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著者	Naritaka Oshita, Masaki Yamada, Masahide
	Yamaguchi
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Compact objects as the catalysts for vacuum decays

Naritaka Oshita^{a,b,*}, Masaki Yamada^c, Masahide Yamaguchi^d

^a Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

^b Department of Physics, Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

^c Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, MA 02155, USA

^d Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan

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ABSTRACT

We discuss vacuum decays catalyzed by spherical and horizonless objects and show that an ultra compact object could catalyze a vacuum decay around it within the cosmological time. The catalytic effect of a horizonless compact object could be more efficient than that of a black hole since in this case there is no suppression of the decay rate due to the decrement of its Bekestein entropy. If there exists another minimum with AdS vacuum in the Higgs potential at a high energy scale, the abundance of compact objects such as monopoles, neutron stars, axion stars, oscillons, Q-balls, black hole remnants, gravastars and so on, could be severely constrained. We find that an efficient enhancement of nucleation rate occurs when the size of the compact object is comparable to its Schwarzschild radius and the bubble radius. © 2019 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license

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1. Introduction

Compact objects are ubiquitous in high-energy physics as well as astrophysics and play significant roles in cosmological history of the Universe. To name a few, monopoles [1], Q-balls [2–11], oscillons [12–22], boson stars (including axion stars) [23–38], gravastars [39,40] (see also [41]), neutron stars, black hole (BH) remnants [42,43], and (primordial) BHs [44–54] are examples that have been studied extensively in the literature for several decades. Pursuing consistency of these objects in cosmology and astrophysics is important to construct a realistic particle physics model and is complementary to high-energy colliders to find a new physics beyond the standard model.

It has been proposed that BHs may be objects catalyzing vacuum decays around them [55–65], which was pioneered by Hiscock [66]. The abundance of the catalyzing objects should be small enough to avoid the nucleation of AdS vacuum bubble within our observable Universe until present. Actually, this is particularly important in the standard model of particle physics [61,67,68], where the Higgs potential could develop a AdS vacuum at a high energy scale because of the running of quartic coupling [69–83]. According to their result, even a single BH within our observable Universe leads to the bubble nucleation if its mass is small enough.

* Corresponding author.

E-mail address: naritaka@resceu.s.u-tokyo.ac.jp (N. Oshita).

One may wonder what property of BHs contributes to the promotion of a vacuum decay around it. Gregory, Moss, and Withers found [60,62] that the exponential factor of a vacuum decay rate around a BH is determined by two factors, $\Gamma \propto e^{-B + \Delta S}$, where Γ is the vacuum decay rate, ΔS is the change of Bekenstein entropy of the BH, and B is an on-shell Euclidean action depending on the Euclidean dynamics of a bubble wall. Then they found that the decrement of B due to gravity of a BH overwhelms the entropy decrement. Although they found an extremely large enhancement of bubble nucleation rate around a BH, it has been discussed that the main effect comes from the thermal fluctuation due to the Hawking radiation [64]. This implies that the nucleation rate is overestimated because the same effect generates a thermal potential that tends to stabilize the Higgs at the symmetric phase [64,65,84] or because the thermal effect of Hawking radiation should be small for a large bubble. Thus, though the bubble nucleation rate is still enhanced around a BH because of the effect of gravity, it is not so large as expected before. If gravity of a BH mainly contributes to the promotion of a vacuum decay, it is meaningful to consider the catalyzing effect even around horizonless objects. The absence of horizons is equivalent to the absence of the suppression factor due to the change of Bekenstein entropy $e^{\Delta S}$, and therefore, horizonless compact objects may be more important candidates of catalyzing objects for vacuum decays. In this manuscript, we discuss such a vacuum decay around a spherical horizonless object as a catalyzing one.





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This paper is organized as follows. In Sec. 2.1, we explain the formalism to calculate the bubble nucleation rate around a generic compact object. We use a Gaussian density function for the object as an example to calculate the nucleation rate in Sec. 2.2. We will see that the efficient enhancement occurs if the radius of the compact object, its Schwarzschild radius, and the radius of the nucleated bubble are of the same order with each other. We then discuss a parameter region for a t'Hooft-Polyakov monopole that is excluded because of the nucleation of AdS vacuum in Sec. 2.3. In Sec. 3, we discuss differences from the bubble nucleation around a black hole. We will see that the nucleation rate is more enhanced around a horizonless compact object than around a black hole with the same total mass. Our conclusions are summarized in Sec. 4.

