One Approach for Analysis of Fuzzy Linear Hybrid Automata

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Abstract

The article deals with the stability of a new class of continuous-discrete systems - fuzzy linear hybrid impulse automata. Sufficient conditions of stability are obtained. Stability and stability resistance on the part of variables are proved. The cases of cyclic are described. Research conducted by the method of Lyapunov's functions.

Keywords: Lyapunov's method, stability, hybrid automata, L-stability.

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Introduction

As suggested by the work of Lyapunov functions method applies to fuzzy hybrid automatic cyclic. Resistance distinct hybrid machines studied by many authors [1-6]. To investigate the stability offered to check the condition of Lyapunov functions to reduce interchanges machine. In [7, 8] the approach does not require finding solutions hybrid machine. This approach is developed to study the stability of fuzzy impulsive hybrid automata. Fuzzy is considered in terms of [9, 10].

Key definitions and preliminary results

We introduce the following notation.

$\|\cdot\|$ - - Euclidean norm in the space R^d (notation same for all d);

$\nabla_f V(y_0) = V'(y_0) f(y_0)$, if $V:\mathrm{R}^d \to R$ i $f:\mathrm{R}^d \to \mathrm{R}^d$ or $f:\mathrm{R}^d \to \mathrm{R}^{d\times l}$.

1. Definition Fuzzy hybrid machine with fuzzy switching (FHMFS) is a tuple $HA = (Q, Y, PS, g, w, h, Inv, Init, Jump)$, in which

- Q - finite set of discrete states;
- $Y = \mathrm{R}^d$ - a set of continuous states;
- $PS = (X, 2^X, P)$ - space of capabilities with the normalized degree opportunities;
- $w:\mathrm{R}^+ \times X_+ \to \mathrm{R}^l$ - process fuzzy walk in space PS ;
- $g:Q\times\mathrm{R}^d \to \mathrm{R}^d$, $h:Q\times\mathrm{R}^d \to \mathrm{R}^{d\times l}$ - partially defined functions that help given by continuous machine behavior while in the discrete state; and will mark $g_q = y \mapsto g(q,y)$ i $h_q = y \mapsto h(q,y)$;
- $Inv: Q \to Y \setminus \{\varnothing\}$ - a function that defines a plurality of discrete invariance condition;
- $Init \subseteq Q\times Y$ - a set of initial conditions;
- $Jump: Q\times Y \times X \to 2^{Q\times Y}$ - display, which specifies the transition between discrete states;

and the condition $Inv(q) \subseteq Domg_q \cap Domh_q$ for all $q \in Q$.

2. Definition. Orbit phase, which allowed by FHMFS HA is procession $\chi = (\tau, q, y, x)$, in which $x \in X_+$, $\tau = (I_i)_{i=0}^N \in \mathrm{H}T$, $q: \langle\tau\rangle \to Q$ - display i $y = (y^i)_{i\in\langle\tau\rangle}$ - indexed family of continuous maps $y^i : I_i \to Y$, such that:

1) $y^i(t) \in Inv(q(i))$ for all $t \in [\tau_i, \tau_i')$, if $i \in \langle\tau\rangle$ and besides, $y^i(\tau_i') \in Inv(q(i))$, if $i = N(\tau)$ i $\tau_i' \in U(\tau)$;

2) $(q(i+1), y^{i+1}(\tau_{i+1})) \in Jump(q(i), y^{i}(\tau_i'), x)$ for all $i \in \langle \tau \rangle \setminus \{N(\tau)\}$;

3) function y^i locally absolutely continuous on I_i and satisfies the equation $\dot{y}^i(t) = g(q(i), y^i(t)) + h(q(i), y^i(t))\dot{w}(t,x)$ for almost all $t \in I_i$;

4) $(q(0), y^0(0)) \in Init$. Note that in paragraph 3 features $t' \mapsto g_{q(i)}(y^i(t'))$ i $t' \mapsto h_{q(i)}(y^i(t'))$ is determined to $I_i \setminus \{\tau_i'\}$, because $y^i(t) \in Inv(q(i))$ for all $t \in [\tau_i, \tau_i')$ in paragraph 1.

Consider fixed FHMFS $HA = (Q, Y, PS, g, w, h, Inv, Init, Jump)$.

