Министерство образования и науки
Российской Федерации
Национальный исследовательский
Томский государственный университет
Философский факультет

INITIA:
АКТУАЛЬНЫЕ ПРОБЛЕМЫ
СОЦИАЛЬНЫХ НАУК
(26–27 апреля 2019 г)

Материалы XXI Международной
конференции молодых ученых

Томск
2019

«SCHOLARLY WRITING AND PRESENTATION»

VARYING DOMAIN FIRST-ODER MODAL LOGIC

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How do we think? That is one of the most significant questions of philosophy. Our thinking is expressed in our language. In logic, we abstract from the content of our thoughts in an attempt to grasp their structure. Logic is a formal theory using which we can formalize statements of natural or artificial languages. Different logics can be used to formalize different types of statements. The purpose of the presentation is to present Varying Domain First-Oder Modal Logic and its advantages. For this purpose, I will explicate the concept of a varying domain first-order modal model in comparison with the concept of extensional first-order model.

First of all, we should consider modal formal languages and their properties. There are some differences between languages of varying domain first-order modal logic and ones of classical first-order logic. The differences are due to using modals in varying domain first-order modal logic. That is why the vocabulary of modal logic extends that of classical first-order logic by using two new unary operators, □ (necessarily) and ◊ (possibly). Using modal operators, we can formalize such statements as «It is necessary that P» and «It is possible that P». Let me give you a few examples of formalizing statements of this type:

1. It is possible that Pushkin is bald. $\Diamond Bp$
2. It is possible that there is life on Mars. $\Diamond \exists x(Lx\&Mx)$
3. It is possible that there is a five-legged cat. $\Diamond \exists x(Cx\&Fx)$
4. It is necessary that the God is all good. $\Box Ag$
5. It is necessary that Providence exists. $\Box \exists x(x=p)$
6. It is necessary that the God is all good and that Providence exists. $\Box (Ag\&\exists x(x=p))$

As a result, the syntax of formal modal languages is extended by a new clause in definition of formulas. This rule is «If X is a formula, so are □X and ◊X».

Now we can give the definition of the varying domain first-order modal model. A modal model (model for short) is an ordered quadruple (G, R, D, I), where: a) G is a non-empty set whose members are called possible worlds. Possible worlds are alternative conceivable states of the actual world, and we take them into account when using modalities. We will use Greek letters to refer to them b) R is a binary relation on G, called accessibility relation, which is the way in which possible worlds are connected to each other. If worlds B and Ω are in relation R, we write BRΩ. In natural language this means that Ω is accessible from B, in other words, Ω is an alternative world to B; c) D – is a function mapping members of G to non-empty sets, also called domain function. In other words, this function assigns to every possible world the set of objects existing in it. This means that in a varying domain first-order modal logic it makes sense to talk about the existence or non-existence of objects in possible worlds. Also this means that using this formal language we can formalize statements about objects that do not exist in actual world indicating the fact of their non-existence. An example is «Socrates is wiser than anyone alive». d) Interpretation is a function that assigns to each n-place relation symbol R, and to each possible world ΓϵG, some n-place relation on the union of all domains of possible worlds of the model. Now we can compare extensional models used in classical first-order logic and modal models. As we can see, a modal model is more complicated than extensional model represented just by domain, which is simply a set of objects and by an interpretation, i.e. by a function that assigns to each n-place predicate an n-place relation on the set of objects.

