Quantum phase transitions in atomic nuclei

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Macroscopic/Classical Phase Transitions

Definition of phase and phase transition

- Phase: state of matter that is uniform throughout, not only in chemical composition but also in physical properties.
- Phase Transition: abrupt change in one or more properties of the system.

The phase of the system

- Most stable phase of matter is the one with the lowest thermodynamical potential Φ. This is a function of some parameters that are allowed to change (F(T,V), F(T,B); G(T,p), G(T,M)).
- Φ is analogous to the potential energy, V(x), of a particle in a one dimensional well. The system looks for the minimum energy going into the bottom of the potential.

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Macroscopic/Classical Phase Transitions

- Control parameter: variable that affects the system, it can be changed smoothly and “arbitrarily”.
- Order parameter: observable that changes as a function of the control parameter and that defines the “phase” of the system.
- Ordered and disordered phases correspond to a value of the order parameter equal and different from zero, respectively.
- Order of a phase transition: order of the first derivative of the Gibbs potential with respect to the control parameter that first experiences a discontinuity: first, continuous (second order).

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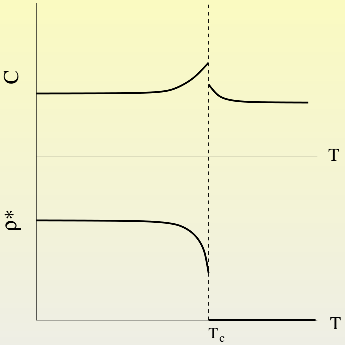
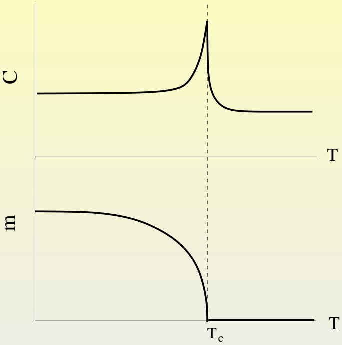
Examples of Macroscopic Phase Transitions

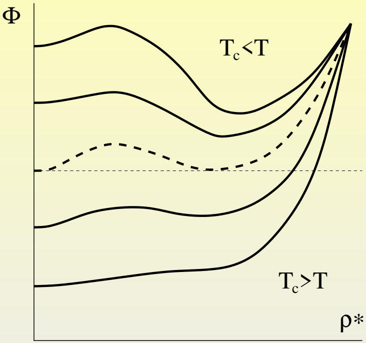


First order phase transition.
Liquid-gas

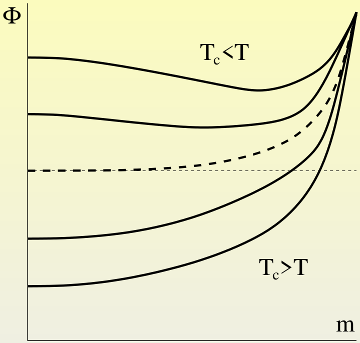


Second order phase transition.
Paramagnetic-ferromagnetic

What is happening at the phase transition point?



First order phase transition



Second order phase transition

Φ in the Landau theory

$$\Phi = A(T, ...)\beta^4 + B(T, ...)\beta^2 + C(T, ...)\beta$$

Quantum Phase Transitions

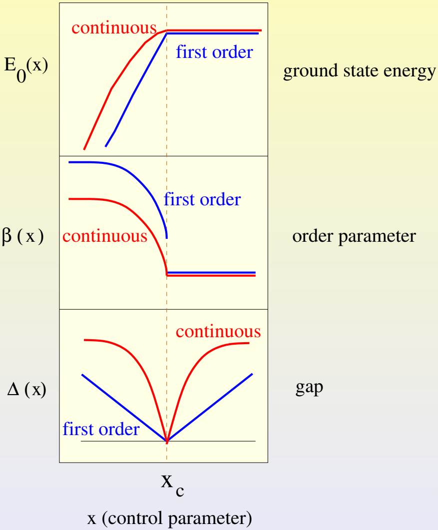
QPT occurs at some critical value, x_c, of the control parameter x that controls an interaction strength in the system's Hamiltonian $H(x)$. It is implicit a zero temperature.

