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Nuclear Physics at the border: shape coexistence and quantum phase transitions

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in collaboration
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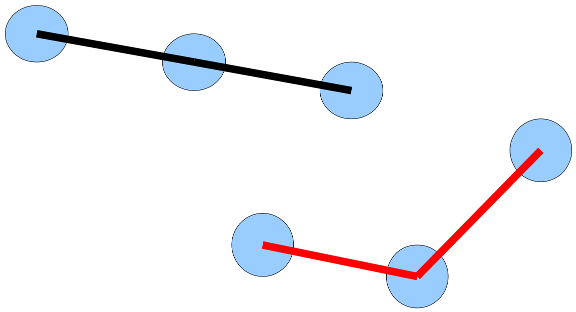
What is shape coexistence?

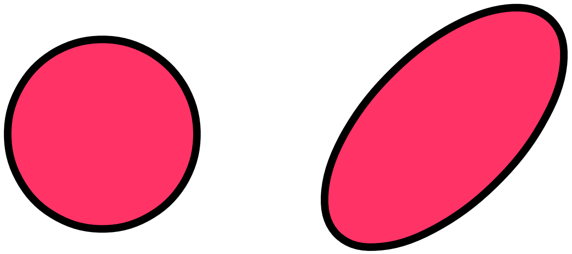
It appears in quantum systems where eigenstates with very different shapes coexist.

Therefore, it is implicit the existence of a geometric interpretation.

Molecules

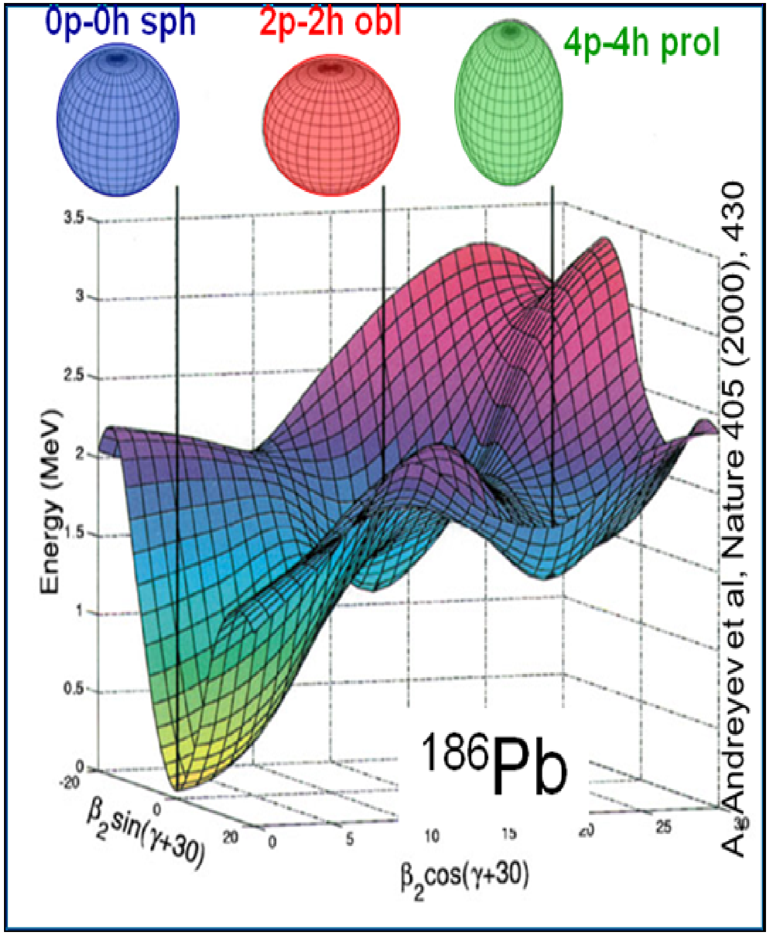


Nuclei

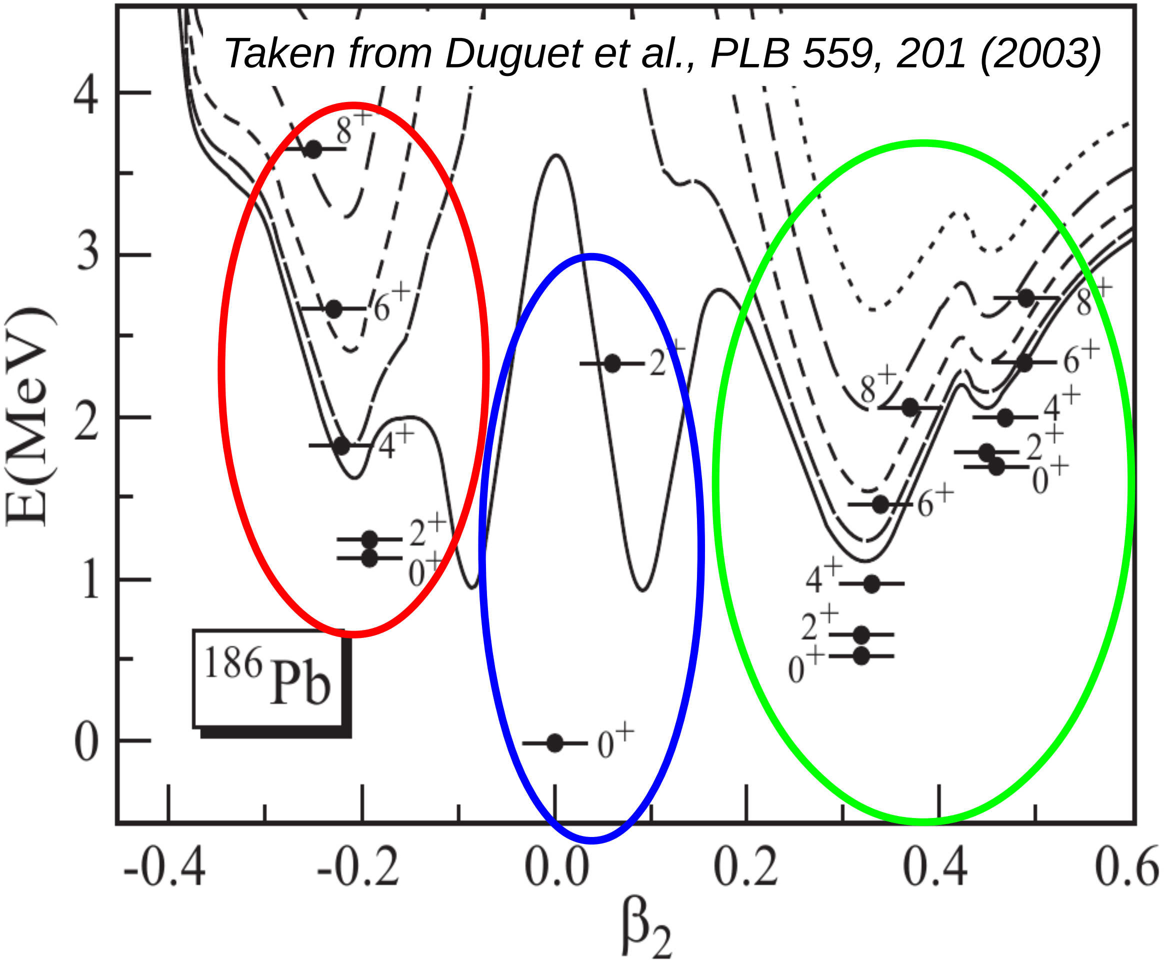


Shape coexistence

It appears in quantum systems where eigenstates with very different density distribution coexist.



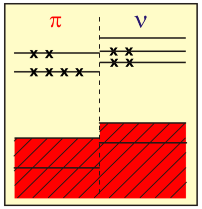
A. Andreyev et al, Nature 405 (2000), 430

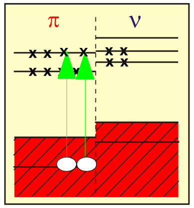


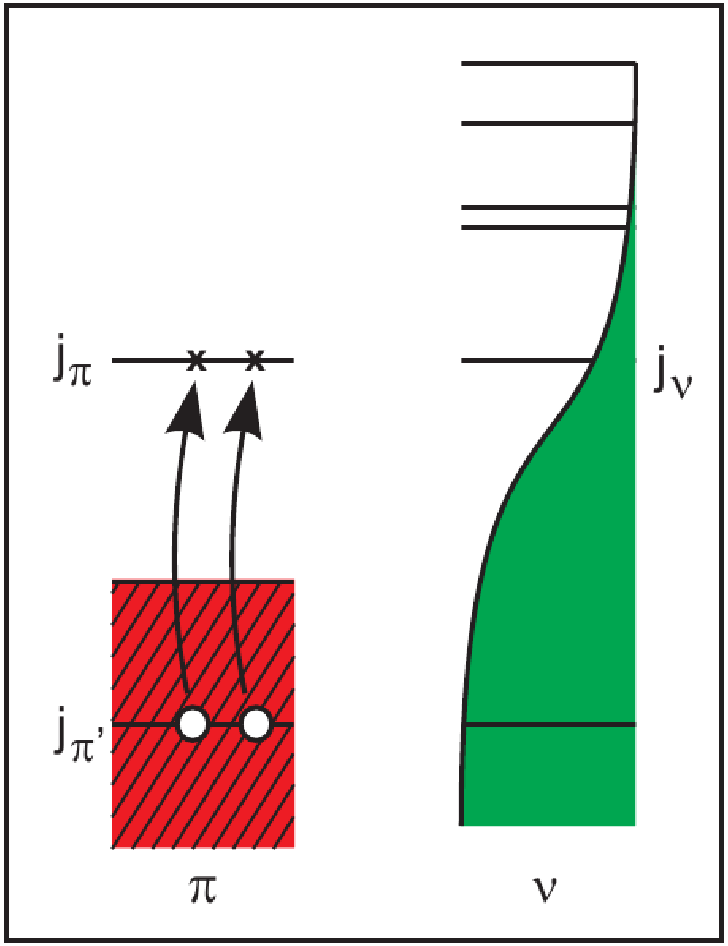
Taken from Duguet et al., PLB 559, 201 (2003)

Shell Model

- For nuclei near to closed shells, either for neutrons or for protons, it can be energetically favorable to have excitations of 2p-2h, 4p-4h … crossing the energy gap.
- The np-nh excitations have a lower excitation energy than expected due to the correlation energy: pairing and deformed correlations.
- Restricted to light and medium-heavy nuclei, at present.

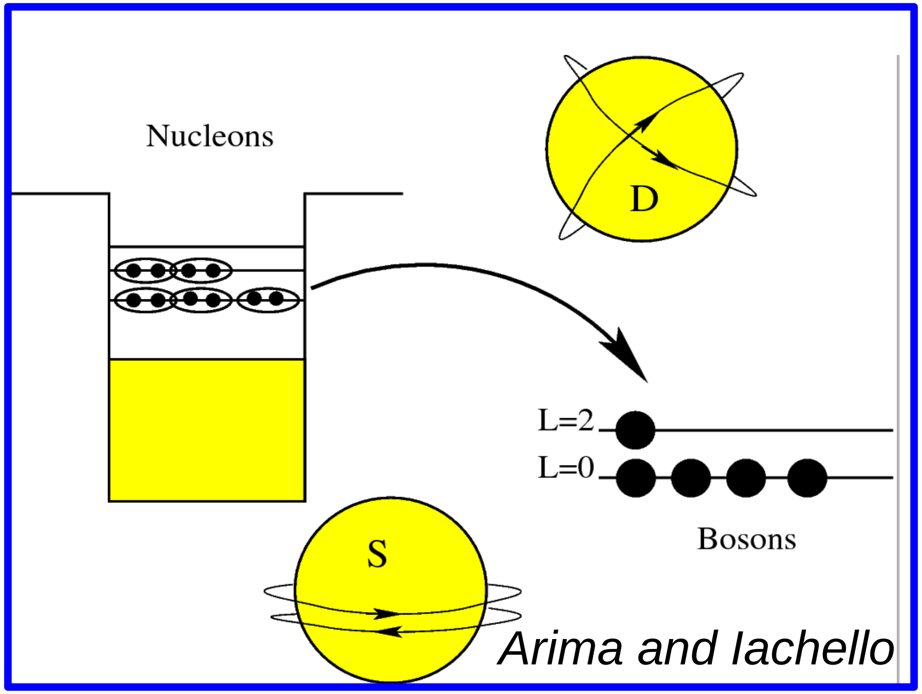
$\phi(J,M)=a(J,M)$ 

$+\ b(J,M)$ 



In heavy nuclei the huge model space imposes some kind of truncation: symmetry dictated truncation.

