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A study on position estimation algorithm using global positioning system satellites data

Ashutosh Srivastava*

Geoinformatics Department, Indian Institute of Remote Sensing, Dehradun 248 001, India

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An investigation is carried, to study the position estimation algorithm by a receiver using GPS satellites. The RINEX version 2.11 data were used for the analysis as input to test and establish an algorithm. The obtained position from the mathematical algorithm was compared with the reference position. Dilution of Precision (DOP) estimation is also carried out to analyse the geometry of global positioning system (GPS) satellites as observed by the receiver and its effect on position estimation. It is observed that, using a position estimation algorithm that is developed for this study, a good estimate of location is obtained and is found matching with the reference values closely and exhibited better estimates if compared with portable GPS receiver measurements. The results obtained after processing RINEX data shows the difference close up to 6th decimal places or up to centimeter-level for latitude and longitude.

Keywords: Mathematical algorithm, GPS, Position estimation

1 Introduction

The main components of Geospatial technology are to store and to manage large quantities of spatially distributed data for various geospatial analyses. The association of these data corresponds to their respective geographic features. For example, air quality data would be associated with a sampling site, represented by a point or polygon which consists of many points. Forest data will be connected with forest cover area represented on a map by polygons. In Georefrencing, position coordinates are assigned to each pixel of the raster data. These coordinates can be collected using a navigation device. It is required to know the precise geo-location in various geospatial analyses. Different navigation systems are in operation like global positioning system (GPS), global navigation satellite system (GLONASS), and Indian regional navigation satellite system (IRNSS), etc. These systems provide position information precisely. These constellations transmit their position and velocity to the ground-based receiver which is used further to estimate the receiver position. GPS is one of the available and oldest navigation satellite constellations to provide position information and plays a very important role in the management of geospatial requirements. It is a little complex technology for which several interface techniques have come up to make it more user's friendly. Very simple, low cost and easy handling devices are available within accuracies of 10 to 20 meters. If the device is more complex and designed for high precision requirements then it will cost more. These devices can provide very high-level accuracies. Because of advancement and development in technology more options are available to select GPS receiver. The Global Positioning System was set up by the U.S. Department of Defense. This was primarily used for military applications and further, it was extended to civilian use. This constellation consists of continuously orbiting satellites in medium earth orbits (MEO) that transmit radio signals. Receivers on the ground use these signals and estimate the position on the Earth's surface accurately. From each GPS satellite, radio signals are transmitted to Earth which contains specific information. The information includes the exact position of the satellite which is called the ephemeris information, date and time the signal was sent, and the information related to satellite health. The control segment observes the health of satellites, the orbital parameters, and signals strength.

Finally, the user segment of the GPS has different types of GPS receivers. The receiver will receive and process the signals received from all visible GPS satellites. For processing, by the receiver, at least four GPS satellites must be visible. GPS satellites

*Corresponding Author (E mail:asrivastava@iirs.gov.in)

communicate the receiver using two radio frequency links which are L1 (1575.42 Mhz) and L2 (1227.60 Mhz). Using these links GPS satellites send ephemeris information to the receiver which is used to estimate the receiver position. GPS transmits three types of ranging codes which are precision (P) code, Y-code, and coarse acquisition (C/A) codes. Precision (P) code is the principal navigation ranging code. Y-code is used in place of the P-code when an anti-spoofing mode of operation is initiated. Information is transmitted from the GPS satellites to the receiver. This is one-way communication only.

Many researchers have carried out modeling to determine the position of the receiver using different methodologies and to correct the position accuracies in their area of studies. Thompson[1] discussed the mathematics of GPS receiver in his work. Sigrist et al.[2] have worked out the impact of the forest canopy on the quality and accuracy of GPS measurements. The role of the Kalman filter in the modeling of GPS errors is studied by Malleswari et al.[3]. Rigorous modeling of GPS residuals errors was investigated for precise point positioning[4]. The event-based conceptual model was studied for context-aware movement analysis[5]. Algorithm for location-based services using indoor positioning techniques was determined[6]. Martikainen et al.[7] discussed the outlier-robust estimation of GPS satellite clock offset in their work. Hedgecock et al.[8] have worked for high accuracy differential tracking of low-cost GPS receivers. The study of carrier phase difference positioning with Kalman filter was carried out by Su et al.[9]. Mosavi et al.[10] has used least squares techniques for GPS receivers positioning filters using pseudo-range and carrier phase measurements. Wang et al.[11] have discussed generating routable road maps from vehicle GPS traces. Parthasarathy[12] also discussed about the positioning and navigation using GPS. Recently Varma and Nithiyananthan[13] have worked out the Matlab simulations based identification model for various points in the global positioning system. The main components of obstruction in GPS signals are ionospheric and tropospheric delays. These delays are investigated by various investigators. Van der et al.[14] has worked out the tropospheric delay in GPS navigation. Wang et al.[15] have studied about tropospheric delay estimation for pseudolite positioning. Allain and Mitchell[16] have worked out the ionospheric delay corrections for single-frequency GPS receiver over Europe using tomographic mapping. Ionospheric propagation effect on GNSS signals has been studied by Hoque and Jakowski[17].

