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PAPR Analysis in OFDM-IQ-IM Systems

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By

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Abstract

One of the key disadvantages of OFDM system, implemented already in 4G and 5G is high PAPR. For this reason, it is very important to evaluate the PAPR performance of any potential multiplexing technique candidate for upcoming generations. Due to the superior performance over OFDM considering BER performance, spectral efficiency, energy efficiency, OFDM-IQ-IM is one of the promising multiplexing techniques for upcoming generations of wireless technology. Therefore, the PAPR performance of OFDM-IQ-IM system has been analysed here.

In deterministic approach, subcarriers are considered to be modulated by symbols with highest power and the upper limit of the PAPR of OFDM-IQ-IM system has been formulated. Using statistical distribution, a probabilistic approach has been taken to determine the PAPR performance of the OFDM-IQ-IM and OFDM-IM systems. The distribution of PAPR of OFDM-IQ-IM and OFDM-IM systems has been evaluated considering the discrete time baseband signals for both in-phase and quadrature components as independent Gaussian random variables.

A comparative analysis of the PAPR of OFDM, OFDM-IM and OFDM-IQ-IM systems has been made in both deterministic and probabilistic approach. Thus improved PAPR performance has been noticed in OFDM-IQ-IM system compared to OFDM-IM and OFDM systems for same spectral efficiency.

Dedication

To my dear parents:

Father: Md. Jahangir Alam

Mother: Farhana Alam

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List of Abbreviations

ACE	Active Constellation Extension
ADC	Analog to Digital Converter
ADSL	Asymmetric Digital Subscriber Line
BER	Bit Error Rate
bps	Bits Per Second
bps/Hz	Bits Per Second per Hertz
CCDF	Complementary Cumulative Distribution Function
CDF	Cumulative Distributive Function
CDMA	Code-Division Multiple Access
CLT	Central Limit Theorem
CP	Cyclic Prefix
D2D	Device-to-Device
DAB	Digital Audio Broadcasting
DAC	Digital to Analog Converter
dB	Decibels
DFT	Discrete Fourier Transform
DVB-T	Digital Video Broadcasting - Terrestrial

ETSI	European Telecommunication Standards Institute
FDMA	Frequency-Division Multiple Access
FFT	Fast Fourier Transform
GI	Guard Interval
HDSL	High-bit-rate Digital Subscribe Line
HPA	High Power Amplifiers
ICI	Inter Carrier Interference
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Discrete Fourier Transform
IID	Independent-Identically Distributed
IM	Index Modulation
IMM	Index Modulation Mapping
ISB	Index Selector Bits
ISI	Inter Symbol Interference
LLR	Log-Likelihood Ratio
LO	Local Oscillators
M2M	Machine-to-Machine
MCM	Multi-Carrier Modulation
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MM-OFDM-IQ-IM	Multiple Mode OFDM-IQ-IM
OFDM	Orthogonal Frequency Division Multiplexing
OFDM-HIQ-IM	OFDM Hybrid In-phase/Quadrature Index Modulation

OFDM-IM	OFDM with Index Modulation
OFDM-IQ-IM	OFDM with In-phase/Quadrature Index Modulation
PAM	Pulse Amplitude Modulation
PAPR	Peak to Average Power Ratio
PDF	Probability Density Function
PMF	Probability Mass Function
PRT	Peak Reduction Tone
PSD	Power Spectral Density
PSK	Phase Shift Keying
PTS	Partial Transmit Sequence
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature PSK
RF	Radio Frequency
SAP	Subcarrier Activation Pattern
SE	Spectral Efficiency
SI	Side Information
SIM	Subcarrier Index Modulation
SIM-OFDM	Subcarrier Index Modulation OFDM
SLM	Selected Mapping
SM	Spatial Modulation
SNR	Signal to Noise Ratio
STC	Space-Time Coding
TDMA	Time-Division Multiple Access

TI	Tone Injection
TR	Tone Reservation
UWA	Underwater Acoustic
WiMAX	Worldwide interprobability Microwave Access
WLAN	Wireless Local Area Networks

Chapter 1

Introduction

During the last few decades, wireless technology has evolved so much and has become an integral part of our day to day life. It is very certain that in coming years, it will rule out wired communications completely. Wireless communications offer more scalability, speed, accuracy and cost effectiveness. 5G is already deployed in various parts of the world. Modern technologies and techniques require higher data rate, thus in ever increasing need of bandwidth. To keep pace with this constantly evolving needs, wireless technology has now become one of the most highlighted area of research.

1.1 Research background

1.1.1 Multiplexing techniques

In telecommunications and computer networks, multiplexing is a method by which multiple analog or digital signals are combined into one signal over a shared medium. So, multiplexing is a technique that is used to divide the communication channel such as a cable or an optical fiber into several logical channels and use each of them to transmit a separate message or data stream. With the evolvement of wireless technology several multiplexing techniques where introduced based on requirements.

Generations	1G	2G	3G	4G	5G
Deployment	1970	1990	2004	2010	Soon
Data Bandwidth	2 Kbps	64 Kbps	2Mbps	1 Gbps	20 Gbps
Multiplexing	FDMA	TDMA	CDMA	OFDM	Modified OFDM
Primary Services	Analog Phone Calls	Digital phone calls and messaging	Phone calls, messaging, data	All- IP Service (including voice messages)	High speed, high capacity and provide large broadcasting of data in Gbps
Key Differentiator	Mobility	Secure, Mass adoption	Better internet experience	54.67 Faster broadband internet, Lower latency	Better coverage and no dropped calls, much lower latency, better performance

Table 1.1: Evolution of multiplexing techniques with generations [1]

Here in Table 1.1 we can see, in case of 1G the data bandwidth we had is 2 Kbps and the multiplexing technique we used is Frequency Division Multiple Access (FDMA). For 2G and 3G the data bandwidth increased to 64 Kbps and 2 Mbps and the multiplexing techniques we used are Time-Division Multiple Access (TDMA) and Code-Division Multiple Access (CDMA) accordingly. But for 4G and 5G we can see the main focus is on High speed internet, better coverage and better performance. So the data bandwidth for 4G and 5G increased to 1 and 20 Gbps accordingly which are much higher than that of 1G, 2G or 3G. And, to have that amount of data rate, Orthogonal Frequency Division Multiplexing (OFDM) and different modified version of OFDM are used or proposed as multiplexing techniques.

1.1.2 FDMA, TDMA and CDMA

If we have multiple users using FDMA we can allocate portion of the frequency spectrum to each user so they can transmit data at the same time (Figure1.1(a)). If we are using TDMA then, multiple users can share a frequency spectrum each having a separate time slot to transmit their data (Figure1.1(b)). In case of CDMA, a special coding scheme was introduced so that multiple signals from different users can be transmitted over the same frequency band at the same time (Figure1.1(c)).

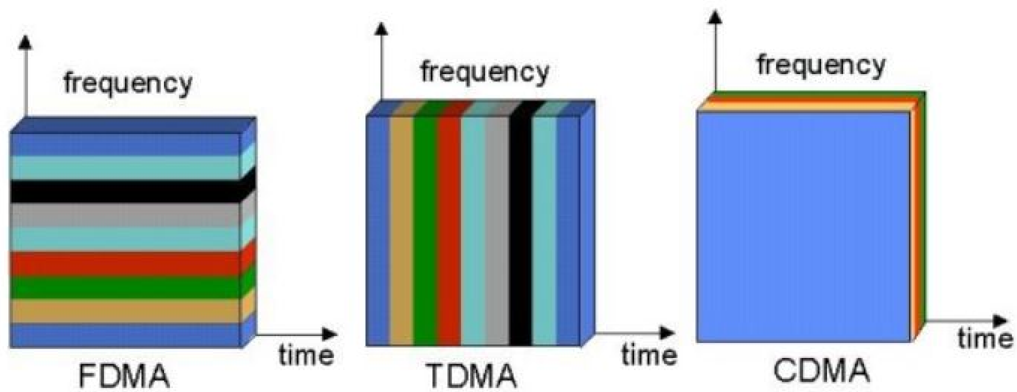


FIGURE 1.1: Different multiplexing techniques (a) FDMA, (b) TDMA, (c) CDMA

1.1.3 Difference between FDMA and OFDM

FDMA allows multiple users to share one link by dividing available bandwidth into different non-overlapping sub-carriers. A guard-band is a narrow frequency range, inserted between adjacent sub-carriers so that there is no overlapping between them. The guard-bands don't pass any information. Although the guard-bands are used to avoid interference, it results in wastage of useful and scarce frequency resources. Therefore, the full capacity of the system is not utilised.

OFDM was used as multiplexing technique in 4G. It is a variation of FDMA. Here, the sub-carriers are closely spaced with no guard-bands between them. For the same available bandwidth OFDM can transmit more data than FDMA. In other words, OFDM system uses less amount of bandwidth compared to FDMA to send equal amount of data which can be seen from Figure 1.2 where N represents the total number of sub-carriers. As

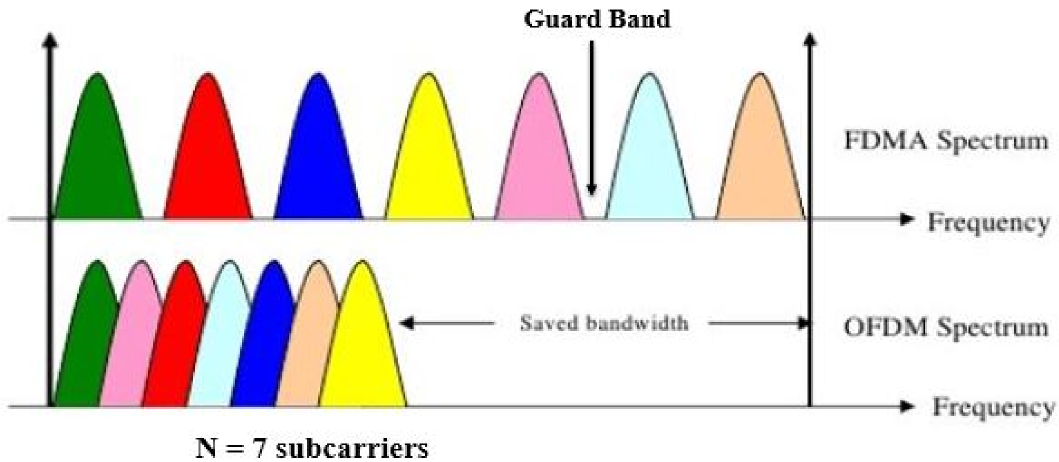


FIGURE 1.2: Subcarrier allocation in FDMA and OFDM system

the sub-carriers are overlapping one another there might be interference. The effect of interference is avoided in OFDM, as the sub-carriers are designed to be orthogonal to each other. Orthogonal depicts that each of the subcarriers in OFDM system works without interference and dependence on on another. Here in Figure 1.3, when the green signal reaches it's peak all the other signals are at their 0 point or null. In OFDM system, subcarriers are multiplexed in a way where one subcarrier has its peak when the rest of the subcarriers are null at that point. This orthogonal feature helps the demultiplexer in the receiver to separate them from one another.

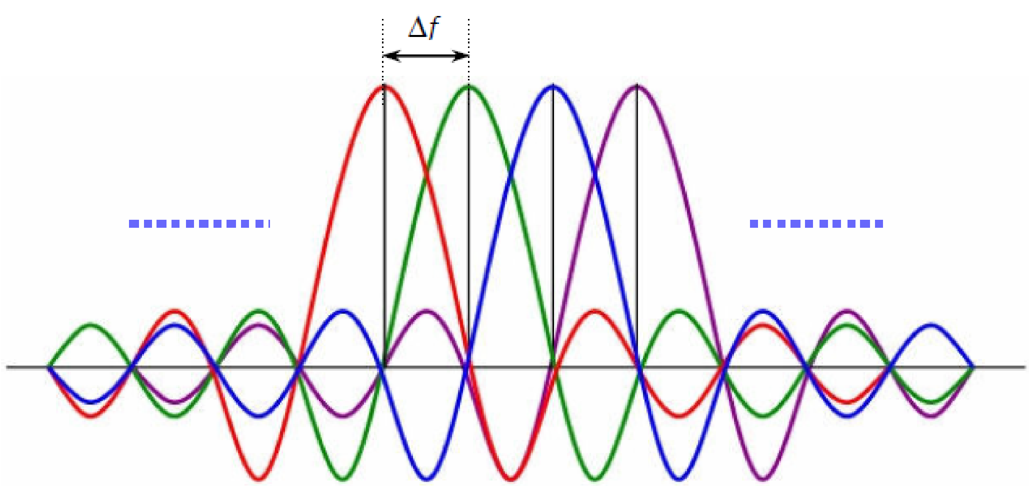


FIGURE 1.3: Orthogonal subcarriers in OFDM system

1.1.4 OFDM development and applications

OFDM basically divides high speed data stream into several low speed data streams. Then those data streams are being modulated on subcarriers which are orthogonal to each other. In OFDM system, small scale fading and Inter Symbol Interference (ISI) is avoided as the symbol period is larger than delay spread [2]. The subcarriers in OFDM system have significant overlap in frequency domain which results in, very high spectral efficiency [3]. In 1965, R.W. Chang proposed the concept of OFDM technique for the first time [4]. Then the performance of OFDM system was analyzed by Saltzberg in 1967 [5]. In 1970, OFDM technology was patented at the USPD. OFDM was used for high-frequency military communications in the 1970s [6]. The Discrete Fourier Transform (DFT) technique was accumulated with Multi-Carrier Modulation (MCM) technique by S.B. Weinstern and P.M. Ebert in 1971 [7]. The complexity of OFDM modulation and demodulation and the overall cost of the system was reduced by Fast Fourier Transform (FFT) implementation of DFT. And to assure orthogonality among subcarriers Cyclic Prefix (CP) was inserted into OFDM signals by Peled and Ruiz in 1980 [8]. Introduction of CP reduced the ISI caused by multi-path channel significantly.

Since the 1990s usage of OFDM has been very popular in wideband communication systems. In 1991, OFDM was used in High-bit-rate Digital Subscribe Line (HDSL) applications [9]. Also in 1991, OFDM was accumulated in Asymmetric Digital Subscriber Line (ADSL), which provided 8 Mbits/second data rate in 1 MHz bandwidth [10]. In 1995, OFDM was deployed for the first time in Digital Audio Broadcasting (DAB) standard which was adopted by European Telecommunication Standards Institute (ETSI) [11]. In 1997, OFDM was adopted in Digital Video Broadcasting - Terrestrial (DVB-T) standard which is used extensively for terrestrial broadcasting of digital television in Europe and many other countries [12]. OFDM was selected as access scheme of physical layer in Wireless Local Area Networks (WLAN) standard IEEE802.11a (Wi-Fi) in 1999 [13]. It could provide a data rate, ranging between 6 Mbps and 54 Mbps. In 2003, OFDM was also used in WLAN IEEE802.11g standard [14] which resulted in increased data rate using the same bandwidth as IEEE802.11a. OFDM was adopted in Worldwide interprobability Microwave Access (WiMAX) standard for broadband for wireless metropolitan area networks in 2004 [15]. In 2009, OFDM was used as the multiplexing technique in IEEE802.11n standard which increased the data rate furthermore using multiple antennas [16]. This is the standard for next generation wireless LAN. OFDM is adopted in LTE and LTE Advanced 4G mobile networks, combined with other techniques such as adaptive resource allocation, adaptive coding, Space-Time Coding (STC), Multiple Input Multiple Output (MIMO).

1.1.5 Advantages and disadvantages of OFDM

According to [17] and [18], some of the advantages that made OFDM to replace CDMA technique in 4G LTE networks are as follows:

- **High spectral efficiency:** OFDM system uses subcarriers which are orthogonal to each other. It allows them to overlap one another and thus a distinguishable amount of bandwidth is saved and the frequency resources can be utilised properly.

- **Simple implementation using FFT/IFFT:** OFDM system uses IFFT and FFT for modulation and demodulation process. So, high frequency oscillators are not used anymore. As the high frequency oscillators needed to be synchronised precisely, the overall complexity and cost of the system is reduced significantly.

- **Frequency diversity:** OFDM can cope with frequency selective fading as it has inherent frequency diversity.

- **Robustness again ISI and multipath propagation:** In OFDM, high speed data is stream is split into several low speed parallel data streams which results in longer symbol period. The longer symbol period results in proportional decrease of effects caused by time-varying fading. Therefore, OFDM system is less sensitive to time synchronization errors. Also, Guard Interval (GI) and CP is included in OFDM system, which eliminated ISI and Inter Carrier Interference (ICI) completely.

- **Robustness against narrowband cochannel interference:** In single carrier modulation, a highly complex equalizer is needed in the receiver to compensate the corresponding effect caused by frequency selective fading. On the other-hand in OFDM each of the subacrrier is modulated by low speed parallel data streams and each of them undergoes flat fading. For this reason, OFDM can be deployed using only a single-tap equalizer for each subcarrier. So, OFDM can be used un severe channel conditions without using complex time-domain equalization. That may result into fading of some sabcarriers, but that can be solved using several coding and interleaving techniques.

But there are some disadvantages of OFDM, which limits the system performance significantly. Those are:

- **Sensitive to carrier offset and drift:** In OFDM system, it is a must for the subcarriers to be orthogonal, thus require strict frequency synchronization. The Bit Error Rate (BER) performance can be easily degraded by introduction of a little phase noise and frequency offset, as it will effect the orthogonality of the subcarriers.
- **High Peak to Average Power Ratio (PAPR):** The PAPR of MCM systems such as OFDM is much higher than that of single carrier systems. In ideal OFDM system, all the subcarriers are active with the same phase. When the signals overlap at any instant, the instantaneous amplitude of the OFDM signal will be much larger than the average power of the subcarriers thus causes high PAPR. To compensate this high PAPR, Radio Frequency (RF) amplifiers will need to have very large dynamic range. And to operate the these RF amplifiers, high power is needed and for that the battery life will reduce faster and also will increase the cost of the overall system. Moreover, BER performance degradation and out-of-band radiation is caused by the distorted signals due to instantaneous power exceeding linear region.

These limitations should be addressed before using OFDM in practice for upcoming generations of wireless technology.

1.2 Research scope

1.2.1 Different PAPR reduction schemes in OFDM system

To resolve the high PAPR problem in OFDM system different methods have been applied over years. They can be sectioned in three parts: limiting class, signal scrambling and encoding approaches. The method that can be easily implemented is limiting class. In this method a threshold is specified and those signals whose peak are larger than that threshold are clipped. After the clipping they are sent to Digital to Analog Converter (DAC) or RF power amplifier. The scrambling technique reduces the probability PAPR signals to occur without clipping them. It basically represents a data sequence in many different combinations and among them the one with lowest PAPR is being chosen. In encoding approach, signals are encoded using code sets and the ones that are less than a threshold are chosen to transmit through the channel. Though the PAPR reduction schemes reduces the PAPR of the system but has some trade-offs. So, before adopting any reduction scheme, it is mandatory to analyze all possible trade-offs and select the most optimum one.

1.2.2 Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM)

For 5G and and upcoming generations of wireless networks, OFDM and its modified versions are potential candidates. One of them is Orthogonal Frequency Division Multiplexing with Index Modulation (OFDM-IM), which is also a multi-career transmission scheme. It is basically based on the concept of Spatial Modulation (SM) [19], [20]. Here, the incoming bit stream is divided into two parts. One is index selection bits which are used to determine which subcarriers will be active and passing signals. And then the other part modulation bits which will be used to modulate the active subcarriers, similar to OFDM system. OFDM-IM system gives significantly better BER performance than classical OFDM system in a specific configuration [19]. In OFDM-IM, bits are transmitted using IM, which results in better error performance and is more effective at high SNR region. Using the index bits, OFDM-IM can be adjusted to get desired spectral efficiency. The system design is flexible. Considering ICI, OFDM-IM is more robust in nature than classical OFDM. OFDM-IM shows consider-able advantages over conventional OFDM system in high mobility environment and also in case of Device-to-Device (D2D), Machine-to-Machine (M2M), Underwater Acoustic (UWA) communication systems [21]. A deterministic approach was taken to formulate the PAPR of OFDM and OFDM-IM system [22]. It shows that for same configuration, PAPR will be reduced for OFDM-IM than conventional OFDM. However, the PAPR of OFDM-IM is still high. Various reduction schemes are proposed recently to reduce the PAPR of OFDM-IM system.

1.3 Objective and organization of the thesis

The deterministic approach gives us the upper bound of PAPR. When no. of subcarriers N is very high, there is very less probability for the system to attain theoretically highest PAPR [23]. In that scenario, the upper bound may not characterise the PAPR of OFDM signal properly [24]. For this reason, based on conventional analysis, several researchers derived statistical distribution of the PAPR of OFDM [25],[26],[27]. Here in the thesis, statistical distribution of the PAPR of OFDM-IM has been derived. Another modified OFDM multiplexing scheme was proposed in [28],[29], which applied IM into both IQ components independently and jointly. The system was named Orthogonal Frequency Division Multiplexing with In-phase/Quadrature Index Modulation (OFDM-IQ-IM) in [30]. OFDM-IQ-IM system works with smaller subcarrier group and constellation size thus allowing more flexibility while designing the system. It provides better error performance compared to OFDM-IM system at the same spectral efficiency [30]. Utilizing the orthogonality of the in-phase and quadrature components, OFDM-IQ-IM system offers better spectral efficiency than OFDM-IM system [31]. Also, OFDM-IQ-IM system provides improved robustness against doppler effects and channel correlation [32]. Here in this thesis, the PAPR of the OFDM-IQ-IM system is formulated utilising the deterministic approach. Also, statistical derivation of the PAPR of OFDM-IQ-IM system has been done and a fair comparison of PAPR performance among OFDM, OFDM-IM and OFDM-IQ-IM systems has been analysed.

In chapter 2, the configuration of OFDM, OFDM-IM, OFDM-IQ-IM systems and various PAPR reduction schemes of OFDM and OFDM-IM systems are discussed.

In chapter 3, the formulations of PAPR of OFDM and OFDM-IM systems in deterministic approach are discussed and a deterministic formula of the PAPR of OFDM-IQ-IM system is derived.

In chapter 4, the probabilistic approach or statistical derivation considering conventional analysis of the PAPR of OFDM system is discussed and the statistical derivations of the PAPR of OFDM-IM and OFDM-IQ-IM system is presented.

In chapter 5, general comparison between the three systems considering deterministic approach of PAPR and probabilistic approach of PAPR have been studied.

Lastly, in chapter 6, we conclude by summarizing the thesis and providing suggestions for future work.

Chapter 2

System Overview

In this chapter, system overview of OFDM, OFDM-IM and OFDM-IQ-IM is given. Also, how high PAPR effects the system is being highlighted and several PAPR reduction techniques of the PAPR of OFDM and OFDM-IM system have been studied.

2.1 OFDM system

OFDM is the most bandwidth efficient multiplexing scheme that is used in cellular technology. It allows overlapping of the subcarriers without any ICI which results in better spectral efficiency. OFDM system has an added advantage of being less susceptible to impulsive noise than other MCM variants, because of each subcarrier having a lower information rate and data symbol intervals for being longer. OFDM signals are easy to demodulate as the subcarriers are orthogonal to each other over the symbol interval.

2.1.1 OFDM system structure

The basic transceiver of OFDM system has been shown in Figure 2.1. It has two sides: transmitter and receiver. Incoming bits or input data is processed in every stage to reach from the sender to the receiver. Each of the building blocks of the diagram has their own functionality.

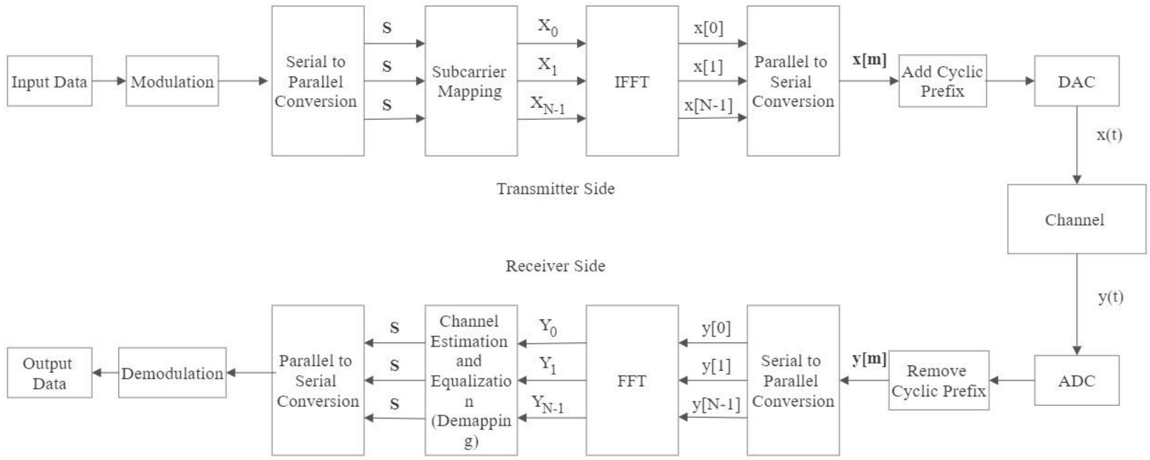


FIGURE 2.1: Block diagram of an OFDM transceiver

- **Modulation and parallel to serial conversion:** The input data is basically binary bit streams containing 1s and 0s. The bits are then digitally modulated to convert them into symbols representing constellation points. It can be done before the serial to parallel conversion or after it. Each constellation symbol will consist of $b = log_2(M)$ bits. There will be total 2^b possible combinations of constellation symbols. Here, M is the modulation index and the value of M depends on the digital modulation scheme that is used as shown in table : The serial to parallel converter is used to convert

Modulation Scheme	M	Modulation Scheme	M
BPSK	2	32 QAM	32
BASK	2	64 QAM	64
QPSK	4	128 QAM	128
4 QAM	4	256 QAM	256
16 QAM	16	1024 QAM	1024

TABLE 2.1: Modulation index M for different digital modulation schemes

serial high speed data stream to N parallel low speed data streams. Then the symbols S are sent for subcarrier mapping.

- **Subcarrier mapping:** Now the symbols coming out after subcarrier mapping are represented as X_l. Here, X_l is a complex symbol which is basically a constellation symbol from M-ary Phase Shift Keying (PSK) or M-ary Quadrature Amplitude Modulation (QAM) constellation diagram being modulated on l-th subcarrier. Based on the very basic model of OFDM system presented in [18], the transmitted OFDM signal was represented as:

$$x(t) = \sum_{l=0}^{N-1} X_l \; exp(j2\pi f_l t), \qquad 0 \leq t \leq T, \tag{2.1}$$

where $f_l = f_0 + \frac{l}{T}$ and $T = NT_S$. Here, T is OFDM symbol interval whereas, T_S is the data symbol period. The amplitude and the phase of the subcarriers may vary due to different complex data symbols in practice. Now, if we consider the subcarrier spacing as a multiple of $1/T$, the subcarriers are mutually orthogonal, as

$$\frac{1}{T}\int_0^T exp(j2\pi f_l t) exp(-j2\pi f_m t) dt = \begin{cases} 1, & m = n, \\ 0, & m \neq n, \end{cases} \tag{2.2}$$

where $|f_l - f_m| = \frac{k}{T}, \quad k = 0, ..., N-1$.

- **IFFT/FFT in OFDM system:** In basic OFDM configurations, Local Oscillators (LO) were used to achieve orthogonality among the subcarriers. However, for large number of subcarriers this approach is very impractical as having so many oscillators will increase the complexity and cost of the system. To resolve this issue, FFT was introduced, which is an efficient computational tool to perform the DFT. We can synthesize the OFDM signal in 2.1 by using Inverse Discrete Fourier Transform (IDFT). OFDM signal $x(t)$ will be sampled at a rate T/N, will result into following samples

$$x[m] = x(mT/N) = \sum_{l=0}^{N-1} X_l \;\; exp(j\frac{2\pi}{N} lm), \qquad m = 0, ..., N-1, \tag{2.3}$$

Equation 2.3 represents the IDFT of the original data symbols X_L. So, at the receiver side, by performing the DFT of the received samples, we can recover original data symbols X_l, as

$$X_l = \sum_{m=0}^{N-1} x[m] \;\; exp(-j\frac{2\pi}{N} lm), \qquad l = 0, ..., N-1, \tag{2.4}$$

Using these two operation IDFT and DFT, the implementation of the modulation and demodulation processes in OFDM are simplified significantly. In practice, IFFT/FFT is used instead of IDFT/DFT, because of reduced computational complexity. A N-

point IDFT needs N_2 complex multiplications whereas the Radix-2 Inverse Discrete Fourier Transform (IFFT) needs only $(N/2)log_2N$ complex multiplications [33]. So, IFFT is more efficient approach due to less computational complexity.

