Available online at www.sciencedirect.com

ScienceDirect

Procedia Structural Integrity 28 (2020) 752–763

Structural Integrity Procedia

www.elsevier.com/locate/procedia

1st Virtual European Conference on Fracture

An extension of the Equivalent Material Concept applied to fracture of U-notched solids

F.J. Gómez[a]*, S. Cicero[b], A.R. Torabi[c]

[a] *ADVANCED MATERIAL SIMULATION SL, C/Elcano 14, Bilbao 48008, Spain*
[b]*Laboratory of Materials Science and Engineering, University of Cantabria, E.T.S. de Ingenieros de Caminos, Canales y Puertos, Av/Los Castros 44, Santander 39005, Spain*
[c]*Fracture Research Laboratory, Faculty of New Science and Technologies, University of Tehran, Tehran, Iran*

Abstract

The Equivalent Material Concept is a tool that allows the fracture behavior of notched components in elastoplastic materials to be analyzed by transforming them into fictitious linear elastic ones, defining a fictitious failure stress. The combination of a failure criterion with this approach offers a methodology for predicting the maximum load of notched solids with elastoplastic behavior. The validity limits have been established experimentally by a logistic regression using fracture data of linear elastic, small scale yielding and fully plastic material. This paper also proposes an extension of the Equivalent Material Concept, applying the fictitious transformation in a more complete form to tensile, toughness and notch tests. New magnitudes as the fictitious stress, the fictitious toughness and the fictitious notch stress intensity factors have been defined. The final proposal is a partial application of the fictitious transformation that incorporates all of the experimental data gathered.

Keywords: Fracture; EMC; U-notch; failure criteria.

* Corresponding author.
E-mail address: javier.gomez@amsimulation.com

10.1016/j.prostr.2020.10.087

1. Introduction

The presence of U-notches and round V-notches is common in structural elements and can be the origin of a critical failure. In order to guarantee the integrity of these critical points, an assessment methodology is required. When the critical defect is a crack in linear elastic materials, Linear Elastic Fracture Mechanics (LEFM) states that the maximum load is reached when the applied stress intensity factor is equal to the material fracture toughness. This criterion is still valid in elastoplastic materials when the plastic zone is limited to a region close to the crack tip (Irwin, 1957).

Nomenclature

E	elastic modulus
f_t	Fracture strength
K_I	Stress intensity factor
K_{IC}	Fracture toughness
$K_{IC,elas}$	Apparent fracture toughness
$K_{IC,f}$	Fictitious fracture toughness
$K_{IC,t}$	Interpolated fracture toughness
K_{IC}^R	Critical notch stress intensity factor
$K_{IC,f}^R$	Fictitious notch stress intensity factor
$K_{IC,t}^R$	Interpolated notch stress intensity factor
K_{IC}^{R*}	Non-dimensional critical notch stress intensity factor
K_I^R	Notch stress intensity factor
l_{ch}	Characteristic length
L_r	Ratio between the maximum load and plastic collapse load
$L_{r,max}$	Maximum ratio between the maximum load and plastic collapse load
m	initial slope
P	Fictitious fracture load
P_f	Applied load
P_{max}	Maximum load
R	Notch radius
SED	Strain energy density
$SED_{necking}$	Strain energy density under necking
u	Displacement of the load point
α_i	Fitting coefficients.
ε	Strain
ε_f	Failure strain
ε_u	Strain under necking
ε^*_f	Fictitious failure strain
σ	Stress
σ_f	Fictitious fracture strength
σ_t	Interpolated fracture strength
σ_{max}	Elastic stress at the tip of the notch
σ_u	Ultimate tensile strength
σ_y	Elastic limit

In U-notched solids, there is no stress singularity at the notch root in linear-elastic materials. However, the approximate expression of the stress field near the notch root given by Creager and Paris (1967) can be used to perform a similar assessment based on the notch stress intensity factor (Glinka and Newport 1987), i.e. failure occurs when the notch stress intensity factor reaches a critical value that depends on the notch root radius. In elastoplastic materials, the formulation of Creager and Paris is no longer valid. One possibility to overcome this limitation is to apply tensional corrections as suggested by Neuber (1958) or Glinka (Molski and Glinka 1981, Glinka 1985). Torabi, one of the authors of the present communication, has proposed a similar correction: the Equivalent Material Concept, EMC (Torabi 2012 and 2013). Following EMC, the real material is substituted by a fictitious one with the same elastic modulus, fracture toughness, and an equivalent failure stress.

The EMC can be combined with various failure criteria for predicting the maximum load of components with U-notches such as the critical strain energy density, the theory of critical distances (Fuentes et al 2018) or the cohesive zone model (Gómez and Torabi 2018). Gomez and Torabi applied the Equivalent Material Concept combined with the dimensionless formulation of the critical U-notch curve and proposed a first validity limit of the methodology based on the ratio between the applied load and the plastic collapse load.

