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Effect of Dust Particles on Rotating Micropolar Fluid Heated From Below Saturating a Porous Medium

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Abstract

This paper deals with the theoretical investigation of the effect of dust particles on a layer of rotating micropolar fluid heated from below saturating a porous medium. A dispersion relation is obtained for a flat fluid layer contained between two free boundaries using a linear stability analysis theory and normal mode analysis. The principle of exchange of stabilities is found to hold true for the micropolar fluid saturating a porous medium heated from below in the absence of dust particles, rotation and micropolar heat conduction parameter. The oscillatory modes are introduced due to the presence of the dust particles and rotation, which were non-existence in their absence. The presence of micropolar heat conduction parameter may also introduce oscillatory modes. For the case of stationary convection, the effect of various parameters like medium permeability, rotation, dust particles, coupling parameter, micropolar coefficient (A) and micropolar heat conduction parameter has been analyzed. The thermal Rayleigh number for the onset of instability is also determined numerically and results are depicted graphically. In the present paper, an attempt is also made to obtain the sufficient conditions for the non-existence of overstability.

Keywords: Micropolar Fluid; Thermal Convection; Porous Medium; Dust Particles; Rotation

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1. Introduction

A general theory of micropolar fluids was originally introduced by Eringen (1966). According to him, a subclass of microfluids [Eringen (1964)] which exhibit the micro-rotational effects and micro-rotational inertia is the micropolar fluids. Certain anisotropic fluids, e.g., liquid crystals which are made up of dumb bell molecules are of this type. In fact, animal blood happens to fall into this category. Other polymeric fluids and fluids containing certain additives may be represented by the mathematical model underlying micropolar fluids. Compared to the classical Newtonian fluids, micropolar fluids are characterised by two supplementary variables, i.e., the spin, responsible for the micro-rotations and the micro-inertia tensor describing the distributions of atoms and molecules inside the fluid elements in addition to the velocity vector. Liquid crystals, colloidal fluids, polymeric suspension, animal blood, etc. are few examples of micropolar fluids [Lebon and Perez-Garcia (1981)]. Kazakia and Ariman (1971) and Eringen (1972) extended this theory of structure continue to account for the thermal effects.

Micropolar fluids abound in engineering science and some common examples are human blood, plasma, sediments in rivers, drug suspension in pharmacology, liquid crystals, etc. The past four decades have seen an incredible interest emerge in applications of these fluid theories to numerous problems in engineering sciences ranging from biofluid mechanics of blood vessels to sediment transport in rivers and lubrication technology. It has wide applications in the developments of micropolar biomechanical flows and as such is both an engineering science one.

The more applications of micropolar fluid may include lubrication theory, boundary layer theory, short waves for heat conducting fluids, hydrodynamics of multicomponent media, magnetohydrodynamics and electrohydrodynamics, biological fluid modelling, etc. Hydrodynamics of micropolar fluids has significant applications to a variety of different fields of physics and engineering such as synovial lubrication, knee cap mechanics, arterial blood flows, and cardio-vascular flows, cervical flows (spermatozoa propulsion dynamics), pharmacodynamics, of drug delivery and peristaltic transport of micropolar fluid pumping. We are highlighted some key areas of applications in Figure 1.

The micropolar fluid theory has been successfully applied by many authors [Rosenberg (2003), Shukla and Isa (1975), Sinha and Singh (1982a, b), Sinha et al. (1982a, b), Chandra and Philip (1996), Power (1998)] to the modeling of the bone, micropolar hip-joint lubrication model, modeling the CSF (cerebral spinal fluid), to study various problems in lubrication, etc. A large number of references about modeling and applications aspect of micropolar fluids has been given in the book [Gezregorz (1999)]. In the review paper [Ariman (1973)], the most comprehensive discussion of applications of fluids with microstructure, micropolar fluids in particular are tabularized. The literature concerning applications of micropolar fluids in engineering sciences is vast and still quickly growing.

The theory of thermomicropolar convection began with Datta and Sastry (1976) and interestingly continued by Ahmadi (1976). Labon and Perez-Garcia (1981), Bhattacharya and Jena (1983), Payane and Straughan (1989), Sharma and Kumar (1995, 1997) and Sharma and Gupta (1995). The above works give a good understanding of thermal convection in micropolar fluids. The Rayleigh-Benard instability in a horizontal thin layer of fluid heated

from below is an important particular stability problem. A detailed account of Rayleigh-Benard instability in a horizontal thin layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar (1981). Perez-Garcia et al. (1981) have extended the effects of the microstructures in the Rayleigh-Benard instability and have found that in the absence of coupling between thermal and micropolar effects, the Principle of Exchange of Stabilities (PES) holds good.

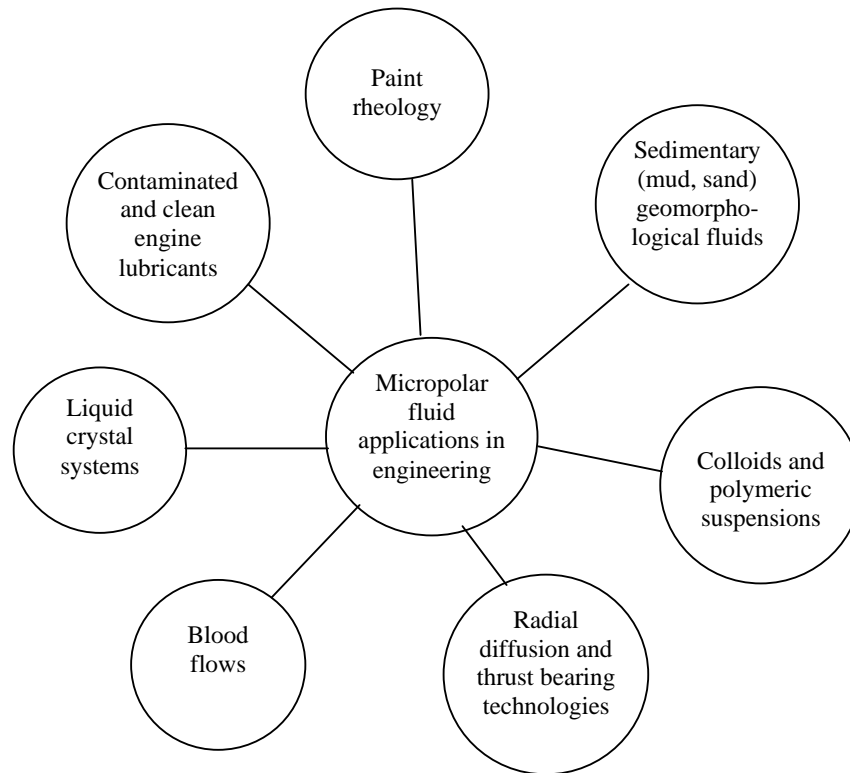


Figure 1.

Perez-Garcia and Rubi (1982) have shown that when coupling between thermal and micropolar effect is present, the Principle of Exchange of Stabilities (PES) may not be fulfilled and hence oscillatory motions are present in micropolar fluids. The effect of rotation on thermal convection in micropolar fluids is important in certain chemical engineering and biochemical situations Sunil et al. (2006a). Qin and Kaloni (1992) have considered a thermal instability problem in a rotating micropolar fluid. They found that the rotation has a stabilizing effect. The effect of rotation on thermal convection in micropolar fluids has also been studied by Sharma and Kumar (1994), whereas the effect of rotation on thermal convection in micropolar fluids in porous medium has been considered by Sharma and Kumar (1998). The study of thermal convection in a rotating layer of a fluid heated from below in porous media is motivated both theoretically as also by its practical applications in engineering.

Among the applications in engineering disciplines, one find the food process industry, chemical process industry, solidification and centrifugal casting of metals. With growing importance of micropolar fluids in modern technology and industries, the investigation on such fluids in porous media are desirable. Generally, it is accepted that comets consists of a dusty 'snowball' of a mixture of frozen gases which is the process of their journey changes from solid to gas and vice versa. The physical properties of comets, meteorites and inter-planetary dust strongly suggests that importance of porosity in the astrophysical context [McDonnel (1978)].

The porous medium of very low permeability allows us to use the generalized Darcy's model [Walter (1977)] including the inertial forces. This is because for a medium of very large stable particle suspension, the permeability tends to be very small justifying the use of the generalized Darcy's model including the inertial forces. This is also because the viscous drag force is negligibly small in comparison with the Darcy's resistance due to the presence of large particle suspension. In the last decade, stability problems on micropolar fluids have been studied in porous and non-porous medium by many authors [Sunil et al. (2006a), Qin and Kaloni (1992), Sharma and Kumar (1994), Sharma and Kumar (1998)].

More recently, Reena and Rana (2008) have studied the effect of rotation in micropolar fluid permeated with variable gravity field in porous medium with thermal effect and Mittal and Rana (2008) have studied the thermal convection of rotating micropolar fluid in hydromagnetics saturating a porous medium. A comprehensive review of the literature concerning convection in porous medium is available in the book of Nield and Bejan (1998).

In all the above studies, fluid has been considered to be clean (i.e., free from dust particles). In many geophysical situations, the fluid is often not pure but contains suspended/dust particles. The motivation for the present study is also due to the fact that micropolar fluid particle mixtures are not commensurate with their scientific and industrial importance and the effect of dust particles on stability problems of micropolar fluid through porous medium finds its usefulness in several geophysical situations, chemical technology, biomechanics and industry.

Contribution of Saffman (1962) in this direction is immemorable. He has considered the stability of laminar flow of a dusty gas. Scanlon and Segal (1973) have considered the effect of dusty particles on the onset of Benard convection, whereas, Sharma et al. (1976) have studied the effect of suspended particles on the onset of Benard convection in hydromagnetics and found that the critical Rayleigh number is reduced because of the heat capacity of the particles. The thermal instability of fluids in porous medium in the presence of suspended particles and rotation has been studied by Sharma and Sharma (1982) and found that suspended particles destabilize the layer and rotation has stabilizing effect on the system. Prakash and Manchanda (1996) studied the effect of magnetic viscosity and suspended particles on the thermal instability of a plasma in porous medium and showed that the effect of suspended particles is destabilizing. Palaniswamy and Purushotham (1981) have studied the stability of shear flows of stratified fluids with fine dust and found that the fine dust increases the region of instability.