Global Positioning System (GPS) plays an important role in geospatial technology as it is required in surveying and mapping which require precise position. Most of the places GPS continuously operating reference stations (CORS) are established for the support of these activities. CORS continuously collects the position information at the desired interval. A CORS was established in the Indian Institute of Remote Sensing (IIRS) campus which provides position information at 1-sec interval. Raw data are also converted into the receiver independent exchange (RINEX) format of different versions. In the present study, RINEX 2.11 data were taken from the CORS for the analysis. A mathematical algorithm was established to estimate the receiver position using RINEX data. RINEX data are available in three files, viz. Navigation message file, Observation data file and Meteorological data file. In Navigation RINEX data file, GPS satellite information is available. Since GPS ephemeris is updating every two hours, so the latest GPS satellite information as per the estimation requirement from the navigation data file is taken. In RINEX observation file, receiver end information is stored which contains frequency record, channel record, visible GPS satellite ranging information at the different channel, and at a different time interval. In the present study, C1 and P2 types of ranging information from the L1 and L2, which are available in the observation data file, were used to estimate the receiver position. In this work, the first GPS satellite position was estimated using navigation data in Cartesian coordinates. At the time of propagation satellite clock error, receiver clock error and signal travel time were accounted for. Once GPS satellite positions and its range of information from the receiver are retrieved, a linear least square method is used to estimate the receiver position. This receiver position was converted from Cartesian coordinates to its geolocation in latitude and longitude. The receiver information was computed at every 1-second interval. Finally, averaged value of all computed latitude and longitude is estimated after one-hour estimation. The GDOP value was also calculated to observe the effect of the geometry of the satellites on the receiver position. For the validation and accuracy assessment of obtained results, these were compared with CORS and portable receiver derived results. To retrieve the measurements from a portable receiver it is kept very close to the CORS receiver. Among all the outputs, mathematical algorithm showed a close matching with CORS receiver results and gave a better estimate if compare with portable receiver's results.

2 RINEX (Receiver Independent Exchange) data

Receiver Independent Exchange Format (RINEX) is a data interchange format in which navigation data are stored. This data were used for post-processing to obtain a more accurate results. This data can be processed along with other data like better models of the atmospheric parameters (temperature, pressure, and humidity) at the time of measurements. After processing of the RINEX data position of the receiver obtained. This was based on several observations taken from more than one satellite. The format of RINEX data was updated from time to time to include new types of measurements. Presently the most common version is 2.11. The latest version of RINEX format is 3.03. The RINEX version 2.11 format consists of three types of ASCII files, viz- Navigation, Observation and Meteorological data files. The contents of each file were categorized into two parts header section and a data section. In the header section, complete information for the entire file is stored. The header section file will be at the starting of the file followed by measurements. The RINEX data of 15 June 2016 were taken for the current study.

3 Mathematical formulation

3.1 GPS week number and seconds conversion from observation file

In observation file, ranging time is given in UTC, which is converted into GPS timing to bring GPS satellite ephemeris epoch and ranging epoch at common platform because GPS ephemeris in navigation files are given in GPS weeks and GPS seconds using the relation GPST = UTC + leap seconds. Leap second is a correction in UTC which is calculated based on Earth rotation time. At the time of this analysis, week number 1901 was running. GPS seconds information was also started to make out a given epoch within a week. GPS second information started from midnight on Saturday and Sunday. At the same time, new GPS week also started.

3.2 Conversion and propagation of satellite position

The GPS satellites position in Cartesian coordinates were estimated using following relations:

$$x_k = x'_k cos\Omega_k - y'_k cosi_k sin\Omega_k \quad \dots (1)$$

$$y_k = x'_k sin\Omega_k - y'_k cosi_k cos\Omega_k \quad \dots (2)$$

$$z_k = y'_k sini_k \quad \dots (3)$$

where, x_k, y_k and z_k are Earth-fixed coordinates, x'_k and y'_k are positions in orbital planeand Ω_k and i_k corrected longitude of ascending node and inclination respectively, which were estimated using the Keplerian elements and corrections provided in navigation data file.

