5-2019

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Recebido: 18-01-2019 | Aprovado: 16-04-2019 | DOI: https://doi.org/10.23882/MJ1902

The Dynamic Integration of Content and Language during Bilingual Discussion of Mathematics Word Problems

A Integração Dinâmica de Conteúdo e Língua durante Discussão Bilíngue de Questões Matemáticas com Enunciado

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Abstract: This study explores the discourse strategies adopted by a bilingual tutor to integrate content and second language learning, and support a student with limited English proficiency while discussing a set of word problems. Our findings reveal an integrated curriculum space wherein instructional focus oscillates between language and content. Pedagogical scaffolding was accomplished through strategic interspersing of pedagogical supports characterised by a linguistic focus (attention to words exchanged) or an epistemic focus (attention on mathematical ideas). Central to linguistically focused strategies was the deployment of oral translation techniques (literal translation, borrowing, modulation, and equivalence). By contrast, epistemically focused strategies entailed metaphorical gesticulation of abstract concepts in a more concrete spatialised form. It is argued that learning content in a second language requires more than vocabulary instruction at the beginning of content lessons. A more flexible approach is needed wherein English learners are provided with just-in-time support as new language naturally emerges.

Keywords: bilingual discussion, content-language integrated learning, high school mathematics, word problem

Resumo: Este estudo explora as estratégias adotadas por um mentor bilíngue, com a finalidade de integrar a aprendizagem de conteúdo e uma segunda língua, bem como, dar apoio pedagógico a um aluno com proficiência limitada na língua inglesa durante a discussão de questões matemáticas com enunciado. Os nossos resultados revelam um espaço curricular integrado no qual o foco pedagógico oscila entre a língua e o conteúdo. O suporte pedagógico é caracterizado por um foco linguístico (atenção a palavras utilizadas) ou um foco epistêmico (atenção a ideias matemáticas). As estratégias linguísticas envolvem a tradução oral. Em contrapartida, as estratégias epistêmicas envolvem uma gesticulação metafórica de conceitos abstratos, numa forma espacial mais concreta. Também se argumenta que a aprendizagem do conteúdo, numa segunda língua, requer o ensino de vocabulário antes de iniciar o conteúdo propriamente dito. Uma abordagem mais flexivel é necessária, na qual os estudantes recebam o apoio de uma forma natural.

Palavras-chave: aprendizagem integrada de conteúdo e língua, discussão bilíngue, questões matemáticas com enunciado, matemática de segundo grau

In our globalised world, no longer can content educators afford to see themselves simply as instructors of "non-language content" or expect non-native students to simply learn academic language (vocabulary, grammar, genre) first in a separate language-dedicated classroom before coming to their classrooms to learn content, that is, conceptual knowledge accumulated in academic fields like mathematics, chemistry, history, etc. The rapid growth in multilingualism among content learners has created a pressing need for *content-language integration*. Central to content-language integration is a shift from a separate language-dedicated classroom to wider curriculum spaces wherein language and content are integrated and coexist harmoniously (Cross, 2016; Fritzen, 2011). Rather than receiving language instruction separately before engaging with academic content, non-native speakers learn a new language inside the content classroom as they participate in content activities. Content serves as a space for non-native speakers to learn an additional language in contextualised and purposeful ways.

Faced with such a daunting task, content teachers have turned to pedagogical models of content-based instruction such as SIOP (Sheltered Instruction Observation Protocol) (Echevarria, Vogt, & Short, 2008) and SDAIE (Specially Designed Academic Instruction in English) (Cline & Nicochea, 2003). However, familiarity with these models has led to instruction wherein content-language integration is minimal and separation continues to persist, mostly in the form of vocabulary instruction in the beginning of content lessons (a practice known as front-loading language). Teachers mechanically follow a prescribed and fixed sequence of instructional steps, rather than flexibly adapting the models to the situational conditions that exist in their classrooms and dynamically responding to their students' specific language needs (providing them with just-in-time support). As a result, they are unable to reach higher levels of content-language integration in their classroom. The present study takes a step in this direction by examining the discourse strategies adopted by a bilingual tutor to support a student with limited English proficiency while discussing a set of word problems.

Teaching Language Learners Mathematics

Mathematics educators have resorted a variety of pedagogical strategies to simultaneously support content and linguistic proficiency of mathematics learners who speak multiple languages (Moschkovich, 2005). Many have favored linguistically focused strategies such as translation (Demski, 2009); use of cognates (Gomez, 2010); word squares (Winsor, 2007); labeling classroom artifacts, chalkboards, whiteboards, and bulletin boards (Cipriano, 2011); introduction to mathematics vocabulary (DfEE, 2000); and small-group mixing of students with varied proficiency levels (Gutierrez, 2002). In contrast, others have favored strategies with a stronger epistemic focus (i.e., focused mainly on mathematical content) such as careful scaffolding and elicitation of students' ideas and understandings (Razfar, Licon Khisty, & Chval, 2011; Walqui, 2006). Designed to meet language learners' need for both linguistic and epistemic support, these teaching strategies have been increasingly used to help students learn not only mathematics but also how to engage in mathematical meaning-making in a second language (Gibbons, 2003).