According to Sunil et al. (2005e), the multiphase fluid systems are concerned with the motion of a liquid or gas containing immiscible inert identical particles. Of all multiphase fluid systems observed in nature, blood flows in arteries, flow in rocket tubes, dust in gas cooling systems to enhance the heat transfer, movement of inert solid particles in the atmosphere, sand or other particles in sea or ocean beaches are the most common examples of multiphase fluid systems. Studies of these systems are mathematically interesting and physically useful for various good reasons.

The effect of dust particles on visco-elastic rotating fluid in porous medium for variable gravity field has been found by Sharma and Rana (2002). The effect of dust particles on non-magnetic fluids has been investigated by many authors [Sharma and Sunil (1994), Sharma et al. (2002), Sunil et al. (2003), Sunil et al. (2004)].

The effect of dust particles on ferromagnetic fluids has been investigated by many authors [Sunil et al. (2005a, b, c, d, e, 2006b)]. The main result of all these studies is that dust particles are found to have a destabilizing effect. Unfortunately not much work has been done on the stability of dusty flow for micropolar fluids.

In view of the above investigations and keeping in mind the importance and applications of micropolar fluids, in geophysics, chemical engineering, astrophysics, biomechanics and industry, it is attempted to discuss the effect of dust particles on thermal convection in a micropolar rotating fluid in porous medium, using generalized Darcy’s model including the inertial forces. Many papers are devoted to various stability problems of micropolar fluids, but to the best of our knowledge and belief, this problem has not been investigated yet. The present study can serve as a theoretical support for experimental investigations, e.g., evaluating the influence of impurities in a micropolar fluid on thermal convection phenomena.

2. Mathematical Formulation of the Problem

Here, the stability of an infinite horizontal layer of an incompressible micropolar fluid of thickness d embedded in dust particles in porous medium of porosity ε and permeability k_1 acted on by a uniform vertical rotation $\Omega = (0, 0, \Omega)$ and gravity $\mathbf{g} = (0, 0, -g)$ is considered (Figure 2).

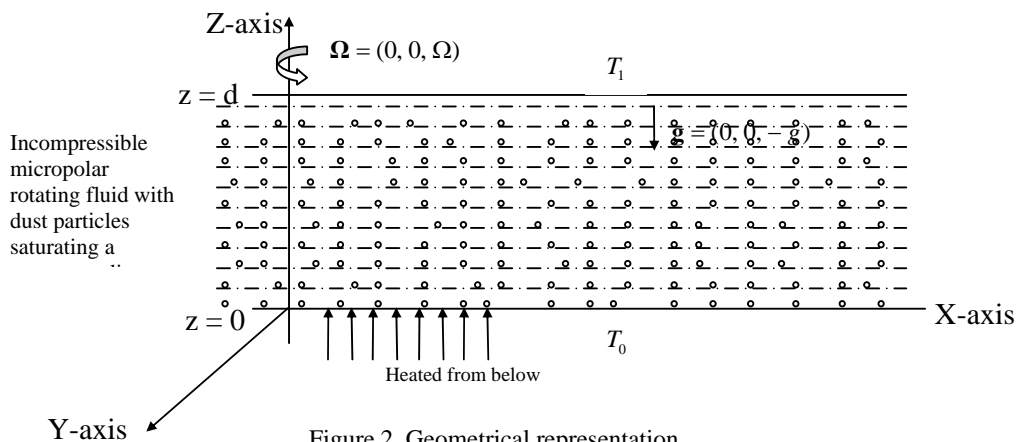


Figure 2. Geometrical representation

The vertical axis is taken as the z -axis. This fluid layer is heated from below and remains at rest until a critical (steady) adverse temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ between the lower and upper limiting surfaces is maintained. The critical temperature gradient depends upon the bulk properties and boundary conditions of the fluid. Both the boundaries are taken to be free and perfect conductors of heat.

The mathematical equations governing the motion of a micropolar fluid saturating a porous medium following Boussinesq approximation for the above model are as follows [Eringen (1966), Grzegorz (1999)].

The continuity equation for an incompressible fluid is

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

The momentum and internal angular momentum equations for the generalized Darcy model including the inertial forces are

$$\frac{\rho_0}{\varepsilon} \left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q} \cdot \nabla) \right] \mathbf{q} = -\nabla p + \rho \mathbf{g} - \frac{1}{k_1} (\mu + k) \mathbf{q} + k \nabla \times \mathbf{v} + \frac{2\rho_0}{\varepsilon} (\mathbf{q} \times \boldsymbol{\Omega}) + \frac{K' N}{\varepsilon} (\mathbf{q}_d - \mathbf{q}), \tag{2}$$

and

$$\rho_0 j \frac{\partial \mathbf{v}}{\partial t} = (\varepsilon' + \beta') \nabla (\nabla \cdot \mathbf{v}) + \gamma' \nabla^2 \mathbf{v} + \frac{k}{\varepsilon} \nabla \times \mathbf{q} - 2k \mathbf{v}, \tag{3}$$

where $\rho, \rho_0, \mathbf{q}, \mathbf{v}, \mu, k, p, \varepsilon', \beta', \gamma', j, t, \mathbf{q}_d(X, t)$ and $N(X, t)$ are the fluid density, reference density, filter velocity, spin (microrotation), shear kinematic viscosity coefficient (constant), coupling viscosity coefficient or vortex viscosity, pressure, bulk spin viscosity coefficient, shear spin viscosity coefficient, micropolar coefficient of viscosity, microinertia constant, time, filter velocity and number density of dust particles of the micropolar fluid, respectively. $\mathbf{X} = (x, y, z)$ and $K' = 6\pi\mu\eta$, η being the particle radius, is the Stokes drag coefficient. When the fluid flows through a porous medium, the gross effect is represented by Darcy's law. As a result, the usual viscous terms in the equations of fluid motion is replaced by the resistance term $-\left[\frac{\mu + k}{k_1} \right] \mathbf{q}$, where k_1 is the medium permeability.

Assuming a uniform particle size, a spherical shape and small relative velocities between the fluid and dust particles, the presence of dust particles adds an extra force term in the equation of motion (2), proportional to the velocity difference between the dust particles and the fluid. The effect of rotation contributed two terms:

(a) centrifugal force $-\frac{\rho_0}{2} \text{grad} |\boldsymbol{\Omega} \times \mathbf{r}|^2$, and

(b) coriolis force $\frac{2\rho_0}{\varepsilon} (\mathbf{q} \times \boldsymbol{\Omega})$.

In equation (2), $p = p_f - \frac{1}{2} \rho_0 |\boldsymbol{\Omega} \times \mathbf{r}|^2$ is the reduced pressure, whereas p_f stands for fluid pressure. The temperature equation for an incompressible micropolar fluid is

$$\left[\rho_0 C_v \varepsilon + \rho_s C_s (1 - \varepsilon) \right] \frac{\partial T}{\partial t} + \rho_0 C_v (\mathbf{q} \cdot \nabla) T + m N C_{pt} \left(\varepsilon \frac{\partial}{\partial t} + \mathbf{q}_d \cdot \nabla \right) T = K_T \nabla^2 T + \delta' (\nabla \times \mathbf{v}) \cdot \nabla T, \quad (4)$$

where $C_v, C_s, C_{pt}, \rho_s, K_T, T$ and δ' are the specific heat at constant volume, specific heat of solid (porous matrix) material, specific heat of dust particles, density of solid matrix, thermal conductivity, temperature and micropolar heat conduction coefficient (coefficient giving account of coupling between the spin flux and heat flux), respectively. mN is the mass of dust particles per unit volume.

The density equation of state is

$$\rho = \rho_0 [1 - \alpha(T - T_a)], \quad (5)$$

where α and T_a are coefficient of thermal expansion and average temperature given by $T_a = (T_0 + T_1)/2$, where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the fluid layer, respectively.

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles [Sharma et al. (1976)]. Interparticle reactions are ignored for we assume that the distances between particles are quite large compared with their diameter. The effects due to pressure, gravity and Darcian force on the dust particles are negligibly small and hence ignored. Under the above assumptions, the equations of motion and continuity for the suspended particles are

$$mN \left[\frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q}_d \cdot \nabla) \right] \mathbf{q}_d = K' N (\mathbf{q} - \mathbf{q}_d), \quad (6)$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla \cdot (N \mathbf{q}_d) = 0, \quad (7)$$

In the initial (quiescent) state, the solution of (1)-(7) is

$$\mathbf{q} = \mathbf{q}_b = (0, 0, 0), \mathbf{q}_d = (\mathbf{q}_d)_b = (0, 0, 0), \mathbf{v} = \mathbf{v}_b = (0, 0, 0), \rho = \rho_b(z) \text{ defined as}$$

$$\rho = \rho_0 [1 + \alpha \beta z],$$

$$p = p_b(z), T = T_b(z) = -\beta z + T_a, \text{ where } \beta = \frac{T_1 - T_0}{d}, N = N_b = N_0 \text{ (constant)} \quad (8)$$

where the subscript b denotes the basic state.

3. Perturbation Equations

The stability of the basic state can be analyzed by introducing small perturbations around the basic state as follows:

Let $\mathbf{u} = (u_x, u_y, u_z)$, $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$, $\mathbf{u}' = (u'_x, u'_y, u'_z)$, $\delta p, \delta \rho, \theta$ and N' denote respectively the perturbations in micropolar fluid velocity \mathbf{q} , spin \mathbf{v} , particle velocity \mathbf{q}_d , pressure p , density ρ , temperature T and suspended particle number density N_0 .

Now, the procedure for finding the non-dimensional form of linearized perturbation equations is given in Appendix-A.