This paper analyses more precisely the validity region using the complete experimental programme of U-notched specimens tested at the University of Cantabria by Cicero and collaborators (Fuentes et al 2018, Madrazo et al 2018). The programme included linear elastic materials as PMMA at low temperature, the notched geometries of an aluminium alloy, Al7075-T651, as structural steels as S275 and S355, where fracture occurred in fully plastic conditions. This paper also proposes an extension of the Equivalent Material Concept, applying the fictitious transformation defined at the EMC in a more complete form to tensile, toughness and notch tests. The final proposal is a partial application of the fictitious transformation.

2. Application of Equivalent Material Concept to U-notched solids

In linear elastic materials until failure, the maximum load that the cracked components support under mode I loading is obtained by equaling the stress intensity factor K_I, which depends on the geometry and the applied load, to the fracture toughness K_{IC}, function of the material:

$$K_I = K_{IC}(material) \tag{1}$$

When failure starts in a U-notch, the stress field at the tip of the notch is not singular, but it can be approximated by the Creager and Paris expression (Creager and Paris 1967). The stress field depends on the notch radius, R, and a stress parameter, called the notch stress intensity factor, K_I^R. A similar fracture criterion can be formulated equaling the notch stress intensity factor to a critical value, K_{IC}^R, a generalized toughness that is a function of the material and the radius (Glinka and Newport, 1987).

$$K_I^R = K_{IC}^R(material, R) \tag{2}$$

The critical function, K_{IC}^R, can be obtained experimentally, applying failure criteria or damage models as the theory of critical distances (Seweryn and Lukaszewicz 2002, Susmel and Taylor 2008), the cohesive zone model (Gómez et al 2000), the strain energy density criterion (Sih 1974), the averaged strain energy density criterion (Lazzarin and Berto, 2005) or the Finite Fracture Mechanics (Leguillon 2002). These theories can be summarized in an approximated form using the following non-dimensional formulation (Gómez et al 2005):

$$\frac{K_I^R}{K_{IC}} = K_{IC}^{R*}\left(\frac{R}{l_{ch}}, material\right) \tag{3}$$

$$l_{ch} = \left(\frac{K_{IC}}{f_t}\right)^2 \tag{4}$$

where f_t is the fracture strength. The dependence of expression (3) on the material is relatively weak. Gomez and Elices (Gomez et al. 2005 and 2006) applying this non-dimensional form to ceramics and polymeric materials, showed the secondary level of influence of the material and failure criteria.

$$\frac{K_I^R}{K_{IC}} \approx K_{IC}^{R*}\left(\frac{R}{l_{ch}}\right) \tag{5}$$

