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Introduction to the mathematical theory of knowledge conceptualization: Conceptual systems and structures

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Abstract. The paper departs from the general problem of knowledge integration and the basic strategies that can be adopted to confront this challenge. With the purpose of providing a sound meta-theoretical framework to facilitate knowledge conceptualization and integration, as well as assessment criteria to evaluate achievements regarding knowledge integration, the paper first reviews the previous work in the field of conceptual spaces. It subsequently gives an overview of structural tools and mechanisms for knowledge representation, recapped in the *modal stratified bond model* of global knowledge. On these groundings, a novel formalized representation of conceptual systems, structures, spaces and algebras is developed through a set of definitions which goes beyond the exploration of mental knowledge representation and the semantics of natural languages. These two components provide a sound framework for the development of the glossaLAB international project with respect to its two basic objectives, namely (i) facilitating knowledge integration in general and particularly in the context of the general study of information and systems; (ii) facilitating the assessment of the achievements as regards knowledge integration in interdisciplinary settings. An additional article tackles the solutions adopted to integrate these results in the elucidation of the conceptual network of the general study of information and systems.

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Keywords: Knowledge conceptualization, knowledge integration, conceptual systems, conceptual structures, conceptual spaces, conceptual algebras.

1 Introduction

Knowledge of people in general and scientific knowledge in particular has become an extensively large system, which is continuously growing. Even various big knowledge domains, such as mathematics, physics or information sciences, are so huge that an expert in one subdomain (say, group theory) does not know and understand what is going in another subdomain (say, probability theory). This brings scientists to the problem of *knowledge integration*. There are different approaches to this problem. One of the most powerful approaches to knowledge integration is *abstraction*. Another useful method of knowledge integration is *algorithmization*. The third tactic is *conceptual knowledge integration*. It consists of two major steps: *domain knowledge conceptualization* and *systemic integration*. In domain knowledge conceptualization, the existing domain knowledge is transformed into the conceptual form. There are three key goals of this process. The first goal is to bring diverse forms of knowledge representation to a synthesized description in a unified format. The second goal is simplification of existing advanced knowledge making it comprehensible to a much wider audience in comparison with the top experts in a specific domain. The third goal is preparation of knowledge for the subsequent knowledge integration.

The glossaLAB project, to which the present work is contributing, aims at enabling knowledge integration through the utilization of *interdisciplinary glossaries* [23, 24, 25]. In these glossaries, not to be confused with usual glossaries, the meaning of concept names used in interdisciplinary settings (particularly devoted to the study of systems and information, as well as application problems that requires strong interdisciplinarity) are simultaneously elucidated from the different disciplinary perspectives summoned. Its ultimate purpose is contributing to the development of sound *transdisciplinary* settings in which the integration of knowledge is effectively achieved (*ibidem*). In the first place, the commitment to the study of systems and information offers good groundings for knowledge integration in general, in virtue of the abstraction of the concepts involved. On the other hand, its form, as an online hypertext in continuous development by an interdisciplinary community of users, offers the means to carry out the theoretical work required for knowledge integration and meta-theoretical assessment of the integration effectively achieved. To both purposes, we need modelling conceptual systems as well as addressing the problem of knowledge integration. We concentrate our attention here in the first part, the modelling of conceptual systems, while the specific problems of knowledge integration have been analyzed in previous works [22, 23] and is further addressed in another work [24].

To build, explore and utilize a mathematical model of this process, we apply the *representational model of a concept* [10], *hypertexts*, *hypermedia* [6, 44] and *named set theory* [13, 18]. In a conceptual knowledge system, each concept is represented by a *hypertext* (hypermedia) and is mathematically modeled by syntactic and model logical varieties [8, 12], which is a system of interconnected named sets. Here we construct the base for knowledge conceptualization and integration developing formalized representation of conceptual systems, structures, spaces and algebras. That is why in Section 2, we give an overview of the previous work in this area. However, our overall goal, in which this paper is only the first step, is to develop a theory of conceptual knowledge integration, which demands taking into account all forms of knowledge representation and not only the conceptual one. That is why in Section 3, we give an overview of structural tools and mechanisms of knowledge representation coming to the conclusion that although these tools and mechanisms are highly developed in contemporary epistemology, methodology and AI, they do not pay enough attention to the conceptual level of knowledge systems. This demands developing new tools for knowledge conceptualization and integration. Construction of the first level of tools and mechanisms for knowledge conceptualization and integration is the main purpose of this paper.