Another strategy recommended for the support of language learner participation in mathematics discussion is to "talk" with the hands in front of the classroom. It is believed that the visual cues portrayed by teachers' use of hand movement while also verbalising content knowledge can improve language learners' understanding of mathematical content knowledge. Shein's (2012) exploration of mathematics teachers' gesturing revealed three categories of gestural supports: (1) pointing with a finger, hand, or writing object to bring attention to a person, place, or object that may or may not have been apparent in the verbal communication; (2) representational movements that serve to elaborate on abstract or concrete ideas communicated verbally; and (3) writing on a worksheet or white board while simultaneously speaking about a concept. Teachers' gestural scaffolds can have a linguistic focus (they communicate word referents) as well as an epistemic focus (they communicate mathematical concepts).

Generally speaking, the existing literature provides consensual endorsement of pedagogical approaches characterised by dialogical meaning-making (discussion

rather than lecture) and multimodality (combining texts with imagery, gesticulation, manipulatives, etc.). Nonetheless, its scope is limited to isolated pedagogical strategies and fixed series of instructional steps, typically overlooking the fluid and highly dynamic nature of pedagogical action. How to effectively design language-content curriculum spaces wherein language learners are provided with *just-in-time support* (Bransford, Brown, & Cocking, 2002) in the form of well-placed interjections and vocabulary prompts has received little attention. This is precisely what we set out to illuminate in the present paper.

Methodology

This study adopted a qualitative research approach (Bogdan & Biklen, 2003; Creswell, 2003), being aligned with socio-constructivist perspectives on human interaction (Robson, 2002) aimed not at uncovering a presumably objective reality, but at generating a trustworthy interpretation of a social situation co-constructed by participants. More specifically, we conducted a microethnographic case study of tutor-student interactions as they discussed a set of mathematics word problems. As Patton (1990) states, case studies are used when "one needs to understand some special people, particular problems, or unique situation in great depth" (p. 54). In the present case, descriptive data systematically collected through open-ended research methods (video-recorded observations) and analyzed inductively to build a naturalistic account (Lincoln & Guba, 1985) of how a content-language curriculum space unfolds micro-sequentially from on turn to the next as two bilingual speakers discuss mathematics word problems written in a second language. This research design is reflective of our goal of achieving a more advanced theory-based understanding of the educational phenomenon of "language-content integration." Our primary aim is analytical depth and theorising, not inductive generalization to larger populations or educational contexts.

Data Collection

Our dataset was comprised of ten hours of video-recordings of bilingual exchanges (Portuguese/English) between the first author (Alan) and a female student (Cassie) during one-on-one mathematics tutoring sessions. A total of two sessions of approximately 1.5 hour each were recorded. Although Alan and Cassie shared Portuguese as their native language, Alan was originally from Brazil and had already resided in the US for nearly ten years teaching and completing a doctoral program in education. Cassie, on the other hand, had recently arrived from Angola, a Portuguese-speaking country in West Africa, where she completed her high school education. A language learner with relatively less developed proficiency in English and the mathematical register (specialized jargon), Cassie was preparing to take the Scholastic Assessment (SAT) exam – college entrance examination -- while attending a program of English as a Second Language (ESL) at a large Midwestern university in the USA. At the time of this study, Cassie's English proficiency was identified as being at an *Intermediate* level – two out of the four possible levels (Beginner, Intermediate, Advanced, and Proficient) in her ESL program. Further, Cassie's educational status was that of a transitioning higher-education international student. As Jindal-Snape and Rienties (2016) describe, "international students experience multiple contextual transitions – moving to a new country, moving to a new educational system, and moving to programmes for a higher education degree" (p. 2). These contextual transitions were directly linked to Cassie's linguistic and epistemic needs and were carefully taken into account as part of Alan's efforts to integrate mathematics and English instruction.

Because Alan was bilingual, had previously taught school mathematics for several years in Brazil, and had taken two years of calculus as part of his bachelors' degree in chemistry, he was hired by the university as a mathematics tutor. The goal of these tutoring sessions was to prepare Cassie specifically for the mathematics portion of the SAT examination, and potentially increase her chances of obtaining a score high enough for her to gain acceptance into a US college. Further, he sought to create a content-language curriculum space that could enhance Cassie's experiences transitioning to US higher education.

It should be noted that, although Portuguese is the official language of both Brazil and Angola, there are noticeable differences in accent. These differences are particularly evident in more informal contexts such as casual exchanges due to variations in colloquialism (local slangs and expressions). For instance, in Angola, is common for speakers to use *Calão* (a creolised version of Portuguese that incorporates terms from local languages/dialects) in the course everyday conversations. However, overall, there are more similarities than differences between Brazilian and Angolan Portuguese. Further, the formal context in which the tutoring sessions took place helped reduce use of colloquialism and thus minimise miscommunication.