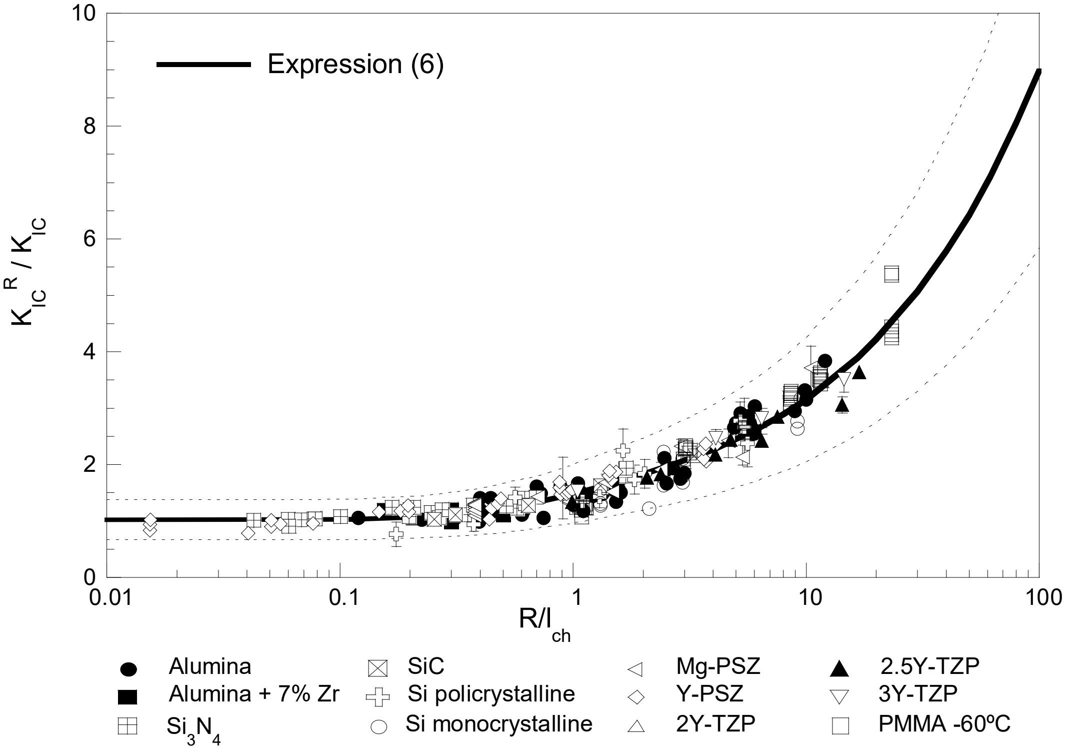

Fig. 1 Non-dimensional notch stress intensity factor in linear elastic materials (Gomez and Elices 2006)

Figure 1 collects the critical non-dimensional notch stress intensity factor of alumina, silicon nitride, monocrystalline and polycrystalline silica, zirconia partially stabilized with magnesia, zirconia partially stabilized with yttria, tetragonal zirconia fully stabilized with yttria, and PMMA at -60°C. The experimental results were fitted to the following expression which itself constitutes a phenomenological failure criterion (Gomez et al 2005). An experimental band has been plotted at the figure enclosing the elastic data.

$$\frac{K_I^R}{K_{IC}} = \sqrt{\frac{1 + 0.47392(R/l_{ch}) + 2.1382(R/l_{ch})^2 + \pi/4(R/l_{ch})^3}{1 + (R/l_{ch})^2}} \tag{6}$$

The Creager and Paris expression is not valid for plastic materials, and a revision of the failure criteria based on the notch stress intensity factor concept is needed. The maximum stress in a tensile test is affected by the necking hindering its measuring, and the non-dimensional formulation of the notch stress intensity factor must be redefined. Approximate expressions formulated by Neuber (1958) or Glinka (1985) lead to the concept of an equivalent linear elastic material. Torabi proposed the Equivalent Material Concept based on the strain energy density, similar to Glinka, establishing the equivalency at the necking. Following Torabi, the fictitious elastic material has the same elastic properties, E and ν, the same toughness K_{IC} and a different failure stress obtained equating the strain energy density of a tensile test under maximum load of a real material with a fictitious one.

$$(SED)_{necking} = \frac{\sigma_f^2}{2E} \tag{7}$$

The fictitious failure stress, σ_f, is obtained with expression (7) and is used to calculate the non-dimensional notch stress intensity factor. This formulation has been applied to plastic materials and specimen sizes where failure of a U-notched solid is reached under small scale yielding (Gomez and Torabi 2018) as PMMA at room temperature (Gomez et al 2000), polycarbonate (Nisitani and Hyakutake 1985), vessel steel at -196ºC (Lee et al 2002) and Al7075-T651 aluminum alloy at directions TL and LT Al7075-T651 and aluminum alloy at directions TL and LT (Madrazo et al 2018).

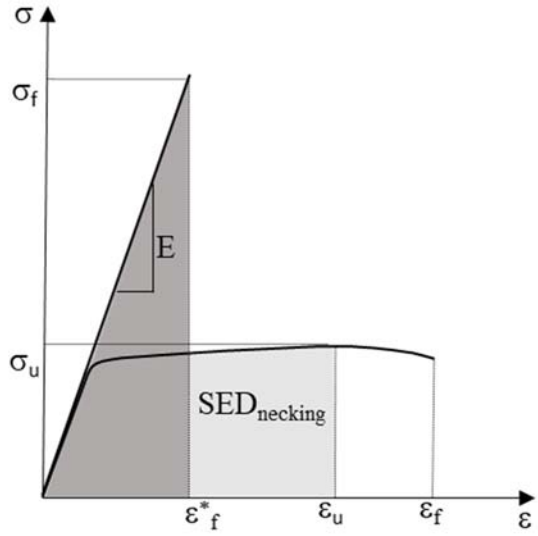

Fig. 2 Stress-strain curve at the real and fictitious material.

A more complete experimental program of notch failure tests in fully plastic materials was performed by Cicero and co-workers at the University of Cantabria. All details with a complete description can be found in (Fuentes et al 2018, Madrazo et al 2018). They studied the influence of the notch radius on the fracture toughness of two steels, S355 and S275, at different temperatures. The experimental program consisted of compact specimens with a notch radius from 0 to 2.0 mm. The stress-strain curve of the materials and the values of the plastic collapse loads can be found in the original work and the fictitious fracture strength has been calculated using expression (7). The notch stress intensity factors of the geometries have been determined using expression (8) and appear in Figure 3.

$$K_I^R = \frac{\sigma_{max}}{2}\sqrt{\pi R} \tag{8}$$

where σ_{max} is the stress at the tip of the notch. Figure 3 summarizes all the dimensionless factors obtained corresponding to linear elastic, small scale yielding and fully plastic materials. The proposed methodology is not valid for some of the structural steel samples.