With this in mind, in Section 4, we build a mathematical model of conceptual systems in the form of conceptual spaces and algebras. It is necessary to remark that our theory is essentially different from the previous research due to the following reasons. The aim of the conceptual spaces constructed before was exploration of mental knowledge representation and reconstruction of conceptual semantics. Our goal is exploration of textual knowledge representation and construction tools, structures and mechanisms for conceptual knowledge integration in the textual form. Finally, a conclusive section summarizes results as regards the objectives of the work.

2 Conceptual spaces as tools for learning theory and the semantics of natural languages

The concept of *conceptual space* was introduced and studied by Peter Gärdenfors in his theory of conceptual representation in the mind, aiming at building foundations of learning theory and the semantics of natural languages [32, 33, 30, 29]. In his works, Gärdenfors further developed the approach of Osgood, Suci and Tannenbaum to the measurement of meaning [45].

According to Gärdenfors, *conceptual spaces* are defined as systems of quality dimensions of different types. Quality dimensions based on perception can be, for instance, temperature, brightness, color, weight, pitch and the three ordinary spatial dimensions. The dimensions represent perceived similarity and dissimilarity in the following sense: the closeness between any two points of the conceptual space corresponds to the similarity judged through perception. Points in a conceptual space denote objects, while regions of similar points represent concepts [29].

This makes it natural to call these structures by the name *attributive conceptual spaces*. In contrast to this, spaces introduced and studied in this paper are called *structural conceptual spaces*.

Properties correspond to convex regions in the conceptual space. For instance, a sour taste corresponds to a convex region in the five-dimensional space of basic tastes. As a result, a concept, according to Gärdenfors, is a bundle of properties joined to information about the way how these properties correlate to each other [32]. For instance, the concept of an orange encompasses properties corresponding to regions of color space, shape space, taste space, nutrition space, and other spaces.

One of the most important applications of conceptual spaces concerns the modeling of the semantic processes involved in language acquisition. According to the main assumption in this application new words are never learnt separately as single words, but rather as words within the same domain. For example, once the child learns a color, other color words are learnt at the same time. On the other hand, they learn them in connection to objects which are colored, and through the uttering of color relations while stating sentences about the world, be it real or imagined. These links establish correlations among dimensions of the concept space which determine the convex region of the concepts involved.

Based on his theory, Gärdenfors describes a linguistic learning process in the following way.

"What is it that you know when you know a language? Certainly, you know many words of the language (its lexicon), and you know how to put the words together in an appropriate way (the syntax). More important, you know the meaning of the words (the semantics of the language). If you do not master the meaning of the words you are using, there is no point in knowing the syntax (unless you are a parrot). You can communicate in a foreign language with some success just by knowing some words and without using any grammar. In this sense, semantic knowledge precedes syntactic knowledge." [34].

Operations in conceptual spaces such as concept composition were formalized and studied in [48, 1, 41]. Method of utilization of grammatical structures within conceptual spaces in categories were developed and studied in [7].

All this clearly demonstrates that the existing theory of conceptual spaces is essentially oriented on mental processes of conceptual information processing. At the same time, conceptual information and knowledge representation in encyclopedia and dictionaries as well as text formation for these sources have dissimilar regularities and conditions. That is why to develop a theory having in mind our purpose, we need different types of conceptual spaces and other structures, which we construct in this paper.

3 Structural tools and mechanisms of knowledge representation

There are three main areas where researchers explored and developed knowledge representation with the modern emphasis on formal structures, methods and techniques: epistemology as field of philosophy, methodology of science and artificial intelligence [17].

Studies of knowledge started with the beginning of philosophy. Great Greek philosophers Plato and Aristotle paid considerable attention to problems of knowledge. Knowledge was enthusiastically studied in ancient Indian teachings and doctrines such as Samkhya, Nyaya, Jaina, Buddhist and other school of Hindu philosophy [47].

However, philosophers made the main emphasis on mundane knowledge developing logic as a tool for cognition in the form of knowledge acquisition and justification. Researchers started to study and develop knowledge representation when people tried to make their computers intelligent, teaching computers to solve problems people can solve. The reason was that mundane knowledge was represented by notions and texts of natural languages, which computers were not able to understand. Orientation on computers brought force formal methods with orientation on structural tools and mechanisms of formalized knowledge representation.

As a consequence, the most widespread model of knowledge is the *standard (positivist or logical) model*, in which knowledge is represented by logical propositions and/or predicates (cf. [20, 46, 51]). Another popular approach is the *structuralist model* *of knowledge*, in which knowledge is represented by collections of models utilizing means of set theory (cf. [52, 4, 54]).

