Gua¯nyu¯ yo¯uxia¯n, cuòwù hé zhèngquè – ga¯o dé nà, 'ga¯o dé nà', Wu wénjùn, jí suànfa

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论吴文俊的数学史业绩

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内容提要

本书为纪念吴文俊先生诞辰一百周年的文集。本文集共收入36篇文章，从不同视角论述吴文俊先生在中国数学史研究领域的学术思想及其影响，回忆、缅怀吴文俊先生支持数学史事业的各种事迹，颂扬吴文俊先生对中国数学史事业做出杰出贡献的高尚品德，表达数学史学界对吴文俊先生的敬爱与景仰之情。

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关于优先、错误和正确

——高德纳、"高德纳"、吴文俊，以及算法

◎ 延斯·霍伊鲁普(Jens Høyrup)(丹麦)

(丹麦罗斯基尔德大学)

首先必须声明我不会中文。我在此谈论的是一个可以通过我所熟知的语言来研究的主题。确切地说，本文所依据的材料的语言是英语和法语。①

我从胡吉瑞(Jiri Hudecek)所著吴文俊政治传记中的几段话谈起：[Hudecek 2014：117*f*]

> 尽管希腊几何学是公理化的，但公理化并不以其在现代数学中的同样方式来作为希腊几何学的工作方法。也许是因为这些问题，吴在其后来的作品中引入了证明与算法的对立来作为替代。② 计算机科学家高德纳(Donald Knuth)的作品是这一转变中的关系重大的影响因素。吴研究过高德纳所编写的计算机教材《计算机程序设计的艺术》(*The Art of Computer Programming*，首卷问世于1968年，其后1969年第二卷，1973年第三卷)，该教材由带有评注的算法组成，正如中国古代数学经典由问题-解答及其方法所组成。高德纳还发表过一篇关于古巴比伦算法的文章[Knuth 1972]。尽管吴文俊本人并没有引用高德纳的这篇文章，吴文俊的年轻同事李文林和袁向东[1982]在与吴文俊发表过文章[1982c]的同一卷文集内提到了该文。李文林，以及曾于1980年代初在北京学习中国古代数学的林力娜(Karine Chemla)，都向我证实了该文对吴文俊想法的影响。[个人通信]
>
> 吴也可能从高德纳的卷首语中得到了启发："帮助计算机科学得到尊敬的方法之一是展示它深深地根植于历史、并不是一个年幼的现象"。吴文俊本人转向历史有着类似的动机，尽管也可以说吴的行动方向与高德纳是相反的：通过展示那些根植于中国古代数学的东西来使得中国古代数学获得尊敬。高德纳对古巴比伦数学与计算机科学作了一系列类比：古巴比伦数学的六十进制计数法实际上是第一个浮点计数法；其代数算法是"机器语言"，对立于现代代数的"符号语言"；古巴比伦数学使用数值代数而不论其物理或几何意义。高德纳还将某些古巴比伦算法与"栈式计算机"或"宏展开"作对比。高德纳的这篇文章并不是严肃的算法史论文，它更是对于计算机科学之基本技术的久远先辈的一个提醒。但高德纳文章的最后一段对吴文俊来说一定非常有提醒性：
>
> > 其他那些发展又是怎样的呢？古埃及人数学不错，考古学家也已经发掘出一些与巴比伦泥板几乎同样古老的纸草书。古埃及的乘法，本质上基于二进制系统，(……)尤其有趣(……)。然后是古希腊人，他们虽着重几何学，但也有欧几里得算法这样的东西；而该算法是最古老的非平凡算法。(……)还有古印度人和中国人；显然有多得多的东西可以讲。(Knuth 1972：676)
>
> 相比高德纳，吴仅强调了中国数学，显得狭隘了。也许甚至可以猜想，吴随后尝试证明一些中国古代算法相较西方算法在计算上的优越性的同时也是在试图剥夺古代文明"对手"们的一些荣耀。

① 本文之引文均为英文，都由译者一并译为中文—译者注。

② 即替代了"公理化-机械化"的两分法，胡吉瑞又或吴文俊现在或过去已经视此两分"从历史角度来看不甚令人满意"(胡吉瑞语"not quite satisfactory from the historical perspective")。

