1995ASPC...75..238M

Multi-feed Systems for Radio Telescopes
ASP Conference Series, Vol. 75, 1995
Darrel T. Emerson and John. M. Payne (eds.)

DAMMANN GRATINGS FOR LOCAL OSCILLATOR BEAM MULTIPLEXING

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ABSTRACT In submillimeter-wave heterodyne imaging systems, optical coupling provides the most efficient way of combining local oscillator power with the array of signals from the telescope. For systems limited to one local oscillator source an ideal optical-coupling scheme would produce an array of appropriately scaled images of the local oscillator feed at the detectors without any loss in power. One candidate for a beam multiplexing system is the combination of an interferometric coupler with a type of binary phase grating known as a Dammann grating. In this paper, we consider in some detail the feasibility of such a system.

INTRODUCTION

Dammann gratings (DG) are binary phase gratings which produce a finite array of diffraction spots of equal intensity when illuminated by a (coherent) plane wave. Dammann gratings were developed for use at optical frequencies (Jahns *et al.* 1989), and are binary in the sense that the optical path length through the grating takes on just two values. The original goal of Dammann was to obtain multiple images of a single coherently illuminated object (Dammann and Klotz 1977). In the case of a local oscillator(LO) multiplexing system we wish, in a similar manner, to produce multiple images of the LO source feed, such that the array of images couple well to the array of mixer feeds when used in combination with an interferometric coupler. This approach is similar to other multiplexing schemes reported by Jacobsson *et al.* 1990 and Belousov *et al.* 1991.

By producing a number of images of the LO feed, the DG potentially guarantees very high coupling efficiencies. Thus, a DG would be particularly useful in situations where the total LO power per array element available through ordinary means is limited (such as in the cases of an array with a large number of elements, a sparsely filled array or at high frequencies where LO power is severely limited). A DG is a passive device so the amount of power to each element is fixed and not variable; the repeatability, however, of SIS junction characteristics produced in the same manufacturing process now suggests that

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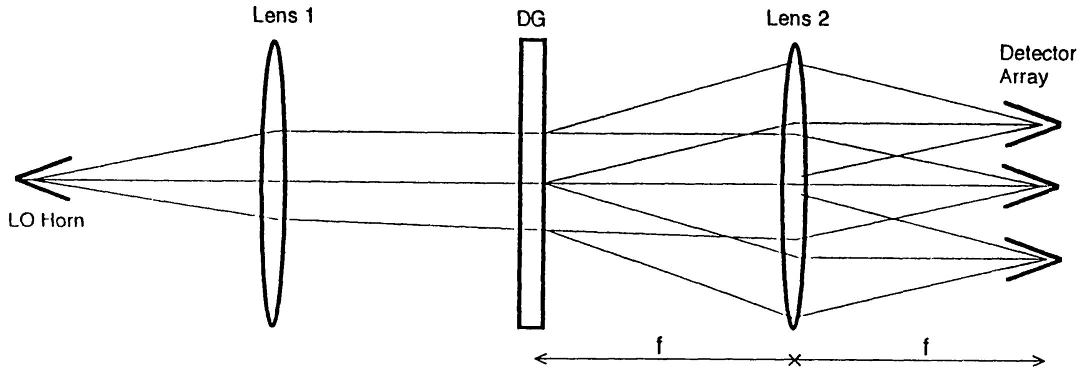

FIGURE 1 Optical set-up for beam multi-plexing with DG.

an array of mixers with the same LO power requirement is feasible.

THEORY

Consider the case of a DG illuminated by an incident field, $\psi(x, y)$, produced by the local oscillator source feed, as shown in Fig. 1. Plane-wavefront illumination of the grating located on the common focal plane of L_1 and L_2 is assumed; this will be the case if the phase center of the LO feed is at the focal point of the first lens L_1. The grating consists of an infinite array of basic cells with the transmission function of the on-axis cell being given by the aperture function $t(x, y)$, say. An example of the basic cell (one period of the transmission function for the grating) for a one-dimensional grating is shown in Fig. 2. For the case where the binary phase steps are equivalent to either 0 and π, the transmission function for the grating at (x, y) is given by $t(x, y) = \pm 1$. The transmission of an infinite grating with rectangular symmetry is given by the infinite double sum: $g(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} t(x - m\Delta x, y - n\Delta y)$, where Δx and Δy are the periods of the grating in the x and y directions, respectively. At the output focal plane of lens L_2, the field due to the grating is given by the usual Fourier transform relationship:

$$E(u, v) = \int\int g(x, y)\psi(x, y)e^{-i2\pi(ux+vy)}dxdy, \qquad (1)$$

where the spatial frequencies u and v are related to the coordinates in the Fourier output plane through $u = x_o/\lambda f$ and $v = y_o/\lambda f$, where f is the focal length of lens L_2.