Interacting Boson Model (IBM)



Arima and Iachello

Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D). Those pairs are then described by means of bosons: s and d.

$$s^\dagger, d^\dagger_m (m = 0, \pm 1, \pm 2)$$

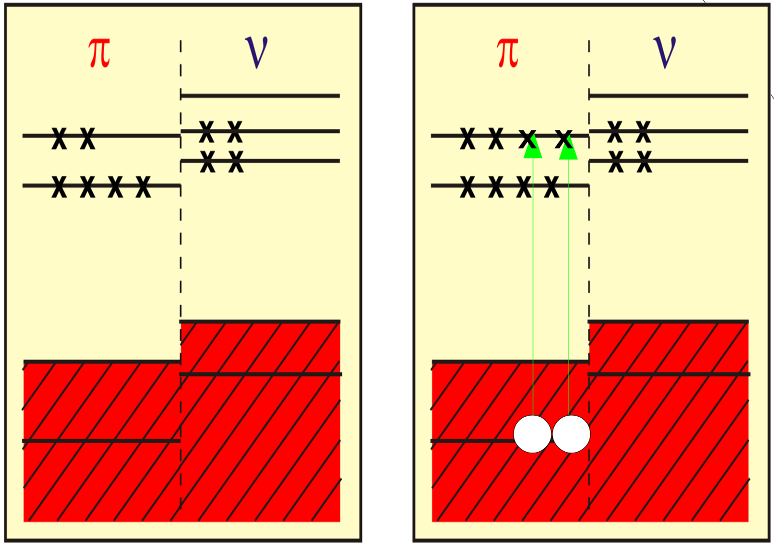
$$s, d_m (m = 0, \pm 1, \pm 2)$$

$$\boxed{\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L}}$$

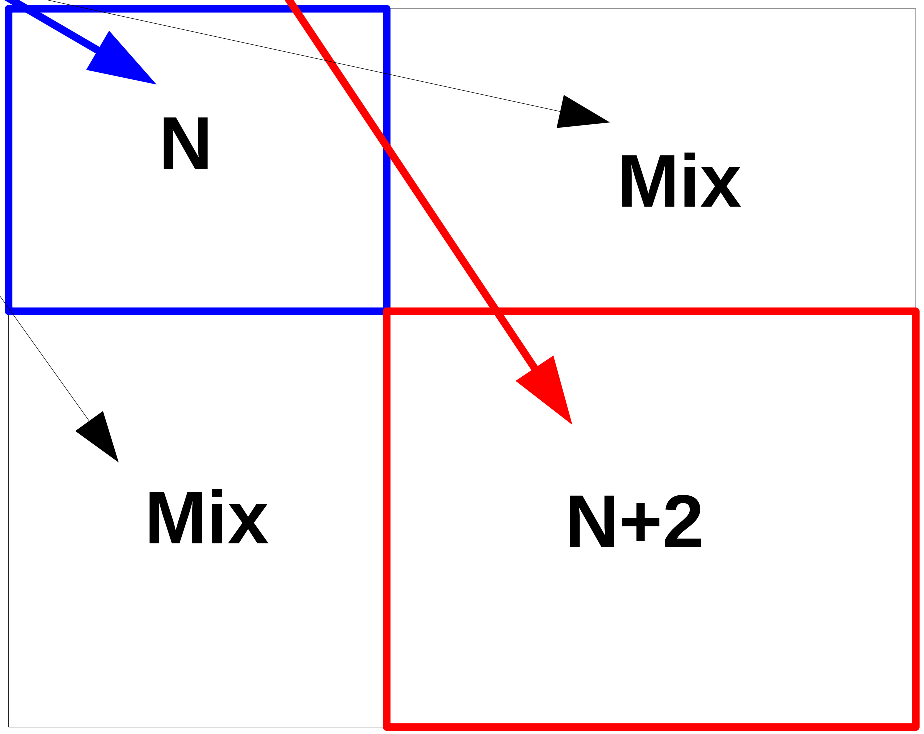
Interacting Boson Model

(configuration mixing)

$$\hat{H} = \hat{P}_N^\dagger \hat{H}_{\rm ECQF}^N \hat{P}_N + \hat{P}_{N+2}^\dagger \left(\hat{H}_{\rm ECQF}^{N+2} + \Delta^{N+2}\right) \hat{P}_{N+2} + \hat{V}_{\rm mix}^{N,N+2},$$



N **N+2**



N	**Mix**
Mix	**N+2**

Macroscopic/Classical Phase Transitions

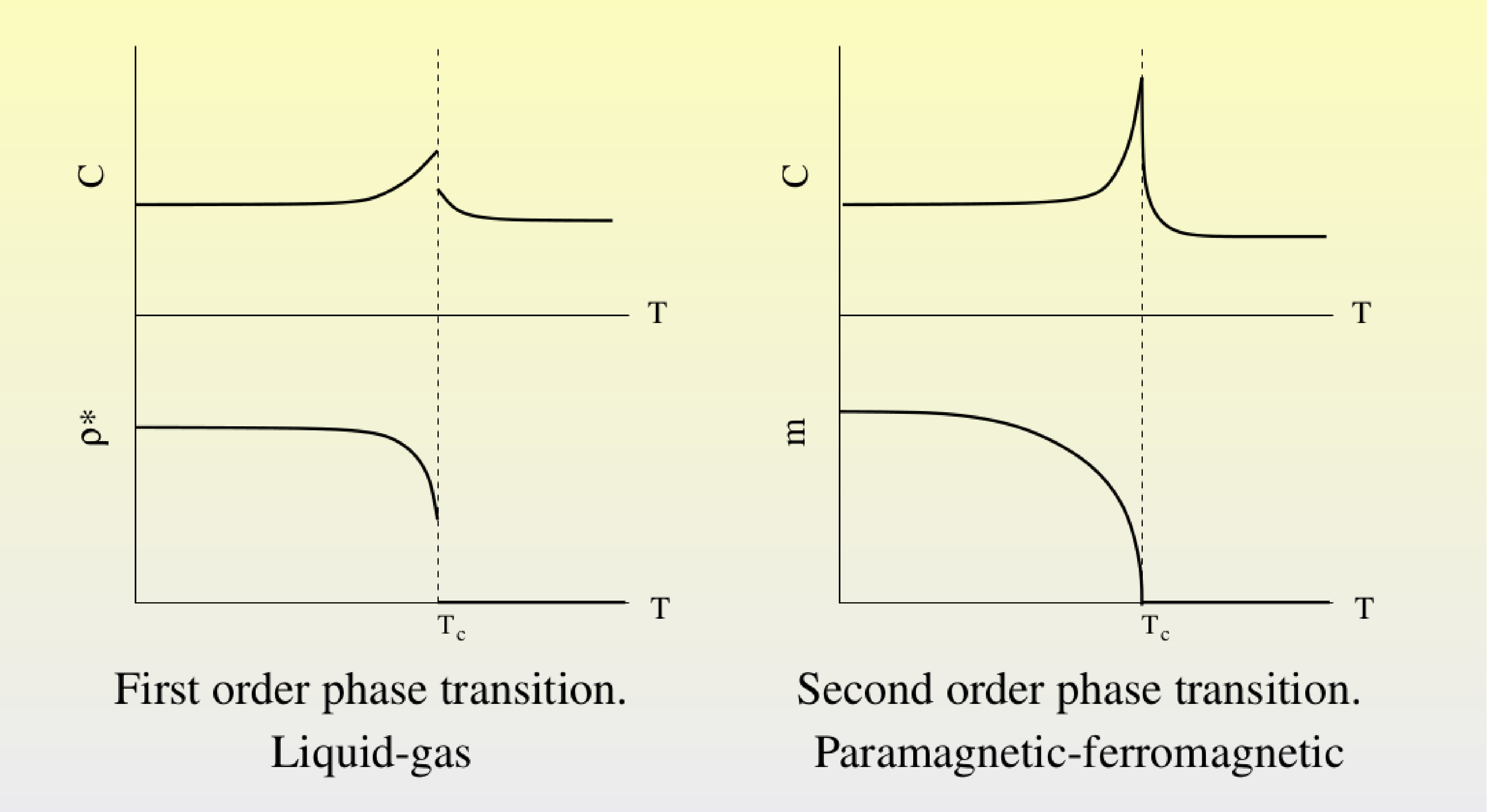
Definition of phase and phase transition

- Phase: state of matter that is uniform throughout, not only in chemical composition but also in physical properties.
- Phase Transition: abrupt change in one or more properties of the system.

The phase of the system

- Most stable phase of matter is the one with the lowest thermodynamical potential Φ. This is a function of some parameters that are allowed to change (F(T,V), F(T,B); G(T,p), G(T,M)).
- Φ is analogous to the potential energy, V(x), of a particle in a one dimensional well. The system looks for the minimum energy going into the bottom of the potential.

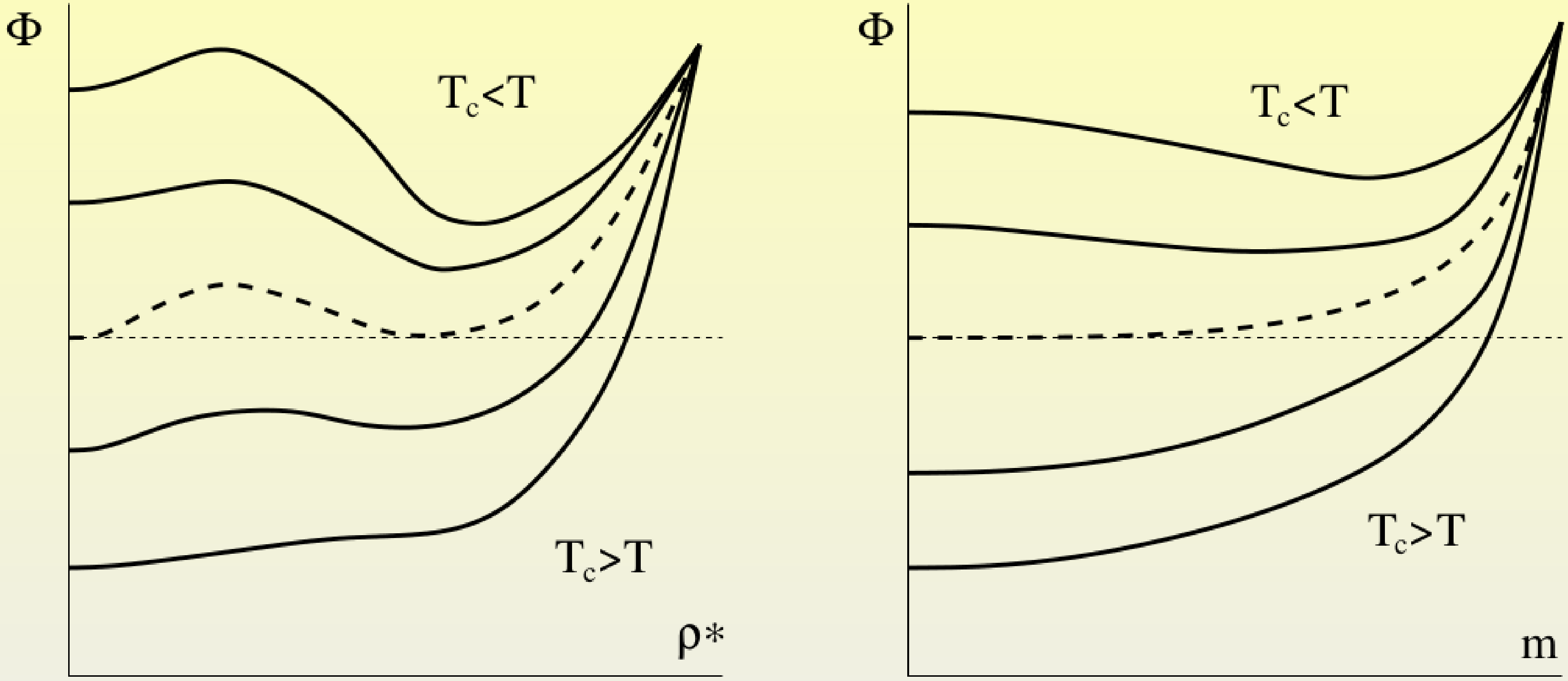
Examples of Macroscopic Phase Transitions



First order phase transition.
Liquid-gas

Second order phase transition.
Paramagnetic-ferromagnetic

What is happening at the phase transition point?



