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MEDIATING EFFECT OF SELF-EFFICACY ON THE RELATIONSHIP BETWEEN INSTRUCTION AND STUDENTS' MATHEMATICAL REASONING

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Abstract

Literature is well-stocked with studies confirming that an instructional approach, self-efficacy, and mathematical reasoning skills are critical for enhancing students' conceptual understanding and achievement in mathematics. However, there has been little emphasis on establishing whether being able to reason mathematically depends only on the instructional approach or students' self-efficacy beliefs about mathematics also play a hidden role. A quasi-experimental study involving 301 grade 11 students from six public secondary schools in one district was carried out to investigate the mediating effect of self-efficacy on the relationship between instruction and students' mathematical reasoning. Participants of the study were selected using the cluster random sampling method. Data were collected before and after the intervention via a mathematical reasoning test and a mathematics self-efficacy beliefs questionnaire. A Parallel Multiple Mediator Model in SPSS using the PROCESS custom dialogue version 3.4 was employed for data analysis. Findings suggest that mathematics self-efficacy and task-specific self-efficacy beliefs collectively and significantly mediate the effect of the instructional approach on students' mathematical reasoning. The Student Teams-Achievement Division (STAD) was found to be an effective approach for enhancing students' mathematical reasoning alongside self-efficacy beliefs. These findings provide evidence on the need to select an instructional approach that does not only focus on developing students' cognitive abilities such as mathematical reasoning but also fosters students' affective attributes such as maths self-efficacy beliefs.

Keywords: instructional approach, mathematical reasoning, self-efficacy beliefs, STAD

Abstrak

Sejumlah penelitian menegaskan bahwa pendekatan instruksional, *self-efficacy*, dan keterampilan penalaran matematika sangat penting untuk meningkatkan pemahaman konseptual dan prestasi siswa dalam matematika. Namun, terdapat sedikit penekanan pada penetapan apakah mampu bernalar secara matematis hanya bergantung pada pendekatan instruksional atau keyakinan *self-efficacy* siswa tentang matematika juga memainkan peran tersendiri. Sebuah studi eksperimen semu yang melibatkan 301 siswa kelas 11 dari enam sekolah menengah umum di satu distrik dilakukan untuk menyelidiki efek mediasi dari self-efficacy pada hubungan antara instruksi dan penalaran matematis siswa. Sampel penelitian dipilih dengan menggunakan metode cluster random sampling. Data dikumpulkan sebelum dan sesudah intervensi melalui tes penalaran matematis dan angket keyakinan *self-efficacy* matematika. Sebuah model *Mediator Multiple Parallel* di SPSS yang menggunakan *PROCESS custom dialogue* versi 3.4 digunakan untuk menganalisis data. Hasil penelitian menunjukkan bahwa *self-efficacy* matematika dan keyakinan *self-efficacy* tugas khusus secara kolektif dan signifikan memediasi efek pendekatan instruksional pada penalaran matematika siswa. Model *Student Teams-Achievement Division* (STAD) ditemukan menjadi pendekatan yang efektif untuk meningkatkan penalaran matematis siswa di samping keyakinan *self-efficacy*. Hasil ini memberikan bukti tentang kebutuhan untuk memilih pendekatan instruksional yang tidak hanya fokus pada pengembangan kemampuan kognitif siswa seperti penalaran matematika tetapi juga menumbuhkan atribut afektif siswa seperti keyakinan *self-efficacy* matematis.

Kata kunci: pendekatan instruksional, penalaran matematis, keyakinan *self-efficacy*, STAD

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Enhanced mathematical reasoning does not only bring about students' improved performance in mathematics but also leads to increased application of mathematical knowledge to real-world experiences. There is a need to put more emphasis on teaching practices that "enable students to master a concept, while at the same time learning the mathematical reasoning behind it" (Ross, 1998, p.253) as opposed to that which is crammed or learned by memorisation of isolated mathematical facts. This is why many scholars from different parts of the world (e.g. Jäder et al., 2016; Jeannotte & Kieran, 2017; Mata-Pereira & da Ponte, 2017; Saleh et al., 2018) have stressed a need for teachers to turn their attention on developing mathematical reasoning among learners of school mathematics. Despite such calls, developing students' mathematical reasoning has not been an easy task for most teachers of school mathematics across the world.

A considerable body of mathematics education literature has demonstrated that both secondary school students and university students exhibit limited mathematical reasoning skills. While several reasons for this quality landscape in some Zambian school mathematics classrooms are known (Mukuka et al., 2019; Mukuka, Balimuttajjo, et al., 2020b) other contributing factors are not fully explored and understood. A commonly cited reason for students' inadequate mathematical reasoning in the existing literature is related to the instructional and assessment approaches that teachers adopt in their mathematics lessons. For instance, Tejeda and Gallardo (2017) observed that the teaching and assessment methodologies that are being adopted by most teachers are not moving quite fast enough to bring about the desired teaching practices. Teachers' continuous hold to traditional instructional methods of teaching and giving of routine tasks when assessing learners has made it difficult for them to develop the much-desired mathematical competencies (such as conjecturing, justifying, and mathematising) among students.

Besides the instructional and assessment approaches, it has been observed that most of the studies that have embarked on developing students' cognitive abilities like mathematical reasoning have not incorporated other attributes such as affective and psychomotor skills. Based on the existing literature (e.g. Czocher et al., 2019; Grigg et al., 2018; Kohen et al., 2019; Öztürk et al., 2019; Pajares & Kranzler, 1995), it has been noted that students' beliefs or judgements about their ability to perform mathematical tasks is critical for their academic success. Research has also shown that self-efficacy beliefs do affect students' academic achievement (Ozgen & Bindak, 2011), and choices of their future careers and job selection (Waller, 2006). This shows that self-efficacy is one of the affective learning attributes that deserve consideration in a bid to attain success in mathematics education.

In trying to build students' mathematical reasoning and self-efficacy beliefs, there is a need for teachers to provide students with opportunities to be actively engaged in doing mathematics. However, developing such competencies among students of varying aptitudes requires appropriate instructional and assessment approaches. Active learning/learner-centered approaches such as cooperative learning appear to offer more support to this kind of environment than the traditional teacher-centered methodologies. Among other models of cooperative learning that have been extensively researched and discussed in previous studies (e.g. Butera & Buchs, 2019; Ghaith, 2018; Slavin, 2015), Student Teams-Achievement Division (STAD) is more likely to strengthen students' mathematical reasoning and self-efficacy beliefs.

According to Slavin (2015), STAD is a model of cooperative learning in which students work in small heterogeneous groups to learn the rules and procedures as they listen to the whole class presentation by the teacher, complete group work/worksheets, take an exercise/quiz/test without help from others, and then get feedback on their group achievement. In line with the social constructivist approach, the STAD model of cooperative learning is anchored on the premise that learners construct new knowledge through interactions with others. Unlike the behaviorist/traditional classroom in which a teacher "tells, shows, models, demonstrates, and/or teaches the skill to be learned" (Baumann, 1988, p.714), social constructivist theorists argue that knowledge develops as one engages in dialogue with others (Palincsar, 1998).