$$\hat{H} = x\,\hat{H}_1 + (1-x)\,\hat{H}_2$$

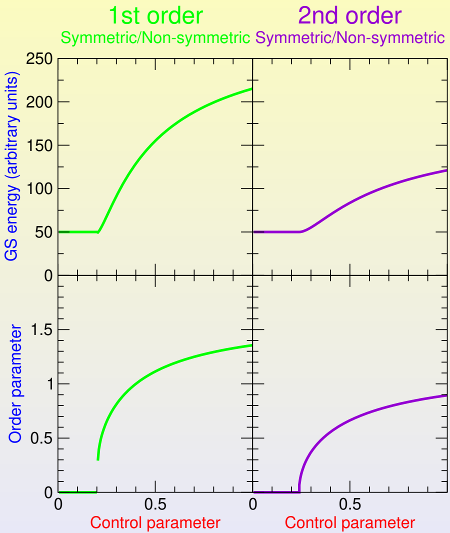
At the critical point:

- The ground state energy is nonanalytic.
- The gap Δ between the first excited state and the ground state vanishes.

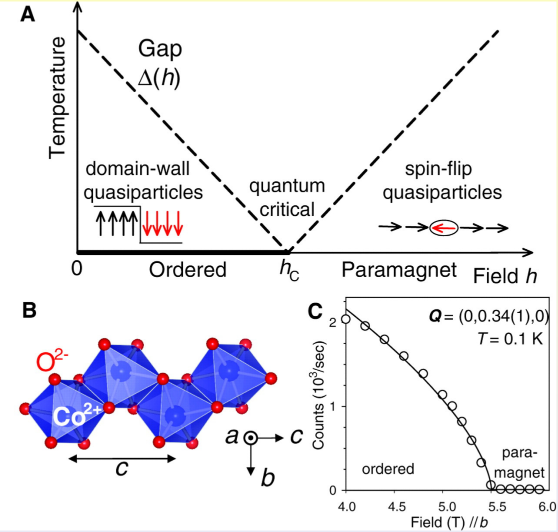
Quantum Phase Transitions



The variation of the order parameter



QPT: experimental example for an Ising chain



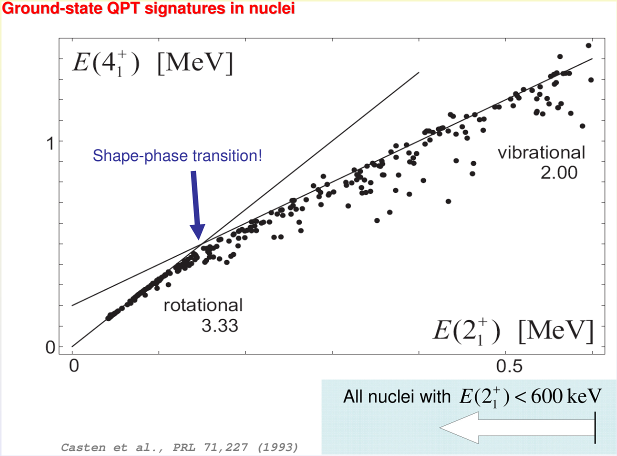
R. Coldea et al., Science 327, 177-180 (2010).

Challenges

- It is a finite system, therefore abrut changes, if any, are smoothed out.
- There is not a true control parameter.
- How can we define an order parameter?
- How can we define the phases of the system?
- The phase transition does not characterize a single nucleus, but is a property of a entire region.
- We are treating with a quantum system.

Fingerprints

Energy ratios.