高德纳的《计算机程序设计的艺术》是否关系重大的影响因素，对这一点我没有任何看法，而且它也与本文主题无关，无论如何它看起来只关乎从"机械化"到"算法"的术语转变。① 但是说"(高德纳对巴比伦数学的算法化解释)对吴文

① 我突然想到达尔文《物种起源》第六版提到斯宾塞言"最适者生存"的那些话[1872：49]——即达尔文说斯宾塞的这一表达"更精确，并且有时候同样合宜"。这一暧昧的赞扬对于达尔文自己的理论当然没有任何影响，甚至不值得列入索引。

胡吉瑞更早的时候[Hudecek 2012：55]谈到吴文俊使用机械化方法来发展现代数学知识的努力时，只是说"吴也许看到了计算机科学家高德纳的著作这一积极范例"，并没有宣称高德纳著作是关系重大的影响因素。

俊思想的影响"得到确证,倒是引起了我的怀疑。首先,根据胡吉瑞自己的一些引证[Hudecek 2014: 116-119],吴并不吝于承认从他人获得的教益。其次,吴文俊强烈怀疑将"代数"归功于古巴比伦人的观点,①以及斯特洛伊克(Dirk Struik)的这一信念:存在一体而不可分的"东方"数学。[Struik 1948: I, xii and *passim*]而高德纳既不怀疑古巴比伦代数的真实性,又暗示古巴比伦、埃及、印度和中国数学共属于同种(见前文胡吉瑞引高德纳语)。吴文俊当然很可能在某时已经知道高德纳1972年的文章,但在此事上,他看起来遵循了这样一个原则:用我的老朋友泰斯巴克(Marinus Taisbak)写给我的私人信件里的话来说,就是"为世界和平之利益,就不引述我所不同意的那些了"。无论如何,吴文俊自传里的一段话[Wu 2017]表明他对中国古代数学的算法化(一开始称为"机械化")本质的洞见应当早于他被认为接触到高德纳文章的时间:

> 文化大革命期间,我被派到一个工厂制造计算机。我一开始震撼于计算机的能力。我还花时间研究中国古代数学并开始理解中国古代数学真正是什么。中国古代数学思想和方法的深刻和强大令我深为震惊。在这一影响下,我开始考察用机械化方法证明几何定理的可能性。

巴 比 伦

接下来让我们更仔细地看看高德纳关于古代算法说了什么,以判断其论述的质量,首先将其置于所引材料的年代背景来看,其次简短地从目前的算法观念(不过这多少有点时代错置)来评价他的论述。

如胡吉瑞所说,高德纳的文章并不是严肃的历史写作(我们还可以补充说既不是算法的历史也不是其他什么东西的历史);而高德纳本来就没打算把这文章写成历史论文。他开门见山地说:[1972: 671]

> 帮助计算机科学得到尊敬的方法之一是展示它深深地根植于历史、并

① 如果吴文俊当时能获得诺伊格鲍尔(Neugebauer)或丹然(Thureau Dangin)的《古巴比伦数学文书》编本,他的立场也许会更不那么激烈。但这只是一个假设的历史,只是为了强调他当时读不到这些本子。

> 不是一个年幼的现象。因此自然要去看看存世的最古老的处理计算的文献,研究几乎四千年前的人们是怎么处理这一课题的。

高德纳的核心论点依托于对诺伊格鲍尔译文(当时可得的最佳译文——高德纳为了使其更易理解又做了一些调整)中的一些逐步计算的描述,在这一描述中,巴比伦文本看起来在指示一个单纯由算术步骤构成的过程。这些被用来说明:[p. 672]

> 巴比伦数学家[…]擅于解许多类型的代数方程。但是他们没有和我们所使用的一样显明的代数符号;他们用一列逐步的估值规则来表达每一个公式,即用计算该公式的一个算法来表达该公式。实际上,他们用公式的"机器语言"表达来工作,而不是用符号语言。

同时,高德纳抱怨说他的例子[p. 674]只表达了

> "直线式的"计算,而不涉及任何的分支和决策。为了构造计算机科学家看来真正的非平凡算法,我们需要有一些影响控制流的操作。

高德纳忽略了这一点:恰恰是这一直线性允许他将一个计算——必然是无分支的——与他认为该计算所表达的算法视作同一。① 为了得到某种像带有停止标准的循环的东西,高德纳指向了关于复利的那些问题,高德纳说[Knuth, p. 674],在复利问题里,"冗长而相当笨拙的过程读起来近乎类似一个宏展开"——显然只在熟悉"宏"的人眼中看来如此。巴比伦计算者只不过一再重复同一计算而已。