First order phase transition

Second order phase transition

Φ in the Landau theory

$$\Phi = A(T,...)\beta^4 + B(T,...)\beta^2 + C(T,...)\beta$$

What is a Quantum Phase Transition (QPT)?

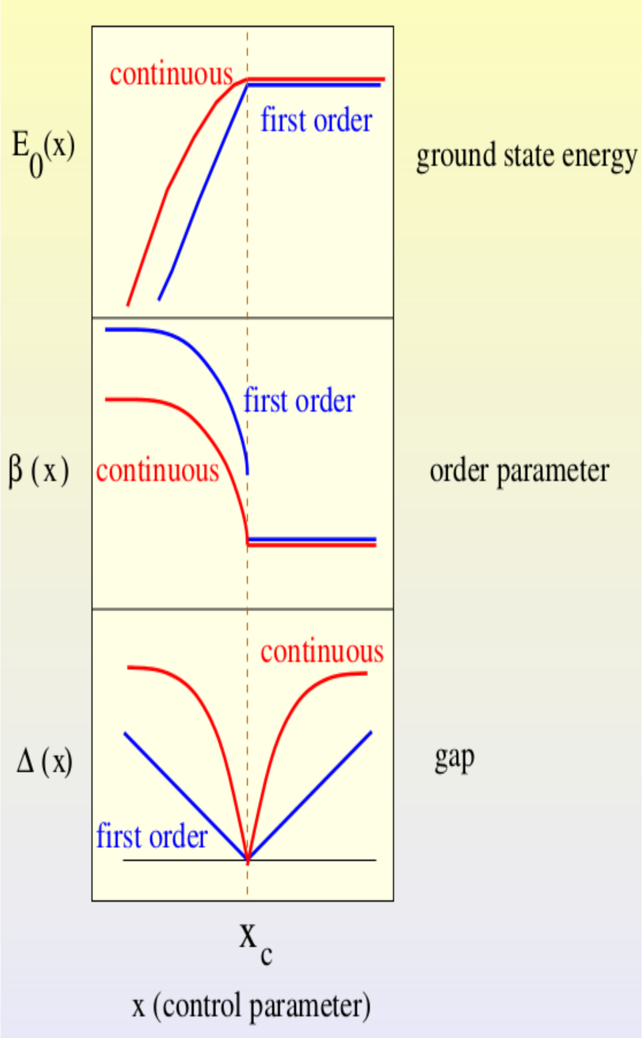
QPT occurs at some critical value, $\boldsymbol{x_c}$, of the control parameter *x* that controls an interaction strength in the system's Hamiltonian *H(x)*. It is implicit zero temperature.

$$H = x H_1 + (1-x) H_2$$

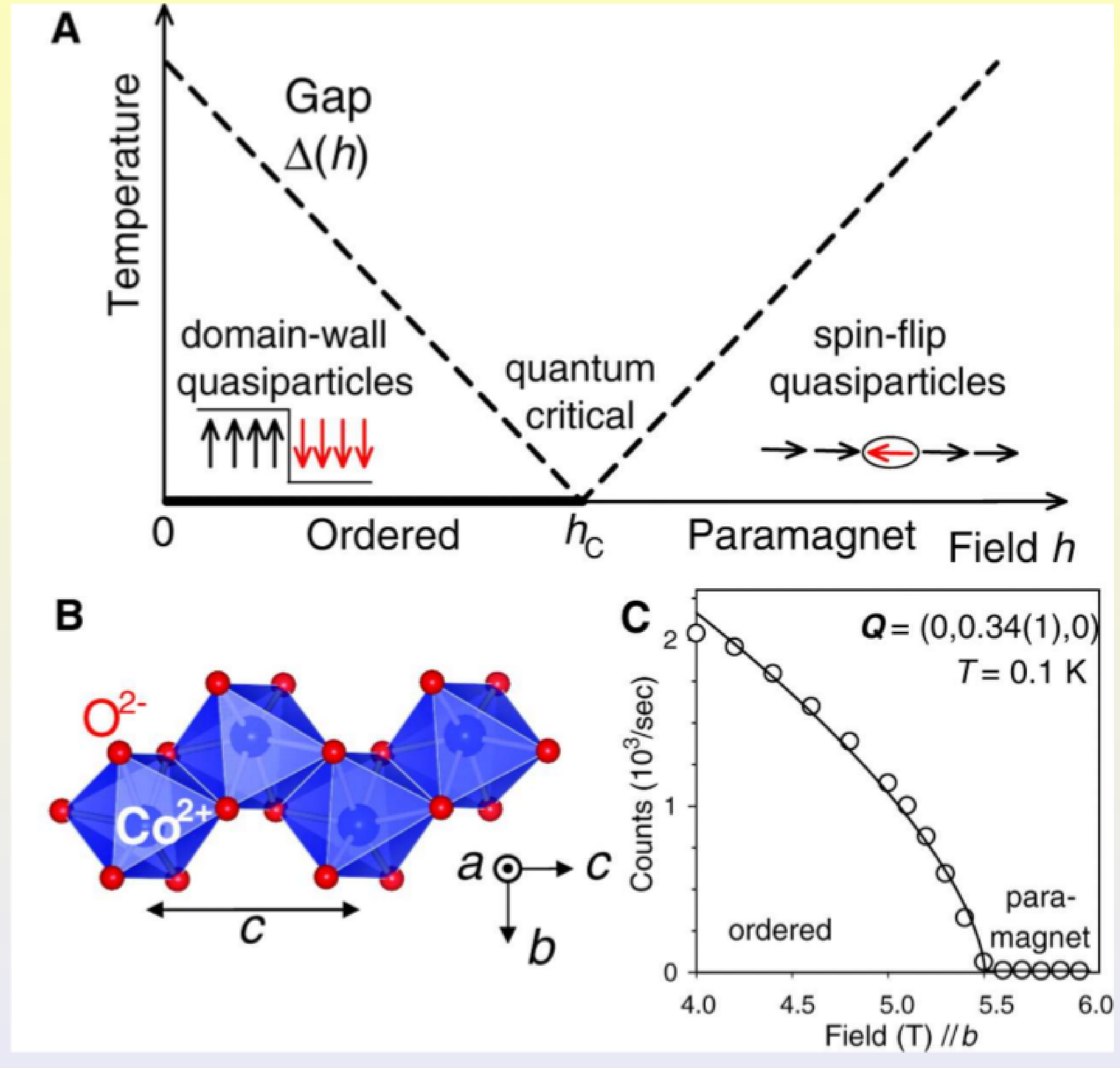
The value of the *order parameter (deformation)* suddenly changes when crossing the critical point.

At the critical point:

- The ground state energy is nonanalitic.
- The energy gap between the first excited state and the ground state vanishes.

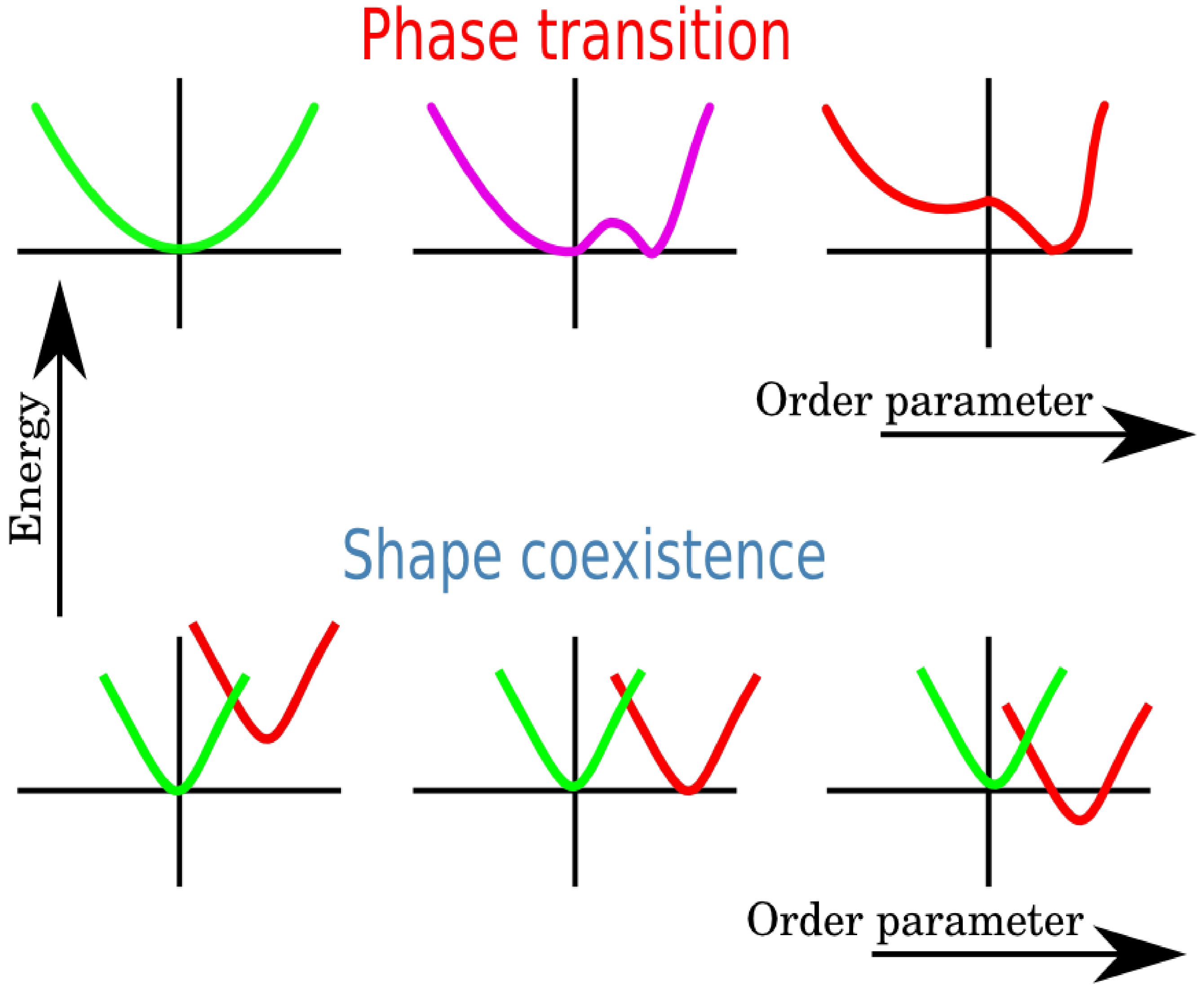


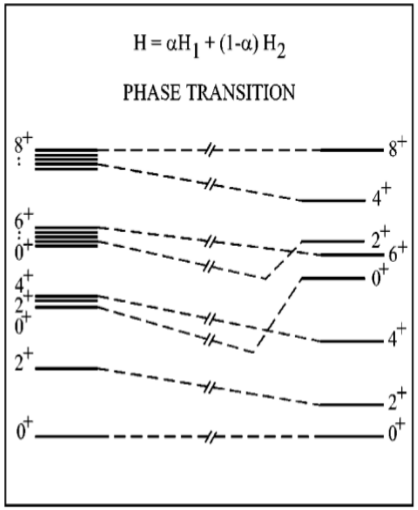
QPT: experimental example for an Ising chain