The dimensionless boundary conditions are

$$u_z = 0, \quad \frac{\partial^2 u_z}{\partial z^2} = 0, \quad \boldsymbol{\omega} = 0, \quad \theta = 0, \quad \zeta_z = 0 \quad \text{at } z = 0 \text{ and } z = 1, \tag{9}$$

where, $\zeta_z = (\nabla \times \mathbf{u})_z$ is the z -component of vorticity.

Here, we consider the case where the boundaries are free and perfectly heat conducting. On a free surface, shear stress is zero and the velocity normal to the surface is zero. For micro-rotation boundary conditions, we assume the micro-rotation to be zero on the surface.

Now, analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[u_z, \Omega'_z, \zeta_z, \theta] = [U(z), G(z), Z(z), \Theta] \exp(ik_x x + ik_y y + \sigma t), \tag{10}$$

where k_x, k_y are the wave numbers along with x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and σ is the stability parameter which is, in general a complex constant.

For solutions having the dependence of the form (10), equations (A.10)-(A.13) become solutions (see Appendix-B). (B.1)-(B.4) must be sought, which satisfy the boundary conditions.

$$\begin{aligned} U = 0 = D^2 U, \quad DZ = 0 \\ G = 0, \quad \Theta = 0 \quad \text{at } z = 0 \text{ and } z = 1. \end{aligned} \tag{11}$$

Using (11), equations (B.1)-(B.4) give

$$D^2 \Theta = 0, \quad D^2 G = 0 \quad \text{at } z = 0 \text{ and } z = 1, \tag{12}$$

and it can be shown from equations (B.1)-(B.4) and boundary conditions (11), (12) that all even order derivatives of U vanish on the boundaries. Thus the exact solution of the system subject to the boundary conditions (11) and (12) characterizing the lowest mode is

$$U = U_0 \sin \pi z, \tag{13}$$

where U_0 is a constant.

Now for finding the dispersion relation, eliminating Θ, Z and G from equation (B.1)-(B.4), we obtain

$$\begin{aligned}
 (D^2 - k^2) & \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right]^2 [l\sigma + 2A - (D^2 - k^2)] [E_1 p_1 \sigma - (D^2 - k^2)] U \\
 & = -Rk^2 \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right] [l\sigma + 2A - (D^2 - k^2)] (1+\tau\sigma + b') \\
 & \quad + \bar{\delta}\varepsilon^{-1} A (D^2 - k^2) (1+\tau\sigma)] U - KA\varepsilon^{-1} (D^2 - k^2)^2 [E_1 p_1 \sigma - (D^2 - k^2)] (1+\tau\sigma) \\
 & \quad \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right] U - 4\Omega^2 \varepsilon^{-2} (1+\tau\sigma) [E_1 p_1 \sigma - (D^2 - k^2)] (1+\tau\sigma) \\
 & \quad [l\sigma + 2A - (D^2 - k^2)] D^2 U \tag{14}
 \end{aligned}$$

Substituting (13) in (14), we get

$$\begin{aligned}
 b & \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right]^2 [l\sigma + 2A + b] [E_1 p_1 \sigma + b] \\
 & = Rk^2 \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right] [l\sigma + 2A + b] (1+\tau\sigma + b') - \bar{\delta}\varepsilon^{-1} Ab(1+\tau\sigma) \\
 & \quad + KA\varepsilon^{-1} b^2 [E_1 p_1 \sigma + b] (1+\tau\sigma) \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1+K) \right\} (1+\tau\sigma) + \frac{f}{\varepsilon}\sigma \right] \\
 & \quad - 4\Omega^2 \varepsilon^{-2} (1+\tau\sigma) [E_1 p_1 \sigma + b] (1+\tau\sigma) [l\sigma + 2A + b], \tag{15}
 \end{aligned}$$

where, $b = \pi^2 + k^2$.

Equation (15) is the required dispersion relation studying the effect of medium permeability, rotation and suspended particles. In the absence of suspended particles, ($\tau = 0, b' = 0, f = 0$), equation (15) reduces to

$$\begin{aligned}
 b & \left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K) \right]^2 [l\sigma + 2A + b] [E p_1 \sigma + b] \\
 & = Rk^2 \left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K) \right] [l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1} Ab] + KA\varepsilon^{-1} b^2 [E p_1 \sigma + b] \left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K) \right] \\
 & \quad - 4\Omega^2 \varepsilon^{-2} \pi^2 [E p_1 \sigma + b] [l\sigma + 2A + b], \tag{16}
 \end{aligned}$$

a result derived by [Sharma and Kumar (1998), equation (31)]. In the absence of rotation ($\Omega = 0$), equation (16) reduces to

$$b \left[\sigma\varepsilon^{-1} + \frac{1}{P_l}(1+K) \right] [E p_1 \sigma + b] [l\sigma + 2A + b] = Rk^2 [l\sigma + 2A + b - \bar{\delta}\varepsilon^{-1} Ab] + KA\varepsilon^{-1} b^2 [E p_1 \sigma + b],$$

a result derived by [Sharma and Gupta (1995), equation (28)].

4. Stability of the System and Oscillatory Modes

Here, we examine the possibility of the oscillatory modes, if any, on stability problem due to the presence of rotation, suspended particles, medium permeability and micropolar parameters. Multiplying equation (B.1) by U^* , the complex conjugate of U and integrating w.r.t. z between the limits $z=0$ to $z=1$ and making use of equations (B.2)-(B.4) with the help of boundary conditions (11) and (12), we get

$$\left[\left(\frac{\sigma}{\varepsilon} + \frac{\overline{1+K}}{P_l} \right) (1 + \tau\sigma) + \frac{f}{\varepsilon} \sigma \right] I_1 - (1 + \tau\sigma) \left[\left\{ \frac{Rk^2}{(1 + \tau\sigma^* + b^*)} (1 + \tau\sigma^*) (E_1 p_1 \sigma^* I_2 + I_3 + \bar{\delta} \int_0^1 G^* \Theta dz) \right\} \right. \\ \left. + K \varepsilon A^{-1} \{ (l\sigma^* + 2A) I_5 + I_6 \} - \frac{I_4}{(1 + \tau\sigma^*)} \left\{ (1 + \tau\sigma^*) \left(\frac{\sigma^*}{\varepsilon} + \frac{\overline{1+K}}{P_l} \right) + \frac{f}{\varepsilon} \sigma^* \right\} \right] = 0, \quad (17)$$

where,

$$I_1 = \int_0^1 (|DU|^2 + k^2 |U|^2) dz, \\ I_2 = \int_0^1 |\Theta|^2 dz, \\ I_3 = \int_0^1 (|D\Theta|^2 + k^2 |\Theta|^2) dz, \\ I_4 = \int_0^1 |Z|^2 dz, \\ I_5 = \int_0^1 |G|^2 dz, \quad \text{and} \\ I_6 = \int_0^1 [|DG|^2 + k^2 |G|^2] dz.$$

The integrals I_1 to I_6 are all positive definite. Now, putting $\sigma = i\sigma_i$ in equation (17), where σ_i is real and taking imaginary parts only, we get

$$\sigma_i \left[\left\{ \left(\frac{1}{\varepsilon} + \frac{f}{\varepsilon} \right) + \left(\frac{1+K}{P_l} \right) \tau \right\} I_1 - Rk^2 \left(\frac{1 + \tau^2 \sigma_i^2}{b_1^2 + \tau^2 \sigma_i^2} \right) (\tau I_3 - E_1 p_1 b_1 I_2) - K \varepsilon A^{-1} \{ \tau (2A I_5 + I_6) - I_5 \} \right. \\ \left. + I_4 \frac{1}{(1 + \tau^2 \sigma_i^2)} \left\{ - \left[\left(\frac{1}{\varepsilon} + \frac{f}{\varepsilon} \right) + \left(\frac{1+K}{P_l} \right) \tau \right] (1 - \tau^2 \sigma_i^2) + \left(\frac{\overline{1+K}}{P_l} - \frac{1}{\varepsilon} \tau \sigma_i^2 \right) 2\tau \right\} \right] = 0, \quad (18)$$

where $b_1 = 1 + b' = 1$, in the absence of suspended particles. Here, for the sake of convenience, we have taken $\bar{\delta} = 0$ (i.e., absence of coupling between spin and heat flux). Equation (18) implies that σ_i may be either zero or non-zero, meaning that modes may be either non-oscillatory or oscillatory.

Limiting Case:

In the absence of dust particles, we obtain the result as

$$\sigma_i \left[\frac{1}{\varepsilon} (I_1 - I_4) + Rk^2 Ep_1 I_2 + K \varepsilon A^{-1} I_5 \right] = 0, \tag{19}$$

a result derived in our earlier paper [Reena and Rana (2008)], equation (43). Now, in the absence of rotation, equation (19) becomes

$$\sigma_i \left[\frac{1}{\varepsilon} I_1 + Rk^2 Ep_1 I_2 + K \varepsilon A^{-1} I_5 \right] = 0. \tag{20}$$

The expression inside the bracket of equation (20) is always positive definite. Hence, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities (PES) is satisfied for the present problem.

Thus from equation (18), we conclude that the oscillatory modes are introduced due to the presence of dust particles and rotation, which were non-existence in their absence. The presence of micropolar heat conduction parameter ($\bar{\delta}$) may also bring the oscillatory modes.

5. Case of Overstability

Since σ is, in general, a complex constant, so we put $\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real. The marginal state is reached when $\sigma_r = 0$; if $\sigma_r = 0$ implies $\sigma_i = 0$, one says that principle of exchange of stability (PES) is valid otherwise we have overstability and then $\sigma = i\sigma_i$ at marginal stability. Putting $\sigma = i\sigma_i$ in equation (15), equating real and imaginary parts and eliminating R between them, we get

$$f_4 C_1^4 + f_3 C_1^3 + f_2 C_1^2 + f_1 C_1 + f_0 = 0. \tag{21}$$

The coefficients f_3 and f_4 are given in Appendix-C.

The coefficients f_2, f_1 and f_0 being quite lengthy and not needed in the discussion of overstability, have not been written in Appendix-C.