For a rectangular array of LO beams in the image plane, rectangular symmetry holds for the grating and we write $t(x, y) = t_x(x)t_y(y)$. In the example of the basic period of a one dimensional grating shown in Figure 2, the free parameters are $\pm x_1, \pm x_2, \pm x_3$ etc. If the Fourier transform of $t(x)$ (where we have dropped the subscript x) be given by $T(u)$, then the Fourier transform of the DG transmission function $g(x)$ $(= \sum_{m=-\infty}^{\infty} t(x - m\Delta x) = t(x) * \sum_{m=-\infty}^{\infty} \delta(x - m\Delta x))$

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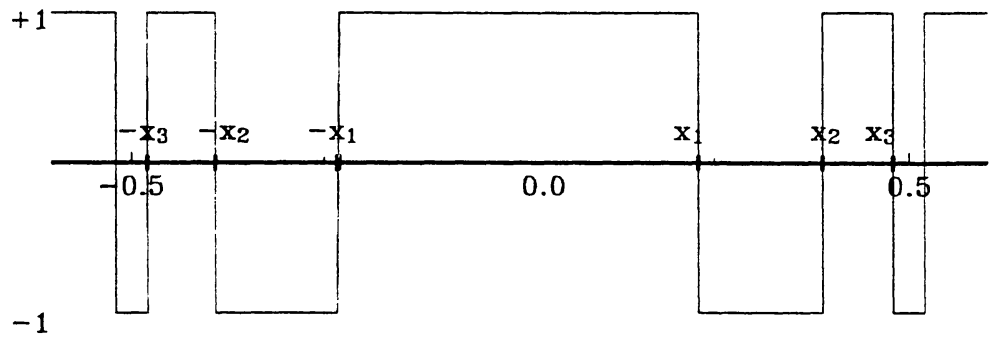

FIGURE 2 One-dimensional, symmetric binary function having values of +1 and -1 only.

is given by: $G(u) = T(u).\sum_{m=-\infty}^{\infty} \delta(u - m\Delta u)$, and an array of point-like spots (or infinitesimally narrow fringes in the one dimensional case) separated by a common distance of $\Delta u = 1/\Delta x$ will be obtained. Physically, these correspond to the different diffraction orders of a *uniformly illuminated grating.* The amplitude of the different orders is determined by the Fourier transform (or equivalently, the Fraunhofer diffraction pattern) of the basic grating period $t(x)$. The different diffraction grating orders occur when $u = m\Delta u$, where m is some integer. Thus, $G(u) = 0$ unless $u = m\Delta u$; this is equivalent, of course, to the usual grating formula: $n\lambda = d\sin\theta = \Delta x.(x_o/f)$.

For a DG illuminated by a field $\psi(x)$, with Fourier transform field $\Psi(u)$ on the output plane, the pattern obtained in the output plane is given by:

$$E(u) = G(u) * \Psi(u) = [T(u).\sum_{m=-\infty}^{\infty} \delta(u - m\Delta u)] * \Psi(u), \qquad (2)$$

an array of spots convolved with the output LO beam that would have been obtained directly in the absence of the DG. Note $\Psi(u)$ will be an image of the (virtual) field at the LO feed phase center.

If we require a $2M + 1$ array of non-overlapping images of the LO phase center field, of equal intensity, then $G(u)$ should consist of $2M + 1$ point-like spots of equal intensity, with all higher diffraction orders of negligible intensity. Thus, an ideal DG will satisfy:

$$|G(u)| \propto \sum_{m=-M}^{M} |\delta(u - m\Delta u)|. \qquad (3)$$

The single aperture diffraction pattern of the basic grating period, $T(u)$, must therefore be such that $|T(m\Delta u)| = |T(0)|$ for m between $-M$ and M (giving $2M + 1$ diffraction orders of equal intensity for $G(u)$), and $T(m\Delta u) \approx 0$, otherwise (ensuring most of the power is in the central orders). To ensure this in terms of the DG we must choose the values of x_1, x_2, x_3 etc. mentioned above for a single period of the grating. The values chosen should ensure maximum

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power is diffracted into the $(2M+1)$ array of spots (thus guaranteeing little power into higher diffraction orders).