For the tutoring sessions, we were provided by the university with free copies of the textbook *SAT Subject Test: Math Level 1* (Wolf, 2008). While our curricular material was written exclusively in English, discussions were bilingual with Portuguese as the *dominant language* (Jiménez, Garcia, & Pearson, 1996), that is, the predominantly used linguistic code (Jakobson, 1960; Saville-Troike, 2003) when speaking. As such, our video-recordings offer a unique opportunity to examine discourse strategies deployed by bilingual speakers to communicate mathematics interliguistically with the instructional goal of helping a language learner perform well on her SAT examination. Examples of word problems discussed and solved during the mathematics tutoring sessions can be found in Table 1.

Problem	Transcript
1	Elaine has 50 coins, all nickels and dimes that have a total value of $3.40. How many of her coins are nickels? (A) 18; (B) 24; (C) 25; (D) 28; (E) 32
2	The length of each side of a square is 3 more than the length of each side of a regular pentagon. If the perimeters of the square and pentagon are equal, how long is each side of the pentagon? (A) 6; (B) 12; (C) 10; (D) 15; (E) 18
3	What is the measure of the angle formed by the minute and hour hands of a clock at 1:50? (A) 90 º; (B) 95 º; (C) 105 º; (D) 115 º; (E) 120º
4	A, B, and C are points on a line with B between A and C. Let M and N be the midpoints of AB and BC, respectively. If AB : BC = 3 : 1, what is MN : BC? (A) 2/3; (B) 1/1; (C) 3/2; (D) 2/1; (E) 3/1
5	If the difference between the measures of the two acute angles of a right triangle is 20º, what is the measure, in degrees, of the smaller one? (A) 20 º; (B) 35 º; (C) 45 º; (D) 55 º; (E) 80º
6	The lengths of the sides of a triangle are 3, 5, and x. How many possible values are there, if x must an integer? (A) 1; (B) 5; (C) 7; (D) 8; (E) Infinitely many

Table 1—Mathematics Word Problems Resolved during Tutoring Sessions.

Our study was designed to maximise the potential for new insights on the pedagogical integration of content and language. First, it focuses on a cognitive-oriented learning situation wherein a student engages with a content-based task whose main purpose is the acquisition of new knowledge (conceptual information) in a foreign language. The new language is learned through immersive participation in the specialised discourse of an expert community (mathematical discourse) rather than engagement in metalanguage (discussion of the new language itself). Secondly, our choice of this particular instructional site was partially informed by Cross' (2016) findings that content-language integrated lessons are often characterised by minimal integration (separation persists within lessons). Rather than oscillating back and forth between language and content, these lessons begin typically with a deliberate focus on vocabulary and grammar and then quickly shift to content (attention to language is limited to the beginning of the lesson and separation persists). Our focus on a bilingual mathematics instructor who spoke the student's mother tongue and used it support of the student's mathematics learning *throughout* allowed us to examine a more dynamic curriculum space with a higher degree of integration wherein the instructional focus oscillated constantly between language and content.

Our specific focus was also motivated by recent research showing that, although popular among mathematics instructors, word problems can be ineffective and problematic for language learners. Characterised by unique linguistic features (Gerofsky, 1996; Verschaffel, Greer, & de Corte, 2000), word problems have been shown to pose considerable comprehension difficulties for readers when written in a second language (not natively spoken at home). Common difficulties include confusing mathematical terminology, unclear syntactic structures, problem statements without an explicitly identified unknown at the beginning of the text, and premises with double negations (Mestre, 1988); and, comparative syntactic patterns (e.g., "as… as"), prepositional phrases (e.g., "divided into"), passive voice, connectors with implicit logical relations (e.g., "if and only if"), and linguistic constructions that lack a one-to-one correspondence between words and symbols (Spanos, Rhodes, Dale, & Crandall, 1988). Struggle with these features is often a direct consequence of nonnative readers' tendency to translate problem statements word-by-word, sequentially, and literally from left to right, a text-processing practice that often leads to cognitive difficulties such as the variable-reversal error (Mestre, 1988); that is, treating variables as labels.

Data Analysis

To examine the mathematics instructor's dynamic integration of content and language, we first transcribed the video-recordings of the tutoring sessions and then conducted a *microethnography*, that is, a study of language-mediated social interaction in minute detail through an up-close and exhaustive examination of how people use language and other forms of communication to realise the social work of their daily lives (Erickson, 1996). Gee and Green (1998) define it as "a research language that describes the ways members of a social group construct the structures of daily life" (p. 132). Like other forms of discourse analysis guided by an ethnographic perspective, its goal is to describe, explain, and understand local systems of meaning-making in depth. Unlike macroethnography, which typically takes the form of longitudinal studies conducted mainly through participant observation for longer periods of time, microethnography is concerned with the moment-by-moment, real-time unfolding of speech. Its focus is more particularistic (providing insight into localised patterns of communicative behavior in particular social contexts) than generalising (producing general theories of human communication).

Our microethnographic analysis was conducted at the level of *episodes* or *key cultural scenes* (Erickson, 1996); short stretches of naturally occurring discursive interactions (tutor-student dialogues) with variable numbers of utterances. Put differently, we conducted an in-depth and up-close analysis of the communicative strategies used by the mathematics tutor and the student to jointly comprehend or "navigate" mathematics word problems. This microethnographic analysis initially revealed a recurrent, three-stage pattern in the participants' collaborative problem-solving

approach: aloud translation, mathematical transcription, and mathematical calculation. More specifically, the tutor and tutee first translated each word problem aloud from English to Portuguese, then transcribed the word problem into an equation or diagram (drawn on a large whiteboard in front of the classroom), and lastly made their mathematical calculations both orally and in writing. Based on this initial finding, a written report was then produced which described and illustrated the main linguistic and epistemic strategies in each of the three different stages of their bilingual problem-solving approach.

