P(D) = 0.09, since this is the value of P(D) for which a priori the manager changes the decision from action DN to action CB. In addition, it is important to remind that the formulation introduced in Section 3 is based on the assumption that a priori it is always convenient to choose action a_2, i.e. CB for this case study: this corresponds to having P(D) > 0.16, since in this way both the agents agree on choosing action CB a priori, i.e. their index R is < 1.

Consider the case of the monitoring system installed. In this case the decision maker can rely on the monitoring data ε, and then the preposterior expected utilities can be evaluated, in formula:

$$u_{\mathrm{CB}|\varepsilon} = u(\mathrm{z}_{\mathrm{DT}}), \qquad u_{\mathrm{DN}|\varepsilon} = u(\mathrm{z}_{\mathrm{F}}) \cdot \mathrm{p}(\mathrm{D}|\varepsilon). \tag{37a,b}$$

Note that the preposterior expected utilities of action DN depends on the posterior probability of having the bridge damaged p(D|**ε**), that can be calculate as in Equation (33). The resultant preposterior expected utilities u_{pp} are presented in Figure 11(b), in term of P(D), both in the unconditional and conditional form. It is possible to notice that the conditional outcome, i.e. $^{(O|M)}u_{pp}$, has again a discontinuity, this time for P(D) = 0.53, which corresponds to the value of P(D) for which a posteriori the manager changes his decision from action DN to action CB.

Finally, the *VoI* is simply the difference between the preposterior expected utility and the prior expected utility. Both the *unconditional* and *conditional VoI* can be calculated, according respectively to Equation (4) and Equation (7). Figure 11(c) shows the results, always in term of P(D). As regards the *unconditional VoI,* i.e. ${}^{(O)}uVoI$ and ${}^{(M)}uVoI$, it is possible to observe that they are maximum exactly at the value of P(D) for which it becomes convenient a priori to close the bridge, i.e. if P(D) = 0.16 for the owner and P(D) = 0.09 for the manager; these are the values which corresponds to having R = 1. In addition, it is possible to verify that it is never possible to find a negative *unconditional VoI*, according to the principle introduced in Section 2 that “information can’t hurt”. In addition, it can be noticed that, as expected since in this case the manager is less risk seeking than the owner, it is not possible to find any value of P(D) for which the *conditional VoI*, i.e. ${}^{(O|M)}VoI$, becomes negative. This happens because, due to Malcolm’s risk adverse behaviour, he would always choose to close the bridge a posteriori sooner than Ophelia, i.e. ${}^{(M)}\bar{\varepsilon} < {}^{(O)}\bar{\varepsilon}$, and then a smaller positive *uVoI* may be obtained, but it is not possible to achieve what has been defined as *Loss for Disagreement* (*LfD*): consequently, it is impossible to get a negative *VoI*, as expected.