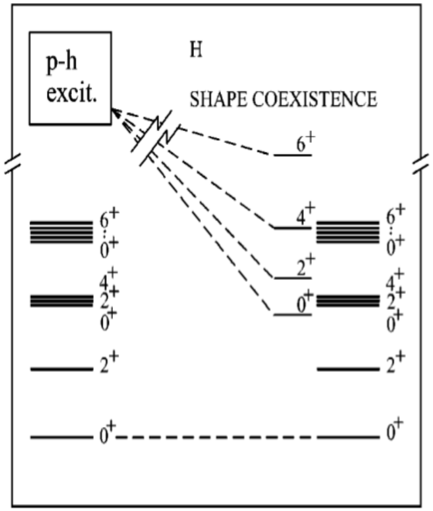


R. Coldea et al., Science 327, 177-180 (2010).

A schematic comparison

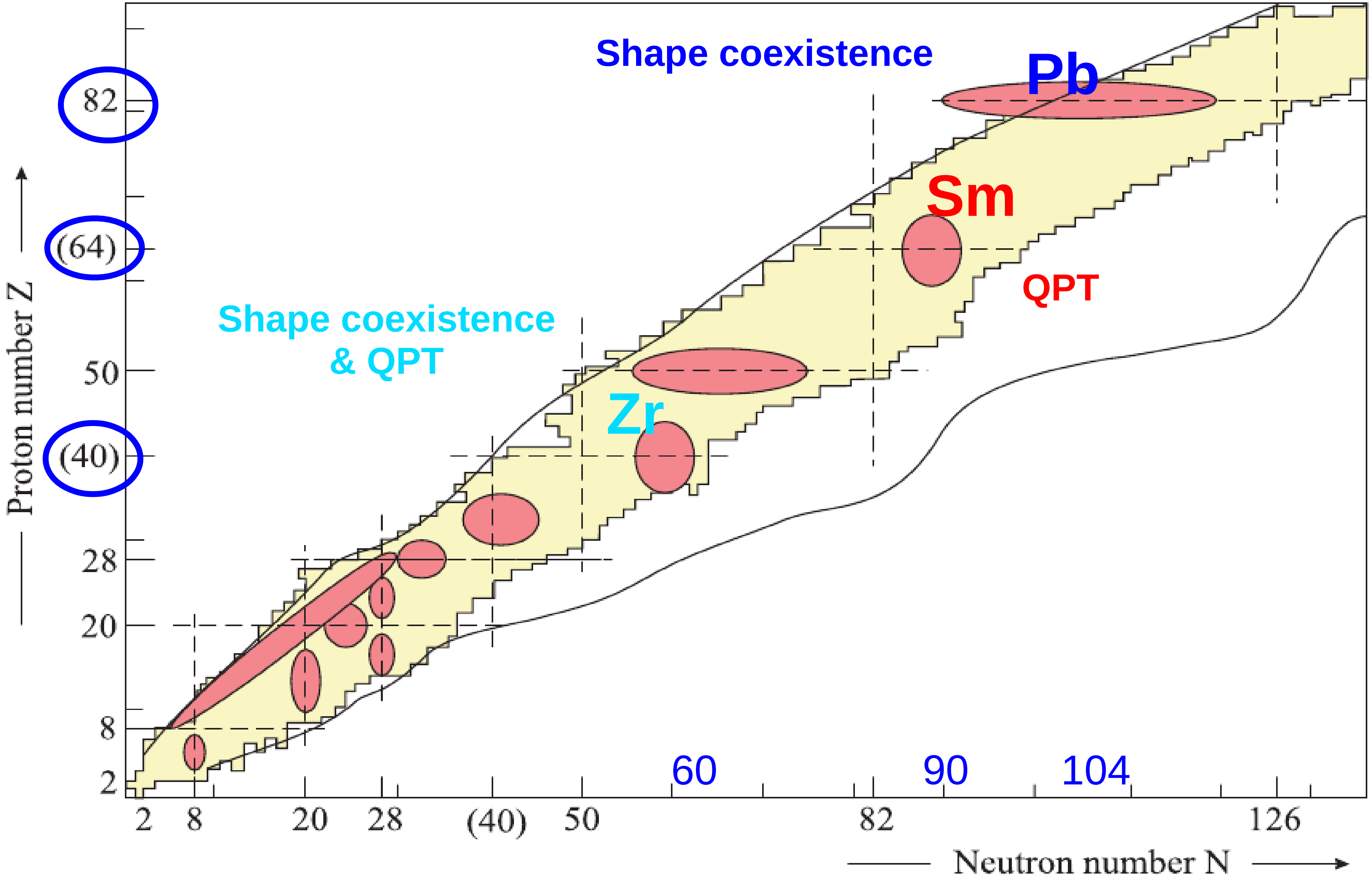






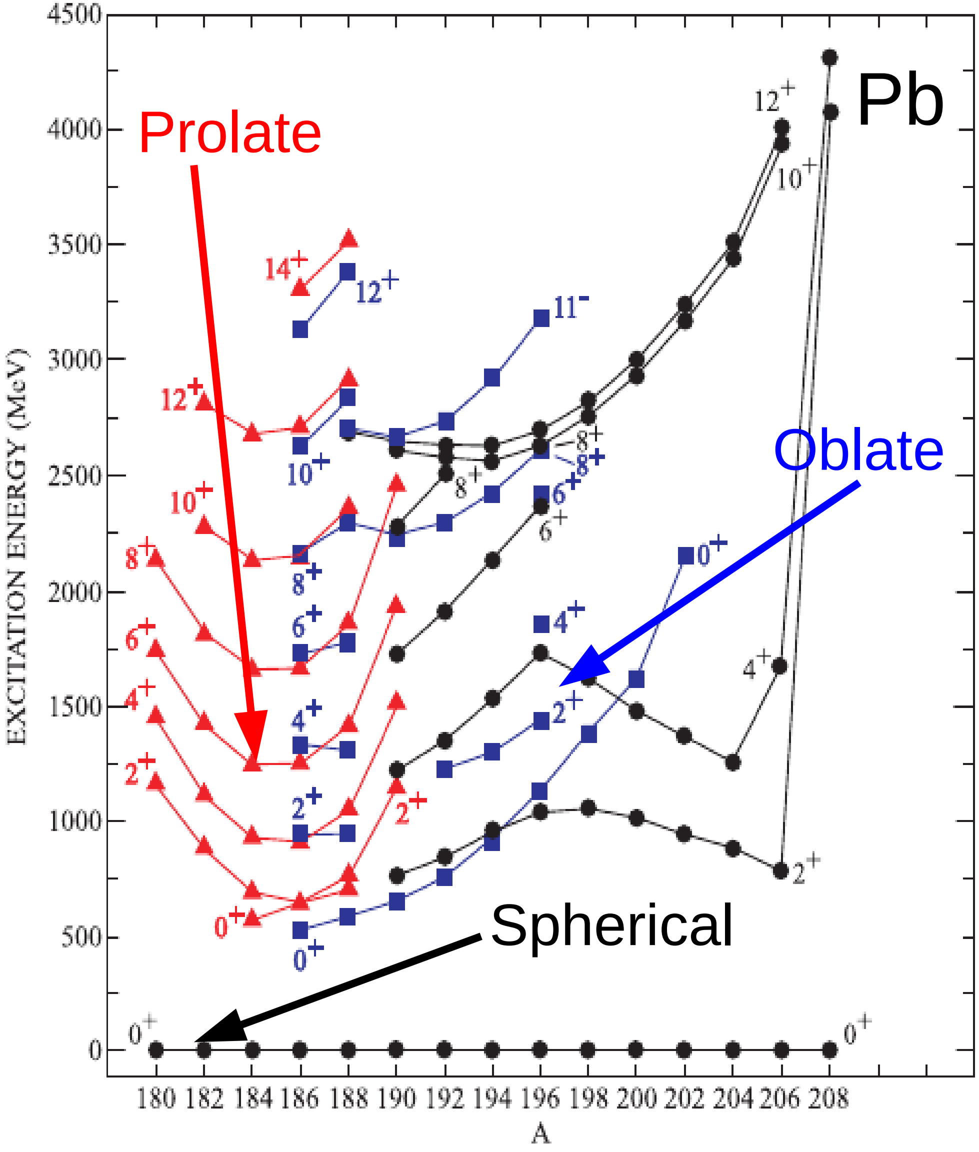
PRC 69, 054304 (2004)