Since σ_i is real for overstability, the true values of $C_1 (= \sigma_i^2)$ in equation (21) are positive. So, the sum of roots of equation (21) is positive, but this is impossible if $f_4 > 0$ and $f_3 > 0$ (since the sum of roots of equation (21) is (f_3/f_4)). Thus, $f_3 > 0$ and $f_4 > 0$ are the sufficient conditions for the non-existence of overstability.

It is clear from equations. (C.1) and (C.2) that f_3 and f_4 are always positive if

$$L_2 > b_1 / \varepsilon, KE_1 p_1 < 2, E_1 p_1 L_0 > b / \varepsilon, 0 < \bar{\delta} \varepsilon^{-1} A < 1, f(1 - \bar{\delta} \varepsilon^{-1} A) > b' \text{ and } Akb^2 > 4\pi^2 \Omega^2 l,$$

which implies that

$$C_v (1 - \bar{\delta} \varepsilon^{-1} A) > C_{pr}, \frac{bP_1}{\varepsilon} < KE_1 p_1 < 2, 0 < \bar{\delta} \varepsilon^{-1} A < 1, \text{ and } \Omega < \left(\frac{b}{2\pi} \right) \sqrt{\left(\frac{AK}{l} \right)}, \tag{22}$$

and the other inequality $L_2 > b_1 / \varepsilon$ giving $f > b'$, which automatically satisfied in

$$f (1 - \bar{\delta}\varepsilon^{-1}A) > b'.$$

Thus, for $C_v (1 - \bar{\delta}\varepsilon^{-1}A) > C_{pt}$, $\frac{bP_l}{\varepsilon} < KE_1 p_1 < 2$, $0 < \bar{\delta}\varepsilon^{-1}A < 1$ and $\Omega < \left(\frac{b}{2\pi}\right) \sqrt{\left(\frac{AK}{l}\right)}$, overstability can not occur and the principle of exchange of stabilities (PES) is valid. Hence, the above conditions (22) are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

Particular Cases

1. In the absence of micropolar heat conduction parameter ($\bar{\delta} = 0$), conditions as expected reduces to $\frac{bP_l}{\varepsilon} < KE_1 p_1 < 2$ and $C_v > C_{pt}$ (i.e., the specific heat of the fluid at constant volume is greater than the specific heat of dust particles).
2. In the absence of suspended particles ($\tau = 0, h = 1, E_1 = E$), equation (21) reduces to a quadratic equation

$$f_2 C_1^2 + f_1 C_1 + f_0 = 0. \tag{23}$$

The coefficients f_1 and f_2 are given in Appendix-D. Equation (23) is similar to the result derived by Reena and Rana [(2008), equation (54)], which gives the sufficient conditions of non-existence of overstability as

$$0 < \bar{\delta} < \frac{\varepsilon}{A}, \quad \Omega < \frac{(1+K)\varepsilon\sqrt{b}}{2\pi P_l}, \quad KEp_1 < 2, \tag{24}$$

the violation of which does not necessarily imply the occurrence of overstability. The same result derived by Reena and Rana [(2008), equation (58)]. If in addition to absence of suspended particles, the fluid is also non-rotating ($\Omega = 0$), the condition (24) reduces to

$$0 < \bar{\delta} < \frac{\varepsilon}{A}, \quad KEp_1 < 2. \tag{25}$$

The same result derived by Reena and Rana ((2008), equation (59)) and in addition to absence of suspended particles and rotation ($\Omega = 0$), the micropolar heat conduction parameter ($\bar{\delta}$) is also absent, the conditions (25) reduce to

$$KEp_1 < 2. \tag{26}$$

If we take medium permeability very-very large ($P_l \rightarrow \infty$), then in the absence of suspended particles, rotation and micropolar heat conduction parameter, the condition of non-existence of overstability is also $KEp_1 < 2$, which is in good agreement with the result obtained earlier by Sharma and Gupta (1995). Thus, the presence of suspended particles,

rotation and micropolar heat conduction parameter may bring overstability. The medium permeability may also bring overstability.

6. The Case of Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$; hence putting $\sigma = 0$ in equation (15), the Rayleigh number is given by

$$R = \frac{b^3 \left(\frac{1+K}{P_l} \right) \left\{ \left(\frac{1+K}{P_l} \right) - KA\varepsilon^{-1} \right\} + b^2 \left\{ 2A \left(\frac{1+K}{P_l} \right)^2 + 4\Omega^2 \varepsilon^{-2} \pi^2 \right\} + b(8\Omega^2 \varepsilon^{-2} \pi^2 A)}{k^2 \left(\frac{1+K}{P_l} \right) \{ (2A+b) b_1 - \bar{\delta} \varepsilon^{-1} Ab \}}, \tag{27}$$

where $b_1 = 1 + b' = 1$ in the absence of dust particles, which leads to the marginal stability curve in stationary conditions. Equation (27) expresses the Rayleigh number R as a function of the dimensionless wave number k , medium permeability P_l (Darcy number), rotation (Ω), dust particles parameter b_1 , coupling parameter K (coupling between vorticity and spin effects), micropolar coefficient A (the ratio of the micropolar viscous effects K to micropolar diffusion effects C_0) and micropolar heat conduction parameter $\bar{\delta}$ (coupling between spin and heat flux). The parameters K and A measure the micropolar viscous effect and micropolar diffusion effect, respectively. Since the marginal state dividing stability from instability is stationary, this shows that at the onset of instability, there is no relative velocity between particles and fluid and hence no particles drag on the fluid. Therefore, the Rayleigh number is reduced solely because the heat capacity of the clean fluid is supplemented by that of the suspended (dust) particles. This explains the physics and the role of dust parameters, Sunil et al. (2005c).

To investigate the effects of medium permeability, rotation, dust particles, coupling parameter K , micropolar coefficient A and micropolar heat conduction parameter $\bar{\delta}$, we examine the behavior of $\frac{dR}{dP_l}$, $\frac{dR}{d\Omega}$, $\frac{dR}{db_1}$, $\frac{dR}{dK}$, $\frac{dR}{dA}$ and $\frac{dR}{d\bar{\delta}}$ analytically. Equation (27) gives

$$\frac{dR}{dP_l} = - \frac{b(b+2A) \left[b \left(\frac{1+K}{P_l} \right)^2 - 4\Omega^2 \varepsilon^{-2} \pi^2 \right]}{k^2 (1+K) \{ 2Ab_1 + b(b_1 - \bar{\delta} \varepsilon^{-1} A) \}}, \tag{28}$$

which is always negative when

$$P_l < \frac{\varepsilon(1+K)\sqrt{b}}{2\pi\Omega} \quad \text{and} \quad \bar{\delta} < \frac{\varepsilon b_1}{A}. \tag{29}$$

It implies that the medium permeability has a destabilizing effect when condition (29) holds. In the absence of suspended particles ($b_1 = 1$) condition (29) reduces to

$$P_l < \frac{\varepsilon(1+K)\sqrt{b}}{2\pi\Omega} \quad \text{and} \quad \bar{\delta} < \frac{\varepsilon}{A},$$

which is in good agreement of earlier results derived by Reena and Rana (2008).

In the absence of micropolar heat conduction parameter ($\bar{\delta} = 0$) and rotation ($\Omega = 0$), equation (28) yields that $\frac{dR}{dP_l}$ is always negative, implies that the medium permeability always has a destabilizing effect on the system for stationary convection in porous medium, i.e., Rayleigh number decreases with an increase in medium permeability. Medium permeability may have a dual role in the presence of rotation. The medium permeability has a stabilizing effect, if $\Omega > \frac{\varepsilon(1+K)\sqrt{b}}{2\pi P_l}$. Thus, for higher values of rotation, the stabilizing/destabilizing effect of medium permeability has been predicted.

Thus, in micropolar rotating fluid heated from below saturating a porous medium, there is a competition between the destabilizing role of medium permeability and stabilizing role of micropolar heat conduction parameter and rotation, but there is complete destabilization in the above inequalities given by (29). Equation (27) also gives

$$\frac{dR}{d\Omega'} = \frac{4\varepsilon^{-2} \pi^2 b (b + 2A)}{k^2 \left(\frac{1+K}{P_l}\right) [2Ab_1 + b(b_1 - \bar{\delta}\varepsilon^{-1}A)]}, \tag{30}$$

where we have taken $\Omega' = \Omega^2$, which is always positive if

$$\bar{\delta} < \frac{b_1\varepsilon}{A}. \tag{31}$$

This shows that the rotation has stabilizing effect when condition (31) holds. In the absence of micropolar heat conduction parameter ($\bar{\delta}$), equation (30) yields that $\frac{dR}{d\Omega'}$ is always positive, i.e., the Rayleigh number increases with an increase in rotation, implying thereby the stabilizing effect of rotation, but in the presence of micropolar heat conduction parameter, rotation has destabilizing role if $\bar{\delta} > \frac{\varepsilon(2A+b)b_1}{Ab}$.

Thus, for sufficiently higher values of micropolar heat conduction parameter, the destabilizing role of rotation has been predicted. Thus in the presence of micropolar heat conduction parameter, rotation may have stabilizing/destabilizing effect but in the absence of micropolar heat conduction parameter, rotation always has stabilizing effect. Equation (27) also yields

$$\frac{dR}{db_1} = - \frac{(2A+b) \left[b^3 \left(\frac{1+K}{P_l}\right) \left\{ \left(\frac{1+K}{P_l}\right) - KA\varepsilon^{-1} \right\} + b^2 \left\{ 2A \left(\frac{1+K}{P_l}\right)^2 + 4\Omega^2\varepsilon^{-2}\pi^2 \right\} + b(8\Omega^2\varepsilon^{-2}\pi^2 A) \right]}{k^2 \left(\frac{1+K}{P_l}\right) [(2A+b)b_1 - \bar{\delta}\varepsilon^{-1}Ab]^2}, \tag{32}$$

which is always negative, if

$$\frac{1}{P_l} > \frac{A}{\varepsilon}, \tag{33}$$

which implies that dust particles are found to have a destabilizing effect when condition (33) holds. Here, we also observe that in a non-porous medium, $\frac{dR}{db_1}$ is always negative, implying thereby the destabilizing effect of dust particles.