Some researchers treat knowledge as a collection of devices for the formulation and resolution of problems, modeling knowledge through systems of propositions and queries, sometimes including several types of problem representation, as well as, guidelines to address the problems using erotetic logic in order to analyze problems thoroughly and to solve them. This representation was consolidated in the *interrogative model of knowledge* (cf., for example, [35]). Another approach treats knowledge as assemblies of highly organized guidelines, concepts and solutions to problems [53].

The first unified approach to formalized knowledge representation was achieved in the *structure-nominative model* [11]. The unified character of this model resides in the fact that other formalized models of knowledge became subsystems of the structure-nominative model, at the time the structures employed within each model are named sets of systems of named sets [18]. In this vein, the aforementioned standard model of knowledge corresponds to a logic-linguistic subsystem of the unified model, while the structuralist model corresponds to a model-representing subsystem.

3.1 The modal stratified bond model of global knowledge

Later on this unified model was broadened and improved by one of the authors [18] moving further in the modeling of global knowledge. Nowadays, the state-of-the-art to this regard is the **modal stratified bond model of global knowledge** developed by the same author [17], virtually capable to embrace any other system model of scientific knowledge and knowledge in general.

In accordance with this model, general knowledge has three basic dimensions – the modal, the systemic and the hierarchical dimension – although separate knowledge items may only have one or two of these dimensions.

The ***modal dimension*** reflects the three types of knowledge modality referred to in table 1.

Table 1. Modal dimension of global knowledge.

Knowledge modality	Its epistemic structures are explicit or implicit expression of:
Assertoric	being knowledge
Hypothetic or heuristic	being possible knowledge
Erotetic	knowledge deficit

We can find examples of assertoric knowledge in logical propositions or statements, for instance, “the Sun is a star”. Examples of hypothetic knowledge can be found among beliefs whose certainty is weak, for instance, “there is a lot of water in the shadowed craters of the moon.” On the other hand, questions and problems, such as “how can the water be conserved on the moon surface?”, constitute basic forms of erotetic knowledge.

Knowledge modalities shape strata of knowledge systems determining their *horizontal structure*, while its *vertical structure* is determined by the ***hierarchical dimension*** whose levels are referred to in table 2. These levels are in themselves comprised of layers and sections.

Table 2. Hierarchical dimension of global knowledge.

Knowledge level	It is comprised of:
Componential	elements and groups of elements upon which structures of the attributive level of knowledge are assembled.
Attributive	the static knowledge structures which are built up by the elements of the componential level.
Productive	the dynamic knowledge structures which includes the means of acquiring, producing and transmitting knowledge

Finally, the ***systemic dimension*** of knowledge is composed by the three categories referred to in table 3.

Table 3. Systemic dimension of global knowledge.

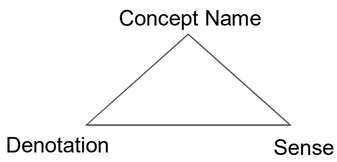
Knowledge category	It is comprised of:	Examples
Descriptive (declarative or propositional)	knowledge about properties and relations of objects	"a cat is white" "two is less than three"
Representational	The set of representations (through knowledge structures) of an object	The image of Jose that Alex has of his friend
Operational	Knowledge about how to perform actions, how to organize behavior and how to regulate and control system's operation	algorithms, regulations, instructions, procedures, guidelines, etc.

4 Conceptual systems, structures, spaces and algebras

In this section, we build a formalized representation of conceptual systems. The basic element (block) of this representation is the concept, which itself has a certain structure. Construction of conceptual spaces and algebras is based on definite models of concepts. Let us consider some of these models as a basis for the subsequent consideration of conceptual systems, spaces, algebras and structures.

4.1 Models of concepts

The first known model of a concept belongs to Gottlob Frege, who treated concepts as ways of thinking of objects, properties and relations [28]. Formally, this is represented by the following diagram.

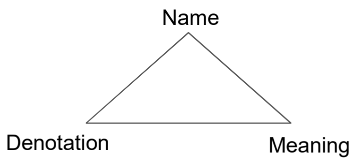


(1)

Fig. 1. Frege's *Concept Triangle*

In this triadic structure, the *denotation* of a concept is a set of all objects denoted by the concept, while the *sense* of a concept accounts for its cognitive significance being the way by which people conceive of the denotation of this concept.

Bertrand Russell conceived concepts as constituents of propositions, whereas his model of concept is similar to the model of Frege [49].



(2)

Fig. 2. Russel's *Concept Triangle*

In this triadic structure, the *denotation* of a concept consists of particular exemplifications of the concept, while the *meaning* comprises the set of propositions describing the concept.