- **Guard interval and cyclic prefix:** One of the major advantages offered by OFDM is it is less effected by the spread delay caused by multipath propagation. The period of an OFDM symbol is N times of the input data symbol period. And due to that the corresponding ratio of the spread delay to the symbol period reduces N times. A GI with duration of T_g is inserted into the OFDM symbol to eliminate ISI. T_g has to be longer than the maximum time delay spread. But if there is no sample during the GI, the sub-channels may have interference among themselves as an effect of multipath propagation. This hampers the orthogonality among the subcarriers. For this reason, CP is added into duration of GI. CP is basically a copy of the data within the last T_g duration of each OFDM symbol. As a result of adding CP, signals will not suffer ICI during the demodulation process because of time spread delay being less than T_g [34].

- **Digital to analog converter:** After the CP insertion, the OFDM signal is upconverted using a high frequency component to pass it through the wireless channel. This is called the bandpass signal $x(t)$.

- **Receiver side:** When the RF signal $y(t)$ is received at the receiver side, it is passed through the Analog to Digital Converter (ADC). Then, the timing and synchronization are performed and CPs are removed. After that the parallel samples $y[m]$ are passed through the FFT block and transforms from time domain to frequency domain Y_l. And then the demapping is performed using the constellation diagram to obtain the parallel data symbols. Lastly, the decoding is performed and the original serial data stream is recovered.

2.1.2 OFDM PAPR reduction schemes

It requires very High Power Amplifiers (HPA) with large linear region to minimize the effect of very high PAPR. As a result, the power efficiency decreases. Also, amplifiers and ADCs with large linear region is needed on the receiver side. So, high PAPR significantly hampers the overall system performance. To resolve this several researchers proposed various PAPR reduction schemes for OFDM system over the years.

- **Clipping and filtering:** It is the most easy and straightforward way of reducing PAPR. This technique is implied in the time domain. The amplitude of the signals are kept under a desirable level, using a soft limiter. Though clipping method keeps the PAPR under a certain threshold but it may cause in-band and out-of-band distortion. Also, the noise created by clipping which degrades the BER performance. To solve this problem, the clipped time-domain signals are first transformed into frequency-domain signals using FFT and out-of-band signals are set to 0 to perform the filtering. And then the filtered signals are again transformed into time-domain by IFFT [34]. Some peak regrowth may be caused by filtering and as a result of that, signals after clipping and filtering might exceed the clipping threshold. To resolve this issue, an iterative method was proposed in [35], [36] which will take number of iterations to reach a desirable amplitude level. But this increases the system complexity. To filter out the out-of-band noise optimized filter H has been proposed in [37]. But due to the convex optimization problem the overall computational complexity of the system increases. Several windowing techniques such as Gaussian, Kaiser and cosine filters are used to reduce the peak value by adding a window function. But this eventually leads to the increase of the bandwidth of the signals.

- **Companding:** Nonlinear strategies to reduce high PAPR have also been proposed by researchers over the years. Using these, the unwanted noise elements created while passing through DAC and HPA could be avoided. Companding is another scheme to reduce PAPR, which will compress signals with high peak and expands the signals with small amplitude. Decompanding is performed in the receiver side reverse the action. But this way the signals get distorted [38]. A non-linear companding with variable companding parameters to achieve a good PAPR reduction with less distortion was proposed in [39]. The performance of band-limited OFDM system is improved by using non-symmetric decompanding [40].

- **Coding:** Coding technique to reduce PAPR involves encoding input data into a codeword with low PAPR. If an OFDM system has four sub-carriers, the PAPR of the system can be reduced by mapping three bits input data into four bits codeword. In frequency domain, the last bit is the parity bit. The code with minimum PAPR is chosen [41]. Golay complementary sequences have zero autocorrelation and non-zero delay shift and thus makes them a good selection for coding. OFDM signals will have PAPR of atmost 3dB using golay complementary codewords. Specific subset of golay codes along with decoding techniques that ensures PAPR reduction and forward error correction is presented in [42]. But there are no suitable codes presented yet, if the OFDM system has subcarriers more than 64 [43]. Also, the code length is high and it will result in drastic rate loss.

- **Partial transmit sequence:** Partial Transmit Sequence (PTS) is another method to reduce PAPR. In this technique [26], after the IFFT operation time domain vectors are multiplied by phase rotation factors. This will result into multiple different candidate of PTS OFDM signals. All the candidates are compared and the one with the minimum PAPR is chosen to transmit through the wireless channel. But the computational complexity is very high for this scheme as the system has to perform exhaustive search to find the best phase rotation factors combination. A low complex PTS scheme was suggested in [44] by exploiting the mutual relationship between the phase rotation factors.

- **Selected mapping:** Selected Mapping (SLM) technique is based on probability. The phase rotation factors are used here too to change the distribution of the original signal. In case of PTS the original data are multiplied and rotated by phase rotation factors after the IFFT operation, whereas in SLM scheme, the original data are rotated before IFFT operation. So, there will be many copies of the original data symbol and then they will be transformed from frequency domain to time domain using IFFT. Lastly, the one with minimum PAPR will be chosen to transmit through the channel. SLM was first introduced in [45]. It is also a distortion-less scheme for PAPR reduction. But in case of SLM, Side Information (SI) needs to be transmitted, which reduces the data rate and efficiency. If the computational complexity is fixed for both PTS and SLM scheme SLM will outperform PTS regarding PAPR [46].

- **Tone reservation and tone injection:** Tone Reservation (TR) is also a distortion-less PAPR reduction scheme introduced in [47]. Here, the term "Tone" denotes sub-carriers. A part of the subcarriers termed as Peak Reduction Tone (PRT) are selected. PRT do not contain any data. New signals with lower PAPR are generated by adding the PRTs with the original OFDM signal. The TR scheme for PAPR reduction is incorporated with PRT set and clipping threshold. It creates problem that cannot be solved for practical subcarrier numbers [48]. These problems are elaborately discussed and near optimal PRT algorithms are suggested in [48] and [49]. Though TR scheme reduces the PAPR but it wastes subcarriers which significantly decreases the data rate. To resolve that Tone Injection (TI) is introduced. In TI scheme, the constellation size is increased so that the original constellation points can be mapped into several equivalent points in the extended constellations [50]. Thus reduced PAPR can be achieved, combining the overlapped data signals and peak reduction signals.

There are some other mention able PAPR reduction schemes besides the above mentioned ones. Some of them are interleaving , pilot sequences , m-sequences. These schemes are not so popular for not having efficient PAPR reduction performance. These PAPR reductions are summarised in very precisely and a comparison table is shown. Here, Table

PAPR Reduction Schemes	PRC	API	BER	DRL	CC	OBR
Clipping	Good	No	Yes	No	Low	Yes
Companding	Good	No	Yes	No	Mid	Yes
Coding	Good	No	No	Yes	High	No
PTS	Good	No	No	Yes	High	No
SLM	Good	No	No	Yes	High	No
TR	Good	No	No	Yes	High	No
TI	Good	No	No	Yes	High	No

Table 2.2: Comparison among different PAPR reduction schemes.

2.2 gives a summary of comparison of different PAPR reduction schemes considering PAPR reduction capability (PRC), average power increase (API), BER degradation, data rate loss (DRL), computational complexity (CC) and out-of-bound radiation (OBR). In Figure 2.2, PAPR of different PAPR reduction schemes has been shown with respect to Complementary Cumulative Distribution Function (CCDF) [48].

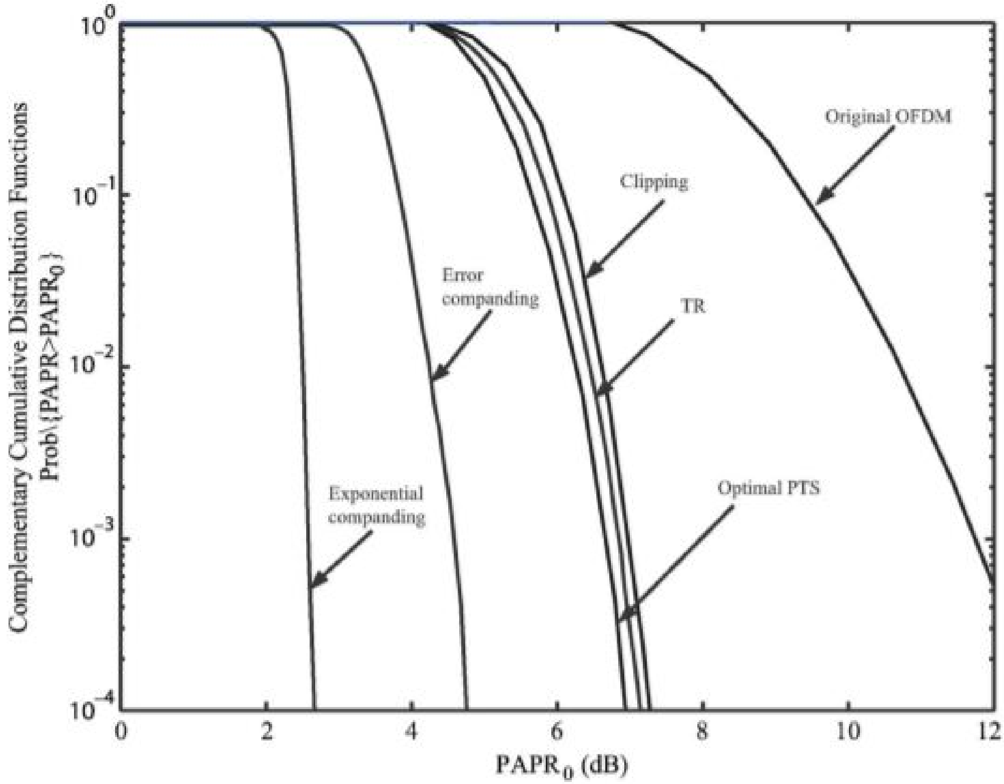


FIGURE 2.2: CCDF vs PAPR graph for several OFDM PAPR reduction schemes [48]

2.2 OFDM-IM system

Index Modulation (IM) technique is a promising techniques for 5G and upcoming generations of wireless networks can utilize the indices of active resources to increase the data rate. A brief study has been done on existing IM techniques in [51]. In [52], IM technique was first incorporated with OFDM system and was termed as Subcarrier Index Modulation (SIM) in that study and OFDM-IM in further studies. OFDM-IM transmits extra information bits along with the data bits that increases the Spectral Efficiency (SE) and improves the BER peroformance than traditional OFDM system [21].

2.2.1 OFDM-IM system structure

The transceiver block diagaram of OFDM-IM system is shown in Figure 2.3. Both the transmitter side and receiver side have some building blocks which are similar to OFDM system block diagram. The functionalities of the common building blocks are similar.

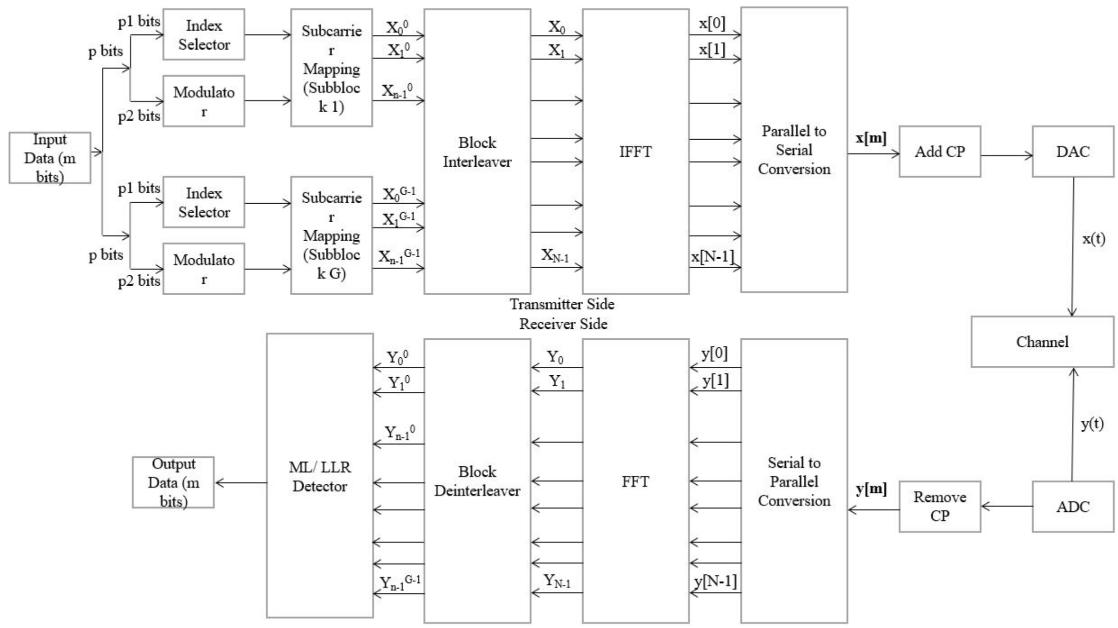


FIGURE 2.3: Block diagram of OFDM-IM transceiver

- **Bit splitting:** In OFDM-IM system all the sub-carriers don't transmit signals. Using the Index Selector Bits (ISB), the number of active subcarrier is chosen. Firstly, the total number of subcarriers N is divided into G subblocks with each subblock consisting of n subcarriers. Using the index bits, k subcarriers among n subcarriers are chosen which will be active. Then, using the modulators, those active subcarriers will be modulated with constellation symbols based on the digital modulation scheme used. The total no. of input bit is m. The total no. of bits that enters into a subblock is denoted by p so that $m = pG$. Now, for each sub-block p bits are divided into two parts: $p1$ bits and $p2$ bits. $p1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits are used to denote which subcarriers are active and are called ISB. And after selection of active subcarriers in the subblock they are modulated with constellation symbols using $p2 = k \log_2 M$ bits.

- **Block interleaver:** After the subcarrier mapping in each subblock the data symbols are denoted by X_d^g where, g denotes the subblock number and n denotes subcarrier number in that sublock. If a subcarrier in a subblock is inactive then corresponding X_d^g for that subcarrier will be incorporated with zero value. Then those symbols go through block interleaver which ensures that subcarriers of a subblock went through uncorrelated wireless fading channels for improving error performance of the detector [53]. Basic OFDM-IM structure didn't have this block [19]. It was added in [53] and [54] where the output symbols of the interleaver were represented by X_l, where $X_{d.G+g} = X_d^g$ and G denotes the total number of subblocks. Then they are converted from frequency domain to time domain using FFT and CPs were added similar to that of conventional OFDM. And finally after using the DAC, OFDM signal $x(t)$ is transmitted through the wireless channel.

- **ML and LLR detector:** In case of OFDM system, the receiver had to determine the M-ary data symbols. But for OFDM-IM system, the receiver needs to determine the index of the active subcarriers and the M-ary data symbols that are modulated into the active subcarriers. At the receiver side, the received OFDM signal goes through ADC, CP removal, FFT operation and block deinterleaving. Subcarrier by subcarrier detection technique cannot be applied in OFDM-IM system as it can be done in OFDM system. As for OFDM-IM system information bits to be detected accurately the detector has to process the indexing informations too. The first detection method of OFDM-IM system is done by using a Maximum Likelihood (ML) detector. ML detector considers both the Subcarrier Activation Patterns (SAPs) and the complex data symbols. The system provides optimum performance but is complex. So, another detection method with lower complexity is by using Log-Likelihood Ratio (LLR) detector which considers each subcarriers independently. In this method, first it is determined if the subcarrier is active and then the symbol that is modulated into that subcarrier. It uses a probabilistic approach to determine if the subcarrier is active or inactive. LLR detection method does not consider the SAP pattern, that is why this detection method is considered near optimal. Also, some low cost, low complexity, sub-optimal detection methods were proposed in [55], [56], [57] with BER performance almost similar to that of the ML method.

2.2.2 OFDM-IM system characteristics

OFDM-IM is a promising candidate for upcoming generation of wireless network. The system characteristics of OFDM-IM has been studied in several literatures and in few cases better performance than conventional OFDM system was found.

- **Spectral efficiency:** In case of OFDM, as all the subcarriers are active so the SE is fixed. For OFDM-IM system, the spectral efficiency is lower than that of the limit for OFDM system $log_2 M$ bits per second per subcarrier (bps/subcarrier) [21]. A different approach was taken in [28], in each sub-carrier the number of active subcarrier was different. This approach resulted in slightly better SE but not significant. In [58], it was shown that if the number of active subcarriers is set one less than the total number of subcarriers in a group, it will result in better spectral efficiency but will lead to poor BER performance. In [59], the effect of modulation index M, the number of subcarriers in a subblock Q, and the number of active subcarriers in a subblock L over the SE of OFDM-IM system has been studied and some limits are provided for better SE. According to this study, the spectral efficiency of OFDM-IM system will be better than that of OFDM system if Q is set greater than or equal to twice of M.

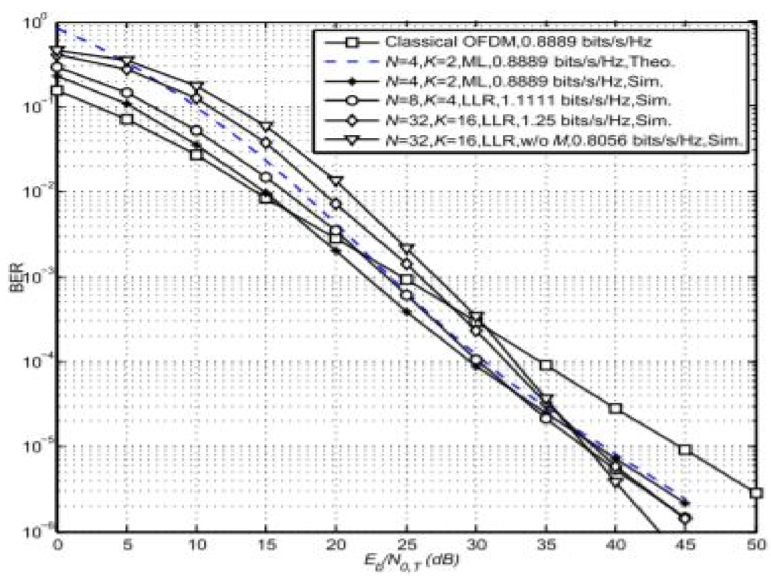


Figure 2.4: BER performance of OFDM-IM System at different configurations considering M=2 and various values of N and K [21]

- **BER performance:** BER performance is one of the most important characteristic of a system. Several studies have been done to evaluate the BER performance of OFDM-IM system. According to [60], [61], [62] OFDM-IM provides significantly better BER performance than OFDM system specially at high Signal to Noise Ratio (SNR). In Figure 2.4, the BER performance of OFDM and OFDM-IM system has been shown for SNR with a range from 0 to 50 decibels(dB) [21]. Here, in the figure, M, N and K represents modulation index, total number of subcarriers in a subblock and total number of active subcarriers in a subblock correspondingly. In [63], BER performance of OFDM-IM system has been evaluated in terms of subcarrier activation ratio. Subcarrier activation ratio is defined as the ratio of number of active subcarriers in a subblock to total number of active subcarriers in a subblock. In that study it has been shown that if the subcarrier activation ratio is smaller, the better BER performance can be achieved in case of OFDM-IM system.

- **Energy/Power saving and ICI:** In case of OFDM system all the subcarriers are active which results into zero energy/power saving. According to [59], more energy/power saving per subcarrier can be achieved by increasing the difference between the total number of subcarriers per subblock and the total number of active subcarriers per subblock in an OFDM-IM system. In [64], it has been reported that OFDM-IM is more robust to ICI than OFDM system. The significantly better performance of OFDM-IM system compared to conventional OFDM system in the presence of ICI has been proved utilising the several ICI cancellation methods in [65].

2.2.3 OFDM-IM PAPR reduction schemes

At first, the PAPR of OFDM-IM scheme was studied in [64] and mention able reduction of PAPR was noticed in OFDM-IM system compared to conventional OFDM system. In [66], the level-crossing rate analysis was done on the PAPR of OFDM-IM system and was concluded that OFDM-IM system will have less PAPR than OFDM system but that will still be very high. And we already know, due to high PAPR while passing through HPA, in-band distortion and out-of-band radiation will be created. Various PAPR reduction methods introduced for OFDM system can't be applied in OFDM-IM system because they did not consider the presence of active and inactive subcarriers based on index modulation [67]. So, not many studies have been done on the reduction of PAPR of OFDM-IM system.

The first time a PAPR reduction scheme considering unique characteristics of OFDM-IM using convex programming, was studied in [67]. In this approach, the inactive subcarriers were incorporated with dither signals while the signals in the active subcarriers weren't changed and to optimize the design of the dither signals convex programming was used. In [68], a low complex PAPR reduction method was suggested utilizing the Active Constellation Extension (ACE) method used for OFDM in [69]. In Figure 2.5, three types of constellation diagram is shown [69]. The ACE constellation diagram is shown where the constellation points were extended without effecting the minimum euclidean distance. In first method, they extended the inactive subcarrier constellation point around zero point with the extension region limited by radius R and the active subcarrier constellation points weren't extended. And in second method, they combined ACE method and first method.

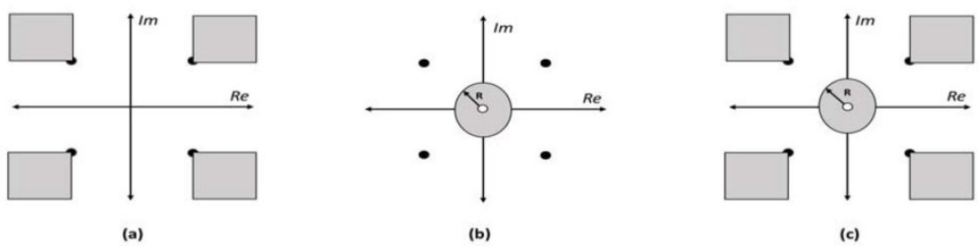


FIGURE 2.5: Signal Constellation Diagram for (a) ACE method, (b) Method I of [69], (c) Method II of [69]

Another PAPR reduction technique for OFDM-IM system was introduced in [70] using multiple mapping rules for index modulation. Instead of using different phase sequences different Index Modulation Mapping (IMM) rules have been used to produce multiple OFDM-IM signals. And then the OFDM-IM signal with the lowest PAPR will be chosen similar to conventional SLM scheme used in OFDM system [71]. In [72], mutilevel dither signals were used to reduce the PAPR of OFDM-IM system. Here, maximum freedom of dither signals in inactive subcarriers is utilized by providing variable amplitude constraints to inactive subcarriers while keeping the index demodulation error in limit. Here in Table all PAPR reduction techniques of OFDM-IM system has been listed with their approximate results and trade offs.

Reference	OFDM-IM PAPR reduction technique and result	Trade offs
2017 [67]	Convex Programming. Reduces PAPR by more than 3 dB from conventional OFDM-IM for same configuration.	The various characteristics of the subblocks if QAM is employed in the active subcarriers is not considered. Even though the index modulation and demodulation in OFDM-IM are performed subblock by subblock.
2018 [68]	Modified Active constellation extension method. Reduces PAPR by almost 3 dB compared to ACE reduction scheme for OFDM.	Slight degradation of BER performance. In case of higher order modulation BER performance is a major concern.
2018 [70]	Multiple Mapping Rules. Reduces PAPR by 3-4 dB for the same configuration than conventional OFDM-IM.	The BER performance degrades compared to conventional OFDM-IM and almost similar to that of conventional OFDM.
2019 [72]	Multilevel Dither Signals. Reduces PAPR by more than 5 dB from conventional OFDM-IM and more than 2 dB from [70].	Slight degradation of BER performance compared to original OFDM-IM.

Table 2.3: Result and tradeoffs of different PAPR reduction schemes for OFDM-IM system.

2.3 OFDM-IQ-IM system

Due to many advantages, OFDM-IM system is one of the most potential candidates to be implemented as the multiplexing scheme in next generations of wireless network. To ensure better performance and to compensate the limitations of OFDM-IM system, several modified OFDM-IM system have been proposed by researchers in recent years. One of the methods where the IM bits were applied on both in-phase channel and quadrature channel jointly and independently, was proposed in [28] and [29]. The system was named as OFDM-IQ-IM in [30].

2.3.1 OFDM-IQ-IM system structure

The transceiver diagram of an OFDM-IQ-IM system has been shown in Figure 2.6. Like the former ones, this system also has two sides: transmiiter and receiver. The OFDM-

IQ-IM transceiver diagram is almost similar to that of the OFDM-IM system with slight improvisation in the transmitter side.

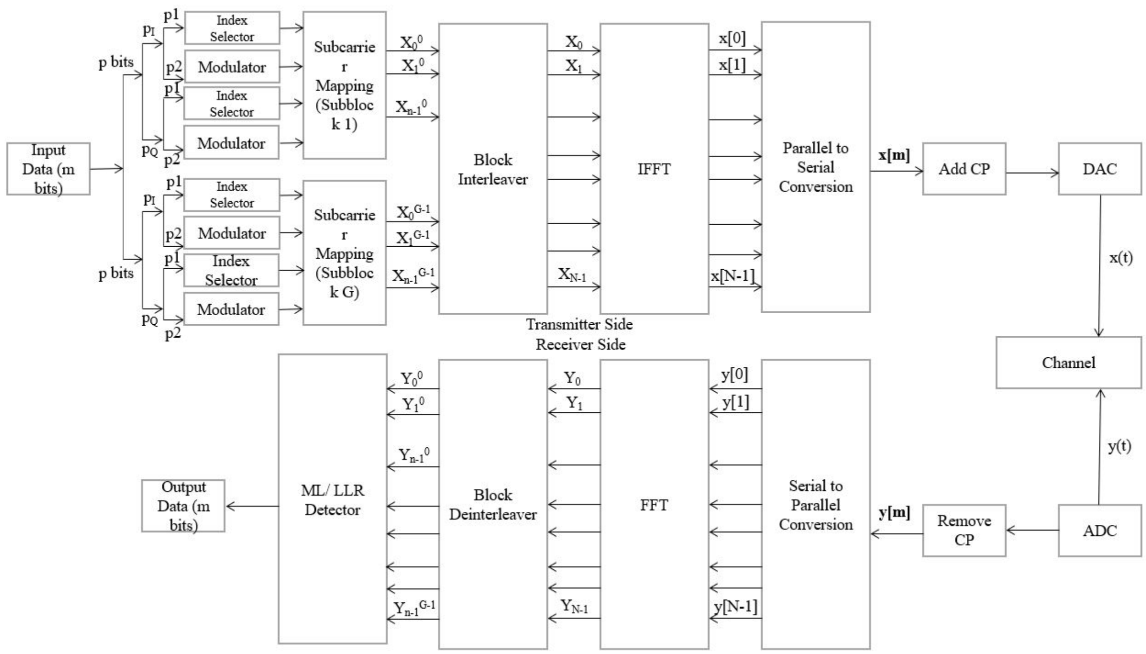


FIGURE 2.6: Block diagram of OFDM-IQ-IM transceiver

- **Transmitter side:** OFDM-IQ-IM system chooses the subcarriers that will be active using the ISBs similar to OFDM-IM system. The total number of subcarriers N is divided into G subblocks. Each subblock will have n subcarriers. The total serial input m bits is divided into parallel bit streams, each having p bits. So, it can be written $m = pG$. In OFDM-IQ-IM system, bits entering into each subblock p is divided into two equal parts p_I and p_Q bits which are transmitted into in-phase and quadrature channel correspondingly. Then p_I and p_Q bits both are divided into $p1$ bits and $p2$ bits. For in-phase channel, $p1 = \lfloor \log_2 \binom{n}{k} \rfloor$ index bits will denote k subcarriers in the subblock which will be active and then using the $p2 = k \log_2 M$ modulation bits those active subcarriers will be modulated with constellation points from M-ary Pulse Amplitude Modulation (PAM) diagram. The subcarriers $n - k$ subcarriers will be inactive for in-phase channel. Similarly for quadrature channel, $p1 = \lfloor \log_2 \binom{n}{k} \rfloor$ bits are used to denote the k subcarriers that will be active and be modulated with constellation points from M-ary PAM diagram. For OFDM-IQ-IM system index modulation of in-phase channel and quadratrue channel is done independently. A subacrrier for which both in-phase and quadrature channels are active they will transmit M^2-ary QAM symbol. So, the complex data symbol of l-th subcarrier of g-th subblock is written as $X_l^g = I_l^g + jQ_l^g$, where I_l^g and Q_l^g denotes the real data symbols of the in-phase channel and quadrature channel of the corresponding subcarrier. After that the data symbols go through interleaving, IFFT operation, CP insertion, DAC and finally upconverted to $x(t)$ to transmit through the channel.

- **Receiver side:** On the receiver side, the received signal passes through the ADC, CP removal, FFT operation and deinterleaving. After that the received data symbols can be represented by Y_l^g where, l denotes the subcarrier index and g denotes the subblock index. After that, those data symbols are transmitted through ML detector. According to [73], for in-phase channel the SAP for the subblock which denotes the index of the active subcarriers in a subblock is determined using ML method. The LLR method [28] can be used too which will reduce the complexity and will produce near optimal output. Once the SAP is determined then the data symbols carried by the in-phase channel of a subcarrier in a subblock is determined using ML method. The similar approach is taken for the quadrature channel. And lastly the ISBs found using the SAP sequence and demodulation of the data symbols of in-phase and quadrature channel will give us the bits that were transmitted.

2.3.2 OFDM-IQ-IM system characteristics

In OFDM-IQ-IM system, IM can be implemented on both the in-phase channel and quadrature channel independently. So, the system characteristics of the OFDM-IQ-IM system compared to that of the OFDM-IM system were studied in few literatures in recent years.