3. Limits of the methodology

To analyze the limits of the methodology, the ratio between the maximum load and the plastic collapse load, L_r, is introduced in the study. This magnitude has been extensively used in structural integrity assessment, and a complete definition and tabulation for the plastic collapse load of different geometries can be found in FITNET (2007).

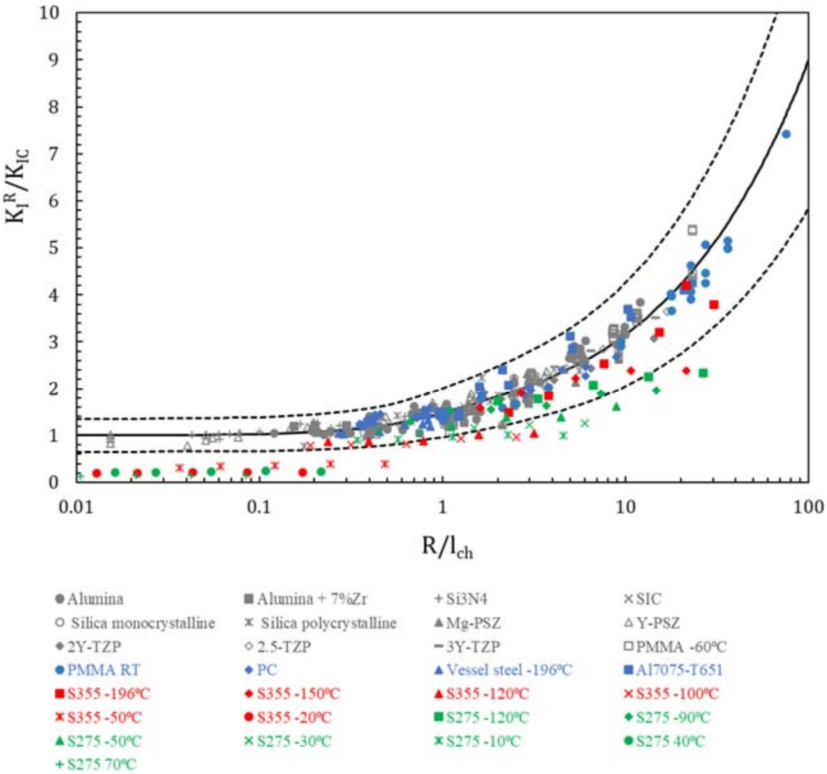

Fig. 3 Non-dimensional notch stress intensity factor. Application of the Equivalent Material Concept.

In order to establish a limit, the dash lines of Figure 1 have been considered the boundary of the validity region. Outside these lines, data are invalid. The analysis could be carried out in a parameter space (R/l_{ch}, L_r), or to avoid the material dependence in a parameter space (R/l_{ch}, $L_r/L_{r,max}$).

$$L_{r,max} = \frac{\sigma_u + \sigma_y}{2\sigma_y} \tag{9}$$

where σ_u is the ultimate tensile stress and σ_y the elastic limit of the material. The experimental values of $L_r/L_{r,max}$ versus R/lch have been plotted for the aluminum alloy A7075-T651, the steel S275JR and the steel S355J2 in Figure 4. The red crosses correspond to the notched specimen where the EMC approximation is not valid and the blue circles those that fall within the range of validity shown in Figure 3. Invalid values are concentrated in the upper area of figure, where failure starts at fully plastic conditions.

To define the separation border between the two families of points, a logistic regression has been performed. As a first approximation, the separation boundary is assumed to be linear. The continuous and dashed green lines in Figure 4 represent the probability of 50% and 95%, respectively. The points below the continuous line have a probability greater than 0.5 of being valid and those that are below the dashed line a probability greater than 0.95. The methodology proposed, combining the Equivalent Material Concept with linear elastic notch failure criteria, can be applied when the following condition is met