Multiple strategies were adopted during data collection and analysis to enhance construct validity and reliability. First, as the mathematics tutor facilitating the discussion, Alan was actively involved with the phenomenon being researched. This ensured familiarity and an insider perspective on the bilingual context of instruction. Second, *peer debriefing sessions* with external researchers (not directly involved with the tutoring sessions) were frequently held during data analysis. During these sessions, key scenes were examined collectively, individual analyses shared, and interpretations discussed extensively. The emergent account was gradually adjusted to include any variation that surfaced from this reflective group interpretation. These sessions were frequently held to triangulate emerging interpretations of the data and guard against individual researcher biases (Robson, 2002). A systematic effort was made not to allow Alan's direct involvement in the tutoring sessions to become a source of researcher bias.

Findings

Our analysis revealed a dynamically and highly integrated curriculum space wherein instructional focus oscillated constantly between language and content through strategic deployment of a variety of linguistic and epistemic strategies. Linguistically-oriented strategies were characterised by an English-Portuguese focal alignment wherein the two bilingual speakers sought to jointly comprehend the textual contents of word problems by collaboratively negotiating a level of *linguistic equivalence* (House, 1997). Put simply, the tutor and tutee sought to identify equivalent words, expressions, or meanings in both English and Portuguese. This negotiation of interlinguistic equivalence typically took the form of "label quests" (Martin, 1999; Rollnick, 2000), that is, dialogic searches for equivalent labels in a second language.

By contrast, epistemically-oriented strategies had a Portuguese-Mathematics focal alignment, typically entailing intralinguistic changes such as simplified rewording within a single language (e.g., switching between academic and colloquial Portuguese). Rather than translating, the two speakers sought to make mathematical ideas clearer and clear, and explicit by restating them in their own words, without shifting to a second language. By rewording within the same language, the two speakers fostered shared comprehension of the mathematical content under deliberation. Central to this epistemic process was *conceptual convergence* (Oliveira, & Sadler, 2008; Roschelle, 1992) – participants' understandings of mathematical concepts gradually converged and became publically shared through discussion.

A particularly helpful example that can clarify the distinction made between a linguistic focus and an epistemic focus can found in a previous study in which we compared teacher-student interactions in two different science classrooms (Oliveira & Weinburgh, 2019). In the first classroom, Mary (a science teacher) and Darren (a student with limited English proficiency) discuss how the treads on a tire (referred as "bumps") serve as a source of friction. When Darren mispronounces the word "bumps", Mary follows up with a clarification request ("Are you saying bombs like B-O-M-B-S?"). In the second classroom, Carol (a science teacher) and Tony (a student with limited English proficiency) discuss the reintroduction of wild wolves at Yellowstone Park. When Tony claims to have recently seen a wolf, Carol replies "Maybe that was a coyote. I don't think there are any wolves in New York State". In both classrooms, students make a problematic word choice ("bombs" rather than "bumps," and "wolf" rather than "coyote") when trying to participate in the discussion. However, Mary and Carol's foci differ. Mary focuses on a perceived mispronunciation of the word "bump" as "bomb." Her main concern is clarifying the word itself rather than the conceptual idea behind this particu-

lar word choice (linguistic focus). By contrast, Carol seeks to make the orally articulated scientific idea more precise, clear, and explicit. Her focus is on the concepts of wolf and coyote as she highlights to students the need for careful consideration of geographical location when scientifically naming similar species of wild canine organisms. What is under consideration in this case is a faulty idea (a misconception) behind the student's word choice (epistemic focus).

Linguistic Strategies

Aloud Translation. Various translation techniques were used during aloud reading of word problems (the first stage of bilingual word-problem solving). For instance, while translating word problem 1 aloud, Alan and Cassie spent considerable time discussing the lexical meaning of "nickels" and "dimes," two words for which there are no one-to-one equivalent translations in Portuguese:

Alan: **Nickels and dimes**, que que isso? **Coins?**
Cassie: **Dimes**, umm, tem alguma coisa haver com diamantes?
Alan: Com o que?
Cassie: Diamantes? Com pedras?
Alan: Não, não é **diamonds**. Isso é **dimes**, não. **Nickels**, **dimes**
Cassie: Ah, damas? Não?
Alan: São as moedinhas. São as moedas, não é diamante não. **Coins**, essa palavra **coins** é moedas.
Cassie: Umhm, e **nickels**?
Alan: Você sabe que cada moeda nos Estados Unidos tem um nome, né? A amarelinha de um centavo é o **penny**, ou você não sabe?
Cassie: Tem amarelinha?
Alan: Nunca viu um **penny**?
Cassie: Ou é a vermelhina?
Alan: A vermelhinha, aquilo lá é um **penny**, certo?
Cassie: Ah, okay. Certo.
Alan: O **nickel** é o de cinco centavos, um pouco maior, o prateado, e dime é a menorzinha de dez centavos.
Cassie: E **quarter** é o de vinte e cinco?
Alan: É o de vinte e cinco. Ou seja cada moeda tem um nome aqui.
Alan: O que que é a estória da Elaine?
Cassie: A Elaine tem 50 **coins**, moedas, todas **nickels** e **dimes**, tem o total de valor de 3.40 dólares, quantos **coins** ela tem, quantos coins ela tem são nickels?
Alan: Umhm p