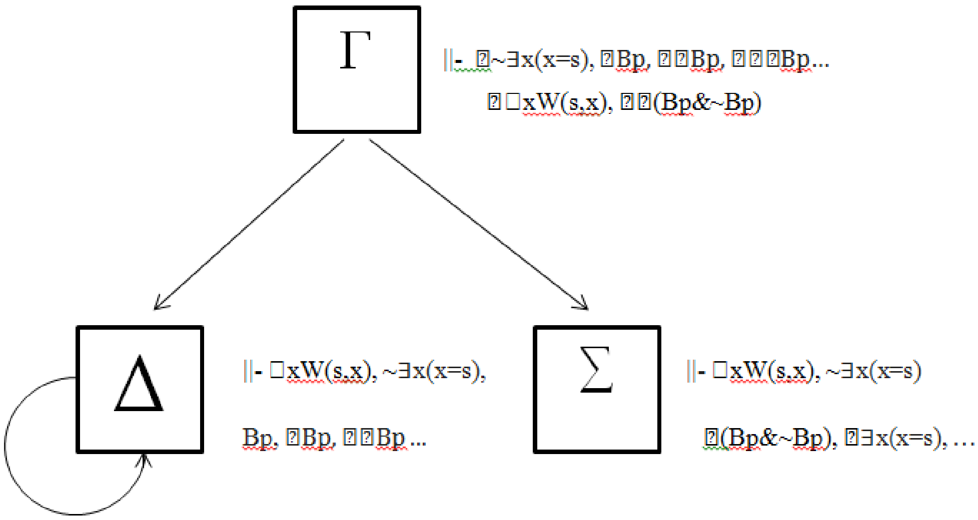
Thus, in the varying domain first-order modal logic the truth value of a formula depends on the possible worlds. Formula X can be true at one world and false at another. There is a definition of truth in a model:

[Truth in a Model] Let M = (G, R, D, I) be a varying domain first-order modal model. For each ΓϵG, and each valuation v in D(M):

1. If R is an n-place relation symbol, M, $\Gamma||\text{-}_v R(x_1, \ldots ,x_n)$ provided $(v(x_1), \ldots , v(x_n)) \epsilon I(R, \Gamma)$.
2. M, $\Gamma||\text{-}_v \sim X \leftrightarrow$ M, Γ||-/X.
3. M, $\Gamma||\text{-}_v(X\&Y) \leftrightarrow$ M, $\Gamma||\text{-}_v$ X and M, $\Gamma||\text{-}_v$ Y.
4. M, $\Gamma||\text{-}_v \Box X \leftrightarrow$ for every ΔϵG, if ΓRΔ then M, $\Delta||\text{-}_v X$.
5. M, $\Gamma||\text{-}_v \Diamond X \leftrightarrow$ for some ΔϵG, ΓRΔ and M, $\Delta||\text{-}_v X$.
6. M, $\Gamma||\text{-}_v(\forall x)\Phi \leftrightarrow$ for every x-variant w of v at Γ, M, $\Gamma||\text{-}_w \Phi$.
7. M, $\Gamma||\text{-}_v(\exists x)\Phi \leftrightarrow$ for some x-variant w of v at Γ, M, $\Gamma||\text{-}_w \Phi$.

Let me give an example. On the picture 1 in the form of a graph we will depict the model M for a language L. Language L contains two predicate symbols (B means *bald*, and W means a two-place relation *wiser then*), and two constant symbols (s means Socrates, p means Pushkin) The set G contains possible worlds Γ, Δ and Σ. Accessibility relation between worlds is shown in the graph using arrows. Let D(Γ)={a, Socrates}, D(Δ)={a, Pushkin}, and D(∑)={b, Pushkin}. Let I(B, Γ) be the empty set, I(B, Δ)={a, Pushkin}, and I(B, ∑)={a,b}. Let I(W, Γ)={(Socrates, a)}, I(W, Δ)={(Socrates, Pushkin), (Socrates, a), (Pushkin, a)}, and I(W, ∑)={(Socrates, Pushkin), (Socrates, b), (b, Pushkin)}. We can see that the statement ∀xW(s,x) is true at Δ and ∑, but Socrates exists at the world Γ. The formula □ ∀xW(s,x) is true at Γ because at every world accessible from Γ, the formula ∀xW(s,x) is true, according to the clause 4 of our definition. The formula ~∃x(x=s) is false at Γ, but □ ~∃x(x=s) is true at Γ because at every world accessible from Γ, the formula ~∃x(x=s) is true. The formula ◊Bp will be true at Γ, because in some worlds accessible from Γ, in our example in Δ, the formula Bp is true, according to the clause 5 of our definition. As the world Δ is an alternative world of itself and there is no other world accessible from it, formulas □Bp, □□Bp, □□□Bp and so on are true at Δ. Consequently, at the world Γ formulas ◊□Bp,

◊□□Bp and so on are true. The world Σ hasn't any accessible world. We will call such a world a dead end. Following the definition, every formula, for example □(Bp&~Bp), will be necessarily true at such worlds, but no one will be possible. Consequently, formulas like ◊□(Bp&~Bp), will be true at Γ.



Picture 1

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