$$L_r \leq 1.0438 - 0.006508\frac{R}{l_{ch}} \tag{10}$$

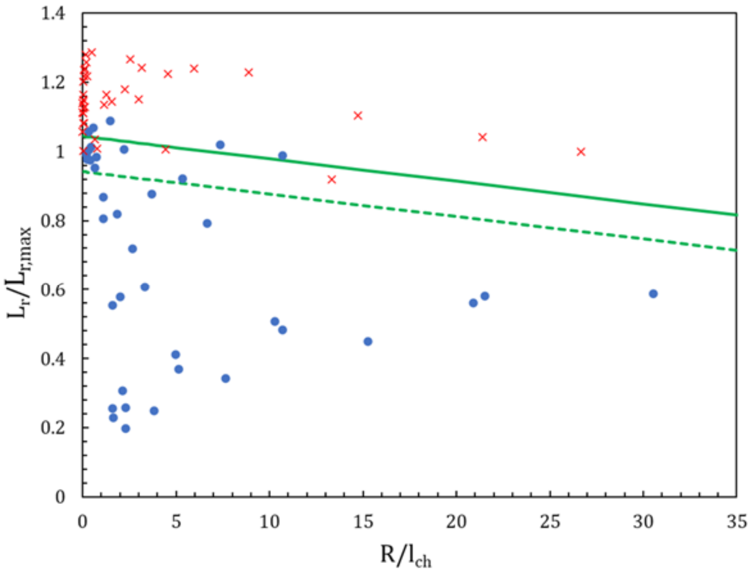

Fig. 4 Validity region of the Equivalent Material Concept.

Table 1. Real and fictitious properties for the fully plastic materials studied.

Material	Temperature (ºC)	σ_u (MPa)	σ_f (MPa)	K_{IC} (MPam$^{0.5}$)	$K_{IC,elas}$ (MPam$^{0.5}$)
S275JR	-120	613.8	5633.9	48.80	48.84
	-90	597.3	5381.0	62.73	62.78
	-50	564.7	5376.7	80.60	70.36
	-30	548.6	5504.3	100.70	81.64
	-10	536.3	5860.8	122.80	79.82
	40	504.8	5260.8	504.78	108.97
	70	492.8	5046.5	777.33	107.37
S355J2	-196	922.9	3864.7	31.28	31.32
	-150	757.9	6264.9	60.57	58.75
	-120	671.6	5825.0	146.60	117.69
	-100	646.5	5608.0	157.48	113.88
	-50	602.7	5806.4	372.13	115.14
	-20	587.7	5893.5	635.63	117.91

4. Extension of the EMC

One of the reasons for the lack of validity, shown in Figure 4, is the toughness value used in the dimensionless expression (6). When the radius tends to zero, the notch stress intensity factor must tend to the stress intensity factor, but clearly S275 and S355 do not follow this behavior for higher temperatures. The toughness has been obtained using the Standard ASTM E-1820 (1999), and the plastic behavior of cracked compact specimens was important. The value proceeds from the J-integral, a conversion that cannot be applied to U-notches.

An alternative is to use the apparent toughness $K_{IC,elas}$ calculated from the maximum load multiplying by the elastic shape factor of the stress intensity factor. These values are shown in Table 1 for the two fully plastic materials studied.

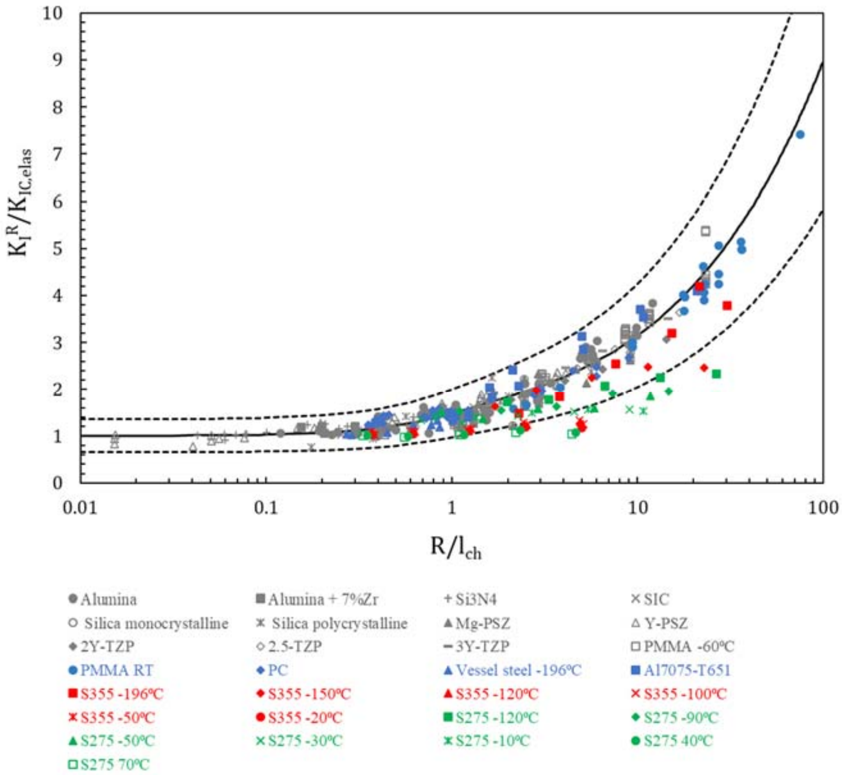

Fig. 5 Non-dimensional notch stress intensity factor: values obtained with the apparent toughness.