Thus, the medium permeability and porosity have significant role in developing conditions for the destabilizing behavior of dust particles. Also, it can be easily shown that in the absence of micropolar heat conduction parameter, there is no change in the effect of dust particles on the system. Thus, the destabilizing behavior of dust particles is independent of presence of micropolar heat conduction parameter ($\bar{\delta}$).

It can easily be derived from equation (27) that

$$\frac{dR}{dK} = \frac{b \left[\left\{ b^2 \left(\frac{1+K}{P_l} \right)^2 - 4\Omega^2 \varepsilon^{-2} \pi^2 (b+2A) \right\} \frac{1}{P_l} + Ab \left(\frac{2}{P_l} - \frac{b}{\varepsilon} \right) \left(\frac{1+K}{P_l} \right)^2 \right]}{k^2 \left(\frac{1+K}{P_l} \right)^2 [(2A+b)b_1 - \bar{\delta}\varepsilon^{-1}Ab]}, \tag{34}$$

which is always positive, if

$$K > \frac{2\Omega\pi P_l \sqrt{(b+2A)}}{\varepsilon b}, \quad \frac{2}{P_l} > \frac{b}{\varepsilon}, \quad \bar{\delta} < \frac{b_1 \varepsilon}{A}. \tag{35}$$

This shows that coupling parameter has a stabilizing effect when condition (35) holds. In the absence of rotation and in a non-porous medium, equation (34) yields that $\frac{dR}{dK}$ is always positive, i.e., Rayleigh number R increases with increase in permeability parameter K , implying thereby the stabilizing effect of coupling parameter K .

Again it follows from equation (27) that

$$\frac{dR}{dA} = \frac{\left[b^4 \left(\frac{1+K}{P_l} \right)^2 \varepsilon^{-1} \left\{ (\bar{\delta} - b_1 P_l) + \frac{b_1 P_l}{(1+K)} \right\} + 4\Omega^2 \varepsilon^{-3} \pi^2 \bar{\delta} \right]}{k^2 \left(\frac{1+K}{P_l} \right)^2 \left\{ (2A+b)b_1 - \bar{\delta}\varepsilon^{-1}Ab \right\}^2}, \tag{36}$$

which is always positive, if

$$\bar{\delta} > b_1 P_l. \tag{37}$$

This shows that micropolar coefficient A has a stabilizing effect when condition (37) holds. In the absence of micropolar heat conduction parameter, equation (36) yields that $\frac{dR}{dA}$ is

always negative, implies that Rayleigh number R decreases with an increase in micropolar coefficient A implying thereby that the micropolar coefficient A always has a destabilizing effect in the absence of micropolar heat conduction parameter $\bar{\delta}$.

In the absence of dust particles ($b_1 = 1$) condition (37) reduces to $\bar{\delta} > P_l$, i.e., the micropolar heat conduction parameter is greater than medium permeability, which is in good agreement with the results obtained earlier [Reena and Rana (2008)]. Equation (27) also yields

$$\frac{dR}{d\bar{\delta}} = \frac{b\epsilon^{-1}A \left[b^3 \left(\frac{1+K}{P_l} \right) \left\{ \left(\frac{1+K}{P_l} \right) - KA\epsilon^{-1} \right\} + b^2 \left\{ 2A \left(\frac{1+K}{P_l} \right)^2 + 4\Omega^2 \epsilon^{-2} \pi^2 \right\} + b(8\Omega^2 \epsilon^{-2} \pi^2 A) \right]}{k^2 \left(\frac{1+K}{P_l} \right) \left\{ (2A+b) b_1 - \bar{\delta} \epsilon^{-1} Ab \right\}^2},$$

which is always positive, if

$$\frac{1}{P_l} > \frac{A}{\epsilon}. \tag{38}$$

This shows that micropolar heat conduction parameter $\bar{\delta}$ has a stabilizing effect when condition (38) holds. In a non-porous medium $\frac{dR}{d\bar{\delta}}$ is always positive, implies that micropolar heat conduction parameter always has a stabilizing effect in a non-porous medium. Also, it is clear from (38) that stabilizing effect of micropolar heat conduction parameter $\bar{\delta}$ is always independent of presence of dust particles. Thus, porosity and medium permeability have significant role in developing condition for the stabilizing behavior of micropolar heat conduction parameter.

7. Numerical Computation

Here in this section, the values of thermal Rayleigh number (R) for the onset of instability are determined for various values of permeability parameter (Darcy number) P_l , rotation parameter Ω , dust particles parameter b_1 and micropolar parameters K, A and $\bar{\delta}$, using equation (27) and variation of R with various parameters are illustrated in Figure 3-9.

Figure 3 illustrates that as permeability parameter P_l increases, Rayleigh number R always decreases for small values of rotation parameter Ω , whereas for higher values of Ω , R decreases for lower values of P_l and then increases for higher values of P_l , implying thereby that medium permeability has a destabilizing effect for lower values of Ω whereas for sufficiently higher values of Ω , medium permeability may have a destabilizing or a stabilizing effect which can also be observed from Figure 4.

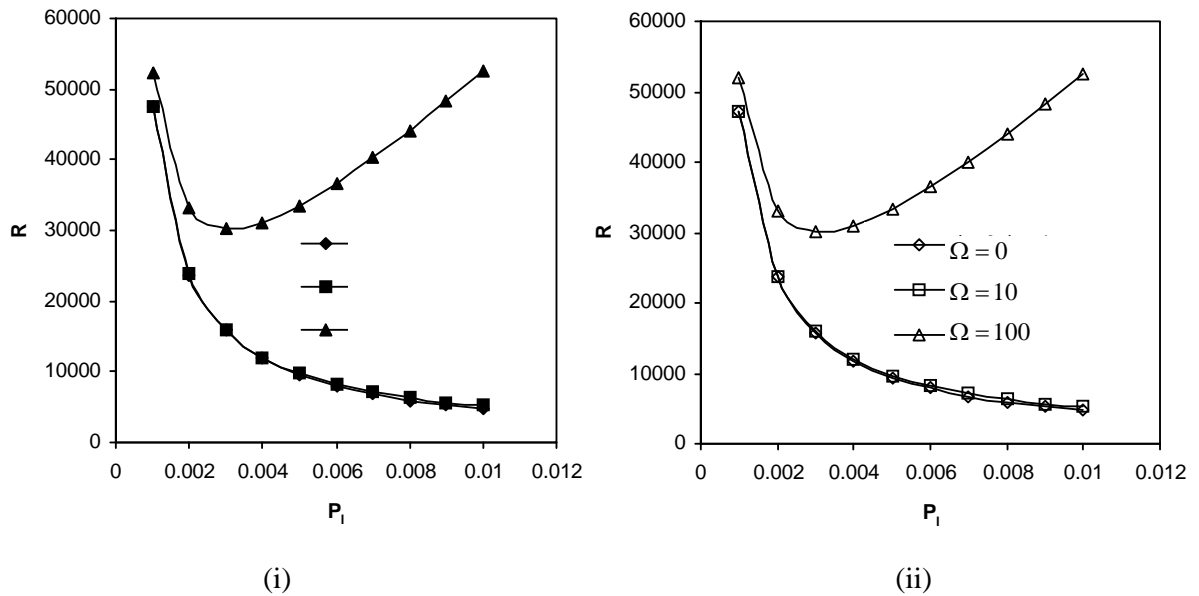


Figure 3. Marginal instability curve for variation of Rayleigh number R versus P_1 for $\varepsilon = 0.5, A = 0.1, K = 0.2, b_1 = 3, k = 1$ and (i) $\bar{\delta} = 0.05$, (ii) $\bar{\delta} = 0$.

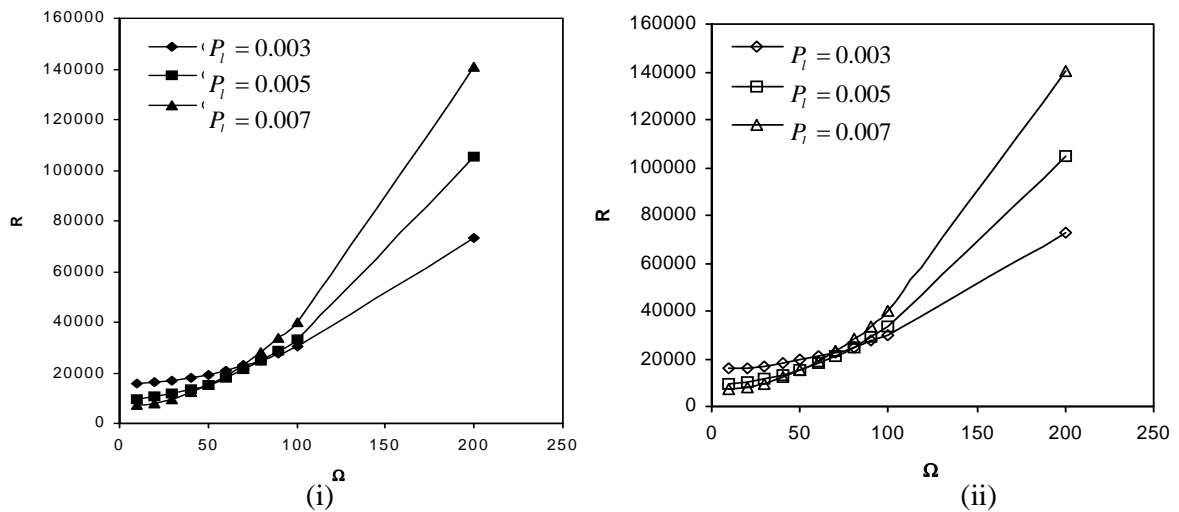


Figure 4. Marginal instability curve for variation of Rayleigh number R versus Ω for $\varepsilon = 0.5, A = 0.1, K = 0.2, b_1 = 3, k = 1$ and (i) $\bar{\delta} = 0.05$, (ii) $\bar{\delta} = 0$.