There is a multitude of evidence confirming that the STAD model of cooperative learning is not only ideal for improving students' conceptual understanding and achievement in mathematics (Ardiyani et al., 2018; Kramarski & Mevarech, 2003; Mukuka, Balimuttajjo, et al., 2020a) but also for improving students' social interactions, motivation, self-confidence and attitude towards mathematics (Hossain & Ahmad, 2013; Slavin, 1987; 2015). Additionally, the STAD learning model has been found useful at all levels of school mathematics since it is anchored on the premise that all students, regardless of their mathematical abilities, can contribute equally to the group success. Its emphasis on group success motivates group members to teach others or learn from others to improve their performance as a group. In the long run, those seeking clarifications from others and those who try to explain concepts to their groupmates all tend to increase their conceptual understanding.

Although the STAD learning model has been found beneficial to students' social interactions and learning outcomes, some studies have pointed out that it is rarely used in mathematics classrooms due to several challenges associated with its implementation (see Buchs et al., 2017; Gillies & Boyle, 2010; Mukuka et al., 2019). For instance, Baines et al (2015) pointed out that students may sit together in a group but fail to work as a group. Nurlaily et al. (2019) also found that teachers had encountered a lot of difficulties in dividing time for guiding group discussions as most of the students tended to wait for a teacher's explanation even when the given tasks were meant to be completed through group work.

Based on the background highlighted above, this research investigated the effects of the Student Teams-Achievement Division (STAD) on students' mathematical reasoning and self-efficacy beliefs. The question of whether students' self-efficacy beliefs could explain why an instructional approach significantly affects/improves their mathematical reasoning has also been addressed in this paper.

METHOD

Research Design

This study employed a quantitative research approach, utilising a quasi-experimental research design. Among the different types of quasi-experiments, this study employed a "non-equivalent (pretest and posttest) control-group design" (Creswell, 2014, p.220). This research design was deemed appropriate for the study because it involved intact classes of grade 11 students from different schools in one district. Fraenkel, Wallen, and Hyun (2006) also indicated that this design is often preferred in educational research especially when the experimental and comparison groups constitute the naturally assembled groups or intact classes. Therefore, a quasi-experimental research design was deemed appropriate for this study, especially that it was meant to fit into naturally/already existing school structures/schedules.

Research Participants

Six public secondary schools from Ndola District in the Copperbelt province of Zambia were available for the study. These schools were selected from a list of 20 public secondary schools using cluster random sampling. The three clusters were based on school average performance of the 2018 national examinations – two low performing schools (below 50% pass rate), two moderate performing schools (from 50% to 75% pass rate), and two high performing schools (Above 75% pass rate). Of the two schools from each cluster, one was randomly allocated to the experimental group while the other was allocated to the control group. After random allocation of the schools to the two groups (experimental and control), one grade 11 class was randomly chosen from each school and all the students from each selected classroom were included in the sample. This means that six teachers of mathematics were also included in the study sample.

Grade 11 students were considered suitable for the study because they were not only a non-examination class but had also learned secondary school mathematics for over three years. It was further noted that these students had already been exposed to several mathematics topics that were a pre-requisite for quadratic equations and quadratic functions. After carrying out all the procedures explained above, 301 grade 11 students of ages ranging from 14 to 20 (M =16.3, SD =1.00) were selected including the six mathematics teachers for the selected classrooms. Table 1 gives a summary of the other sample characteristics.

All the six teachers (2 females and 4 males) involved in the study were holders of a Bachelor's degree in education with a major in mathematics. Each group (experimental and control) consisted of 1 female and 2 male teachers. The average years of teaching experience in the experimental group was 11 while that of the control group was 12.3. This implies that the average teaching experience and qualifications of teachers in the experimental group were not significantly different from that of the control group. This gave the impression that teacher participants were almost at the same level in terms of teaching experience and professional qualifications. Furthermore, it had already been established that students' prior reasoning ability for both the control group and the experimental group were equivalent. The Chi-square test performed during the baseline study (Mukuka, Balimuttajjo, et al., 2020b) further established that students' mathematical reasoning abilities (conjecturing, justification, and validation of algebraic statements/arguments) were independent of the school average performance.

Table 1. Sample Characteristics

			Group		Total
			Control	***Experimental***	
School Mean Performance	Low	*Count*	71	26	97
		% within group	47.3%	17.2%	32.2%
	Moderate	*Count*	43	80	123
		% within group	28.7%	53.0%	40.9%
	High	*Count*	36	45	81
		% within group	24.0%	29.8%	26.9%
Total		*Count*	150	151	301
		% within group	100.0%	100.0%	100.0%

Note. Percentages are calculated within each group (control and experimental).

Experimental Conditions

Two groups from the sample were formed and exposed to two different instructional approaches. Students who were assigned to the control group were taught using an expository instructional approach while those assigned to the experimental group were taught using the STAD model of cooperative learning. All the 6 classes involved were taught the same topics (quadratic equations and quadratic functions) for six weeks using the same teaching and learning materials. These topics were selected for different reasons. First, both the quadratic equations and quadratic functions have been quite challenging for most secondary school students not only in Zambia (Examinations Council of Zambia, 2012; 2016; 2018; Mukuka, Balimuttajjo, et al., 2020b) but also in other contexts (Hong & Choi, 2014; Santia et al., 2019; Zaslavsky, 1997). Second, these two topics are considered necessary and as pre-requisites for various advanced courses in mathematics and other physical sciences at tertiary levels of education. Third, graph sketching and interpretation on quadratic functions is central to the study of mathematics and other physical sciences. Besides that, quadratic equations and quadratic functions were among the topics that were to be taught in all the selected schools during the time of data collection for the present study.

In all the selected classrooms, mathematics was taught at least 3 times a week translating into six to seven 40-minute class periods (3 double periods or 3 double and 1 single period) as prescribed by the Zambian curriculum for secondary school mathematics (Curriculum Development Centre, 2013). While the experimental group was taught using a STAD model of cooperative learning, the control group was taught using expository methods of teaching. The expository method of teaching was characterised by lectures, chalk and talk, and the question and answer techniques. The commonly observed practice was having students sit in rows and columns while listening to a teacher's lecture after which a class exercise was given followed by the evaluation of students' work. In cases where time was not enough to give and mark a classwork/activity, homework was given and students' work was to be checked before the presentation of the next lesson.

In the STAD learning mode (experimental group), every lesson began with a teacher's short presentation of the new material to the entire class. These short presentations were usually aided by the use of a projector (for PowerPoint presentations), flip charts, or chalkboard. It was during this stage a mathematics software (Graphmatica) was utilised to enable students to observe, within a short time, how the shapes of quadratic functions ($f(x) = ax^2 + bx + c$) were changing with changes in the coefficient of the x^2 term.