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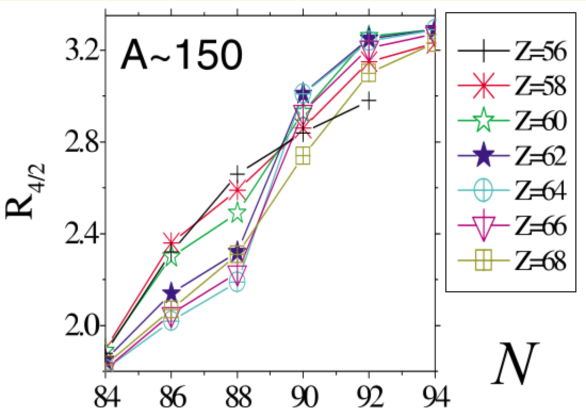
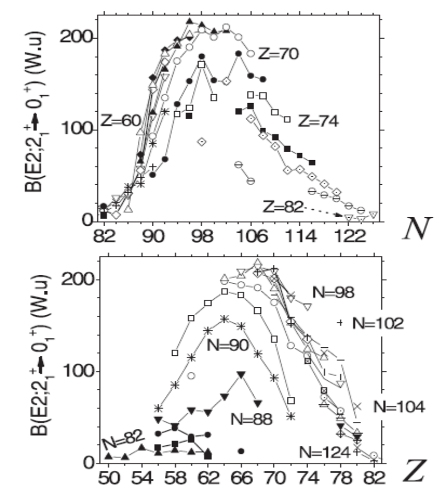
Fingerprints

More about energy ratios.



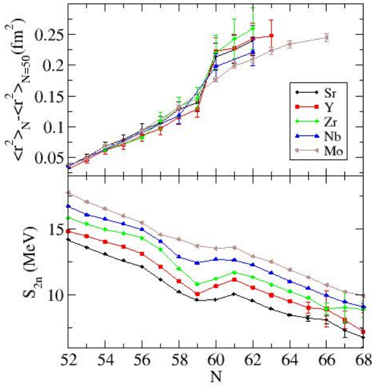
Fingerprints

Transition rates.



Fingerprints

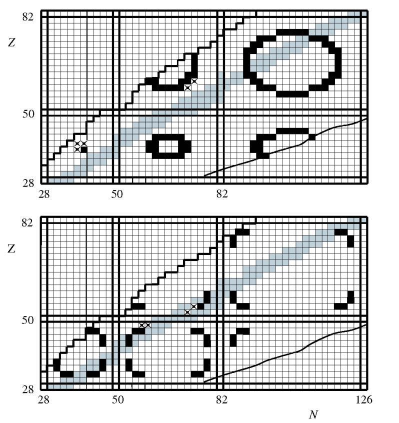
Two-neutron separation energies and radii



Fingerprints

Where to look for? The emergence of collective behaviour in the complex many-body system is driven by the competence between the proton-neutron interaction and the like-nucleon pairing interaction.

$$P = \frac{N_p N_n}{N_p + N_n} \rightarrow P \sim 2.5; 4-5$$



R. Casten and E A McCutchan,

J. Phys. G **34** R285, (2007).

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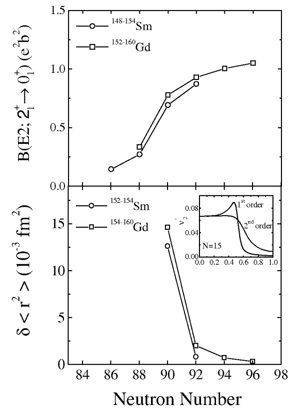
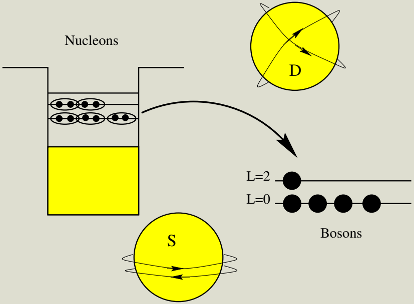
Finite-N effects

For finite particle number N, there is not a true singularity but rather a well defined scaling behaviour of the relevant quantities towards a singular large$-N$ limit.



