3-18-2021

MODELING AND ANALYSIS OF NONSTATIONARY GAS FILTRATION UNDER GAS DYNAMIC PARAMETERS VARIATION

N Ravshanov
"Bulletin of TUIT: Management and Communication Technologies", ab.shaxnoza84@gmail.com

Elmira Nazirova
elmira_nazirova@mail.ru

Recommended Citation

Ravshanov, N and Nazirova, Elmira (2021) "MODELING AND ANALYSIS OF NONSTATIONARY GAS FILTRATION UNDER GAS DYNAMIC PARAMETERS VARIATION," *Bulletin of TUIT: Management and Communication Technologies*: Vol. 4 , Article 3.
DOI: https://doi.org/10.51348/tuitmct423
Available at: https://uzjournals.edu.uz/tuitmct/vol4/iss2/3

MODELING AND ANALYSIS OF NONSTATIONARY GAS FILTRATION UNDER GAS DYNAMIC PARAMETERS VARIATION

Ravshanov N., Nazirova E. Sh.

Abstract. A mathematical model that describes a partial differential equation with boundary, internal and initial conditions was developed in the article, to study the gas-dynamic parameters of the gas filtration process in a porous medium under isothermal conditions. The study was performed based on the reviews of research works related to mathematical modeling in recent years. Computational experiments (CE) were conducted on a computer to determine the response of the main parameters on the process of gas filtration in a porous medium on the basis of the developed mathematical tool (model, numerical algorithm, and software). The results of numerical calculations were presented in the form of tables and graphical objects for the purpose of developing oil and gas fields and increasing oil and gas recovery. With the analysis of the numerical calculations performed, it was established that when the parameters and properties of gas filtration are considered as functions of pressure, then the process of gas filtration in porous media can be adequately described, as a whole, and it correctly reflects the main point of the object under research. It could be concluded that with the mathematical tool developed, it became possible to conduct a comprehensive study of the process of gas filtration in a porous medium.

Keywords. Mathematical model, differential equation, gas filtration, porous medium, computational experiment, mine gallery of wells.

Introduction. An increase in production efficiency is associated, first, with the improvement of technologies and their application at specific facilities that require a comprehensive study using one or another mathematical tool. To accelerate the development of oil and gas fields, improve their technical and economic indices, maximize the product recovery from old oil and gas deposits, it is necessary to conduct comprehensive studies using easily implemented mathematical tools.

One of the effective mathematical tools, which makes it possible to predict and control this process, is a mathematical model, numerical algorithm, and software complex for implementing the task and conducting a CE on a computer, under various input technological parameters of the object under research.

It should be noted that the issues of analyzing the functioning and research of complex oil- and gas-dynamic systems, which include hydrocarbon deposits and their management, are in the field of interest of specialists both in the mechanics of continuous media and in the theory of optimal control of the above-mentioned processes, which have great scientific and practical value.

As follows from the analysis of the sources, the process of mass transfer in porous media is characterized by a significant number of interconnected hydrodynamic, technological and economic parameters that change during oil and gas systems operation.

The problems of mathematical modeling of the process of gas and fluid filtration in porous media were considered in many publications and substantial and significant results of a theoretical and applied nature were obtained.

In [1], a numerical study of single-phase and two-phase filtration of gas and liquid in naturally fractured reservoirs was conducted in the small-scale simulation. According to the authors, the results of the calculations made it possible to establish previously unknown features of filtration flows in fractured-porous reservoirs, including the nature of the exchange of flows between interbedding media. It was stated that no currently existing mathematical models of two-phase filtration take into account physical phenomena occurring at the boundaries of the pore blocks of the reservoir.

An attempt at a more adequate description of the two-phase incompressible fluid filtration in a fractured porous medium was done in [2]. A new mathematical statement of a three-dimensional problem was proposed, based on a system of first-order differential equations written in terms of "velocity-pressure-saturation". In that article, for the numerical integration of the problem, an explicit finite-difference scheme was used, in which the spatial approximation was implemented by a finite element mixed method with low-order elements on grids of parallelepipeds. A parallel implementation of the proposed method was developed using MPI for numerical simulation on modern supercomputers.

In [3], a mathematical model of mixed dimension was given to simulate filtration problems in fractured-porous media (built-in fracture model). The mathematical model was described by a system of parabolic equations: d-dimensional one for a porous medium and (d - 1)-dimensional one for a system of fractures. The system of equations is linked by setting a special flow function. According to the authors of that article, the model allows the use of grids for the matrix of the porous medium, independent of the grid for fractures. An approximation is constructed for the numerical solution, using the finite element mixed method. The results of a model problem numerical solution are presented, showing the efficiency of the proposed method for modeling a flow in fractured porous media.

Applied problems of oil and gas production were numerically studied in [4], using mathematical models of multiphase fluid flows in porous media. The basic model includes continuity equations and Darcy's laws for each phase, as well as an algebraic expression for the sum of saturations. Primary computational algorithms for such problems were implemented using the pressure equation. In that article, the authors highlighted the main properties of the pressure problem and discussed the need to perform them at a discrete level. A non-self-adjoint operator characterizes the resulting elliptic problem for the pressure equation. Possibilities of an approximate solution of an elliptic problem by iterative methods were

considered. Particular attention is paid to numerical algorithms for calculating pressure on parallel computers.

As stated in [5], significant amounts of proven hydrocarbon reserves are contained in the fields of varying degrees of fracturing. Fractures have high conductivity, which means they substantially affect the process of oil production from the reservoir. In this regard, it is necessary to study the processes of filtration of a mixture of water and oil in fractured porous reservoirs. Based on the above, that article poses the task of choosing options for the designation of injection and production wells in a zonal-heterogeneous fractured-porous medium for the most complete oil recovery from the reservoir. The influence of the water injection rate on the efficiency of oil recovery from the fractured porous reservoir was also studied. The tasks were solved using their own three-dimensional two-phase hydrodynamic simulator based on the porosity-permeability double model.

On the basis of the fundamental laws of the conservation of energy, non-stationary processes of filtration of two-phase fluids in multilayer reservoirs in the bottom hole zone of a well are considered in [6]. In this study, the number of layers, fluid pressure in these reservoirs, reservoir permeability, oil viscosity, etc. are taken into account in the mathematical statement of the problem of filtration of two-phase fluids in multilayer reservoirs. Plane-parallel and axisymmetric cases were investigated. In the numerical solution, an unstructured grid was used, and the grid compacted approaching the well. The time integration step is determined by the generalized Courant inequality; as a result of numerical solutions, there are no large fluctuations. Oil flow rates, Poisson's ratios, D-well diameters, filter height, filter permeability, as well as the total filter cake thickness and area were taken as the main input data in the numerical study and modeling of non-stationary two-phase filtration processes.

In [7], a new numerical method was proposed for solving the problem for a two-dimensional system of equations describing the filtration of a multiphase fluid in a porous

medium with appropriate initial and boundary conditions. For this, a special auxiliary task was introduced, which has some advantages over the main task. Based on the auxiliary problem, an efficient and accurate numerical algorithm for solving the problem was derived. Besides, the authors presented some results of numerical experiments on related subjects of physics.