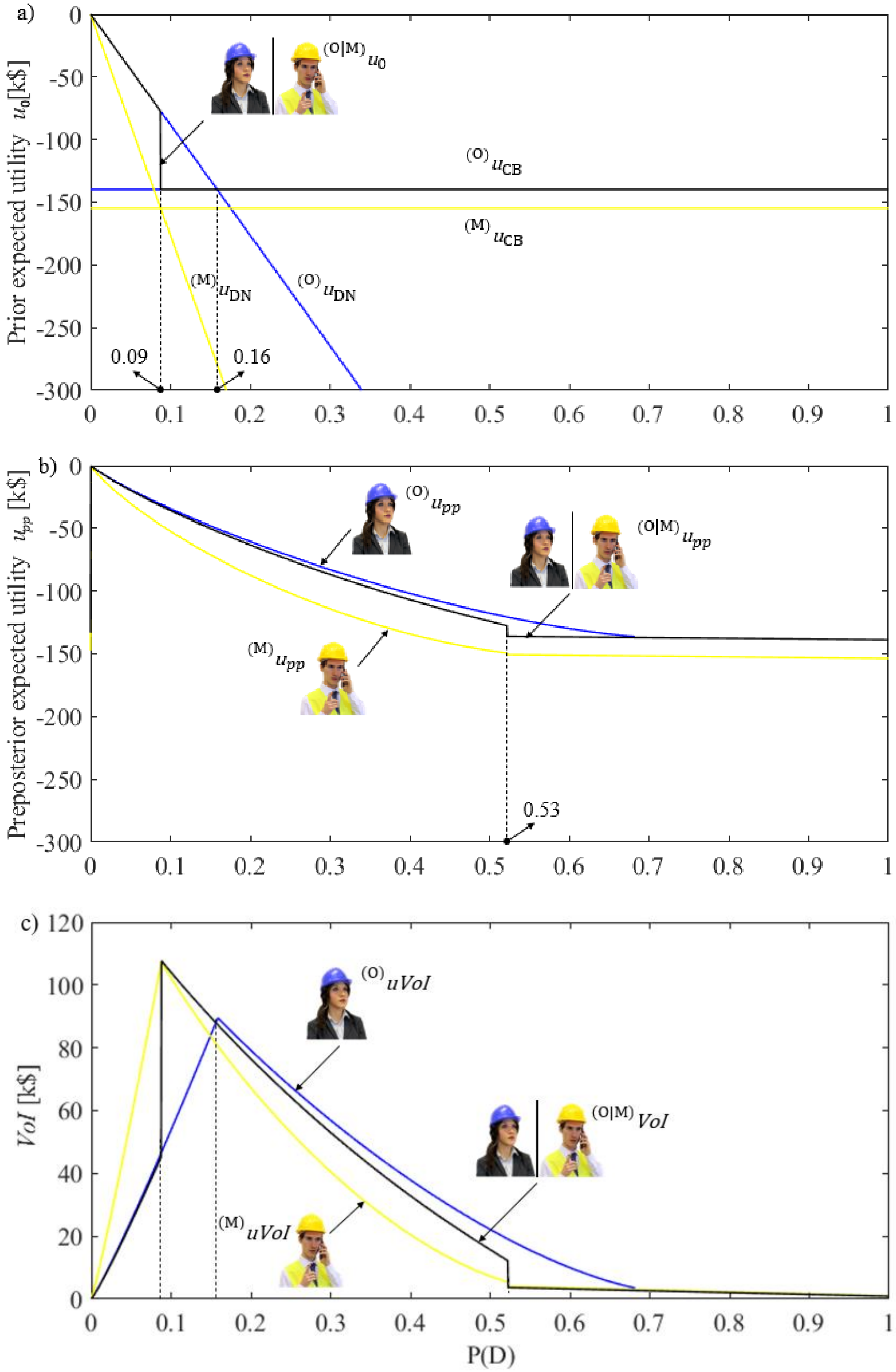

Figure 11. Prior expected utilities u_0 (see Equation 2 and 5) (a), preposterior expected utilities u_{pp} (see Equation 3 and 6) (b), *VoI* (see Equation 4 and 7) (c).

4.5 Case 2: Malcolm the manager risk seeking

In this second case, Ophelia the owner is considered still risk neutral, while Malcolm the manager is now risk seeking. This corresponds to the case where, according to the developed formulation of Section 3, it should be possible to find a negative *conditional VoI*. The goal is then to find out for which values of P(D), and consequently of the term q, this happens. The procedure followed is the same as in Section 4.4.

The expected utilities u_0 a priori are evaluated, i.e. in the case of a monitoring system not installed. In this case, as shown in Figure 12(a), for Ophelia is again convenient a priori to close the bridge if P(D) > 0.16, since she is still risk neutral, while for Malcolm it becomes P(D) > 0.37, that is clearly higher because of his risk seeking behaviour. As a consequence, the two agents agree on choosing a priori action CB if P(D) > 0.37.