For a phase step difference of π ($t(x) = \pm 1$) for the grating with $2N$ steps, one can show that $T(u)$ is given by:

$$T(u) = \frac{1}{\pi u} \sum_{n=0}^{N} (-1)^n (\sin 2\pi u x_{n+1} - \sin 2\pi u x_n). \tag{4}$$

We choose the minimum number of steps by setting $N = M$, to give the grating minimum complexity (Dammann and Klotz 1977). Thus, we solve for $|T(m\Delta u)| = T(0)$ for $|m| \leq M$. Various solutions are tabulated in Dammann and Klotz (1977) for different values of M.

If one wants a grating with an even number of output spots then neighbouring elements must be out of phase by π. This will cause the grating maxima to lie not in the direction given by $n\lambda = d \sin\theta$, but rather $(n + \frac{1}{2})\lambda = d \sin\theta$, and an even number of equal intensity diffraction spots is obtained (Morrison 1992).

LO BEAM MULTIPLEXING

Now we consider applying the above theory to an example 25 element array system arranged on a square grid of 5×5 elements. We assume a scalar feed (corrugated horn) for the LO source, and diagonal horn feeds for the mixer array. To a good approximation the resulting propagating LO beam can be considered to have a simple Gaussian field distribution, whose form does not change as the beam propagates. At the grating we can write the incident field as $\psi(x,y) = \exp(-(x^2+y^2)/W_G^2)$, where W_G is the usual Gaussian beam waist radius, while at the output Fourier plane $\Psi(u,v) = \exp(-(u^2+v^2)/w_F^2) = \exp(-(x_o^2+y_o^2)/W_F^2)$, where W_F is the waist radius at the Fourier plane. Using the usual results for Gaussian beam propagation: $W_F = \lambda f/\pi W_G$, or $w_F = 1/\pi W_G$(Goldsmith 1982). If both lenses have the same focal length then of course W_F equals the beam waist radius at the LO horn phase center.

For a 5×5 array $M = 2$ for the grating; the optimum power coupling solution for the positions of the edges of the grooves in the grating for the one dimensional basic period $t(x)$ are given by $x_1 = \pm 0.132\Delta x$, and $x_2 = \pm .480\Delta x$, where Δx is the grating period. What we require is that the positions of the diffraction orders coincide with the centers of the mouths of the individual diagonal horns making up the mixer feed array. Assuming a closely packed array of diagonal horns of side length a, with horn walls of negligible thickness, implies a diffraction order spacing Δx_o on the output Fourier plane of $\Delta x_o = a = \lambda f \Delta u = \lambda f/\Delta x$, corresponding to the grating period of Δx.

The best fit Gaussian beam to the field at the mouth of a diagonal horn has a radius given by: $W = 0.43a$ (Withington and Murphy 1992). For a horn of finite length the beam waist radius at the horn phase centre will be somewhat less than this, a typical value for a horn of moderate length being: $W_o = 0.38a$. Thus, if we wish to match this radius to the incident LO beams ($W_F = W_o$), the corresponding beam width W_G at the grating must have been:

$$W_G = \lambda f/\pi W_F = \lambda f/\pi(0.38a) = 0.837\Delta x,$$

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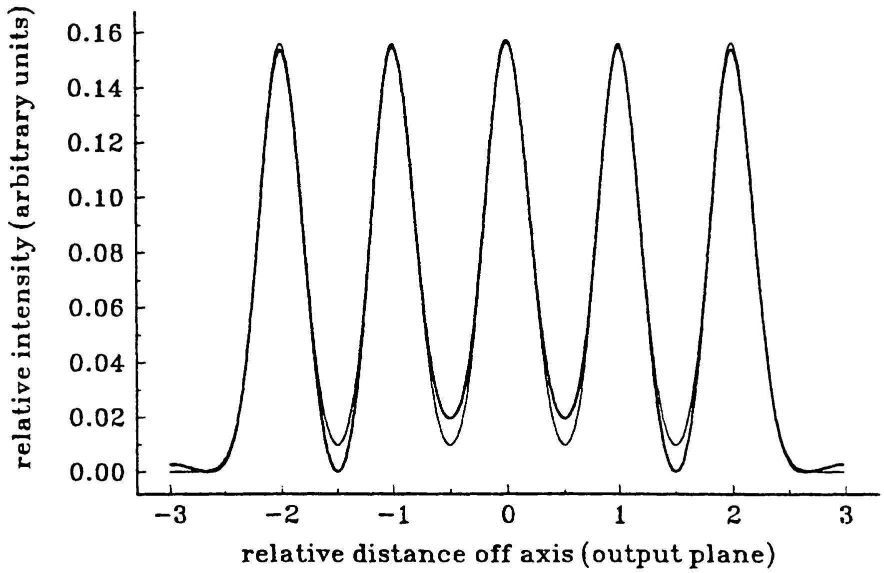