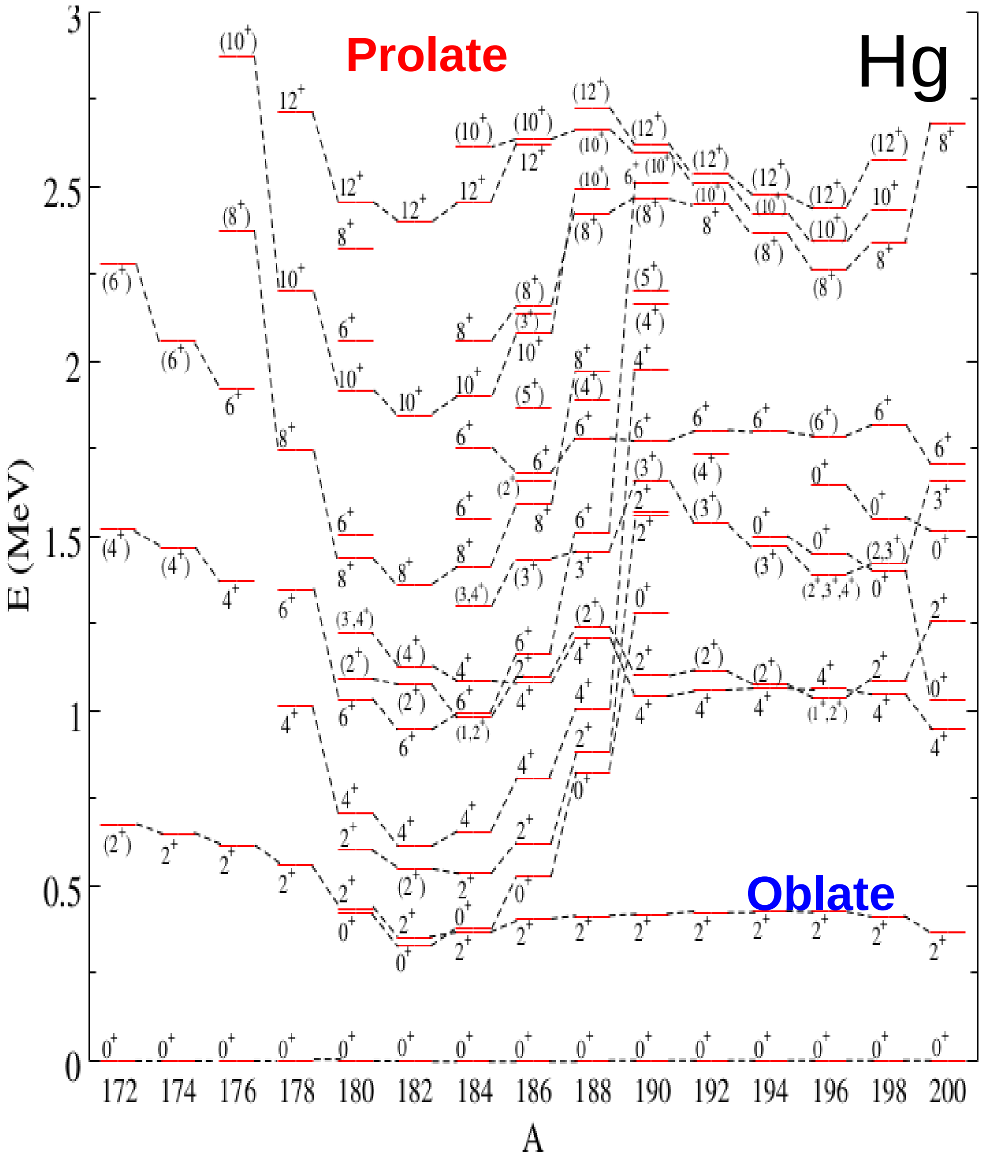
Regions of reference



K. Heyde and J. L. Wood, Rev. Mod. Phys. 83, 1467 (2011).

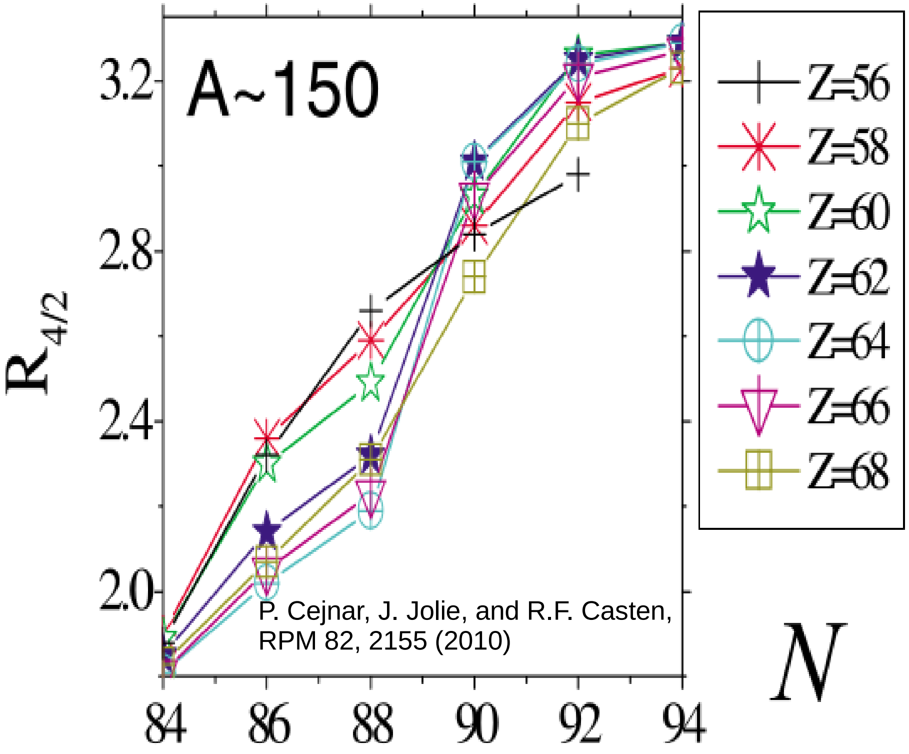
Shape coexistence key indicators



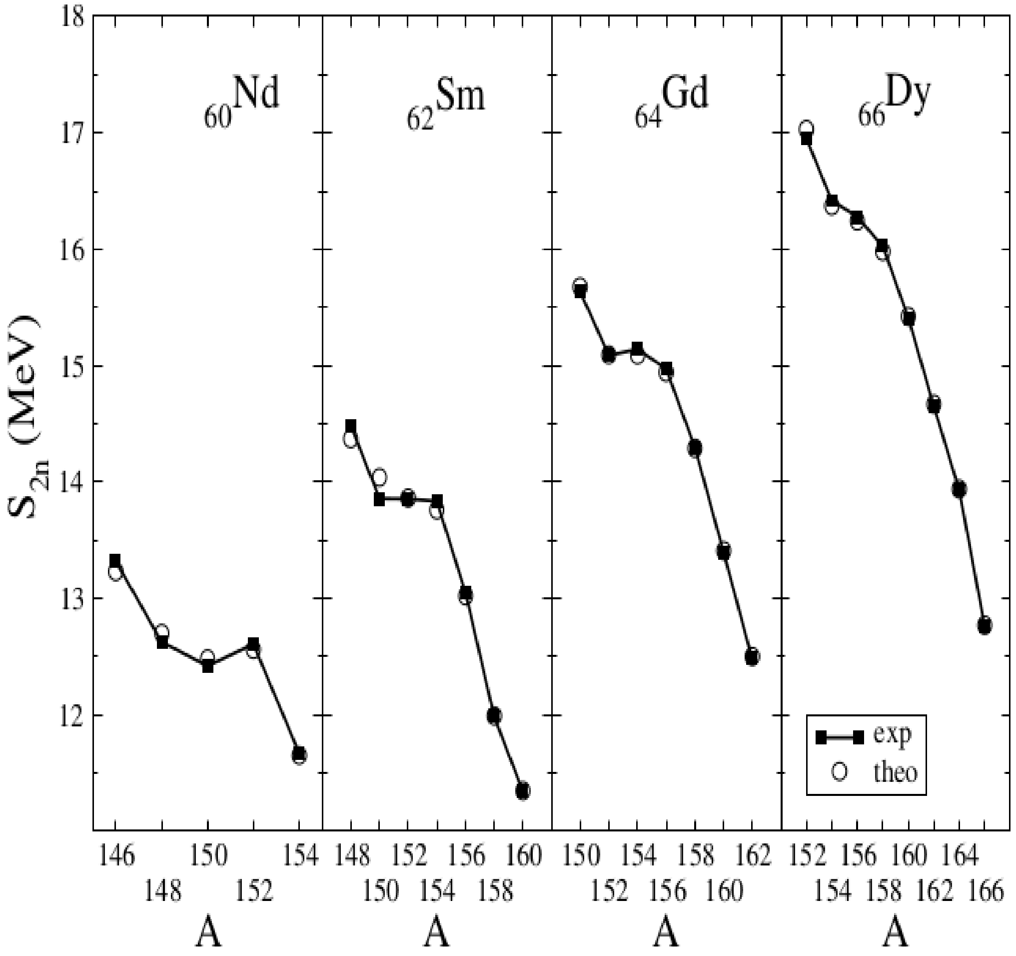


JEGR and K. Heyde, PRC 89, 014306 (2014)

QPT key indicators

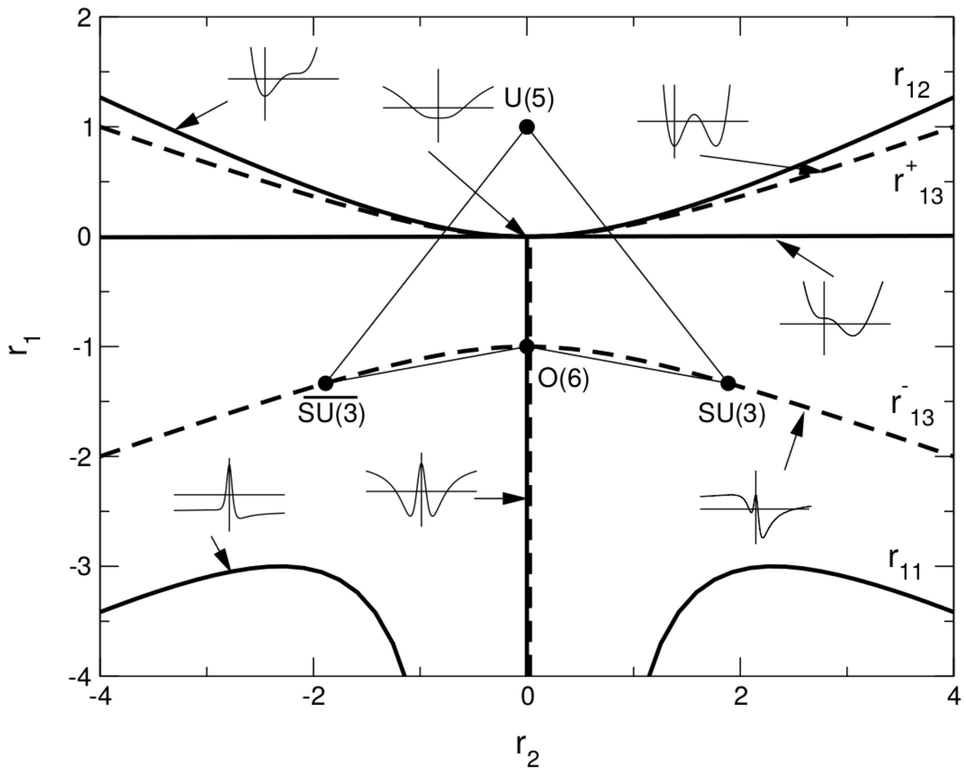


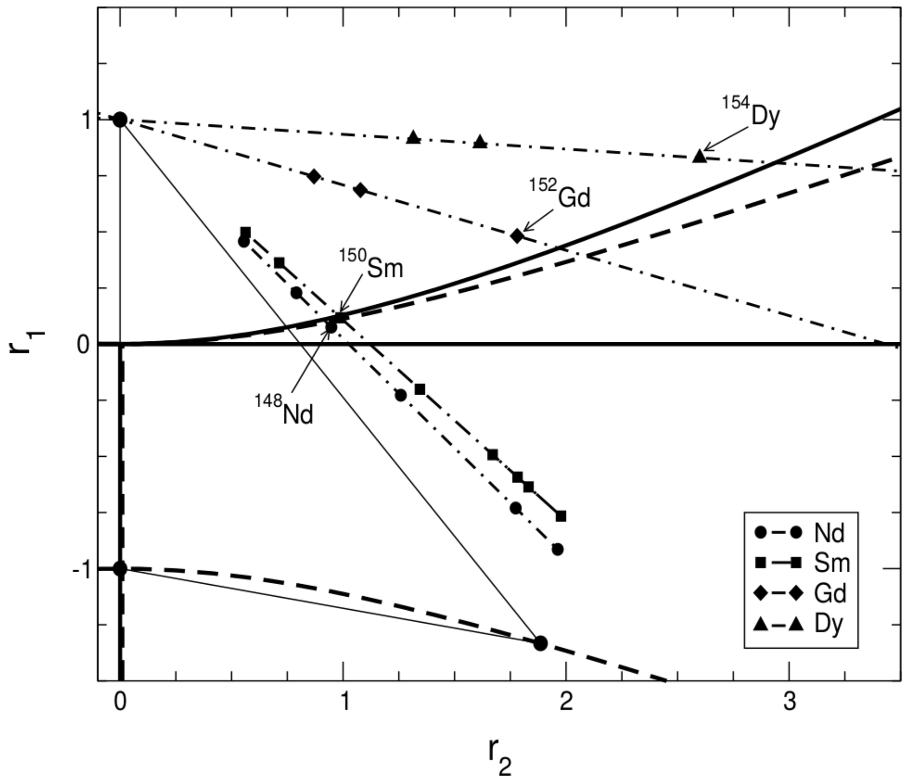
P. Cejnar, J. Jolie, and R.F. Casten, RPM 82, 2155 (2010)



E(4+)/E(2+) can be used as an order parameter and, therefore is a key observable to find where a QPT develops.