After the teacher's short presentation, students were split into small groups of 4 or 6 depending on the class size. These groups were heterogeneous in terms of students' mathematical ability and gender. The purpose of constituting heterogeneous groups was to enable the less-able students to seek clarifications from the more knowledgeable students in a familiar language. Each student was allowed to justify their ideas and reasoning to peers until consensus among the group members was reached. While working cooperatively in heterogeneous groups, students were faced with contextualized problems and hands-on activities. Group activities were characterised by working with multiple representations (concrete, visual, and abstract/symbolic) of particular algebraic concepts on quadratic equations and quadratic functions. In line with the socio-constructivist perspectives, the STAD model of cooperative learning was meant to enable students to construct new knowledge based on existing ideas with support from peer and social interactions.

After group discussions, students were given tasks in terms of class exercises homework, or quizzes. Quizzes were given fortnightly while class exercises and/or home works were given for every lesson. During quizzes, students were not allowed to help one another because a student's quiz score was to be compared with his/her previous score to determine whether there was an improvement or not. Based on the "improvement score conversion table" and the "test score sheet" proposed by (Li & Lam, 2013, p.16), members of a particular team earned or lost points for their respective groups. After summing up points for each group, a group with the highest average or groups that attained the prescribed and desired performance level earned an award or recognition.

Instruments

The data reported in this paper were collected using two research instruments namely the Mathematics Self-efficacy Beliefs Questionnaire (MSBQ) and the Mathematical Reasoning Test (MRT) on quadratic equations and quadratic functions. Upon completion of the MSBQ, students were requested to take a mathematical reasoning test whose some of the items were also included in the MSBQ. This was done to determine whether students' confidence in answering specific tasks was a true reflection of reality.

Mathematics Self-Efficacy Beliefs Questionnaire

Mathematics Self-efficacy Beliefs Questionnaire (MSBQ) items had been framed based on a "Mathematics Self-efficacy and Anxiety Beliefs Questionnaire" developed by May (2009, p. 70). Some questionnaire items were modified to suit the purpose and setting of the present study. Additionally, the questionnaire was subjected to a pilot study for validation and reliability analysis.

Sixty-one grade 11 students from one secondary school in Kitwe District, participated in the pilot study. Of this number, 26 were male, 30 were female and 5 did not indicate their gender. This sample was considered sufficient and representative of the intended sample especially that the actual study had to involve grade 11 students from a nearby district of Ndola, the provincial capital. The *Cronbach's Alpha* and the *Exploratory Factor Analysis* were used for reliability analysis and the establishment of construct validity. Besides demographic information, the questionnaire comprised two other sections: mathematics self-efficacy and anxiety beliefs, and task-specific mathematics self-efficacy.

Self-efficacy and anxiety beliefs were measured through general self-assessment of a student's ability to understand the subject following a 5-point Likert scale from 0(never) to 4(always). Students were requested to indicate how often they felt confident to succeed in various aspects of mathematical learning. On the other hand, task-specific mathematics self-efficacy was assessed through student ratings of their confidence in answering each of the 11 mathematical reasoning questions on quadratic equations and functions using a 5-point Likert scale from 0(not confident at all) to 4 (very confident). Further details about the development and validation of the MRT items are given in the next sub-section.

An exploratory factor analysis (EFA) was performed using the principal component analysis (PCA) extraction method. This was done to determine the number of categories (factors) that could be extracted from the 26 items of the initial MSBQ excluding task-specific self-efficacy items. To optimize the number of factors, the default number in SPSS given by Kaiser's criterion (eigenvalue > 1) was used and 7 components were extracted. However, only two components had at least 3 items with factor loadings greater than 0.4. Based on what is recommended in the literature (Child, 2006; Field, 2013; Guadagnoli & Velicer, 1988), two factors were fixed and PCA was re-run with all the 26 items since none of them had a communality score of less than 0.2

After a re-run, two items were removed based on having communality scores of less than 0.2 and factor loadings less than 0.4. The procedure was repeated for the remaining 24 items, 11 of which loaded more on component 1 while 12 items loaded more on component 2. Only one item had a factor loading of less than 0.4 on both components and so it was removed. A qualitative analysis of the items was done and it was found that all the 11 items that loaded on component 1 were related to anxiety beliefs while the other 12 items that loaded on component 2 were related to self-confidence beliefs. The 11 items that loaded on component 1 were retained under the sub-category named 'Student's anxiety beliefs in mathematics' and the other 12 items that loaded on component 2 were retained under the sub-category named 'Student's self-confidence beliefs in mathematics'.

The same procedure was followed in the third section of the questionnaire (Task-specific self-efficacy). A test for multicollinearity based on the inter-item correlation matrix was also carried out. Since no pair of the retained items had a correlation coefficient greater than 0.8, all the items were considered independent and none had violated the multicollinearity assumption.

After ensuring that the multicollinearity assumption was not violated for all the retained items under each sub-category, *Cronbach's alpha (α)* was then generated. As it is advisable to assign alpha scores on specific scales that have been verified to be unidimensional (Gardner, 1995), the following alpha values were generated for each sub-category of the retained questionnaire items:

1. Student's self-confidence beliefs in mathematics (α = 0.88)
2. Student's anxiety beliefs in mathematics (α = 0.80)
3. Task-specific self-efficacy (α = 0.86)

These results reflect that the degree to which different categories of questionnaire items would produce similar measures was reasonable especially that each of the alpha values was above the recommended threshold of 0.70. Cronbach's alpha was computed for each sub-category as it is considered more valuable to measure the internal consistency of questionnaire items for a single construct rather than measuring different constructs at once (Adams & Wieman, 2010; Taber, 2018). For data analysis purposes, students' anxiety beliefs were reversed at the data entry stage. This means that the first two sub-scales were merged to form one category referred herein as maths self-efficacy. Thereafter, an average score for each respondent had been generated for both maths self-efficacy and task-specific self-efficacy beliefs.

Mathematical Reasoning Test (MRT)

Before administration to the intended research participants, researcher-formulated mathematical reasoning test items on quadratic equations and quadratic functions were assessed and validated. Assessment and validation were done to determine the suitability of the test items in each mathematical reasoning dimension (conjecturing, justifying, and mathematising) in terms of sufficiency, clarity, coherence, and relevance. Based on expert ratings and comments, all the MRT items were refined and later administered to the intended respondents. Further details about the MRT item validation and allocation to the three mathematical reasoning dimensions are available in the supplementary files associated with a published paper (Mukuka, Mutarutinya, et al., 2020) at https://doi.org/10.1016/j.dib.2020.105546.