In [8], the computer modeling of filtration processes based on the laws of hydrodynamics was considered. The adequacy of the developed models was verified by a series of computational experiments. The created mathematical programs and software serve the purposes of research, forecasting, and decision-making in the development and design of oil and gas fields

In [9], the mathematical statement of the problem of non-stationary fluid filtration in heterogeneous porous media, which differ from each other in hydrogeological characteristics, is considered. To solve this problem, an approximate analytical method of straight lines was developed, which reduces a multidimensional partial differential equation describing the process of fluid filtration in layered porous media to solving a one-dimensional differential equation. A calculation formula was obtained to determine the process of mass transfer between layers, depending on the operating modes of production wells and the hydrogeological parameters of the filtration layers. The developed model and algorithm can serve as a software for the development of oil and gas fields.

A mathematical model and a numerical algorithm for studying the process of oil and gas filtration in a porous medium at piston extrusion are developed in [10]. To solve this problem, a numerical algorithm was developed based on the phase-front straightening method, the integro-interpolation method and the use of a conservative finite-difference scheme. The adequacy of the proposed mathematical apparatus was verified by conducting a series of CE. The created mathematical program and software make it possible to analyze the parameters of the filtration process in a reservoir system and to forecast and make decisions in the development and design of oil and gas fields.

In [11], a generalized nonlinear mathematical model of filtration in multilayer inhomogeneous low-permeability porous media was developed, in which the total rate of fluid injection into the porous medium is constant. As the authors of the article note, the number of layers in the model can be arbitrary, and the generalized model is suitable for describing one-dimensional characteristics of the flow (non-Darcy flow), in reservoirs with low permeability and high heterogeneity. In that article, using the similarity transformation method, an exact analytical solution was obtained for a model with multiple moving boundaries, where the formula for the substrate of fluid injection into each layer is given. The paper substantiates the evidential and theoretical bases for solving problems of fluid filtration in porous media related to non-Darcy flow, where they can be very useful for rigorous verification of the results of numerical modeling. In that article, based on the comprehensive study conducted, some design recommendations and optimization were proposed for the development of heterogeneous reservoirs with low permeability and reservoirs with heavy oil to increase oil recovery.

The problem of fluid filtration through layered porous media is considered in [12], and a mathematical model and a universal generalized algorithm for controlling the flow through a finite number of layers are developed. For the numerical solution of the problem, a finite-difference scheme of the third order of accuracy was developed to calculate the rate and shear stress at the interfaces between the layers.

The study of the fluid flow characteristics in three-layer porous layers, when it is believed that the two outer porous layers have infinite width, and the middle porous layer has a finite width, is considered in [13]. As the authors note, the mathematical model of fluid filtration in the middle area can be described as a fully developed laminar flow and is assumed to be governed by the Brinkman equations, while the flow through the upper and lower porous media is governed by Forchheimer's equations. From

the statement of the problem, it is seen that in two areas of the boundary between the middle porous layer of finite width and the outer infinite porous layers, the continuity of the velocity and shear stress is ensured, and it was established that the flow rate is affected by two parameters: the Reynolds number and the Darcy number. The effect of these parameters on the profiles of the flow rate through the filtration area is investigated and presented in the article.

Lattice Boltzmann Model (LB) is proposed in [14], for simulating fluid flow in multi-scale porous media, allowing aggregates of smaller pores and solid particles to be considered as "equivalent media", where a partial return scheme is used to simulate the resistance of each aggregate, represented in the model of gray lattice, to the fluid flow. The authors of the article argue that the law of conservation of mass is fulfilled even when filtration of inhomogeneous media is considered.

The study in [15] presents a new method for solving the problem of fluid filtration in porous media - a multilayer boundary value problem. The proposed method is based on the effective adaptation of the classical shooting method, in which boundary value problems are solved by solving a sequence of initial problems. An illustration of the method is presented in the application of the process of filtration of a fluid flow through a multilayer porous medium; the test examples given indicate that the method is reliable and accurate.

In [16], a numerical Gray Lattice Boltzmann model is presented for modeling fluid flow in permeable porous media. In contrast to the majority of the above mathematical models, the model proposed by the authors makes it possible to simulate fluid filtration into a multilayer permeable porous medium, while maintaining the continuity of the macroscopic field of displacement velocities. The developed mathematical model was tested using available analytical solutions and an analytical expression was obtained for the case of variable porosity of the medium and, in addition, the authors investigated the importance of introducing a

transition layer with a given porosity function near the boundaries and interface areas.

A steady flow of a viscous incompressible fluid through a porous channel consisting of two porous layers of different thicknesses is studied in [17]; the layers are considered isotropic, but with different values of permeability and porosity; the channel ends with impermeable solid walls on top and on bottom; the cohesion condition is imposed on the walls. To calculate the velocity distribution in the layers, it is assumed that the interface between them is discontinuous, and the corresponding conditions for matching the velocity and shear stress are used to obtain analytical expressions. An expression is obtained for the velocity at the interface, which shows the dependence of this velocity on the Darcy numbers, porosity, basic viscosity of the flowing fluids, viscosity factors and the thickness of the layers involved. The article describes a four-step procedure to obtain the values of the parameters involved and to determine the velocity at the interface.

A new nonlinear system reaction-diffusion-convection associated with a system of ordinary differential equations, which simulates the combustion front in a multilayer porous medium, is presented in [18]. The developed mathematical model takes into account heat transfer between layers and heat loss to the external environment. To simplify the model of the object, several assumptions are made, for example, the non-compressibility of the porous medium, the temperature and heat concentration in each layer are considered unknown. When the heat concentration in each layer is a known function, the authors proved the existence and uniqueness of the classical solution to the initial and boundary value problem for the corresponding system. The proof of the authors' theorems is further used for a new approach to the problems of combustion in porous media. The authors constructed monotonic iterations of the upper and lower solutions and proved that the iterations converge to a unique solution to the

problem, first locally, and then, over time, globally.

In [19], a mathematical model is given for the problem of gas filtration in a porous medium, described by a nonlinear partial differential equation and the corresponding boundary and internal conditions. The article presents the main stages of constructing a mathematical model of gas filtration in porous media, taking into account changes in the hydrodynamic parameters of the object under research. Solving the problem, locally-one-dimensional schemes and schemes of longitudinal-transverse direction were used. To solve a nonlinear problem, computational iterative algorithms were developed, and a series of computational experiments on a computer, their analysis and conclusions were presented to study the responses of the main parameters of the process.

A mathematical model and an effective numerical algorithm are considered in [20], to solve the problem of filtration of multiphase media (water and oil) in a porous medium, to research, monitor, and forecast the main indices of the process and make management decisions. A numerical algorithm for solving the problem is based on the concepts of differential-difference schemes and the differential sweep method, and makes it possible to conduct computer experiments. When modeling the object of research, the process of displacement of one fluid by another fluid in a porous medium was considered.

In [21] the problem of gas filtration in a porous medium is considered, where a mathematical model of an object is described by a nonlinear partial differential equation and the corresponding boundary and internal conditions. The article presents the main stages of constructing a mathematical model of the gas filtration process in porous media, taking into account the change in the hydrodynamic parameters of the object under research; for the solution of the problem, numerical methods, locally-one-dimensional schemes, and schemes of the longitudinal-transverse direction are used.

In [22], a mathematical model and an effective numerical algorithm were developed to perform numerical calculations on a computer, considering such factors as the deposition rate of small particles in porous media, changes in porosity, and filtration coefficients over time. The mathematical model was reduced to the simultaneous solution of a system of differential equations of parabolic type, which describes the filtration processes in reservoirs separated by a low-permeability bulkhead under corresponding initial and boundary conditions. Computational experiments for various reservoir parameters and flow rates of two producing wells are presented.