FIGURE 3 Array of Gaussian beams (thin line) superimposed on DG beam array (thick line).

the Gaussian beam width is of order the grating period! The implications of this for the phase array is we need only a *small number* of periods in the grating, and thus it should not be too complicated to manufacture. The resulting set of image Gaussian beams is shown in Fig.3, for the case where the total number of periods in the DG equals 4×4. For comparison, a set of 5 Gaussian beams is shown in intensity and, as can be seen, the coupling is high. Since about 77.4% of the power is contained in the central orders for the one-dimensional case (Dammann and Klotz 1977), the total LO coupling loss is of the order of 40% (in two dimensions). This power is lost into higher diffraction orders than 2, and so some power spills round the side of the array of diagonal horns. However, this power can be easily terminated using an absorbing microwave material around the array. It should also be possible to reduce loss by having a basic grating cell with more grooves (more degrees of freedom).

GRATING BANDWIDTH

As with all binary optics the DG will only operate correctly over a finite bandwidth. In many astronomical submillimeter receivers, however, the bandwidth is of order 15%. We therefore investigate a frequency detuning of this order. Two effects occur if the wavelength of the LO beam is not at the design wavelength of the grating:

(i) The actual inter-beam diffraction order separation in the output plane will change implying some of the beams will no longer couple well to the detector array as they will be misaligned with the horn centres. Realignment, however, can be achieved easily by designing some variable magnification into the LO path optics.

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(ii) The two binary phase delays for the grating are wavelength dependent, and so the grating will only operate as expected at the design frequency.

A groove depth of l corresponds to a relative phase difference of $\phi = (n-1)2\pi l/\lambda$. The relative phase difference equals the relative change in wavelength $\Delta\phi/\phi = \Delta\lambda/\lambda$. The effect is to change $T(u)$:

$$T(u) = \left[\frac{(1+e^{i\Delta\phi})}{2\pi u}\sum_{n=0}^{N}(-1)^n(\sin 2\pi u x_{n+1} - \sin 2\pi u x_n)\right] - \frac{(1-e^{i\Delta\phi})\sin\pi u}{2\pi u}. \quad (5)$$

The effect of the last term is to cause the grating function at the Fourier plane, $|G(0)|$, to become greater than $|G(n)|$, so that more power is coupled to the central order and less to the other orders.

If we are not operating at the design wavelength and we assume the worst case scenario of $\Delta\lambda/\lambda = 0.063$ (corresponding to a frequency displacement of 20 GHz for a 345GHz receiver system), then $\Delta\phi \approx 0.18$. We can then calculate the effect of this phase error on the DG for a 5×5 array. For the example in question $|G(n\Delta u)|/|G(0)| \approx .97$, thus an imbalance in power coupling of about 6 % will occur between the central pixel and other pixels in a one-dimensional array. In the case of the 2-D array a corner pixel will receive 12 % less LO power. SIS mixer sensitivity to local oscillator power for optimised performance is usually not so critical that such a variation would cause a significant deterioration in performance across the array.

CONCLUSIONS

In this paper we have presented a theoretical study of the feasibility of using Dammann gratings for LO beam multiplexing in array receiver systems at millimeter/submillimeter wavelengths. We have shown that at the design wavelength we get good coupling with efficiencies of better than 60% for a 5×5 array, with the missing power being channelled into higher diffraction orders which can easily be terminated. The bandwidth of such a grating would be of the order of 15%, which would be adequate for astronomical array receivers operating in the submillimeter waveband. Such an LO multiplexing scheme would work best for the case where mixer characteristics are very similar, so that the LO requirement of individual mixers is the same. The scheme is particularly suitable for sparse arrays.

We are now investigating the practical feasibility of producing a 5×5 grating at submillimeter wavelengths for the scenario referred to above. Since with the transmission grating there may be some reflection loss, we are also looking into reflection grating designs. These would have to be operated off-axis to prevent beam vignetting.

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