- **Spectral efficiency:** In OFDM-IQ-IM the index bits are used separately for in-phase and quadrature channel. Same data rate can be provided by OFDM-IQ-IM system as OFDM-IM system using smaller subblocks and lower modulation index [32]. This assures more flexibility while designing the system than OFDM-IM. The spectral efficiency of the OFDM-IQ-IM system has been studied in [74]. In this study, the spectral efficiency of OFDM-IQ-IM compared to OFDM-IM system was simulated for

one subblock and and for the equal active subcarrier number OFDM-IQ-IM will give better spectral efficiency than OFDM. From [74], we can get the formula for spectral efficiency of OFDM, OFDM-IM and OFDM-IQ-IM.
Spectral efficiency of OFDM system,

$$\zeta_{OFDM} = \log_2 M, \tag{2.5}$$

where M is the modulation index of the digital modulation scheme used (QAM,PSK).
Spectral efficiency of OFDM-IM system,

$$\zeta_{OFDM-IM} = \frac{G}{N}\left(\left\lfloor \log_2 \binom{n}{k} \right\rfloor + k \log_2 M\right), \tag{2.6}$$

where N is the total number of subcarriers. G represents the total number of subblocks. n and k represents total number of subcarriers in a subblock and total number of active subcarriers in a subblock accordingly. Lastly M is the modulation index of the digital modulation scheme used (QAM,PSK).
Spectral efficiency of OFDM-IQ-IM system,

$$\zeta_{OFDM-IQ-IM} = \frac{G}{N}\left(2\left\lfloor \log_2 \binom{n}{k} \right\rfloor + 2k \log_2 M\right), \tag{2.7}$$

where N is the total number of subcarriers. G represents the total number of subblocks. n and k represents total number of subcarriers in a subblock and total number of active subcarriers in a subblock accordingly for each of the in-phase and quadrature channel. Lastly M is the modulation index of the digital modulation scheme used (PAM) in each of the channel. Here, in Table2.4 the SE of OFDM, OFDM-IM and OFDM-IQ-IM have been determined considering $N = 128$. Here, k is is considered as $n-1$ for OFDM-IM and OFDM-IQ-IM systems. For OFDM-IM and OFDM-IQ-IM the flexibility is more to compute a desired SE, while for OFDM the value of SE is fixed for any value of M. And from 2.4 we can clearly conclude for the same configuration OFDM-IQ-IM system provides better spectral efficiency than both OFDM-IM and OFDM.

M	16 for OFDM, OFDM-IM and 4 for OFDM-IQ-IM			
N	128			
G	1	2	4	8
n	128	64	32	16
k	127	63	31	15
$\zeta_{OFDM-IM}$	4.023	4.031	4.031	4
$\zeta_{OFDM-IQ-IM}$	4.078	4.125	4.1875	4.25
ζ_{OFDM}	4			

Table 2.4: Spectral efficiency of OFDM, OFDM-IM and OFDM-IQ-IM systems for specific configuration

- **BER performance:** The BER performance of OFDM-IQ-IM system has been first studied in [28]. In this study OFDM-IQ-IM was termed as generalized scheme of OFDM-IM and has been shown that it has significantly improved BER performance than OFDM and OFDM-IM. According to [30], for the same spectral efficiency OFDM-IQ-IM system has better coding gain than OFDM and OFDM-IM systems. However, if the modulation order is increased the coding gain will decrease. In [74], it was mentioned that BER performance of OFDM-IQ-IM system outperforms OFDM and OFDM-IM systems in low SNR region. However, in high SNR region due to higher number of modulation order the BER performance gets worse. Even for V2X communications, OFDM-IQ-IM can provide better BER performance than OFDM-IM and OFDM system and it is sensitive to modulation index and subblock size [32]. Multiple Mode OFDM-IQ-IM (MM-OFDM-IQ-IM) was introduced in [75], which outperforms BER performance of all the already mentioned systems with a trade off of increasing computational complexity. A modified OFDM-IQ-IM system named OFDM Hybrid In-phase/Quadrature Index Modulation (OFDM-HIQ-IM) was proposed in [76]. Here Figure 2.7 is taken from [76] where we can clearly see for the low SNR region OFDM-IQ-IM outperforms OFDM-IM and classical OFDM system for specified configuration. Also the proposed system OFDM-HIQ-IM in that research outperforms the conventional OFDM-IQ-IM BER performance. Precoding technique is incorporated with MIMO-OFDM, MIMO-OFDM-IM and MIMO-OFDM-IQ-IM in [77] where the best BER performance can be achieved by using MIMO-OFDM-IQ-IM with precoding technique than any other systems. Besides the superior BER performance of OFDM-IQ-IM system has been highlighted in [78], [79], [73], [80].

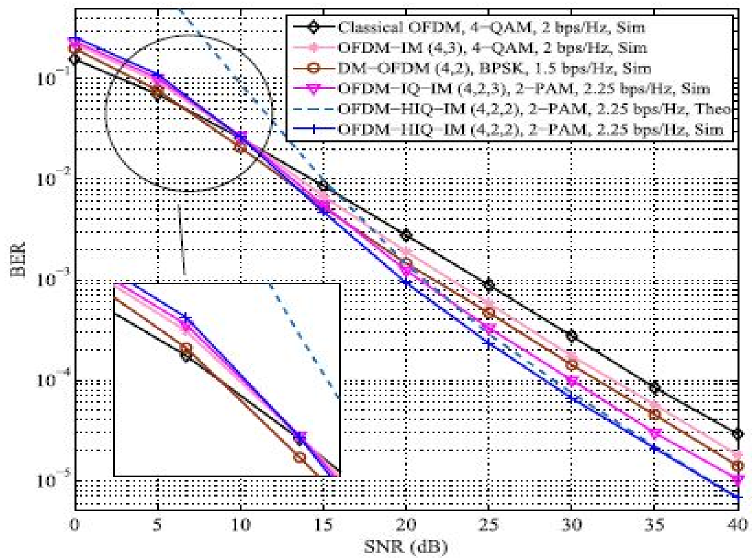


Figure 2.7: Comparison of BER performance of OFDM, OFDM-IM, OFDM-IQ-IM and OFDM-HIQ-IM systems [76]

- **Robustness against doppler effects and channel correlation:** According to [32], OFDM-IQ-IM system is more robust against doppler effects and channel correlation than OFDM-IM system due to reduced subblock size.

- **Flexibility and Energy Efficiency:**Due to reduced subblock size or the capability of controlling the status (active or inactive) of subcarrier in-phase and quadrature components independently it allows more flexibility while designing the system. Also due to the inactive subcarriers or subcarriers modulated by lower modulation order, it provides more energy efficiency.

- **PAPR performance:** According to author's best knowledge after extensive literature study, no study have been done evaluating the PAPR performance of OFDM-IQ-IM system. Considering the superiority over other multiplexing schemes based on BER performance and SE, the PAPR performance of OFDM-IQ-IM is an important research topic to evaluate the overall performance of OFDM-IQ-IM system and this thesis will be focusing on that.

Chapter 3

PAPR of OFDM-IQ-IM in Deterministic Approach

3.1 What is PAPR?

High PAPR is one of the major disadvantages of OFDM system. Even for OFDM-IM system PAPR is still very high. Various reduction schemes have been proposed for both the systems. OFDM-IQ-IM which is a modified version of OFDM-IM system has been proposed and BER performance and SE of OFDM-IQ-IM have been studied by researchers in recent years. According to those studies, OFDM-IQ-IM exhibits better performance than OFDM and OFDM-IM system considering BER performance and SE. Therefore, in this thesis, the PAPR of OFDM-IQ-IM has been determined.

The simplest definition of PAPR is the ratio of maximum power, P_{max} to average power, P_{avg}.

$$PAPR = \frac{P_{max}}{P_{avg}}, \tag{3.1}$$

The PAPR of an OFDM signal maximum instantaneous power of a sample in an OFDM signal divided by the average power of all the samples of that OFDM signal. The output bandpass OFDM signal, $x(t$ can be represented as,

$$x(t) = \Re\{\tilde{x}(t)\, e^{j2\pi f_c t}\}, \tag{3.2}$$

where $\tilde{x}(t)$ is the complex baseband signal. If the complex baseband signal $\tilde{x}(t)$ is defined over a time interval $t \in [0, T_S]$ then the PAPR of the baseband OFDM signal can be defined as,

$$PAPR = \frac{max_{0 \leq t \leq T_S} |\tilde{x}(t)|^2}{E\{|\tilde{x}(t)|^2\}}, \tag{3.3}$$

where $E\{|\tilde{x}(t)|^2\}$ determines the average power of the continuous time baseband OFDM signal. And the power of the continuous time baseband OFDM signal at an instant when the amplitude of the signal reaches maximum envelope value.

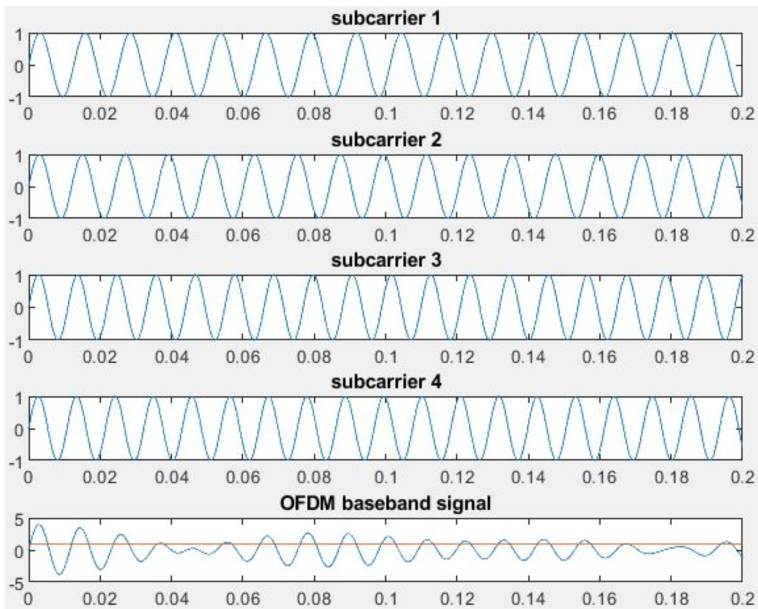


FIGURE 3.1: OFDM baseband signal considering $N = 4$

In Figure 3.1, we have generated an OFDM bandpass signal considering total subcarrier number, $N = 4$. This signal is a constructive signal. The red line indicates the average power. And the highest peak of the envelope indicates the instantaneous peak power. We can see in Figure 3.1 the highest peak of the OFDM baseband signal which is much higher than the average power. It varies depending on the number of subcarriers and modulation index used. As there are large number of independently modulated subcarriers in the OFDM system, the PAPR of the system is very high.

In OFDM system, typical PAPR is the magnitude of $10dB$. It means in order to transmit $0.2\,W$ power on an average the power amplifier used in OFDM system must be capable to handle peaks of about $2\,W$, which is ten times higher. And this introduces some major problems such as,

- Reduced efficiency of the RF power amplifier.
- If very high power amplifier is used to handle large peak powers, the cost of the overall system increases.
- Increases complexity in the analog to digital and digital to analog converters.
- If the highest peak is clipped to solve the problem, it will introduce various distortions in the transmitter chain.
- Degradation of system performance such as BER performance.
- The high PAPR will result in high battery power consumption. This is a significant disadvantage in cellular communication as part of the units are powered by battery. So, the battery power will drain fast and repetitive charging of mobile phone would be needed after every short interval.

3.2 PAPR of OFDM system

PAPR of an OFDM system is formulated using discrete time OFDM baseband signals in [22]. If we sample our continuous time baseband signal, $\tilde{x}(t)$ then each of the sample is denoted by $x[m]$ where, $m = 0, ..., N-1$. In Figure 2.1, $x[m]$ after the parallel to serial conversion of the IFFT samples is called is called the discrete time baseband equivalent OFDM signal. Here, each of the sample of discrete time baseband equivalent OFDM signal , $x[m]$ is basically the m-th sample of the IFFT block. According to [22], we can write,

$$x[m] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}, \qquad m = 0, 1, ..., N-1, \tag{3.4}$$

where $l = 0, ..., N-1$ denotes the subcarrier number and X_l denotes the modulated data symbols on l-th subcarrier.

If we have two complex numbers with equal power their summation will result to the highest possible power when both the numbers are same. Here in Figure 3.2 from [22], we

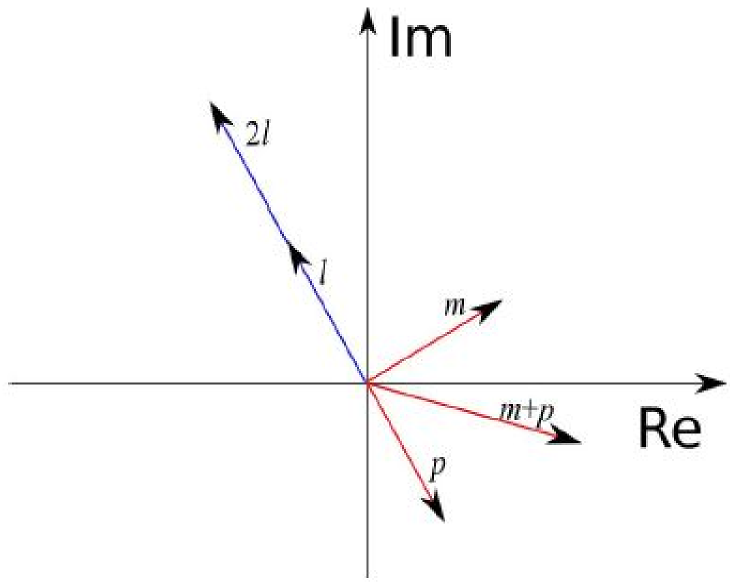


FIGURE 3.2: Graphical representation of summation of complex numbers with equal power $-l, m, p \in \mathbb{C}$ [22]

can see three vectors l, m, p with equal power which means, $|l|^2 = |m|^2 = |p|^2$. Now $m+p$ in Figure 3.2 represents the sum of two different vectors m and p whereas, $2l$ represents the sum of two same vectors l and l. We can clearly state from the vector diagram that $|l|^2 \geq |m+p|^2$.

According to [22], the peak power of conventional OFDM will occur at discrete time sample a if we consider,

$$X_l = de^{\frac{-j2\pi la}{N}}, \qquad d \in \mathbb{C}, \tag{3.5}$$

So, for that sample time a we can rewrite (3.4) as,

$$x[a] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi la}{N}}, \tag{3.6}$$

Now, putting the value of X_l from (3.5) into (3.6) will give us,

$$\begin{aligned} x[a] &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} de^{-\frac{j2\pi la}{N}} e^{\frac{j2\pi la}{N}} \\ &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} d \end{aligned}$$

The result is the summation of N equal complex numbers d. So, the peak value at $x[a]$ can be achieved when all the complex numbers are same. So, we can incorporate the similar concept in (3.4), if we consider the sample time $m = 0$. We can rewrite (3.4) as,

$$\begin{aligned} x[0] &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi l0}{N}} \\ &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l \end{aligned}$$

Maximum power at $x[0]$ can be achieved if all the subcarriers are modulated with the same constellation symbol which has the highest possible power. If we denote the constellation symbol with highest possible power as S_{HP}, then we can write $x[0]$ as,

$$\begin{aligned} x[0] &= \frac{1}{\sqrt{N}} N S_{HP} \\ &= \sqrt{N} S_{HP} \end{aligned}$$

Maximum power P_{max} can be found if we multiply $x[0]$ with it's complex conjugate,

$$P_{max} = x[0]x[0]^*, \tag{3.7}$$

And the product of $x[0]$ with it's complex conjugate can be represented as N-times of the product of the constellation symbol with highest power, S_{HP} and it's complex conjugate,

$$x[0]x[0]^* = N S_{HP} S_{HP}^*, \tag{3.8}$$

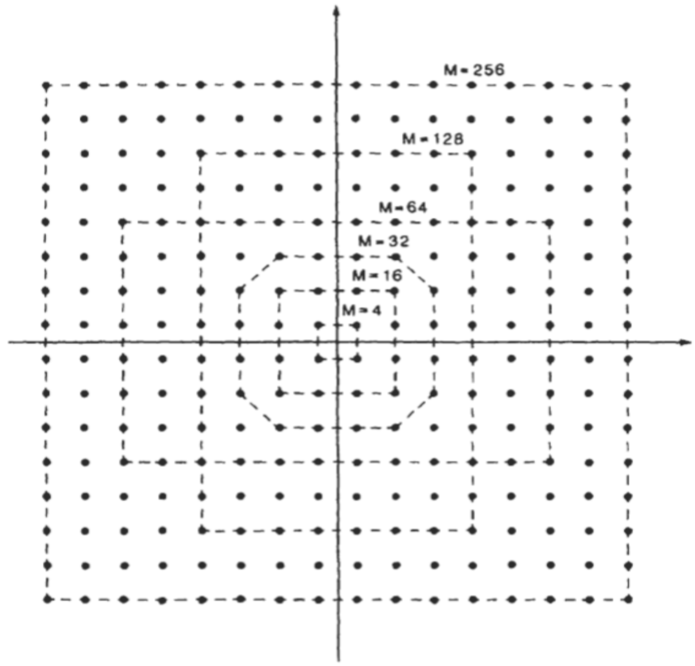


FIGURE 3.3: Sqaure QAM constellations from [82]

Now here the symbol constellation is considered square QAM. If (I_i, Q_i) are independent integers which denotes a symbol position in the constellation diagram. According to [81], the minimum values of (I_i, Q_i) coordinates are $(\pm 1, \pm 1)$ and the coordinates can have any value of any element from the $L \times L$ matrix as shown below,

$$[I_i, Q_i] = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \dots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \dots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \dots & (L-1, -L+1) \end{bmatrix} \tag{3.9}$$

where $L = \sqrt{M}$ and $i = 1, 2, \dots, M$. So, if we consider 16-QAM digital modulation scheme, $M = 16$, $L = 4$ and the matrix of (3.9) can be written in this case,

$$[I_i, Q_i] = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix} \tag{3.10}$$

But we know $M = 2^n$, where n determines the number of bits in a symbol. If $n = 5$ then L will not be an integer. So, we won't be able to define a QAM using the matrix of (3.9). So in Figure 3.3 from [82], different modified matrix were shown for different QAM.

Now from (3.9) the maximum values of (I_i, Q_i) coordinates are $(L-1, L-1)$. So, the symbol with highest possible power can be written as,

$$S_{HP} = (L-1) + j(L-1), \tag{3.11}$$

If we put the value of $L = \sqrt{M}$ in (3.11), the for a square M-QAM modulation the symbol with highest power can be written as,

$$S_{HP} = (\sqrt{M}-1) + j(\sqrt{M}-1), \tag{3.12}$$

Now multiplying symbol with highest power S_{HP} with it's complex conjugate will give us [81],

$$\begin{aligned} S_{HP}S_{HP}^* &= \Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big((\sqrt{M}-1) - j(\sqrt{M}-1)\Big) \\ &= (\sqrt{M}-1)^2 - j^2(\sqrt{M}-1)^2 \\ &= 2(\sqrt{M}-1)^2 \end{aligned}$$

Putting the value of $S_{HP}S_{HP}^*$ into (3.8) we get,

$$x[0]x[0]^* = 2N(\sqrt{M}-1)^2, \tag{3.13}$$

So, from (3.7) and (3.13) the maximum power of OFDM system can be written as [22],

$$P_{max} = 2N(\sqrt{M}-1)^2, \tag{3.14}$$

Now the average power can be found by dividing the average frame energy by number of points in the time domain and the average frame energy can be found by multiplying average energy per subcarrier and number of subcarriers [22]. And the average energy per subcarrier is basically the average energy of an M-QAM symbol.

$$
\begin{aligned}
P_{avg} &= \frac{E_{avg}}{N} \\
&= \frac{E_{avg}^{QAM} N}{N} \\
&= E_{avg}^{QAM}
\end{aligned}
$$

According to [81] the average energy of an M-QAM symbol can be formulated as,

$$E_{avg}^{QAM} = \frac{2(M-1)}{3}, \tag{3.15}$$

The average power of a baseband OFDM signal can be defined in terms of the M-ary modulation scheme used as,

$$P_{avg} = \frac{2(M-1)}{3}, \tag{3.16}$$

Now putting (3.14) and (3.16) into (3.1) would give us,

$$
\begin{aligned}
PAPR &= \frac{2N(\sqrt{M}-1)^2}{\frac{2(M-1)}{3}} \\
&= 3N\frac{(\sqrt{M}-1)(\sqrt{M}-1)}{(M-1)} \\
&= 3N\frac{(\sqrt{M}-1)(\sqrt{M}-1)}{(\sqrt{M}+1)(\sqrt{M}-1)}
\end{aligned}
$$

So, the PAPR of an OFDM system in deterministic approach or the upper limit of the PAPR of an OFDM system can be defined as [22],

$$PAPR = 3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}, \tag{3.17}$$

where N denotes the total number of subcarriers and M denotes the modulation index of the digital modulation scheme that has been used for mapping purpose.

3.3 PAPR of OFDM-IM system

In chapter 2, we have discussed about OFDM-IM system. In OFDM-IM system index bits have been introduced. Index bits are part of the message bits which are used to denote the index of the active subcarriers. Several study on the PAPR of OFDM-IM system have already been discussed. In this section, the PAPR formula for OFDM-IM system has been derived using the same methodology used in [22] for Subcarrier Index Modulation OFDM (SIM-OFDM) system.

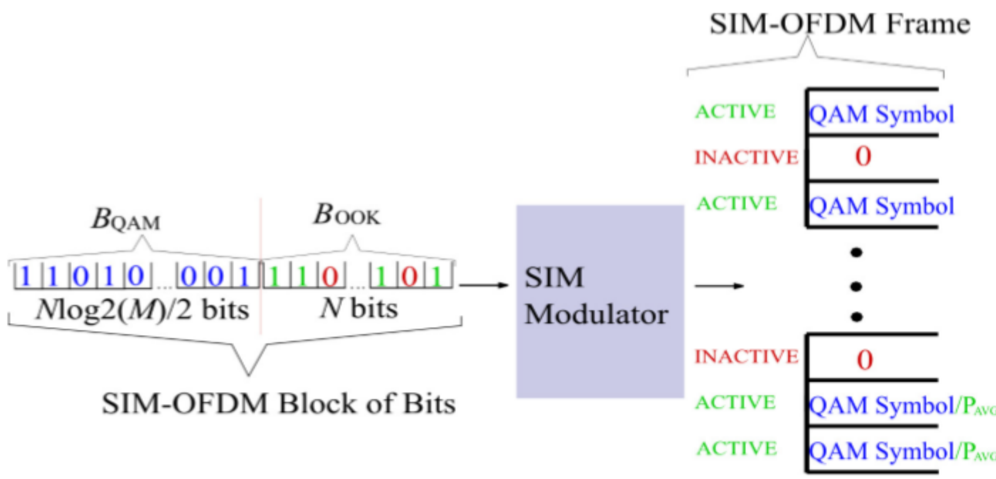


FIGURE 3.4: SIM-OFDM career modulation scheme from [22]

In SIM-OFDM system the total number of bits transmitted in one frame are defined as $N + \frac{Nlog_2(M)}{2}$ bits. Here, first N bits are termed as B_{OOK} and before the transmitting the bits the majority bits are investigated. Based on the majority bits the active subcarriers are decided. In Figure 3.4 the majority bit is 1. So, N subcarriers will be associated with N bits of B_{OOK}. The subcarriers associated with majority bits will be active and the rest will be inactive. And $\frac{Nlog_2(M)}{2}$ bits are used to modulate the first $\frac{N}{2}$ subcarriers using M-QAM digital modulation scheme to encode B_{QAM}. The rest of the active subcarriers are allocated with average power of the M-ary QAM symbols and will be used to signal the receiver about the majority bits in the B_{OOK}.

On the receiver side in order to demodulate the M-QAM symbols without any error, it is necessary to detect the subcarrier state and also the state of the previous subcarriers state correctly. A coherent OOK detector was used for this purpose. As one incorrect detection will result into the alteration of the order of the bits in B_{QAM} [22]. So, when the number of subcarriers N is very high, the detection becomes more complex and thus impact the BER performance.

In OFDM-IM system subcarriers are divided into G groups and look-up table or combinatorial method is used for data symbol mapping as explained in chapter 2. In OFDM-IM system, each of the subblocks will have n subcarriers so that $N = nG$ and among those n subcarriers k subcarriers will be chosen as active using the index bits. So, we denote the total number of active subcarriers as $N_a = kG$.

In case of OFDM-IM all the subcarriers are not active. So, the inactive subcarriers can not be used to modulate a symbol. Hence, to accumulate all the signal energy at a point like OFDM system is not possible for OFDM-IM system. Now, the sample of discrete time baseband OFDM-IM signal, $x[m]$ or the m-th sample of the IFFT block of OFDM-IM system can be represented similar to (3.4) and considering sample time $m = 0$ will give us,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l, \tag{3.18}$$

Now in case of OFDM-IM before the IFFT operation interleaver is used. Before the interleaver the data symbols are denoted as X_d^g where d denotes the subcarrier index within a subblock and g denotes the subblock index. So, for OFDM-IM system we can rewrite (3.18) as,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \sum_{d=0}^{n-1} X_d^g, \tag{3.19}$$

Now, in each subblock we know among n subcarriers only k subcarriers will be active. The rest of the subcarriers will be inactive and they will not be modulated with constellation symbol. To get the peak value all those active subcarriers in a subblock will have to be modualted with the symbol with highest possible power denotated by S_{HP} and $x[0]$ becomes,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} kS_{HP}, \tag{3.20}$$

All the subblocks will have equal number of active subcarriers. And to ensure the maximum value all the subcarriers of all the subblocks have to be modulated with the symbol with the highest power, S_{HP}. So the final expression for $x[0]$ for OFDM-IM system to ensure maximum power is,

$$x[0] = \frac{1}{\sqrt{N}} kGS_{HP}, \tag{3.21}$$

Now, if we multiply $x[0]$ with its complex conjugate $x[0]^*$ we get,

$$x[0]x[0]^* = \frac{1}{N} k^2 G^2 S_{HP} S_{HP}^*, \tag{3.22}$$

For a fair comparison we will consider square M-QAM modulation for OFDM-IM system. So, for a square M-QAM modulation symbol with highest power is $S_{HP} = (\sqrt{M} - 1) + j(\sqrt{M} - 1)$ as shown in (3.12). If we now multiply S_{HP} with its complex conjugate,

$$\begin{aligned} S_{HP} S_{HP}^* &= \Big((\sqrt{M} - 1) + j(\sqrt{M} - 1)\Big)\Big((\sqrt{M} - 1) - j(\sqrt{M} - 1)\Big) \\ &= (\sqrt{M} - 1)^2 - j^2(\sqrt{M} - 1)^2 \\ &= 2(\sqrt{M} - 1)^2 \end{aligned}$$

Now, putting this value into (3.22) would give us,

$$x[0]x[0]^* = \frac{2}{N} k^2 G^2 (\sqrt{M} - 1)^2, \tag{3.23}$$

So, from (3.7) and (3.23) the maximum power of OFDM-IM system can be written as,

$$P_{max} = \frac{2}{N} k^2 G^2 (\sqrt{M} - 1)^2, \tag{3.24}$$

Now the average power for OFDM-IM system can be found by dividing the average frame energy by number of points in the time domain similar to OFDM system. But the average frame energy for OFDM-IM will be different as all subcarriers are not active. Inactive subcarriers will not contribute. So, the average frame energy for OFDM-IM system can be found by multiplying average energy per subcarrier and number of active subcarriers, N_a. The average energy per subcarrier can be defined as the average energy of an M-QAM symbol. So, we can express the average power for OFDM-IM system as,

$$\begin{aligned} P_{avg} &= \frac{E_{avg}}{N} \\ &= \frac{E_{avg}^{QAM} N_a}{N} \end{aligned}$$

Now from [81] and also shown in (3.15) the average energy of an M-QAM symbol can be formulated as $E_{avg}^{QAM} = \frac{2(M-1)}{3}$. So putting the value of E_{avg}^{QAM}, the average power of a baseband OFDM-IM signal can be defined in terms of the M-ary modulation scheme used as,

$$P_{avg} = \frac{2(M-1)N_a}{3N}, \tag{3.25}$$

Here, N_a denotes the total number of active subcarriers. So if put $N_a = kG$ into (3.25) we get,

$$P_{avg} = \frac{2(M-1)kG}{3N}, \tag{3.26}$$

Now putting (3.24) and (3.26) into (3.1) would give us,

$$\begin{aligned} PAPR &= \frac{\frac{2}{N} k^2 G^2 (\sqrt{M}-1)^2}{\frac{2(M-1)kG}{3N}} \\ &= \frac{2k^2G^2(\sqrt{M}-1)^2}{N} \times \frac{3N}{2(M-1)kG} \\ &= \frac{2k^2G^2(\sqrt{M}-1)(\sqrt{M}-1)}{N} \times \frac{3N}{2kG(\sqrt{M}+1)(\sqrt{M}-1)} \end{aligned}$$

So, the PAPR of an OFDM-IM system in deterministic approach or the upper limit of the PAPR of an OFDM-IM system can be defined as,

$$PAPR_{IM} = 3N_a \frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}, \tag{3.27}$$

where $N_a = kG$ denotes the total number of active subcarriers and M denotes the modulation index of the digital modulation scheme that has been used for mapping purpose.