Figure 12(b) presents the unconditional and conditional preposterior expected utilities, needed in order to evaluate the *VoI*, which is instead illustrated in Figure 12(c). As regards the *unconditional VoI*, i.e. $^{(O)}uVoI$ and $^{(M)}uVoI$, it can be observed again that they are maximum exactly at the value of P(D) for which it becomes convenient a priori to close the bridge, i.e. if P(D) = 0.16 for the owner and P(D) = 0.37 for the manager, and that it is never possible to find a negative *unconditional VoI*. Conversely, it is clearly possible to find some values of P(D) for which the *conditional VoI*, i.e. $^{(O|M)}VoI$, becomes negative: 0.58 < P(D) < 0.87. This happens because, due to his risk seeking behaviour, Malcolm would always choose to close the bridge a posteriori later than Ophelia, i.e. $^{(M)}\bar{\varepsilon} > {}^{(O)}\bar{\varepsilon}$, and therefore the so-defined *LfD* is obtained. Since there are some values of P(D), i.e. 0.58 < P(D) < 0.87, for which this the *LfD* is bigger than the *uVoI*, the consequence is that a negative *conditional VoI* is achieved.

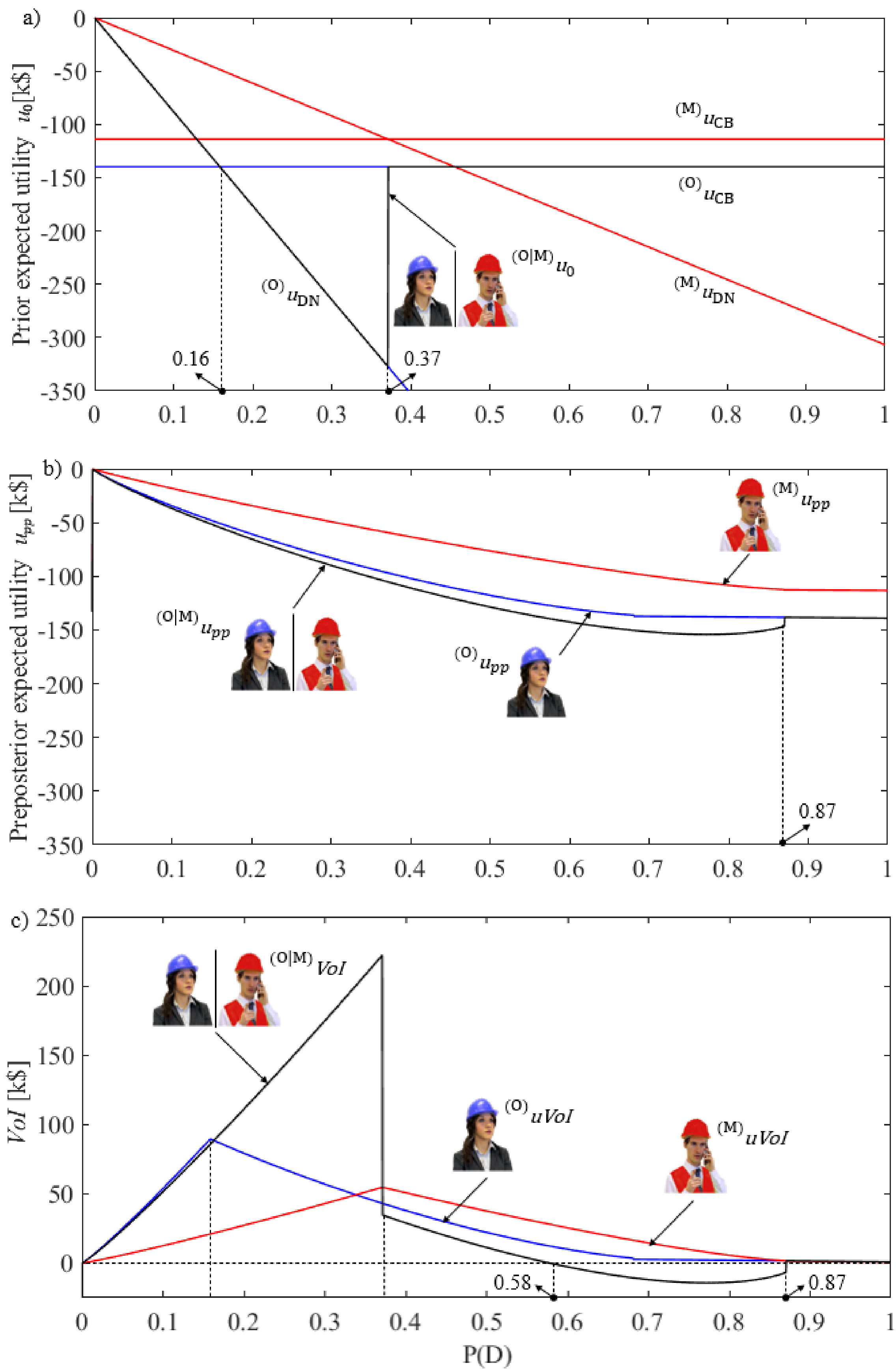

Figure 12. Prior expected utilities u_0 (see Equation 2 and 5) (a), preposterior expected utilities u_{pp} (see Equation 3 and 6) (b), VoI (see Equation 4 and 7) (c).

4.6 Discussion about negative conditional VoI

In the previous subsections has been demonstrated that, as expected, it is possible to obtain a negative *conditional VoI* only when the manager is more risk seeking than the owner, which agrees with the conclusions obtained theoretically in Section 3. In this specific case study, it happens when 0.58 < P(D) < 0.87, which corresponds to 1.38 < q < 6.69.

One specific case in this range is now analysed: for instance, P(D) = 0.65 is chosen, i.e. q = 1.86. In this case, indexes R and the thresholds for the two agents are:

$$^{(\mathrm{M})}R = 0.32 > {}^{(\mathrm{O})}R = 0.10. \tag{38}$$

$$^{(\mathrm{M})}\bar{\varepsilon} = 247\ \mu\varepsilon > {}^{(\mathrm{O})}\bar{\varepsilon} = -84\ \mu\varepsilon. \tag{39}$$