2. Bubble nucleation around a compact object

2.1. Formalism

We consider a nucleation of a thin wall vacuum bubble around a spherical object. If we assume that the system is static, the metric inside and outside of the bubble can be written as

$$ds^{2} = -C_{\pm}(r_{\pm})dt_{\pm}^{2} + D_{\pm}(r_{\pm})dr_{\pm}^{2} + r_{\pm}^{2}d\Omega_{2}^{2},$$
(1)

where C_{\pm} and D_{\pm} are determined by the Einstein equation and will be specified later. The quantities associated with the outer and inner region are labeled by the suffix "+" and by "-", respectively.

The thin wall vacuum bubble can be characterized by its energy density, σ , and pressure, p. The ratio of p to σ , $w \equiv p/\sigma$ (equation-of-state parameter), is assumed to be a constant. We here choose the scale of radial coordinates r_{\pm} so that $r_{+} = r_{-} \equiv R$ on the wall. A schematic picture showing the vacuum decay process we here assume is depicted in Fig. 1.

Introducing the extrinsic curvature on the outer (inner) surface of the wall, $K_{AB}^{(+)}$ ($K_{AB}^{(-)}$), the energy-momentum tensor (EMT) of wall, S_{AB} , and the induced metric on the wall, h_{AB} , the dynamics of the thin wall with $\xi^A = (\tau, \theta, \phi)$ is described by the Israel junction conditions as

$$K_{AB}^{(+)} - K_{AB}^{(-)} = -8\pi G \left(S_{AB} - \frac{1}{2} h_{AB} S \right),$$
(2)

$$\sqrt{C_{\pm}D_{\pm}}K_{AB}^{(\pm)} = \operatorname{diag}\left(-\frac{d\beta_{\pm}}{dR}, \beta_{\pm}R, \beta_{\pm}R\sin^{2}\theta\right),\tag{3}$$

$$S^{A}{}_{B} \equiv \operatorname{diag}\left(-\sigma, p, p\right), h_{AB} \equiv \operatorname{diag}\left(-1, R^{2}, R^{2} \sin^{2} \theta\right), \qquad (4)$$

where

$$\beta_{\pm} \equiv \epsilon_{\pm} \sqrt{C_{\pm} + C_{\pm} D_{\pm} (dR/d\tau)^2},\tag{5}$$

and τ is the proper time of the wall and ϵ_{\pm} is the sign of spatial components of extrinsic curvature. We here simply neglect the interaction between the horizonless object and the bubble except for their gravitational interaction. The case with such an interaction being taken into account will be discussed elsewhere (see also Refs. [85–87] in the context of Q-ball in supersymmetric models without taking gravity effects into account).

We are interested in the decay of Higgs vacuum, where the metastable vacuum has a negligibly small vacuum energy and the true vacuum has a negative vacuum energy $\rho_{\rm V} < 0$. We also introduce a compact object at the origin of the spatial coordinate, which modifies the metric because of the nonzero mass density $\rho_{\rm c}(r)$. For simplicity, we here assume the EMT of the object which gives the following static solutions of the Einstein equation:

$$C_{\pm} = D_{\pm}^{-1} = f_{\pm}(r_{\pm}) \equiv 1 - 2GM_{\pm}(r_{\pm})/r_{\pm} + H_{\pm}^2 r_{\pm}^2, \tag{6}$$



Fig. 1. A schematic picture showing a vacuum decay catalyzed by a static and spherical object.

with

$$H_{+} = 0, \ H_{-}^{2} \equiv -\frac{8\pi G}{3}\rho_{\rm v}, \tag{7}$$

$$M_{\pm}(r_{\pm}) \equiv \int_{0}^{r_{\pm}} d\bar{r}_{\pm} 4\pi \bar{r}_{\pm}^{2} \rho_{\rm c}^{(\pm)}(\bar{r}_{\pm}).$$
(8)

If we use an arbitrary mass density, $\rho_c^{(\pm)}(r)$, the compact object does not satisfy the static Einstein equation unless an appropriate EMT for the chosen $\rho_c^{(\pm)}(r)$ exists. Although in this case the metric (1) cannot be used, we expect that we can use it to capture a qualitative result. To be more rigorous, in the Appendix, we calculate the vacuum decay rate around a gravastar-like object, which is constructed to be (approximately) static. We specify its interior EMT and use the metrics consistent with the specified EMT. Then one could find that the aforementioned assumption for the metrics, (1), does not qualitatively change our results and main conclusions.