QPT: energy surfaces and phase diagram

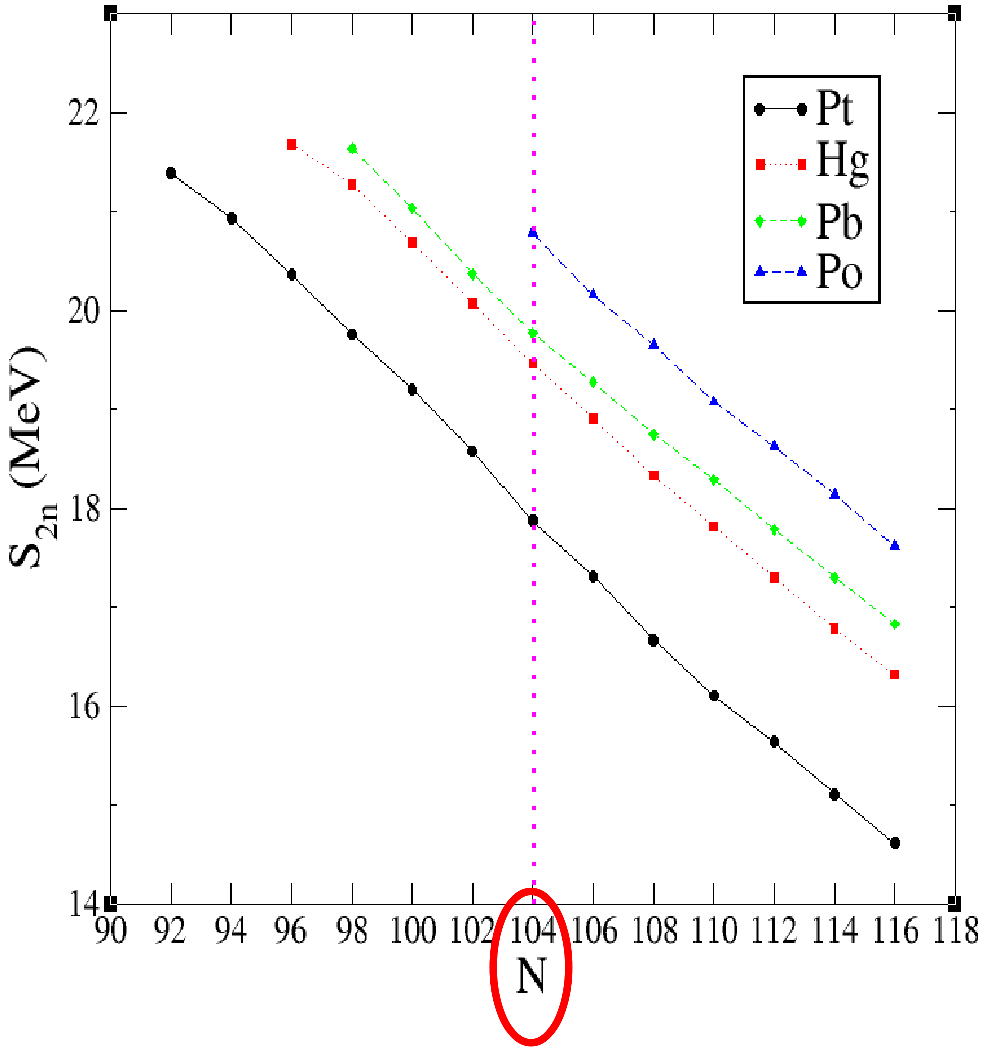


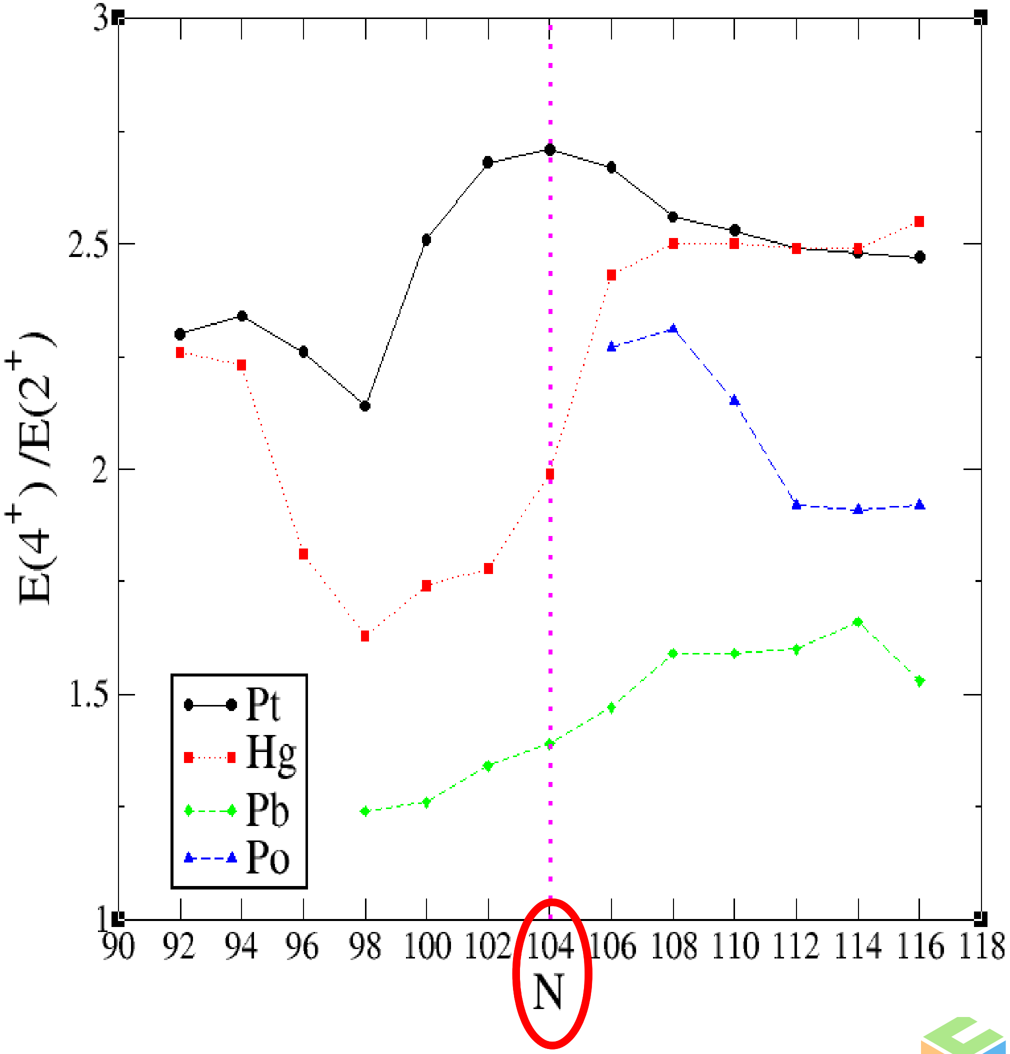


Some open questions

- In both cases there is a rapid change in the deformation of certain states.
- Are both approaches compatible in an unified way?
- Can a Quantum Phase Transition be described in terms of the onset of intruder configurations?
- Is shape coexistence always present “before” a Quantum Phase Transition sets in?

Pb region

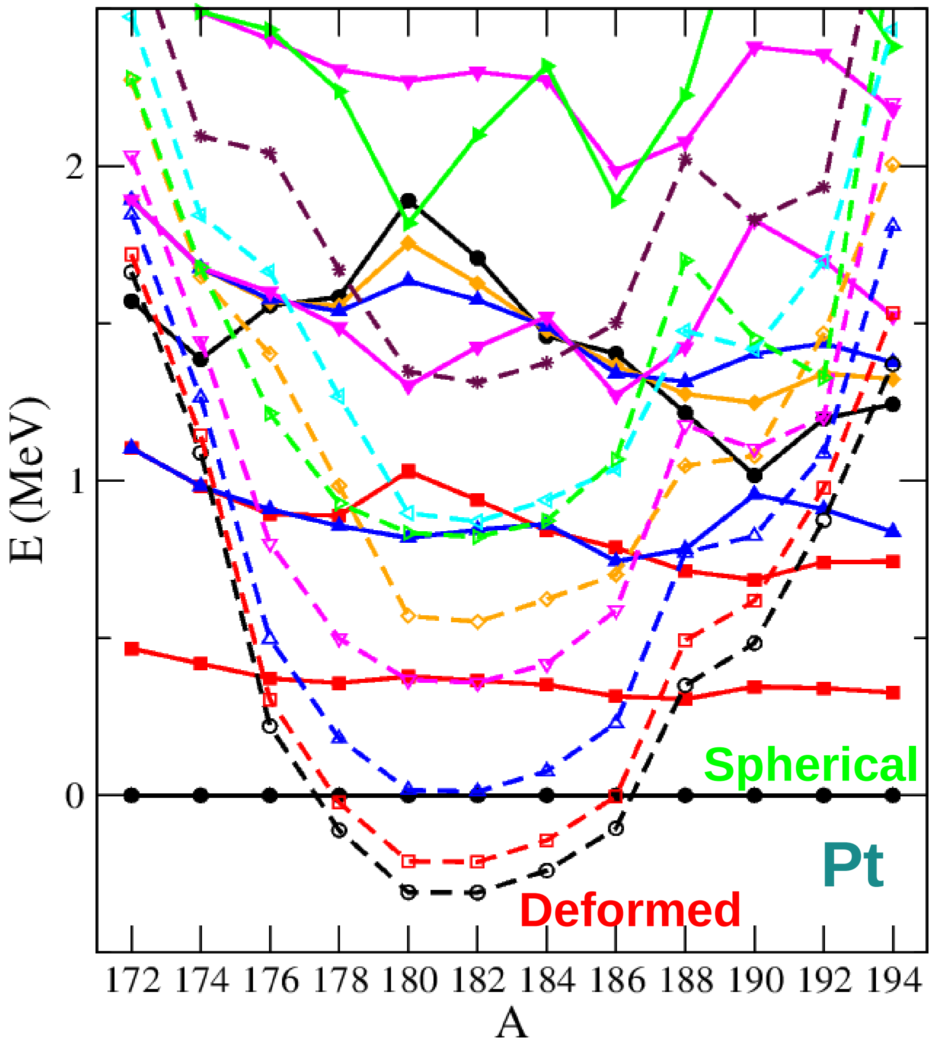

S_{2n} (MeV)
14
16
18
20
22
90 92 94 96 98 100 102 104 106 108 110 112 114 116 118
N
Pt
Hg
Pb
Po


$E(4^+)/E(2^+)$
1
1.5
2
2.5
3
90 92 94 96 98 100 102 104 106 108 110 112 114 116 118
N
Pt
Hg
Pb
Po

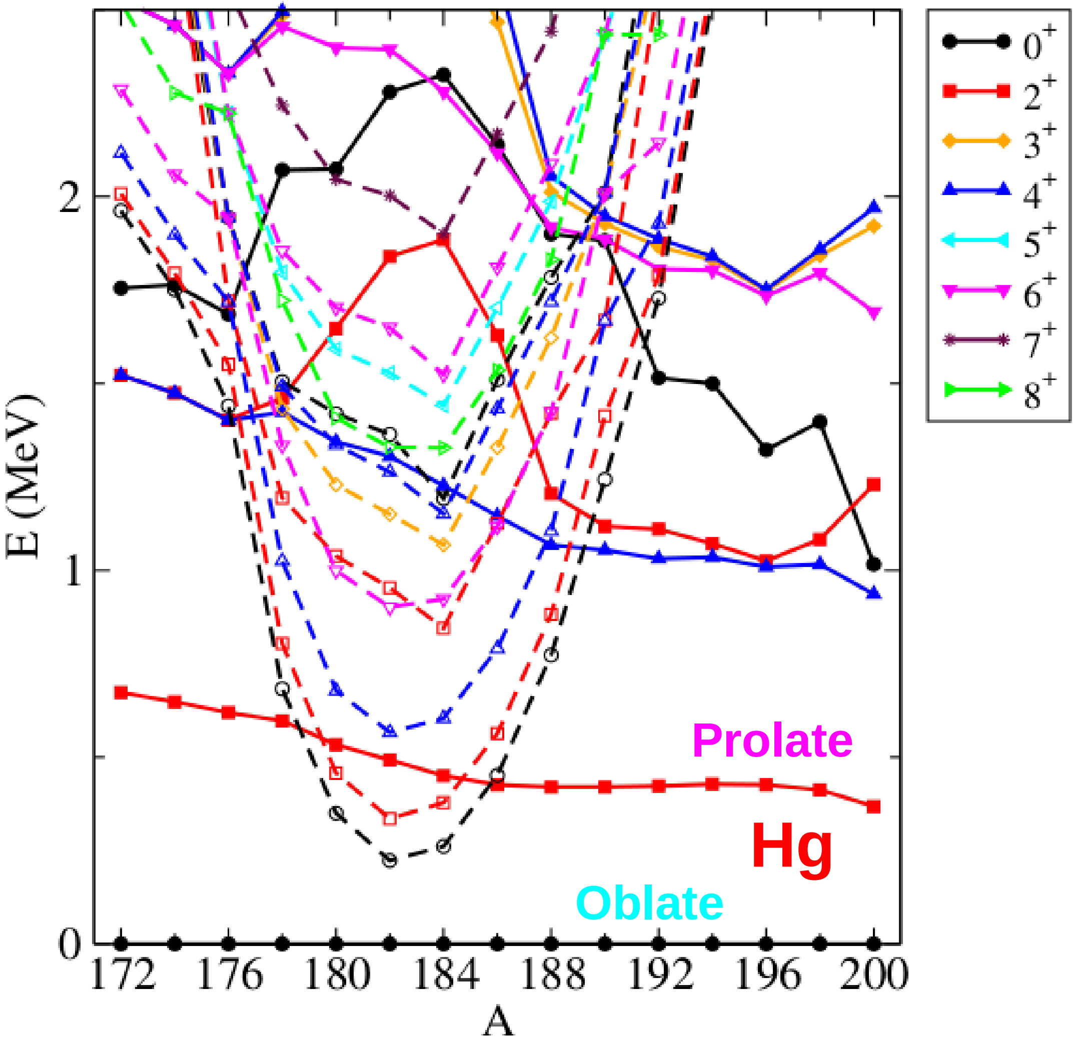
Unperturbed energies (IBM-CM)