In mathematical modeling of the process of mass transfer in porous media, the authors of the published studies mainly used the general laws of fluid and gas mechanics, reduced to systems of linear and nonlinear partial differential equations with the corresponding initial, boundary, and internal conditions characterizing the variable state of the system under study.

Analyzing the above, a comprehensive study is conducted in the article related to the change in the hydrodynamic parameters of the process, which depend on the change in gas pressure in a porous medium during the operation of gas fields.

Statement of problem. The process of non-stationary gas filtration in a porous medium with a variation in gas-dynamic parameters by introducing dimensionless variables has the following form

$$\overline{P} = \frac{P}{P_0} , \quad \overline{x} = \frac{x}{L} , \quad \overline{Y} = \frac{Y}{L} , \quad \overline{t} = \frac{K_0 P_0 Z_0}{\mu_0 m_0 L^2} t ,$$

$$\overline{K} = \frac{K}{K_0} , \quad \overline{\mu} = \frac{\mu}{\mu_0} , \quad \overline{Z} = \frac{Z}{Z_0} , \quad \overline{m} = \frac{m}{m_0} ,$$

$$\overline{q} = \frac{C \mu_0 z_0}{K_0 P_0^2 H} q ,$$

a mathematical model has the following form:

$$\frac{\partial}{\partial x}\left[K(x,y,P)\frac{\partial P^2}{\partial x}\right]+\frac{\partial}{\partial y}\left[K(x,y,P)\frac{\partial P^2}{\partial y}\right]=\frac{\partial}{\partial t}\left[M(x,y,P)P\right]$$

(1)

$$K(x,y,P)\frac{\partial P^2}{\partial n}=N\left(P(x,y,t)-\varphi_1(x,y,t)\right)$$

(2)

or

$$P(x,y,t)=\varphi_2(x,y,t),\ (x,y)\in\Gamma,\ t>0$$

(3)

$$\oiint_{\Gamma_\kappa}K(x,y,P)\frac{\partial P^2}{\partial n}dS=q_k(t),\ t>0$$

(4)

or

$$P(x,y,t)=\Psi(t),\ (x,y)\in\Gamma,\ t>0$$

(5)

$$P(x,y,0)=\varphi(x,y),\quad(x,y)\in G$$

(6)

where $k(x,y,P)$ is the reservoir permeability; $z(P,T)$, $\mu(P,T)$ are the viscosity and over-compressibility of gas, respectively; T is the gas temperature;

$m(x,y,\mathrm{P})$ is the reservoir porosity, Γ_κ is the well contour; q_k is the flow rate reduced to normal conditions; $C=\dfrac{P_{AT}T_{n\imath}}{T_0}$, P_{AT} is the atmospheric pressure. $T_{n\imath}$, T_0 are the reservoir and specified temperatures, respectively; P_0, K_0, m_0, μ_0, z_0 are the characteristic values of pressure (for example, initial reservoir pressure), permeability, reservoir porosity, gas viscosity and over-compressibility (for example, for a characteristic pressure value), respectively. *L* is the characteristic length (for example, the diameter of the filtration area).

To take into account the change in gas-dynamic parameters, in the mathematical statement of the problem we expand the functions $\dfrac{m(\mathrm{P})}{z(P)}$ in a series of the neighborhood of a certain pressure value, for example, corresponding to a certain moment, time. Then, limiting ourselves to terms of the first degree, we obtain:

$$\frac{m(\mathrm{P})}{z(P)}=\frac{m(\overline{P})}{z(\overline{P})}\left\{1+\left[\frac{1}{z(P)}\frac{\partial z}{\partial p}(\overline{P})-\frac{1}{m(\overline{P})}\frac{\partial m}{\partial p}(\overline{P})\right]\overline{P}-\left[\frac{1}{z(P)}\frac{\partial z}{\partial p}(\overline{P})-\frac{1}{m(\overline{P})}\frac{\partial m}{\partial p}(\overline{P})\right]\right\}$$

;   (7)

where

$$\overline{P}=P(x_i,y_i,t_{\kappa-1}).$$

Then we obtain the following:

$$\frac{\partial}{\partial t}\left[\frac{m(P)}{z(P)}P\right]=\frac{m(\overline{P})}{z(P)}\left\{1+\left[\frac{1}{m(\overline{P})}\frac{\partial m}{\partial p}(\overline{P})-\frac{1}{z(\overline{P})}\frac{\partial z}{\partial p}(\overline{P})\right]P\right\}\frac{\partial P}{\partial t}+$$

$$+\left\{\left[\frac{1}{z(\overline{p})}\frac{\partial z}{\partial p}(\overline{P})-\frac{1}{m(\overline{P})}\frac{\partial m}{\partial p}(\overline{P})\right](P-\overline{P})\right\}\frac{\partial P}{\partial t}$$

(8)

If to ignore the expression in curved brackets, we could make the same error that is incorporated in the differential equation of non-stationary gas filtration in a porous medium (the inertial terms are zero).

Function $\dfrac{K(\mathrm{P})}{\mu(P)}\dfrac{1}{z(P)}$ can be similarly expanded in a series of suppression. Then we obtain:

$$\frac{K(P)}{\mu(P)}\frac{1}{z(P)}=\frac{K(\overline{P})}{\mu(P)}\frac{1}{z(\overline{P})}\left\{1+\left[\frac{1}{K(\overline{P})}\frac{\partial K}{\partial p}(\overline{P})-\frac{1}{z(\overline{P})}\frac{\partial z}{\partial p}(\overline{P})\right](P-\overline{P})\right\}$$

(9)

Introduce the notation:

$$\overline{\alpha}=\frac{1}{m(\overline{p})}\frac{\partial m}{\partial p}(\overline{P})-\frac{1}{z(\overline{P})}\frac{\partial z}{\partial p}(\overline{P}),$$

$$\overline{\beta}=\frac{1}{K(\overline{P})}\frac{\partial m}{\partial p}(\overline{P})-\frac{1}{\mu(\overline{P})}\frac{\partial M}{\partial p}(\overline{P})-\frac{1}{z(\overline{P})}\frac{\partial z}{\partial p}(\overline{P})$$

$$\overline{K}=K(\overline{P}),\ \overline{\mu}=\mu(\overline{P}),\ \overline{z}=z(\overline{P}),\ \overline{m}=m(\overline{P})$$

Then, instead of differential equation (1), we obtain

Ravshanov N., Nazirova E. Sh.

$$\frac{\overline{m}}{z}\frac{(1+\overline{\alpha}P)}{P}\frac{\partial P^2}{\partial t} = \frac{\partial}{\partial x}\left\{\frac{\overline{k}}{\mu}\frac{1}{z}\left[1+\overline{\beta}(P-\overline{P})\right]\frac{\partial P^2}{\partial x}\right\}+$$

$$+\frac{\partial}{\partial y}\left\{\frac{\overline{k}}{\mu}\frac{1}{z}\left[1+\overline{\beta}(P-\overline{P})\right]\frac{\partial P^2}{\partial y}\right\}$$

(10)

Solution methods. The numerical solution of the differential equation (10) can be obtained by applying the computational scheme outlined in [23]; if we introduce the notation:

$$K = \{\frac{\overline{k}}{\mu}\frac{1}{z}\left[1+\overline{\beta}(P-\overline{P})\right]\}, \quad M = \frac{\overline{m}}{z}\frac{(1+\overline{\alpha}P)}{P}.$$

We will assume further that the functions characterizing the filtration properties are calculated from the average gas pressure over the previous time layer.