Now to find the relation between the PAPR of OFDM and OFDM-IM system, we will divide (3.27) by (3.17),

$$
\begin{aligned}
\frac{PAPR_{IM}}{PAPR} &= \frac{3N_a\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}}{3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}} \\
&= \frac{3N_a(\sqrt{M}-1)}{(\sqrt{M}+1)} \times \frac{(\sqrt{M}+1)}{3N(\sqrt{M}-1)}
\end{aligned}
$$

The PAPR of OFDM-IM system can be written in terms of the PAPR of OFDM system as,

$$PAPR_{IM} = \frac{N_a}{N} PAPR, \tag{3.28}$$

We know, N_a will always be less than N. So, the ratio $\frac{N_a}{N}$ will always be less than 1. Therefore, the upper limit of PAPR of OFDM-IM system will always be less than the upper limit of PAPR of OFDM system. If $N_a = N$, all the subcarriers will be active and the system will behave like OFDM system thus the upper limit for the PAPR of both the systems will be same.

3.4 PAPR of OFDM-IQ-IM system

In chapter 2, we have discussed about OFDM-IQ-IM system which shows better performance than OFDM and OFDM-IM system in terms of BER peformance and spectral analysis. Now, we will derive the formula for the PAPR of OFDM-IQ-IM broadband signal in deterministic approach using the methodologies we used for determining the PAPR for OFDM and OFDM-IM system in previous sections.

In OFDM-IQ-IM system the subcarriers are divided into G subblocks. Each of them have n subcarriers so that $N = nG$. In case of OFDM-IQ-IM system index modulation is done independently and jointly on both in-phase channel and quadrature channel. In OFDM-IQ-IM system total number of bits transmitted in a subblock p is divided into two equal parts p_I and p_Q and transmitted into in-phase channel and quadrature channel. For in-phase channel, index bits, $p1$ will determine the k out of n subcarriers that will be active. The active subcarriers will be modulated with M-PAM constellation symbol using the modulation bits, $p2$. The $n-k$ inactive subcarriers will not transmit any signal. Similarly for quadrature channel, also p_Q bits is divided into index bits $p1$ and modulation bits $p2$. Index bits, $p1$ will denote which k subcarriers are active among the n subcarriers in a subblock and $p2$ bits will be used to modulate those active subcarriers with M-PAM constellation symbol. The sample of discrete time baseband OFDM-IQ-IM signal, $x[m]$ or the m-th sample of the IFFT block of OFDM-IQ-IM system can be expressed by (3.4) and taking sample time $m = 0$ will give us,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l, \tag{3.29}$$

Now in OFDM-IQ-IM system, the k subcarriers can be active independently and jointly for in-phase channel and quadrature channel. So for a subcarrier can have four states in OFDM-IQ-IM system. They are:

- A subcarrier can be active for both in-phase and quadrature channel.
- A subcarrier can be active for in-phase channel but inactive for quadrature channel.
- A subcarrier can be active for quadrature channel but inactive for in-phase channel.
- A subcarrier can be inactive for both in-phase and quadrature channel.

When a subcarrier will be active for any one channel it will be modulated with a symbol from M-PAM constellation diagram. And when a subcarrier will be active for both the channel it will adopt M^2-ary QAM constellation [73].

Subcarrier State	X_d^g
A subcarrier can be active for both in-phase and quadrature channel.	$I_l^g + jQ_l^g$
A subcarrier can be active for in-phase channel but inactive for quadrature channel.	$I_l^g + j0$
A subcarrier can be active for quadrature channel but inactive for in-phase channel.	$0 + jQ_l^g$
A subcarrier can be inactive for both in-phase and quadrature channel.	$0 + j0$

TABLE 3.1: Different data symbol for different subcarrier state in OFDM-IQ-IM system

In OFDM-IQ-IM system interleaver is also used. Before the interleaver the data symbols are denoted by X_d^g where d and g denotes the subcarrier index in a subblock and the subblock index accordingly. For different subcarrier state X_d^g can be expressed in different way and that is shown in Table 3.1. Here, I_l^g and Q_l^g denotes the the M-PAM constellation point modulated on the l-th subcarrier of the g-th subblock. So, the (3.29) can be rewritten as,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \sum_{d=0}^{n-1} X_d^g, \tag{3.30}$$

Now in a subblock if we consider r subcarriers are active for both in-phase channel and quadrature channel and are modulated by M^2-QAM constellation symbols. And, $k - r$ subcarriers are active for only in-phase channel and will be modulated with M-PAM constellation symbol. Also, $k-r$ subcarriers are active for only quadrature channel and will be modulated with M-PAM constellation symbol. The rest of the $n-2k+r$ subcarries will be inactive and will not be modulated to transmit any signal. So, for a subblock we can write,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \{(k-r)(I_l^g + j0) + (k-r)(0 + jQ_l^g) + r(I_l^g + jQ_l^g)\}, \tag{3.31}$$

To get the peak value, the $k-r$ subcarriers, active for only in-phase channel will be multiplied by the symbol of highest power of the M-PAM constellation denoted by $S_{HP}{}'$. To get the peak value, the $k-r$ subcarriers, active for only quadrature channel will be multiplied by the symbol of highest power of the M-PAM constellation denoted by $S_{HP}{}'$. Lastly to get peak value of r subcarriers, active for both in-phase and quadrature channel will be multiplied by the symbol of highest power of the M^2-QAM constellation denoted by S_{HP}. So, we can modify (3.31) as,

$$x[0] = \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \{(k-r)S_{HP}{}' + (k-r)jS_{HP}{}' + rS_{HP}\}, \tag{3.32}$$

If we consider, (I_i, Q_i) are independent integers which denotes a symbol position in the constellation diagram. The minimum values for (I_i, Q_i) coordinates are $(\pm 1, \pm 1)$ and the coordinates can have any value of any element from the $1 \times L$ matrix as shown below,

$$[I_i, Q_i] = \Big[(-L+1, 0) \quad (-L+3, 0) \quad \ldots \quad (L-1, 0)\Big] \tag{3.33}$$

where $L = M$ and $i = 1, 2, \ldots, M$. So, if we consider 4-PAM digital modulation scheme, $M = 4$, $L = 4$ and the matrix of (3.33) can be written in this case,

$$[I_i, Q_i] = \Big[(-3, 0) \quad (-1, 0) \quad (1, 0) \quad (3, 0)\Big] \tag{3.34}$$

Now from (3.33) the maximum values of (I_i, Q_i) coordinates are $(L-1, 0)$. So, the symbol with highest possible power of the M-PAM constellation can be written as,

$$S_{HP}{}' = (L-1) + j0, \tag{3.35}$$

For an even power of 2, square QAM is a constellation whose points are spaced on a grid in the form of a square. A square M-QAM is formed by a product of two $\sqrt{M}$-PAM, one on I axis and the other on Q axis. For example, a 16-QAM constellation is formed by two 4-PAM constellations as drawn in Figure 3.5,

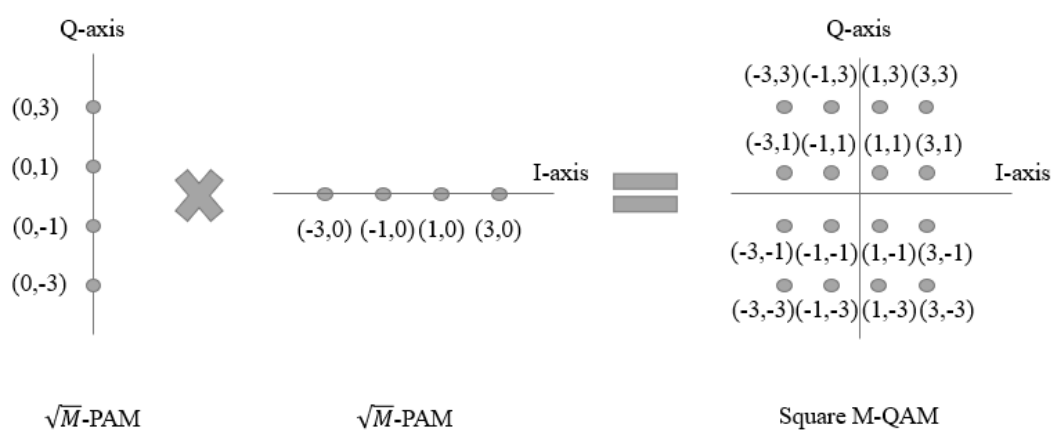


Figure 3.5: PAM to square QAM conversion

So, for a $\sqrt{M}$-PAM constellation we can write (3.35) as,

$$S_{HP}' = (\sqrt{M} - 1), \tag{3.36}$$

From (3.12) we can see, for a square M-QAM modulation the symbol with highest power can be written as,

$$S_{HP} = (\sqrt{M} - 1) + j(\sqrt{M} - 1),$$

Putting the value of (3.12) and (3.36) into (3.32) we get,

$$\begin{aligned}
x[0] &= \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \Big\{(k-r)(\sqrt{M}-1) + (k-r)j(\sqrt{M}-1) + r\Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big\} \\
&= \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \Big\{(r-r)(\sqrt{M}-1) + (r-r)j(\sqrt{M}-1) + k\Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big\} \\
&= \frac{1}{\sqrt{N}} \sum_{g=0}^{G-1} \Big\{k\Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big\}
\end{aligned}$$

All the subblocks will have equal number of active subcarriers. And to ensure the maximum value the final expression of $x[0]$ for OFDM-IQ-IM system can be written as,

$$x[0] = \frac{1}{\sqrt{N}} kG\Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big), \tag{3.37}$$

Now, multiplying $x[0]$ from (3.37) with its complex conjugate we get,

$$x[0]x[0]^* = \frac{1}{N} k^2 G^2 \Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big((\sqrt{M}-1) - j(\sqrt{M}-1)\Big), \tag{3.38}$$

Now if we put the value of $\Big((\sqrt{M}-1) + j(\sqrt{M}-1)\Big)\Big((\sqrt{M}-1) - j(\sqrt{M}-1)\Big) = 2(\sqrt{M}-1)^2$ into (3.38) we get,

$$x[0]x[0]^* = \frac{2}{N} k^2 G^2 (\sqrt{M}-1)^2, \tag{3.39}$$

So, from (3.7) and (3.39) the maximum power of OFDM-IQ-IM system can be written as,

$$P_{max} = \frac{2}{N} k^2 G^2 (\sqrt{M} - 1)^2, \tag{3.40}$$

Now the average power for OFDM-IQ-IM system can be found by dividing the average frame energy by number of points in the time domain similar to OFDM and OFDM-IM system. But the average frame energy for OFDM-IQ-IM will be different from OFDM as all subcarriers are not active. Inactive subcarriers will not contribute. Also, the average frame energy for OFDM-IQ-IM will be different from OFDM-IM as the total number of active subcarriers is not constant for all the subblocks. For OFDM-IQ-IM system few subcarriers will be modulated with M-QAM constellation symbol and few of the subcarriers will be modulated with $\sqrt{M}$-PAM constellation symbol. The average frame energy for OFDM-IQ-IM system can be found by multiplying average energy per subcarrier and number of active subcarriers. If we assume r subcarriers in a subblock which are active for both in-phase and quadrature channel, are modulated with M-QAM symbols. The average energy per subcarrier for those r subcarriers can be defined as the average energy of an M-QAM symbol. For $k - r$ subcarriers of that subblock which are only active for in-phase channel, they will be modulated with $\sqrt{M}$-PAM symbols. The average energy per subcarrier for those $k - r$ subcarriers can be defined as the average energy of an $\sqrt{M}$-PAM symbol. And for $k - r$ subcarriers of that subblock which are only active for in-phase channel, they will be modulated with $\sqrt{M}$-PAM symbols. The average energy per subcarrier for those $k - r$ subcarriers can be defined as the average energy of an $\sqrt{M}$-PAM symbol. So, we can express the average power for OFDM-IQ-IM system as,

$$\begin{aligned} P_{avg} &= \frac{E_{avg}}{N} \\ &= \frac{\left(E_{avg}^{QAM} r + E_{avg}^{PAM}(k-r) + E_{avg}^{PAM}(k-r)\right) G}{N} \end{aligned} \tag{3.41}$$

Now from [81] and also shown in (3.15) the average energy of an M-QAM symbol can be formulated as $E_{avg}^{QAM} = \frac{2(M-1)}{3}$. Now, we will determine the average energy of an $\sqrt{M}$-PAM symbol.

We consider a symmetric M-PAM constellation diagram as shown in Figure 3.6,

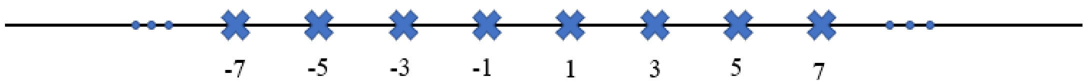


FIGURE 3.6: Symmetric M-PAM constellation diagram

Here, signal amplitudes have equal distance among them and they are symmetric about 0. We can denote the M-PAM constellations symbols as,

$$A_m = (2m - 1 - M),\ m = 1, 2, \ldots, M, \tag{3.42}$$

For an example, if we consider $M = 4$ then $A_1 = -3, A_2 = -1, A_3 = 1, A_4 = 3$. Average energy per symbol for an M-PAM constellation is defined as,

$$\begin{aligned} E_{avg}^{PAM} &= \frac{E_g}{M}\sum_{m=1}^{M}(2m-1-M)^2 \\ &= \frac{1}{M}\frac{M(M^2-1)E_g}{3} \\ &= \frac{E_g}{3}(M^2-1) \end{aligned}$$

Here, E_g denotes the energy of the symbol with lowest amplitude, $|A_m|$. So, it will always be 1. So, putting the value of $E_g = 1^2 = 1$ we get,

$$E_{avg}^{PAM} = \frac{1}{3}(M^2 - 1), \tag{3.43}$$

In OFDM-IQ-IM system we are using $\sqrt{M}$-PAM constellation. Considering $\sqrt{M}$-PAM constellation we can rewrite (3.42) as,

$$E_{avg}^{PAM} = \frac{1}{3}(M - 1), \tag{3.44}$$

Putting the values of E_{avg}^{QAM} and E_{avg}^{PAM} into (3.41) we get,

$$
\begin{aligned}
P_{avg} &= \frac{\left(r\frac{2(M-1)}{3} + (k-r)\frac{(M-1)}{3} + (k-r)\frac{(M-1)}{3}\right)G}{N} \\
&= \frac{\left((2r-r-r)\frac{(M-1)}{3} + (k+k)\frac{(M-1)}{3}\right)G}{N} \\
&= \frac{\left(2k\frac{(M-1)}{3}\right)G}{N}
\end{aligned}
$$

Here, k the number of active subcarriers for each channel in a subblock will be same for all the subblocks. So, the average power of a baseband OFDM-IQ-IM signal can be defined as,

$$
P_{avg} = \frac{2(M-1)kG}{3N}, \tag{3.45}
$$

Now putting (3.40) and (3.45) into (3.1) would give us,

$$
\begin{aligned}
PAPR &= \frac{\frac{2}{N}k^2G^2(\sqrt{M}-1)^2}{\frac{2(M-1)kG}{3N}} \\
&= \frac{2k^2G^2(\sqrt{M}-1)^2}{N} \times \frac{3N}{2(M-1)kG} \\
&= \frac{2k^2G^2(\sqrt{M}-1)(\sqrt{M}-1)}{N} \times \frac{3N}{2kG(\sqrt{M}+1)(\sqrt{M}-1)}
\end{aligned}
$$

So, the PAPR of an OFDM-IQ-IM system in deterministic approach or the upper limit of the PAPR of an OFDM-IQ-IM system can be defined as,

$$
PAPR_{IQ} = 3kG\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}, \tag{3.46}
$$

where G denotes the total number of subblocks and k denotes the number of active subcarriers for each channel in a subblock. Lastly, $\sqrt{M}$ denotes the modulation index of PAM modulation scheme used in each channel for mapping purpose.

Now to find the relation between the PAPR of OFDM and OFDM-IQ-IM system, we will divide (3.46) by (3.17),

$$\frac{PAPR_{IQ}}{PAPR} = \frac{3kG\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}}{3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}}$$
$$= \frac{3kG(\sqrt{M}-1)}{(\sqrt{M}+1)} \times \frac{(\sqrt{M}+1)}{3N(\sqrt{M}-1)}$$

The PAPR of OFDM-IQ-IM system can be written in terms of the PAPR of OFDM system as,

$$PAPR_{IQ} = \frac{kG}{N} PAPR, \tag{3.47}$$

We know, kG will always be less than N. So, the ratio $\frac{kG}{N}$ will always be less than 1. Therefore, the upper limit of PAPR of OFDM-IQ-IM system will always be less than the upper limit of PAPR of OFDM system. If $kG = N$, all the subcarriers of all the subblocks will be active for both in-phase and quadrature channel and the system will behave like OFDM system thus the upper limit for the PAPR of both the systems will be same.

If we denote k_1 as the number of active subcarriers in each subblock for OFDM-IM system, G_1 as the number of subblocks for OFDM-IM system, k_2 as the number of active subcarrier components (in-phase or quadrature) in each subblock for each each channel for OFDM-IQ-IM system and G_2 as the number of subblocks for OFDM-IQ-IM system then from (3.46) and (3.27) we can write the PAPR of OFDM-IQ-IM in terms of OFDM-IM system as,

$$PAPR_{IQ} = \frac{k_2 G_2}{k_1 G_1} PAPR_{IM}, \tag{3.48}$$

From [32] and explained in Chapter 2 we can say for lower value of k and G, OFDM-IQ-IM system will outperform OFDM-IM system regarding BER performance and SE. So considering that k_2G_2 will be less than k_1G_1 thus the upper limit of PAPR of OFDM-IQ-IM system will be less than the upper limit of PAPR of OFDM-IM system. If $r = k$, all the active subcarriers of all the subblocks will be active for both in-phase and quadrature channel and the system will behave like OFDM-IM system thus the upper limit for the PAPR of both the systems will be same. For same SE the PAPR of OFDM, OFDM-IM and OFDM-IQ-IM systems will be analysed using numerical analysis in Chapter 5.

Chapter 4

PAPR of OFDM-IM and OFDM-IQ-IM in Probabilistic Approach

4.1 Limitation of deterministic approach of finding PAPR

In deterministic approach we modulated all the subcarriers with the symbol with highest power. The upper limit of the PAPR of OFDM system for a M-QAM system is $3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}$. But if the number of subcarrier, N is very high the probability of occurrence of the theoretical maximum PAPR becomes negligible [23]. The problem will be more visible if we consider a M-PSK system. For a M-PSK OFDM system we can write the IFFT sample at time m as,

$$x[m] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}, \tag{4.1}$$

The average power of the discrete time baseband OFDM signal for a M-PSK system can be shown as,

$$P_{avg} = E\{|x[m]|^2\}, \tag{4.2}$$

Using the value of $x[m]$ from (4.1) we can find the value of $E\{|x[m]|^2\}$,

$$\begin{aligned}
E\{|x[m]|^2\} &= E\Big\{\Big|\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}\Big|^2\Big\} \\
&= \frac{1}{N}\sum_{l=0}^{N-1} E\{|X_l|^2\}E\{|e^{\frac{j2\pi lm}{N}}|^2\} \\
&= \frac{1}{N}\sum_{l=0}^{N-1} E\{|X_l|^2\} \\
&= \frac{1}{N}\sum_{l=0}^{N-1} a^2 \\
&= \frac{1}{N} N a^2
\end{aligned}$$

Here, $e^{\frac{j2\pi lm}{N}}$ is a phase factor so the expected value of this term will be 1. From Figure 4.1 we can see for all the PSK symbols the amplitude will be constant, a. Thus, all the symbols will have equal energy, a^2. Now putting the value of $E\{|x[m]|^2\}$ into (4.2) we get the average power of OFDM baseband signal for M-PSK system as,

$$P_{avg} = a^2, \tag{4.3}$$

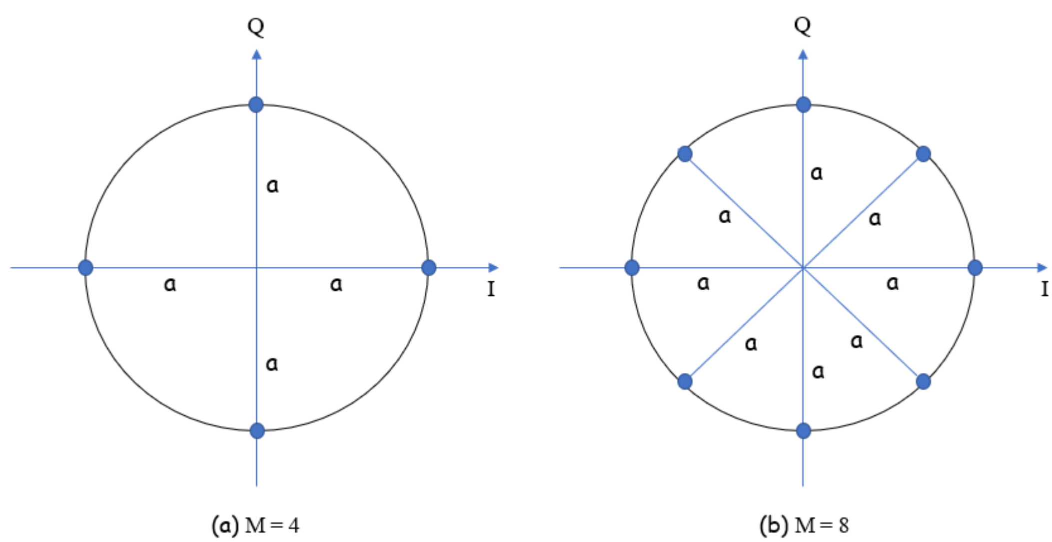


FIGURE 4.1: (a) $M = 4$ PSK and (b) $M = 8$ PSK constellation diagram

To get the peak power we will take the IFFT sample at sample time $m = 0$. Now, from (4.1) we can write,

$$
\begin{aligned}
x[0] &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^0 \\
&= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l \\
&= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} a \\
&= \frac{1}{\sqrt{N}} N a \\
&= \sqrt{N} a
\end{aligned}
$$

Multiplying $x[0]$ with it's complex conjugate we get,

$$x[0]x[0]^* = Na^2, \tag{4.4}$$

The peak power can be defined as the product of $x[0]$ and it's complex conjugate $x[0]^*$. Now from (4.4) we can write the peak power of OFDM baseband signal for M-PSK system as,

$$P_{max} = Na^2, \tag{4.5}$$

If we divide the peak power by the average power we get the PAPR of the system. From (4.5) and (4.3) we can write the upper limit of the PAPR of an OFDM baseband signal for M-PSK system as,

$$PAPR = N, \tag{4.6}$$

From (4.6) we can see the theoretical upper limit of the PAPR of the system will be N for N number of subcarriers. It is possible to have M^2 maximum pattern which will return the value of highest PAPR N. So, the probability of getting the highest PAPR is $\frac{M^2}{M^N} = M^{2-N}$ [24]. If we consider a Quadrature PSK (QPSK) system where $M = 4$ and total number of subcarriers $N = 32$, then the probability of observing the theoretically maximum PAPR is only 8.7×10^{-19}. If we consider the symbol period of an OFDM symbol is $200\mu s$

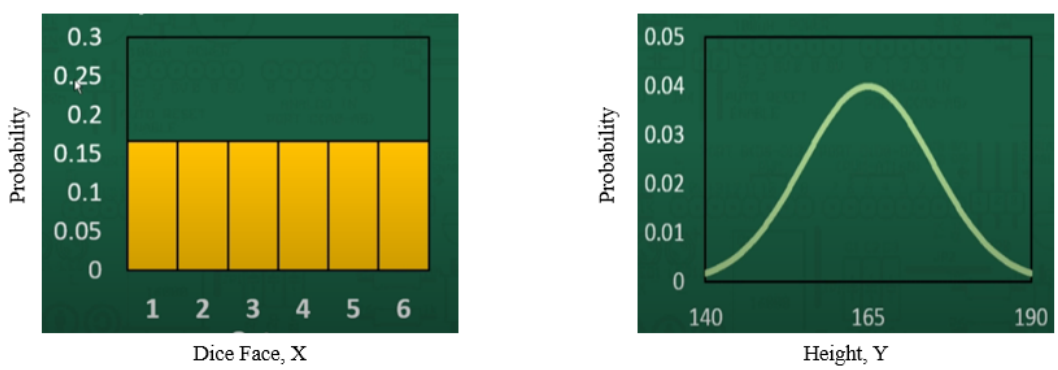
then the theoretically maximum PAPR can be observed once in 7.3 million years. So, the upper limit we got using the deterministic approach is not sufficient to properly characterise the PAPR of an OFDM system. Thus the statistical distribution of the PAPR in terms of CCDF came into discussion.

4.2 Characterising PAPR as CCDF function

In the deterministic approach we have considered all the subcarriers will be modulated by the constellation symbol with highest power. But in practical scenario, the symbol by which a subcarrier will be modulated depends on the upcoming bit stream. We know the bits generated are random. Symbols will be random in nature and the resultant peak power depending on the combination of these random symbols will be random. Therefore, PAPR of the system will be a random variable. Random variables are defined by characteristic function.

4.2.1 Cumulative Distributive Function (CDF)

A random variable can be discrete or continuous in nature. For an example, if we through a dice the face that will come up can have any discrete number from 1 to 6. Whereas, if we measure average height of a person that can have any value within a range.



(a) PMF of X

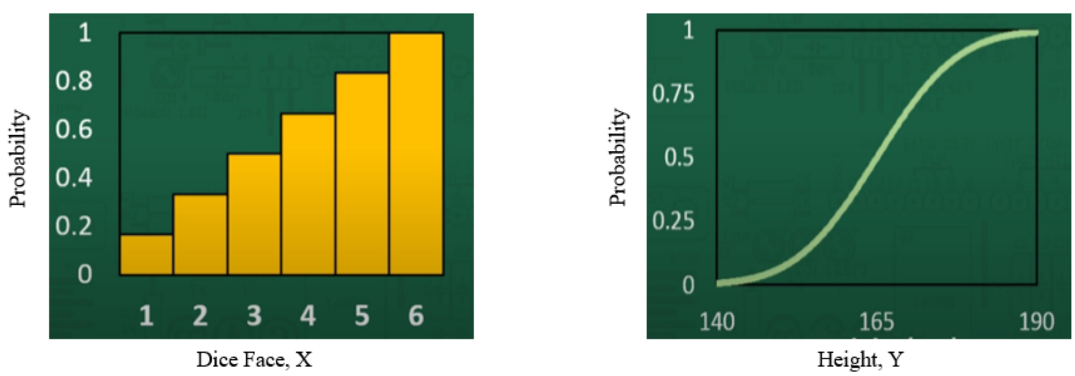
(b) PDF of Y

FIGURE 4.2: Distribution function: (a) PMF of discrete random variable X and (b) PDF of continuous random variable Y

We consider the number that will come up as a random variable, X and the height of a person as a random variable, Y. In Figure 4.2 the Probability Mass Function (PMF) for discrete random variable, X and Probability Density Function (PDF) of continuous random variable, Y is shown. PMF expresses the probability of each discrete value of a random variable where as the PDF shows the probability of a continuous random variable. CDF of a random variable Z evaluated at z, defines the probability that Z will take a value less than or equal to z. If we denote the CDF of a random variable Z as $F_Z(z)$ then we can write,

$$F_Z(z) = P(Z \leq z), \tag{4.7}$$

Figure 4.3 shows the CDF of random variables X and Y. We can see the CDF plots start from 0 and ends at 1. If we write $F_X(6)$ it donates the probability of the dice face that will come to have a value less than or equal to 6. This is a certain event and will give us the probability 1. If we write $F_Y(168.8)$, it denotes the area of the PDF below the height 168.8cm considering the whole area equals to 1. So, CDF is useful to show the probability of a random variable taking a value less than or equal to a fixed threshold.