高德纳还评述说:[p. 674]

> 我们找不到像"Go to step 4 if x<0"这样的测试语句,因为巴比伦人没有负数;我们甚至找不到像"Go to step 5 if x=0"这样的条件测试语句,因为他们并不把零当作数!没有这样的测试语句,取而代之地,事实上有分别的算法来对应不同的情况。(例如,[MKT I, 312-314]中的一个算法与另一个实际上步步相同,只不过其中一个因参数之一为零而更简单。)

其实我们也找不到任何像"Go to step 4 if x>10"的测试语句——没有数字零与负

① 如果一系列计算对应于一个实际上有分支的算法,那么通过提及在分支点应该做什么的决策准则,就可以对所采用的路径做出解释。这样的语言是存在的——巴比伦数学文本中规律性地出现这样的话来解释一些步骤:"因为他[设计问题的师傅]说过",只不过这样的语言解释没有一例是对应于分支点决策情况的。

数并不是巴比伦数学文本不含决策或分支的原因。没有就是没有。而存在"分别的算法来对应不同的情况"表明高德纳的算法概念用于巴比伦文献时是空洞的,并且掩盖了相关文本的真正特征。巴比伦文本所包含的例题既要是示范性的,也要以必然的灵活性来应用。① 一些文本甚至展现了不同的解法是可能的—于是古巴比伦文书 AO 8862[MKT I, 108 - 117]的头三题本可以用同一方法来解,但文本展示了三种不同技巧分别来解这三题。

高德纳不可能知道的是巴比伦文书并不是如诺伊格鲍尔和丹然的译文所示那样"指示单纯由算术步骤构成的过程",所谓的"代数"文本实际上指示了方形几何面积的割补操作,在这一操作中,计算过程的正确性与基于符号的简单方程代数一样直观显然—具体例子可参阅[Høyrup 2017]。不过这是另一件事,与今天所谈问题无关。高德纳在 1972 年时可能已经知道的是公元前一千纪晚期的巴比伦数理天文学中描述行星运动的"之字形函数"(zigzag functions)应该是使用(带有分支的)固定算法来计算的—参阅[ACT I, 30 - 32]。星表本身并没有解释相关算法,的确,只是宣称数字由相关算法算得。不过,天文"程序文本"解释了毫无疑问可称为算法的东西,经常带有"DO ... WHILE"类型的决策语句;一些例子(大部分是残篇)见于[ACT I, 186 - 276]。一个保存得相当好的样本由布莱克-贝恩松(Lis Brack-Bernsen)和洪格尔(Hermann Hunger)发表[2008]。因此在较晚的时期写作了这些文本的巴比伦天文学家-僧侣充分能够算法化地思考—但这并不必然要求比他们早 1200 到 1700 年的书吏-教师也如此。②

下面这段话也不能被采纳为古埃及、印度和中国数学包含算法的论据:[Knuth 1972: 676]

> 其他那些发展又是怎样的呢?古埃及人数学不错,考古学家也已经发掘出一些与巴比伦泥板几乎同样古老的纸草书。古埃及的乘法,本质上基于二进制系统,(尽管他们的计算是十进制的,使用类似罗马数字的东西)尤其有趣;但在其他方面,他们使用笨拙的"单位分数"使他们远远落后于巴比伦人(…)。还有古印度人和中国人;显然有多得多的东西可以讲。

① 任何想要尝试引入"灵活算法"之类概念的人(高德纳可没这么做!),不妨试试在不使用明确分支的情况下写出一个"灵活的"计算机程序。
算法,根据定义,是非灵活的(not flexible),用吴文俊的原话来说,是机械的(mechanical)[Hudecek 2014: 117*f*]。

② [Høyrup 2018]更细致地按今天的算法概念讨论了巴比伦数学中的算法这一问题。

也不能认为高德纳掌握更多的信息而只是选择了不具体展示。巴比伦文书集成[MCT, MKT, TMB]除外,其参考文献里谈到历史的条目只有高中生读本艾伯伊(Asger Aaboe)的《早期数学史片段》(*Episodes from the Early History of Mathematics*)[1964],以及两本科普书:诺伊格鲍尔的《古代精密科学》(*Exact Sciences in Antiquity*)[1957]和范德瓦尔登(B. L. van der Waerden)的《科学苏醒》(*Science Awakening*)[1954],这三本书都无法为他在相关问题上提供更多的信息。①