As expected, the threshold of Malcolm the manager is bigger than the one of Ophelia the owner, since Malcolm is more risk seeking than Ophelia. Consequently, there is a very wide range of values, from -84 με to 247 με, whereby Malcolm would keep the bridge open in disagreement with Ophelia, who instead believes this is a dangerous practice which can potentially result in a big loss. She is then forced to keep the bridge open for ε = [-84 με ÷ 247 με], even if it would be more convenient for her to close it: this causes a *LfD*, as shown in Figure 13(b). Since this negative area is bigger than the one of *uVoI*, the resultant *conditional VoI* is negative:

$$^{(\mathrm{O|M})}VoI = -11.61\ k\$. \tag{40}$$

This means that in this case Ophelia perceives the monitoring information as damaging: in summary, a negative *VoI* corresponds exactly to the amount of money Ophelia the owner is willing to pay to prevent Malcolm the manager using the monitoring system.

In conclusion, it has been proved that, for the prototype decision problem analysed in this contribution, the achievement of a negative *conditional VoI* depends on a combination between how much more seeking is the manager in comparison to the owner, and the choice of prior probabilities. While the development of the case study has been conditioned by the choice of specific risk appetites of the agents, i.e. fixed θ, in order to have a final verification of the conclusions achieved, it is interesting to investigate how the *conditional VoI* varies according to both the prior damage probability P(D), i.e. in term of index q, and the ARA coefficient θ of the manager, i.e. in term of his appetite for risk. Figure 14 shows graphically the results, with both a top view and a 3D view: it is clear that a negative *conditional VoI*, i.e. the dark blue area, can be achieved only for some specific combinations of high P(D) and positive θ, which indeed corresponds to a manager who is more risk seeking in comparison to the owner (who instead is defined to be risk neutral, i.e. $\theta = 0$). In the top view of Figure 14(a), the specific case analysed in this section is highlighted, i.e. $\theta = 3.034$ M^{-1} for the manager and P(D) = 0.65: it allows one to verify that, as calculated in Equation (40), this case falls into the area where a negative *conditional VoI* is achieved.