Let $Orb(HA)$ - set of phase orbit machine HA, φ_w - distribution function process fuzzy walk w.

3. Definition. Steady state HA called point $y_* \in Y$, that:

1) $Jump(q, y_*, x) \subseteq Q \times \{y_*\}$ fir all $q \in Q$ i $x \in X_+$;

2) for each $\chi = (\tau, q, y, x) \in Orb$, that $y^0(\tau_0) = y_*$, all display y^i, $i \in \langle \tau \rangle$ are constant and taking values y_*. Let $St(HA)$ - set of stationary states HA.

4. Definition. Steady state $y_* \in St(HA)$ called persistent with level of α, where $\alpha : (0,+\infty) \to [0,1)$ - function defined in a neighborhood of zero, if for any number $\varepsilon > 0$ is a number $\delta > 0$, such that, for all phase orbit $\chi = (\tau, q, y, x) \in Orb(HA)$, where $y = (y^i)_{i \in \langle \tau \rangle}$ i $\tau = (I_i)_{i \in \langle \tau \rangle}$, that $\|y^0(\tau_0) - y_*\| < \delta$ i $P\{x\} > \alpha(\varepsilon)$, the condition $\|y^i(t) - y_*\| < \varepsilon$ for all $i \in \langle \tau \rangle$ i $t \in I_i$.

Let $St(HA)$ - set of stationary states HA.

Let $DF(\mathsf{R}^d)$ - class of all partially defined continuous functions $f : \mathsf{R}^d \to \mathsf{R}^+$, which are continuously differentiable on the inside of your domain.

For each partially defined function $f : \mathsf{R} \to \mathsf{R}$ denoted $Infinv(f)$ partially defined function $g : \mathsf{R} \to \mathsf{R}$, given the conditions

- $g(y) = \inf\{x \in \mathsf{R} \mid f(x) \le y\}$, if $y \in \mathsf{R}$ and a set $\{x \in \mathsf{R} \mid f(x) \le y\}$ not empty and bounded above;
- $g(y)$ not determined otherwise.

Let $HLo(HA, y_*)$ set of tuples $\left(\alpha, (V_q)_{q \in Q}, (\nu_q)_{q \in Q}\right)$, in which

- $\alpha : (0,+\infty) \to [0,1)$ - function defined in a neighborhood of zero,
- $(V_q)_{q \in Q}$ - family of functions indexed class $DF(\mathsf{R}^d)$,
- $(\nu_q)_{q \in Q}$ - indexed family of predefined functions $v_q : \mathsf{R}^+ \to \mathsf{R}^+$, defined in a neighborhood of zero (Including zero), and the conditions in which $\psi = Infinv(\alpha)$:

Lo1) $\{y_*\} \cup O_q \subseteq DomV_q$ for all $q \in Q$, where O_q - some open superset $Inv(q)$, and if $(q_2, y_2) \in Jump(q_1, y_1, x)$ for some $q_1, q_2 \in Q$, $y_1, y_2 \in Y$ and $x \in X_+$, then $y_1 \in DomV_{q_1}$ i $y_2 \in DomV_{q_2}$;

Lo2) $v_q(0) = 0$ i $v_q(w) > 0$ for all $w \in Domv_q \setminus \{0\}$;

Lo3) $V_q(y_*) = 0$ for all $q \in Q$;

Lo4) if $V_q(y) \le \nu_q(w)$ for some $y \in DomV_q$ and $w \in Domv_q$, then $\|y_* - y\| \le w$;

Lo5) for all elements $q \in Q$, $x \in X_+$ and $y \in Inv(q)$, such that $P\{x\} \in Dom\psi$ and $V_q(y_1) > \nu_q(\psi(P\{x\}))$, defined by the inequality

$$\nabla_{g_q} V_q(y) \le -\frac{1}{\kappa_w}\sqrt{\varphi_w^{-1}(P\{x\})}\left\|\nabla_{h_q} V_q(y)\right\|.$$

We introduce the following notation (where $q_1, q_2 \in Q, u \in (0,1]$):