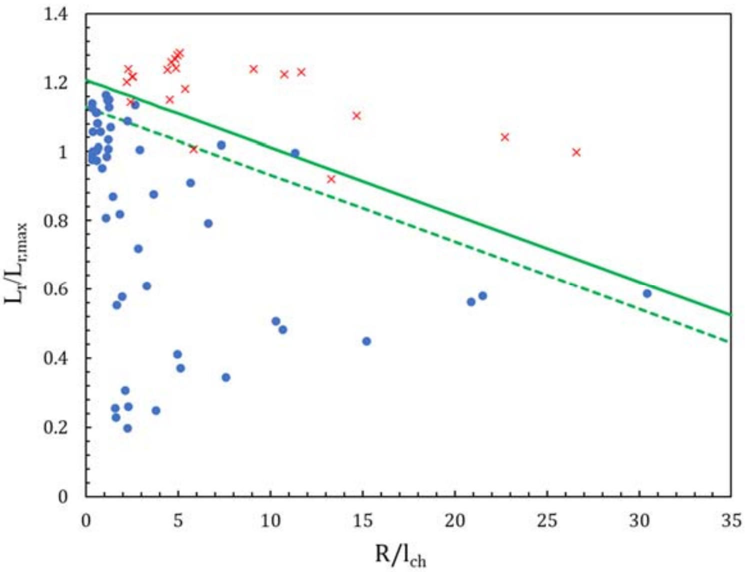

Fig. 6 Validity region: values obtained with the apparent toughness.

The new approach increases the number of valid tests as can be observed in Figure 5. A new validity region can be analyzed as shown in Figure 6 and a new boundary is obtained.

$$L_r \leq 1.2068 - 0.01951\frac{R}{l_{ch}} \tag{11}$$

The apparent approach slightly improves the EMC but there are still some notched tests where the results are not valid even for plastic collapse load ratios less than 1 when the radius is relatively great. In order to improve this number, a new idea is formulated. Torabi et al. proposed an extension of the EMC, called the modified EMC (MEMC) applying the same strain energy equivalency to the crack test (Torabi and Kamyab 2019). The fictitious material has the same elastic properties as the real one but a different fracture toughness. The new fictitious maximum load is calculated assuming that the work developed before the maximum load is the same (Figure 6). Using this value, an equivalent fracture is obtained.

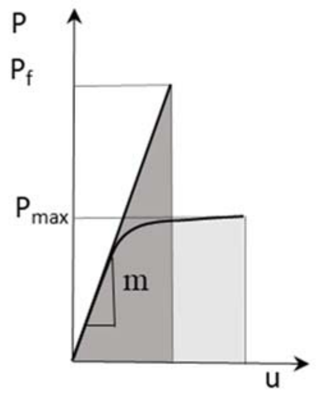

Fig. 7 Load displacement curve at the real and fictitious material.

The same idea can be applied to all U-notch tests determining the fictitious maximum load of all notched specimens and a corresponding fictitious notch stress intensity factor, $K^R_{IC,f}$. Moreover, the fracture conditions can be defined partially considering three coefficients interpolating between the real material and the fictitious one, defined with the subscript f.

$$\begin{aligned} \sigma_t &= \alpha_1\sigma_u + (1-\alpha_1)\sigma_f \\ K_{IC,t} &= \alpha_2 K_{IC,elas} + (1-\alpha_2)K_{IC,f} \\ K^R_{IC,t} &= \alpha_3 K^R_{IC} + (1-\alpha_3)K^R_{IC,f} \end{aligned} \tag{12}$$

The new extension of the equivalent material concept can be called the Partial Equivalent Material Concept (PEMC).

5. Validation and results

The coefficient α_1, α_2 and α_3 have been determined by fitting. The elastic fracture data has no influence on the new fitting as the fictitious material is the same as the real one. This data has been used only to obtain the phenomenological critical notch stress intensity factor curve defined in expression (6).