Cohen and Murphy consider four types of existing concept models: *extensional*, *fuzzy set-theoretical*, *prototypical* and *semantic* models [21].

Hampton describes that five broad classes of models have been proposed by different researchers [36]:

1. The *classical model* takes concepts as clearly and entirely determined by a system of necessary and sufficient features as the system attributes, which later were divided into two groups: *defining features* and *characteristic features*.
2. The *prototype model* represents a concept by an object with the most common attributes of the category or the system of these attributes, which is called the prototype. Objects are denoted by the concept if the similarity to the prototype suffices.
3. The *exemplar model* is similar to the prototype model but instead of being based on one prototype, it is based on a set of exemplifications.
4. The *theory-based model* has the form of a structured frame or schema, comprising theoretical knowledge about the relations between these attributes, as well as their causal and explanatory links.
5. The *psychological essentialism model* agrees with the classical "core" definition of concept, though admitting among its determinations empty "place holders", as it can be, for instance, further attributes which are still unknown.

It is also worth mentioning the novel *quantum model* of concepts, which has the form of the triad state-context-property. Here the states of a concept correspond to unitary vectors within a Hilbert space, affected by a linear operator modelling the influence of the context [2].

The representational model. Currently the most general model is the *representational model* of a concept which was co-introduced and studied by one of the authors [10] and was later developed by the same author [15]. Within this model a particular type of named sets or fundamental triads constitute its surface structure [18]. In this model, whose higher specification level is illustrated in Fig.3, the concept name can either be a word or a text.

As it has been proven, this model, as a structure of a higher abstraction degree, comprises virtually any other concept model [15]. In this regard, the aforementioned concept model from Frege (Fig.1) can be derived from the representational model considering "sense" and "denotation" as conceptual representatives. Similarly, the concept model from Rusell (Fig.2) can be derived from the representational model considering "meaning" and "denotation" as conceptual representatives.

Fig. 3. Higher specification level of a concept in the *representational model*.

In a lower specification level, the conceptual representative is differentiated in the three components referred to in table 4.

Table 4. Components of the *conceptual representative* in the representational model.

Component	corresponds to	Equivalent to
Concept Domain (D_C)	the domain of reality referred to by the concept *C*	*Denotation* in the models of Frege and Russell
Meaning (or *broad-spectrum concept knowledge*)	knowledge about the concept domain D_C	*Sense* in Frege's concept model
Representation	a set of representations of the knowledge about the concept domain D_C	(it includes) *meaning* in Russell's concept model

In addition, the collection of all knowledge about the concept domain D_C is called the *abundant domain knowledge*.

In the case of being "knowledge" the name of the concept at stake, an article about knowledge within an encyclopedia or a dictionary is a representation of the concept knowledge. In other words, while one concept may have several representations, the union of representations of this concept is also a representation of the same concept.

4.2 Conceptual Systems

However, concepts, as consider in the previous paragraphs, do not exist separately but form conceptual systems.

Definition 4.1. A *conceptual system* is composed of concepts and relations of different levels.

Within the conceptual systems, we can find concept of three different types, systemic, emphasized and background concepts, as referred to in table 5.:

Table 5. Types of concept within a conceptual system.

Concept type	Consist of	They have descriptions within the conceptual system (definitions)
Primary or *Systemic*	knowledge items from a given knowledge domain	Yes
Secondary or *Emphasized*	Concepts which are used in descriptions of primary/systemic concepts	Yes
Tertiary or *Background*	Concepts which are used in descriptions of primary/systemic concepts	No

Considering concepts, it is necessary to distinguish properties of concepts and properties that define concepts. For instance, states of concepts in the quantum model are properties of concepts [2] while quality dimensions in the model of Gärdenfors are properties that define concepts [32].

4.3 Conceptual spaces, algebras and structures

To define conceptual spaces and algebras, we discern relations of two types: *pure relations* and *operational relations*.

For instance, properties are pure relations. Relations between elements in a network are also pure relations. Operational relations define operations. In essence, any operation can be presented by a relation. For instance, we can define $R_+(a, b, c)$ is true if and only if: $a + b = c$. In particular, $R_+(2, 2, 4)$ is true while $R_+(2, 3, 4)$ is false in the conventional arithmetic.

Definition 4.2. A conceptual system is a *conceptual space* or more exactly, a *structural conceptual space*, if it is a logical model, i.e., it has only pure relations.

In other words, a structural conceptual space consists of formal (abstract) representations of concepts, e.g., of prototype, exemplar or representational models, and formal relations between these models.

For instance, in the quantum model of concepts, referred to above, concepts are mapped by vectors within a Hilbert space [2].