Equation (2) now reduces to the following equations

$$\frac{d}{dR}(\beta_{-} - \beta_{+}) = -8\pi G (\sigma/2 + p), \qquad (9)$$

$$(\beta_{-} - \beta_{+}) = 4\pi \, G\sigma(R)R. \tag{10}$$

One obtains $\sigma = m^{1-2w}R^{-2(1+w)}$ by solving (9), where *m* is the typical energy scale of the wall, and we can rewrite (10) as

$$\left(\frac{dz}{d\tau'}\right)^2 + V(z) = -1,\tag{11}$$

$$V(z) \equiv -\frac{a_{+}}{z} - \frac{z^{2}}{4} \left[\frac{\Delta a \bar{m}^{2w-1} z^{2w-1}}{4\pi \bar{H}^{2w+1}} + \frac{z^{2(1+w)} \bar{m}^{2w-1}}{4\pi \bar{H}^{2w+1}} - \frac{4\pi \bar{H}^{2w+1}}{z^{2(1+w)} \bar{m}^{2w-1}} \right]^{2} \leq 0,$$
(12)

where we re-defined the following non-dimensional variables and parameters:

$$z \equiv H_{-}R, \ \tau' \equiv H_{-}\tau, \ a_{\pm} \equiv 2GM_{\pm}H_{-}, \Delta a \equiv 2G(M_{+} - M_{-})H_{-}, \ \bar{m} \equiv m/M_{\text{Pl}}, \ \bar{H} \equiv H_{-}/M_{\text{Pl}}.$$
(13)

The parameters \bar{m} and \bar{H} are the ones in the Planck units $M_{\text{Pl}} = 1/\sqrt{G}$. Implementing the Wick rotation, $\tau = -i\tau_{\text{E}}$, (11) gives the bounce solution that describes the bubble nucleation process. In the following, we assume that matter fields forming a compact object has no interaction with another matter field which eventually undergoes the phase transition. In this case, the transition would not change the mass of the object, and therefore, $\Delta a = 0$, that is, $M_+(R) = M_-(R) \equiv M(R)$, is valid in (12). Even if this is not the case, $\Delta a = 0$ is a good approximation, provided that $|M_+ - M_-| \ll (4\pi/3)R^3|\rho_V|$, for which the first term in the square-bracket in (12) is negligible compared to the second term.

The Euclidean action, B_{co} , can be calculated from the bounce solution with the following integration [60]:

$$B_{\rm co} = \frac{1}{4G} \int d\tau_{\rm E} (2R - 6GM + 2GM'R) \left(\frac{\beta_+}{f_+} - \frac{\beta_-}{f_-}\right). \tag{14}$$

The transition rate, Γ_D , can be estimated as

$$\Gamma_{\rm D} \sim R_{\rm CDL}^{-1} \sqrt{\frac{B_{\rm co}}{2\pi}} \exp(-B_{\rm co}),\tag{15}$$

where we estimate the prefactor by taking a factor of $\sqrt{B_{co}/2\pi}$ for the zero mode associated with the time-translation of the instanton and we use the light crossing time of the bubble, R_{CDL} , as a rough estimate of the determinant of fluctuations, which will be defined more precisely below.

2.2. Results for Gaussian mass function

As an example, we consider the case where the density distribution of the horizonless object is given by the Gaussian form:

$$\rho_{\rm c}(r) = \rho_0 e^{-r^2/\xi^2},\tag{16}$$

where ρ_0 and ξ represent the typical mass density and the size of the compact object, respectively, and $\rho_c(R) \equiv \rho_c^{(+)}(R) = \rho_c^{(-)}(R)$. Motivated by the Higgs vacuum decay, we take $\bar{H} = 10^{-6}$, $\bar{m} = 6 \times 10^{-4}$, and w = -1 throughout this manuscript.¹ Here, we implicitly assume that the Higgs potential is supplemented by a non-renormalizable ϕ^6 term as considered in Ref. [62] so that we can use the thin-wall approximation. We also take $\xi = 10^3 M_{\rm Pl}^{-1}$ as an example.

Effective potentials governing the wall position (*V*(*z*)) for the above parameters are plotted in Fig. 2-(a). The dashed line represents the case of Coleman De-Luccia (CDL) tunneling, where $M_{\text{tot}}/M_{\text{Pl}} = 0$ with $M_{\text{tot}} \equiv \int_0^\infty dr' 4\pi r'^2 \rho_{\text{c}}(r')$. In this case, a bubble is nucleated at the point P_0 i.e. with the radius $R \simeq \alpha H_-^{-1} \equiv R_{\text{CDL}}$ for $\alpha \equiv 8\pi G m^3/H_- \ll 1$ [60,88]. As we increase $M_{\text{tot}}/M_{\text{Pl}}$, the effective potential becomes lower. We plot the cases of $M_{\text{tot}}/M_{\text{Pl}} = 400$ (a black solid line), 872.6 (a black dashed-dotted line), and 952.1 (a blue solid line).