3.3 Corrections

Corrections are incorporated while estimating the receiver position using GPS satellites data. More common corrections applied on data are given here as:

3.3.1 Onboard clock correction coefficients

The correction coefficients were transmitted in RINEX navigation message file. These correction coefficients account for the clock error characteristics in terms of bias and drift and satellite implementation characteristics in terms of group delay. The time was corrected using following equation

$$t = t_{sat} - \Delta t_{sat} \text{ (in GPS seconds)} \quad \dots (4)$$

where, t = GPS system time, t_{sat}= message transmission time, Δt_{sat} = time corrections

3.3.2 Atmospheric corrections

The velocity of GPS signals is affected by free electrons that are moving in the Ionosphere. When the GPS signal passes from this medium, it cannot travel at the same speed as it travels in a vacuum environment. Signal delay is proportional to the inverse of the square of the carrier frequency $\frac{1}{f^2}$. Klobucher model was used to correct ionospheric delay in ranging data. After Ionosphere when the signal travels through troposphere then also its velocity is affected by the medium. This interruption is due to variations in temperature, pressure, and humidity. The RINEX meteorological data file can be used to retrieve the meteorological parameters. In the present study, these values were incorporated using a tropospheric correction model.

3.4 Position estimation

To estimate the receiver location minimum four satellites are required, to compute the location. Suppose four GPS satellites location are

(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) *and* (x_4, y_4, z_4). Satellite transmit signal at time t and after some time receiver receives this signal at time t_1. Satellite pseudorange can be estimated by multiplying velocity of light into the travel time $(t_1 - t)$. Mathematically, this pseudorange includes geometric range, clock corrections, and atmospheric delays. Suppose the observed ranges for four satellites are r_1, r_2, r_3 *and* r_4. A least square method is applied to compute the receiver position (x, y, z).

4 Accuracy assessment

In the present analysis, accuracy assessment was carried out on two levels, firstly the position of the receiver obtained from using the established algorithm which is compared with the reference values at various levels. Secondly, DOP values were tested against values that can be considered as standard values for good estimation.

4.1 Comparison with receiver derived values

Receiver derived position available in RINEX output files were taken for comparison. The comparisons were made at following two levels:

- ✓ X, Y and Z coordinates in ECEF frame
- ✓ Latitude and longitude comparison

4.2 Dilution of Precision

The covariance matrix obtained from computing $(A^T A)^{-1}$ gives information about the geometric quality of the receiver position. If dilution of precision is smaller, then computed position will be more accurate. DOP values are dependent on the position of GPS satellites in space. Since the satellites are constantly in motion the DOP values keep on changing. Using following equations GDOP value can be obtained:

Covariance matrix is given by

$$(A^T A)^{-1} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{Ycdt} \\ \sigma_{YX} & \sigma_Y^2 & \sigma_{YZ} & \sigma_{Ycdt} \\ \sigma_{ZX} & \sigma_{ZY} & \sigma_Z^2 & \sigma_{Ycdt} \\ \sigma_{cdtX} & \sigma_{cdtY} & \sigma_{cdtZ} & \sigma_{cdt}^2 \end{bmatrix} \quad \ldots (5)$$

✓ Geometric Dilution of Precision (GDOP) is given by

$$\text{GDOP} = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + \sigma_{cdt}^2} \quad \ldots (6)$$

In similar way, Position Dilution of Precision (PDOP), Horizontal Dilution of Precision (HDOP), Vertical Dilution of Precision (VDOP) and Time Dilution of Precision (TDOP) were estimated.

5 Results and Discussion

A study on the position estimation algorithm was carried out to analyse the receiver functioning. The RINEX data of 15 June 2016 from the CORS was taken for the processing and to estimate the position. The results of the study are very well compared with the receiver generated results.

5.1 GPS satellites position

It can be observed that positions estimated in the current study are close to the IGS products which shows a good estimation. From the Fig. 1, numbers of satellites are shown which were initially visible to the receiver. As time elapsed, some of these satellites will disappear from the receiver and some new satellites will come into the receiver network but in any case, at least a minimum of four satellites should be visible to the receiver for its position estimation. Comparative results for the GPS satellite G16 on 15 June 2016 at 2:00 UTC are given in Table 1.

5.2 Receiver position

The receiver position was estimated using C1 and P2 type of ranging data observations. The analysis, indicated that, the estimated receiver position from the present study was very much close to the receiver position given in the RINEX data file (receiver derived results). When comparison was made between

