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Kinetics of the vacuum e^-e^+ plasma in a strong electric field and problem of radiation

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We consider a quantum kinetic equation for e^-e^+ plasma created from vacuum under the action of a strong time-dependent linearly polarized electric field. Simplification of the collision integral for photon emission along the polarization direction of the field is discussed.

Keywords: Strong electric field; electron-positron plasma; kinetic equations; radiation.

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1. Introduction

A kinetic equation (KE) approach to e^-e^+ plasma (EPP) creation from vacuum under the action of a strong linearly polarized time-dependent electric field is well recognized and widely applied for studying the pair creation process in a strong laser field and in heavy ion collisions.[1,2] Generalizations, e.g., to an arbitrary polarization of the external field[3] and/or to the account for backreaction,[4] have been under active development.

Recently, within the framework of the BBGKY hierarchy approach, a generalized system of KEs was derived[3,5] for a EPP coupled to a quantized electromagnetic field (photon reservoir) with an internal plasma field governed self-consistently by the Maxwell equations. In a second order of perturbation theory with respect to the quantized electromagnetic field this includes the processes of photon emission/absorption and pair photoproduction/single photon annihilation in a combined external and internal background field. In the absence of the field these channels are closed due to energy-momentum conservation, but are activated in its presence with possible observable consequences: photon radiation, depletion of the external field and corresponding suppression of the net pair production.[6] Here we discuss a simplification of the collision integral for photon emission directed parallel to the polarization axis of a linearly polarized time dependent electric field.

2. Photon Kinetic Equation in the Emission Channel

For brevity, let us focus on the photon emission channel in the photon KE[5]

$$\dot{F}(\mathbf{k},t) = \int \frac{d^3p_1}{(2\pi)^3} \int \frac{d^3p_2}{(2\pi)^3} \int^t dt' K^{(\gamma)}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}; t, t')\{f(\mathbf{p}_1, t')[1 - f(\mathbf{p}_2, t')] \times [1 + F(\mathbf{k}, t')] - f(\mathbf{p}_1, t')[1 - f(\mathbf{p}_2, t')]F(\mathbf{k}, t')\} = S^{(\gamma)}(\mathbf{k}, t), \tag{1}$$

where $F(\mathbf{k},t)$ and $f(\mathbf{p},t)$ are the distribution functions of photons and electrons (positrons), respectively. The kernel in the collision integral (CI) in the r.h.s. of Eq. (1) reads:

$$K^{(\gamma)}(\mathbf{p}, \mathbf{p}_1, \mathbf{k}; t, t') = \frac{(2\pi)^3 e^2}{2k} \delta(\mathbf{p} - \mathbf{p}_1 - \mathbf{k}) \mathrm{Re}\,\{[\overline{u}u]^{r+}_{\alpha\beta}(\mathbf{p}, \mathbf{p}_1, \mathbf{k}; t) \times [\overline{u}u]^{r}_{\alpha\beta}(\mathbf{p}, \mathbf{p}_1, -\mathbf{k}; t') \cos \int_{t'}^{t} d\tau [\varepsilon(\mathbf{p}, \tau) - \varepsilon(\mathbf{p}_1, \tau) - k]\}, \tag{2}$$

where $[\overline{u}u]^{r}_{\alpha\beta}(\mathbf{p}, \mathbf{p}_1, \mathbf{k}; t) = \overline{u}_\beta(\mathbf{p}, t)\gamma^\mu u_\alpha(\mathbf{p}_1, t) e^r_\mu(\mathbf{k})$, $u(\mathbf{p}, t)$ is the spinor in the presence of an external field in the leading order of the WKB approximation, which differs from a free spinor by the substitution $\mathbf{p} \mapsto \mathbf{P}(t)$,[3,5] $\varepsilon(\mathbf{p}, t) = \sqrt{m^2 + P^2(t)}$ is quasiparticle energy, $\mathbf{P}(t) = \mathbf{p} - e\mathbf{A}(t)$ is kinetic momentum, $\mathbf{A}(t)$ is vector potential of a homogeneous time-dependent external electric field, $e^r_\mu(\mathbf{k})$ is the polarization 4-vector of the emitted or absorbed photon, $k = |\mathbf{k}|$ is the photon energy. Integral over $\mathbf{p}_1$ is actually removed by δ-function in (2) which reflects momentum conservation in a homogeneous field. The distribution function $f(\mathbf{p}, t)$ should be found as solution of a coupled KE for EPP.[5,7]

In the absence of external field the CI in KE (1) describes a relaxation process with slowly varying distribution functions. Then, after integration over t', the cosine factor in the r.h.s. of Eq. (2) turns into δ-function $\delta(\varepsilon_0(\mathbf{p}) - \varepsilon_0(\mathbf{p}_1) - k)$, which vanishes identically in virtue of incompatibility of the conservation laws of momentum $\mathbf{p} - \mathbf{p}_1 - \mathbf{k} = 0$ and energy $\varepsilon_0(\mathbf{p}) - \varepsilon_0(\mathbf{p}_1) - k = 0$ (here $\varepsilon_0 = \varepsilon|_{A=0}$). Thus $S^{(\gamma)}_0 = 0$.