Then the quantities $\overline{K}$, $\overline{\mu}$, $\overline{Z}$, $\overline{\alpha}$ and $\overline{\beta}$ have corresponding constant values for each calculated time layer, as a result, the differential equation is simplified and has the following form:

$$\frac{2m\overline{\mu}}{\overline{k}}(1+\overline{\alpha}P)\frac{\partial P}{\partial t} = \frac{\partial}{\partial x}\{[1+\overline{\beta}(P-\overline{P})]\frac{\partial P^2}{\partial x}\}+\frac{\partial}{\partial y}\left\{\left[1+\overline{\beta}(P-\overline{P})\right]\frac{\partial P^2}{\partial y}\right\}$$

(10)

In solution (10), it is necessary to introduce the notation

$$K = 1+\overline{\beta}(P-\overline{P}),$$

$$M = \frac{\overline{m}\,\overline{\mu}}{\overline{k}}(1+\overline{\alpha}P)\frac{1}{P}.$$

If the values of $\overline{K}, \overline{\mu}, \overline{z}, \overline{\alpha}, \overline{\beta}$, in contrast to the previous case, are determined by the initial reservoir pressure, then they will be constant for the entire development period.

For numerical integration of the problem posed, we introduce spatial and temporal grids in the following way:

$$\omega_{h_i h_j} = \left\{x_i = x_{i-1}+h_i, i=\overline{1,n_j}, y_j = y_{j-1}+h_j, j=\overline{1,n_i}\right\}$$

$$; \omega_{\tau k} = \left\{t_k = t_{k-1}+\tau_k, k=1,2,...\right\}$$

where h_i, h_j are the values of the

spatial grid steps corresponding to the node

with coordinates x_i, y_i; τ_k is the value of the time grid step for the time instant t_k.

We assume that the outer boundary of the filtration area can be approximated in a stepwise form, as in [23]. Then the discrete filtering area can be considered as a class of nodes of the family of straight lines $C_{1i}, j=\overline{1,n_2}$ parallel to the *OX* axis or as a class of nodes of the family of straight lines $C_{2i}, i=\overline{1,n_1}$ parallel to the *OY* axis.

Discussion of results. Let us give the comparison of calculation results on a computer with the solution of the original differential equation without simplifications, and with the simplifications mentioned above, for the case of non-stationary filtration in a square area of a system of 5 wells symmetrically located relative to the center of the area. The calculations were conducted for the following values of the initial data: initial reservoir pressure of 277 atm, typical values of reservoir permeability are - $K_0 = 0,5$; over-compressibility - $Z_0 = 1$, dynamic viscosity - $\mu_0 = 0,01212$, reservoir porosity $m_0 = 0,2$.

Let us now proceed to the gas-dynamic analysis of the influence of individual parameters characterizing the filtration properties as a function of pressure on the pattern of the filtration process. The results of calculations on a computer were obtained for the values of the above example when solving the original differential equation.

We have tested various cases:

a) - $K = K(P)$, μ, z, m - the constants;

b) - $\mu = \mu(p)$, K, z, m - the constants;

c) - $z(p), k, \mu, m$ - the constants;

d) - $m = m(p), k, \mu, z$ - the constants.

The results of the performed numerical calculations are shown in tables 1 - 4, and Figs. 1-11.

Table 1. Change in gas pressure in wells and its neighboring nodes at t = 1779.172 *h*

Gas pressure in wells	1	2	3	4	5

P (cent. well)	223.932	224.423	224.423	224.423	224.423
P (1st well)	224.122	224.591	224.592	224.635	224.635
P (2nd well)	224.122	224.591	224.592	224.635	224.635
P (3rd well)	224.122	224.592	224.591	224.635	224.635
P (4th well)	224.122	224.593	224.591	224.635	224.635
P (below the cent. well)	225.251	225.244	225.245	224.244	225.251
P (to the left of cent. well)	225.338	225.334	225.331	224.331	225.338
P (above the cent. well)	225.251	225.443	225.245	224.244	225.251
P (to the right of cent. well)	225.338	225.333	225.331	224.331	225.338

From the analysis of the performed numerical calculations (Tables 1, 2), it follows that, basically, a decrease in the gas pressure occurs in the central well and its neighboring nodes. With an increase in the operation time of the wells, the gas pressure becomes equal in all wells, except the wells located to the left and to the right of the central well.

Table 2. Change in gas pressure in wells and its neighboring nodes at t = 49580.233 h

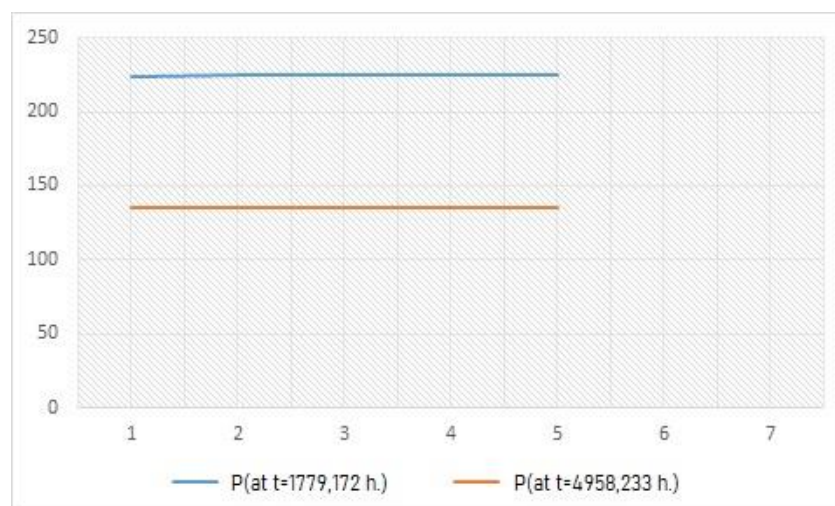| Gas pressure in wells	1	2	3	4	5
P (cent. well)	135.042	135.652	135.652	135.652	135.652
P (1st v well)	135.244	135.826	135.826	135.882	135.882
P (2nd well)	135.244	135.826	135.826	135.882	135.882
P (3rd well)	135.244	135.826	135.826	135.882	135.882
P (4th well)	135.244	135.826	135.826	135.882	135.882
P (below the cent. well)	135.642	136.635	136.637	136.635	135.642
P (to the left of cent. well)	137.011	136.993	136.998	136.993	137.011
P (above the cent. well.)	135.642	136.635	136.637	136.635	135.642
P (to the right of cent. well)	137.011	136.993	136.998	136.993	137.011

Fig. 1. Gas pressure changes in the central well

Fig. 2. Change in gas pressure in
wells and in neighboring corners
(t = 1779.172 *h*)

As seen from the curves in Fig. 1, the
gas pressure in the central well and its
neighboring nodes decreases proportionally
with time. From the analysis of numerical
calculations, it follows that the decrease in
the gas head in neighboring nodes is affected
by the operation of other wells located around
the central one.