(a) CDF of X

(b) CDF of Y

FIGURE 4.3: CDF of discrete random variable X and (b) CDF of continuous random variable Y

4.2.2 Complementary Cumulative Distribution Function (CCDF)

CCDF is the complement of CDF. CCDF starts from 1 and goes to 0. It means for higher thresholds the probability of the random variable being greater than the thresholds is progressively smaller. CCDF can be expressed in terms of CDF as,

$$CCDF = CDF - 1, \tag{4.8}$$

CCDF of a random variable Z evaluated at z, defines the probability that Z will take a value greater than z. If we denote the CCDF of a random variable Z as $\bar{F}_Z(z)$ then we can write,

$$\bar{F}_Z(z) = P(Z > z), \tag{4.9}$$

Now as for OFDM system PAPR is a random variable. CCDF of PAPR will give us the probability of the system PAPR greater than a certain threshold. If we consider the threshold as ρ then the CCDF of PAPR can be written as,

$$\bar{F}_{PAPR}(\rho) = P(PAPR > \rho), \tag{4.10}$$

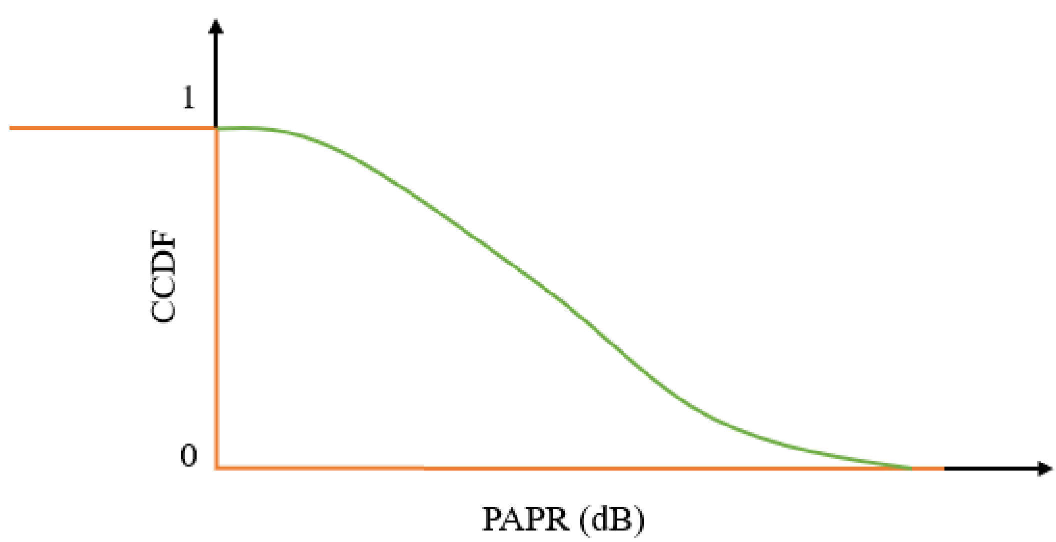


FIGURE 4.4: CCDF vs PAPR characteristics curve

Here in Figure 4.4, the CCDF vs PAPR characteristics curve has been shown. The orange curve represents the ideal scenario for PAPR of single carrier systems. Where the probability of PAPR greater than $0dB$ is always 0 and the probability of PAPR less than $0dB$ is always 1. That's why we want our PAPR to be close to $0dB$ for our systems. But in OFDM the PAPR is very high so it deviates the curve from $0dB$ as shown by the green curve. This is called the statistical distribution of the PAPR.

4.3 PAPR distribution of OFDM system

We will now determine the statistical distribution of PAPR of the OFDM system. OFDM is an MCM system where a wide-band single career bandwidth W is divided into N subcarriers. The bandwidth of each subcarrier will be $\frac{W}{N}$. And each of them will have subcarrier frequency, $f_i = i\left(\frac{W}{N}\right)$ and the spacing between the subcarriers is $\Delta f = f_{i+1} - f_i$. If the entire OFDM frame is sent at a rate, $r = \frac{1}{T}$, then each data symbol will be sent at a rate, $r_S = \frac{r}{N}$ Hz and and each data symbol will have symbol duration, $T_S = NT$. Thus we can write, $r = \frac{1}{T} = \frac{N}{T_S} = W$. So, in OFDM system N symbols will be sent parallelly at a slower rate thus minimizing ISI and multipath fading effects.

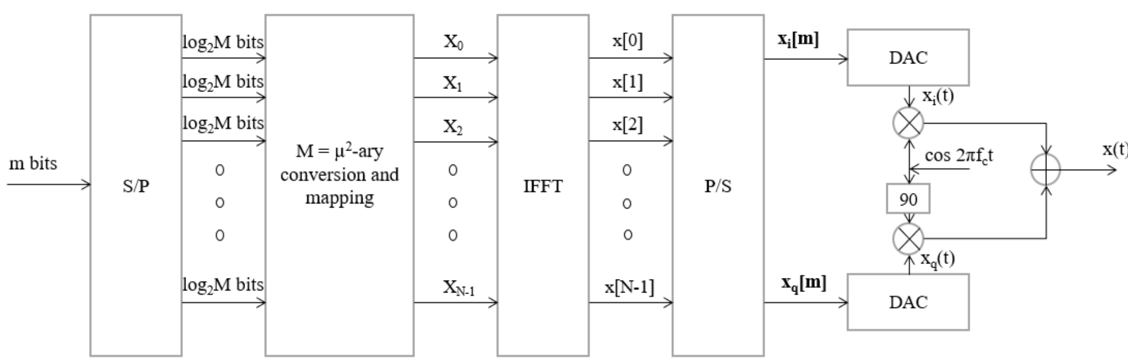


FIGURE 4.5: OFDM transmitter block diagram

In Figure (4.5) we have shown a OFDM transmitter diagram. Here total N number of subcarriers were used and each subcarrier will be modulated by a symbol. Each symbol will have log_2M bits. M will depend on the modulation scheme used. There will be total 2^{log_2M} number of symbol combinations. If m is the total number of bits transmitted per

OFDM frame then we can write,

$$m = Nlog_2M, \tag{4.11}$$

If we consider $N = 4$ and $4 - QAM(M = 4)$ modulation scheme for an OFDM system then total number of bits in one frame, $m = 4log_24 = 8$ bits. Each symbol will have $log_24 = 2$ bits and there will be total $2^2 = 4$ different combinations of symbol.

In an M-ary QAM OFDM system the M-ary is converted into μ-ary digit streams so that $M = \mu^2$. This results in decrease of symbol time and increase of symbol rate. In Figure 4.6, we have shown the bits to constellation point conversion and the constellation diagram for a 4-QAM OFDM system where $M = 4$.

Grey Code	Constellation Point
00	-1-j1
01	-1+j1
11	1+j1
10	1-j1

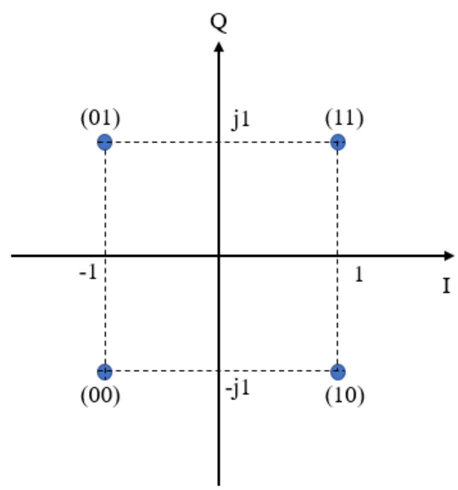


FIGURE 4.6: Bit to symbol conversion and constellation diagram for $M = 4$-QAM ODFM system

Here, grey coding is used to facilitate the error detection process on the receiver side. It is because due to grey coding adjacent symbols and constellation points will differ by only one bit. So, it reduces the error probability.

Grey Code	Constellation Point
0	-1
1	1

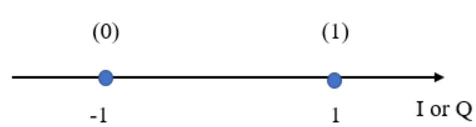


FIGURE 4.7: Bit to symbol conversion and constellation diagram for $\mu = 2$ (BPSK) for an ODFM system

Now if the system is converted into $\mu = \sqrt{M}$-ary the constellation diagram will be similar to BPSK as shown in Figure 4.7. If we consider an incoming bit stream 00011010 then how the bits will be split without using μ-ary conversion and using μ-ary conversion is shown in Figure 4.8.

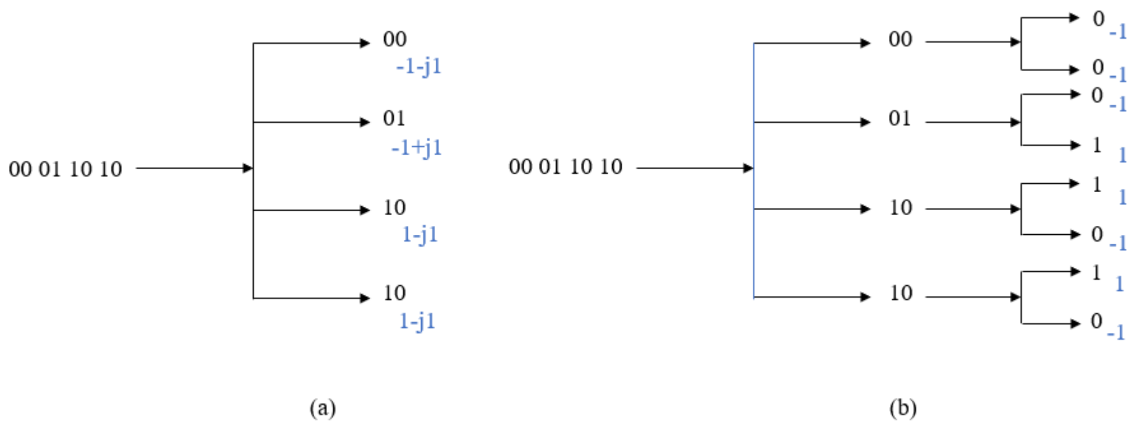


FIGURE 4.8: Bit splitting in OFDM system (a) without using μ-ary conversion (b)using μ-ary conversion

If we consider the outputs after μ-ary conversion of an M-QAM symbol as I_l and Q_l for in-phase and quadrature components of the M-QAM symbol accordingly then we can express them as,

$$
\begin{aligned}
I_l &= a_{2l}, \\
Q_l &= a_{2l+1},
\end{aligned}
\tag{4.12}
$$

where $l = 0, 1, \ldots, (N-1)$ denotes the subcarrier number that will be modulated by the symbol. After μ-ary conversion outputs are denoted as a_h such that $a_h \in \{\pm 1, \pm 3, \ldots, \pm(\mu - 1)\}$ where $h = 0, 1, \ldots, (\mu N - 1)$. For the above example we can write,

$$
\begin{aligned}
I_0 &= a_0 = -1 & \qquad Q_0 &= a_1 = -1 \\
I_1 &= a_2 = -1 & \qquad Q_1 &= a_3 = 1 \\
I_2 &= a_4 = 1 & \qquad Q_2 &= a_5 = -1 \\
I_3 &= a_6 = 1 & \qquad Q_3 &= a_7 = -1
\end{aligned}
$$

After the subcarriers are modulated with the constellation symbols the data symbols can be represented as,

$$X_l = I_l + jQ_l, \tag{4.13}$$

After that the IFFT operation is performed and data symbols are converted from frequency domain to time domain. This is called the OFDM baseband signal. So, at a sample time m the discrete time OFDM baseband signal is expressed as,

$$x[m] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}, \qquad m = 0, 1, \ldots, N-1, \tag{4.14}$$

If we now separate the discrete time baseband OFDM signal into real and imaginary components and denote them as $x_i[m]$ and $x_q[m]$ then we can write,

$$\begin{aligned} x_i[m] = \Re\{x[m]\} &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} I_l cos\Big(\frac{2\pi lm}{N}\Big) \\ x_q[m] = \Im\{x[m]\} &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} Q_l sin\Big(\frac{2\pi lm}{N}\Big) \end{aligned} \tag{4.15}$$

When $x_i[m]$ and $x_q[m]$ is passed through the digital to analog converter, the continuous time outputs are represented as $x_i(t)$ and $x_q(t)$ correspondingly. According to [24], $x_i(t)$ and $x_q(t)$ can be written as,

$$\begin{aligned} x_i(t) &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} |X_l| cos\Bigg(2\pi \frac{l-(N-1)/2}{T} t + argX_l\Bigg) \\ x_q(t) &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} |X_l| sin\Bigg(2\pi \frac{l-(N-1)/2}{T} t + argX_l\Bigg) \end{aligned} \tag{4.16}$$

Then the complex continuous time baseband OFDM signal can be defined over time interval $t \in [0, T]$ can be expressed as,

$$\begin{aligned} \tilde{x}(t) &= x_i(t) + jx_q(t) \\ &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{j2\pi \frac{l-(N-1)/2}{T} t} \end{aligned} \tag{4.17}$$

Here, the cyclic prefix addition and pulse shaping filters are not included as for high number of subcarriers pulse shaping is not required [83]. This is because of the Power Spectral Density (PSD) of OFDM bandpass signal is almost rectangular in shape. The baseband signal $\tilde{x}(t)$ is up-converted to a high frequency f_c to transmit the signal in high frequency RF channel. We can express the high frequency continuous time bandpass signal as,

$$x(t) = \Re\{\tilde{x}(t)e^{j2\pi f_c t}\}, \tag{4.18}$$

For most of the wireless applications $f_c >> \frac{1}{T}$. So from (4.18) we can see the term $e^{j2\pi f_c t}$ will be negligible and thus the RF bandpass signals will be almost equivalent to the complex baseband signals [24]. So, the PAPR of the baseband OFDM signals are mainly considered.

Now, I_l and Q_l from (4.12) depends on source digits. Source digits are independent and random. So, I_l and Q_l will be independent and identically distributed random variables with mean zero and variance $\frac{\mu^2-1}{3}$. From (4.15) we can say $x_i[m]$ and $x_q[m]$ are the summation of N Independent-Identically Distributed (IID) random variables with zero mean and variance $\frac{\mu^2-1}{3}$. If N is very high then $x_i[m]$ and $x_q[m]$ will become Gaussian random variables with mean zero and variance $\frac{\mu^2-1}{3}$ according to Central Limit Theorem (CLT). According to [24], samples for $x_i[m]$ and $x_q[m]$ are uncorrelated to each other,

$$E\big[x_i[m]x_i[o]\big] = E\big[x_q[m]x_q[o]\big] = \begin{cases} \frac{\mu^2-1}{3}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases} \tag{4.19}$$

As we know uncorrelated Gaussian random variables are independent to each other. So, $x_i[m]$ and $x_q[m]$ are independent Gaussian random variables. As, we can write $x[m] = x_i[m] + jx_q[m]$ so now we will find the correlation between the samples of $x[m]$,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[(x_i[m] + jx_q[m])(x_i[o] - jx_q[o])\big] \\ &= E\big[x_i[m]x_i[o] - jx_i[m]x_q[o] + jx_i[o]x_q[m] + x_q[m]x_q[o]\big] \\ &= E\big[x_i[m]x_i[o]\big] - jE\big[x_i[m]x_q[o]\big] + jE\big[x_i[o]x_q[m]\big] + E\big[x_q[m]x_q[o]\big] \\ &= E\big[x_i[m]x_i[o]\big] + E\big[x_q[m]x_q[o]\big] \end{aligned}$$

Now, if we consider $m = o$ then,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[x_i[m]^2\big] + E\big[x_q[m]^2\big] \\ &= 0 + \frac{\mu^2 - 1}{3} + 0 + \frac{\mu^2 - 1}{3} \\ &= \frac{2(\mu^2 - 1)}{3} \end{aligned}$$

And if we consider $m \neq o$ then,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[x_i[m]\big]E\big[x_i[o]\big] + E\big[x_q[m]\big]E\big[x_q[o]\big] \\ &= 0 \end{aligned}$$

Thus the samples of $x[m]$ are mutually uncorrelated and statistically independent,

$$E\big[x[m]x[o]^*\big] = \begin{cases} \frac{2(\mu^2-1)}{3}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases} \tag{4.20}$$

We know from Chapter 3, the average power for a M-QAM OFDM baseband signal can be written as,

$$P_{avg} = \frac{2(M-1)}{3} \tag{4.21}$$

For a sample $x[m]$ we can find the peak to average power ratio using the formula,

$$PAPR_{x[m]} = \frac{|x[m]|^2}{P_{avg}} \tag{4.22}$$

Here, $|x[m]|^2$ can be written as,

$$|x[m]|^2 = x_i[m]^2 + x_q[m]^2,$$

Putting the value into (4.22) we get,

$$PAPR_{x[m]} = \frac{x_i[m]^2 + x_q[m]^2}{P_{avg}} \tag{4.23}$$

Here, $x_i[m]$ and $x_q[m]$ are uncorrelated and independent Gaussian random variables with mean zero and variance $\frac{\mu^2-1}{3}$. So, $PAPR_{x[m]}$ is a random variable which will have chi-square distribution and the PDF of random variable $PAPR_{x[m]}$ can be written as,

$$f_{PAPR_{x[m]}}(\rho) = e^{-\rho} \tag{4.24}$$

So, the CDF of the random variable $PAPR_{x[m]}$ can be represented as the probability of the random variable being less than a specific threshold ρ,

$$F_{PAPR_{x[m]}}(\rho) = P(PAPR_{x[m]} < \rho) \tag{4.25}$$

Now to find the value of $P(PAPR_{x[m]} < \rho)$ we will integrate $f_{PAPR_{x[m]}}(\rho)$ for a limit from 0 to ρ which will give us the CDF of the random variables,

$$F_{PAPR_{x[m]}}(\rho) = 1 - e^{-\rho} \tag{4.26}$$

As the samples are mutually uncorrelated and independent so the CDF of the PAPR of baseband OFDM signal can be written as,

$$\begin{aligned} CDF(\rho) &= P\Big(\max_{0 \le m < N-1} PAPR_{x[m]} < \rho\Big) \\ &= P(PAPR_{x[0]} < \rho)P(PAPR_{x[1]} < \rho)\dots P(PAPR_{x[N-1]} < \rho) \\ &= \big(1 - e^{-\rho}\big)^N \end{aligned}$$

So, finally using (4.8) we find the CCDF of the PAPR of baseband OFDM signal,

$$CCDF(\rho) = 1 - \big(1 - e^{-\rho}\big)^N \tag{4.27}$$

4.4 PAPR distribution of OFDM-IM system

Now we will determine the statistical distribution of PAPR of OFDM-IM system. In case of OFDM-IM system the information bits in a frame are used not only for subcarrier modulation purpose but also for subcarrier index selection purpose.

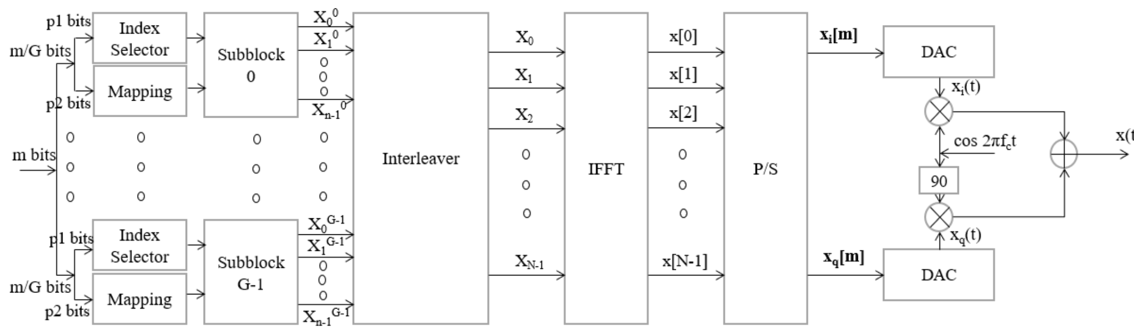


FIGURE 4.9: OFDM-IM transmitter block diagram

Only the transmitter part of the OFDM-IM system have been shown in Figure 4.9. Here, total m bits are transmitted per OFDM-IM frame. Those m bits are divided into G equal parts. Here, G is the total number of subblock. $p = \frac{m}{G}$ bits will enter into into each subblock. Now, in each subblock the p bits entered are divided into two parts $p1$ and $p2$. $p1$ bits are called index bits. Here, $p1 = \left\lfloor log_2\binom{n}{k} \right\rfloor$ bits are used for the selection of the k active subcarriers among n subcarriers in a subblock. On the other hand, mapping bits $p2 = klog_2M$ will be used to modulate the active subcarriers in that subblock. The total number of bits entering into a subblock is, $p = p1 + p2$. So, the total number of bits transmitted per OFDM-IM frame,

$$\begin{aligned} m &= pG \\ &= \left(\left\lfloor log_2\binom{n}{k} \right\rfloor + klog_2M \right) G \end{aligned} \tag{4.28}$$

Here, the $p1$ are termed as ISB. There will be total 2^{p1} different combination of ISB termed as ISB_α where, $\alpha = 1, 2, \ldots, 2^{p1}$. There will be total $\binom{n}{k}$ SAP combinations and among them any 2^{p1} combinations will be associated with ISB. Thus a look up table is formed. The look up table will be available on both the transmitter and receiver side.

Now we take a 4-QAM ($M = 4$) OFDM-IM system with total number of subcarriers $N = 16$, total number of subblocks $G = 4$, total number of subcarriers in a subblock $n = 4$ and total number of active subcarriers in each subblock $k = 2$. For this system total number of index bits, $p1 = \left\lfloor log_2 \binom{n}{k} \right\rfloor = \left\lfloor log_2 \binom{4}{2} \right\rfloor = 2$ bits in a subblock. Also, total number of mapping bits, $p2 = klog_2 M = 2log_2 4 = 4$ bits in a subblock. Total number of bits that will enter into a subblock, $p = p1 + p2 = 6$ bits. So total number of bits transmitted per OFDM-IM frame in this system will be $m = pG = 24$bits.

First, to form the look up table we will associate each of the ISB with a specific SAP combination. There will be total $2^{p1} = 4$ different combinations of ISB. Now associating the ISB with the SAP the look-up table for this system if formed as shown in Table 4.1.

α	ISB_α	SAP
1	00	1001
2	01	1010
3	11	0011
4	10	0110

TABLE 4.1: Look-up table for OFDM-IM system when $n = 4$ and $k = 2$

The SAP basically denotes which of the subcarriers in a subblock will be active. As there are $n = 4$ subcarriers in a subblock so each SAP combination will have 4 bits consisting 1s and 0s. If a subcarrier is associated with value 1 then the subcarrier is active and if the subcarrier is associated with value 0 then the subcarrier is inactive. The mapping bits will now determine the constellation symbols by which the active subcarriers are going to be modulated. As we are using 4-QAM modulation scheme, 2 bits will be in each constellation symbol and there will be total $2^2 = 4$ constellation points. If we denote constellation symbols as S_i, where $i = 0, 1, \ldots, M-1$. Here, Table 4.2 shows the constellation symbol combinations for this example and the constellation diagram of Figure 4.6 will also be applied in this case.

S_i	Grey Code	Constellation Point
S_0	00	-1-j1
S_1	01	-1+j1
S_2	11	1+j1
S_3	10	1-j1

TABLE 4.2: Mapping table for M = 4-QAM OFDM-IM system

Similar to OFDM system the $\mu = \sqrt{M}$-ary conversion is done. And then after that the active subcarriers in a subblock are modulated. The the data symbols after μ-ary modulation of a subcarriers in-phase and quadrature components are represented as I_d^g and Q_d^g where d denotes the subcarrier index in a subblock and g denotes the subblock index. So, the output data symbols of a subblock would be X_d^g such that $X_d^g = I_d^g + jQ_d^g$. I_d^g and Q_d^g both will be assigned 0 values if the d-th subcarrier in g-th subblock is inactive.

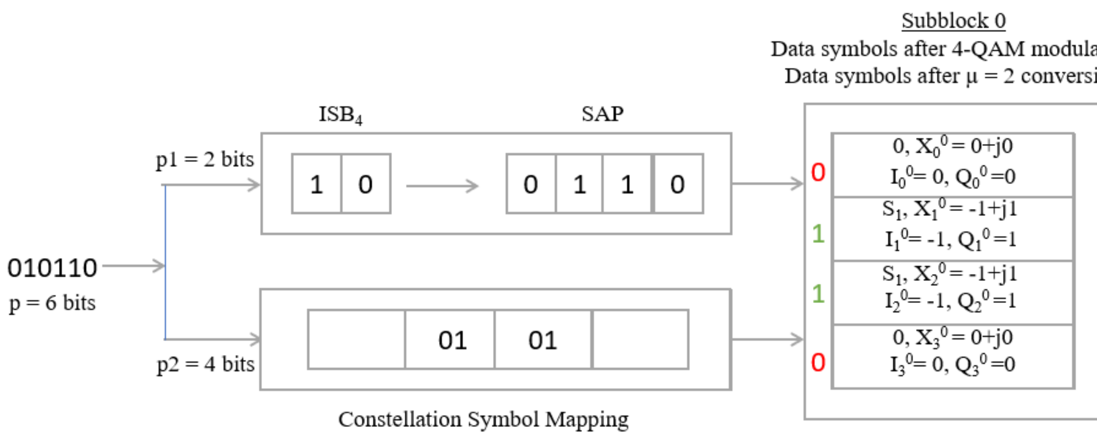


FIGURE 4.10: Bits splitting and subcarrier modulation in $M = 4$-QAM OFDM-IM system

Here in Figure 4.10 we have shown how bits are transmitted and subcarriers modulation is done for a specific subblock, subblock 0 ($g = 0$). According to our system configuration $p = 6$ bits will enter into each subblock. If we consider an arbitrary bit stream 010110, then $p1 = 2$ bits and $p2 = 4$ bits will be separated for indexing and mapping purpose correspondingly. From Table 4.1 we can see 10 bit stream denotes ISB_4 and SAP associated with this is 0110. So the $n = 0$ and $n = 3$ subcarriers in subblock 0 will be inactive and $n = 1$ and $n = 2$ will be active. Inactive subcarriers will not be modulated with constellation symbols and thus will be assigned 0 values such as $X_0^0 = X_3^0 = 0 + j0$. Now the active subcarriers will be modulated with constellation symbol, S_1 from Table 4.2. So, $X_1^0 = X_2^0 = -1 + j1$ are the data symbols after the subcarrier modulation and I_d^g and Q_d^g will be assigned to each active subcarrier from $\mu = \sqrt{M}$-ary conversion. In this process we will get data symbols, X_d^g for all the subblocks.

After that from Figure 4.9 we can see the interleaving is performed. After interleaving is performed the set of OFDM-IM data symbols are represented as,

$$\begin{aligned}\mathbf{X} &= [X_0 \quad X_1 \ldots X_{N-1}] \\ &= [X_0^0 \ldots X_0^{G-1} \ldots X_{n-1}^0 \ldots X_{n-1}^{G-1}]\end{aligned} \tag{4.29}$$

This can be generalised using the expression,

$$X_{d.G+g} = X_d^g, \tag{4.30}$$

where $d = 0, 1, \ldots, n-1$ and $g = 0, 1, \ldots, G-1$. For example if we consider, $d = 0$ and $g = 0$ then after the interleaving the data symbol will be expressed by $X_{0.4+0} = X_0 = X_0^0$ and if we change $g = 1$ and keep $d = 0$ same then the data symbol after the interleaving is expressed by $X_{0.4+1} = X_1 = X_0^1$ it will denote the second element of the data symbol set after interleaving. For our example we considered total number of subblocks, $G = 4$. Now, to find the last element if we consider $d = n-1$ and $g = G-1$ then from (4.30) we can find $X_{(n-1)G+(G-1)} = X_{nG-G+G-1} = X_{nG-1} = X_{N-1} = X_{n-1}^{G-1}$. Here, nG is equal to the total number of subcarriers in the system N. According to [84] the possible N indices of OFDM-IM subcarriers which will be mapped by the elements of g^th subblock are $\{0G+g, 1G+g, \ldots, (n-1)G+g\}$. In Figure 4.11 we have shown for our example how the interleaving is done and how the data symbols X_l are formed from X_d^g symbols.

We have denoted all the active subcarriers in a subblock with green color and all the inactive subcarriers with red color. Based on transmitted bits the status of the subcarriers (active/inactive) is decided. Assuming two active subcarriers and two inactive subcarriers in all the other subblocks the subblock data symbols are interleaved into OFDM-IM block. The table in Figure 4.11 shows the sets of indices of the OFDM-IM block that will be mapped by the corresponding subblock elements. And in the diagram we have shown clearly that after successful mapping there will be total $kG = 8$ active subcarriers and $(n-k)G = 8$ inactive subcarriers in the OFDM-IM block for this example.

The data symbols of the OFDM-IM block are represented as X_l. And from (4.13) we can see $X_l = I_l + jQ_l$. Here I_l and Q_l can be found using $I_{l=d.G+g} = I_d^g$ and $Q_{l=d.G+G} = Q_d^g$ these expressions, where $d = 0, 1, \ldots, n-1$ and $g = 0, 1, \ldots, G-1$.