Figure 3 (ii) also illustrates that in the absence of rotation Ω and micropolar heat conduction parameter $\bar{\delta}$, permeability always has a destabilizing effect. Figure 4 indicates the stabilizing nature of Ω , as the value of R increases as Ω increases for every value of P_1 , in the presence or absence of micropolar heat conduction parameter $\bar{\delta}$, whereas figure 5 indicates that Ω has destabilizing effect for higher values of micropolar heat conduction parameter and stabilizing effect for lower values of micropolar heat conduction parameter, but from Figures 4 and 5, it is clear that in the absence of micropolar heat conduction parameter, rotation Ω always has stabilizing effect, i.e., Rayleigh number R increases as Ω increases.

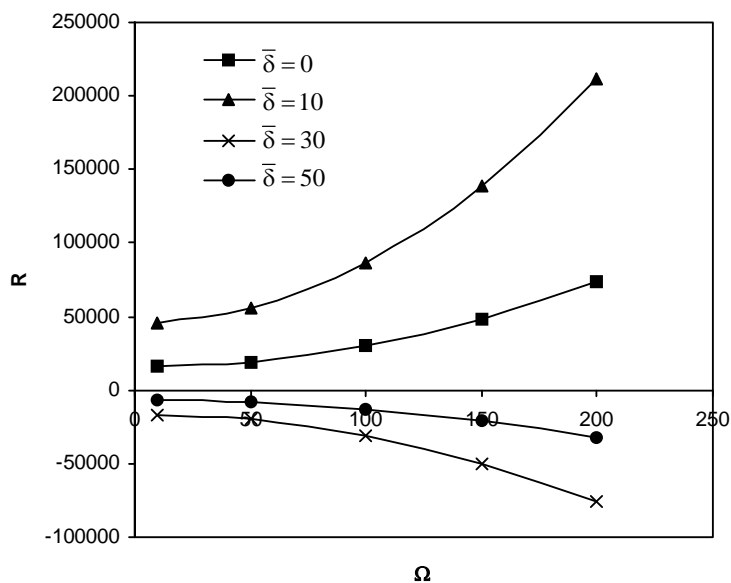


Figure 5. Marginal instability curve for variation of Rayleigh number R versus Ω for $\varepsilon = 0.5, A = 0.1, K = 0.2, b_1 = 3, k = 1$.

Figure 6 shows destabilizing nature of R as b_1 increases, i.e., as the value of dust parameter increases, the Rayleigh number R decreases.

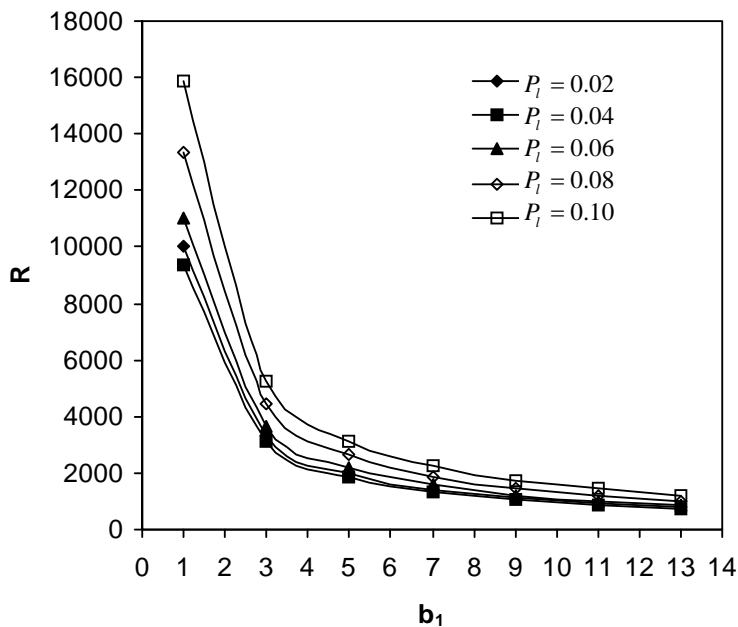


Figure 6. Marginal instability curve for variation of Rayleigh number R versus b_1 for $\varepsilon = 0.5, A = 0.1, K = 0.2, \Omega = 10, k = 1, \bar{\delta} = 0.05$.

It is also observed from figure 6 that in the absence of dust particles ($b_1 = 1$), the thermal Rayleigh number is very high however in the presence of dust particles ($b_1 > 1$), the thermal Rayleigh number is reduced because of the specific heat of the dust particles (because the heat capacity of clean fluid is supplemented by that of the dust particles).

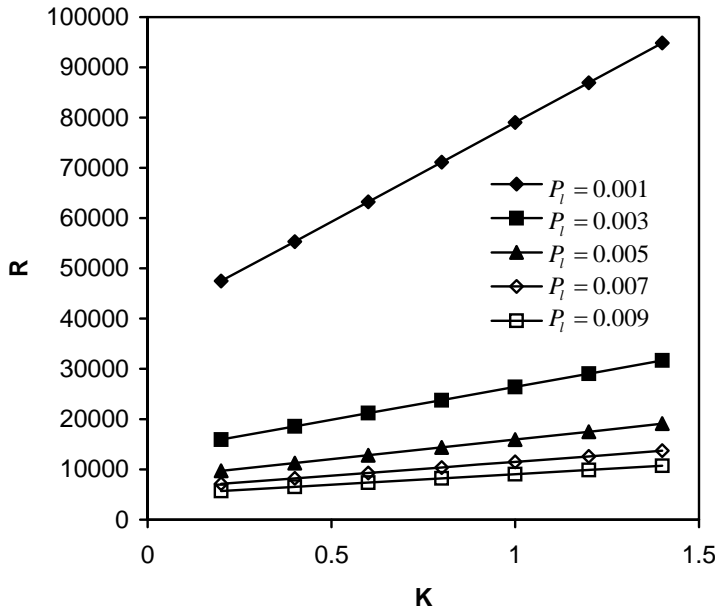


Figure 7. Marginal instability curve for variation of Rayleigh number R versus K for $\varepsilon = 0.5, A = 0.1, k = 1, \Omega = 10, b_1 = 3, \bar{\delta} = 0.05$.

Figure 7 and 9 represent the plots of thermal Rayleigh number R versus coupling parameter K and micropolar heat conduction parameter $\bar{\delta}$ for various values of P_i which indicate that the coupling parameter and micropolar heat conduction parameter has a stabilizing effect, whereas figure 8 represents the plot of thermal Rayleigh number R versus micropolar coefficient parameter (A) in the presence and absence of micropolar heat conduction parameter $\bar{\delta}$ which indicates that the micropolar coefficient parameter (A) have a stabilizing effect on the system in the presence of $\bar{\delta}$ and destabilizing effect in the absence of $\bar{\delta}$.

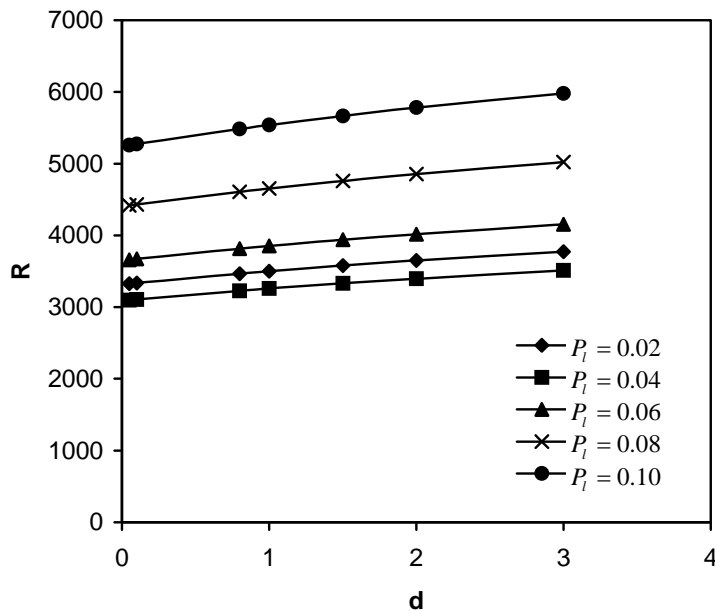


Figure 8. Marginal instability curve for variation of Rayleigh number R versus A for $\varepsilon = 0.5, K = 0.2, b_1 = 3, \Omega = 10, P_l = 0.003$.

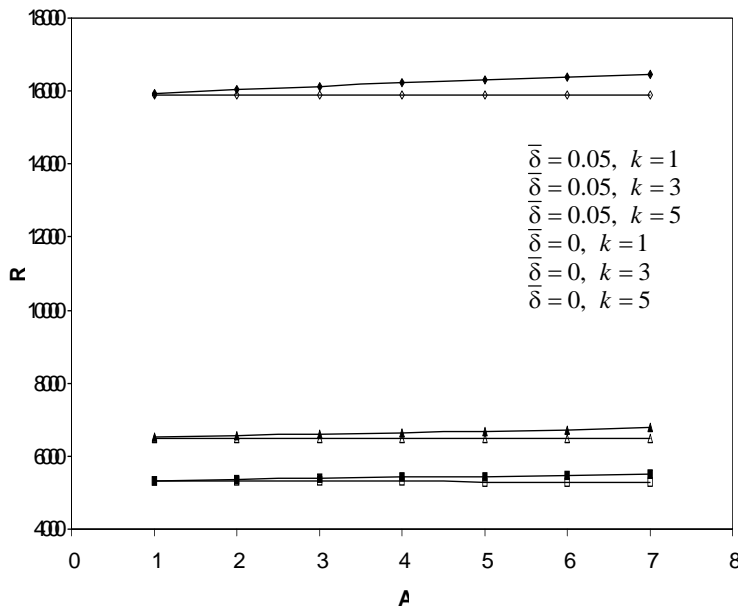


Figure 9. Marginal instability curve for variation of Rayleigh number R versus $\bar{\delta}$ for $\varepsilon = 0.5, A = 0.1, K = 0.2, b_1 = 3, \Omega = 10, k = 1$.