The analysis of the performed
numerical calculations showed that at the
operation time of the well t=1779,172 *h*, gas
pressure in the first, second, third and fourth
wells and neighboring nodes is similar to
each other (Table 1).
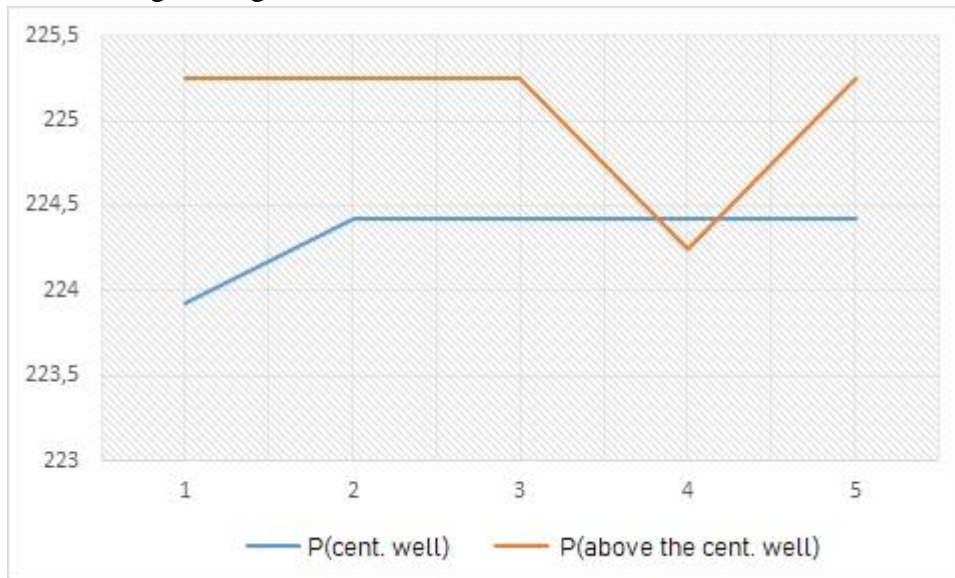
Fig. 3. Change in gas pressure in wells and
in neighboring corners
(t = 1779.172 *h*)

From the numerical calculations
(curves in Fig. 3) it is seen that due to the
operation of the central well, the gas
pressure drops sharply in the upper well and
its neighboring nodes, and the gas head in
the central and 4th wells simultaneously
decreases at t=49580.233 *h* (Fig. 4); in the
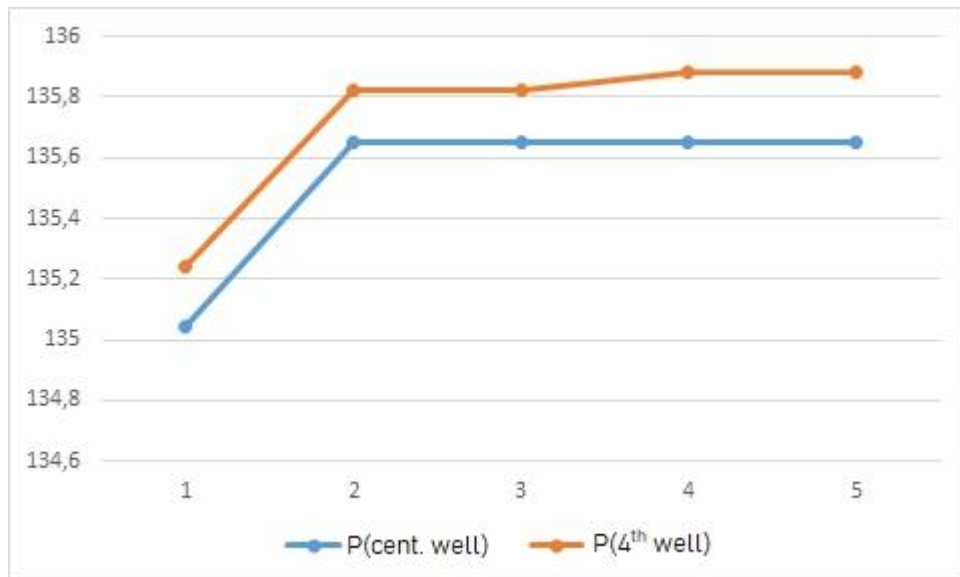well left to the central well, it becomes the
same at all nodes (Fig. 5).

Fig. 4. Change in gas pressure in wells and
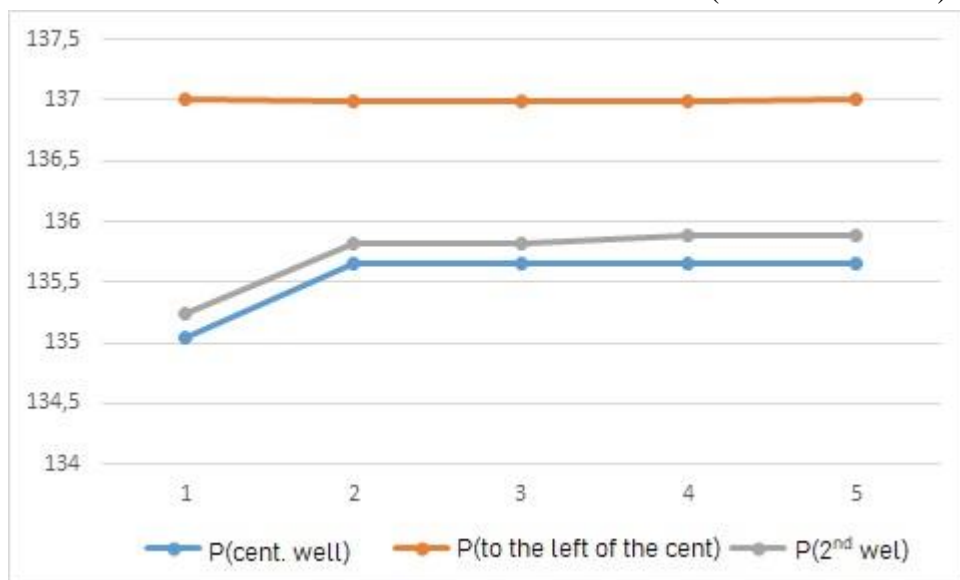in neighboring corners
(at t=49580.233 *h*)

Fig. 5. Change in gas pressure in wells and
in neighboring corners
(at t=49580.233 *h*)

The analysis of numerical calculations showed that the gas pressure to the left and to the right of the central and other wells, at the well development time t=49580.233 *h*, remains high (Table 2, Fig. 5).
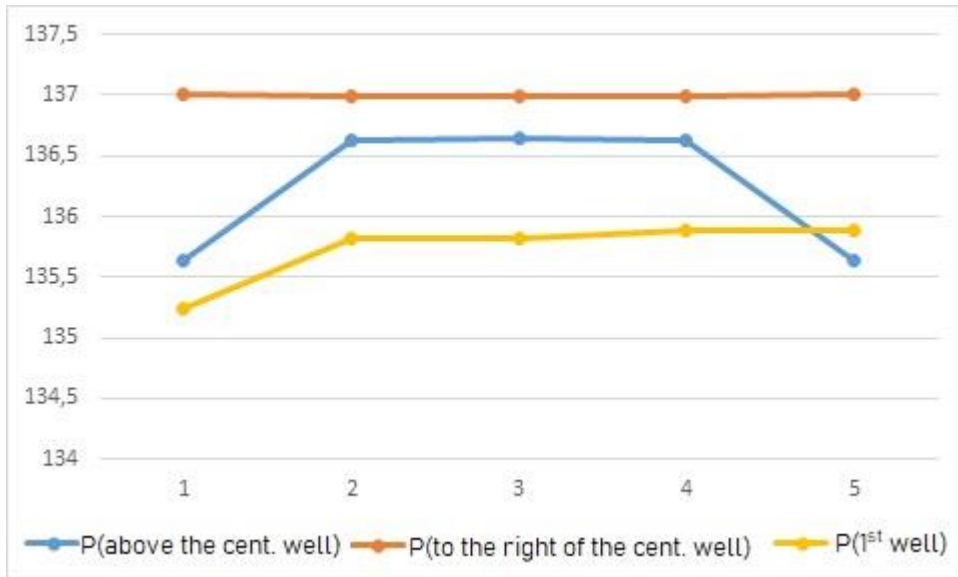
Fig. 6. Change in gas pressure in wells and
in neighboring corners
(at t=49580.233 *h*)