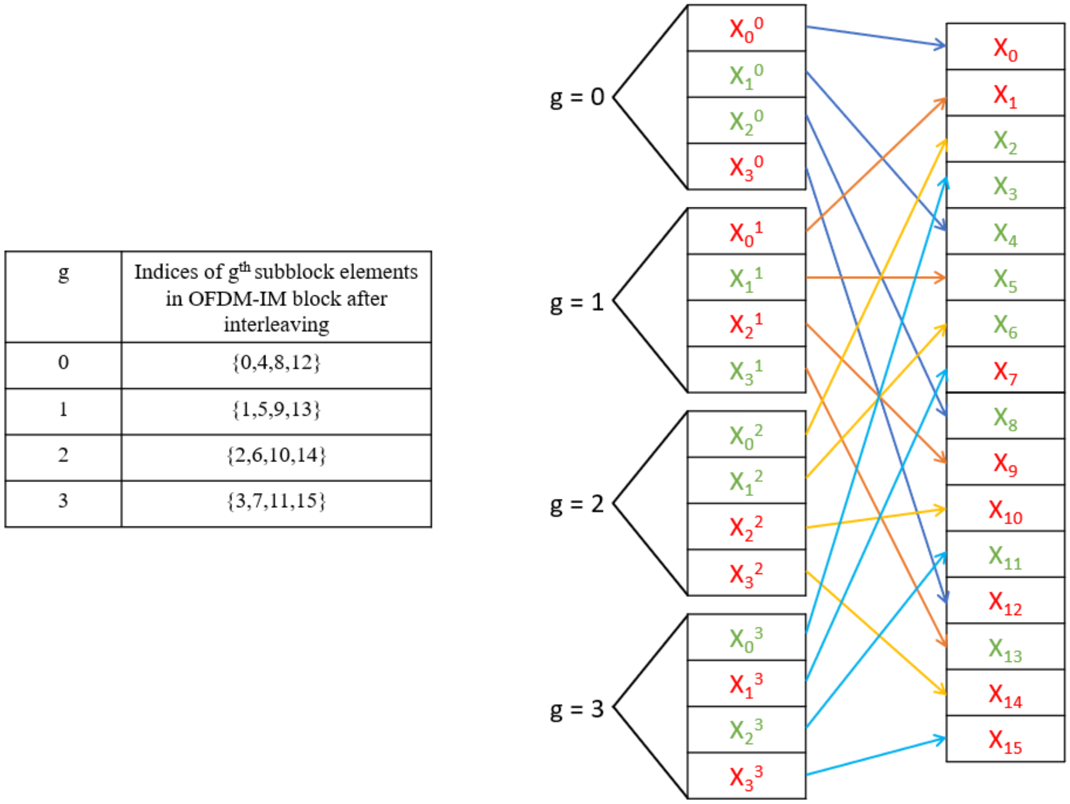


FIGURE 4.11: Interleaving of data symbols from subblock to OFDM-IM block when $n = 4$, $k = 2$, $G = 4$ and $N = 16$

We will denote the set of indices of active subcarriers in an OFDM-IM block as Υ. Now for our example, $\Upsilon = \{2, 3, 4, 5, 6, 8, 11, 13\}$. For, each subblock the indices of the active subcarriers can be termed as the g^th subset of Υ and it can be represented as,

$$\Upsilon^g = \{0.G + g, 1.G + g, \ldots, (n-1).G + g\} \cap \Upsilon \tag{4.31}$$

For our example the set of indices of active subcarriers for each subblock is shown in Table 4.3.

g	Indices of g^{th} subblock elements in OFDM-IM block	Υ^g
0	$\{0, 4, 8, 12\}$	$\{4, 8\}$
1	$\{1, 5, 9, 13\}$	$\{5, 13\}$
2	$\{2, 6, 10, 14\}$	$\{2, 6\}$
3	$\{3, 7, 11, 15\}$	$\{3, 11\}$

TABLE 4.3: Set of indices of active subcarriers in a subblock for OFDM-IM system when $n = 4$, $k = 2$, $G = 4$ and $N = 16$

For OFDM-IM system the length of $|\Upsilon|$ determines the total number of active subcarriers in the OFDM-IM block so that, $|\Upsilon| = kG = N_a$. Alternatively the set of indices of active subcarriers in an OFDM-IM block can be expressed as,

$$\Upsilon = \cup_{g=0,1,\ldots,G-1}\Upsilon^{g} \tag{4.32}$$

After the $\mu = \sqrt{M}$-ary conversion the in-phase of the data symbol I_l can be expressed as, $I_l \in \{\pm 1, \pm 3, \ldots, \pm(\mu - 1)\}$ when, $l \in \Upsilon$. Otherwise I_l will be assigned with 0 value. As, I_l depends on bit streams and bit streams are random. So, I_l is a random variable. Now, we will determine the expected value and variance of random variable I_l.

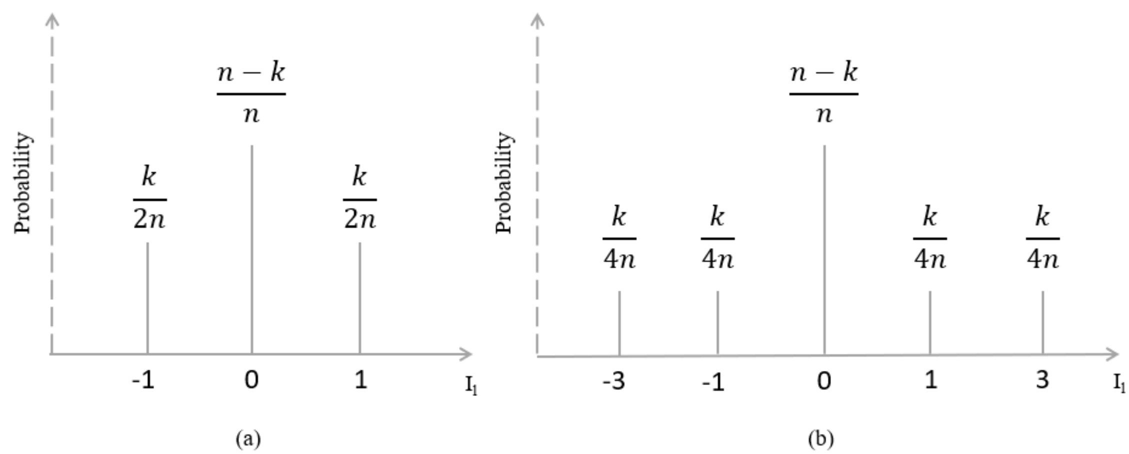


FIGURE 4.12: PMF distribution of discrete random variable I_l for (a) $\mu = 2$, (b) $\mu = 4$

In Figure 4.12 the PMF of I_l is shown. In the first diagram $\mu = 2$ modulation have been done. I_l will be assigned zero values when $l \in \bar{\Upsilon}$ which means the subcarrier is inactive. Now if $l \in \Upsilon$ which means the subcarrier is active and I_l will be assigned any of the constellation symbols from μ-ary modulation. Now for $\mu = 2$, I_l can have two values -1 and 1. The probability of a subcarrier of being active is, $\frac{k}{n}$. So, the probability of a subcarrier of being inactive is, $\frac{n-k}{n}$. Now if the subcarrier is active then we consider I_l equiprobable to get any value of $\{-1, 1\}$ and the probability is given as $\frac{k}{2n}$.

So, the expected value or mean of I_l in this case,

$$\begin{aligned} E[I_l] &= -1 \times \frac{k}{2n} + 0 \times \frac{n-k}{n} + 1 \times \frac{k}{2n} \\ &= 0 \end{aligned}$$

And the variance of I_l in this case,

$$\begin{aligned} Var[I_l] &= E[I_l^2] \\ &= (-1)^2 \times \frac{k}{2n} + 0^2 \times \frac{n-k}{n} + (1)^2 \times \frac{k}{2n} \\ &= 2\frac{k}{2n} \\ &= \frac{k}{n} \end{aligned}$$

For another case in Figure 4.12(b) we considered $\mu = 4$ and if $l \in \Upsilon$ then we assume the equal probability for I_l to get any of the values from $\{-3, -1, 1, 3\}$ and the probability is given by $\frac{k}{4n}$.

In this case for $\mu = 4$ the expected value or mean of I_l,

$$\begin{aligned} E[I_l] &= (-3) \times \frac{k}{4n} + (-1) \times \frac{k}{4n} + 0 \times \frac{n-k}{n} + 1 \times \frac{k}{4n} + 3 \times \frac{k}{4n} \\ &= 0 \end{aligned}$$

And the variance of I_l in this case,

$$\begin{aligned} var[I_l] &= E[I_l^2] \\ &= (-3)^2 \times \frac{k}{4n} + (-1)^2 \times \frac{k}{4n} + 0^2 \times \frac{n-k}{n} + (1)^2 \times \frac{k}{4n} + (3)^2 \times \frac{k}{4n} \\ &= (9+1+1+9)\frac{k}{4n} \\ &= 20\frac{k}{4n} \\ &= 5\frac{k}{n} \end{aligned}$$

So, in general for any μ-ary modulation the mean of random variable I_l is zero and variance is $\frac{(\mu^2-1)}{3}\frac{k}{n}$. By putting the value $\mu = 2$ and $\mu = 4$ we can justify the formula.

For Q_l the PMF will be similar. So, any μ-ary modulation the mean of random variable Q_l is zero and variance is $\frac{(\mu^2-1)}{3}\frac{k}{n}$. Now the data symbols are passed for IFFT operation to convert the data symbols from frequency domain to time domain. This is called baseband OFDM-IM signal. From (4.14) at a sample time m the discrete time OFDM-IM baseband signal is expressed as,

$$x[m] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}, \qquad m = 0, 1, \ldots, N-1,$$

If we now separate the discrete time baseband OFDM-IM signal into real and imaginary components and denote them as $x_i[m]$ and $x_q[m]$ then we can write,

$$x_i[m] = \Re\{x[m]\} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} I_l cos\Big(\frac{2\pi lm}{N}\Big)$$
$$x_q[m] = \Im\{x[m]\} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} Q_l sin\Big(\frac{2\pi lm}{N}\Big)$$

These are similar to conventional OFDM system. The effect of inactive subcarriers would be less number of terms I_l and Q_l will be summed. For inactive subcarriers they will be assigned value zero. As discussed in previous section RF bandpass signals will be almost equivalent to the complex baseband signals. So, here we will mainly consider the PAPR of the discrete time baseband OFDM-IM signal.

Here, I_l and Q_l will be independent and identically distributed random variables with mean zero and variance $\frac{\mu^2-1}{3}\frac{k}{n}$. From (4.15) which is also applicable for OFDM-IM system we can say $x_i[m]$ and $x_q[m]$ are the summation of N IID random variables with zero mean and variance $\frac{\mu^2-1}{3}\frac{k}{n}$. If N is very high then the total number of active subcarriers $|\Upsilon| = N_a$ will be less than N but still high. Thus $x_i[m]$ and $x_q[m]$ will become Gaussian random variables.

Now, we will determine the mean and variance of $x_i[m]$ and $x_q[m]$. Mean of $x_i[m]$ can be expressed as,

$$
\begin{aligned}
E\big[x_i[m]\big] &= E\bigg[\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1} I_l cos\Big(\frac{2\pi lm}{N}\Big)\bigg] \\
&= \frac{1}{\sqrt{N}}E\bigg[\sum_{l=0}^{N-1} I_l\bigg] \\
&= \frac{1}{\sqrt{N}}N_a E[I_l] \\
&= 0
\end{aligned}
$$

Similarly mean of $x_q[m]$, $E\big[x_q[m]\big] = 0$. Considering the phase factor negligible we determine the variance of $x_i[m]$,

$$
\begin{aligned}
var[x_i[m]] &= var\bigg[\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1} I_l\bigg] \\
&= N_a var\bigg[\frac{I_l}{\sqrt{N}}\bigg] \\
&= N_a E\bigg[\Big(\frac{I_l}{\sqrt{N}} - \frac{E[I_l]}{\sqrt{N}}\Big)^2\bigg] \\
&= \frac{N_a}{N} var[I_l] \\
&= \frac{N_a}{N}\frac{k}{n}\frac{(\mu^2-1)}{3} \\
&= \frac{k^2}{n^2}\frac{(\mu^2-1)}{3}
\end{aligned}
$$

Similarly for $x_q[m]$ the variance is $\frac{k^2}{n^2}\frac{(\mu^2-1)}{3}$. For OFDM-IM system the samples for $x_i[m]$ and $x_q[m]$ are uncorrelated to each other,

$$
E\big[x_i[m]x_i[o]\big] = E\big[x_q[m]x_q[o]\big] = \begin{cases} \frac{\mu^2-1}{3}\frac{k^2}{n^2}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases} \tag{4.33}
$$

As we know uncorrelated Gaussian random variables are independent to each other. So, $x_i[m]$ and $x_q[m]$ are independent Gaussian random variables. As, we can write $x[m] =$

$x_i[m] + jx_q[m]$ so now we will find the correlation between the samples of $x[m]$,

$$\begin{aligned}
E\big[x[m]x[o]^*\big] &= E\big[(x_i[m] + jx_q[m])(x_i[o] - jx_q[o])\big] \\
&= E\big[x_i[m]x_i[o] - jx_i[m]x_q[o] + jx_i[o]x_q[m] + x_q[m]x_q[o]\big] \\
&= E\big[x_i[m]x_i[o]\big] - jE\big[x_i[m]x_q[o]\big] + jE\big[x_i[o]x_q[m]\big] + E\big[x_q[m]x_q[o]\big] \\
&= E\big[x_i[m]x_i[o]\big] + E\big[x_q[m]x_q[o]\big]
\end{aligned}$$

Now, if we consider $m = o$ then,

$$\begin{aligned}
E\big[x[m]x[o]^*\big] &= E\big[x_i[m]^2\big] + E\big[x_q[m]^2\big] \\
&= 0 + \frac{\mu^2 - 1}{3}\frac{k^2}{n^2} + 0 + \frac{\mu^2 - 1}{3}\frac{k^2}{n^2} \\
&= \frac{2(\mu^2 - 1)}{3}\frac{k^2}{n^2}
\end{aligned}$$

And if we consider $m \neq o$ then,

$$\begin{aligned}
E\big[x[m]x[o]^*\big] &= E\big[x_i[m]\big]E\big[x_i[o]\big] + E\big[x_q[m]\big]E\big[x_q[o]\big] \\
&= 0
\end{aligned}$$

Thus the samples of $x[m]$ are mutually uncorrelated and statistically independent,

$$E\big[x[m]x[o]^*\big] = \begin{cases} \frac{2(\mu^2-1)}{3}\frac{k^2}{n^2}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases} \tag{4.34}$$

We know from Chapter 3, the average power for a M-QAM OFDM-IM baseband signal can be written as,

$$P_{avg} = \frac{2(M-1)}{3}\frac{kG}{N} \tag{4.35}$$

For a sample $x[m]$ using (4.22) and (4.23) we can write,

$$PAPR_{x[m]} = \frac{x_i[m]^2 + x_q[m]^2}{P_{avg}} \tag{4.36}$$

We can rewrite (4.36) as,

$$PAPR_{x[m]} = ax_i[m]^2 + ax_q[m]^2, \tag{4.37}$$

where $a = \frac{1}{P_{avg}} = \frac{3N}{2(M-1)kG}$. Now, we can generalise (4.37) as a chi-square distribution expressed as,

$$Y = aX^2 \tag{4.38}$$

Here, X is Gaussian with mean zero and variance σ^2. Now we can write the PDF of Y of (4.38) as,

$$\begin{aligned}
f_Y(y) &= \frac{f_X\left[\sqrt{\frac{y}{a}}\right] + f_X\left[-\sqrt{\frac{y}{a}}\right]}{2a\sqrt{\frac{y}{a}}} \\
&= \frac{1}{2\sqrt{ya}}\left[f_X\left[\sqrt{\frac{y}{a}}\right] + f_X\left[-\sqrt{\frac{y}{a}}\right]\right] \\
&= \frac{1}{2\sqrt{ya}}\left[\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{y}{2\sigma^2 a}} + \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{y}{2\sigma^2 a}}\right] \\
&= \frac{1}{\sigma\sqrt{2\pi ya}}e^{-\frac{y}{2\sigma^2 a}}
\end{aligned}$$

Now the characteristic function for Y denoted by $\Psi_Y(j\omega)$ can be expressed as,

$$\Psi_Y(j\omega) = \int_{-\infty}^{\infty} e^{j\omega y} f_Y(y)\,dy$$

Putting the value of $f_Y(y)$ in this equation,

$$\begin{aligned}
\Psi_Y(j\omega) &= \int_{-\infty}^{\infty} e^{j\omega y}\frac{1}{\sigma\sqrt{2\pi ya}}e^{-\frac{y}{2\sigma^2 a}}\,dy \\
&= \frac{1}{(1 - j2\omega\sigma^2 a)^{\frac{1}{2}}}
\end{aligned}$$

If we take $X_1, X_2, \ldots, X_n$ statistically independent Gaussian random variables with zero mean and variance σ^2. Then, we can rewrite (4.38) as,

$$Y = a\sum_{i=1}^{n} X_i^2 \tag{4.39}$$

The characteristic function for (4.39) can be written as,

$$\Psi_Y(j\omega) = \frac{1}{(1 - j2\omega\sigma^2 a)^{\frac{n}{2}}}$$

And the PDF of Y in (4.39) can be expressed as,

$$f_Y(y) = \frac{1}{\sigma^n 2^{\frac{n}{2}} a^{\frac{n}{2}} \Gamma(\frac{1}{2}n)} y^{\frac{n}{2}-1} e^{-\frac{y}{2\sigma^2 a}}, \quad y \geq 0 \tag{4.40}$$

For OFDM-IM system $n = 2$ as $x_i[m]$ and $x_q[m]$ are statistically independent Gaussian random variables with zero mean and variance $\frac{\mu^2-1}{3}\frac{k^2}{n^2}$. For $n = 2$ we can rewrite (4.40) as,

$$f_Y(y) = \frac{1}{\sigma^2 2a} y^0 e^{-\frac{y}{2\sigma^2 a}}, \quad y \geq 0 \tag{4.41}$$

Here, $\Gamma(\frac{1}{2}n) = \Gamma(1) = 1$ and $\sigma^2 = \frac{\mu^2-1}{3}\frac{k^2}{n^2}$. Now putting the value of σ^2 and a into (4.41) we get,

$$\begin{aligned} f_Y(y) &= \frac{1}{2\frac{\mu^2-1}{3}\frac{k^2}{n^2}\frac{3N}{2(M-1)kG}} e^{-\frac{y}{2\frac{\mu^2-1}{3}\frac{k^2}{n^2}\frac{3N}{2(M-1)kG}}} \\ &= \frac{1}{\frac{k}{n}} e^{-\frac{y}{\frac{k}{n}}} \end{aligned}$$

Here, $N = nG$ and $\mu^2 = M$. So, $PAPR_{x[m]}$ is a random variable which will have chi-square distribution similar to random variable Y and the PDF of random variable $PAPR_{x[m]}$ can be written as similar to $f_Y(y)$,

$$f_{PAPR_{x[m]}}(\rho) = \frac{1}{\frac{k}{n}} e^{-\frac{y}{\frac{k}{n}}} \tag{4.42}$$

So, the CDF of the random variable $PAPR_{x[m]}$ can be represented as the probability of the random variable being less than a specific threshold ρ,

$$F_{PAPR_{x[m]}}(\rho) = P(PAPR_{x[m]} < \rho) \tag{4.43}$$

Now to find the value of $P(PAPR_{x[m]} < \rho)$ we will integrate $f_{PAPR_{x[m]}}(\rho)$ for a limit from 0 to ρ which will give us the CDF of the random variable,

$$\begin{aligned} F_{PAPR_{x[m]}}(\rho) &= \int_0^{\rho} \frac{1}{\frac{k}{n}} e^{-\frac{\rho}{\frac{k}{n}}}\, d\rho \\ &= \frac{1}{\frac{k}{n}} \int_0^{\rho} e^{-\frac{\rho}{\frac{k}{n}}}\, d\rho \\ &= \frac{1}{\frac{k}{n}} \big(-\frac{k}{n}\big) e^{-\frac{\rho}{\frac{k}{n}}} \Big|_0^{\rho} \\ &= -\big(e^{-\frac{\rho}{\frac{k}{n}}} - 1\big) \end{aligned}$$

So, the CDF of the random variable $PAPR_{x[m]}$,

$$F_{PAPR_{x[m]}}(\rho) = \big(1 - e^{-\frac{\rho}{\frac{k}{n}}}\big) \tag{4.44}$$

As the samples are mutually uncorrelated and independent so the CDF of the PAPR of baseband OFDM-IM signal can be written as,

$$\begin{aligned} CDF_{IM}(\rho) &= P\Big(\max_{0 \leq m < N-1} PAPR_{x[m]} < \rho\Big) \\ &= P(PAPR_{x[0]} < \rho)P(PAPR_{x[1]} < \rho)\ldots P(PAPR_{x[N-1]} < \rho) \\ &= \big(1 - e^{-\frac{\rho}{\frac{k}{n}}}\big)^N \end{aligned}$$

So, finally using (4.8) we find the CCDF of the PAPR of baseband OFDM-IM signal,

$$CCDF_{IM}(\rho) = 1 - \Big(1 - e^{-\frac{\rho}{\frac{k}{n}}}\Big)^N \tag{4.45}$$

Here, $\frac{k}{n}$ for OFDM-IM system is known as the subcarrier activation ratio.

4.5 PAPR distribution of OFDM-IQ-IM system

Now we will determine the statistical distribution of PAPR of OFDM-IQ-IM system. In case of OFDM-IQ-IM system the information bits in a frame are used for both subcarrier modulation purpose and subcarrier index selection purpose for both in-phase and quadrature channel separately.

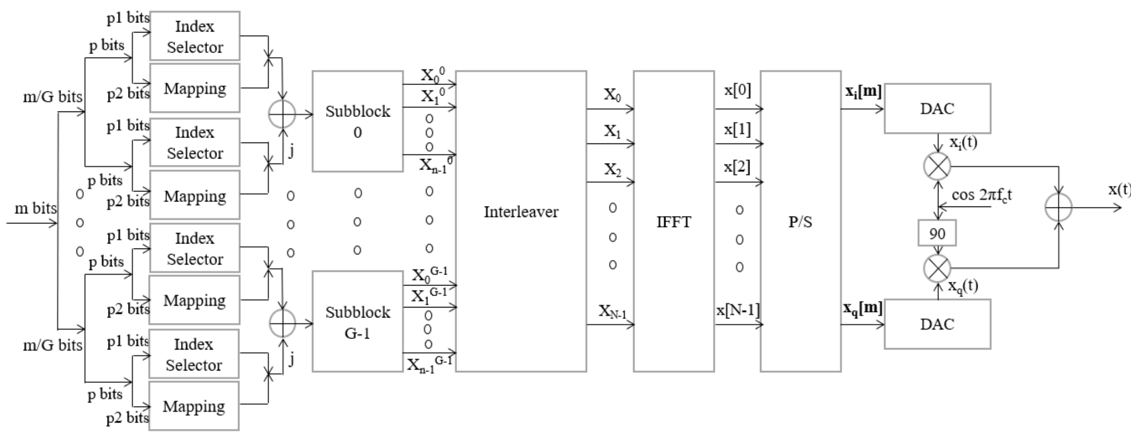


FIGURE 4.13: OFDM-IQ-IM transmitter block diagram

The transmitter of the OFDM-IQ-IM system is shown in Figure 4.13. Here, total m bits are transmitted per OFDM-IQ-IM frame. Those m bits are divided into G equal parts. Here, G is the total number of subblock. $2p = \frac{m}{G}$ bits will enter into into each subblock. Now, in each subblock $2p$ bits are divided into two equal parts for in-phase components and quadrature components. The p bits entering are then divided into two parts $p1$ and $p2$. $p1$ bits are called index bits. Here, $p1 = \left\lfloor log_2 \binom{n}{k} \right\rfloor$ bits are used for the selection of the k active subcarriers among n subcarriers for in-phase components and quadrature components independently in a subblock. On the other hand, mapping bits $p2 = klog_2\sqrt{M}$ will be used to modulate the active subcarriers for each component in that subblock with $\sqrt{M}$-PAM modulation. The total number of bits entering into a subblock is, $2p = 2(p1 + p2)$. So, the total number of bits transmitted per OFDM-IQ-IM frame,

$$\begin{aligned} m &= 2pG \\ &= \left(2\left\lfloor log_2 \binom{n}{k} \right\rfloor + 2klog_2\sqrt{M}\right)G \end{aligned} \tag{4.46}$$

Similar to OFDM-IM system here the $p1$ bits are termed as ISB. There will be total 2^{p1} different combination of ISB termed as ISB_α where, $\alpha = 1, 2, \ldots, 2^{p1}$. There will be total $\binom{n}{k}$ SAP combinations and among them any 2^{p1} combinations will be associated with ISB. Thus a look up table is formed. The look up table will be available on both the transmitter and receiver side.

In case of OFDM-IQ-IM system in-phase components and quadrature components of the subcarriers are modulated separately using $\sqrt{M}$-PAM digital modulation scheme. Here for our example we consider $\sqrt{M} = 4$-PAM digital modulation scheme for both in-phase and quadrature components of the subcarriers, total number of subcarriers $N = 16$, total number of subblocks $G = 4$, total number of subcarriers in a subblock $n = 4$ and total number of active subcarriers in each subblock $k = 2$. For this system total number of index bits for each of the in-phase and quadrature components of the subcarriers, $p1 = \left\lfloor log_2\binom{n}{k}\right\rfloor = \left\lfloor log_2\binom{4}{2}\right\rfloor =$ 2 bits in a subblock. Also, total number of mapping bits for each of the active in-phase components or quadrature components of the subcarriers, $p2 = klog_2\sqrt{M} = 2log_2 4 = 4$ bits in a subblock. Total number of bits that will enter into a subblock, $2p = 2p1 + 2p2 = 12$ bits. So total number of bits transmitted per OFDM-IQ-IM frame in this system will be $m = 2pG = 48$bits.

First, to form the look up table we will associate each of the ISB with a specific SAP combination. There will be total $2^{p1} = 4$ different combinations of ISB. Now associating the ISB with the SAP the look-up table for this system if formed as shown in Table 4.4.

α	ISB_α	SAP
1	00	1100
2	01	0101
3	11	1010
4	10	1001

Table 4.4: Look-up table for OFDM-IQ-IM system when $n = 4$ and $k = 2$

The SAP basically denotes which of the subcarrier components in a subblock will be active. As we already know for OFDM-IQ-IM system in-phase and quadrature components can be active independently so for each sub-block ISB to SAP mapping will be done separately for in-phase components and quadrature components of the subcarriers in that subblock. As there are $n = 4$ subcarriers in a subblock so each SAP combination will have

4 bits consisting of 1s and 0s. If an in-phase or quadrature component of a subcarrier is associated with value 1 then the subcarrier component is active and if associated with value 0 then the subcarrier component is inactive. The mapping bits will now determine the constellation symbols by which the active subcarrier components are going to be modulated. As we are using 4-PAM modulation scheme, 2 bits will be in each constellation symbol and there will be total $2^2 = 4$ constellation points. If we denote constellation symbols as S_i, where $i = 0, 1, \ldots, M-1$. Here, Table 4.5 shows the constellation symbol combinations for this example.

S_i	Grey Code	Constellation Point
S_0	00	-3
S_1	01	-1
S_2	11	1
S_3	10	3

TABLE 4.5: Mapping table for 4-PAM OFDM-IQ-IM system

And the constellation diagram used for our example,

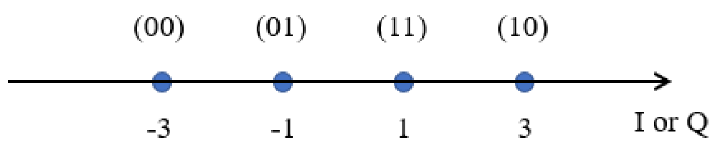


FIGURE 4.14: Constellation diagram for 4-PAM digital modulation scheme

In OFDM-IQ-IM system, the active subcarrier in-phase and quadrature components will be modulated independently with $\sqrt{M}$-PAM constellation symbol. And after the modulation in-phase and quadrature components can be denoted by I_d^g and Q_d^g where d denotes the subcarrier index in a subblock and g denotes the subblock index. So, the output data symbols of a subblock would be X_d^g such that $X_d^g = I_d^g + jQ_d^g$. In OFDM-IQ-IM both in-phase and quadrature components can be active or in-active independently.