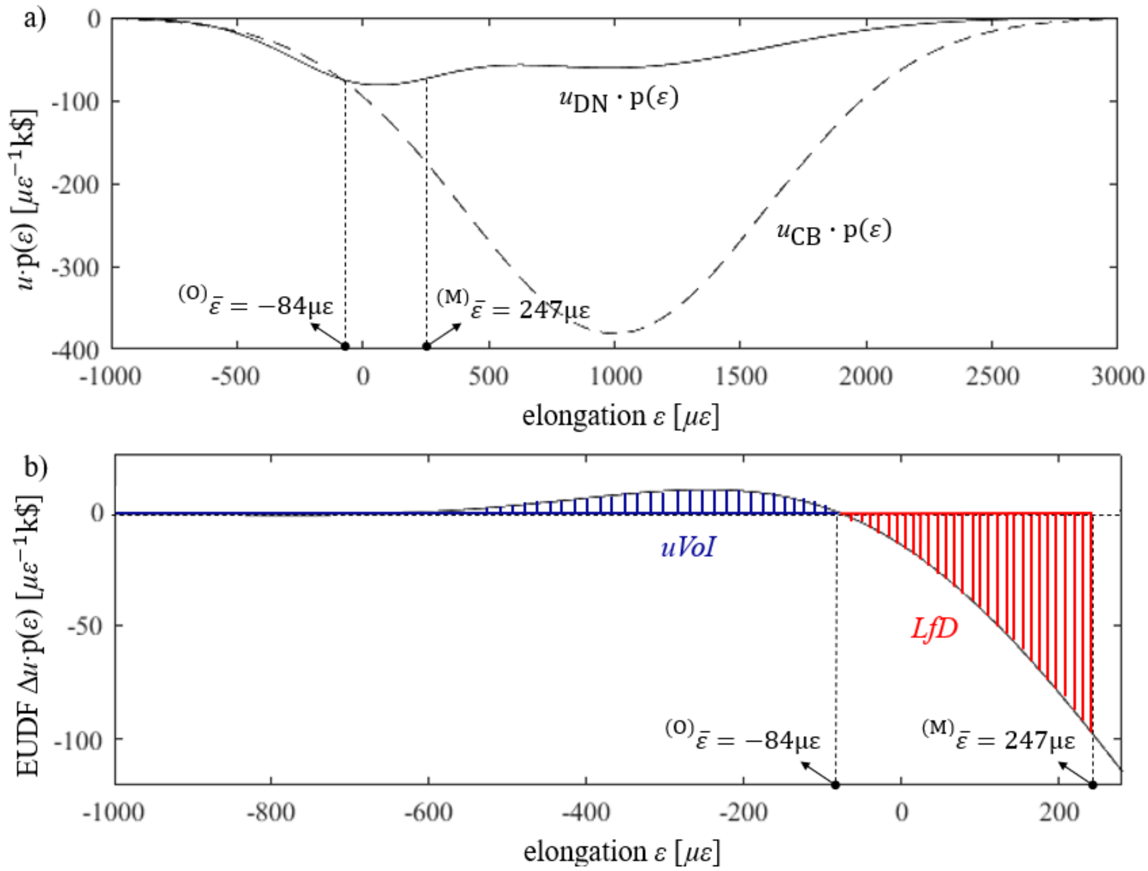

Figure 13. Analysis of the *conditional VoI*: density function of the two expected utilities (a); expected utility density function (EUDF), zoom in the values of interest (b) (it corresponds to Figure 4(f)).