Alan: **Nickels and dimes**, what are these? **Coins?**
Cassie: **Dimes**, umm, does it have anything to do with diamonds?
Alan: With what?
Cassie: Diamonds? With stones?
Alan: No, it's not **diamonds**. This is **dimes**, no. **Nickels**, **dimes**
Cassie: Ah, damsels? No?
Alan: They are little coins. They are coins, not diamond. **Coins**, this word **coins** means coins.
Cassie: Umhm, and **nickels**?
Alan: You do know that each coin in the United States has a name, right? The little yellow, one-cent one is **penny**, or do you not know that?
Cassie: Is there a little yellow one?
Alan: Haven't you ever seen a **penny**?
Cassie: Isn't that the little red one?
Alan: The little red one, that is the **penny**, right?
Cassie: Ah, okay. Right.
Alan: **Nickel** is the five-cent one, a bit bigger, silver, and **dime** is the smaller ten-cent one.
Cassie: And **quarter** is the 25-cent one?
Alan: It is the 25-cent one. In other words, each coin has a different name here.
Alan: What is Elaine's story?
Cassie: Elaine has 50 **coins**, coins, all **nickels** and **dimes**, with the total value of 3.40 dolars, how many **coins** does she have, how many of her coins are nickels?
Alan: Umhm

Alan begins by posing the query "You do know that each coin in the United States has a name, right?" which serves not only as a request for the student to explicitly confirm her comprehension, but also as a source of critical cultural knowledge for the student to comprehend the book writer's intended meanings (i.e., it is both a prompt and a hint). This cultural hint is then elaborated upon by Alan who proceeds to disclose the mathematical meanings behind each term used for informal reference to particular coins in English, even terms not included in the problem such as "penny" and "quarter." Further, Alan resorts to *borrowing* (taking words from a first language when no equivalent exists in a second) (Fawcett, 2003; Oliveira, Colak, & Akerson, 2009; Quigley, Oliveira, Curry, & Buck, 2011; Vinay & Darbelnet, 1996), a translation technique usually deployed when *literal translation* (i.e., explicit word-for-word translation of unknown terms from English to Portuguese) is not possible.

Pointing. Aloud translation was often paralleled by pointing. During aloud translation of word problem 3, the expression "minute and hour hands of a clock" was initially mistranslated aloud as "minute and hour in the hands." This translation difficulty stems from the fact that Portuguese speakers use the expression "pointers of a clock" rather than "hands of a clock." In other words, literal translation produces a problematic Portuguese rendition:

Cassie: Qual é a medida do ângulo formado por minuto e horas, horas nas mãos? Há mãos em horas?
Alan: Lê tudo primeiro, vê se você entende?
Cassie: Ah, qual o minuto, qual a medida do ângulo formado pelo, por minutos e horas depois da mão? **Clock** é de pulso [aponta pro pulso]? Que marca uma hora e cinqüenta minutos?
Alan: Lembra que o seu objetivo não é traduzir palavra por palavra, é entender a situação, né? Então o que que tem a situação? Tá um relógio…
Cassie: Umhm, o relógio de pulso, não? O relógio esta a marcar, o de pulso…
Alan: Está marcando 1 e 50.
Cassie: E ele está a pedir, qual a medida do ângulo formado por minutos e horas? Marca depois da mão [bate no pulso]?
Alan: É como se fosse aquele relógio lá [aponta pro relógio] marcando 1 e 50, né? Ele quer o ângulo entre o que? Que que é essa mão aí que ele tá falando? Olha no relógio, o que que o relógio tem?
Cassie: [faz círculos com a mão]
Alan: Nun tem um ponteiro? É um ponteiro que ele tá falando.
Cassie: É um ponteiro? Mão é um ponteiro?
Alan: Umhm, de horas e minutos. É o tipo de coisa que você tem que deduzir. Você não pode traduzir literalmente que não dá certo. Assim, tem que imaginar o relógio com duas pontas né [usa mãos como ponteiros]? Num é um ângulo aquilo lá que ele tá fazendo entre a hora e o minuto?
Cassie: Ah, **yeah**! **Yeah**.