A nucleated vacuum bubble with $0 \le M_{tot}/M_{Pl} \le 872.6$ initially has its wall radius between P_1 and P_0 (black open circles in Fig. 2) and would expand soon after its nucleation. A nucleated bubble around the horizonless object with $872.6 \le M_{tot}/M_{Pl} \le 952.1$ would be trapped between P_2 and P_3 , where the gravitational force and bubble tension are balanced, and then, it may eventually tunnel to a larger bubble, whose wall is in between P_4 and P_1 . If the mass is larger than or equal to $952.1M_{Pl}$, one has $f_+ = 0$ (a black filled circle in Fig. 2-(b)), that is, a BH forms.

One finds that the effective potential can be drastically distorted because of the gravitational effect of the horizonless object, which makes a bubble wall nucleated around a catalyzing object smaller compared to a CDL bubble (P_0 in Fig. 2-(a)). The distortion of the potential largely enhances the nucleation rate of vacuum bubble and the nucleation of bubbles could occur within the cosmological time as will be shown in the following.



Fig. 2. The effective potential (a) and f_+ (b) for the horizonless object with $M_{\text{tot}}/M_{\text{Pl}} = 0$ (CDL solution), 400 (a black solid line), 872.6 (a black dashed-dotted line), and 952.1 (a blue solid line) and for a BH with $M_{\text{tot}}/M_{\text{Pl}} = 952.1$ (a black dotted line) are shown.



Fig. 3. A plot of the ratio of the decay rate, Γ_D , to the inverse of the cosmological time, Γ_C , as a function of the mass and compactness of the horizonless object. The contour of $\Gamma_D = \Gamma_C$ (red solid line) and contours of ξ/R_{CDL} (white dashed lines) are marked for reference. In the case of $c \le c_{crit} \simeq 0.525$ (gray shaded region), the object inevitably collapses to a BH since a function $f_+(r)$ has zero points there.

In Fig. 3, the ratio of the vacuum decay rate, Γ_D , to the inverse of the cosmological time, $\Gamma_C \equiv H_C \simeq 10^{-61} M_{\rm Pl}$, is shown in the range of $1 \leq M_{\rm tot}/M_{\rm Pl} \leq 15000$ and of $c \leq 5$, where we define the compactness parameter² as

$$c \equiv \xi / (2GM_{\text{tot}}). \tag{17}$$

In our setup, we find that the existence of even a single horizonless object with M_{tot}/M_{Pl} and c within the region enclosed by the red line in the figure (i.e., $10^3 \lesssim M_{tot}/M_{Pl} \lesssim 10^4$ and with $c \lesssim 2$) would be excluded since a bubble would be nucleated around it within the cosmological time.

¹ We assume that the absolute value of true-vacuum energy density, $\rho_{\rm v}$, and the height of the Higgs potential barrier, $V_{\rm max}$, are of the order of the GUT scale $\sim 10^{-12} M_{\rm Pl}^4$. The vacuum expectation value of the true-vacuum state, ϕ_0 , is assumed to be $\phi_0 \sim 10^{-3} M_{Pl}$. This gives $\bar{H} \sim 10^{-6} M_{\rm Pl}$ and $\bar{m} = (\sigma / M_{\rm Pl}^3)^{1/3} \sim \left(\kappa \sqrt{V_{\rm max}/M_{\rm Pl}^4} \times \phi_0 / M_{\rm Pl}\right)^{1/3} \sim 6 \times 10^{-4}$, where the constant κ depends on the details of the Higgs potential [62] and we here take $\kappa \sim 0.1$.

² Although there is an ambiguity in the definition of the radius because of the thick boundary of the Gaussian mass distribution, this ambiguity just changes the scale of c-axis in Fig. 3 and does not affect the result shown there.

We show the contours of ξ/R_{CDL} as white dashed lines in Fig. 3. They indicate that an efficient enhancement occurs only when the radius of the nucleated bubble (which is of the same order with the CDL radius) is comparable to that of the compact object. An efficient enhancement also requires a small compactness so that the gravity effect is efficient around the dense compact object. Therefore, we conclude that the bubble nucleation rate is drastically enhanced around a compact object if the size of the horizonless object is comparable with the radius of CDL bubble and its compactness is of the order of unity.