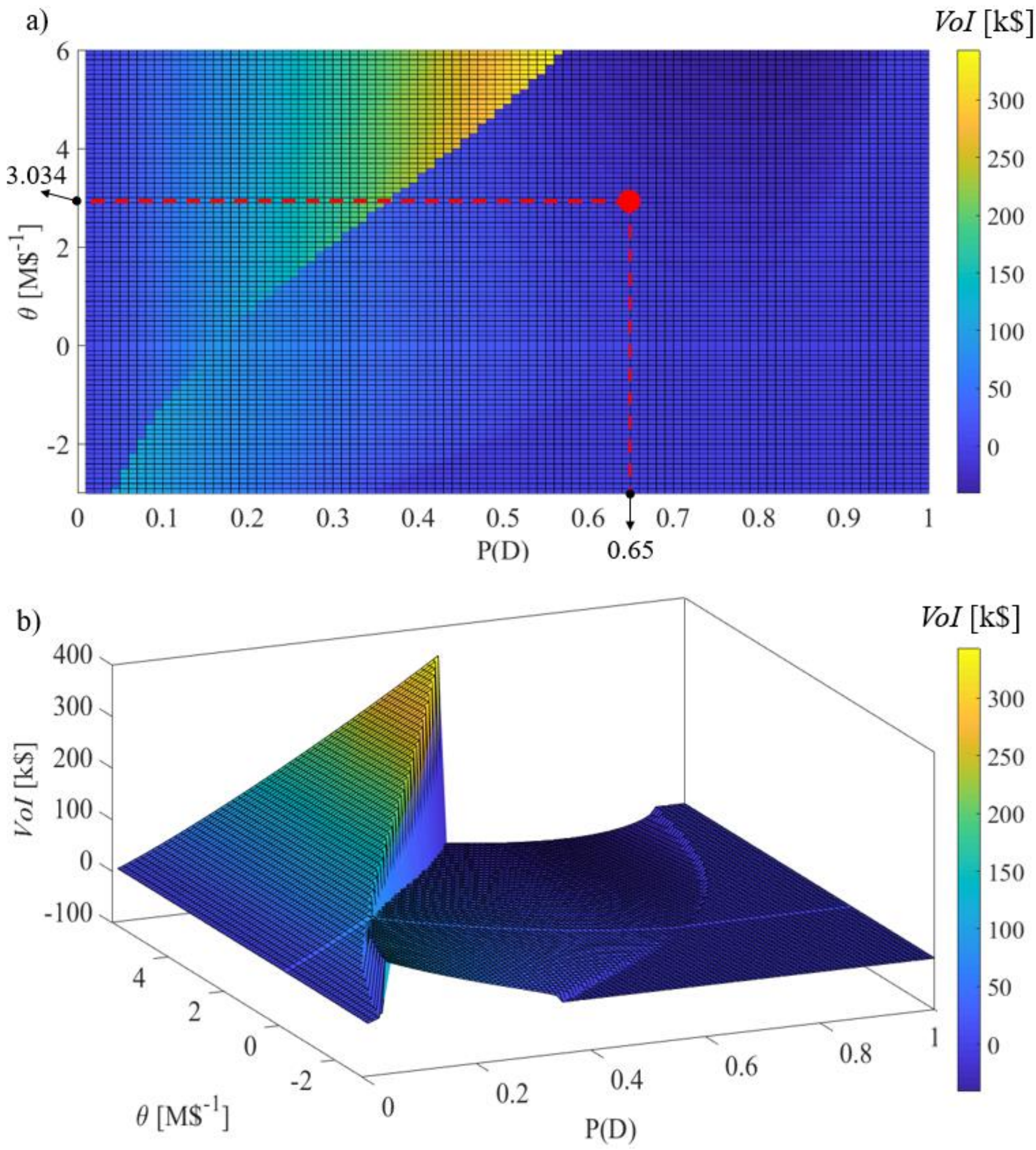

Figure 14. Graphical representation of the *conditional VoI* in function of both the prior damage probability P(D) and the ARA coefficient θ: top view (the red point corresponds to the case analysed in Equation 38, 39 and 40) (a) and 3D view (b).