Procedures

Before the implementation of the intervention, a survey was conducted to determine the prevailing mathematics teaching practices in all the selected schools. Thereafter, randomly selected classes were randomly assigned to the two groups whose conditions have already been highlighted. About 6 weeks from the time a survey was conducted, all the students from the selected classes took a pretest after which teachers assigned to the experimental group were subjected to a 3-day training workshop on effective implementation of STAD. Teachers in the control group were encouraged to continue using expository methods as earlier observed during a baseline survey (*see* Mukuka et al., 2019; Mukuka, Balimuttajjo, et al., 2020b).

Teachers in both conditions were requested to focus more on enhancing students' mathematical reasoning skills. Similar examples, test items, and other learning materials that were given to teachers in the experimental group were also shared with teachers in the control group. The two topics (quadratic equations and quadratic functions) were taught for 6 weeks after which a posttest was administered to all the students who took part in the study. The role of a researcher and the two research assistants during the intervention was to observe lessons and provide technical assistance to teachers in the experimental group, and to ensure that teachers in the control group adhered to the principles of expository methods of teaching.

Data Analysis

The primary purpose of this analysis was to determine whether the instructional approach can predict students' mathematical reasoning regardless of their mathematics self-efficacy beliefs. Before running a mediation analysis, correlations among the three variables were computed for each group (control and experimental). This was done to establish the extent to which the three variables were related to one another for both the pretest and posttest. Thereafter, a "Parallel Multiple Mediator Model" developed by Hayes (2018, p. 149) was used for the analysis of the mediating role of self-efficacy on the relationship between an instructional approach and students' mathematical reasoning. This model was employed using the PROCESS custom dialogue version 3.4 in SPSS version 20. The PROCESS command is not part of SPSS but a custom dialogue that can be installed in SPSS following the download and installation guidelines is available on Haye's website at http://www.afhayes.com/spss-sasand-mplus-macros-and-code.html.

An independent variable, X (group) was modeled as influencing the dependent variable, Y (students' mathematical reasoning) both directly and indirectly through two mediators, M_1 (Maths self-efficacy beliefs after the intervention, abbreviated as *Post_SE*) and M_2 (Maths task-specific self-efficacy beliefs, abbreviated as *Post_TSE*). The causal relationship between the two mediators was also explored. A conceptual diagram of the Parallel Multiple Mediator Model used in this study is illustrated in Figure 1. This model shows the two mediators M_1 and M_2 as both dependent (predicted by X) and independent (predicting Y) variables. The following least squares regression equation incorporated M_1 and M_2 as dependent variables and X as an independent variable:

$$M_i = a_{0i} + a_i X + \varepsilon_i \qquad i = 1, 2.$$

Both the direct and indirect effects of the instructional approach, X (group) on students' mathematical reasoning, Y (MR) were modeled by the following multiple regression equation:

$$Y = b_0 + c'X + \sum_{1}^{k} b_i M_i + \varepsilon_Y$$

In this case, a_{0i} and b_0 represent the intercepts, a_i estimates the effect of X on M_i whereas b_i estimates the effect of M_i on Y while controlling for X and the other mediator variable. The regression coefficient c' estimates the effect of X on Y while holding all mediator variables (M_1 and M_2) constant.

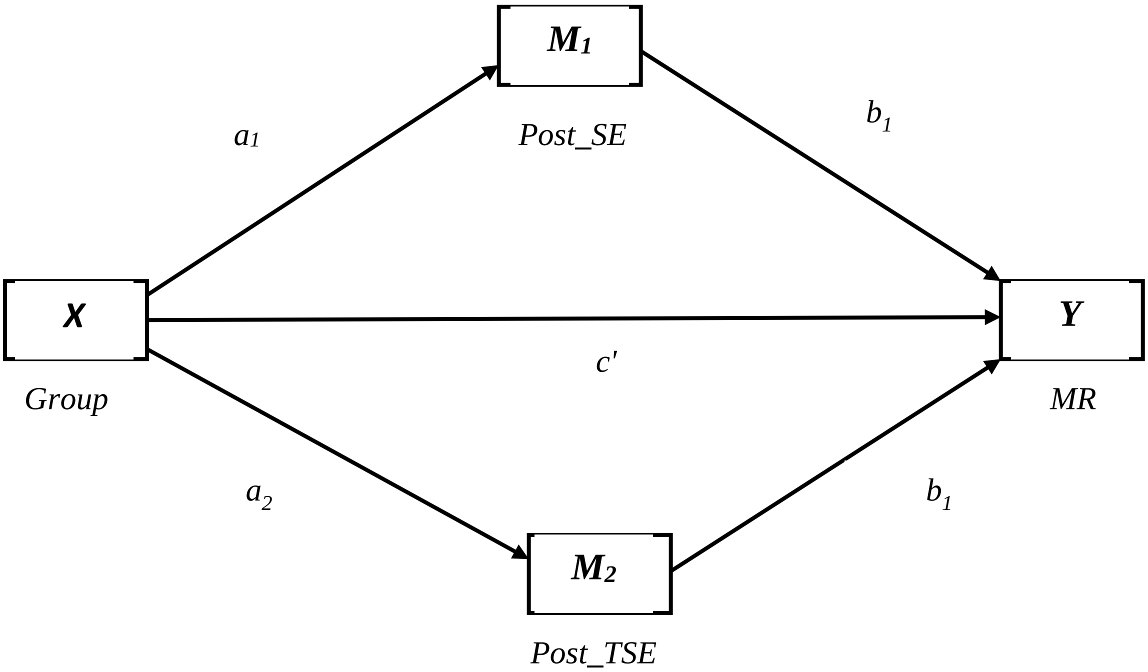

Figure 1. A Conceptual Diagram of a Parallel Multiple Mediator Model (adapted from Hayes, 2018, p.150).

Although the PROCESS command generated the *Sobel test* for the indirect effects, the results from this test have not been reported in this paper owing to some flaws that have made some scholars like Hayes (2018, p.97) not to confidently recommend its use. Its low power to generate accurate confidence intervals and the normality assumption for the sampling distribution of a_ib_i effects make it less useful compared to the bootstrap confidence intervals. The total effect c, the direct effect c' and the indirect effects a_ib_i of the instructional approach on students' mathematical reasoning, have been reported alongside their respective statistical significance tests using *t-tests* and the bootstrap confidence intervals. The amount of variance that each model explains has also been reported in terms of the coefficient of determination, R^2 alongside their respective results from *F-tests*.

RESULTS AND DISCUSSION

Correlations among Variables

Respondents were requested to complete a self-efficacy beliefs questionnaire and also to take a mathematical reasoning test (MRT) before and after the intervention. Table 2 displays the correlations among the three variables of interest. One inference drawn from Table 2 results is that all the three variables were positively correlated regardless of the teaching method to which students were exposed. This implies that students who expressed higher confidence in performing mathematical tasks were associated with higher mathematical reasoning ability. On the other hand, those who expressed lower self-efficacy beliefs were associated with lower mathematical reasoning ability. However, the extent to which each pair of variables were related varied across groups for both the pretest and the posttest measures.