2.3. Constraint on the abundance of compact objects

The nucleation of the anti-de Sitter vacuum bubble, whose origin could be the Higgs instability, within the cosmological time is obviously conflict with the present Universe not filled by the negative vacuum energy. It would expand with speeds comparable with the light speed soon after its nucleation, which would lead to the Universe filled by the negative vacuum energy. Since a compact object could be a catalyst for the vacuum decay, we can put the constraint on the abundance of horizonless objects in the Universe.

For instance, depending on parameters characterizing magnetic monopoles, the monopoles could be ultra compact. Suppose that there is a (hidden) non-Abelian gauge field that is spontaneously broken by a (hidden) Higgs field. If the vacuum manifold has a non-trivial second homotopy group, monopoles arise at the spontaneously symmetry breaking. Introducing the vacuum expectation value (VEV) of the hidden Higgs field, v, the mass and size of a t'Hooft-Polyakov monopole, denoted by M_{mono} and R_{mono} , respectively, can be estimated as

$$M_{\rm mono} \sim \nu / \sqrt{\alpha_{\rm G}}$$
 (18)

$$R_{\rm mono} \sim (\sqrt{\alpha_{\rm G}} \nu)^{-1},\tag{19}$$

where $\alpha_{\rm G}$ is the running gauge coupling constant for the non-Abelian gauge interaction. Imposing the ultra compact condition, $c \simeq R_{\rm mono}/(2GM_{\rm mono}) \sim 1$, one obtains $v \sim M_{\rm Pl}$ and $R_{\rm mono} \sim \ell_{\rm Pl}/\sqrt{\alpha_{\rm C}}$. Therefore, as long as the Higgs potential accommodates a second lower minimum due to the Higgs instability, parameter regions which realize $v \sim M_{\rm Pl}$ and $R_{\rm CDL} \sim R_{\rm mono} \sim \ell_{\rm Pl}/\sqrt{\alpha_{\rm G}}$ should be excluded in order to be consistent with the present Universe not filled by the anti-de Sitter vacuum. Since $R_{\rm CDL} = 8\pi Gm^3/H_{-}^2 \simeq 5 \times 10^3$, $\alpha_{\rm G}$ should be as small as 3×10^{-8} to nucleate the Higgs vacuum bubble.

3. Comparison with the catalyzing effect of black holes

Now we compare our results with the case of bubble nucleation around a BH, which has been extensively discussed in the literature. In Ref. [60], Gregory, Moss, and Withers pointed out that the Bekenstein entropy of a BH with mass M_{tot} may contribute to the vacuum decay rates as

$$\Gamma_{\rm D} \sim R_{\rm CDL}^{-1} \sqrt{\frac{I_{\rm E}}{2\pi}} e^{-I_{\rm E}} = R_{\rm CDL}^{-1} \sqrt{\frac{I_{\rm E}}{2\pi}} e^{-B_{\rm bh} + \Delta S},\tag{20}$$

$$B_{\rm bh} \equiv \frac{1}{4G} \oint d\tau_{\rm E} (2R - 6GM_{\rm tot}) \left(\frac{\beta_+}{f_+} - \frac{\beta_-}{f_-}\right),\tag{21}$$

where $I_{\rm E}$ is the total Euclidean action and $B_{\rm bh}$ is the bulk component of the on-shell Euclidean action depending on the Euclidean dynamics of a vacuum bubble. Contributions from the conical singularities on the Euclidean manifolds before and after the vacuum decay lead to a factor of ΔS , which is equivalent to the change of the Bekenstein entropy of a catalyzing BH.



Fig. 4. The vacuum decay rates around a BH with mass M_{tot} (a dotted line) and that around a horizonless compact object, whose mass is M_{tot} and compactness is fixed with $\xi/2GM_{\text{tot}} = 1$, (a solid line) are shown. Red and black points show the decay rate of the CDL solution and a critical static solution, respectively.