8. Discussion of Results

In this paper, the effect of dust particles on a micropolar rotating fluid heated from below saturating a porous medium has been studied. Here, the simplest boundary conditions, namely, free-free, no-spin, isothermal and perfectly heat conducting has been chosen which make the problem analytical practical and serve the purpose of providing a qualitative insight

into the problem. The case of two free boundaries is of little physical interest, but it is mathematically important because one can derive an exact solution whose properties help our analysis. We have investigated the effect of medium permeability, rotation, dust particles, micropolar coupling parameter (coupling between vorticity and spin effects), micropolar coefficient (the ratio of the micropolar viscous effects to micropolar diffusion effects) and micropolar heat conduction parameter (coupling between spin and heat flux) on the onset of convection. The major results from the analysis of this paper are as follows:

- (i) The principle of exchange of stabilities (PES) is found to hold true for a micropolar rotating fluid saturating a porous medium heated from below in the absence of dust particles, rotation and micropolar heat conduction parameter. The oscillatory modes are introduced due to the presence of the dust particles and rotation, which were non-existence in their absence. The presence of micropolar heat conduction parameter may also introduce oscillatory modes.
- (ii) The conditions, i.e., $\Omega < \left(\frac{b}{2\pi}\right) \sqrt{\left(\frac{AK}{l}\right)}$, $\frac{bP_l}{\varepsilon} < KE_1 p_1 < 2$, $f(1 - \bar{\delta}\varepsilon^{-1}A) > b'$ and $\bar{\delta}\varepsilon^{-1}A < 1$ are sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. Also it is found that in the absence of micropolar heat conduction parameter conditions as expected, reduce to $\frac{bP_l}{\varepsilon} < KE_1 p_1 < 2$ and $C_v > C_{pt}$ (i.e., the specific heat of the fluid at constant volume is greater than the specific heat of dust particles). In the absence of suspended particles, rotation, micropolar heat conduction parameter and for medium permeability very-very large ($P_l \rightarrow \infty$) the above conditions, as expected, reduces to $KEp_1 < 2$ which is in good agreement with the previous results obtain earlier by Sharma and Kumar (1998) and Sharma and Gupta (1997).
- (iii) For the case of stationary convection, the dust particles have a destabilizing effect, whereas rotation has a stabilizing effect on the system under certain conditions. In the absence of rotation and micropolar heat conduction parameter, the destabilizing effect of the medium permeability is depicted, but in the presence of rotation, medium permeability may have a destabilizing or a stabilizing effect on the onset of convection. The destabilizing role of the medium permeability in the absence of rotation can be observed from equation (28).

In the absence of micropolar heat conduction parameter, rotation always has stabilizing effect. In a non-porous medium, dust particles always have destabilizing effect on the system. The medium permeability and porosity have significant role in developing conditions for the destabilizing behavior of dust particles but the destabilizing behavior of dust particles is independent of presence of micropolar heat conduction parameter. We also observe that the thermal Rayleigh number R is reduced in the presence of dust particles because of the specific heat of the dust particles. The destabilizing effect of dust particles on fluids has been accounted earlier and is found to be valid for micropolar fluid also.

The effect of micropolar parameters has been also analyzed and found that coupling parameter, micropolar heat conduction parameter and micropolar coefficient (A) has a stabilizing effect under certain conditions but in the absence of micropolar heat conduction parameter, micropolar coefficient (A) has destabilizing effect, in the absence of rotation and

in a non-porous medium, coupling parameter always has stabilizing effect and in a non-porous medium, micropolar heat conduction parameter always has a stabilizing effect on the system. Also, it is observed from equation (38) that the stabilizing behavior of micropolar heat conduction parameter is always independent of presence of dust particles.

All of the above results for stationary convection are determined numerically and depicted graphically in Figures. 3-9. In order to investigate our results, the physical explanations behind our results found from the calculations are as follows:

- It is well known that the rotation introduces vorticity into the fluid in case of Newtonian fluid [Chandrasekhar (1981)]. Then due to this motion, the fluid moves in the horizontal planes with higher velocities, the velocity of the fluid perpendicular to the planes reduces and hence delays the onset of convection. On account of this, rotation has stabilizing effect. When we consider that the fluid layer is to be flowing through an isotropic and homogeneous medium, free from rotation or a small rate of rotation, then the medium permeability has a destabilizing effect. This is because, as medium permeability increases, the void space increases and on account of this, the flow quantities perpendicular to the planes will obviously be increased. Thus, increase in heat transfer is responsible for early onset of convection. Thus, increase in p_f leads to decrease in R . In case the stabilizing effect of coupling parameter (K). The physical explanation behind this is that as K increases of high rotation, the motion of the fluid prevails essentially in the horizontal planes. As medium permeability increases, this motion is increased. Thus, the component of velocity perpendicular to the horizontal planes reduces, leading to delay in the onset of convection. Hence, affect of medium permeability converts to stabilizing effect in case of high rotation.
- Figure 6 shows the destabilizing nature of R as the value of dust parameter (b_1) increases. The thermal Rayleigh number R shows a drastic decrease in the presence of dust particles because the heat capacity of clean fluid is supplemented by that of the dust particle.
- Figure 7 shows, concentration of micro elements also increases and on account of this, a greater part of the energy of the system is consumed by these elements in developing gyration velocities in the fluid, leading to delay in the onset of convection.
- Figure 8 represents the stabilizing behavior of the micropolar coefficient ($A = K / C_0$), i.e., destabilizing effect of spin-diffusion (couple-stress) parameter. As A increases, the couple stress of the fluid increases. Which causes the microrotation to decrease, rendering the system towards the instability?
- Figure 9 indicates the stabilizing effect of micropolar heat conduction parameter ($\bar{\delta}$). The physical concept behind it is, when $\bar{\delta}$ increases, the heat induced in the fluid due to microelements is also increased, thus inducing the heat transfer from the bottom to the top. The decrease in heat transfer is responsible for delaying the onset of convection. Thus increase in $\bar{\delta}$ lead to increase in R , thereby $\bar{\delta}$ stabilizes the flow.

9. Conclusion

From the above analysis, we conclude that the oscillatory modes are introduced due to the presence of dust particles and rotation. The presence of micropolar heat conduction parameter may also introduce oscillatory modes. The thermal Rayleigh number is reduced solely in the presence of dust particles as the heat capacity of the clean fluid is supplemented by the dust particles. The results show that for the case of stationary convection, the medium permeability, dust particles and spin diffusion parameter has a destabilizing effect under certain condition(s), whereas rotation and micropolar parameters (coupling parameter and micropolar heat conduction parameter) has a stabilizing effect under certain condition(s).

Finally we conclude that the micropolar parameters, rotation, dust particles and permeability have a deep effect on the onset of convection in porous medium. It is believed that the present work will serve for understanding more complex problems including the various physical effects investigated in the present problem.

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Appendix-A

The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\delta\rho = -\rho_0\alpha\theta \tag{A.1}$$

Then the linearized perturbation equations of the micropolar fluid become

$$\nabla \cdot \mathbf{u} = 0, \tag{A.2}$$

$$L_1 \frac{\rho_0}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t} = L_1 \left[-\nabla(\delta p) - \frac{1}{k_1}(\mu + k)\mathbf{u} - \rho_0\alpha\theta\mathbf{g} + k\nabla \times \boldsymbol{\omega} + \frac{2\rho_0}{\varepsilon}(\mathbf{u} \times \boldsymbol{\Omega}) \right] - \frac{mN_0}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t}, \tag{A.3}$$

$$\rho_0 j \frac{\partial \boldsymbol{\omega}}{\partial t} = (\varepsilon' + \beta') \nabla(\nabla \cdot \boldsymbol{\omega}) + \gamma' \nabla^2 \boldsymbol{\omega} + \frac{k}{\varepsilon} \nabla \times \mathbf{u} - 2k\boldsymbol{\omega}, \tag{A.4}$$

$$L_1 \left[(E + b'\varepsilon) \frac{\partial \theta}{\partial t} \right] = L_1 \left[x_T \nabla^2 \theta - \frac{\delta'}{\rho_0 C_v} (\nabla \times \boldsymbol{\omega})_z \beta + \beta u_z \right] + b' \beta u_z \tag{A.5}$$

where, $b' = \frac{mN_0 C_{p'}}{\rho_0 C_v}$, $x_T = \frac{K_T}{\rho_0 C_v}$ (stands for the thermal diffusivity),

$$E = \varepsilon + \frac{\rho_s C_s}{\rho_0 C_v} (1 - \varepsilon), \quad L_1 = \left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right)$$

In writing all the perturbation equations in linearized form, we have neglected all the non-linear terms like $(\mathbf{u} \cdot \nabla) \mathbf{u}$, $(\mathbf{u} \cdot \nabla) \theta$, $\nabla \theta \cdot (\nabla \times \boldsymbol{\omega})$, $(\mathbf{u} \cdot \nabla) \boldsymbol{\omega}$, $(\mathbf{u}' \cdot \nabla) \mathbf{u}'$ and $(\mathbf{u}' \cdot \nabla) \theta$, since the perturbations applied on the system are assumed to be small, the second and higher order perturbations are negligibly small and only linear terms are retained.