The *I*nteracting *B*oson *M*odel

- The IBM is a model which describes the low lying collective states of medium mass and heavy nuclei.
- It can be considered as an approximation to the Shell Model. Two steps are necessary: truncation of the Shell Model space and bosonization of the nucleon pairs.



- The IBM can also be considered as the second quantization of the shape variables of the Geometric Collective Model.

Algebraic structure of the IBM

$$\begin{array}{c} s^\dagger, d_m^\dagger(m=0,\pm1,\pm2) \\ s, d_m(m=0,\pm1,\pm2) \end{array} \longrightarrow \begin{array}{c} \gamma_{lm}^\dagger, \gamma_{lm} \\ (l=0,2; \quad -l \le m \le l) \end{array}$$

$$[\gamma_{lm}, \gamma_{l'm'}^\dagger] = \delta_{ll'}\delta_{mm'}, [\gamma_{lm}^\dagger, \gamma_{l'm'}^\dagger] = 0, [\gamma_{lm}, \gamma_{l'm'}] = 0$$

- The dynamical algebra of the IBM is U(6).
  Generators $U(6)$: $\hat{G}_{ij} = \gamma_i^\dagger \gamma_j$, with $i, j = 1, \ldots, 6$.

$$[\hat{G}_{ij}, \hat{G}_{kl}] = \hat{G}_{il}\delta_{jk} - \hat{G}_{jk}\delta_{il}$$

- Every dynamic operator can be written in terms of $U(6)$ generators.

$$\hat{H} = \sum_{ij} \varepsilon_{ij}\gamma_i^\dagger\gamma_j + \sum_{ijkl} V_{ijkl}\gamma_i^\dagger\gamma_j^\dagger\gamma_k\gamma_l$$

$$\hat{T} = \sum_{ij} t_{ij}\gamma_i^\dagger\gamma_j$$

Generic Hamiltonian

$$\hat{H} = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \kappa_0 \hat{P}^\dagger \hat{P} + \kappa_1 \hat{L} \cdot \hat{L} + \kappa_2 \hat{Q} \cdot \hat{Q} + \kappa_3 \hat{T}_3 \cdot \hat{T}_3 + \kappa_4 \hat{T}_4 \cdot \hat{T}_4,$$

where $\hat{n}_s$ and $\hat{n}_d$ are the s and d boson number operators, respectively, and

$$\begin{aligned}
\hat{P}^\dagger &= \frac{1}{2} d^\dagger \cdot d^\dagger - \frac{1}{2} s^\dagger \cdot s^\dagger, \\
\hat{L} &= \sqrt{10} (d^\dagger \times \tilde{d})^{(1)}, \\
\hat{Q} &= s^\dagger \tilde{d} + d^\dagger \tilde{s} + \chi (d^\dagger \times \tilde{d})^{(2)}, \\
\hat{T}_3 &= (d^\dagger \times \tilde{d})^{(3)} \\
\hat{T}_4 &= (d^\dagger \times \tilde{d})^{(4)}.
\end{aligned}$$

Intrinsic state

- The trial wave function

$$|c\rangle = \frac{1}{\sqrt{N!}}(\Gamma_c^\dagger)^N|0\rangle,$$

where

$$\Gamma_c^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left(s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\,(d_2^\dagger + d_{-2}^\dagger)\right).$$