A conceptual space in the theory of Gärdenfors, referred to above, is a multidimensional feature space, in which vectors denote objects and regions denote concepts [32, 31]. The basis of a conceptual space is comprised of *quality dimensions*, which denote basic features, such as *weight*, *color*, *taste* and so on, used for the definition and comparison of concepts and objects of concepts and objects.

A different kind of semantic spaces found useful applications for building a theory of meaning and its measurement [45].

Different kinds of operators are acting in conceptual spaces. For example, in the quantum model, linear operators acting on the Hilbert space of concepts model contextual impact [2].

Operations with concepts convert conceptual spaces into conceptual algebras.

Definition 4.3. A conceptual system is a *conceptual algebra* if it is a conceptual space with operations.

Note that concepts can include physical elements. For instance, if we take the model of Frege, then the concept *dog* includes all real dogs as its denotation. However, theoretically we study only formal concepts, in which form names are used instead of physical or mental objects.

Definition 4.4. A *conceptual structure* consists of formal concepts and abstract relations of different levels.

It means that on the top level with only binary relations a conceptual structure is a *network of concepts* [37, 39, 40, 42, 5, 25].

Structural conceptual spaces and conceptual algebras are special cases of conceptual structures.

Because properties (features) form intermediate structures, in accordance with the general theory of structures [15], attributive conceptual spaces of Gärdenfors [32, 31] also are special cases of conceptual structures. Another special case of conceptual structures was studied in the theory of meaning and its measurement [45].

At the same time, abstract conceptual spaces form an important type of semantic spaces [17]. As a result, *networks of concepts* represent a principal category of *semantic networks* [50, 9, 14].

In turn, semantic spaces are a significant form of knowledge spaces [3, 26, 38, 19, 17]. In this case, semantic networks shape a noteworthy sort of *knowledge networks* [55]. Interestingly, as the authors have argued elsewhere [22], the network of knowledge agents represents a counterpart of the semantic networks [43].

There are two categories of conceptual spaces: mixed and abstract conceptual spaces.

Definition 4.5. A conceptual system is a *mixed conceptual space* if it has physical and/or mental elements.

Mental spaces are examples of mixed conceptual spaces. They consist of small conceptual bundles associated, on the one hand, to long-term schematic knowledge, named “frames”; on the other to long-term specific knowledge [27].

Conceptual spaces have their structures.

Definition 4.6. A conceptual structure is an *abstract conceptual space* if it is a logical model, i.e., it has only pure relations.

Abstract conceptual spaces are used for exploration of arbitrary conceptual spaces and belong to the World of Structures [15]. All conceptual spaces studied in scientific literature are abstract.

There are two sorts of conceptual algebras: mixed and abstract conceptual algebras.

Definition 4.7. A mixed conceptual space with operations is a *mixed conceptual algebra*.

This shows that it would be useful to study not only mental spaces as in [27] but also mental algebras for the modeling and investigation of human mentality and mental conceptualization of knowledge.

Definition 4.8. A conceptual structure is an *abstract conceptual algebra* if it is an algebraic system, i.e., it contains both pure and operational relations.

There are various operations in (abstract) conceptual algebras. Many of them are used for formation of new concepts or transformation of existing concepts. Operation of the first type is conceptual combination. For instance, concepts *Pet* and *Bird* are combined into the conjunction *Pet-Bird*. Operation of the second type is conceptual abstraction. For instance, the concept *Cat* is transformed into the concept *Animal*.

These componential and abstraction procedures (among conceptual levels) are actually widespread in science and global knowledge, and include more complex process than the stated above, as it is, for instance, the case of metaphorization. Indeed, they represent some of the most fruitful processes in knowledge creation (at the productive level). For its analysis, it is worth mentioning the utilization of category theory as proposed by several authors who have developed formal approaches to analyze the general problem of abduction and metaphorization [56].

To conclude, it is necessary to remark that conceptualization converts assorted knowledge systems described in Section 3 into conceptual spaces and algebras, which provide better knowledge comprehension and more efficient knowledge integration [22, 24].

5 Conclusion

Thus, we have explored the field of conceptual knowledge representation and constructed new tools, structures and mechanisms as the base (foundation) for knowledge conceptualization and conceptual knowledge integration in the textual form such as conceptual structures, structural conceptual spaces and conceptual algebras. It is demonstrated that these tools give and adequate picture of textual knowledge representation in the form of concept networks, which pave the way for the integration of knowledge aimed at the glossaLAB project. The next step is exploration of existing and creating new operators in structural conceptual spaces and operations in conceptual algebras.

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