5. Conclusions

The benefit of SHM can be quantified using the concept of the Value of Information (*VoI*), which is the difference between the utilities of operating the structure with and without the monitoring system. In its calculation, a commonly understood assumption is that the individual who decide on the installation of the monitoring system, i.e. the owner, is the same rational agent who will later use it, i.e. the manager. In the real word, these two agents involved in the decision chain are often different individuals. In this paper, the conditions for which it is possible that the *VoI* becomes negative are investigated, in the case of a specific prototype decision problem. A *negative VoI* means that the owner perceives the idea of monitoring as damaging, to the point they are willing to pay to prevent the manager using the monitoring information. It can be concluded that:

- The *VoI* is never negative when manager and owner are the same rational individual, consistently to the principle that "information can't hurt"; this ideal value is labelled *unconditional VoI* (*uVoI*).
- When manager and owner are not the same individual, the *VoI* is equal or less than the *uVoI*.
- The smaller *VoI* originates from a disagreement between manager and owner on what is the most convenient action to take based on the information from the monitoring system.
- This disagreement produces an overall negative expected utility, labelled *Loss for Disagreement* (*LfD)*.
- When the disagreement is such that the *LfD* exceed the *uVoI*, the value *VoI* become negative.
- The predominant factor that determines a negative *VoI* is the different risk appetite of the two decision makers, owner and manager.
- A necessary, but not sufficient, condition for a negative *VoI* is that the manager is more risk seeking than the owner.

- Other influential factors are the shape of the likelihood distributions and the values of prior probabilities.
- The general mathematical conditions whereby the *VoI* is negative are encoded in Equation (26).

To conclude, one may wonder whether a negative *VoI* is a “good” or a “bad” thing. The answer is that it is not possible to say if it is always a good or a bad thing because, as everything discussed into this paper, it depends on the perspective: if the perspective is the owner’s, clearly the owner optimal situation is to have a manager with the same scale of values and the same risk appetite. As discussed in Section 4.3, this strongly depends not only on the level of education, but also on their role, responsibility and reliability into the decision chain. In principle, by providing strict guidance in form of a standard or procedure for making decision could compel the manager to align to the owner perspective standpoint, resulting in the maximum utility for the owner perspective.

Acknowledgements

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Appendix A: Notation list

Symbol	Definition
VoI	Value of Information
U	Utility
u	Expected utility
u_{pp}	Expected utility of operating the structure with the monitoring system
u_0	Expected utility of operating the structure without the system
S	Structural state
y	Output of the monitoring system
a	Action
a_{opt}	Optimal action
z	Consequences of an action
P(S)	Prior probability
p(y|S)	Likelihood distribution
F(y|S)	Cumulative distribution of the likelihood
p(y)	Evidence
P(S|y)	Posterior probability
E_y	Expected value operator of y
u^*	Expected utility of the manager
a^*	Action of the manager
a^*_{opt}	Optimal action of the manager
$\bar{y}$	Threshold of the owner
$\bar{y}^*$	Threshold of the manager
R	Index that expresses the optimal action a priori from the owner perspective
R^*	Index that expresses the optimal action a priori from the manager perspective
r	Index that expresses the subjective risk appetite of the owner

r^*	Index that expresses the subjective risk appetite of the manager
q	Index that expresses the prior odds
g	Ratio between the likelihoods of two states
G	Ratio between the cumulative distributions of two likelihoods
Δu	Utility gain resulting from changing a decision a posteriori
EUDF	Expected utility density function
uVoI	Unconditional value of information
LfD	Loss for Disagreement
n	Polynomial degree
O	Owner
M	Manager
$^{(O\|M)}VoI$	Conditional value of information
D	State of the case study: damaged
U	State of the case study: undamaged
DN	Action of the case study: do nothing
CB	Action of the case study: close the bridge
z_F	Cost in the case study: failure cost
z_{DT}	Cost in the case study: 1-month downtime cost
ε	Output of the monitoring system in the case study: optical sensor reading
$\bar{\varepsilon}$	Threshold in the case study
μ	Mean value
σ	Standard deviation
θ	Coefficient of Absolut Risk Aversion (ARA)