Here in Figure 4.10 we have shown how bits are transmitted and subcarriers modulation is done for a specific subblock, subblock 0 ($g = 0$). According to our system configuration $p = 12$ bits will enter into each subblock. If we consider an arbitrary bit stream 100101010110, then the bits will be divided into two equal parts each consisting 6 bits for

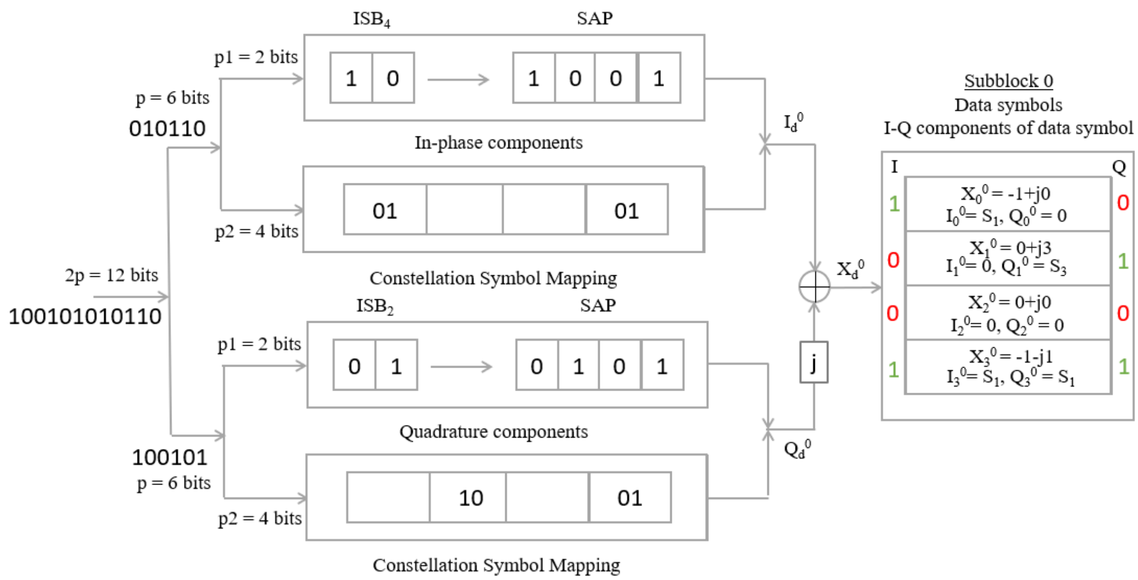


FIGURE 4.15: Bit splitting and subcarrier modulation in $\sqrt{M}$ = 4-PAM OFDM-IQ-IM system

in-phase and quadrature components. Then in each of that $p = 6$ bits will be divided into $p1 = 2$ bits and $p2 = 4$ bits for indexing and mapping purpose correspondingly. According to Table 4.4, for in-phase components ISB 10 will correspond to SAP 1001 and for quadrature components ISB 01 will correspond to SAP 0101. For the first subcarrier in subblock 0 in-phase component is active and will be modulated with S_1 constellation symbol represented by 01 from 4.5 but the quadrature component will be inactive so the data symbol results in $X_0^0 = -1 + j0$. For the second subcarrier in subblock 0 quadrature component is active and will be modulated with S_3 constellation symbol represented by 10 from 4.5 but the in-phase component will be inactive so the data symbol results in $X_1^0 = 0 + j3$. For the third subcarrier both the in-phase and quadrature component will be inactive and will be associated with zero values. This subcarrier will be an inactive subcarrier and the data symbol is expressed as $X_2^0 = 0 + j0$. For the forth subcarrier in the subblock both the subcarriers will be active and thus we know if we modulate in-phase and quadrature components of a subcarrier with $\sqrt{M}$-PAM modulation scheme that will represent a symbol from square M-QAM constellation diagram. Such as for this subcarrier the in-phase and quadrature component is modulated with 4-PAM constellation symbol S_1 thus the data symbol is represented by $-1 - j1$ which represents a symbol in the constellation diagram of 16-QAM. In this process we will get data symbols, X_d^g for all the subblocks.

After that interleaving is performed. After interleaving is performed the set of OFDM-IQ-IM data symbols are represented as,

$$\begin{aligned}\mathbf{X} &= [X_0 \quad X_1 \ldots X_{N-1}] \\ &= [X_0^0 \ldots X_0^{G-1} \ldots X_{n-1}^0 \ldots X_{n-1}^{G-1}]\end{aligned}$$

This can be generalised using the expression,

$$X_{d.G+g} = X_d^g, \tag{4.47}$$

where $d = 0, 1, \ldots, n-1$ and $g = 0, 1, \ldots, G-1$. The interleaving process for OFDM-IQ-IM system is similar to OFDM-IM system as described in previous section. But in OFDM-IQ-IM system the total number of active subcarriers in the OFDM-IQ-IM block is random. To show how after interleaving the final data symbols are found we have separated X_d^g into I_d^g and Q_d^g. We have used $I_{l=d.G+g} = I_d^g$, $Q_{l=d.G+g} = Q_d^g$ and $X_l = I_l + jQ_l$ to show the interleaving properly.

We have denoted all the active subcarrier components in a subblock with green color and all the inactive subcarrier components with red color. Based on transmitted bits the status of the subcarrier components (active/inactive) is decided. Assuming two active in-phase and quadrature subcarrier components and two inactive in-phase and quadrature subcarrier components subcarriers in all the other subblocks the subblock data symbols are interleaved into OFDM-IQ-IM block. The Figure 4.16 shows that after successful mapping the total number of active subcarriers for OFDM-IQ-IM system is not constant like OFDM-IM system, it can be any random value depending on the bits transmitted.

We will denote the set of indices of active subcarriers in an OFDM-IQ-IM block as Υ. Now for our example, $\Upsilon = \{0, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14\}$. For, each subblock the indices of the active subcarriers can be termed as the g^th subset of Υ and it can be represented as,

$$\Upsilon^g = \{0.G + g, 1.G + g, \ldots, (n-1).G + g\} \cap \Upsilon \tag{4.48}$$

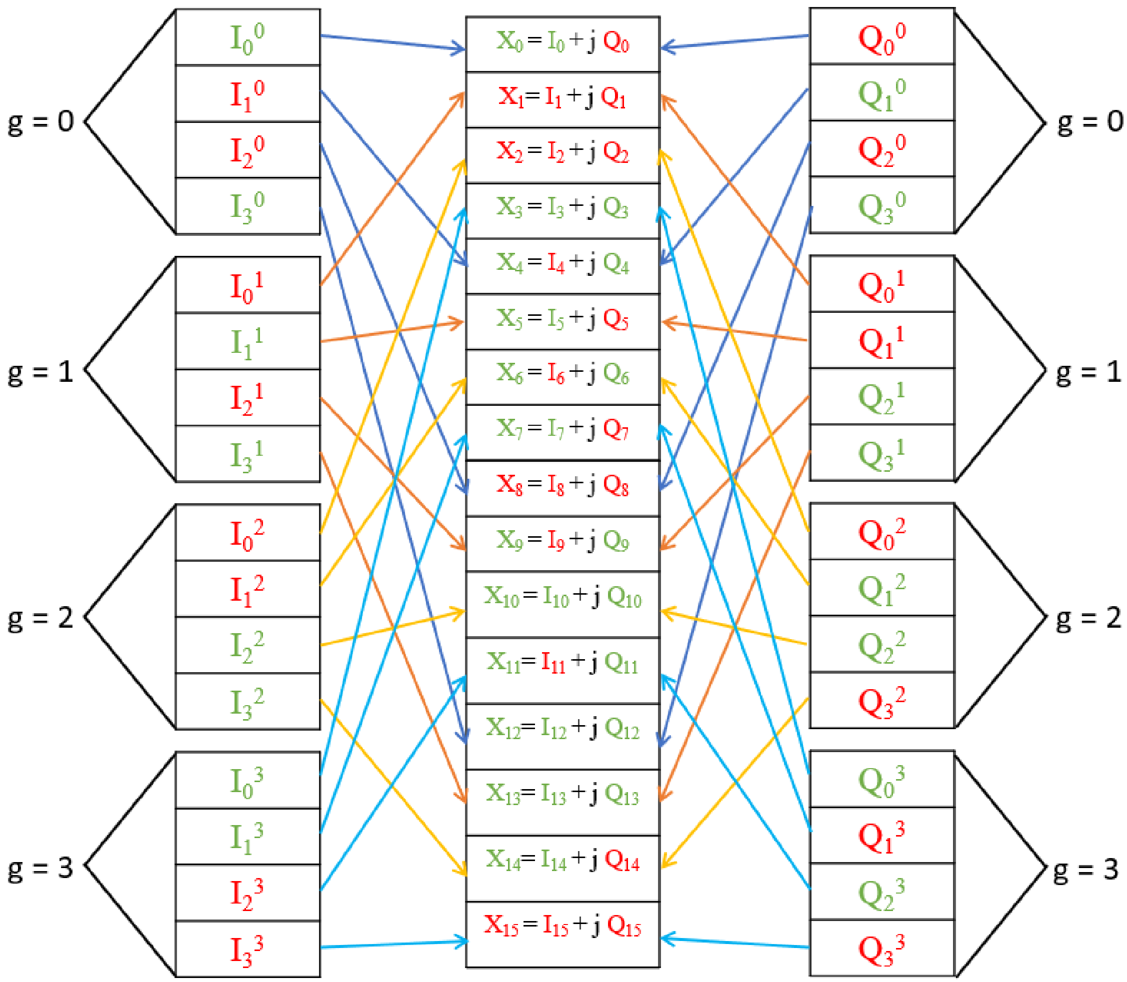


FIGURE 4.16: Interleaving of data symbols from subblock to OFDM-IQ-IM block when $n = 4$, $k = 2$, $G = 4$ and $N = 16$

Also for each subblock the indices of the active subcarriers, $\Upsilon^g = \Upsilon_i^g \cup \Upsilon_q^g$ where Υ_i^g is the set of active in-phase components of subcarriers in $g^t h$ subblock and Υ_q^g is the set of active quadrature components of subcarriers in $g^t h$ subblock. For our example the set of indices of active subcarriers for each subblock is shown in Table 4.6.

g	Indices of g^{th} subblock elements in OFDM-IM block	Υ_i^g	Υ_q^g	Υ^g
0	$\{0, 4, 8, 12\}$	$\{0, 12\}$	$\{4, 12\}$	$\{0, 4, 12\}$
1	$\{1, 5, 9, 13\}$	$\{5, 13\}$	$\{9, 13\}$	$\{5, 9, 13\}$
2	$\{2, 6, 10, 14\}$	$\{10, 14\}$	$\{6, 10\}$	$\{6, 10, 14\}$
3	$\{3, 7, 11, 15\}$	$\{3, 7\}$	$\{3, 11\}$	$\{3, 7, 11\}$

TABLE 4.6: Set of indices of active subcarriers in a subblock for OFDM-IQ-IM system when $n = 4$, $k = 2$, $G = 4$ and $N = 16$

For OFDM-IQ-IM system the length of $|\Upsilon|$ determines the total number of active subcarriers in the OFDM-IQ-IM block. Alternatively the set of indices of active subcarriers in an OFDM-IQ-IM block, $\Upsilon = \cup_{g=0,1,\ldots,G-1}\Upsilon^g$ similar to that of OFDM-IM in (4.32).

The in-phase component of the data symbol I_l is achieved after $\sqrt{M}$-PAM modulation so it can be expressed as, $I_l \in \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\}$ when, $l \in \Upsilon_i$. Here, $\Upsilon_i = \cup_{g=0,1,\ldots,G-1}\Upsilon_i^g$ Otherwise I_l will be assigned with 0 value. As, I_l depends on bit streams and bit streams are random. So, I_l is a random variable. Now, we will determine the expected value and variance of random variable I_l.

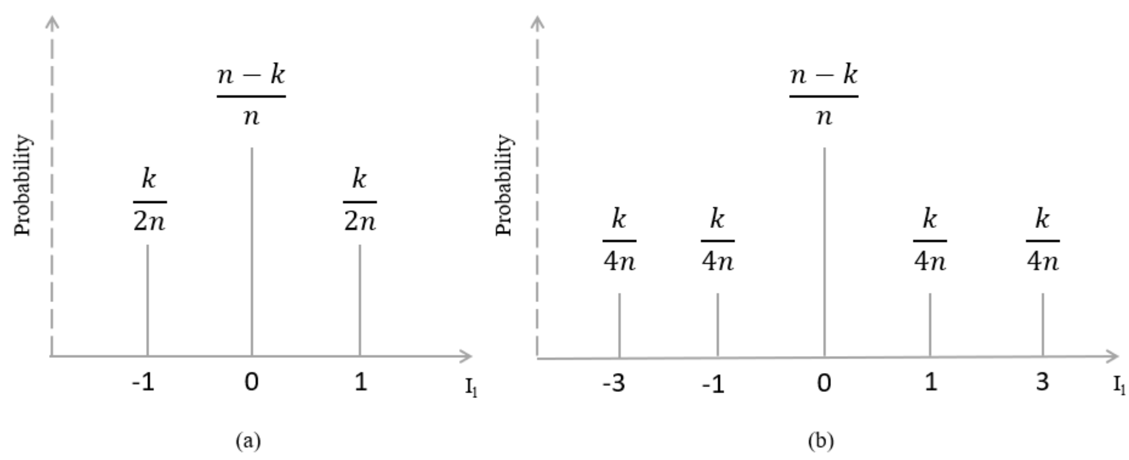


FIGURE 4.17: PMF distribution of discrete random variable I_l for (a) 2-PAM modulation, (b) 4-PAM modulation

In Figure 4.17 the PMF of I_l is shown. In the first diagram $\sqrt{M} = 2$ modulation have been done. I_l will be assigned zero values when $l \in \bar{\Upsilon}_i$ which means the in-phase component of the subcarrier is inactive. Now if $l \in \Upsilon_i$ which means the subcarrier is active and I_l will be assigned any of the constellation symbols from $\sqrt{M}$-PAM modulation. Now for $\sqrt{M} = 2$, I_l can have two values -1 and 1. The probability of an in-phase component of a subcarrier being active is, $\frac{k}{n}$. So, the probability of an in-phase component of a subcarrier of being inactive is, $\frac{n-k}{n}$. Now if the in-phase component of l^th subcarrier is active then we consider I_l equiprobable to get any value from $\{-1, 1\}$ and the probability is given as $\frac{k}{2n}$.For another case in Figure 4.17(b) we considered $\sqrt{M} = 4$ and if $l \in \Upsilon_i$ then we assume the equal probability for I_l to get any of the values from $\{-3, -1, 1, 3\}$ and the probability is given by $\frac{k}{4n}$. For both the case the expected value and variance have been evaluated in previous section.

So, in general for any $\sqrt{M}$-PAM modulation the mean of random variable I_l is zero and variance is $\frac{(M-1)}{3}\frac{k}{n}$. By putting the value $\sqrt{M} = 2$ and $\sqrt{M} = 4$ we can justify the formula. For Q_l the PMF will be similar. So, any $\sqrt{M}$-PAM modulation the mean of random variable Q_l is zero and variance is $\frac{(M-1)}{3}\frac{k}{n}$. Now the data symbols are passed for IFFT operation to convert the data symbols from frequency domain to time domain. This is called baseband OFDM-IQ-IM signal. From (4.14) at a sample time m the discrete time OFDM-IM baseband signal is expressed as,

$$x[m] = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} X_l e^{\frac{j2\pi lm}{N}}, \qquad m = 0, 1, \ldots, N-1,$$

If we now separate the discrete time baseband OFDM-IQ-IM signal into real and imaginary components and denote them as $x_i[m]$ and $x_q[m]$ then we can write,

$$x_i[m] = \Re\{x[m]\} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} I_l cos\Big(\frac{2\pi lm}{N}\Big)$$
$$x_q[m] = \Im\{x[m]\} = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} Q_l sin\Big(\frac{2\pi lm}{N}\Big)$$

These are similar to conventional OFDM system. The effect of inactive subcarriers would be less number of terms I_l and Q_l will be summed. If for any value of l the component is inactive then they will be assigned value zero. As mentioned in previous sections RF bandpass signals will be almost equivalent to the complex baseband signals. So, here we will mainly consider the PAPR of the discrete time baseband OFDM-IQ-IM signal.

Here, I_l and Q_l will be independent and identically distributed random variables with mean zero and variance $\frac{M-1}{3}\frac{k}{n}$. From (4.15) which is also applicable for OFDM-IQ-IM system we can say $x_i[m]$ and $x_q[m]$ are the summation of N IID random variables with zero mean and variance $\frac{M-1}{3}\frac{k}{n}$. If N is very high then the total number of active subcarriers for OFDM-IQ-IM system will be less than N but still high. Thus $x_i[m]$ and $x_q[m]$ will become Gaussian random variables.

Now, we will determine the mean and variance of $x_i[m]$ and $x_q[m]$. Mean of $x_i[m]$ for OFDM-IQ-IM system can be expressed as,

$$\begin{aligned} E\big[x_i[m]\big] &= E\Big[\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1} I_l cos\Big(\frac{2\pi lm}{N}\Big)\Big] \\ &= \frac{1}{\sqrt{N}}E\Big[\sum_{l=0}^{N-1} I_l\Big] \\ &= \frac{1}{\sqrt{N}}|\Upsilon_i|E[I_l] \\ &= 0 \end{aligned}$$

Here, $|\Upsilon_i| = kG$. Similarly mean of $x_q[m]$, $E\big[x_q[m]\big] = 0$. Considering the phase factor negligible we determine the variance of $x_i[m]$,

$$\begin{aligned} var[x_i[m]] &= var\Big[\frac{1}{\sqrt{N}}\sum_{l=0}^{N-1} I_l\Big] \\ &= |\Upsilon_i| var\Big[\frac{I_l}{\sqrt{N}}\Big] \\ &= kGE\Big[\Big(\frac{I_l}{\sqrt{N}} - \frac{E[I_l]}{\sqrt{N}}\Big)^2\Big] \\ &= \frac{kG}{N} var[I_l] \\ &= \frac{kG}{nG}\frac{k}{n}\frac{(M-1)}{3} \\ &= \frac{k^2}{n^2}\frac{(M-1)}{3} \end{aligned}$$

Similarly for $x_q[m]$ the variance is $\frac{k^2}{n^2}\frac{(M-1)}{3}$. For OFDM-IQ-IM system the samples for $x_i[m]$ and $x_q[m]$ are uncorrelated to each other,

$$E\big[x_i[m]x_i[o]\big] = E\big[x_q[m]x_q[o]\big] = \begin{cases} \frac{M-1}{3}\frac{k^2}{n^2}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases}$$

Here, $x_i[m]$ and $x_q[m]$ are independent Gaussian random variables as they are uncorrelated. As, we can write $x[m] = x_i[m] + jx_q[m]$. Now we will find the correlation between the samples of $x[m]$,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[(x_i[m] + jx_q[m])(x_i[o] - jx_q[o])\big] \\ &= E\big[x_i[m]x_i[o]\big] + E\big[x_q[m]x_q[o]\big] \end{aligned}$$

Now, if we consider $m = o$ then,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[x_i[m]^2\big] + E\big[x_q[m]^2\big] \\ &= \frac{2(M-1)}{3}\frac{k^2}{n^2} \end{aligned}$$

And if we consider $m \neq o$ then,

$$\begin{aligned} E\big[x[m]x[o]^*\big] &= E\big[x_i[m]\big]E\big[x_i[o]\big] + E\big[x_q[m]\big]E\big[x_q[o]\big] \\ &= 0 \end{aligned}$$

For OFDM-IQ-IM system the samples of $x[m]$ are mutually uncorrelated and statistically independent,

$$E\big[x[m]x[o]^*\big] = \begin{cases} \frac{2(M-1)}{3}\frac{k^2}{n^2}, & \text{for } m = o \\ 0, & \text{for } m \neq o \end{cases} \tag{4.49}$$

We know from Chapter 3 (3.45), the average power for a $\sqrt{M}$-PAM OFDM-IQ-IM baseband signal can be written as,

$$P_{avg} = \frac{2(M-1)kG}{3N} \tag{4.50}$$

For a sample $x[m]$ from (4.36) and (4.37) we can write,

$$\begin{aligned} PAPR_{x[m]} &= \frac{x_i[m]^2 + x_q[m]^2}{P_{avg}} \\ &= ax_i[m]^2 + ax_q[m]^2 \end{aligned}$$

where $a = \frac{1}{P_{avg}} = \frac{3N}{2(M-1)kG}$. Now, we take a chi-square distribution random variable $Y = aX^2$ where X is Gaussian with mean zero and variance σ^2. Now as shown in previous section we can write the PDF of Y of (4.38) as,

$$\begin{aligned} f_Y(y) &= \frac{1}{2\sqrt{ya}}\left[\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{y}{2\sigma^2 a}} + \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{y}{2\sigma^2 a}}\right] \\ &= \frac{1}{\sigma\sqrt{2\pi ya}}e^{-\frac{y}{2\sigma^2 a}} \end{aligned}$$

And the characteristic function for Y denoted by $\Psi_Y(j\omega)$ can be expressed as,

$$\begin{aligned} \Psi_Y(j\omega) &= \int_{-\infty}^{\infty} e^{j\omega y}\frac{1}{\sigma\sqrt{2\pi ya}}e^{-\frac{y}{2\sigma^2 a}}\,dy \\ &= \frac{1}{(1-j2\omega\sigma^2 a)^{\frac{1}{2}}} \end{aligned}$$

If we take $X_1, X_2, \ldots, X_n$ statistically independent Gaussian random variables with zero mean and variance σ^2. Then, we can rewrite random variable, $Y = aX^2$ as,

$$Y = a\sum_{i=1}^{n} X_i^2$$

And characteristic function can be expressed as,

$$\Psi_Y(j\omega) = \frac{1}{(1-j2\omega\sigma^2 a)^{\frac{n}{2}}}$$

And the PDF can be expressed (4.40) as,

$$f_Y(y) = \frac{1}{\sigma^n 2^{\frac{n}{2}} a^{\frac{n}{2}}\Gamma(\frac{1}{2}n)} y^{\frac{n}{2}-1} e^{-\frac{y}{2\sigma^2 a}}, \quad y \geq 0$$

For OFDM-IQ-IM system $n = 2$ as $x_i[m]$ and $x_q[m]$ are statistically independent Gaussian random variables with zero mean and variance $\frac{M-1}{3}\frac{k^2}{n^2}$. For $n = 2$ we can rewrite $f_Y(y)$ as,

$$f_Y(y) = \frac{1}{\sigma^2 2a} y^0 e^{-\frac{y}{2\sigma^2 a}}, \quad y \geq 0 \tag{4.51}$$

Here, $\Gamma(\frac{1}{2}n) = \Gamma(1) = 1$ and $\sigma^2 = \frac{M-1}{3}\frac{k^2}{n^2}$. Now putting the value of σ^2 and a into (4.41) we get,

$$\begin{aligned} f_Y(y) &= \frac{1}{2\frac{M-1}{3}\frac{k^2}{n^2}\frac{3N}{2(M-1)kG}} e^{-\frac{y}{2\frac{M-1}{3}\frac{k^2}{n^2}\frac{3N}{2(M-1)kG}}} \\ &= \frac{1}{\frac{k}{n}} e^{-\frac{y}{\frac{k}{n}}} \end{aligned}$$

So, $PAPR_{x[m]}$ is a random variable which will have chi-square distribution similar to random variable Y and the PDF of random variable $PAPR_{x[m]}$ as shown in (4.42),

$$f_{PAPR_{x[m]}}(\rho) = \frac{1}{\frac{k}{n}} e^{-\frac{\rho}{\frac{k}{n}}}$$

The CDF of the random variable $PAPR_{x[m]}$ can be represented as the probability of the random variable being less than a specific threshold ρ as shown in (4.43),

$$F_{PAPR_{x[m]}}(\rho) = P(PAPR_{x[m]} < \rho)$$

Now to find the value of $P(PAPR_{x[m]} < \rho)$ we will integrate $f_{PAPR_{x[m]}}(\rho)$ for a limit from 0 to ρ which will give us the CDF of the random variable,

$$\begin{aligned} F_{PAPR_{x[m]}}(\rho) &= \int_0^{\rho} \frac{1}{\frac{k}{n}} e^{-\frac{\rho}{\frac{k}{n}}}\, d\rho \\ &= \frac{1}{\frac{k}{n}} \big(-\frac{k}{n} \big) e^{-\frac{\rho}{\frac{k}{n}}} \Big|_0^{\rho} \\ &= \big(1 - e^{-\frac{\rho}{\frac{k}{n}}} \big) \end{aligned}$$

As the samples are mutually uncorrelated and independent so the CDF of the PAPR of baseband OFDM-IQ-IM signal can be written as,

$$\begin{aligned} CDF_{IQ}(\rho) &= P\Big(\max_{0 \le m < N-1} PAPR_{x[m]} < \rho\Big) \\ &= P(PAPR_{x[0]} < \rho)P(PAPR_{x[1]} < \rho)\ldots P(PAPR_{x[N-1]} < \rho) \\ &= \Big(1 - e^{-\frac{\rho}{\frac{k}{n}}}\Big)^N \end{aligned}$$

So, finally using (4.8) we find the CCDF of the PAPR of baseband OFDM-IQ-IM signal,

$$CCDF_{IQ}(\rho) = 1 - \Big(1 - e^{-\frac{\rho}{\frac{k}{n}}}\Big)^N \tag{4.52}$$

We have to be very careful, as the ratio $\frac{k}{n}$ in OFDM-IQ-IM system denotes the subcarrier component (in-phase or quadrature) activation ratio, not subcarrier activation ratio.

Chapter 5

Results and Discussion

In this chapter we will compare the PAPR performance of OFDM, OFDM-IM and OFDM-IQ-IM system in both deterministic approach and conventional distribution using numerical data and simulations.

5.1 PAPR comparison in deterministic approach

In this section we will compare the PAPR performance of OFDM, OFDM-IM and OFDM-IQ-IM system in deterministic approach. To make a fair comparison we will evaluate the PAPR performance of the three systems for the same spectral efficiency. The unit of SE is Bits Per Second per Hertz (bps/Hz).

5.1.1 System specifications

We will first consider an OFDM system with total number of subcarriers $N = 256$ and $M = 16$-QAM modulation scheme. For this configuration we can determine the SE of OFDM system using (2.5),

$$\begin{aligned}\zeta &= \log_2 M \\ &= \log_2 16 \\ &= 4 \quad bps/Hz\end{aligned}$$

For OFDM-IM system if we take modulation scheme less than $M = 16$-QAM modulation then it is not possible to get same SE like that of OFDM. From (2.6) the formula to find the SE of OFDM-IM system,

$$\zeta_{IM} = \frac{G}{N}\left(\left\lfloor \log_2 \binom{n}{k} \right\rfloor + k \log_2 M\right), \tag{5.1}$$

Now we will vary system parameters such as total subblock G, total subcarriers in a subblock n, total active subcarriers in a subblock k to find different OFDM-IM system configurations to find similar SE like OFDM in Table 5.1.

N	M	SE (bps/Hz)	G	n	k
256	16	4	2	128	Not possible
			4	64	56
			8	32	30
			16	16	15
			32	8	Not possible
			64	4	Not possible

TABLE 5.1: OFDM-IM system configurations to get $SE = 4bps/Hz$

Here in Table 5.1 we have used different combinations of G and n based on the formula $N = nG$. Then we put all the values in the rights side of (5.1) and changed the value of k from 1 to $n-1$ for each combination of G and n. We can see only in three specific configurations the SE of OFDM-IM was equal to the SE of OFDM system.

Now for OFDM-IQ-IM system both in-phase and quadrature components have to be modulated with $\sqrt{M}$-ary PAM constellation to have a fair comparison in terms of SE with an OFDM system that uses M-QAM constellation [73]. The formula to find the SE of OFDM-IQ-IM system for $\sqrt{M}$-ary PAM modulation for each component,

$$\zeta_{IQ} = \frac{G}{N}\left(2\left\lfloor \log_2 \binom{n}{k} \right\rfloor + 2k \log_2 \sqrt{M}\right), \tag{5.2}$$

Now we will vary system parameters such as total subblock G, total subcarriers in a subblock n, total active subcarriers in a subblock k to find different OFDM-IQ-IM system configurations to find similar SE like OFDM in Table 5.2.

N	$\sqrt{M}$	SE (bps/Hz)	G	n	k
256	4	4	2	128	Not possible
			4	64	34
			8	32	18
			16	16	10
			32	8	6
			64	4	3

TABLE 5.2: OFDM-IQ-IM system configurations to get $SE = 4bps/Hz$

Here in Table 5.2 we have used different combinations of G and n based on the formula $N = nG$. Then we put all the values in the rights side of (5.1) and changed the value of k from 1 to $n-1$ for each combination of G and n. We can see only now in five specific configurations the SE of OFDM-IQ-IM was equal to the SE of OFDM system. Also in case of OFDM-IM system when the subblock sizes are small which means when n is small then it was not possible to have the same spectral efficiency like OFDM system. But in case of OFDM-IQ-IM system even for lower n values we can have same spectral efficiency as that of OFDM system.

5.1.2 PAPR computation and comparison

Now we will compute the PAPR of each of the systems in deterministic approach considering the above system configurations. Then we will compare the PAPR of the three systems for same SE.

For OFDM system considering $N = 256$ and $M = 16$-QAM from (3.17) we can find the PAPR value as,

$$\begin{aligned} PAPR &= 3N\frac{(\sqrt{M}-1)}{(\sqrt{M}+1)} \\ &= 3 \times 256 \times \frac{4-1}{4+1} \\ &= 460.8 \end{aligned}$$

Now to find the PAPR of OFDM-IM we will use the formula (3.27),

$$PAPR_{IM} = 3N_a \frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}, \tag{5.3}$$

Here, $N_a = kG$ determines the total number of active subcarriers in an OFDM-IM block. We will compute PAPR for the OFDM-IM system configurations for which the same SE as OFDM system was achieved from Table 5.1.

N	M	G	n	k	$PAPR$
256	16	4	64	56	403.2
		8	32	30	432
		16	16	15	432

TABLE 5.3: PAPR computation for OFDM-IM system configurations to get $SE = 4bps/Hz$

Now to find the PAPR of OFDM-IQ-IM we will use the formula (3.46),

$$PAPR_{IQ} = 3kG \frac{(\sqrt{M}-1)}{(\sqrt{M}+1)}, \tag{5.4}$$

Here, $|\Upsilon_i| = |\Upsilon_q| = kG$ determines the total number of active in-phase or quadrature components in an OFDM-IQ-IM block. We will compute PAPR for the OFDM-IQ-IM system configurations for which the same SE as OFDM system was achieved from Table 5.2.