Cassie: What's the measure of the angle formed by the minute and hour in the hands? Are there hands in hours?
Alan: Read everything first, see if you can understand.
Cassie: Oh, what's the minute, what's the measure of the angle formed by, by minutes and hours after the hand? **Clock** is the wrist one [points to wrist]? That marks 1 hour and 50 minutes?
Alan: Remember that your objective is not to translate word by word, it is to understand the situation, ok? So, what's up with this situation? There's a clock…
Cassie: Umhm, a wrist one, no?
The clock is showing, the wrist one…
Alan: It's showing 1 and 50.
Cassie: And it's asking, what's the measure of the angle formed by minutes and hours? Marking after the hand [taps her own wrist]?
Alan: It is as if it were that clock over there [points to clock] showing one and fifty, okay? It wants the angle between what? What is this hand that it is talking about? Look at the clock, what does it
have?
Cassie: [makes "air circles" with her hand]
Alan: Doesn't it have a pointer? A pointer is what he is talking about.
Cassie: It's a pointer? Hand is a pointer?
Alan: Umhm, of hours and minutes. It is the type of thing that you need to deduce. You cannot translate literally because it does not work out. So, you have to imagine a clock with two pointing ends [uses both hands as pointers], ok? Isn't it an angle what it is making over there between the hour and the minute?
Cassie: Oh, **yeah**! **Yeah**.

Cassie immediately follows her inaccurate translation with the query "are there hands in hours?" However, rather than simply providing Cassie with a direct translation of the non-literal meaning of the word "hands," Alan encourages her to independently deduce its meaning from further examination of the text (focusing on the larger text rather than isolated words) and from direct examination of a real clock in the immediate classroom context. Alan uses his index finger to designate the contextual location of the referent of unknown words or expressions. His pointing serves as an *illustrator*, that is, a type of gesture designed to physically illustrate the meaning of accompanying speech (Burgoon, Buller, & Woodall, 1989). By withholding and delaying his provision of a direct translation ("A pointer is what he is talking about"), Alan encourages Cassie to actively and independently translate the unknown word based on the available information and context rather than being passively told its meaning in the second language.

The above instance of translation was performed multimodally through cooperative co-deployment of *deictic gestures* (McNeill, 2005) and *equivalence*, an adaptive translation technique commonly deployed by translators when literal translation is not possible (Ni, 2009; Vinay & Darbelnet, 1996). Instead of a one-to-one correspondence among literal terms, Alan resorted to an equivalent concept (replacement of a different term with a similar lexical meaning) in the second culture, hence changing the original semantics and producing an adapted translation ("clock hands" is adapted to "clock pointers").

Epistemic Strategies

Concept Rewording. By far, the epistemic strategy most commonly used by the two bilingual speakers was concept rewording. For instance, after translating word problem 4 aloud, Alan (standing by the board with a marker) and Cassie (speaking from her seat) engaged in a discussion about how to express its textual information mathematically (in symbolically terms):

Cassie: Deixa M e N ser o ponto médio [reler questão]
Alan: Então M e N respectivamente, então o M é o ponto médio de quem?
Cassie: De AB e BC.
Alan: Não, cuidado. Ele tá falando respectivamente, então um coisa está pra primeira coisa, e outra coisa está pra segunda coisa, tá? [risos]
Cassie: Ah!
Alan: Eu to falando meio vago mas é porque é o M e o N, um é o ponto médio de um, e o N é o ponto médio do outro.
Cassie: Ah, respectivamente!
Alan: Respectivamente, é o primeiro pro primeiro, e o segundo pro segundo.
Cassie: M é sobre AB.

Cassie: Let M and N be the midpoint [rereads statement].
Alan: So, M and N respectively, so M is whose midpoint?
Cassie: AB's and BC's.
Alan: No, you need to be careful. It is saying respectively, so one thing is to the first thing, and another thing is to a second thing, ok? [laughs]
Cassie: Oh!
Alan: I'm talking a bit vague but that's because it's M and N, one is the midpoint of the first one, and N is the midpoint of the other.
Cassie: Oh, respectively!
Alan: Respectively, it is first to first and second to second.
Cassie: M is in between AB.

When Alan poses the query "so M is whose midpoint?" Cassie initially replies "AB's and BC's," a mathematically inaccurate interpretation. Rather than correcting Cassie directly (, Alan immediately follows Cassie's response with commentary in which the word "respectively" is rearticulated in considerably simpler forms: "one thing is to the first thing, and another thing is to a second thing," "one is the midpoint of the first one, and N is the midpoint of the other" and "first to first and second to second." By re-wording in everyday Portuguese terms, Alan seeks to ensure shared understanding of the mathematical concept of "respectively."

Another instance of rewording took place during mathematical calculation of the angles in a geometrical diagram. Alan noticed that Cassie attempted to do her mathematical calculation through visual inspection despite the note "figure not drawn to scale":

Alan: Tá vendo a observação abaixo da figura?	Alan: Do you see the note below the figure?
Cassie: Não está desenhado na escala? [lendo alto]	Cassie: It's not drawn to scale? [translates aloud]
Alan: Exatamente, é isso que tá falando mesmo.	Alan: Exactly, that's indeed what it's being stated.
Cassie: Então está a dizer que?	Cassie: So, what it's saying is that?
Alan: Voce não pode concluir nada com relacão a figura, né? Por que é só um esqueminha, nun tá medido.	Alan: You cannot conclude anything with regard to the figure right? Because it's just a little scheme, it's not measured.
Cassie: Então, quer dizer que o ângulo não está a tocar na reta?	Cassie: So, it means that the angle is not touching the line?
Alan: Tá, tá tocando, só que você não pode falar quem é o maior deles só olhando no tamanho.	Alan: It is, it is touching, but you cannot say which is the bigger one just by looking at the size.