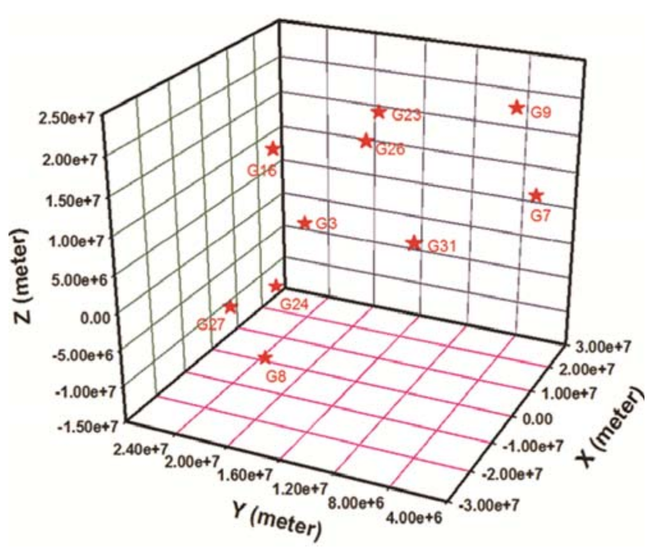

Fig. 1 — GPS satellite position at beginning of estimation process

the position computed from a present estimation using C1 type of ranging data which corresponds to 11:00 UT RINEX data file and the reference values, the difference between average values of X was found to be 1.829 meter. Similarly, for Y and Z value differences were -4.367 meter and -0.549 meter respectively. If P2 type of ranging data were used than the difference in X, Y and Z observed 1.464 meters, 2.448 meters and 2.587 meters respectively. The differences in latitude and longitude were estimated at the centimeter level for C1 type of ranging data, while for P2 type of ranging data this difference is observed at meter level. More observations are available for C1 type of data as compared to P2 type of range data which is also affecting the DOP values as well as the position accuracy. Results obtained from the algorithm, reference results from RINEX data file along with the difference between them using both ranging sources are represented in Table 2 to Table 6. From Table 2 and Table 3 it was observed that difference varied from fifth to sixth decimal places. From Table 4, Table 5 and Table 6 the difference was observed at meter level between both the sources. This shows that the results obtained from the current study are much close to reference parameters. A comparative study is also carried out between the observations taken from a portable GPS receiver and the measurements obtained from other sources, given in Table 2 to Table 6. This shows that measurements taken from the portable receiver have a slightly more deviation from the CORS measurements. In this case, estimated measurements were much closer to CORS measurements which provide a better accuracy.

Latitude, Longitude, and Cartesian position coordinates (X, Y, and Z) of the receiver are graphically represented in Fig. 2 to Fig. 5. For this, receiver position was estimated at 1-sec interval for

Table 1 — GPS satellite position comparison

	X (meter)	**Y(meter)**	**Z(meter)**
Estimated using RINEX data	-1880354.427	20940870.147	15880239.557
Reference position from IGS	-1880355.424	20940871.427	15880240.385
Difference	0.997	-1.28	-0.828

a. Estimated from RINEX data file
b. Mean value obtained from portable receiver
c. Mean value estimated from Mathematical algorithm used in present study
a-b. Difference between estimated values from RINEX data file and and mean value from portable receiver
a-c. Difference between estimated values from RINEX data file and and mean value estimated using mathematical model

Table 2 — Latitude comparison

Ranging data	Latitude (deg.)				
	a	b	c	a-b	a-c
C1	30.3410763527	30.3411121300	30.3410685073	-0.0000357773	0.0000078454
P2	30.3410763527	30.3411121300	30.3410628849	-0.0000357773	0.0000134678

Table 3 — Longitude comparison

Ranging data	**Longitude (deg.)**				
	a	**b**	**c**	**a-b**	**a-c**
C1	78.0465587998	78.0465297400	78.0465684276	0.0000290598	-0.0000096278
P2	78.0465587998	78.0465297400	78.0465868171	0.0000290598	-0.0000280173

Table 4 — X-component comparison

Ranging data	**X (meter)**				
	a	**b**	**c**	**a-b**	**a-c**
C1	1141177.1809	1141180.000	1141175.3515	-2.8191	1.8294
P2	1141177.1809	1141180.000	1141175.7162	-2.8191	1.4647

Table 5 — Y-component comparison

Ranging data	**Y(meter)**				
	a	**b**	**c**	**a-b**	**a-c**
C1	5390351.1840	5390349.00000	5390355.5516	2.1840	-4.3676
P2	5390351.1840	5390349.00000	5390348.7355	2.1840	2.4485

Table 6 — Z-component comparison

Ranging data	Z (meter)				
	a	b	c	a-b	a-c
C1	3203395.8732	3203400.0000	3203396.4222	-4.1268	-0.5490
P2	3203395.8732	3203400.0000	3203393.2857	-4.1268	2.5875