As seen from the curves in Fig. 6, the
gas flow rate in the central well affects the
decrease in the gas head in the upper

(relative to the central well) well and
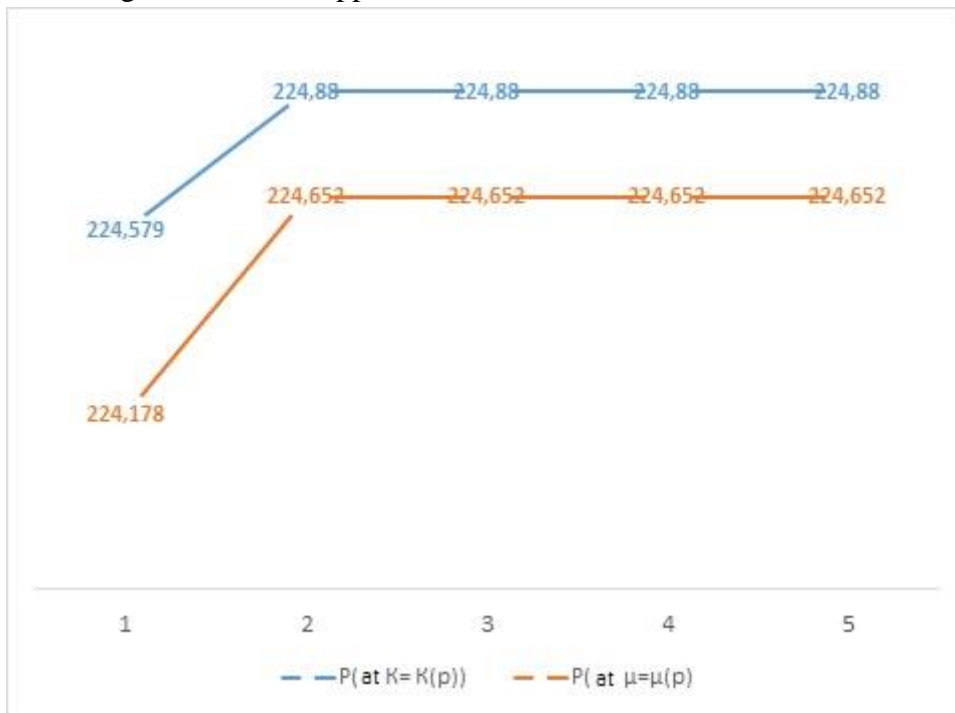neighboring nodes. Gas pressures in the
uppermost well and the fourth node are P =
135.58 atm and 135.56 atm at t=49580.233
h, and in other nodes it is 136.55 atm.

Fig. 7. Change in gas pressure in the central
well
at t=1779,172 *h*

As seen from the curves in Fig. 7, the
dynamics of the change in gas pressure in

the wells is substantially influenced by the
hydrodynamic parameters of the object, for
example, when the formation permeability
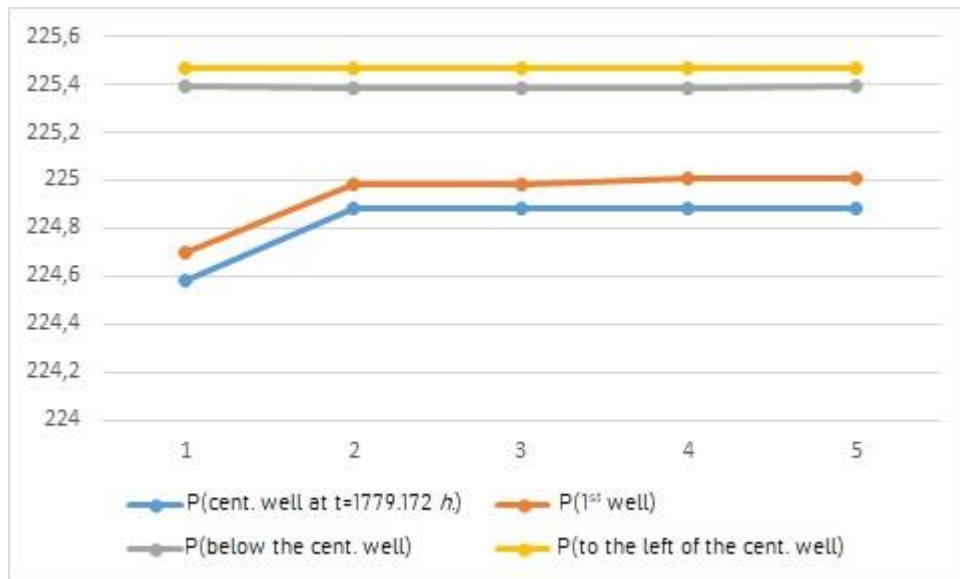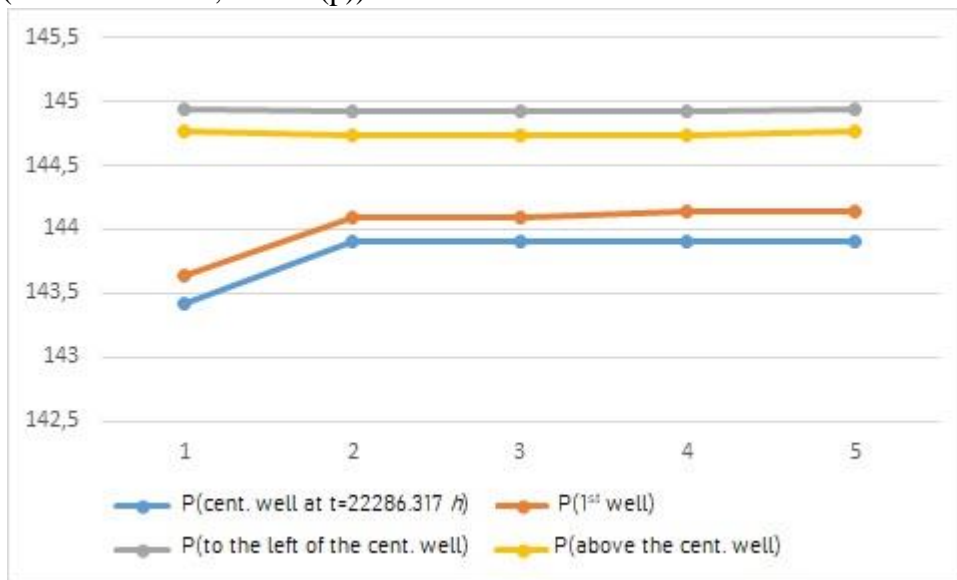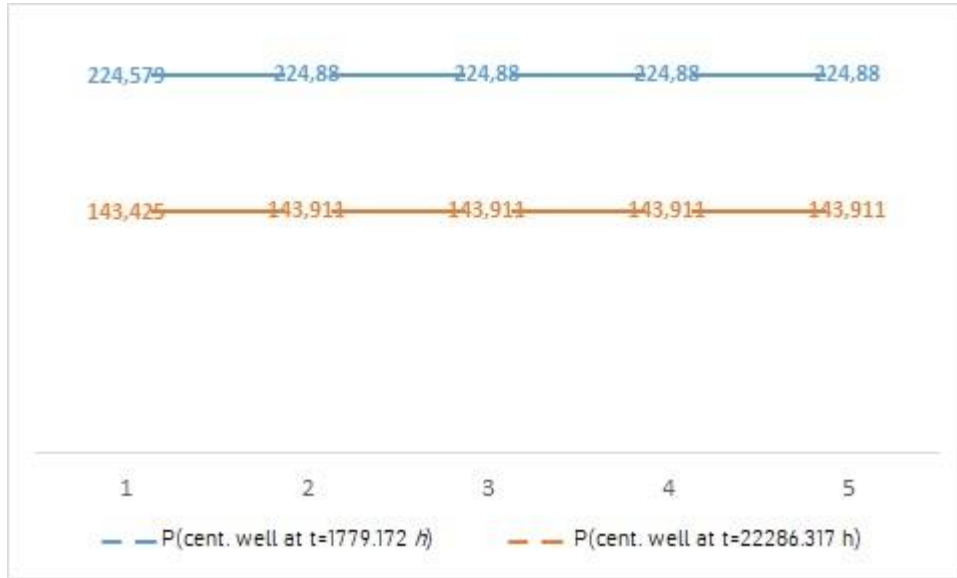in a porous medium depends on the gas
pressure.