After Cassie's aloud translation of the mathematical expression "not drawn to scale" (a literal translation), Alan restates the mathematical register in simpler and more comprehensible lay terms: "it's just a little scheme, it's not measured" and "it's not measured, they did not measure it carefully." These simplified rewordings in everyday parlance served to promote collective comprehension of the mathematical idea being expressed, not its linguistic form (verbal representation in a given language).

Metaphoric Gestures. Concept rewording was frequently combined with metaphoric hand gestures (epistemic gesticulation). One instance of metaphoric gesticulation took place during mathematical calculation of word problem 6. While solving this problem, Alan and Cassie engaged in a short discussion about the triangle inequality theorem using a generic triangle drawn on the board (Figure 1).

As can be seen on Figure 1, Alan combines verbal analogy ("think of it as walking from point 1 to point 2") with iconic gesticulation (gestural representation in terms of spatiotemporal correspondence). Alan's hand movements combine two different types of illustrators: *pictographs* (static construction of shape through gestures with high degree of similarity in appearance to abstract concepts or systems) and *kinetographs* (dynamic gestural depictions of unfolding human action) (Allen, 2000). Not only is the triangle inequality theorem communicated in simple lay terms, but it is also made visually accessible through metaphorical spatialization, promoting collective mathematical comprehension.

Figure 1—Epistemic accommodation of the triangle inequality theorem.

Another instance of metaphoric gesticulation occurred during mathematical transcription of word problem 2. Alan and Cassie deliberated why the mathematical statement "the length of each side of a square (represented by the letter L) is 3 more than the length of each side of a regular pentagon (represented by the letter P)" should be transcribed as "L = P + 3." As can be seen in Figure 2, Alan uses his arms and hands to iconically express mathematical equality in terms of vertical spatial relations (up/down). These sort of iconic *gestures* share some sort of spatiotemporal correspondence (e.g., trajectory, direction) with abstract ideas or processes being representationally communicated (McNeill, 2005).

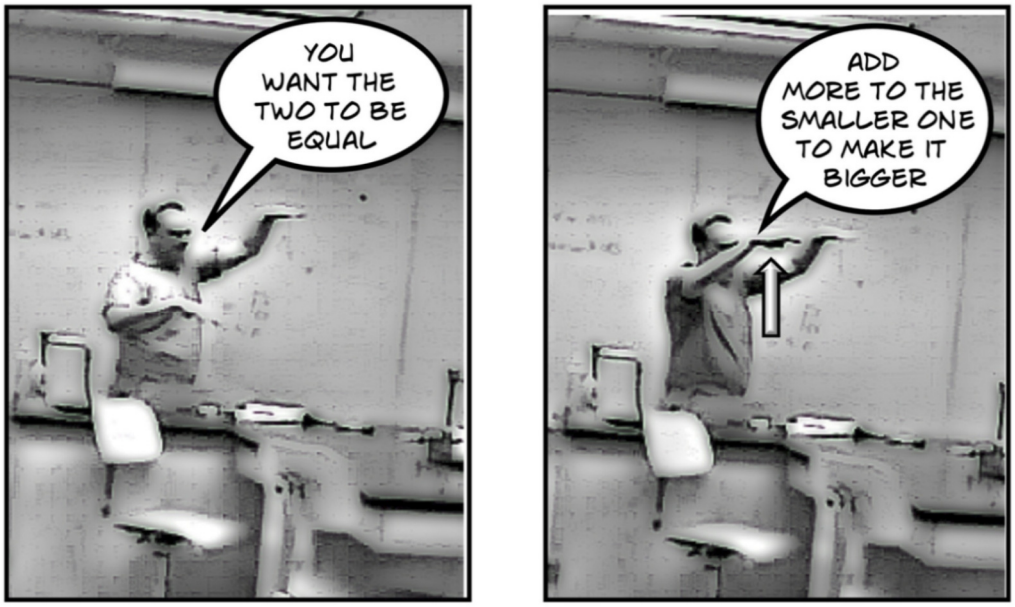

Figure 2—Epistemic accommodation of algebraic equality.

Through his arm movements Alan spatialises equality as a state wherein both of his hands (each representing one side of a mathematical equation) are at equal heights. Achieving this state requires either lowering the higher hand (i.e., subtracting three units from one side of the square) or raising the lower hand (i.e., adding three units to one side of the pentagon). While continuing to make iconic movements with his two arms, Alan elaborated upon this initial articulation of the notion of algebraic equality by uttering the following:

> Because, if you add 3 to L which is already bigger, what will happen? [raises his higher hand further] It won't be equal, right? Another way to say it is to take from L, L minus 3 equals P [lowers his higher hand] so, instead of you adding to the smaller one, you take from the bigger one, does it make sense? You are trying to get the two [L and P] to be equal [waving his hands at same height] you either add more to the smaller one to make it bigger or you take from the bigger one to make it smaller, the thing is to make them equal.