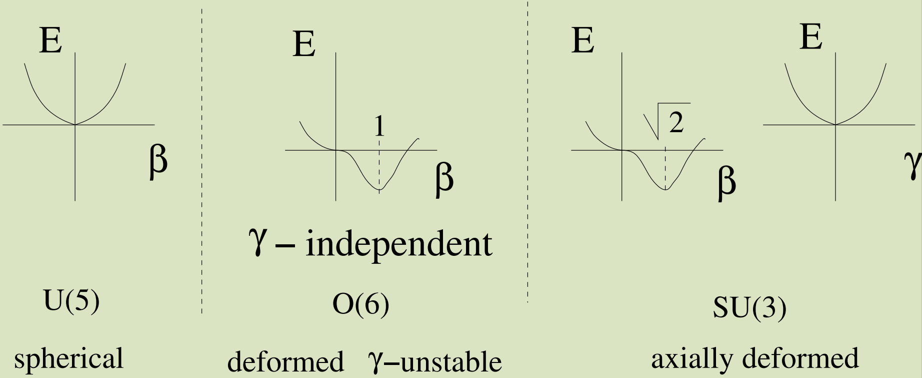
- The energy surface

$$\begin{aligned}\langle c|H|c\rangle &= \frac{N}{5(1+\beta^2)}\Big(5\,\varepsilon_s + 25\,\kappa_2 + \beta^2\,(5\,\varepsilon_d - 3\,\kappa_1 + 5\,\kappa_2 + 5\,\chi^2\,\kappa_2 - 7\,\kappa_3 + 9\,\kappa_4)\Big)\\ &+ \frac{N(N-1)}{140(1+\beta^2)^2}\Big(35\,\kappa_0 + \beta^2(-70\,\kappa_0 + 560\,\kappa_2)\\ &\quad -80\,\sqrt{14}\,\beta^3\,\chi\,\cos(3\,\gamma)\,\kappa_2 + \beta^4(35\,\kappa_0 + 40\,\chi^2\,\kappa_2 + 72\,\kappa_4)\Big).\end{aligned}$$

- The symmetry limits
  $U(5)$ limit $\rightarrow \beta = 0$.
  $SU(3)$ limit $\rightarrow \beta = \sqrt{2}, \gamma = 0, \pi/3$.
  $O(6)$ limit $\rightarrow \beta = 1$, γ unstable nucleus.

Intrinsic state

IBM energy surface



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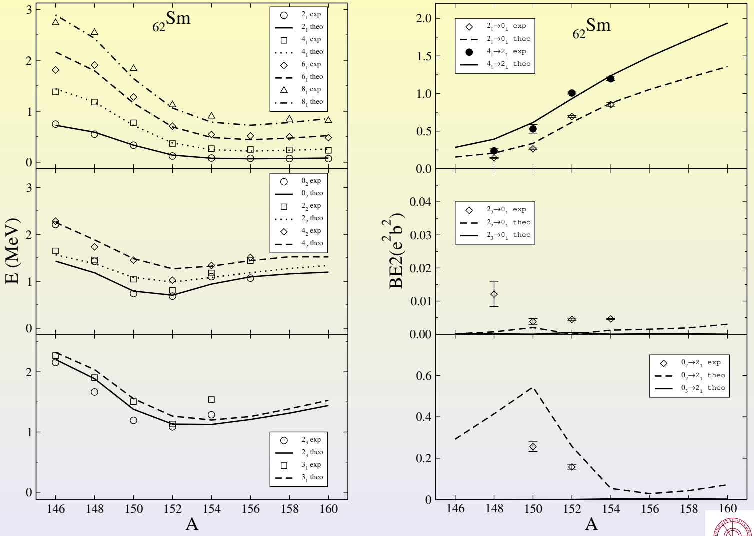
Spectra: experiment vs theory