实际上,高德纳如此明智,必不至于宣称古埃及、印度和中国数学包含算法。最后这段话只不过是在呼应文章的开篇,呼吁人们"去看看存世的最古老的处理计算的文献,研究几乎四千年前的人们是怎么处理这一课题的"。胡吉瑞用来将吴文俊比作狭隘的那个"高德纳"是肤浅理解的产物。

古 代 中 国

现在让我们看看吴文俊是如何研究中国古代数学的。显然,如果像高德纳阅读美索不达米亚文献那样阅读《九章算术》之类的文本,那么包含计算指示的后者因而可以被宣称"含有算法"。胡吉瑞看起来就是在这个层次上理解吴文俊对中国古算的"算法化"观点的——至少胡吉瑞 2014 年[Hudecek 2014: 130]的解释没有超出这个范围。然而,吴文俊提到"中国古代数学思想和方法的深刻和强大令我深为震惊",即便对于那些没有能力阅读他以中文发表的数学史著作的人而言,这也表明了他是在另一个层次上动心和行动的。而当高德纳试图以巴比伦数学来支持现代计算机科学的悠久历史时,抱怨巴比伦数学只有直线式计算的肤浅,这实际上使其不可能深刻也无法特别强大。

① 另一个问题是能否应用算法分析古代美索布达米亚和法老时代的数学文本,里特(Jim Ritter)[2004]和伊穆豪森(Annette Imhausen)[2003]已经令人信服地展示了这是可行的。但两位学者都没有宣称所分析的材料本身是由算法构成。

首先从《九章算术》里陈述计算程序(如开平方和开立方)的方式谈起。如林力娜(总结她更早的工作时)所说[1991: 75],这些陈述有时候"使用了迭代、条件和赋值:这三个办法恰恰被高德纳在《基本算法—计算机程序设计的艺术》中列为基本概念"。并且,与高德纳强解说美索不达米亚数学文本里有条件句相比,条件句真实地存在于中国古代数学文本中—[Chemla 1987]的附录提供了相关文句的译文以及对应于相关段落的流程图。

此处不再深入细节—林力娜在此课题上从不同角度做出了比我所可能做到的好得多得多的工作,而且这不仅仅是因为她能够阅读原文。我将从"算法"问题移步到"算法文化"(algorithmic culture)问题,吴文俊强调的更是后者,尽管他一开始使用了不同的术语。

胡吉瑞复述或解释吴文俊的话语时说,"尽管希腊几何学是公理化的,但公理化并不是希腊几何学的工作方法",此处的"工作方法"指十九世纪初以来的并被布尔巴基符号化了的那种公理化方式(吴文俊作为职业数学家肯定熟知布尔巴基)。从另一个方面看,公元前四世纪希腊几何学中发展起来的公理化文化,自欧几里得时代以来成为具有支配地位的思想观念[Høyrup 2018]。在这之前的数学文化里,虽然必须有论证,但论证的起点只是"(在某)局部显见(的东西)"(locally obvious),而不诉诸绝对的基本原则(first principles)。①

巴比伦数学文化与此不同。它在教学性解释中也会使用"局部显见",但总体来看并不明言(不过也许口头教学中会说)。其主要教学目的,如前所述,是通过示范性例题来训练学生,使他们能够尽可能灵活地运用范例所涉技巧—即高德纳所看见的"不同情况下使用不同的算法"。

而中国古代数学与以上几种都不一样—中国古算之理念,即便是在想传递理解的时候,也是"算法化的"(algorithmic)。这一点可以用《九章算术》第三卷[ed., trans. Chemla & Guo 2004: 280 - 311]来说明。该章处理按"程度"(degrees)—根据刘徽注,也就是按"级别"(ranks)—来进行分配。【《九章算术》卷第三"衰分",刘徽注"衰分,差也"—译者注】

① 如希波克拉底研究月牙形时所使用的两条原理是"毕达哥拉斯定理"和面积比等于特征线之比的平方。这两条原理至少自公元前第二千纪以来就在实用量地几何学里得以应用。参阅[Høyrup 2019]