Change in the ground state structure

No change in the ground state structure

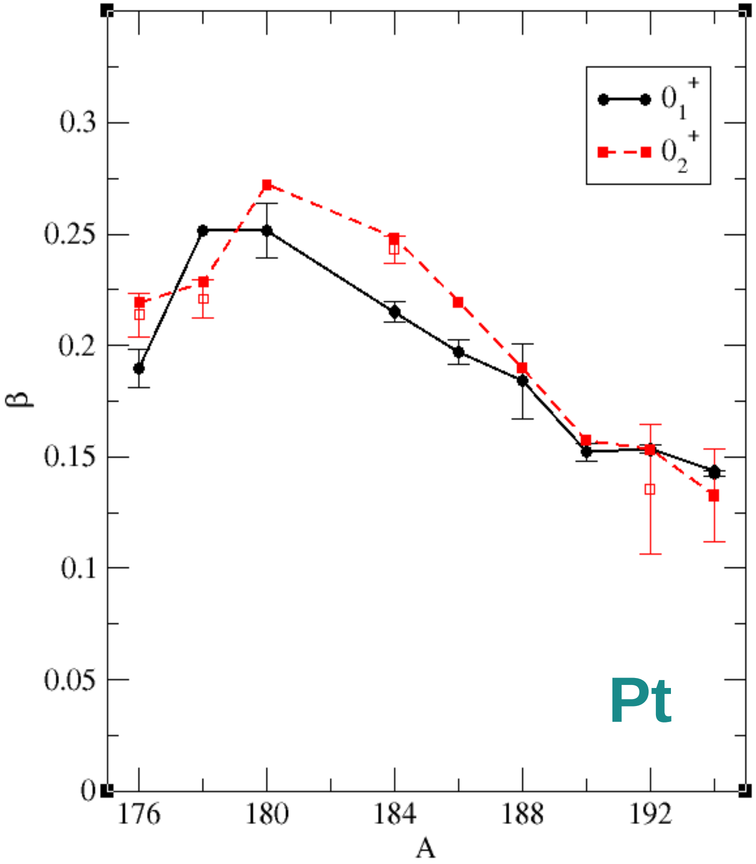


JEGR and K. Heyde, NPA 825, 39 (2009)



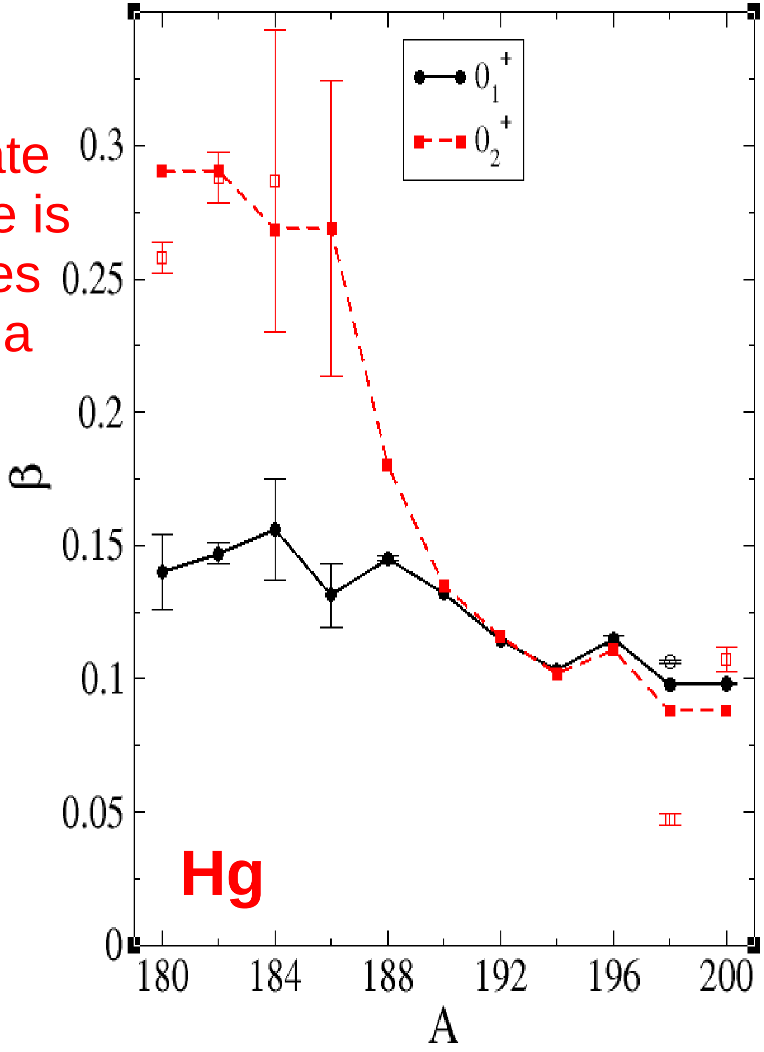
JEGR and K. Heyde, PRC 89, 014306 (2014)

What a about the relation between QPT and shape coexistence in Pb region

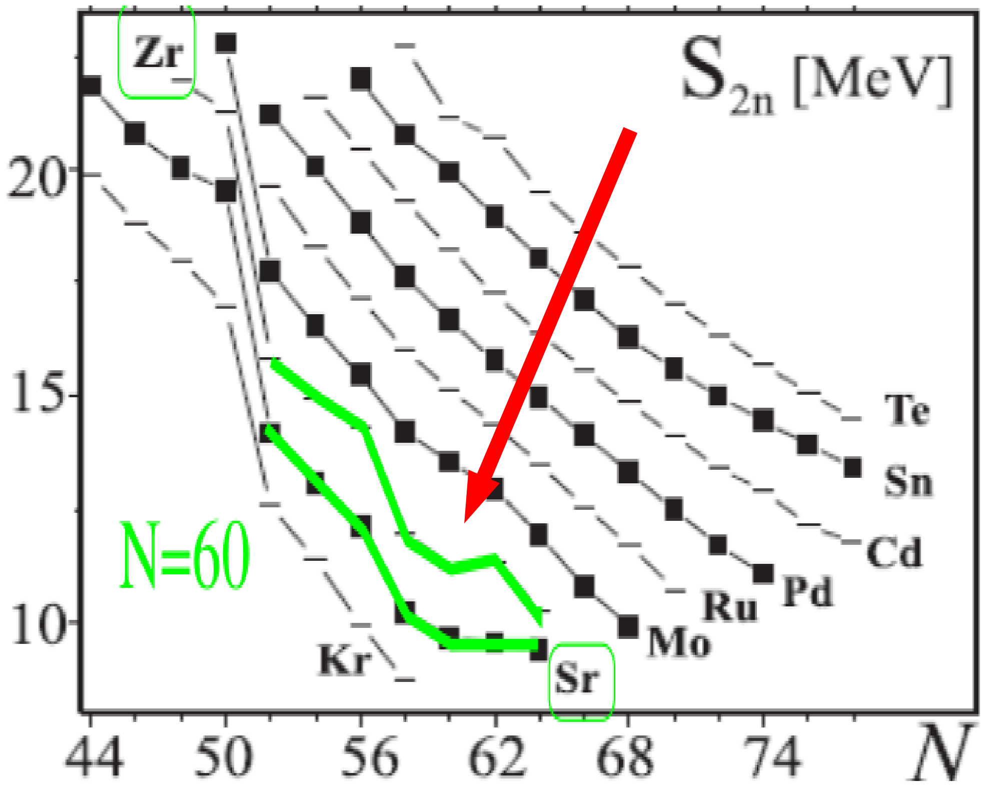


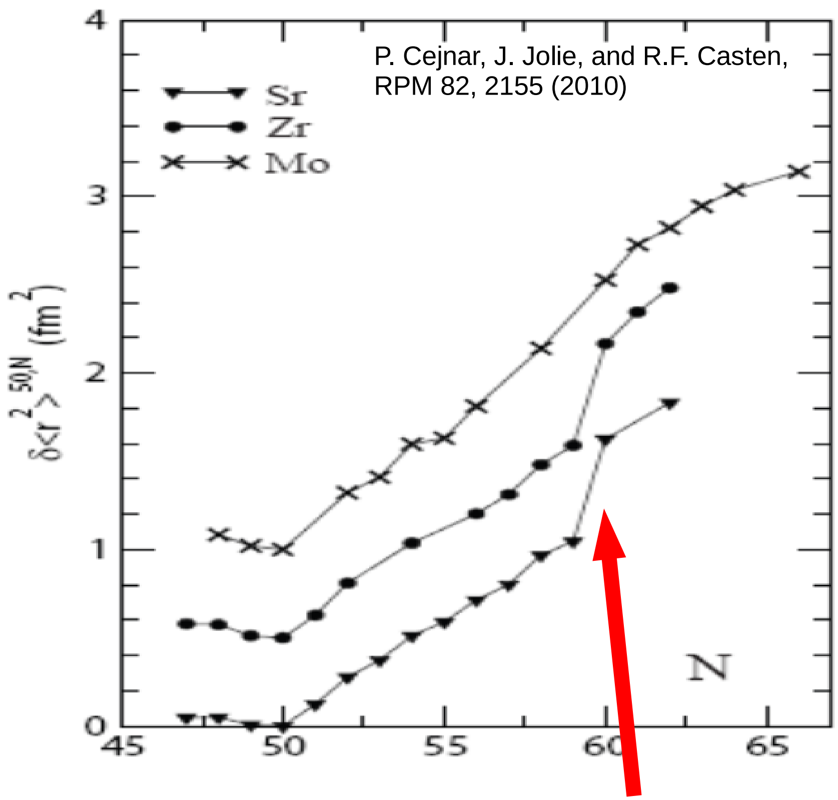
Hg:The ground state never changes its character and the influence of the closest intruder state on the ground state is small: this precludes the appearance of a QPT.

Pt: An intruder becomes ground state and therefore deformation should suddenly change, however, a large mixing between intruder and regular configurations smooth out the effects of any QPT.