$J(q_1, q_2, u) = \{(y_1, y_2) \mid \exists x \in X : P\{x\} >$

$> u \wedge (q_2, y_2) \in Jump(q_1, y_1, x)\}$

$E = \{(q_1, q_2) \in Q \times Q | \exists x \in X : P\{x\} > 0 \wedge$

$\wedge \exists (y_1, y_2) : (q_2, y_2) \in Jump(q_1, y_1, x)\}$

Theorem 1. (the cyclic stability of stationary states**)** Let HA - FHMFS a cyclic discrete switching states $\hat{q}_1 \to \hat{q}_2 \to \ldots \to \hat{q}_n \to \hat{q}_1$. Suppose that for steady state $y_* \in St(HA)$ there is a tuple

$HL = \left(\alpha, (V_q)_{q \in Q}, (\nu_q)_{q \in Q}\right) \in HLo(HA, y_*)$.

Suppose the following conditions:

1) for each arc $(q_1, q_2) \in E$ there is a number $\delta_{q_1 q_2} > 0$ and display $\vartheta_{q_1 q_2} : [0, \delta_{q_1 q_2}] \to \mathsf{R}^+$ such that $\vartheta_{q_1 q_2}(0+) = \vartheta_{q_1 q_2}(0) = 0$ for all elements $u \in D$ and pairs $(y_1, y_2) \in J(q_1, q_2, u)$ performed inequality

- $V_{q_2}(y_2) \le v_{q_2}(\psi(u))$, if $V_{q_1}(y_1) \le v_{q_1}(\psi(u))$,

- $V_{q_2}(y_2) \le \vartheta_{q_1 q_2}(V_{q_1}(y_1))$, if $V_{q_1}(y_1) \in [0, \delta_{q_1 q_2}]$ i $V_{q_1}(y_1) > v_{q_1}(\psi(u))$;

2) $\lambda(\hat{q}_1 \hat{q}_2 .. \hat{q}_{n-1} \hat{q}_n \hat{q}_1) \le_{SD} \lambda(\hat{q}_1)$, there is s'-condition.

Then steady state y_* automaton HA is steady with a level α.

Pulse fuzzy hybrid machines

We prove a theorem on the stability of the stationary states of cyclical pulse FHMFS.

Theorem 2. Let the machine pulse, $(V_q)_{q \in Q}$ - functions family of class $DF_0^\infty(\mathsf{R}^d, y_*)$, such, that $O_q \subseteq DomV_q$ for each $q \in Q$, where O_q - some open superset $Inv(q)$, and for all $q_1, q_2 \in Q$, $y_1, y_2 \in Y$, $x \in X_+$, such that $(q_2, y_2) \in Jump(q_1, y_1, x)$, performed $y_1 \in DomV_{q_1}$ i $y_2 \in DomV_{q_2}$. Let $(\nu_q)_{q \in Q}$ - family of display $v_q : \mathsf{R}^+ \to \mathsf{R}^+$, such that $v_q(0) = 0$ i $0 < v_q(w) < V_q(y)$ for all $w > 0$ i $y \in DomV_q$ such that $\|y_* - y\| > w$.

Let for all $q \in Q$, $x \in X_+$ i $y \in Inv(q)$ such that $P\{x\} \in Dom\psi$ and $V_q(y_1) > v_q(\psi(P\{x\}))$ performed inequality

$$\nabla_{g_q} V_q(y) \le -\frac{1}{\kappa_w}\sqrt{\varphi_w^{-1}(P\{x\})}\left\|\nabla_{h_q} V_q(y)\right\|.$$

Then let, for all arc $(q_1, q_2) \in E$ there is a number $\delta_{q_1 q_2} > 0$ and display $\vartheta_{q_1 q_2} : [0, \delta_{q_1 q_2}] \to \mathsf{R}^+$ such that $\vartheta_{q_1 q_2}(0+) = \vartheta_{q_1 q_2}(0) = 0$ and for all elements $u \in D$ and payers $(y_1, y_2) \in J(q_1, q_2, u)$ inequalities

а) $V_{q_2}(y_2) \le v_{q_2}(\psi(u))$, if $V_{q_1}(y_1) \le v_{q_1}(\psi(u))$,

б) $V_{q_2}(y_2) \le \vartheta_{q_1 q_2}(V_{q_1}(y_1))$, if $V_{q_1}(y_1) \in [0, \delta_{q_1 q_2}]$ i $V_{q_1}(y_1) > v_{q_1}(\psi(u))$.