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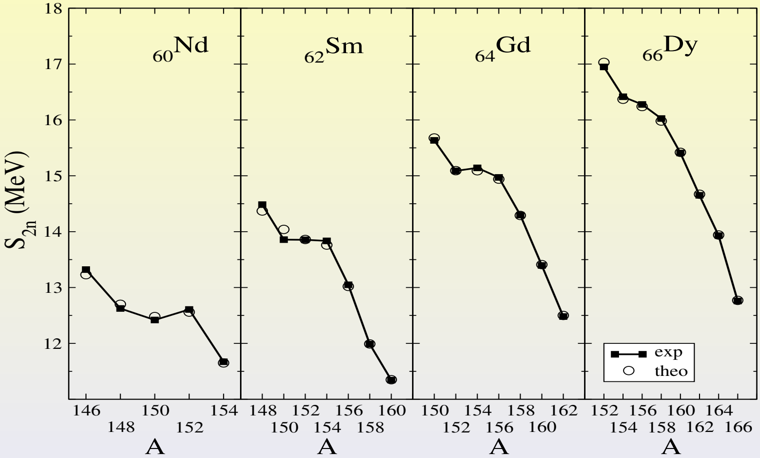
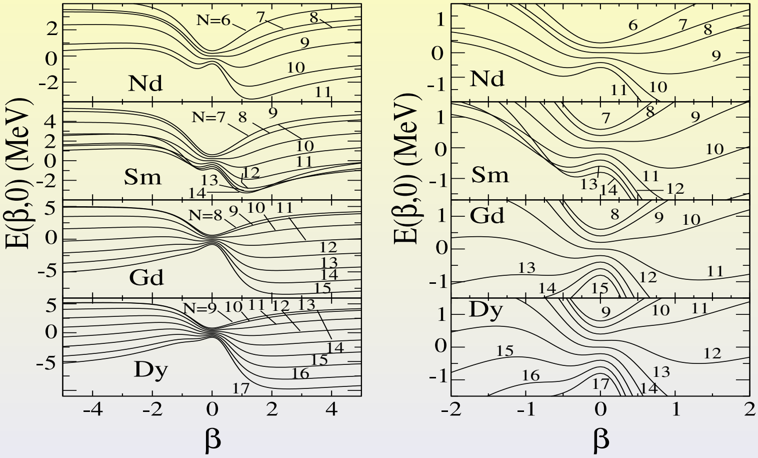
Two neutron separation energy



Energy surfaces



Intrinsic state

$$\langle c|\hat{H}|c\rangle = \frac{\epsilon}{(1+\beta^2)^2}\Big(\beta^4 + r_1\beta^2(\beta^2+2) - r_2\beta^3\cos(3\,\gamma)\Big),$$

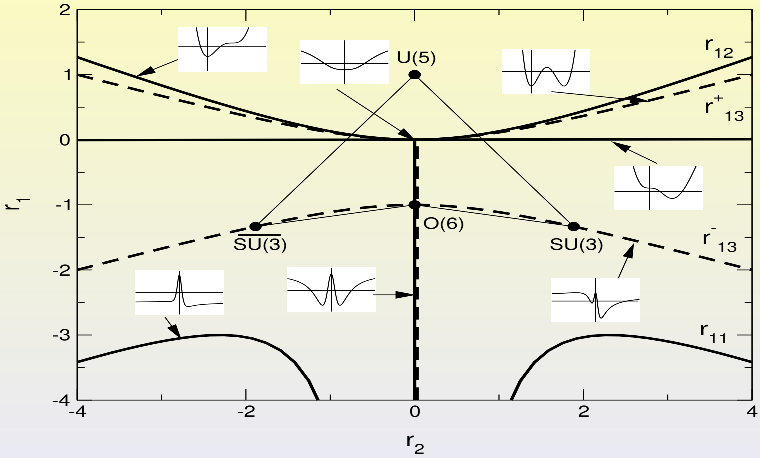
Essential parameters

$$r_1 = \frac{a_3 - u_0 + \tilde{\varepsilon}/(N-1)}{2a_1 + \tilde{\varepsilon}/(N-1) - a_3}, \qquad r_2 = \frac{2a_2}{2a_1 + \tilde{\varepsilon}/(N-1) - a_3},$$