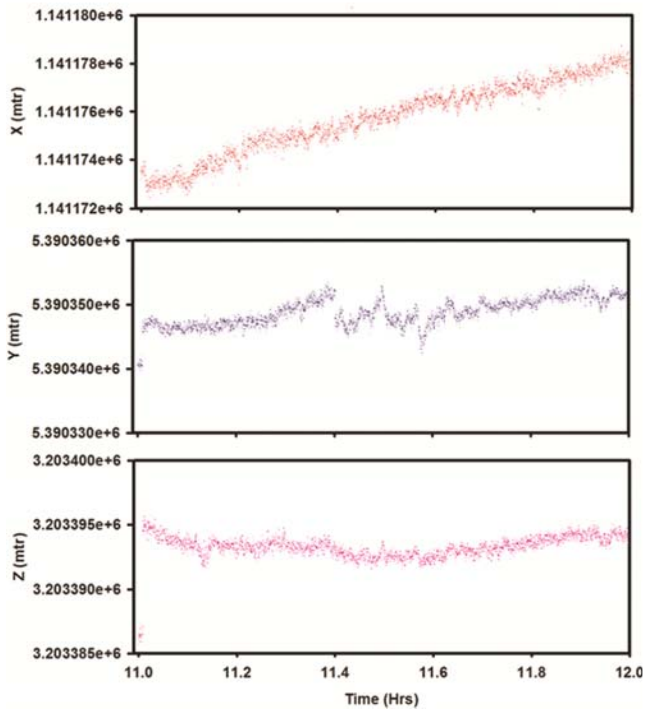

Fig. 2 — Estimated position of Receiver using C1 type of ranging data

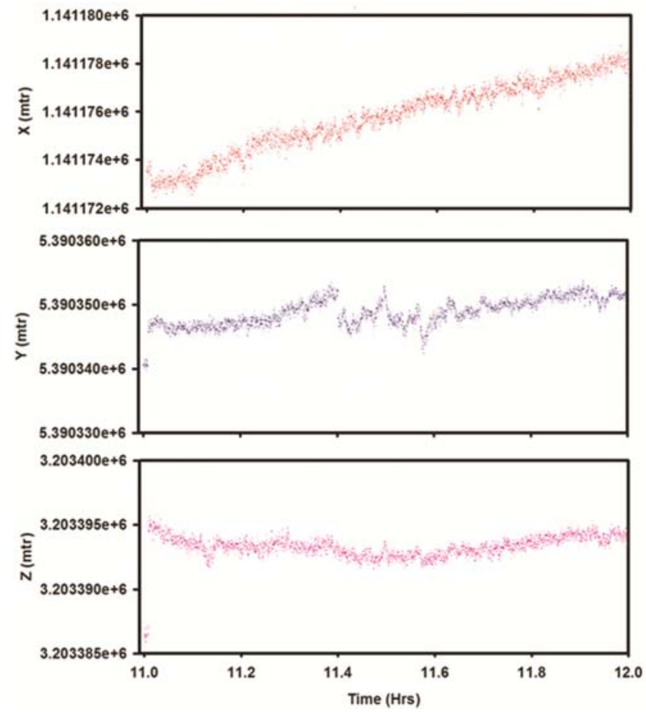

Fig. 3 — Estimated position of Receiver using P2 type of ranging data

the timing 11:00:00 UTC to 12:00:00 UTC. It was clearly observed from Fig. 2 to Fig. 5 that there was a continuity gap in the plot which can be explained by the fact that the number of the visible satellites to the receiver are different. From the Figs 6 and 8, it can be verified that whenever the change was observed from 7 satellites to 8 satellites or in a reverse way that was visible to the receiver, the continuity broken in latitude, longitude, and Cartesian position components. From the figures, it was also observed that at some of the places 9 satellites were observed. The receiver estimated the position using visible satellites. If number of visible satellites is more, the obtained receiver position will be much accurate and if it is less, accuracy will come down but in any case minimum, visible satellites should not be less than four.

5.3 Dilution of precision

From Figs 6 and 8, it was observed that, the GDOP and PDOP values obtained from both the ranging data sources were almost similar and within the range of 5 which reflects that the estimated results are good. The GDOP and PDOP values were near to 2 when the number of GPS satellites visible to the receiver was 8 and if this was 7 the GDOP and PDOP values were near to 3. This shows that if the number of GPS satellites visible to the receiver was more they will create good geometry which is resultant a less GDOP or PDOP values. In the present case, the GDOP and PDOP values were less than 5 in both cases whether visible GPS satellites were 7 or 8. This means satellite forming a good geometry in both the cases, but since GDOP value was less when the visible GPS satellites were 8, so this reflects a better geometry compared to other cases when the visible GPS satellites were 7.