Note that even horizonless compact objects can emit Hawking radiation (see, e.g., Refs. [89–91]) because of the vacuum polarization in a strong gravitational field and its thermal effect on the Higgs potential may have to be taken into account. The details of the Higgs potential are characterized by the parameters (i.e. \bar{H} , \bar{m} , and w) in the thin-wall approximation, and therefore, those parameters could be affected by such a thermal effect in our setup.³

When a BH efficiently catalyzes the vacuum decay, the size of the BH, $2GM_{tot}$, is comparable with the CDL bubble radius, R_{CDL} , and the prefactor in (20) can be rewritten as $(GM_{tot})^{-1}\sqrt{I_E/2\pi}$, which is consistent with the prefactor in Ref. [60]. The Bekenstein entropy decreases because of the vacuum decay since the decrease of the vacuum energy surrounding the BH makes the area of its event horizon smaller. Although the gravity effect is strong around a BH, the vacuum decay rate would be suppressed by the change of Bekenstein entropy:

$$\Delta S = \pi \left[R_{\rm h,-}^2 - (2GM_{\rm tot})^2 \right] < 0, \tag{22}$$

where the horizon radius $R_{h,-}$ after the bubble nucleation is defined by $f_{-}(R_{h,-}) = 0$.

Horizonless compact objects have no Bekenstein entropy, so that they could more efficiently catalyze vacuum decays than BHs do. We here compare the vacuum decay rate around the horizonless compact object, whose mass is M_{tot} and compactness is fixed with c = 1, with that around a BH whose mass is M_{tot} . The result is shown in Fig. 4. Both the BH and horizonless compact object efficiently catalyze the vacuum decay compared to the CDL solution shown as red points. If there were no contribution of Bekenstein entropy on the decay rate around the BH, the exponential factor for the BH would be larger than that for the horizonless object (blue dashed line in Fig. 4-(b)). However, the decay rate with the compact object (black solid line) is larger than that with the BH (black dotted line) thanks to the absence of the decrement of Bekenstein entropy (red dashed-dotted line).

A bubble nucleated around a BH with $M_{\text{tot}} = M_{\text{crit}} \simeq 1045 M_{\text{Pl}}$ is static (black points in Fig. 4) because of the perfect balance be-

³ Although it might be possible that the BH mass changes due to the bubble nucleation [60–62,67], it was argued that it could be closely related to the thermal excitation of bubble due to the Hawking radiation [64,84].

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tween the gravity of the BH and bubble's tension. On the other hand, there is no well-defined Euclidean solution for a BH with $M_{\text{tot}} \ge M_{\text{crit}}$ [60].

4. Conclusions

We have discussed a role of a horizonless compact object as a catalyst for a vacuum decay. As long as the interaction between a bubble and a catalyzing object is negligible, our results do not depend on the details of the object much and its gravity plays an essential role in the catalyzing process. The universality of our result is also discussed in the Appendix. This suggests that one can put some constraints on the abundance of various kinds of horizonless compact objects, such as monopoles, Q-balls, Boson stars, gravastars, BH remnants, and so on. In particular, the Higgs vacuum may decay into an AdS vacuum if there exists even a single compact object whose radius is comparable to its Schwarzschild radius and the CDL bubble radius. For instance, depending on parameters characterizing magnetic monopoles, the monopoles could be ultra compact. As long as the Higgs potential accommodates a second lower minimum due to the Higgs instability, parameter regions which realize the Planck-scale GUT symmetry breaking with $R_{CDL} \sim R_{mono}$ should be excluded in order to be consistent with the present Universe not filled by the anti-de Sitter vacuum. More realistic cases may arise for Boson stars, oscillons, and Q-balls. In addition, in case that a single compact object is not enough to catalyze the Higgs vacuum to decay into the AdS one, multiple ones could do it, which leads to new constraints on the abundance of such a compact object.

It is also interesting to note that the catalyzing effect of horizonless objects is more efficient compared to that of BHs since there is no suppression of vacuum decay rate due to the decrement of Bekenstein entropies. Therefore, if there had been some ultra compact objects in the Universe, they could have played a critical role in the cosmological sense.

Finally, we comment on the case where the compact object has an interaction with the nucleated bubble, namely, the Higgs field. In this case, the mass of the compact object can change due to the bubble nucleation. Because of the conservation of energy, the nucleated bubble can use the mass difference of the compact object and the nucleation rate can be drastically enhanced. This is similar to the case of bubble nucleation in a finite temperature plasma, where a bubble can use the thermal energy to be excited with a finite energy. This is also similar to the case for a bubble nucleation around a black hole with the thermal effect of Hawking radiation, where the mass of black hole changes after the bubble nucleation. However, as the thermal effect stabilize the Higgs potential to the symmetric phase in these cases, the interaction between the compact object and the Higgs field may lead to an effective potential that stabilize the Higgs potential. Still, this results in a more efficient enhancement for the nucleation rate and is an interesting possibility for many particle physics models.