However, in general, by turning an external time-dependent field on, this channel is activated. Let us demonstrate and discuss it for the case of photon emission along the polarization direction of a monochromatic linearly polarized field $\mathbf{A}(t) = \{0, 0, (E_0/\nu)\sin\nu t\}$. As a first step, let us adopt the approximation $\varepsilon(\mathbf{p}, t) \approx \varepsilon_*(\mathbf{p}) = \sqrt{m_*^2 + p^2}$ everywhere in the kernel (2), where $m_* = \sqrt{m^2 + \frac{1}{2}(eE_0/\nu)^2}$ is the effective electron mass in such a field. We also assume slow variation of the electron distribution function $f(\mathbf{p}, t)$ that takes place, e.g., in Markovian approximation.[7] Since photon emission is induced by the field, for a qualitative discussion let us consider only contribution of the terms $\propto e^2\mathbf{A}(t)\mathbf{A}(t')$ originating from the spinor convolutions. Integration of such terms over time results in

$$\int_{-\infty}^{t} dt'\mathbf{A}(t)\mathbf{A}(t')\cos\left[\Delta\varepsilon_*(\mathbf{p},\mathbf{k})(t-t')\right] = \frac{1}{2}\left(\frac{E_0}{\nu}\right)^2 \left\{\pi\sin^2(\nu t)\left[\delta(\Delta\varepsilon_* - \nu) + \delta(\Delta\varepsilon_* + \nu)\right] + \sin(2\nu t)\,\mathrm{V.p.}\,\frac{\nu}{\Delta\varepsilon_*^2 - \nu^2}\right\} \sim \frac{\pi}{4}\left(\frac{E_0}{\nu}\right)^2 \delta(\Omega), \tag{3}$$

where $\Delta\varepsilon_*(\mathbf{p},\mathbf{k}) = \varepsilon_*(\mathbf{p}) - \varepsilon_*(\mathbf{p}-\mathbf{k}) - k$ is the energy deficiency of the process. Here in passing to the final result we took into account that (i) the first δ-function is vanishing identically; (ii) in virtue of slow variation of the distribution functions we can pass to a time average of (3), so that the only contribution comes from the second δ-function expressing the energy conservation law in the presence of the field

$$\Omega(\mathbf{p},\mathbf{k},\nu) = \Delta\varepsilon_*(\mathbf{p},\mathbf{k}) + \nu = 0, \tag{4}$$

which is compatible with momentum conservation in some region of the parameters $\mathbf{p},\ \mathbf{k},\ \nu$.

Integration over electron momentum $\mathbf{p}$ greatly simplifies if we consider emission of soft ($k \ll p$) photons along the direction of the field (i.e., z-axis). Indeed, using the expansion

$$\Delta\varepsilon_*(\mathbf{p},\mathbf{k}) \approx \frac{\partial\varepsilon_*(\mathbf{p})}{\partial\mathbf{p}}\mathbf{k} - k = \left(\frac{p_z}{\varepsilon_*(\mathbf{p})} - 1\right)k,$$

and assuming $k > \nu/2$ we have:

$$\delta(\Omega) \approx \frac{\varepsilon_{*\perp}}{k}\left[\frac{\nu}{k}\left(2 - \frac{\nu}{k}\right)\right]^{-3/2}\delta(p_z - p_*), \tag{5}$$

where $\varepsilon_{*\perp} = (m_*^2 + p_\perp^2)^{1/2}$ is the transverse energy and

$$p_* = \varepsilon_{*\perp}\left(1 - \frac{\nu}{k}\right)\left[\frac{\nu}{k}\left(2 - \frac{\nu}{k}\right)\right]^{-1/2}. \tag{6}$$

Then integration is trivial over p_z and idle over the azimuthal angle, thus we finally obtain:

$$S^{(\gamma)}(\mathbf{k},t) \simeq \frac{\alpha}{8}\left(\frac{m}{k}\frac{m}{\nu}\frac{E_0}{E_c}\right)^2\left[\frac{\nu}{k}\left(2 - \frac{\nu}{k}\right)\right]^{-1/2}\int_0^\infty \frac{dp_\perp p_\perp}{\varepsilon_{*\perp}} f(p_\perp, p_*; t) \times \left[1 - f(p_\perp, p_*; t)\right], \tag{7}$$

where $\alpha = e^2/4\pi$ is the fine structure constant and $E_c = m^2/e$ is the Schwinger critical field. The remaining integral can be evaluated by knowing the distribution $f(\mathbf{p})$ of the EPP subsystem. In the adopted approximation the r.h.s. is non-vanishing strictly above the threshold $k = \nu/2$, which is soft assuming the EPP distribution is effectively localized in momentum space decaying at infinite momenta.

3. Conclusion

Opening of the photon emission and absorption channels is a rather typical example of an external field impact on quantum elementary processes in a many-particle system.[9,10]

Here we have demonstrated how a CI for emission/absorption by EPP[5] of a soft photon along the polarization direction of a linearly polarized time dependent electric field can be simplified analytically. Such kind of simplification is crucial for both qualitative estimates and for development of effective algorithms to solve the KEs numerically. However, further work is needed to justify the assumptions adopted here (in particular, the one of slow variation or splitting of the variation scales of the EPP distribution function), as well as to generalize the approach to other channels of one-photon pair photoproduction and annihilation. Further developments are crucial for analysis of the role of photon reservoir in kinetics of the EPP subsystem[5] and, in particular, of the problem of depletion of the external field[6] in a laser driven EPP.

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