The same signatures are seen from Figs 7 and 9 which represent the HDOP, VDOP, and TDOP values estimated at 1-sec interval in the analysis period. All values were observed near or less than 2. When visible GPS satellites were 7, the observed values

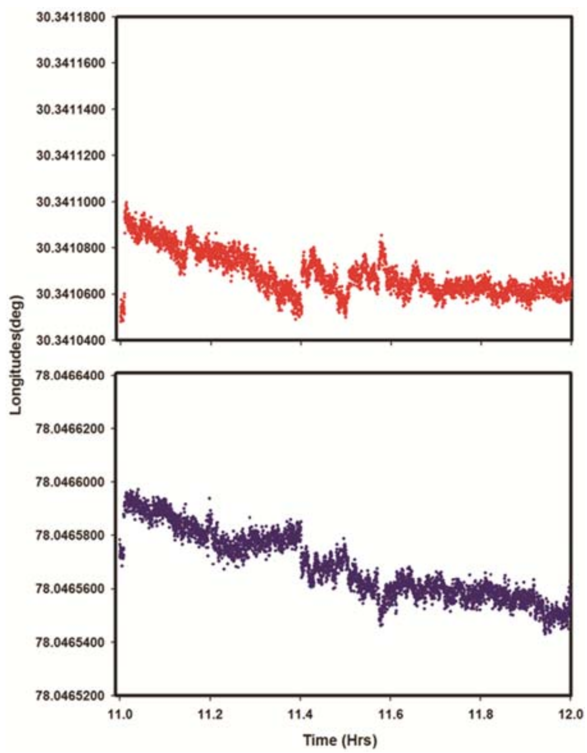

Fig. 4 — Estimated Latitude and Longitude using C1 type of ranging data

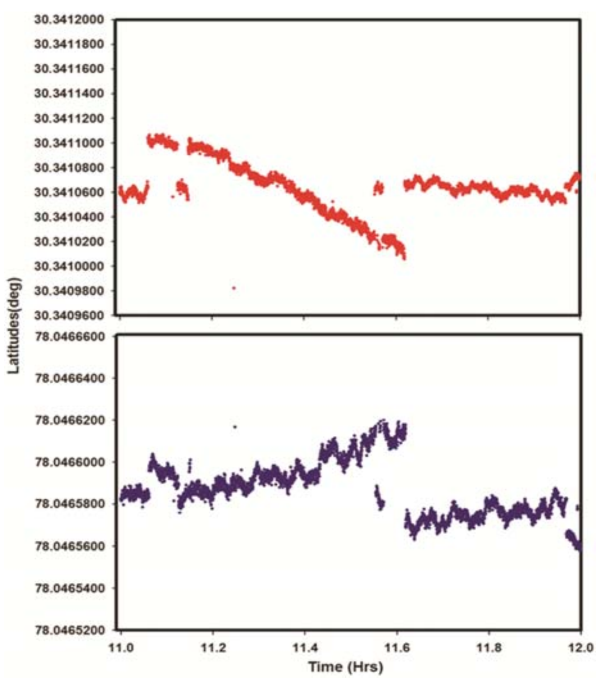

Fig. 5 — Estimated Latitude and Longitude using P2 type of ranging data

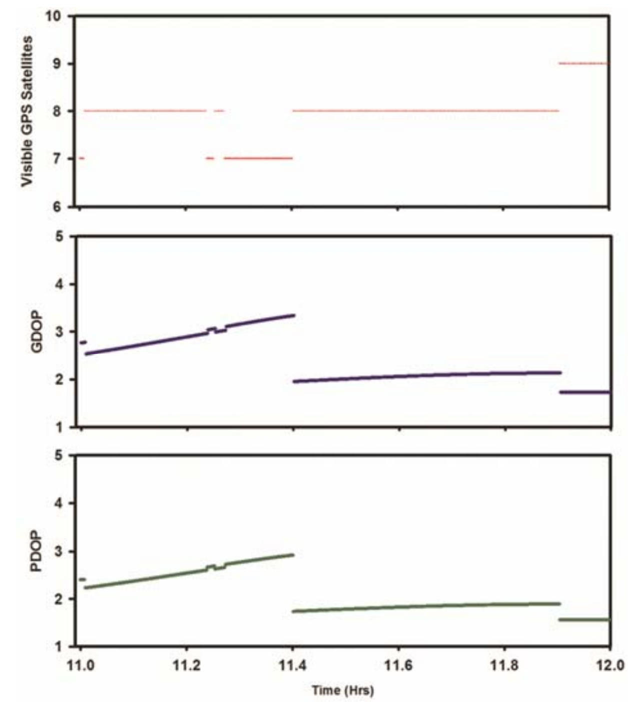

Fig. 6 — Visible GPS satellites and estimated value of GDOP and PDOP using C1type of ranging data