Suppose that values $d_i(u, v) \in \mathsf{R}^+, i = \overline{1, n}$ by all $(u, v) \in D \times [0, v_{\max}]$ and inequality $d_n(u, v) \le \max\{v_{\hat{q}_1}(\psi(u)), v\}$ for all $(u, v) \in D \times [0, v_{\max}]$, where

$$d_1(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_2}(y_2)\,|\,(y_1,y_2)\in J(\hat{q}_1,\hat{q}_2,u)\wedge\\ \wedge V_{\hat{q}_1}(y_1)\le\max\{\nu_{\hat{q}_1}(\psi(u)),v\}\end{matrix}\right\};$$

$$d_2(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_3}(y_3)\,|\,(y_2,y_3)\in J(\hat{q}_2,\hat{q}_3,u)\wedge\\ \wedge V_{\hat{q}_2}(y_2)\le\max\{\nu_{\hat{q}_2}(\psi(u)),d_1(u,v)\}\end{matrix}\right\};$$

$$d_3(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_4}(y_4)\,|\,(y_3,y_4)\in J(\hat{q}_3,\hat{q}_4,u)\wedge\\ \wedge V_{\hat{q}_3}(y_3)\le\max\{\nu_{\hat{q}_3}(\psi(u)),d_2(u,v)\}\end{matrix}\right\};\ \ldots$$

$$d_n(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_1}(y_1)\,|\,(y_n,y_1)\in J(\hat{q}_n,\hat{q}_1,u)\wedge\\ V_{\hat{q}_n}(y_n)\le\max\{\nu_{\hat{q}_n}(\psi(u)),d_{n-1}(u,v)\}\end{matrix}\right\}.$$

Then steady state y_* pulse cycle of FHMFS HA stable with level α.

Proof. We verify the conditions of the theorem (1). Prove that $\left(\alpha,(V_q)_{q\in Q},(\nu_q)_{q\in Q}\right)\in HLo(HA,y_*)$. The condition of the theorem implies that the conditions Lo1, Lo3, Lo5 run. Conditions Lo2 performed as $\nu_q(w)>0$ at $w>0$ i $\nu_q(0)=0$. Condition Lo4 performed, by the theorem if $\|y_*-y\|>w$, it $V_q(y)>\nu_q(w)$.

The condition of the theorem implies the condition (1) Theorem 1.

Verify the condition 2 of the Theorem (1). As mentioned

$d_i(u,v)\in\mathrm{R}^+, i=\overline{1,n}$ for all $(u,v)\in D\times[0,v_{\max}]$, then for all arcs (q_1,q_2), we have

$(\bar{c}(q_1)\cdot\bar{c}(q_1,q_2))(u,v)=$

$\sup s((q_1,q_2),u,\bar{c}(q_1)(u,v))=d_i(u,v)$

where s - display with definition of markup λ.

Then $\begin{matrix}d_n(u,v)=(\bar{c}(\hat{q}_1)\cdot\bar{c}(\hat{q}_1,\hat{q}_2)\cdot\bar{c}(\hat{q}_2)\cdot\bar{c}(\hat{q}_2,\hat{q}_3)\cdot\ldots\cdot\\ \bar{c}(\hat{q}_n)\cdot\bar{c}(\hat{q}_n,\hat{q}_1))(u,v)\le\max\{\nu_{\hat{q}_1}(\psi(u)),v\}\end{matrix}$

for all $(u,v)\in D\times[0,v_{\max}]$. Then $\max\{\nu_{\hat{q}_1}(\psi(u)),d_n(u,v)\}\le\max\{\nu_{\hat{q}_1}(\psi(u)),v\}$, and $c(\hat{q}_1)(u,d_n(u,v))\le c(\hat{q}_1)(u,v)$ where for all $(u,v)\in D\times[0,v_{\max}]$ performed

$\lambda(\hat{q}_1\hat{q}_2..\hat{q}_{n-1}\hat{q}_n\hat{q}_1)(u,v)\le\lambda(\hat{q}_1)(u,v).$

Then, by definition, $\le_{SD}$ (for fixed a $v_{\max}$ and set D), performed

$\lambda(\hat{q}_1\hat{q}_2..\hat{q}_{n-1}\hat{q}_n\hat{q}_1)\le_{SD}\lambda(\hat{q}_1).$

Thus the conditions of the theorem (1) performed as steady state y_* of the machine HA stable with level α.