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Appendix A. Static gravastar-like objects

Throughout the main text we assume that the compact object is static at least during the nucleation process and the metric is given by the static solution (1). This is (approximately) justified for most of the realistic situations, like neutron stars, boson stars, oscillons, monopoles, and Q-balls, and so on. However, the density function $\rho_c(r)$ as well as the metric functions C_{\pm} and D_{\pm} should be carefully chosen so that it is a static solution to the Einstein equation. In this Appendix, we consider a gravastar-like object to show that the result in Fig. 3 does not change qualitatively as long as we choose those functions carefully to (approximately) satisfy the static equilibrium.

We use the following EMT for the gravastar-like object:

$$T_{\nu}^{\mu} = \text{diag}(-\rho(r), p(r), p(r), p(r)), \qquad (A.1)$$

$$\rho(r) \equiv \rho_0 \frac{1 - \tanh\left((r - \xi)/\delta\right)}{2} + \rho_v = -p(r)$$
(A.2)

with r < R, where T_{ν}^{μ} is the bubble interior EMT and δ represents the thickness of the boundary of the gravastar-like object. When $\delta \ll \xi$, one can use the thin wall approximation and the bubble interior energy density, ρ , is written as

$$\rho(r) \simeq \begin{cases}
\rho_0 + \rho_v \equiv \rho_{in} > 0 & \xi > r \\
\rho_v < 0 & \xi < r < R,
\end{cases}$$
(A.3)

where the energy density of the gravastar-like object ρ_0 is constant. Assuming the form of its pressure as $p = -\rho$, the inner metric of the gravastar-like object is given by

$$g_{\mu\nu}^{(\text{in})} = \text{diag}(-f_{\text{in}}(r_{\text{in}}), f_{\text{in}}^{-1}(r_{\text{in}}), r_{\text{in}}^2, r_{\text{in}}^2 \sin^2 \theta),$$
(A.4)

$$f_{\rm in}(r) \equiv 1 - H_{\rm in}^2 r^2, \tag{A.5}$$

where $H_{in}^2 \equiv (8\pi G/3)\rho_{in}$ and r_{in} is the radial coordinate inside the object and we set its scale so that $r_{in} = r_- = \xi$ on the boundary of gravastar-like object.

Although the bulk of gravastar-like object has its static metric, whether or not its boundary is also static should be determined by the Israel junction condition that is available only when the thickness of its boundary is smaller than its radius, $\delta \ll \xi$. In the thin wall approximation, the boundary can be characterized only by its energy density, σ_c , and pressure, p_c . Introducing the equation-of-state parameter, $w_c \equiv p_c/\sigma_c$, one has the Israel junction conditions:

$$\beta_{\rm in} - \beta_- = 4\pi \, G \sigma_c(\xi) \xi, \tag{A.6}$$

$$\frac{d}{d\xi} \left(\beta_{\rm in} - \beta_{-}\right) = -8\pi \, G\sigma_c(\xi) \left(1/2 + w_c\right),\tag{A.7}$$

where $\beta_{in} \equiv \epsilon_{in} \sqrt{f_{in}(\xi) + (d\xi/d\tau_c)^2}$ and τ_c is the proper time on the boundary of the object. Solving (A.6) and (A.7), one has the form of $\sigma_c(\xi) \equiv m_c^{-1-2w_c}\xi^{-2(1+w_c)}$. Substituting $\sigma_c(\xi)$ into (A.6), one has

$$\left(\frac{dz_c}{d\tau_c'}\right)^2 + V_c(z_c) = E_c,\tag{A.8}$$

$$V_c(z_c) \equiv -\frac{4\gamma_c^2}{1+h^2} z_c^2 - z_c^{4w_c} \left(1 - z_c^3 + \frac{\gamma_c^2}{z_c^{1+4w_c}}\right)^2,$$
(A.9)

where we defined the following non-dimensional variables and parameters:

$$h \equiv H_{-}/H_{\rm in},\tag{A.10}$$

$$z_c^3 = \left(\frac{1+h^2}{2GM_{\text{tot}}H_-}\right)H_-^3\xi^3,$$
(A.11)

$$\tau_c' \equiv \frac{\sqrt{1+h^2}}{2\gamma_c} H_- \tau_c, \tag{A.12}$$



Fig. 5. A plot of the effective potential, $V_c(z_c)$, with $w_c = 0.6$, $\gamma_c = 0.1$, and $h = 10^{-3}$.