Alan's iconic gesturing provides an image of mathematical equality, a nonverbal representation. Such gestural practice is consistent with the notion of *metaphorical structuring* (Boroditsky, 2000) wherein an abstract domain is representationally mapped onto the more concrete and experiential domain of space. In the case above, Alan metaphorically structures a mathematical equation (an abstract experience involving two intangible entities) as the lowering or raising of physical objects (a concrete experience involving two tangible entities that could be more easily visualised). His iconic gestures constitute spatialised metaphors (Whorf, 1956), that is, metaphorical hand movements in imaginary space commonly used as spatial representations for thinking about abstract and elusive ideas (Boroditsky & Ramscar, 2002; Gentner, Imai, & Boroditsky, 2002) rather than selecting location in physical space. More importantly, it is designed to make the mathematical concept of equality (as opposed to the word itself) more accessible, thus promoting collective conceptual reception.

Discussion

As described above, to effectively accommodate the language learner in the discussion of word problems, the instructor created a dynamically and highly integrated curriculum space wherein instructional focus oscillated constantly between language and content through strategic deployment of a variety of linguistic and epistemic strategies. This space shared little resemblance with the more traditional content-

language integrated lessons wherein vocabulary/grammar instruction at the beginning is followed by content-focused activities that for the most part overlook language. Instead, the tutor maintained a "constant language/content integrated focus" (Cross, 2016) by skillfully making use of oral translation to support the language learner's participation in bilingual word problem solving.

The above finding speaks directly to Cross' (2016) argument that, in integrated curriculum spaces, there usually continues to exist a separation of language and content *within* lessons. Rather than truly co-existing within a single curriculum space, content and language usually remain separate, being addressed in distinct instructional phases. As the reported findings show, a higher degree of integration that goes beyond separate instructional phases is possible. In this highly integrated curriculum space, an instructor dynamically provides learners with linguistic and epistemic scaffolds that enable simultaneous acquisition of both mathematical knowledge and a foreign language. Such a dual acquisition process involves both lexicalization and conceptualization of classroom experiences. Under the instructor's guidance, the student lexicalises classroom experiences (i.e., put them into words) by reading and translating mathematical texts aloud and describing mathematics objects and operations in multiple languages. New terms are acquired through objective reference and concrete language use such as naming, picking out, labelling, and pointing to objects/texts in the immediate classroom context. Simultaneously, the student conceptualises classroom experiences (i.e., articulates ideas and thoughts) by performing cognitive acts such as analyzing experiences, explaining observations, and inferring interpretations (i.e., making sense of experiences had). Rather than happening in separate phases of the lesson, lexicalization and conceptualization are parallel, being characterised by high degree of translanguaging (Garcia & Wei, 2014) – multilingual meaning-making. Students learn to lexicalise as they learn to conceptualise the world in *both* languages.

Another noteworthy finding was that the mathematics tutor recurrently deployed and combined a variety of language-focused strategies that included literal translation as well as adaptive forms of translation such as borrowing (e.g., the English words "nickels" and "dimes"), modulation (changes in word sequence), and equivalence (word replacements). Because some English words and grammatical structures lacked a precise equivalent in the second language, adaptations such as replacements, deletions, additions, and structural changes were repeatedly needed. Rather than being simply translated word-by-word, the mathematics curriculum underwent constant adaption and adjustment due to the highly metaphorical nature of the mathematics register (Pimm, 1984).

It was also observed that within the integrated curriculum space, the student participated actively and creatively and shared translating/translanguaging responsibility with the instructor. Far from being a passive receiver of translation from a teacher who simply translated for the language learners (like a "talking dictionary"), the student shared "collective cognitive responsibility" (Scardamalia, 2002) over the translation and solving of mathematics word problems. Both instructor and student positioned themselves as "transcreators" (Munday, 2009; Perteghella & Loffredo, 2006) of word problems who collaboratively and creatively transcended interlinguistic and intercultural borders.

Another important finding was that iconic gestures served an important epistemic function in the integrated curriculum space, being used by the tutor for metaphorically structuring abstract mathematical concepts of algebraic equality and the triangle inequality theorem in nonliteral space. This finding is consistent with existing research which shows that professional mathematicians use iconic gesturing to make metaphorical mathematical meanings more accessible and comprehensible to others (McNeill, 2005) and that non-native speakers of a common language can effectively employ gesticulation to overcome communicative difficulties in the course of multilingual exchanges (Gullberg, 2011).

The above finding underscores the epistemic value of gestural scaffolds to content-language integration. The mathematics tutor's gesticulation allowed for *transmediation* (Short, 2004), that is, transfer of meanings across representational systems or media (from written text or speech to physical action). By re

-articulating complex and abstract concepts such as mathematical equality in a more concrete spatialised form, the instructor strategically helped the student decode and interpret grammatically encoded conceptual knowledge. While translation was used in support of lexicalization, gesticulation was deployed mainly to foster conceptualization. By strategically interspersing translation and gesticulation, the tutor was able to maintain a "constant language/content integrated focus", hence giving rise to an integrated curriculum space wherein instruction continuously oscillated back and forth between language and content.

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