Table 2. Correlations among Variables

Group	Variable	Pretest			Posttest		
		1	**2**	**3**	**1**	**2**	**3**
Control	1. MRT score	__			__		
	2. Maths self-efficacy	.279**	__		.147	__	
	3. Task-specific	.136	.419**	__	.281**	.114	__
Experimental	1. MRT score	__			__		
	2. Maths self-efficacy	.245**	__		.278**	__	
	3. Task-specific	.344**	.487**	__	.520**	.518**	__

Note. Correlation outputs for each pair of variables are based on the group for both the pretest and posttest measures.

** $p < 0.01$.

Table 2 results show that the extent to which maths self-efficacy beliefs relate to students' mathematical reasoning ability in the control group reduced from a significant correlation in the pretest, $r = .279$, $p < .01$ to an insignificant one in the posttest $r = .147$, $p = .085$. This implies that students' exposure to the traditional method of teaching (expository) did not improve the strength of the relationship between these two variables. On the other hand, the extent to which students' task-specific self-efficacy beliefs and their mathematical reasoning ability were related in the control group improved from an insignificant correlation in the pretest, $r = .136$, $p = .099$ to a significant correlation in the posttest, $r = .281$, $p <.01$.

For the experimental group, Table 2 results show that the extent of the relationship between students' maths self-efficacy beliefs and their mathematical reasoning ability were significant in both the pretest, $r = .245$, $p < .01$, and the posttest, $r = .278$, $p <.01$. The relationship between students' task-specific self-efficacy beliefs and their mathematical reasoning ability was also significant for both the pretest, $r = .344$, $p < .01$ and the posttest, $r = .520$, $p < .01$. This means that the strength of the relationship between students' mathematical reasoning and each of the two other variables (maths self-efficacy and task-specific self-efficacy) improved after students' exposure to the STAD model of cooperative learning.

Mediation Analysis

Before the intervention, an independent samples *t-test* was performed especially that the normality and equality of variances assumptions were satisfied. This was done to establish the equivalence of the control and experimental groups before implementing the planned intervention. Results showed no significant difference in students' maths self-efficacy beliefs between the control and experimental groups, $t(297) = 1.94$, $p = .053$. It was further revealed that students' task-specific self-efficacy beliefs between the experimental and control groups did not differ significantly, $t(297) = 1.54$, $p = .125$. Similarly, there was no significant difference in students' mathematical reasoning between the control and experimental groups, $t(299) = 0.94$, $p = .35$.

After confirming that the two groups were almost at the same level in terms of their self-efficacy beliefs and mathematical reasoning skills, the experimental group $(X = 1)$ was then exposed to a STAD model of cooperative learning whereas the control group $(X = 0)$ was taught using traditional methods of teaching (mainly expository). Using the PROCESS custom dialogue version 3.4 embedded in SPSS, parameters (regression coefficients) shown in Figure 2 for each path were generated together with various additional statistics that have been summarised in Table 3.

The most relevant information here were the direct and indirect effects of the instructional approach (X) on students' mathematical reasoning (Y). The direct effect, $c' = 16.73$ reflects a higher reasoning ability for students in the experimental group compared to their counterparts in the control group, independent of students' self-efficacy beliefs. Results further indicate that this effect was significant, $c' = 16.73$, $t(284) = 6.56$, $p < .001$, implying that the instructional approach (independent of students' self-efficacy beliefs) is a significant predictor of students' mathematical reasoning.

Considering the paths a_1 and b_1 in Figure 2, results show that students assigned to the experimental group had a higher mathematical reasoning ability (by 3.26 units) than their counterparts in the control group as a result of their maths self-efficacy beliefs. Using maths task-specific self-efficacy as a mediator (a_2b_2), results indicate that students who were assigned to the experimental group were 1.76 units higher in their mathematical reasoning ability than those who were assigned to the control group.

Based on the percentile bootstrap confidence intervals generated from the PROCESS procedure, the claim that maths self-efficacy alone mediates the influence of the instructional approach on students' mathematical reasoning is not significant, 95% CI [-0.18, 6.67]. On the other hand, maths task-specific self-efficacy alone significantly mediates the influence of the instructional approach on students' mathematical reasoning, 95% CI [0.17, 3.73]. These results also confirm the statistical significance tests reported in Table 3 about the predictive roles of self-efficacy beliefs, $b_1 = 4.80$, $t(282) = 1.82$, $p = .07$, and task-specific self-efficacy beliefs $b_2 = 12.60$, $t(282) = 6.81$, $p < .001$on students' mathematical reasoning ability.

The total indirect effect (i.e. $a_1b_1 + a_2b_2 = 5.02$) further indicates that students assigned to the experimental group were, on average, 5.02 units higher in their mathematical reasoning ability than those who were assigned to the control group. Results generated from the PROCESS procedure further indicate that we can be 95% confident that the total indirect effect of the instructional approach through the two mediators (M_1 *and* M_2) were between the values 1.36 and 8.76. Since this percentile bootstrap confidence interval excludes zero, it suffices to conclude that maths self-efficacy beliefs and task-specific self-efficacy beliefs collectively and significantly mediate the effect of the instructional approach on students' mathematical reasoning.