The theorem is proved.

Let $d'\le d$ - natural number, where d - the dimension of space contiguous states HA. Let $L\in\mathrm{R}^{d'\times d}$ - matrix of rank d'. Consequently ρ_L - pseudo metrics on R^d, defined by equity $\rho_L(x,y)=(Lx-Ly)^T(Lx-Ly)$.

Definition 5. Steady state $y_*\in St(HA)$ is L-stability with level α, where $\alpha:(0,+\infty)\to[0,1)$ - function, which is defined in a neighborhood of zero, if for any number $\varepsilon>0$ there is a number $\delta>0$, such, that for all phase orbits $\chi=(\tau,q,y,x)\in Orb(HA)$, where $y=(y^i)_{i\in\langle\tau\rangle}$ and $\tau=(I_i)_{i\in\langle\tau\rangle}$, such that $\|y^0(\tau_0)-y_*\|\le\delta$ i $P\{x\}>\alpha(\varepsilon)$, condition $\rho_L(y^i(t),y_*)\le\varepsilon$ for all $i\in\langle\tau\rangle$ and $t\in I_i$.

Theorem 3. Let HA - FHMFS with a cyclic discrete switching states $\hat{q}_1 \to \hat{q}_2 \to ... \to \hat{q}_n \to \hat{q}_1$. Let for steady state $y_* \in St(HA)$ there is a tuple

$HL = \left(\alpha, (V_q)_{q\in Q}, (\nu_q)_{q\in Q}\right) \in HLo^L(HA, y_*)$.

Suppose the following conditions:

1) for each switch $q_1 \to q_2$ there is a number $\delta_{q_1 q_2} > 0$ and display $\vartheta_{q_1 q_2} : [0, \delta_{q_1 q_2}] \to \mathsf{R}^+$ such that $\vartheta_{q_1 q_2}(0+) = \vartheta_{q_1 q_2}(0) = 0$ and for all elements $u \in Dom\psi$ and payers $(y_1, y_2) \in J(q_1, q_2, u)$ inequality

- $V_{q_2}(y_2) \le \nu_{q_2}(\psi(u))$,if $V_{q_1}(y_1) \le \nu_{q_1}(\psi(u))$,

-$V_{q_2}(y_2) \le \vartheta_{q_1 q_2}(V_{q_1}(y_1))$,if $V_{q_1}(y_1) \in [0, \delta_{q_1 q_2}]$ i $V_{q_1}(y_1) > \nu_{q_1}(\psi(u))$;

2) $\lambda(\hat{q}_1 \hat{q}_2 ..\hat{q}_{n-1} \hat{q}_n \hat{q}_1) \le_{SD} \lambda(\hat{q}_1)$, there is s'-condition.

Thus steady state y_* machine HA L-resistant with level α.

Present conditions L- resistant with level α fuzzy impulsive hybrid automata.

Let $HLo^L(HA, y_*)$ - set of tuples $\left(\alpha, (V_q)_{q\in Q}, (\nu_q)_{q\in Q}\right)$, in which:

- $\alpha : (0, +\infty) \to [0,1)$ - function, which is defined in a neighborhood of zero;

- $(V_q)_{q\in Q}$ - indexed family of functions of class $DF(\mathsf{R}^d)$;

- $(\nu_q)_{q\in Q}$ - - indexed family of functions $\nu_q : \mathsf{R}^+ \to \mathsf{R}^+$, defined in a neighborhood of zero (included zero).

Let conditions Lo1, Lo2, Lo3, Lo5 and in addition, such condition (instead Lo4):

Lo4') if $V_q(y) \le \nu_q(w)$ for some $y \in DomV_q$ and $w \in Dom\nu_q$, then $\|Ly_* - Ly\| \le w.$

Theorem 4. (about L-resistant steady states not cyclical pulse FHMFS). Let automata HA not pulse, $(V_q)_{q\in Q}$ - family of functions of class $DF_0^\infty(\mathsf{R}^d, y_*)$, such that $O_q \subseteq DomV_q$ for each $q \in Q$, де O_q - some open superset $Inv(q)$, and for all $q_1, q_2 \in Q$, $y_1, y_2 \in Y$, $x \in X_+$, such that $(q_2, y_2) \in Jump(q_1, y_1, x)$, performed $y_1 \in DomV_{q_1}$ and $y_2 \in DomV_{q_2}$.