Figure: 8. Change in gas pressure in wells
(at t=1779.172 *h*, K = K (p))

(t=22286.317 *h*, K = K (p))

Figure: 9. Change in gas pressure in wells

in neighboring nodes (at K = K (p))
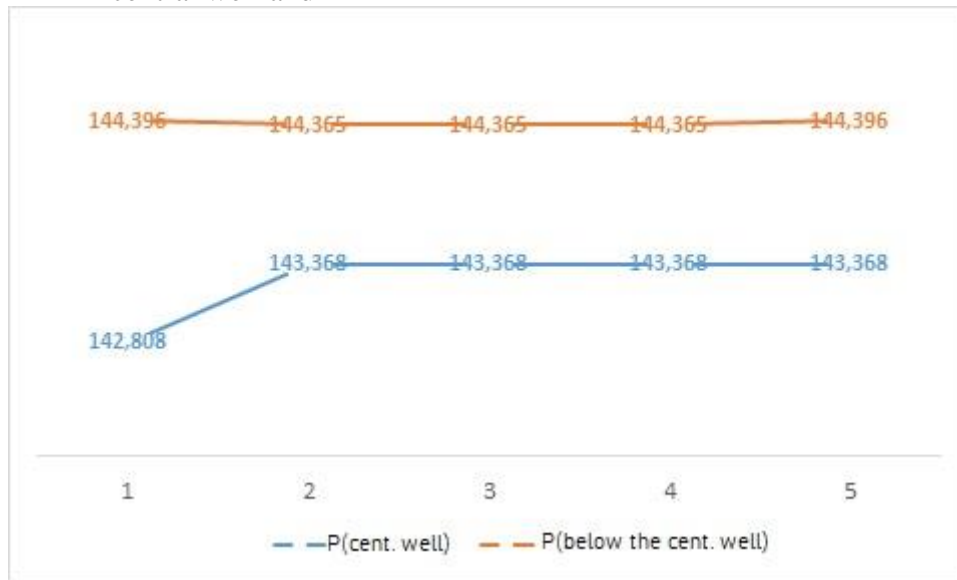
Fig. 10. Change in gas pressure in the
central well and

Fig. 11. Change in gas pressure in wells and
in neighboring nodes (at K = K (p),
t=49580.223 *h*)

Table 3. Gas pressures in wells and
neighboring nodes
at t = 1779.72 *h*, Z = z (p)

Pressure in gas wells and neighboring nodes	1	2	3	4	5
P (cent. well)	225.039	225.251	225.251	225.251	225.251
P (1st well)	225.121	225.323	225.323	225.342	225.342
P (2nd well)	225.121	225.323	225.323	225.342	225.342
P (3rd well)	225.121	225.323	225.323	225.342	225.342
P (4th well)	225.121	225.323	225.323	225.342	225.342
P (below the cent. well)	225.609	225.606	225.606	225.606	225.609
P (to the left of the cent. well)	225.678	225.675	225.675	225.675	225.678
P (above the cent. well)	225.609	225.606	225.606	225.606	225.609
P (to the right of the cent.well)	225.678	225.675	225.675	225.675	225.678

at t = 49580,223 h, , μ=μ(p)

Table 4. Gas pressures in wells and
neighboring nodes

Pressure in gas wells and neighboring nodes	1	2	3	4	5
P (cent. well)	142.175	143.141	143.141	143.141	143.141
P (1st well)	142.918	143.288	143.288	143.329	143.329
P (2nd well)	142.918	143.288	143.288	143.329	143.329
P (3rd well))	142.918	143.288	143.288	143.329	143.329
P (4th well))	142.918	143.288	143.288	143.329	143.329
P (below the cent. well)	143.833	143.804	143.804	143.804	143.833
P (to the left of the cent. well)	144.017	143.989	143.989	143.989	144.017
P (above the cent. well)	143.833	143.804	143.804	143.804	143.833
P (to the right of the cent.well)	144.017	143.989	143.989	143.989	144.017

A detailed analysis of the performed numerical calculations on a computer shows that in some cases the filtration parameters and gas properties, as a function of pressure, can have a substantial effect on the pressure distribution. Under the conditions considered, it turned out that when the hydrodynamic parameters of the object under research depend on pressure, it significantly affects the change in the main indices of the gas filtration process in a porous medium, as compared with other simplifications considered when conducting a computational experiment on a computer (Figs. 7 - 9).

The numerical calculations performed on a computer show that simplifications of the differential equation of non-stationary gas filtration can lead to numerical results that are somewhat different in comparison with the solution of the original equation. Simplification 1 in comparison with the solution of the original equation gives underestimated results with an error at the wells.

The performed numerical calculations show that the permeability coefficient insignificantly affects the process of gas filtration in a porous medium and can be ignored, as it does not affect the calculation accuracy.

An analysis of numerical calculations performed on a computer shows that an account for the effect of changes of filtration parameters and gas properties on pressure can be of appropriate importance in solving some problems of gas-dynamic calculations of deep deposits and deposits with high initial pressures.

The results obtained allow us to conclude that for gas-dynamic calculations of high accuracy, in a number of cases it is necessary to use the differential equations in their original form. Calculations can be performed according to the proposed computational scheme using the principle of matching boundary conditions.

Conclusions. Based on the performed computational calculations, it was established that if the parameters and properties of gas filtration are considered as functions of pressure, then the process of gas filtration in porous media can be more adequately described as a whole and it correctly reflects the essence of the object under research in comparison with other simplifications considered when conducting CE on computer.

It was found that the permeability coefficient insignificantly affects the process of gas filtration in a porous medium and can be practically ignored, as it does not affect the calculation accuracy.

It has been established by numerical experiment that simplifications of the differential equation describing the process of gas filtration in a porous medium can lead to numerical results that are somewhat

different in comparison with the solution of the original problem.