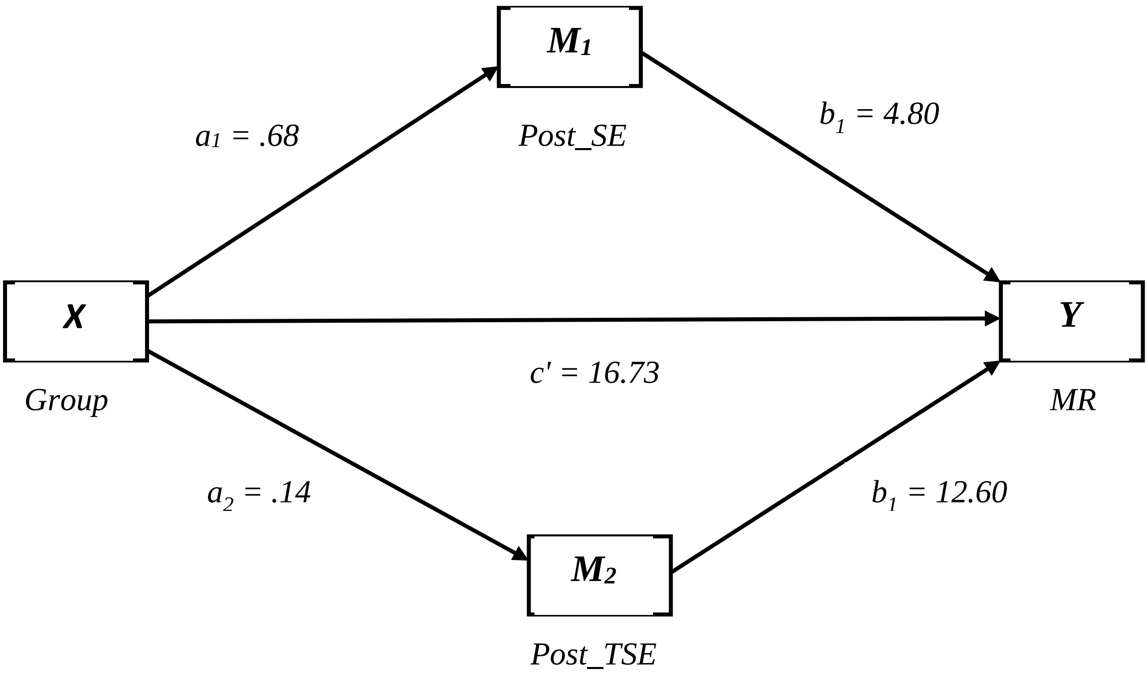

Figure 2. Parallel Multiple Mediator Model for the Teaching Method (Group), Self-Efficacy (M_1&M_2) and Mathematical Reasoning (MR)

In the context of this study, self-efficacy beliefs have been operationalised at two levels: Mathematics self-efficacy and task-specific self-efficacy. The former refers to students' general confidence and anxiety beliefs about mathematics learning and their ability to attain success in mathematics as a whole.

Table 3. Summary of the Mediation and Regression Parameters from the PROCESS Output.

	Consequent											
		M_1(Post_SE)				M_2 (Post_TSE)				*Y* (MR)		
Antecedent		*Coeff.*	*SE*	*p*		*Coeff.*	*SE*	*p*		*Coeff.*	*SE*	*p*
X (Group)	a_1	.68	.05	.000	a_2	.14	.06	.03	*c'*	16.73	2.54	.000
M_1 (Post_SE)	---	---	---	---	---	---	---	---	b_1	4.80	2.64	.07
M_2(Post_TSE)	---	---	---	---	---	---	---	---	b_2	12.60	1.85	.000
Intercept	a_{01}	2.40	.03	.000	a_{02}	2.37	.05	.000	b_0	-19.7	6.45	.003
	$R^2 = .44$					$R^2 = .02$				$R^2 = .42$		
	$F(1, 284) = 223.67$,					$F(1, 284) = 5.03$,				$F(1, 282) = 66.80$		
	$p < .001$					$p = .03$				$p < .001$		

Note. Refer to a detailed PROCESS output and the associated dataset openly available at http://dx.doi.org/10.17632/3472zggczv.1

The latter is concerned with students' confidence or judgement about their ability to solve specific questions on quadratic equations and quadratic functions. Results have shown that maths self-efficacy alone does not significantly mediate the effect of an instructional approach on students' mathematical reasoning. Task-specific self-efficacy has been found to significantly mediate the effect of an instructional approach on students' mathematical reasoning. It has also been established that maths self-efficacy and task-specific self-efficacy beliefs collectively and significantly mediate the effect of an instructional approach on students' mathematical reasoning.

Table 2 results further indicate that the relationship between self-efficacy beliefs and students' mathematical reasoning ability was stronger among the students who were exposed to the STAD model of cooperative learning compared to their counterparts who were taught using the expository methods of teaching. Although correlations do not reveal the cause and effect between two variables, these results partly suggest that students' confidence in performing specific mathematical tasks increased as a result of their enhanced mathematical reasoning ability. These results also confirm those of a study conducted by Öztürk et al.(2019) who found that mathematics self-efficacy perceptions significantly predicted non-routine mathematical problem-solving skills. Although they focused on non-routine mathematical problem-solving skills, their findings are relevant to the findings of this research in the sense that non-routine mathematical problem-solving activities are bound to trigger higher-order thinking and mathematical reasoning among the leaners.

Based on regression parameters displayed in Figure 2 and Table 3, results indicate that the STAD model of cooperative learning does not only enhance students' mathematical reasoning but also improves their maths self-efficacy and task-specific self-efficacy beliefs. Besides that, it has been demonstrated that students who were assigned to the STAD model of cooperative learning had higher maths self-efficacy and task-specific self-efficacy beliefs, which eventually resulted in a higher mathematical reasoning ability as opposed to those who were taught using the expository methods of teaching. These findings have important implications for mathematics education research and practice.

First, results demonstrate a need for mathematics educators and researchers to consider active learning models (such as STAD) that have the potential to develop students' cognitive abilities (such as mathematical reasoning) and their affective skills such as mathematics self-efficacy beliefs. Some previous reviews, meta-analyses, and empirical investigations (e.g., Ardiyani et al., 2018; Hendriana et al., 2018; Johnson & Johnson, 1999; Slavin, 2015) also attest to the benefits that students enjoy when they cooperate with others to achieve a common goal. In line with recommendations by Gillies (2016), the findings of this research demonstrate a need for teachers to provide students with opportunities where they can experience positive interdependence, individual accountability, and multiples ways of solving a given task. Strengthening of such practices in the mathematics classroom is bound to lead to an enhanced mathematical reasoning and self-efficacy beliefs.

Second, it has been established that general self-efficacy beliefs alone may not reveal students' capabilities in performing subject- or concept-specific tasks. It is possible for a student to feel confident about mathematics as a whole but fails to perform specific tasks. This is why some scholars (e.g. Bonne & Lawes, 2016) have stressed the need to focus on something more specific than mathematics as a whole when assessing students' self-efficacy beliefs. This view is in line with that of Bandura (1986), who relates self-efficacy to that which involves someone's judgements about his/her ability to perform specific tasks.

Third, it has been established that apart from employing a suitable instructional approach, students' self-confidence, and anxiety beliefs do have a significant influence on students' understanding of specific mathematical concepts. Previous studies have also established that maths self-efficacy and task-specific self-efficacy beliefs have a significant influence on students' perseverance in solving mathematical tasks (Grigg et al., 2018; Öztürk et al., 2019; Pajares & Kranzler, 1995). In other words, a student's belief and confidence in oneself have a great influence on his/her level of perseverance in exploring and solving mathematical tasks, which could eventually lead to enhanced mathematical reasoning skills.

Fourth, this study has demonstrated that students can, at times over-estimate their confidence in performing specific tasks in mathematics. This is evident by many students who expressed higher confidence in performing certain tasks yet they failed to perform as expected when presented with a mathematical reasoning test. These results echo the findings of a study conducted by Cheema and Skultety (2016) who found that some students "consistently over-estimated" their confidence in performing specific tasks (p.12). Cheema and Skultety (2016) further warned that overestimation or underestimation of one's ability to perform certain tasks may have serious consequences on their psychological or emotional well-being if not addressed. This points to the need for mathematics educators and researchers not to end at only assessing students' judgement about their capabilities in performing certain mathematical tasks, but to engage them with those tasks to get a complete picture of their actual capabilities. It is already known that higher self-efficacy beliefs are associated with higher ability in performing certain tasks. However, this may not always be the case because other factors such as level of education, students' mathematical ability levels, communication skills, and the ability to express themselves in a specific language of instruction may result in under-estimating or over-estimating their capabilities in performing certain tasks.