N	$\sqrt{M}$	G	n	k	$PAPR$
256	4	4	64	34	244.8
		8	32	18	259.2
		16	16	10	288
		32	8	6	345.6
		64	4	3	345.6

TABLE 5.4: PAPR computation for OFDM-IQ-IM system configurations to get $SE = 4bps/Hz$

In Figure 5.1 we have shown the comparison between OFDM, OFDM-IM and OFDM-IQ-IM systems more clearly.

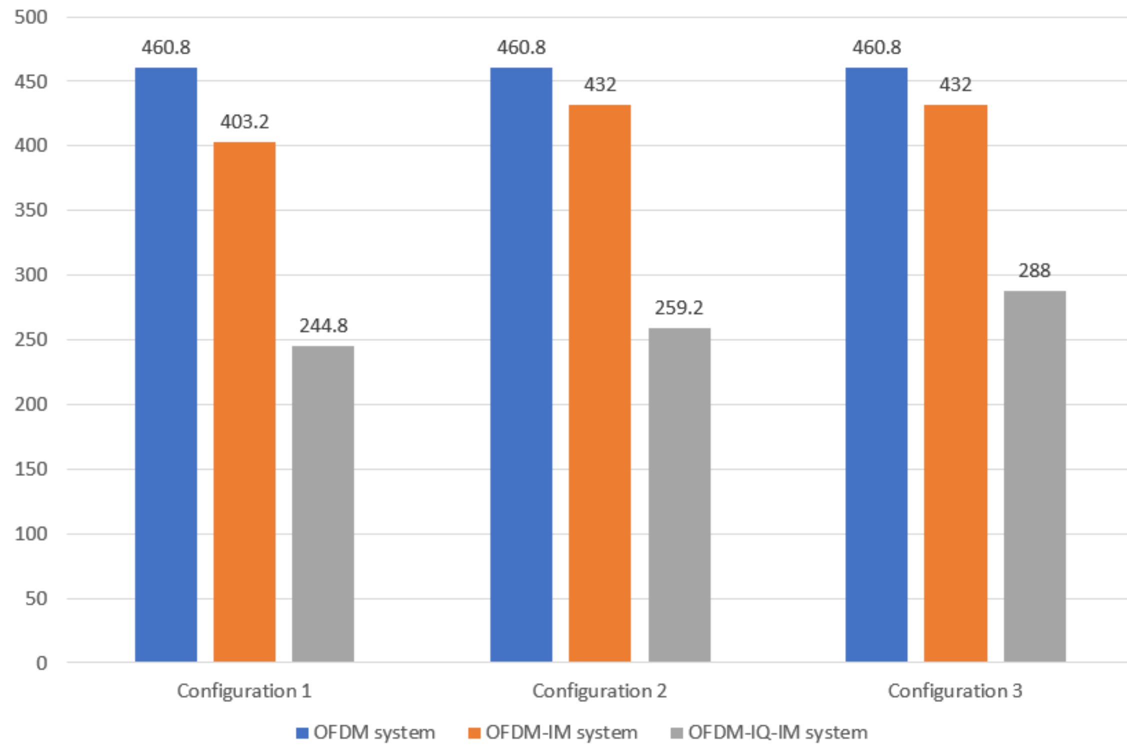


FIGURE 5.1: PAPR comparison between OFDM, OFDM-IM and OFDM-IQ-IM systems for same spectral efficiency in deterministic approach

Here from Figure 5.1 for all three configurations the SE is $4bps/Hz$. For OFDM system the PAPR is same value 460.8 as it has only two variables, N and M which were same ($N = 256$,$M = 16$) for all the configurations. For OFDM-IM and OFDM-IQ-IM we have taken equal number of subcarriers $N = 256$. Now for OFDM-IM system $M = 16$-QAM for all the configurations. And for OFDM-IQ-IM $\sqrt{M} = 4$-PAM is considered for all the configurations. For both OFDM-IM and OFDM-IQ-IM systems ($G = 4, 8, 16$) and $n = 64, 32, 16$ values were taken in configuration 1, configuration 2 and configuration 3 correspondingly. We can see in all the systems the PAPR of OFDM-IM and OFDM-IQ-IM is less than that of conventional OFDM. But OFDM-IQ-IM system reduces the PAPR more significantly in all three configurations than OFDM and OFDM-IM system. Also we can see if the ratio k/n is reduced then PAPR decreases.

5.2 PAPR comparison using probabilistic approach

As we already know from the previous chapter that PAPR in deterministic approach gives us the upper limit for the PAPR of the system. When the total number of subcarriers N is very high then the probability of the highest PAPR to occur is negligible. That is why to know about the PAPR performance of the systems the statistical distribution of the PAPR were derived in previous capture. Now we will show the distribution using simulations and will compare their PAPR performance.

For OFDM system we have derived the CCDF of the PAPR of baseband OFDM signal,

$$CCDF(\rho) = 1 - \left(1 - e^{-\rho}\right)^{N} \tag{5.5}$$

Now using the formula we will simulate the CCDF vs PAPR curves for OFDM system for different values of subcarriers.

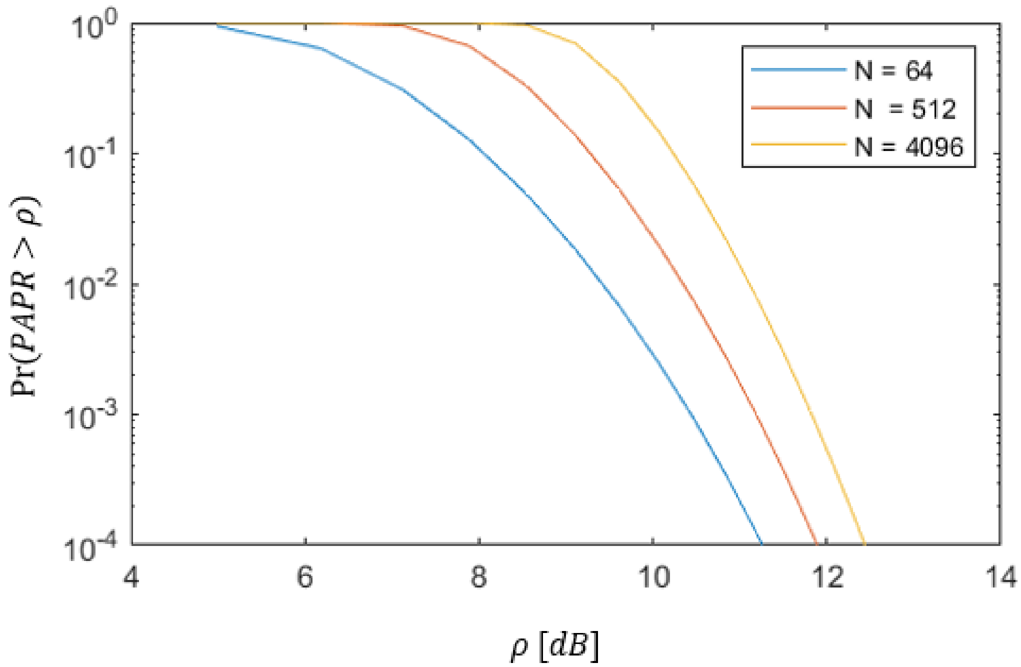


FIGURE 5.2: CCDF of the PAPR of OFDM system with $N = 64, 512, 4096$

Here in Figure 5.2 we can see the CCDF of the PAPR of OFDM system for different number of subcarriers $N = 64, 512, 4096$. We can see with increased number of subcarriers the curve gets deviated from 0 dB so the PAPR of the system increases.

Now for OFDM-IM system we have derived the CCDF of the PAPR of baseband OFDM-IM signal,

$$CCDF_{IM}(\rho) = 1 - \left(1 - e^{-\frac{\rho}{\frac{k}{n}}}\right)^N \tag{5.6}$$

In this formula $\frac{k}{n}$ is the subcarrier activation ratio. Now using (5.6) we will simulate the CCDF vs PAPR curves for OFDM-IM system for different values of subcarriers.

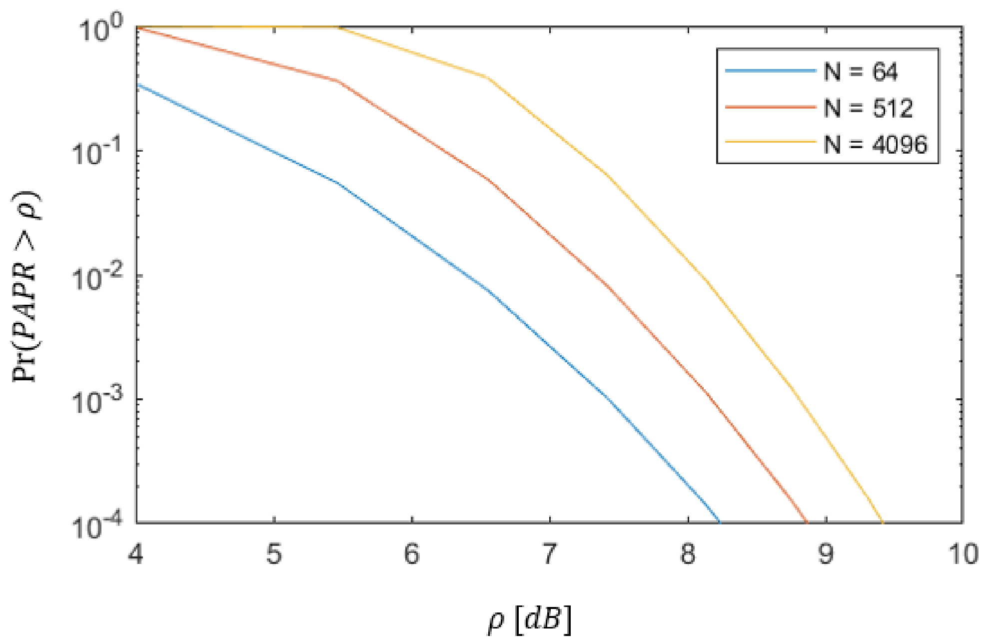


FIGURE 5.3: CCDF of the PAPR of OFDM-IM system with $N = 64, 512, 4096$ and $\frac{k}{n} = 0.5$

Here in Figure 5.3 we can see the CCDF of the PAPR of OFDM-IM system for different number of subcarriers $N = 64, 512, 4096$ for constant $\frac{k}{n} = 0.5$ follows similar trend as the CCDF of the PAPR of OFDM system in Figure 5.2. We can see with increased number of subcarriers the curve gets deviated from 0 dB so the PAPR of the system increases for same CCDF. Now we will keep the total number of active subcarriers fixed and vary the ratio $\frac{k}{n}$.

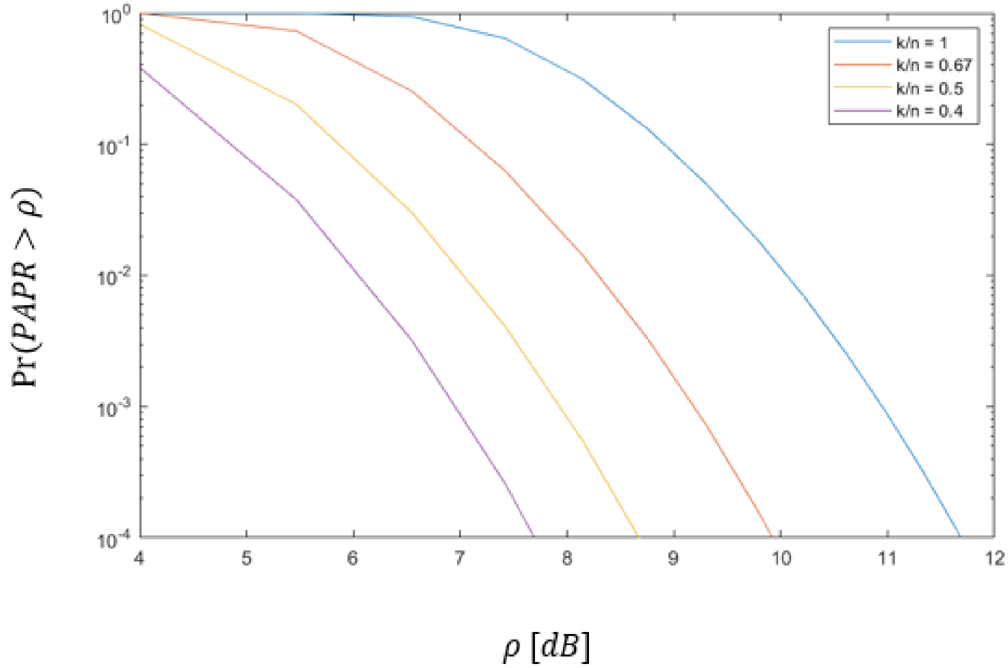


FIGURE 5.4: CCDF of the PAPR of OFDM-IM system with $k/n = 1, 0.67, 0.5, 0.4$ and $N = 256$

Here in Figure 5.4 we can see the CCDF of the PAPR of OFDM-IM system for different number of inverse of subcarrier activation ratios $\frac{k}{n} = 1, 0.67, 0.5, 0.4$ for constant number of subcarriers $N = 256$. We can see with with decreased ratio $\frac{k}{n}$ the curve gets comparatively straighten to 0 dB thus the PAPR of the OFDM-IM system decreases. When, $\frac{k}{n} = 1$ the system will behave like an OFDM system with number of subcarriers $N = 256$. Decreased $\frac{k}{n}$ denotes less number of subcarriers will be active in a subblock (for OFDM-IM) and less number of subcarrier components will be active in a subblock (for OFDM-IQ-IM).

Now for OFDM-IQ-IM system we have derived the CCDF of the PAPR of baseband OFDM-IQ-IM signal,

$$CCDF_{IQ}(\rho) = 1 - \left(1 - e^{-\frac{\rho}{\frac{k}{n}}}\right)^N \tag{5.7}$$

In this formula $\frac{k}{n}$ is the subcarrier component (in-phase or quadrature) activation ratio $\frac{k}{n}$. As (5.6) and (5.7) is similar so the effects of changing the variables will be same for both OFDM-IM and OFDM-IQ-IM system. In case of OFDM-IQ-IM system decreased

$\frac{k}{n}$ denotes less number of subcarrier components (in-phase or quadrature) will be active in a subblock. Another way is to increase the subblock size or the number of total subcarriers in a subblock n thus increasing the total number of subcarrier components (in-phase or quadrature) in a subblock.

Now to make a fair comparison between the PAPR performance of the three systems we will plot CCDF of the PAPR of OFDM, OFDM-IM and OFDM-IQ-IM systems for equal spectral efficiency $4bps/Hz$ also the total number of subcarriers is fixed $N = 256$. Now, from Table 5.1 we can find configurations for OFDM-IM system by which we can get the same spectral efficiency as of OFDM system. And for those configurations the possible unique $\frac{k}{n}$ values are 0.875 and 0.937. From Table 5.2 we can find configurations for OFDM-IQ-IM by which we can get the same spectral efficiency as of OFDM system. And for the first two configurations the possible unique $\frac{k}{n}$ are 0.75, 0.625 taken.

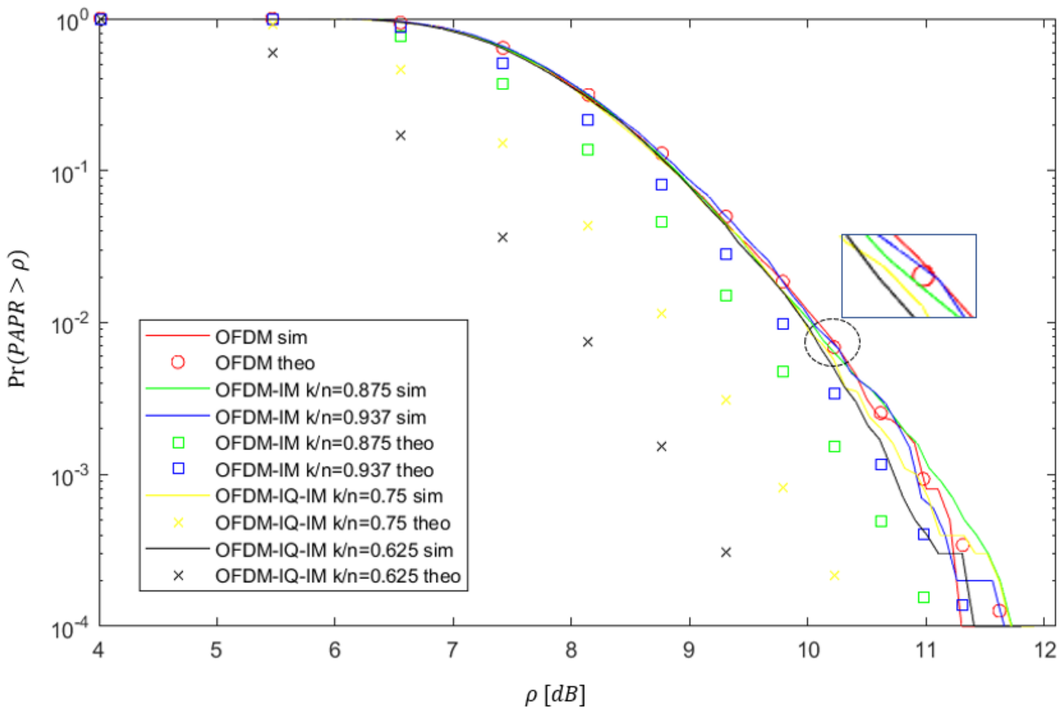


FIGURE 5.5: PAPR performance comparison between OFDM, OFDM-IM and OFDM-IQ-IM system using statistical distribution for SE = $4bps/Hz$

From Figure 5.5 we can compare the PAPR performance of the three systems. Here, we have generated 10000 blocks for each of OFDM, OFDM-IM and OFDM-IQ-IM signal to make a fair comparison between the PAPR of the three systems considering $N = 256$, $M = 4 - QAM$ and $SE = 4bps/Hz$. Here ideal scenario is considered while simulating which means effect of cyclic prefix, pilot and zero padding was ignored for convenience. In the figure the curve marked red represents conventional OFDM and it is most deviated from 0 dB. As we know the further the curve is to the right the higher the PAPR. So, for OFDM system the PAPR is highest. The blue and green curves represents PAPR distribution of OFDM-IM system for $\frac{k}{n} = 0.937$ and $\frac{k}{n} = 0.875$. Finally, the yellow and black curves represents PAPR distribution of OFDM-IQ-IM system for $\frac{k}{n} = 0.75$ and $\frac{k}{n} = 0.625$. Now, to show the comparison more clearly we have taken a fixed value of $CCDF = 0.0067$ and found out the corresponding PAPR values for different systems from the simulation result and the distribution formula that we derived as shown in Table 5.5.

$System$	$PAPRdB(Sim)$	$PAPRdB(Theo)$
OFDM	10.26	10.25
OFDM-IM k/n=0.937	10.25	10.01
OFDM-IM k/n=0.875	10.22	9.72
OFDM-IQ-IM k/n=0.75	10.20	9.106
OFDM-IQ-IM k/n=0.625	10.17	8.206

TABLE 5.5: PAPR computation for OFDM, OFDM-IM and OFDM-IQ-IM system considering $CCDF = 0.0077$ and $SE = 4bps/Hz$

Here in Table 5.5 we can see for CCDF = 0.0067, OFDM system will have the highest PAPR 10.26 dB from simulation and 10.25 dB from (5.5). For CCDF = 0.0067, OFDM-IM system will have the PAPR 10.25 dB from simulation and 10.01 dB from (5.6) considering $k/n = 0.937$. And considering $k/n = 0.875$ we will have PAPR 10.22 dB from simulation and 9.72 dB from (5.6). We can see for both cases decreased k/n results in less PAPR value and thus better PAPR performance. Here, after simulation 0.24 dB and 0.5 dB gain is found for $k/n = 0.937$ and $k/n = 0.875$ correspondingly compared to their theoretical values found from (5.6). For CCDF = 0.0067, OFDM-IQ-IM system will have the PAPR 10.20 dB from simulation and 9.106 dB from (5.7) considering $k/n = 0.75$. And considering $k/n = 0.625$ we will have PAPR 10.17 dB from simulation and 8.206 dB from (5.7). We can see for both cases decreased k/n results in less PAPR value and thus better PAPR performance.

Here, after simulation 1.094 dB and 1.964 dB gain is found for $k/n = 0.75$ and $k/n = 0.625$ correspondingly compared to their theoretical values found from (5.7).

The dB gains of the simulation from the theoretical values can be due to the fact that 10000 blocks for each of OFDM, OFDM-IM and OFDM-IQ-IM signal were generated. Before that 100 blocks and 1000 blocks were generated as shown in Figure . We can see if we increase the number of block the dB gain between the theoretical result and the simulation result decreases. So generating more number of blocks will give more accurate result.

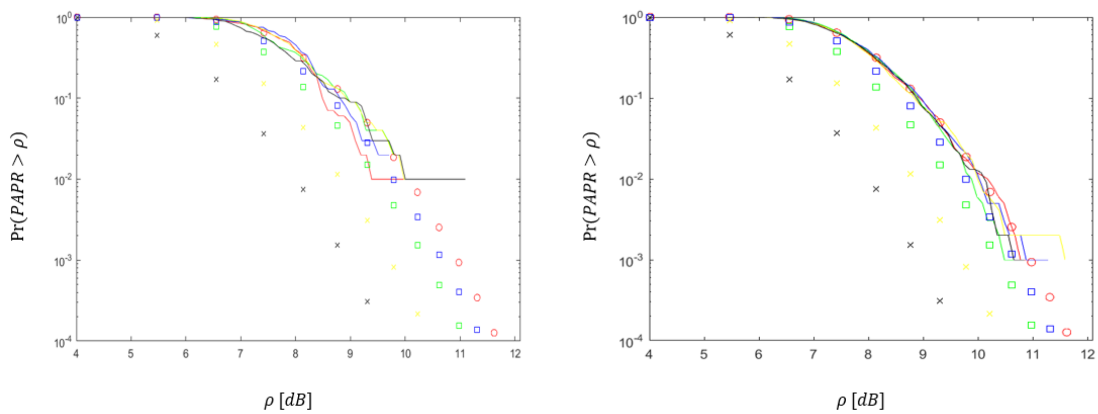


FIGURE 5.6: PAPR performance comparison between OFDM, OFDM-IM and OFDM-IQ-IM system using statistical distribution for SE $= 4bps/Hz$ for (a) 100 blocks and (b) 1000 blocks

Another reason behind the dB gain can be uncertainty. In case of OFDM all the subcarriers will be active, thus there is no uncertainty. But in case of OFDM-IM system a subcarrier can be active or inactive, thus there is uncertainty. And in OFDM-IQ-IM system the subcarrier components (in-phase and quadrature) can be active or inactive independently so there will be more uncertainty thus the deviation is more between the theoretical result and the simulation result.

Chapter 6

Conclusion and Future Work

6.1 Overview

In this thesis the high PAPR which is one of the major drawbacks of OFDM and OFDM-IM systems has been studied and the PAPR of OFDM-IQ-IM system which performs better than OFDM and OFDM-IM systems in terms of BER and SE, has been formulated in both deterministic and probabilistic approach. Using the formulations the PAPR performance of OFDM, OFDM-IM and OFDM-IQ-IM have been compared and in both approaches OFDM-IQ-IM outperforms OFDM and OFDM-IM systems.

In Chapter 1, basic understanding of multiplexing techniques have been presented along with the transformation of multiplexing techniques from 1G to 5G. Also, how OFDM scheme was introduced as a modified version of FDMA scheme has been shown along with the applications, advantages and disadvantages of OFDM system. Here in the disadvantages, the major drawback of having a high PAPR for OFDM system has been mentioned.

In Chapter 2, system structure of OFDM system has been shown, elaborating the functionalities of each building block in the block diagram of an OFDM transceiver. Then the summary has been provided on extensive studies that have been done and various schemes that have been introduced to reduce the high PAPR of OFDM system along the years. Then the modified version of OFDM system that is OFDM-IM system has been introduced along with detailed descriptions of differences in the transceiver diagram of OFDM and OFDM-IM systems. Various system characteristics such as SE, BER performance have been studied for OFDM-IM system along with very few studies on the PAPR performance of OFDM-IM system in recent years. The high PAPR accumulation in OFDM-IM system and very few PAPR reduction techniques that has been recently proposed are highlighted here too. Thus, the system we have focused mainly, OFDM-IQ-IM have been introduced along with its modifications in the transceiver diagram. Brief study has been done and shown on system performances of OFDM-IQ-IM systems based on published research works, which shows noticeable improvement in case of BER and SE compared to OFDM and OFDM-IM systems but according to author's best knowledge no work was done on the PAPR performance of OFDM-IQ-IM system. Thus the motive has been set for this thesis to evaluate the PAPR performance of this system and make a fair comparison with the PAPR performance of OFDM and OFDM-IM system.

In Chapter 3, the concept of PAPR was explained along with the effects on the system due to high PAPR. Then a deterministic approach that was proposed in a study to find the PAPR of OFDM system and SIM-OFDM (from here the index modulation came into scenario) system has been analysed in detail. And the same methodology has been applied to determine the PAPR of OFDM-IM and OFDM-IQ-IM systems. All the steps to reach the final formula to find PAPR for both the systems, were briefly explained. The approach is called deterministic because the bits that were transmitted, were not considered random rather than the symbol with highest power was considered to be transmitted throughout the whole frame.

In Chapter 4, a different and more practical approach was taken to evaluate the PAPR performance of a system. The concept of the deterministic approach that though it gave us an idea about the upper limit for PAPR of OFDM, OFDM-IM and OFDM-IQ-IM systems but is not enough to characterise the overall PAPR performance of a system, was explained. Then elaborations have been done on how the CCDF of the PAPR of a system gives us the proper idea about the PAPR performance in the system. This approach is considered more practical as the transmitted bits were taken as random as in practical scenario. Here while formulating the CCDF of the PAPR for the three systems, the process of bits consisting of $1s$ and $0s$ turning into high frequency bandpass signal, was explained elaborately for all the systems. The CCDF of all three systems have been evaluated considering the discrete time baseband signals both for in-phase and quadrature component as independent Gaussian random variables with zero mean and σ^2 variance. This σ^2 varies from system to system based on how the subcarriers were modulated in the system using transmitted bits.

Lastly in Chapter 5, we have considered numerical data for system specifications keeping the same SE for fair comparison. Based on those numerical data and formulas evaluated in Chapter 3 we computed the PAPR for OFDM, OFDM-IM and OFDM-IQ-IM systems and found out that keeping the SE same, OFDM-IQ-IM will have less PAPR. As we know, the deterministic approach only gives the upper limit for the PAPR of the corresponding system. Thus to strengthen our claim of OFDM-IQ-IM showing better PAPR performance, we have plotted the CCDF of PAPR for all the systems considering same SE for fair comparison. And from the simulations it has been shown that, if we consider same probability for all the system's PAPR to be greater than a threshold value then OFDM will have the highest threshold value. OFDM-IM will have less value than OFDM and OFDM-IQ-IM will have the lowest threshold value. Thus according to both the approaches, OFDM-IQ-IM system shows better PAPR performance compared to OFDM and OFDM-IM systems.

We can see considering BER performance, spectral efficiency, energy efficiency, flexibility and PAPR, OFDM-IQ-IM system performs better than OFDM-IM and OFDM. But in case of OFDM-IQ-IM the receiver complexity increases and that creates delay while detecting the data symbols and thus increasing the latency of the system. But for near field communication it can be used very efficiently. Some of the applications are WiFi, Bluetooth, electronic tickets, key cards etc.

6.2 Suggestions for future studies

This thesis not only evaluates the PAPR performance of a promising multiplexing technique but also paves the way to several other research topics. Few of the ideas are given here and fully acknowledging that there can be several other ideas too.

- As OFDM-IQ-IM and OFDM-IM system has index modulation in common, so the PAPR reduction techniques that has been proposed for OFDM-IM system can be applied into OFDM-IQ-IM system and this might result into significant improvement in the PAPR performance of OFDM-IQ-IM system thus making it more of a potential candidate as the multiplexing technique for future wireless networks.

- Though OFDM system does not consider the unique index modulation characteristics but there have been vast amount of techniques that has been applied to reduce the PAPR of the OFDM system. So researches can be done on if those techniques can be applied in OFDM-IQ-IM system too and how the system behaves after applying the reduction scheme.

- MIMO and multiple access techniques can be accumulated with OFDM-IQ-IM system. Thus the PAPR performance of those systems can be studied using the same methodology used here. And comparison of those systems with MIMO-OFDM and MIMO OFDM-IM system will be another topic of interest for further research.

- Lastly, PAM modulation technique is used in OFDM-IQ-IM system for both in-phase and quadrature components independently. If we can keep the system configuration and methodology same but change the digital modulation technique to QAM or PSK by accumulating dual mode technique or any other method, we might see changes considering the PAPR performance of the system.

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