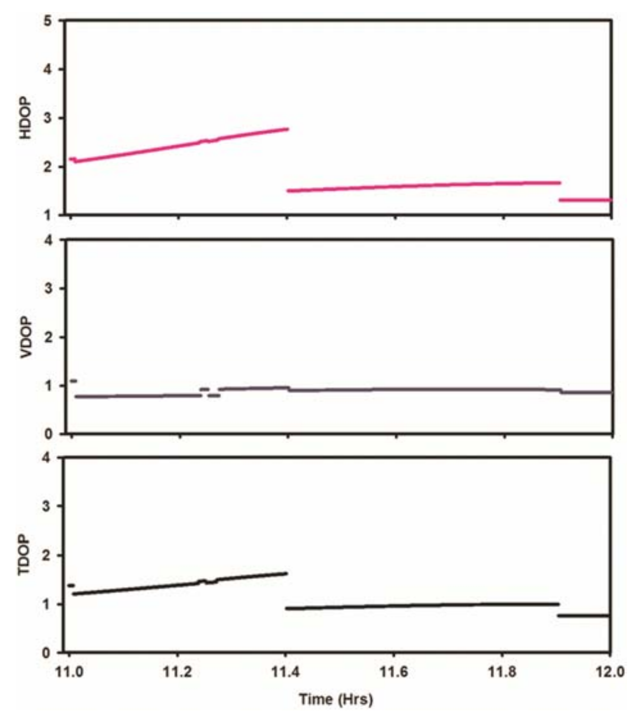

Fig. 7 — Estimated value of HDOP, VDOP and TDOP using C1 type of ranging data

were slightly higher than when the visible GPS satellites were 8. This also indicates the acquisition of good geometry. Estimation of GDOP, PDOP, VDOP, HDOP, and TDOP were done using C1 and P2 type of ranging data are shown in Table 7 and Table 8 respectively.

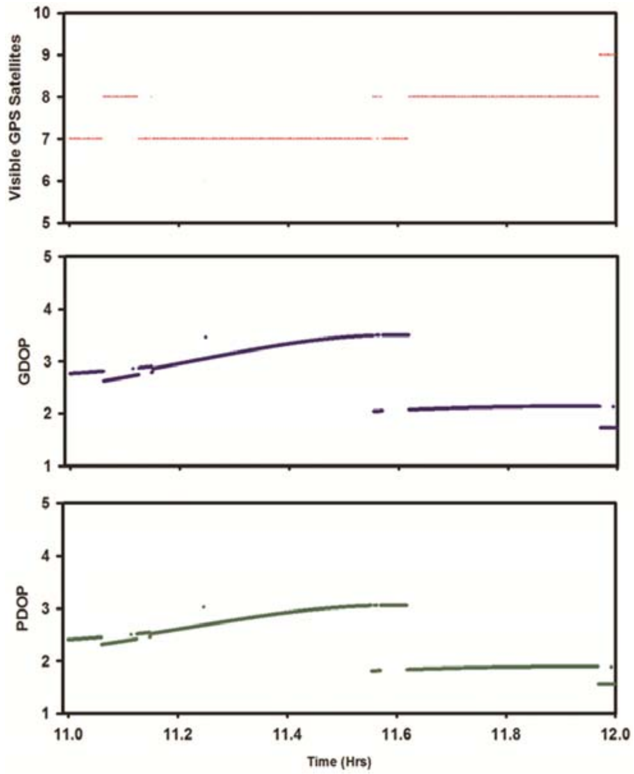

Fig. 8 — Visible GPS satellites and estimated value of GDOP and PDOP using P2 type of ranging data

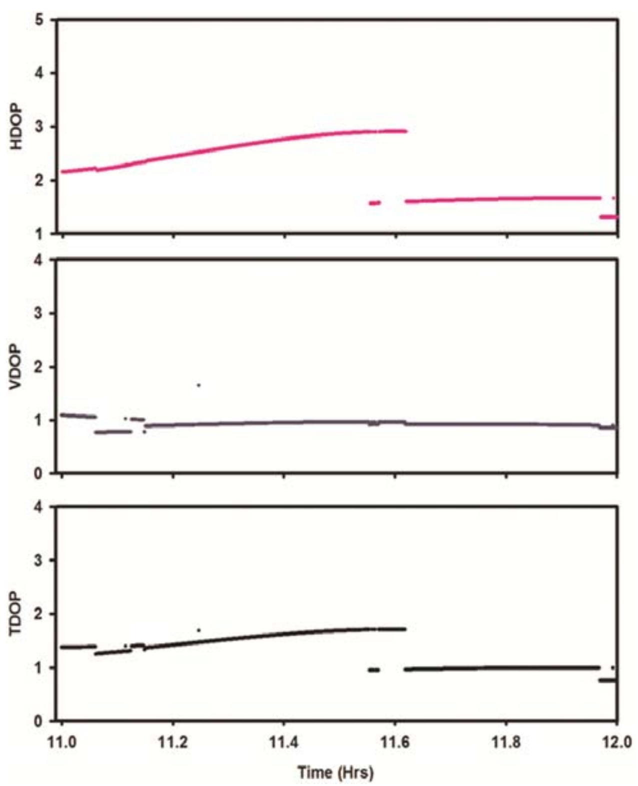

Fig. 9 — Estimated value of HDOP, VDOP and TDOP using P2 type of ranging data

5.4 Altitude

Altitude was also estimated using visible GPS satellites to the receiver. Like earlier two types of signatures were seen which depended on number of visible GPS satellites. The average value of altitude was estimated from C1 and P2 type of ranging data, were 657.694 meter and 664.968 respectively. These are close