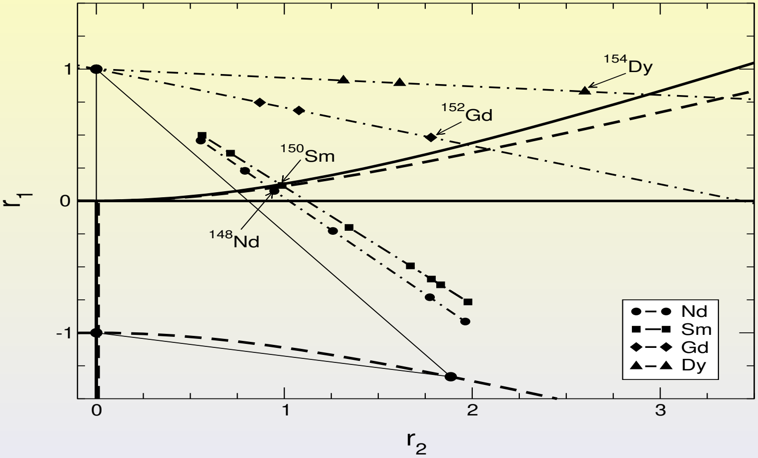
where

$$\tilde{\varepsilon} = \varepsilon_d + 6\,\kappa_1 - \frac{9}{4}\,\kappa_2 + \frac{7}{5}\,\kappa_3 + \frac{9}{5}\,\kappa_4,$$
$$a_1 = \frac{1}{4}\,\kappa_0 + \frac{1}{2}\,\kappa_2 + \frac{18}{35}\,\kappa_4,$$
$$a_2 = 2\sqrt{2}\,\kappa_2,$$
$$a_3 = -\frac{1}{2}\,\kappa_0 + 4\,\kappa_2, \quad u_0 = \frac{\kappa_0}{2}.$$

The phase diagram

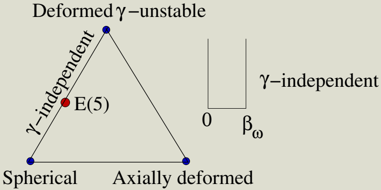


The phase diagram



E(5) critical point symmetry

Iachello introduced in 2000 the concept of critical point symmetry, in the framework of the Bohr Hamiltonian, for the shape transition from spherical to deformed $\gamma-$unstable shapes. The flat potential in the $\beta-$degree of freedom characteristic of the second order phase transition from spherical to deformed-$\gamma-$unstable shape is modeled as a infinite square well.



In this situation eigenvalues and eigenfunctions can be obtained analytically.

E(5)

In cases in which the potential depends only on β, $V(\beta,\gamma) = U(\beta)$, the wave function can be factorized as

$$\Psi(\beta,\gamma,\theta_i) = f(\beta)\Phi(\gamma,\theta_i)$$

where θ_i stands for the three Euler angles, and the Schrödinger equation can be split in two equations,

$$\left[-\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} + \frac{1}{4}\sum_{\kappa}\frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right]\Phi(\gamma,\theta_i) = \tau(\tau+3)\Phi(\gamma,\theta_i)\ ;\ \tau = 0, 1, 2, \ldots\ ,$$

and

$$\left[-\frac{\hbar^2}{2B}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{\tau(\tau+3)}{\beta^2}\right) + U(\beta)\right] f(\beta) = Ef(\beta)\ .$$

If $U(\beta)$ can be modeled as a five dimensional infinite well, the problem is exactly solvable and the corresponding symmetry is called E(5).