Let $(\nu_q)_{q\in Q}$ - family reflections $\nu_q : \mathsf{R}^+ \to \mathsf{R}^+$,

such as $\nu_q(0) = 0$ and $0 < \nu_q(w) < V_q(y)$ for all $w > 0$ i $y \in DomV_q$, such that $\|Ly_* - Ly\| > w$.

Suppose that for all $q \in Q$, $x \in X_+$ and $y \in Inv(q)$, such that $P\{x\} \in Dom\psi$ and $V_q(y_1) > \nu_q(\psi(P\{x\}))$ performed inequality

$$\nabla_{g_q} V_q(y) \le -\frac{1}{\kappa_w}\sqrt{\varphi_w^{-1}(P\{x\})}\left\|\nabla_{h_q} V_q(y)\right\|.$$

Suppose that for all arcs $(q_1, q_2) \in E$, $u \in D$ and $(y_1, y_2) \in J(q_1, q_2, u)$, such that $V_{q_1}(y_1) \le \nu_{q_1}(\psi(u))$, performed $V_{q_2}(y_2) \le \nu_{q_2}(\psi(u))$. Suppose also, that values $d_i(u,v) \in \mathsf{R}^+, i = \overline{1,n}$ definite for all $(u,v) \in D \times [0, v_{\max}]$ and inequality $d_n(u,v) \le \max\{\nu_{\hat{q}_1}(\psi(u)), v\}$ performed for all $(u,v) \in D \times [0, v_{\max}]$, where

$$d_1(u,v) = \sup\left\{\begin{array}{l} V_{\hat{q}_2}(y_2) \mid (y_1, y_2) \in J(\hat{q}_1, \hat{q}_2, u) \wedge \\ V_{\hat{q}_1}(y_1) \le \max\{\nu_{\hat{q}_1}(\psi(u)), v\} \end{array}\right\};\ d_2(u,v) = \sup\left\{\begin{array}{l} V_{\hat{q}_3}(y_3) \mid (y_2, y_3) \in J(\hat{q}_2, \hat{q}_3, u) \wedge \\ \wedge V_{\hat{q}_2}(y_2) \le \max\{\nu_{\hat{q}_2}(\psi(u)), d_1(u,v)\} \end{array}\right\};$$

$$d_3(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_4}(y_4)\,|\,(y_3,y_4)\in J(\hat{q}_3,\hat{q}_4,u)\wedge\\ \wedge V_{\hat{q}_3}(y_3)\le\max\{v_{\hat{q}_3}(\psi(u)),d_2(u,v)\}\end{matrix}\right\};\ldots..$$

$$d_n(u,v)=\sup\left\{\begin{matrix}V_{\hat{q}_1}(y_1)\,|\,(y_n,y_1)\in J(\hat{q}_n,\hat{q}_1,u)\wedge\\ \wedge V_{\hat{q}_n}(y_n)\le\max\{v_{\hat{q}_n}(\psi(u)),d_{n-1}(u,v)\}\end{matrix}\right\}.$$

Thus steady state $y*$ *not pulse cycle* FHMFS HA is L-resistant with level α.

Proof. We verify the conditions of the theorem (3). Prove that $\left(\alpha,(V_q)_{q\in Q},(v_q)_{q\in Q}\right)\in HLo^L(HA,y_*)$. The condition of the theorem implies that the conditions Lo1, Lo3, Lo5 run. Conditions Lo2 performed as $v_q(w)>0$ at $w>0$ i $v_q(0)=0$. Condition Lo4' executed because by theorem if $\|Ly_*-Ly\|>w$, then $V_q(y)>v_q(w)$.

The condition of the theorem implies implementation of the first inequality of condition 1 Theorem (3).