$$\gamma_c^2 \equiv H_-^{2(1+4w_c)/3} \frac{(4\pi \, Gm^{-1-2w_c})^2}{2GM_{\text{tot}}} \left(\frac{1+h^2}{2GM_{\text{tot}}}\right)^{(1+4w_c)/3}, \quad (A.13)$$

$$E_c = -\frac{4\gamma_c^2}{(2GM_{\text{tot}}H_-)^{2/3}(1+h^2)^{1/3}}.$$
(A.14)

Now one obtains stable solutions by appropriately choosing the parameters. An effective potential, $V_c(z_c)$, governing the position of the boundary is plotted in Fig. 5. One finds a stable and static solution (a black filled circle in Fig. 5), at which its radius is $z_c = z_{\min}$ and $E_c = V_c(z_{\min})$. An effective potential governing the dynamics of the boundary before the phase transition is obtained just by taking h = 0 in (A.9). Therefore, the effective potential, $V_c(z_c)$, is almost not affected by the phase transition as long as $h \ll 1$ is hold (see (A.9)). In this case, the gravastar-like object remains almost static even after the bubble nucleation and we can safely use the static metric (1) to calculate the bubble nucleation rate.

Fixing $h(\ll 1)$, γ_c , and w_c , one may obtain a static solution, $dV_c(z_c = z_{\min})/dz_c = 0$, and the total mass and size of the gravastar-like object are given by

$$M_{\text{tot}} = \frac{8\gamma_c^3}{2GH_-(-V_c(z_c = z_{\min}))^{3/2}(1+h^2)^{1/2}},$$
(A.15)

$$\xi = \frac{z_{\min}}{H_{-}} \left(\frac{2GM_{\text{tot}}H_{-}}{1+h^2}\right)^{1/3},\tag{A.16}$$

where we used (A.11) and (A.14).

Appendix B. Vacuum decay rate around the gravastar-like object

Here we calculate the on-shell Euclidean action as a function of $(M_{\text{tot}}, c \equiv \xi/2GM_{\text{tot}})$. Note that we do not take into account a parameter region where $h \ge 0.1$ to approximately keep the gravastarlike object static before and after the phase transition. The mass function, M(r), in (14) should have the form of

$$M(r) = \int_{0}^{r} dr' 4\pi r'^{2} \rho_{c}(r') \simeq \begin{cases} (4\pi/3)r^{3}\rho_{0} & \xi > r\\ (4\pi/3)\xi^{3}\rho_{0} = M_{\text{tot}} & \xi < r, \end{cases}$$
(B.1)

where $\delta \ll \xi$ is hold. This gives the metric on the inner and outer surface of the wall:

$$g_{\mu\nu}^{(\pm)} = \text{diag}(-f_{\pm}(R), f_{\pm}^{-1}(R), R^2, R^2 \sin^2 \theta),$$
(B.2) with

$$f_{+} \simeq \begin{cases} 1 - \frac{2GM_{\text{tot}}}{R} & R > \xi \\ 1 - H_{c}^{2}R^{2} & R < \xi, \end{cases}$$
(B.3)

$$f_{-} \simeq \begin{cases} 1 - \frac{2GM_{\text{tot}}}{R} + H_{-}^2 R^2 & R > \xi \\ 1 - H_{\text{in}}^2 R^2 & R < \xi. \end{cases}$$
(B.4)



Fig. 6. A plot of the ratio Γ_D/Γ_C as a function of the mass and compactness of the gravastar-like object with $\bar{H} = 10^{-6}$, $\delta = 0.01\xi$, $\bar{m} = 6 \times 10^{-4}$, and w = -1. We here only take into account a parameter region corresponding to $h \le 0.1$. The contours of ξ/R_{CDL} (white dashed lines) are marked for reference.

From (14), (15), (B.3), and (B.4), one can calculate the vacuum decay rate. Fig. 6 shows the result of Γ_D/Γ_C . One finds that the result shown in Fig. 6 is qualitatively consistent with our conclusion based on the result in Fig. 3. However, the range of values of compactness, *c*, in which $\Gamma_D > \Gamma_C$ is satisfied, seems to be sensitive to the configuration of the boundary of a catalyzing object. The result in Fig. 6 based on the more concrete set up would be a supporting evidence for the universality of our main proposal, that is, horizonless objects would catalyze vacuum decays when its size is comparable with the size of a CDL bubble and its compactness, $c \equiv \xi/2GM_{tot}$, is of the order of unity, $c \sim O(1)$.

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