References

1. Palatnik, B.M., and T.N. Segin. A New Approach for Small-Scale Simulation of Multiphase Flow in Naturally Fractured Reservoirs. European Petroleum Computer Conference, Stavanger, Norway, May 1992. doi: https://doi.org/10.2118/24263-MS (https://onepetro.org/SPEEPCC/proceedings-abstract/92EPCC/All-92EPCC/SPE-24263-MS/53978)

2. Berveno E.V., Kalinkin A.A. and Laevsky Y.M. Simulation of Incompressible Fluid Filtration in Fractured-porous Oil Field // Conference Proceedings, 6th EAGE Saint Petersburg International Conference and Exhibition, Apr 2014, Volume 2014, p.1 – 5, DOI: 10.3997/2214-4609.20140248 (https://www.earthdoc.org/content/papers/10.3997/2214-4609.20140248)

3. Spiridonov D., Vasilyeva M. Simulation of filtration problems in fractured porous media with mixed finite element method (Embedded Fracture Model) // Mathematical notes of NEFU, 24(3), pp. 100-110. doi:10.25587/SVFU.2018.3.10893. (http://www.mzsvfu.ru/index.php/mz/article/view/simulation-of-filtration-problems-in-fractured-porous-media)

4. Vabishchevich P., Vasilyeva M. Iterative solution of the pressure problem for the multiphase filtration// Mathematical Modelling and Analysis. - 2012. - Vol. 17, Issue 4. - Pp. 532-548. - DOI: 10.3846/13926292.2012.706655 (https://www.tandfonline.com/doi/abs/10.3846/13926292.2012.706655)

5. Pyatkov A.A., Rodionov S.P., Kosyakov V.P. and Musakaev N.G. Study of filtration processes of a two-phase fluid in a zonal-inhomogeneous fractured-porous medium // Journal of Physics: Conference Series. - 2019. - Vol. 1404. - P. 012039. - DOI: 10.1088/1742-6596/1404/1/012039 (https://iopscience.iop.org/article/10.1088/1742-6596/1404/1/012039/meta)

6. Maksat, K., Kalipa, K., Kulyash, B., Orken, M., & Assel, A. (2018). Numerical simulation of two-phase filtration in the near well bore zone, Open Engineering, 8(1), 77-86. doi: https://doi.org/10.1515/eng-2018-0010 (https://www.degruyter.com/view/journals/eng/8/1/article-p77.xml?language=en)

7. Rasulov M., Kul R.H. (2009) The Study of Filtration of Two Phase Fluid in a Porous Medium in a Class of Discontinuous Functions. In: Margenov S., Vulkov L.G., Waśniewski J. (eds) Numerical Analysis and Its Applications. NAA 2008. Lecture Notes in Computer Science, vol 5434. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-00464-3_57 https://link.springer.com/chapter/10.1007/978-3-642-00464-3_57)

8. Kurbonov N., Aminov S. Computer modeling of filtration processes with piston extrusion // Journal of Physics: Conference Series. - 2020. - Vol. 1441. - P. 012147. - DOI: 10.1088/1742-6596/1441/1/012147

9. Ravshanov N., Kurbonov N., Mukhamadiev A. An Approximate Analytical Solution of the Problem of Fluid Filtration in the Multilayer Porous Medium // International Journal of Computational Methods. - 2016. — Vol. 13, № 6. — 1650042 [10 pages] DOI: http://dx.doi.org/10.1142/S0219876216500420

10. Kurbonov N., Ibragimova K. Numerical Modeling of the Filtration Process During Oil Displacement by Gas // International Journal of Advanced Trends in Computer Science and Engineering. - 2020. - Vol. 9, Issue 5. - P. 8526-8532. - DOI: 10.30534/ijatcse/2020/232952020.

11. Liu W. Exact analytical solution of a generalized multiple moving boundary model of one-dimensional non-Darcy flow in heterogeneous multilayered low-permeability porous media with a threshold pressure gradient // Applied Mathematical Modelling. - 2020. — Vol. 81. — Pp. 931-953. - DOI: 10.1016/j.apm.2020.01.028

12. Ford R.A., Abu Zaytoon M.S., Hamdan M. H. Simulation of flow Through Layered Porous Media // IOSR Journal of

Engineering (IOSRJEN). - 2016. — Vol. 06, № 06. — Pp. 48-61.

13. Hajji M.A. Multi-Point Special Boundary-Value Problems and Applications to Fluid Flow Through Porous Media // Proceedings of the International MultiConference of Engineers and Computer Scientists (IMECS 2009). Vol II. - Hong Kong, 2009.— Pp. 18-20.

14. Zhu J., Ma J. An Improved Gray Lattice Boltzmann Model for Simulating Fluid Flow in Multi-scale Porous Media // Advances in Water Resources. - 2013. — Vol. 56. — Pp. 61-76. DOI: 10.1016/j.advwatres.2013.03.001

15. Allan F.M., Hajji M.A. Multi-layer parallel shooting method for multi-layer boundary value problems // 2009 International Conference on Innovations in Information Technology (IIT), Al Ain, 2009, pp. 289-293, doi: 10.1109/IIT.2009.5413402.

16. Yehya A., Naji H., Sukop M.C. Simulating flows in multi-layered and spatially-variable permeability media via a new Gray Lattice Boltzmann model // Computers and Geotechnics. - 2015. — Vol. 70. — Pp. 150-158. DOI: 10.1016/j.compgeo.2015.07.017

17. Almalki W., Hamdan M. H., Kamel M.T. Analysis of flow through layered porous media // Conference: Proceedings of the 12th WSEAS international conference on Mathematical methods, computational techniques and intelligent systems. - 2010. — Pp. 182-189.

18. Batista M.R., Da Mota J.C. Monotone iterative method of upper and lower solutions applied to a multilayer combustion model in porous media// Nonlinear Analysis: Real World Applications - 2021. - Vol. 58. -P.103223. – Doi: 10.1016/j.nonrwa.2020.103223

19. Ravshanov N., Aminov S., and Kravets O.Ja. Mathematical model and numerical algorithms to analyze gas filtration process in a porous medium // Journal of Physics: Conference Series. - 2019. - Vol. 1399, Issue 5. - P. 055036. - DOI: 10.1088/1742-6596/1399/5/055036

20. Ravshanov, N., Nazirova, E.S., Pitolin, V.M. Numerical modelling of the liquid filtering process in a porous environment including the mobile boundary of the oil-water section // Journal of Physics: Conference Series. - 2019. - Vol. 1399, Issue 2. - P. 022021. - DOI: 10.1088/1742-6596/1399/2/022021 (https://iopscience.iop.org/article/10.1088/1742-6596/1399/2/022021)

21. Ravshanov, N., Nazirova, E.S., Aminov, S.M. Mathematical model and numerical algorithms to analyze gas filtration process in a porous medium // TUIT Bulletin. - 2019. - Issue 3(51). - Pp. 45-65. (https://www.researchgate.net/publication/337945326_Mathematical_model_and_numerical_algorithms_to_analyze_gas_filtration_process_in_a_porous_medium)

22. Ravshanov N., Nazirova E. Numerical simulation of filtration processes of strongly polluted oil in a porous medium // Ponte. – 2018. – vol. 74. – № 11/1. – pp. 107-116.

23. Ravshanov N., Orifjonova U. 2020. Modeling the process of fluid filtration in interacting pressure porous layers. Problems of Computational and Applied Mathematics. 6(30): 93-115.