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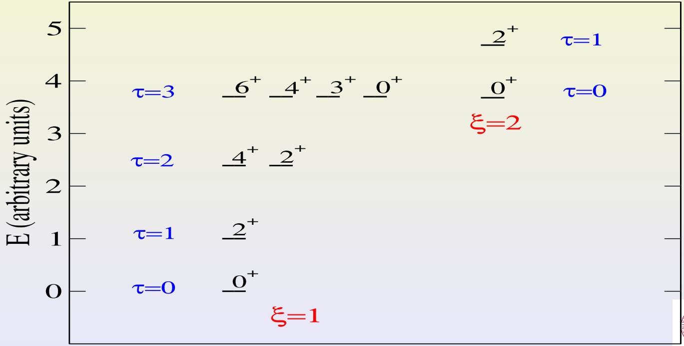
E(5)

The E(5) states are labelled by $|\xi\ \tau\ L\ M\rangle$.
ξ is a label related to the solution of the Schrödinger equation in the β variable as mentioned above,
τ is the label associated to the O(5) algebra
L is the total angular momentum and M its projection on one axis.



E(5): energy ratios

	$\xi = 1$	$\xi = 2$	$\xi = 3$	$\xi = 4$
$\tau = 0$	0.00	3.03	7.58	13.64
$\tau = 1$	1.00	4.80	10.11	16.93
$\tau = 2$	2.20	6.78	12.86	20.44
$\tau = 3$	3.59	8.97	15.81	24.16

Vibrator: $R_{4/2} = 2$
γ-unstable rotor $R_{4/2} = 2.5$

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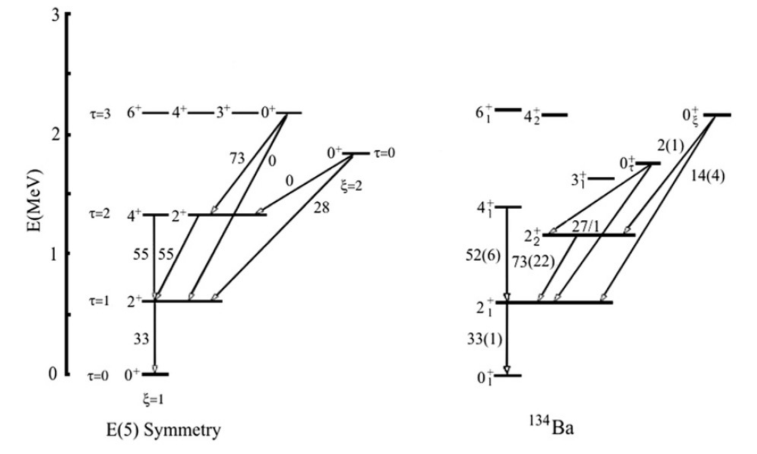
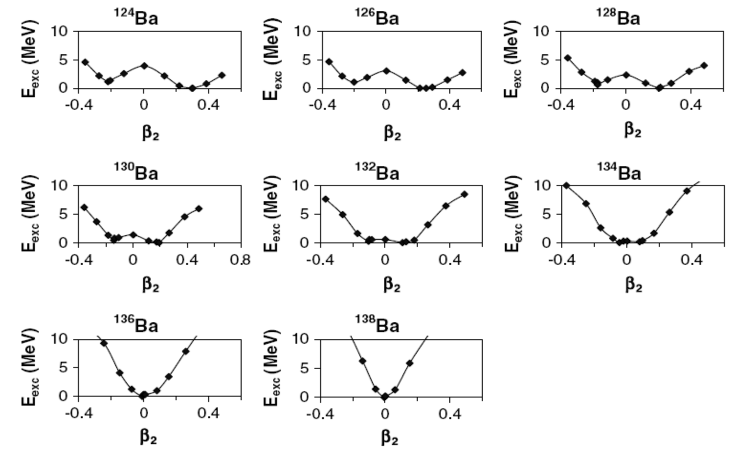
E(5): empirical realizations



E(5): microscopic derivations



Relativistic mean field (Fossion et al., PRC 73, 044310 (2006)).

Summary

- We have presented the concept of Quantum Phase Transitions.
- We have seen some fingerprints of QPT in atomic nuclei.
- We have analyzed QPT in the framework of the IBM
- We have presented the concept of Critical Point Symmetry.

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Muchas gracias

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