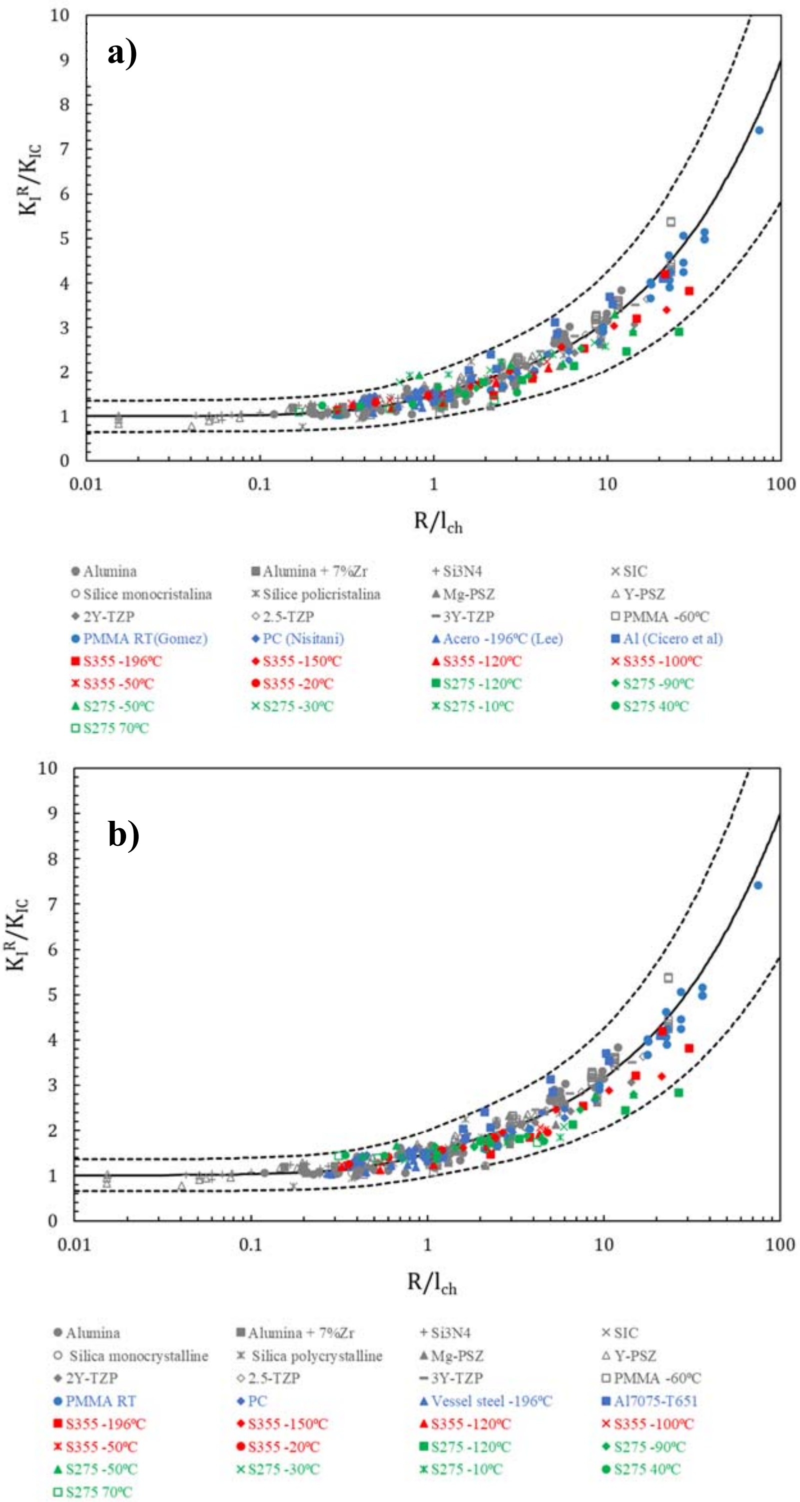

Fig. 7 Non-dimensional notch stress intensity factor: values obtained with the Partial Equivalent Material Concept using a) three coefficients, b) one coefficient.

The small scale yielding data can influence only the value of α_1. The experimental load displacement curve of the notch tests is approximately linear up to fracture. These tests have been removed from the global fitting. Using the materials where plasticity is completely developed under fracture, i.e. the structural steels S275 and S355, the following fitting coefficients are obtained

$$\begin{aligned} \alpha_1 &= 0.018602 \\ \alpha_2 &= 0.93910 \\ \alpha_3 &= 0.85626 \end{aligned} \tag{13}$$

Analyzing the values of expression (13), α_1 is near to zero and α_2 is relatively near to 1. This fact means that the most important fitting parameter is α_3. The Partial Equivalent Material Concept can be reformulated using only one equation as:

$$K_{IC,t}^{R} = \alpha' K_{IC}^{R} + (1-\alpha') K_{IC,f}^{R} \tag{14}$$

Fitting again all the data, the final coefficient obtained is

$$\alpha' = 0.87366$$

The corresponding plot can be seen in Figure 8b. The global quality of the fitting is similar despite using only one coefficient.

6. Conclusions

- A methodology for estimating the maximum load of U-notched components in elastoplastic materials is proposed: The Equivalent Material Concept combined with elastic U-notch failure criteria.
- The validity of the methodology is analyzed in a real parameter space ($L_r/L_{r,max}$, R/l_{ch}) fitting a logistic regression to quantify precisely the boundary of this region.
- Three extensions of the Equivalent Material Concept are proposed and verified. The first extension consists of using an apparent fracture toughness instead of the real one. The boundary of the validity region has been determined again by a logistic regression.
- The second and third approaches have been called Partial Equivalent Material Concept (PEMC), using three and one fitting coefficients respectively. The approaches used have been validated successfully with linear elastic, small scale yielding and fully plastic fracture data.