Table 7 — GDOP and PDOP average values

DOP values			
GDOP		PDOP	
C1	P2	C1	P2
2.3735	2.7121	2.0955	2.3828

Table 8 — HDOP, VDOP and TDOP average values

DOP values					
HDOP		**VDOP**		**TDOP**	
C1	P2	C1	P2	C1	P2
1.8956	2.1904	0.8686	0.9169	1.1140	1.2948

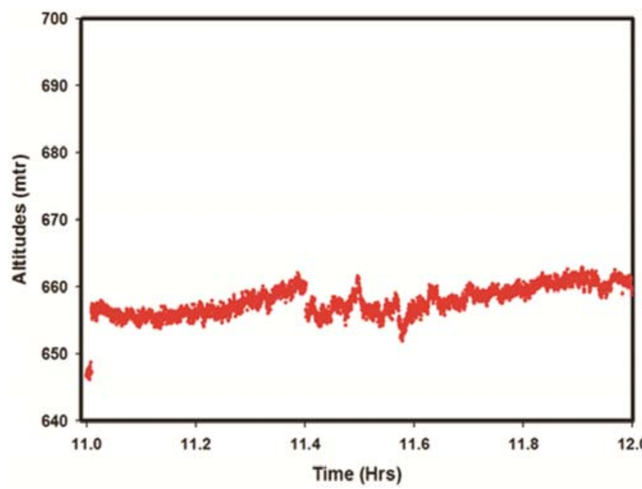

Fig. 10 — Estimated altitude value using C1 type of ranging data

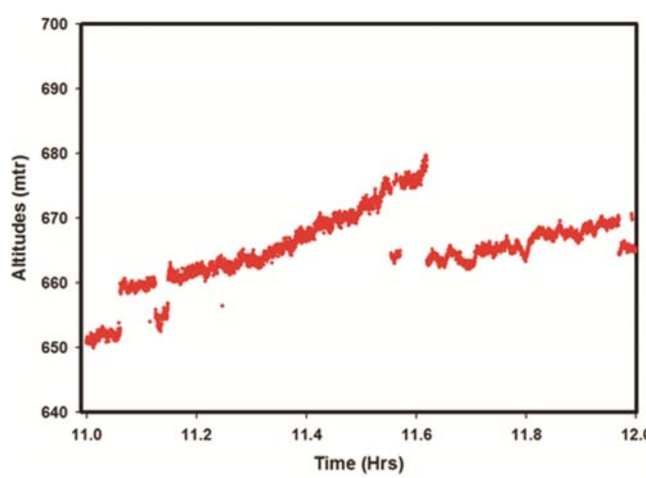

Fig. 11 — Estimated altitude value using P2 type of ranging data

to the reference value 660 meter. Altitudes estimated at 1-sec interval are shown in Figs 10 and 11.

6 Conclusions

A study on position estimation process by a receiver was carried out using GPS satellites data. This algorithm takes input from various RINEX data files and estimates the receiver position. RINEX version 2.11 data were taken for the analysis. The data were acquired from the CORS receiver setup at IIRS, Dehradun. Firstly, GPS satellite position was estimated using RINEX navigation message data for all visible GPS satellites to the receiver at the epoch given in the RINEX observation data file or at the time when the ranging was carried out by the receiver. In present case, ranging was available at 1-sec interval, so GPS satellite position was estimated at 1-sec interval. Once GPS satellite positions are available along with ranging information, the least square estimation method was applied to find the receiver position. This estimation was carried out using both the types of ranging data C1 and P2. Final results were well compared with the existing receiver derived position values available in RINEX data files at different levels for the comparison and validation. Accuracy of latitude and longitude were observed up to 6th decimal places. This accuracy is better if comparison is made with portable receiver measurements. The portable receiver measurements showed more difference with the RINEX data file results as compared to the algorithm results. Results were also verified using geometry-based analysis by evaluating GDOP and PDOP values. If the estimated GDOP and PDOP values were found less than the reference PDOP and GDOP values which is taken 5 then estimated GDOP and PDOP values showed a very good geometry, taken for the analysis and this was obtained by an algorithm. This analysis was carried out to taking one-hour data only which has limitations and subject to further improvement if more data are taken for the analysis. Based on estimated results and its comparison with receiver data results it can be concluded that algorithm used in current study provides good results, so it can be used in the various geospatial analyses for position estimation.

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