Finally, it suffices to state that this research extends prior studies (e.g. Liu et al., 2017) that have experimentally established the mediating effects of self-efficacy beliefs on students' mathematical achievement and different learning models. It also adds to the scarcity of research, especially in the 'global south', particularly Zambia where no study of this nature has been conducted previously.

CONCLUSION

This study involved only grade 11 students from selected public secondary schools in one district. This brings about limitations of the extent to which these results could be generalised to other mathematics classrooms across the world. Amidst this methodological and contextual limitation, the findings of this study provide very important implications for practitioners and researchers in mathematics education. It has been revealed that maths self-efficacy, task-specific self-efficacy, and a well-designed STAD model of cooperative learning, do have a collective and significant effect on students' mathematical reasoning skills. This partly suggests that students' self-efficacy and task-specific self-efficacy beliefs should be taken into consideration when designing instructional and assessment approaches for mathematics classrooms. Therefore, there is a serious need for more research on instructional approaches that do not only focus on developing students' mathematical reasoning skills but also to improve students' confidence in performing specific mathematical tasks.

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REFERENCES

Adams, W. K., & Wieman, C. E. (2010). Development and validation of instruments to measure learning of expert-like thinking. *International Journal of Science Education*, *33*(9), 1289–1312. https://doi.org/10.1080 /09500693.2010.512369

Ardiyani, S. M., Gunarhadi, & Riyadi. (2018). Realistic mathematics education in cooperative learning viewed from learning activity. *Journal on Mathematics Education*, *9*(2), 301–310. http://dx.doi.org/10.22342/jme.9.2.5392.301-310

Baines, E., Blatchford, P., & Webster, R. (2015). The challenges of implementing group work in primary school classrooms and including pupils with special educational needs. *Education 3–13*, *43*(1), 15–29. https://doi.org/10.1080/03004279.2015.961689

Bandura, A. (1986). The explanatory and predictive scope of self-efficacy theory. *Journal of Social and Clinical Psychology*, *4*(3), 359-373. https://doi.org/10.1521/jscp.1986.4.3.359

Baumann, J. F. (1988). Direct instruction reconsidered. *Journal of Reading Behavior*, *37*, 712-718. https://www.jstor.org/stable/40032953

Bonne, L., & Lawes, E. (2016). Assessing students' maths self-efficacy and achievement. *Assessment News*, *Set 2*, 60–63. https://doi.org/10.18296/set.0048

Buchs, C., Filippou, D., Pulfrey, C., & Volpé, Y. (2017). Challenges for cooperative learning implementation: Reports from elementary school teachers. *Journal of Education for Teaching*, *43*(3), 296-306. https://doi.org/10.1080/02607476.2017.1321673

Butera, F., & Buchs, C. (2019). Social interdependence and the promotion of cooperative learning. In K. Sassenberg, & M. Vliek (Eds.). *Social Psychology in Action* (pp. 111-127). Cham: Springer. https://doi.org/10.1007/978-3-030-13788-5_8

Cheema, J. R., & Skultety, L. S. (2016). Self-efficacy and literacy: A paired difference approach to estimation of over-/under-confidence in mathematics- and science-related tasks. *Educational Psychology*, *37*(6), 652–665. https://doi.org/10.1080/01443410.2015.1127329

Child, D. (2006). *The Essentials of Factor Analysis* (3rd ed.). New York: Continuum.

Creswell, J. (2014). *Research Design: Quantitative, Qualitative, and Mixed Methods Approach* (4th ed.). New York: SAGE Publications

Curriculum Development Centre. (2013). *O' level Secondary School Mathematics Syllabus Grade 10-12*. Ministry of General Education. https://www.moge.gov.zm/?wpfb_dl=52

Czocher, J. A., Melhuish, K., & Kandasamy, S. S. (2019). Building mathematics self-efficacy of STEM undergraduates through mathematical modelling. *International Journal of Mathematical Education in Science and Technology*, *51*(6), 807–834. https://doi.org/10.1080/0020739X.2019.1634223

Examinations Council of Zambia. (2012). *Chief Examiner's Report.* https://www.exams-council.org.zm/request-for-statistics

Examinations Council of Zambia. (2016). *Examinations Performance Report for 2015 in Natural Sciences*. https://www.exams-council.org.zm/request-for-statistics

Examinations Council of Zambia. (2018). *2017 Examinations Review Booklet: School Certificate Ordinary Level Chief Examiner's Reports.* https://www.exams-council.org.zm/request-for-statistics

Field, A. (2013). *Discovering Statistics Using SPSS* (4th ed.). New York: SAGE.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2006). *How to Design and Evaluate Research in Education*. Mc Graw Hill.

Gardner, P. L. (1995). Measuring attitudes to science: Unidimensionality and internal consistency revisited. *Research in Science Education*, *25*(3), 283–289. https://doi.org/10.1007/bf02357402

Ghaith, G. M. (2018). Teacher perceptions of the challenges of implementing concrete and conceptual cooperative learning. *Issues in Educational Research*, *28*(2), 385–404. http://www.iier.org.au/iier28/ghaith.pdf

Gillies, R. M. (2016). Cooperative learning: Review of research and practice. *Australian Journal of Teacher Education*, *41*(3), Article 3. https://doi.org/10.14221/ajte.2016v41n3.3

Gillies, R. M., & Boyle, M. (2010). Teachers' reflections on cooperative learning : Issues of implementation. *Teaching and Teacher Education*, *26*(4), 933–940. https://doi.org/10.1016/j.tate.2009.10.034

Grigg, S., Perera, H. N., McIlveen, P., & Svetleff, Z. (2018). Relations among Math Self Efficacy, Interest, Intentions, and Achievement: A Social Cognitive Perspective. *Contemporary Educational Psychology*, *53*(2), 77–86. https://doi.org/10.1016/j.cedpsych.2018.01.007

Guadagnoli, E., & Velicer, W. F. (1988). Relation of a sample size to the stability of component patterns. *Psychological Bulletin*, *103*(2), 265–275. https://doi.org/10.1037/0033-2909.103.2.265

Hayes, A. F. (2018). *Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression-Based Approach* (2nd ed.). New York: A Division of Guilford Publications, Inc.