Acknowledgements

The authors wish to express their gratitude to the European Union's H2020 research and innovation program for their financial support under the LightCoce project (No 814632).

References

ASTM E1820-99, 1999. Standard test methods for measurement of fracture toughness plane stress/plain strain. American Society of Testing and Materials, Philadelphia, USA.

Creager, M., Paris, P.C., 1967. Elastic field equations for blunt cracks with reference to stress corrosion cracking. International Journal of Fracture Mechanics 3, 247–252.

FITNET, 2007. European Fitness-for-Service Network, EU´s Framework 5, Proposal No. GTC1-2001-43049, Contract No. G1RT-CT-2001-05071.

Fuentes, J.D., Cicero, S., Berto, F., Torabi, A.R., Madrazo, V., Azizi, P., 2018. Estimation of Fracture Loads in AL7075-T651 Notched Specimens Using the Equivalent Material Concept Combined with the Strain Energy Density Criterion and with the Theory of Critical Distances. Metals, 8, 87.

Glinka, G., 1985. Energy density approach to calculation of inelastic strain-stress near notches and cracks. Engineering Fracture Mechanics 22, 485–508.

Glinka, G., Newport, A., 1987. Universal features of elastic notch-tip stress fields. International Journal of Fatigue 9, 143–150.

Gómez, F.J., Elices, M., Valiente, A., 2000. Cracking in PMMA containing U-shaped notches, Fatigue Fracture Engineering Material and Structures 23, 795–803.

Gómez, F.J., Guinea, G.V., Elices, M., 2005. Failure criteria for linear elastic materials with U-notches. International Journal of Fracture, 141, 99-113.

Gómez, F.J., Elices, M., 2006. Fracture loads for ceramic samples with rounded notches. Engineering Fracture Mechanics 73, 880–894.

Gómez, F.J., Torabi, A.R., 2019. Application of the equivalent material concept to the study of the ductile failure due to U-notches. International Journal of Pressure Vessels and Piping 172, 65–69.

Irwin, G.R., 1957. Analysis of Stresses and Strain Near the End of a Crack Traversing Plate. Journal of Applied Mechanics 24, 361–364.

Lazzarin, P., Berto, F., 2005. Some expressions for the strain energy in a finite volume surrounding the root of blunt V-notches. International Journal of Fracture 135, 161–185.

Lee, B.W., Jang, J., Kwon, D., 2002. Evaluation of fracture toughness using small notched specimens. Materials Science and Engineering A334, 207–214.

Leguillon, D., 2002. Strength or toughness? A criterion for crack onset at a notch. European Journal of Mechanics-ASolids 21, 51-72.

Nisitani, H., Hyakutake, H., 1985. Condition for determining the static yield and fracture of a polycarbonate plate specimen with notches. Engineering Fracture Mechanics 22, 359–368.

Madrazo, V., Cicero, S., García, T., 2014. Assessment of notched structural steel components using failure assessment diagrams and the theory of critical distances. Engineering Failure Analysis 36, 104–120.

Molski K., Glinka G., 1981. A Method of Elastic-Plastic Stress and Strain Calculation at a Notch Root. Materials Science and Engineering, 50, 93-100.

Neuber, H., 1958. Theory of Notch Stresses: Principles for Exact Calculation of Strength with Reference to Structural form and Material, second ed., Springer Verlag, Berlin.

Seweryn, A., Lukaszewicz, A., 2002. Verification of brittle fracture criteria for elements with V-shaped notches. Engineering Fracture Mechanics 69, 1487–1510.

Sih, G.C., 1974. Strain-energy-density factor applied to mixed mode crack problems. International Journal of Fracture 10(3), 305–321.

Susmel, L., Taylor, D., 2008. The theory of critical distances to predict static strength of notched brittle components subjected to mixed-mode loading. Engineering Fracture Mechanics 75, 534–550.

Torabi, A.R., 2012. Estimation of tensile load-bearing capacity of ductile metallic materials weakened by a V-notch: the equivalent material concept. Material Science and Engineering A 536, 249–255.

Torabi, A.R., 2013. Ultimate bending strength evaluation of U-notched ductile steel samples under large-scale yielding conditions. International Journal of Fracture 180, 261–268.

Torabi A.R., Kamyab, M., 2019. Notch ductile failure with significant strain-hardening: The modified equivalent material concept. Fatigue and Fracture of Engineering Materials and Structures 42, 439–453.