Now, it is usual to write the balance equations in a dimensionless form, scaling as

$$(x, y, z) = (x^*, y^*, z^*) d, \quad t = \frac{\rho_0 d^2}{\mu} t^*, \quad \theta = \beta d \theta^*, \quad \mathbf{u} = \frac{x_T}{d} \mathbf{u}^*, \quad \mathbf{u}' = \frac{x_T}{d} (\mathbf{u}')^*, \quad p = \frac{\mu x_T}{d^2} p^*,$$

$$\boldsymbol{\omega} = \frac{x_T}{d^2} \boldsymbol{\omega}^*, \quad M' = \frac{\rho_0 x_T}{\mu} M'^*, \quad \boldsymbol{\Omega} = \frac{\mu}{\rho_0 d^2} \boldsymbol{\Omega}^*, \quad L_1^* = \tau \frac{\partial}{\partial t^*} + 1, \quad \text{where } \tau = \frac{m\mu}{K' \rho_0 d^2}$$

and then removing the stars for convenience, the non-dimensional form of equations (A.2) – (A.5) become

$$\nabla \cdot \mathbf{u} = 0 \tag{A.6}$$

$$L_1 \frac{1}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t} = L_1 \left[-\nabla(\delta p) - \frac{1}{P_1}(1 + K)\mathbf{u} + R\theta \hat{e}_z + K(\nabla \times \boldsymbol{\omega}) + \frac{2}{\varepsilon}(\mathbf{u} \times \boldsymbol{\Omega}) \right] - \frac{f}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t}, \tag{A.7}$$

$$\bar{j} \frac{\partial \boldsymbol{\omega}}{\partial t} = C_1 \nabla(\nabla \cdot \boldsymbol{\omega}) - C_0 \nabla \times (\nabla \times \boldsymbol{\omega}) + K \left(\frac{1}{\varepsilon} \nabla \times \mathbf{u} - 2\boldsymbol{\omega} \right), \tag{A.8}$$

and $L_1 E_1 P_1 \frac{\partial \theta}{\partial t} = L_1 \left[\nabla^2 \theta - \bar{\delta} (\nabla \times \boldsymbol{\omega})_z + u_z \right] + b' u_z \tag{A.9}$

where, we have taken \hat{e}_z as a unit vector in z -direction and the new dimensionless coefficients are

$$\bar{j} = \frac{j}{d^2}, \quad \bar{\delta} = \frac{\delta'}{\rho_0 C_v d^2}, \quad P_l = \frac{k_1}{d^2}, \quad K = \frac{k}{\mu}, \quad C_0 = \frac{\gamma'}{\mu d^2}, \quad C_1 = \frac{\varepsilon' + \beta' + \gamma'}{\mu d^2}, \quad E_1 = E + b'\varepsilon, \quad f = \frac{mN_0}{\rho_0}$$

together with the dimensionless Rayleigh (R) and Prandtl (p_1) numbers defined as

$$R = \frac{g\alpha\beta\rho_0 d^4}{\mu x_T}, \quad p_1 = \frac{\mu}{\rho_0 x_T}$$

Now, applying curl operator twice to equation (A.7) and taking the z -component, we Get

$$\left[\left\{ \frac{1}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{P_l} (1+K) \right\} L_1 + \frac{f}{\varepsilon} \frac{\partial}{\partial t} \right] \nabla^2 u_z = L_1 \left[R\theta \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + K \nabla^2 \Omega'_z - \frac{2}{\varepsilon} \Omega \left(\frac{\partial \zeta_z}{\partial z} \right) \right] \quad (A.10)$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\Omega'_z = (\nabla \times \boldsymbol{\omega})_z = \left(\frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y} \right)$ and

$$\zeta_z = (\nabla \times \mathbf{u})_z = \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) \text{ is the } z \text{-component of vorticity.}$$

Again applying the curl operator once to equations (A.7) and (A.8) respectively and taking the z -component, we get

$$\left[\frac{L_1 + f}{\varepsilon} \right] \frac{\partial \zeta_z}{\partial t} = L_1 \left[\frac{2\Omega}{\varepsilon} \frac{\partial u_z}{\partial z} - \frac{(1+K)}{P_l} \zeta_z \right] \quad (A.11)$$

$$\text{and } \bar{j} \frac{\partial \Omega'_z}{\partial t} = C_0 \nabla^2 \Omega'_z - K \left[\frac{1}{\varepsilon} \nabla^2 u_z + 2\Omega'_z \right] \quad (A.12)$$

In this equation, the coefficient C_0 and K account for spin diffusion and coupling between vorticity and spin effects respectively.

The linearized non-dimensional form of equation (A.9) is

$$L_1 (E_1 p_1) \frac{\partial \theta}{\partial t} = L_1 \left[\nabla^2 \theta - \bar{\delta} \Omega'_z + u_z \right] + b' u_z \quad (A.13)$$

Appendix-B

$$(D^2 - k^2) \left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l} (1+K) \right\} (1 + \tau\sigma) + \frac{f}{\varepsilon} \sigma \right] U = (1 + \tau\sigma) \left(-Rk^2 \Theta + K(D^2 - k^2)G - \frac{2}{\varepsilon} \Omega DZ \right) \quad (B.1)$$

$$\left[\left\{ \frac{\sigma}{\varepsilon} + \frac{1}{P_l} (1+K) \right\} (1 + \tau\sigma) + \frac{f}{\varepsilon} \sigma \right] Z = \left(\frac{2}{\varepsilon} \Omega D U \right) (1 + \tau\sigma) \quad (B.2)$$

$$[l\sigma + 2A - (D^2 - k^2)]G = -A\varepsilon^{-1}(D^2 - k^2)U, \tag{B.3}$$

and

$$[\{E_1 p_1 \sigma - (D^2 - k^2)\}(1 + \tau\sigma)]\Theta = (1 + \tau\sigma)[U - \bar{\delta}G] + b'U \tag{B.4}$$

where, $l = \bar{j}A/K$, $A = K/C_0$ and $D \equiv d/dz$. Here, A is the ratio between the micropolar viscous effects and micropolar diffusion effects.

Appendix-C

$$C_1 = \sigma_i^2,$$

$$f_4 = l\tau^3\varepsilon^{-2}b \left[\tau\varepsilon^{-1}b \left(l + E_1 p_1 \bar{\delta}\varepsilon^{-1}A \right) + lE_1 p_1 \left(L_2 - \frac{b_1}{\varepsilon} \right) \right] \tag{C.1}$$

$$\begin{aligned} f_3 = & 4\pi^2\Omega^2\varepsilon^{-2}\tau^3l^2 \left[\tau \left(E_1 p_1 L_0 - \frac{b}{\varepsilon} \right) + E_1 p_1 \varepsilon^{-1}(f + b') \right] \\ & + l\tau E_1 p_1 \left[l \left(L_2 - \frac{b_1}{\varepsilon} \right) \left\{ \left(L_0 \tau + \frac{f}{\varepsilon} \right)^2 + \left(\frac{1+2f}{\varepsilon^2} \right) \right\} + \frac{b_1}{\varepsilon} \right] b \\ & + \tau^2 \varepsilon^{-1} \left[(2A\tau + lb') \tau \varepsilon^{-2} A (2 - KE_1 p_1) + \frac{2A\tau}{\varepsilon} E_1 p_1 \left(L_2 - \frac{b_1}{\varepsilon} \right) + E_1 p_1 \bar{\delta} \varepsilon^{-2} A \left\{ \frac{2A\tau}{\varepsilon} + l \left(\frac{1+f}{\varepsilon} \right) \right\} \right] \\ & + l^2 \left\{ \left(L_0 \tau + \frac{f}{\varepsilon} \right)^2 + \left(\frac{2+3f}{\varepsilon^2} \right) + \frac{b'}{\varepsilon} L_2 \right\} + lE_1 p_1 \left\{ L_2^2 + \left(\frac{1+f}{\varepsilon} \right)^2 \bar{\delta}\varepsilon^{-1}A \right\} \\ & + \tau l \bar{\delta}\varepsilon^{-1}A \left\{ \left(\tau L_0 + \frac{f}{\varepsilon} \right) \left(E_1 p_1 L_0 - \frac{b}{\varepsilon} \right) + E_1 p_1 L_0 \frac{f}{\varepsilon} \right\} b^2 \\ & + \tau^3 \varepsilon^{-2} \left[\left\{ \left(L_0 E_1 p_1 + \frac{2A}{\varepsilon} \right) (1 - \bar{\delta}\varepsilon^{-1}A) + \frac{A}{\varepsilon} (2 - KE_1 p_1) \right\} + E_1 p_1 \varepsilon^{-1} \{ f - (f\bar{\delta}\varepsilon^{-1}A + b') \} + KA\varepsilon^{-1}\tau l \right] b^3 \\ & + \tau^4 \varepsilon^{-3} E_1 p_1 b' (AKb^2 - 4\pi^2\Omega^2 l) \bar{\delta}\varepsilon^{-1}A + \tau^4 \varepsilon^{-3} (1 - \bar{\delta}\varepsilon^{-1}A) b^4 \tag{C.2} \end{aligned}$$

where for the sake of convenience, we have put $L_0 = \left(\frac{1+K}{P_1} \right)$, $L_2 = \left[\left(\frac{1+K}{P_1} \right) \tau + \left(\frac{1+f}{\varepsilon} \right) \right]$

Appendix-D

$$C_1 = \sigma_i^2,$$

$$f_2 = [\varepsilon^{-3} l (l + \bar{\delta}\varepsilon^{-1} A E p_1)] b^2 + [\varepsilon^{-2} L_0 E p_1 l^2] b > 0, \tag{D.1}$$

$$f_1 = [\varepsilon^{-3} (1 - \bar{\delta}\varepsilon^{-1} A)] b^4 + \varepsilon^{-2} [E p_1 (1 - \bar{\delta}\varepsilon^{-1} A) (L_0 - K A \varepsilon^{-1}) + \{l \varepsilon^{-1} A (k - L_0 \bar{\delta})\}]$$

$$+ 2 A \varepsilon^{-1} (1 - \bar{\delta}\varepsilon^{-1} A + 1)] b^3 + [2 A E p_1 L_0 \varepsilon^{-2} (1 - \bar{\delta}\varepsilon^{-1} A + 1) + \varepsilon^{-3} (2 - K E p_1) 2 A^2] b^2$$

$$+ l \varepsilon^{-1} (l + E p_1 \bar{\delta}\varepsilon^{-1} A) [L_0^2 b - 4 \Omega^2 \varepsilon^{-2} \pi^2] b + E p_1 L_0 (\varepsilon^{-2} 4 A^2 + L_0^2 l^2) b + 4 \Omega^2 \varepsilon^{-2} \pi^2 E p_1 l^2 L_0$$

(D.2)

There is no need of writing f_0 .