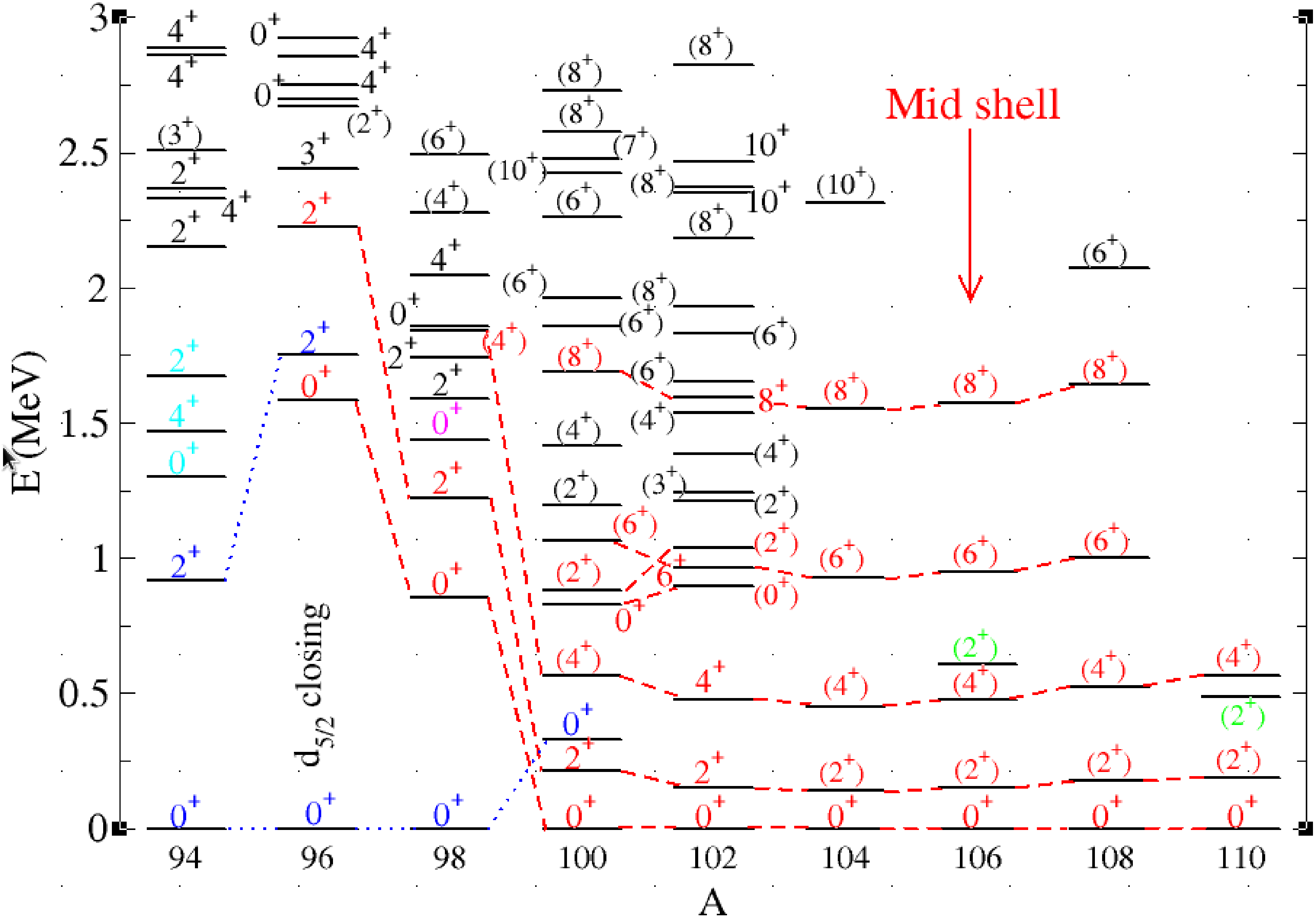
Z=40 subshell closure, A=100





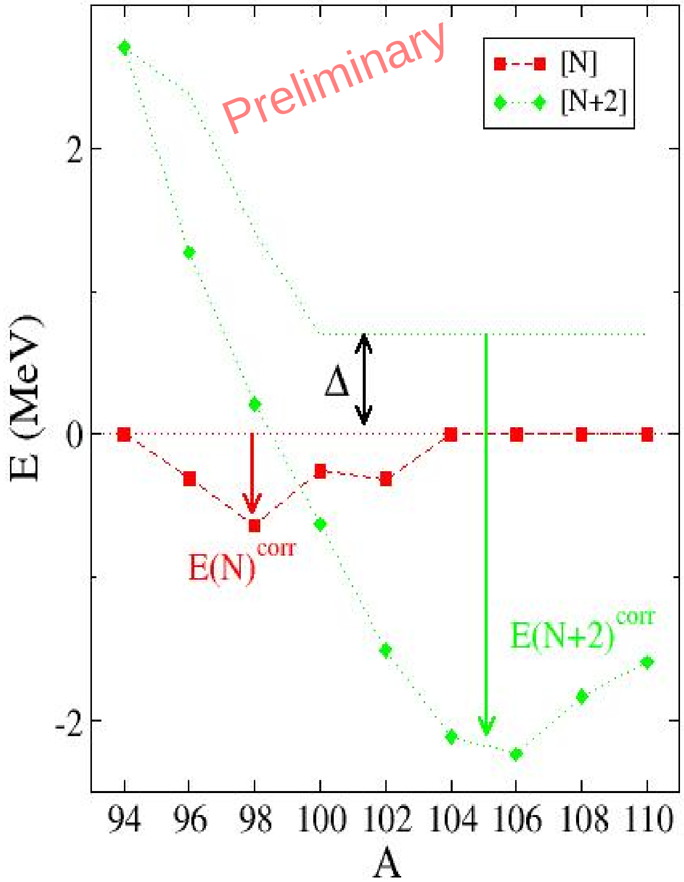
P. Cejnar, J. Jolie, and R.F. Casten, RPM 82, 2155 (2010)

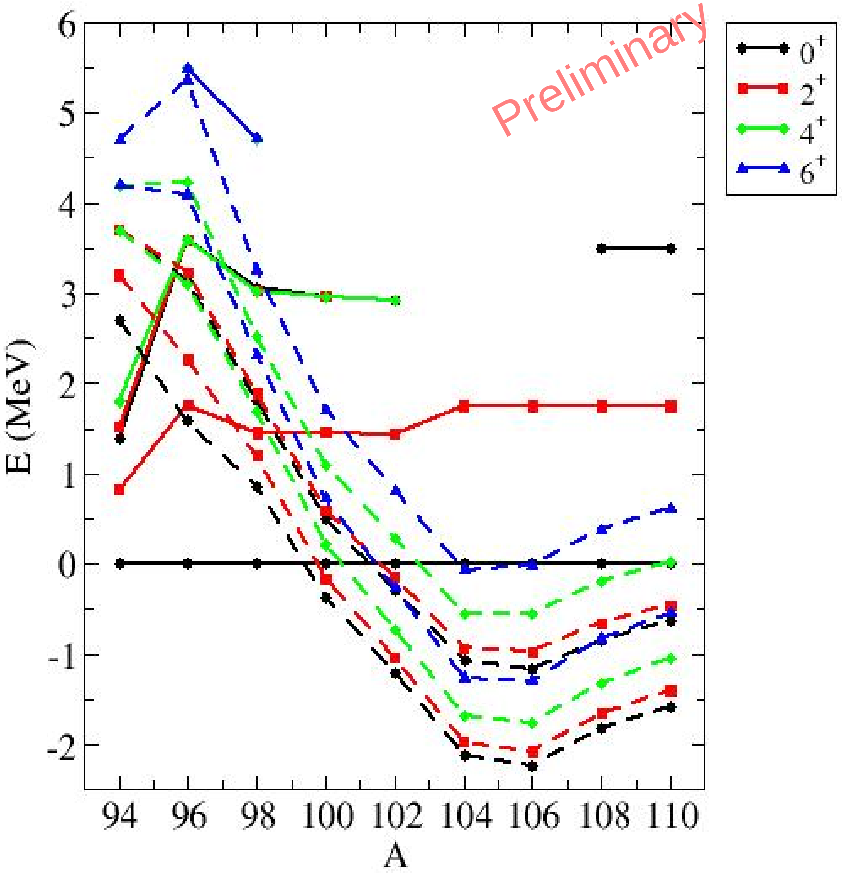
Energy systematics



Shape coexistence

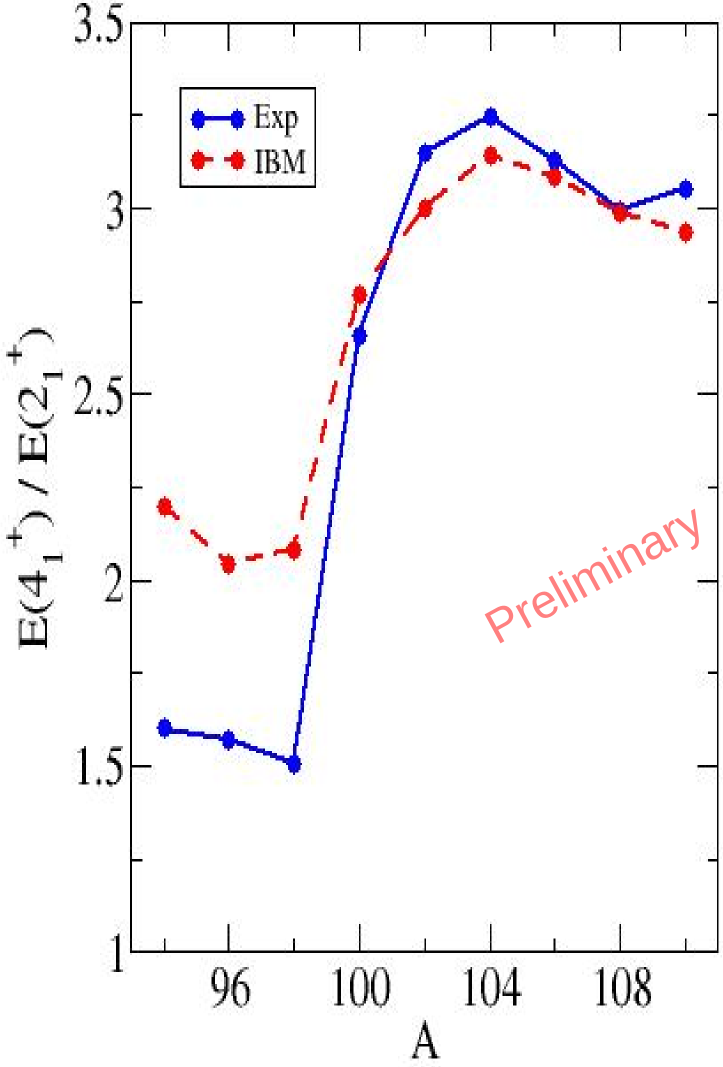
IBM with configuration mixing calculation with two types of configurations (no interaction between the regular and the intruder configuration)

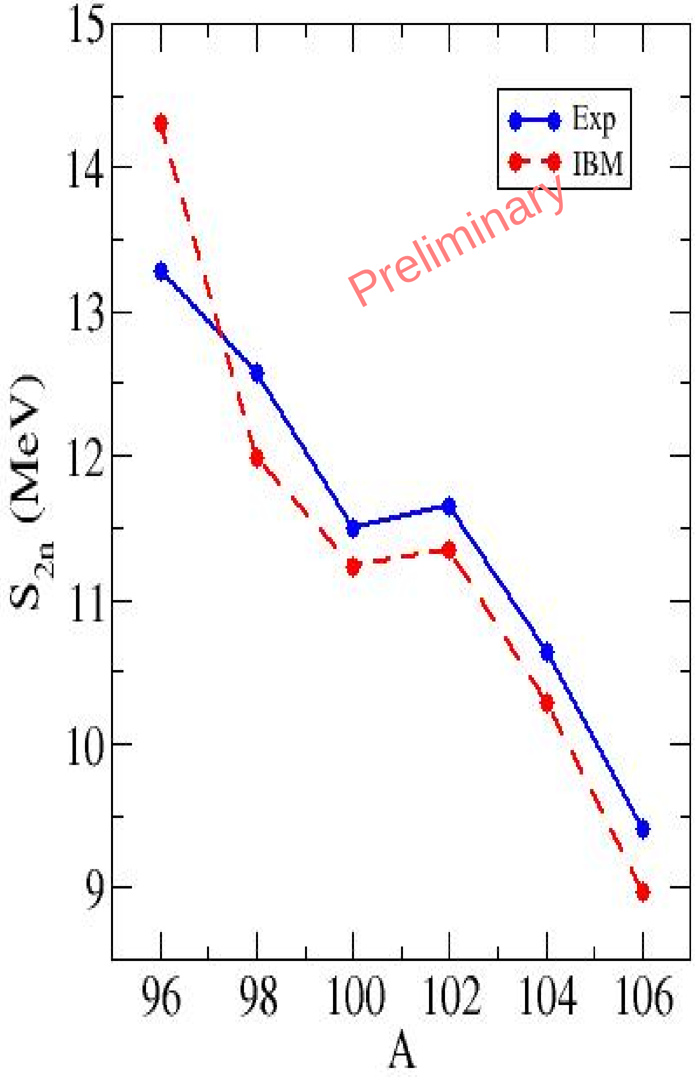




This phenomenon can also be explained in terms of type II shell evolution in *Togashi et al PRL 117, 172502 (2016)*, or in terms of the effect of paring and proton-neutron residual interaction, as in *Heyde et al NPA 466, 189 (1987).*

Quantum Phase Transition





Summary and conclusions

- We have tried to establish a link between two seemingly (un)related phenomena: Shape Coexistence and Quantum Phase Transition.

- In Pt-Pb-Hg two (or even three) configurations coexist, one slightly deformed (or spherical) and of gamma unstable character, corresponding to the regular states and a more deformed one of oblate (prolate) character corresponding to the intruder ones. **No indication of QPTs are observed.**

- In Zr two (or even three) configurations coexist, one of spherical character while the other(s) deformed. **QPT is observed.**

- The crossing of configurations can induce a QPT but the strong interaction between configuration precludes it, in Pt, but not in Zr.

Thank you