Hendriana, H., Johanto, T., & Sumarmo, U. (2018). The role of problem-based learning to improve students' mathematical problem-solving ability and self-confidence. *Journal on Mathematics Education*, *9*(2), 291–300. https://doi.org/10.22342/jme.9.2.5394.291-300

Hong, D. S., & Choi, K. M. (2014). A comparison of Korean and American secondary school textbooks: The case of quadratic equations. *Educational Studies in Mathematics*, *85*(2), 241–263. https://doi.org/10.1007/s10649-013-9512-4

Hossain, A., & Ahmad, R. (2013). Effects of cooperative learning on students ' achievement and attitudes in secondary mathematics. In F. Odabaşı (Ed.), *3rd World Conference on Learning, Teaching and Educational Leadership (WCLTA-2012)* (Vol. 93, pp. 473–477). https://doi.org/10.1016/j.sbspro.2013.09.222

Jäder, J., Sidenvall, J., & Sumpter, L. (2016). Students' Mathematical Reasoning and Beliefs in Non-routine Task Solving. *International Journal of Science and Mathematics Education*, *15*(4), 759–776. https://doi.org/10.1007/s10763-016-9712-3

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, *96*, 1–16. https://doi.org/10.1007/s10649-017-9761-8

Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory into Practice, 38*(2), 67–73. https://doi.org/10.1080/00405849909543834

Kohen, Z., Amram, M., Dagan, M., & Miranda, T. (2019). Self-efficacy and problem-solving skills in mathematics: the effect of instruction-based dynamic versus static visualization. *Interactive Learning Environments*, 1–20. https://doi.org/10.1080/10494820.2019.1683588

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*(1), 281–310. https://doi.org/10.3102/00028312040001281

Li, M. P., & Lam, B. H. (2013). *Cooperative learning: The active classroom*. Hong Kong: The Hong Kong Institute of Education. https://www.csuchico.edu/pedagogy/li,-m.-p.-_-lam,-b.-h.-2013-cooperative-learning.pdf

Liu, R.-D., Zhen, R., Ding, Y., Liu, Y., Wang, J., Jiang, R., & Xu, L. (2017). Teacher support and math engagement: roles of academic self-efficacy and positive emotions. *Educational Psychology*, *38*(1), 3–16. https://doi.org/10.1080/01443410.2017.1359238

Mata-Pereira, J., & da Ponte, J. P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, *96*, 169–186. https://doi.org/10.1007/s10649-017-9773-4

May, D. K. (2009). Mathematics self-efficacy and anxiety questionnaire. *Doctoral dissertation*. Georgia: University of Georgia. https://athenaeum.libs.uga.edu/handle/10724/25886

Mukuka, A., Balimuttajjo, S., & Mutarutinya, V. (2020a). Applying the SOLO taxonomy in assessing and fostering students' mathematical problem-solving abilities. In I.S.P. Vale, L. Westaway, & Z. Nhase (Ed.), *Proceedings of the 28th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 104–112). SAARMSTE

Mukuka, A., Balimuttajjo, S., & Mutarutinya, V. (2020b). Exploring students' algebraic reasoning on quadratic equations: Implications for school-based assessment. In K. K. Mashood, T. Sengupta, C. Ursekar, H. Raval, & S. Dutta (Eds.), *Proceedings of the epiSTEME8 International Conference to Review Research in Science, Technology, and Mathematics Education* (pp. 130–138). Mumbai, India: https://episteme8.hbcse.tifr.res.in/proceedings/

Mukuka, A., Mutarutinya, V., & Balimuttajjo, S. (2019). Exploring the barriers to effective cooperative learning implementation in school mathematics classrooms. *Problems of Education in the 21st Century, 77*(6), 745–757. https://doi.org/10.33225/pec/19.77.745

Mukuka, A., Mutarutinya, V., & Balimuttajjo, S. (2020). Data on students' mathematical reasoning test scores: A quasi-experiment. *Data in Brief, 30,* Article 105546.

https://doi.org/10.1016/j.dib.2020.105546

Nurlaily, V. A., Soegiyanto, H., & Usodo, B. (2019). Elementary school teacher's obstacles in the implementation of problem-based learning model in mathematics learning. *Journal on Mathematics Education*, *10*(2), 229–238. https://doi.org/10.22342/jme.10.2.5386.229-238

Ozgen, K., & Bindak, R. (2011). Determination of self-efficacy beliefs of high school students towards math literacy. *Educational Sciences: Theory & Practice*, *11*(2), 1085–1089. http://oldsite.estp.com.tr/pdf/en/975096b0094167470a5cb5b86b9b1697TAMEN.pdf

Öztürk, M., Akkan, Y., & Kaplan, A. (2019). Reading comprehension, mathematics self-efficacy perception, and mathematics attitude as correlates of students' non-routine mathematics problem-solving skills in Turkey. *International Journal of Mathematical Education in Science and Technology*, 1–17. https://doi.org/10.1080/0020739X.2019.1648893

Pajares, F., & Kranzler, J. (1995). Self-efficacy beliefs and general mental ability in mathematical problem-solving. *Contemporary Educational Psychology*, *20*(4), 426–443. https://doi.org/10.1006/ceps.1995.1029

Palincsar, A. S. (1998). Social constructivist perspectives on teaching and learning. *Annual Review of Psychology*, *49*(1), 345-375. https://doi.org/10.1146/annurev.psych.49.1.345

Ross, K. A. (1998). Doing and proving: The place of algorithms and proof in school mathematics. *The American Mathematical Monthly*, *3*(105), 252–255. https://doi.org/10.1080/00029890.1998.12004875

Saleh, M., Prahmana, R. C. I., Muhammad, I., & Murni. (2018). Improving the reasoning ability of elementary school students through the Indonesian realistic mathematics education. *Journal on Mathematics Education*, *9*(1), 41–54. http://dx.doi.org/10.22342/jme.9.1.5049.41-54

Santia, I., Purwanto, Sutawidjadja, A., Sudirman, & Subanji. (2019). Exploring mathematical representations in solving ill-structured problems: The case of quadratic functions. *Journal on Mathematics Education*, *10*(3), 365–378. https://doi.org/10.22342/jme.10.3.7600.365-378

Slavin, R. (1987). *Cooperative Learning: Student Teams* (2nd ed.). Washington: National Education Association.

Slavin, R. E. (2015). Cooperative Learning in Schools. In *International Encyclopedia of Social & Behavioral Sciences* (2nd. ed., Vol. 4, pp. 881–886). Elsevier. https://doi.org/10.1016/B978-0-08-097086-8.92028-2

Taber, K. S. (2018). The use of Cronbach's Alpha when developing and reporting research instruments in science education. *Research in Science Education*, *48*(6), 1273–1296. https://doi.org/10.1007/s11165-016-9602-2

Tejeda, S., & Gallardo, K. (2017). Performance assessment on high school advanced algebra. *International Electronic Journal of Mathematics Education*, *12*(3), 777–798.

Waller, B. (2006). Math interest and choice intentions of nontraditional African – American college students. *Journal of Vocational Behavior*, *68*(3), 538–547. https://doi.org/10.1016/j.jvb.2005.12.002

Zaslavsky, O. (1997). Conceptual obstacles in the learning of quadratic functions. *Focus on Learning Problems in Mathematics*, *19*(1), 20–44.