Verify the implementation of the other inequality of condition 1of theorem (3). Granted

$\delta_{q_1q_2}=v_{q_1}(0)>0$ and define the mapping $\vartheta_{q_1q_2}:[0,\delta_{q_1q_2}]\to\mathrm{R}^+$ equality

$\vartheta_{q_1q_2}(v)=\sup\{V_{q_2}(y)\,|\,y\in DomV_{q_1}\cap$

$\cap DomV_{q_2}\wedge V_{q_1}(y)\le v\}.$

Note that this value is determined for $y_*\in DomV_{q_1}\cap DomV_{q_2}$ and finite and function V_{q_2} locally limited, and set $V_{q_1}(y)\le v$ limited (for $v>0$) because V_{q_1} allows unlimited extension. Because $V_{q_1}(y)>0$ at $y\ne y_*$, to $\vartheta_{q_1q_2}(0)=0$. Because $\vartheta_{q_1q_2}$ monotonic, then $\vartheta_{q_1q_2}(0+)$ identified. Verify that $\vartheta_{q_1q_2}(0+)=0$. Really $\inf\limits_{\varepsilon>0}\vartheta_{q_1q_2}(v_{q_1}(\varepsilon))=0$, Because function V_{q_1} continuous on its domain i $V_{q_1}(y_*)=0$. But if $\vartheta_{q_1q_2}(0+)\ne0$, then $\vartheta_{q_1q_2}(0+)>0$ and then $\inf\limits_{\varepsilon>0}\vartheta_{q_1q_2}(v_{q_1}(\varepsilon))>0$, it $v_{q_1}(\varepsilon)>0$ at $\varepsilon>0$. Consequently $\vartheta_{q_1q_2}(0+)=0$.

If $(y_1,y_2)\in J(q_1,q_2,u)$ for some $u\in D$ i $V_{q_1}(y_1)\le\delta_{q_1q_2}$, then $y_1=y_2$, automata is not pulse $y_1\in DomV_{q_1}\cap DomV_{q_2}$. Then $V_{q_2}(y_2)\le\vartheta_{q_1q_2}(V_{q_1}(y_1))$ by definition $\vartheta_{q_1q_2}$.

Verify the condition 2 of the Theorem (3). As mentioned $d_i(u,v)\in\mathrm{R}^+, i=\overline{1,n}$ defined for all $(u,v)\in D\times[0,v_{\max}]$, then for all arcs (q_1,q_2),

$(\overline{c}(q_1)\cdot\overline{c}(q_1,q_2))(u,v)=$

$=\sup s((q_1,q_2),u,\overline{c}(q_1)(u,v))=d_i(u,v),$

де s - image from definition markup λ. Then

$d_n(u,v)=(\overline{c}(\hat{q}_1)\cdot\overline{c}(\hat{q}_1,\hat{q}_2)\cdot\overline{c}(\hat{q}_2)\cdot\overline{c}(\hat{q}_2,\hat{q}_3)\cdot\ldots\cdot$

$\overline{c}(\hat{q}_n)\cdot\overline{c}(\hat{q}_n,\hat{q}_1))(u,v)\le\max\{v_{\hat{q}_1}(\psi(u)),v\}$

For all $(u,v)\in D\times[0,v_{\max}]$. Thus $\max\{v_{\hat{q}_1}(\psi(u)),d_n(u,v)\}\le\max\{v_{\hat{q}_1}(\psi(u)),v\}$, and then

$\overline{c}(\hat{q}_1)(u,d_n(u,v))\le\overline{c}(\hat{q}_1)(u,v)$

For all $(u,v)\in D\times[0,v_{\max}]$ performed

$\lambda(\hat{q}_1\hat{q}_2..\hat{q}_{n-1}\hat{q}_n\hat{q}_1)(u,v)\le\lambda(\hat{q}_1)(u,v).$

Then by definition $\le_{SD}$ (for fixed a $v_{\max}$ and set D), performed

$\lambda(\hat{q}_1\hat{q}_2..\hat{q}_{n-1}\hat{q}_n\hat{q}_1)\le_{SD}\lambda(\hat{q}_1).$

Thus conditions of (theorem 3) are satisfied, then steady state y_* automata HA є L-resistant with level α.

The theorem is proved.

Conclusion

The paper examines the properties of fuzzy hybrid machines. For modeling of fuzziness used approach based on the theory of possibilities. The questions of stability and resistance on the part of variables (L-resistance) solutions fuzzy hybrid machines with cyclic changes in local conditions mentioned. Change happens when states achieved a certain trajectory set. The corresponding theorems proved.

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