在法老时代的埃及,此类问题是周知的—莱茵德纸草书第 63 题[trans. Peet 1923: 107]是为一例。计算法则也是一样的。而且该类型问题在中世纪晚期和早期现代欧洲的商业算术里也很常见,被归类在"公司法则"(rule of company)题类中,在这些材料里,解题技法也是一样的。但是《九章算术》有一点与众不同,而这一点恰恰反映了数学文化间的区别。《九章算术》第三卷一上来就用抽象术语给出了解题法则——个(未分支的)算法,而接下来的若干例题,则是该法则之真正应用。这是《九章算术》的数学文化是"算法化的"的第一条线索。

在此意义上,婆罗摩笈多(Brahmagupta)和婆什伽罗二世(Bhaskara II)之间的梵文数学文本也经常(但并不总是)算法化的。但《九章算术》第三章还有更多独特之处。在第一、第三和第五题里,"权重"是立刻给出的,章首所述算法可以如常应用。① 而在第二题里,三者权重并没有直接给出,只告诉说两个比是 1 : 2 和 1 : 2—并且在第四题里,五个权重两两之间的比依次是 1 : 2、1 : 2、1 : 2、1 : 2 和 1 : 2。显然,应该先把各自权重计算出来,即第二题为 1 - 2 - 4,第四题为 1 - 2 - 4 - 8 - 16。但是,两题都没有解释这预先的计算,计算结果直接就被宣布了。也就是说,文本只解释章首算法所覆盖的那部分计算,算法之外的计算也就被排除在解释之外。这样看来,文本的目的是教算法—比教会如何计算更高的目标并不直接可见,但我们可以看到该目标是以算法为中介来进行的。

算法的中心性并不只是《九章算术》这一个作品的特征。尽管《九章算术》是最重要的经典之一,毕竟不能凭一部经典就说是数学文化。中国古代数学是实作(practice),而经典在其中发挥了重要作用。但不能就此将二者混为一谈。不过,编撰于 1060 年的《新唐书》描述了比《九章算术》晚了几乎一千年的科举考试,特别指出了考生们的任务之一是构造算法(construct algorithms)(引文见[Siu & Volkov 1990: 92*f*])。对经典的注疏(如刘徽注)也解释了为什么算法是有效的,而新算法的创造(不特别地属于新的数学知识)是好几个作者吹嘘的东

① 权重并不总是级别,但刘徽已经在注释里解释了当"衰"是家庭人数时如何运用衰分法,即每个人分到的应该相等。

西(引文同上 p. 94)。因此,正如从古希腊风格到现代的数学倾向于将发现定理①视为主要任务,中国古代数学眼中的任务是构造算法—而且显然是可证明为正确的且连贯协调的算法。

这样,从汉至唐的中国古代数学即便在此意义上也是算法化的,而在此意义上,古代印度、美索不达米亚和法老时代(以及中世纪阿拉伯和欧洲)的数学都不是算法化的。

因此,认真的局外人必定得到这样的结论:吴文俊是正确的——不仅仅在于他对中国古代数学特征的概括,还在于他认为此特征是中国古代数学所独有的。

同一个局外人(现在变成局内人了)必须下此结论:高德纳是错误的——批评吴文俊比所谓"高德纳"更狭隘则是错上加错。

以此二结论为条件,谁先谁后就没什么可讨论的了。毫无疑问明希豪森男爵在天上骑着炮弹时就已经是第一了,而怀特兄弟只是第二。②

参考文献

[1] Aaboe, Asger. Episodes from the Early History of Mathematics [M]. New York: Random House, 1964.

[2] ACT: Otto Neugebauer. Astronomical Cuneiform Texts: Babylonian Ephemerides of the Seleucid Period for the Motion of the Sun, the Moon, and the Planets [M]. London: Lund Humphries, 1955.

[3] Brack-Bernsen, Lis, Hermann Hunger. BM 42484 + 42294 and the Goal-Year method [J]. SCIAMUS, 2008, 9: 3 - 23.

[4] Chemla, Karine, Guo Shuchun (eds). Les neuf chapitres. Le Classique mathématique de la Chine ancienne et ses commentaires [M]. Paris: Dunod, 2004.

[5] Chemla, Karine. Should They Read Fortran As If It Were English? [J]. *Bulletin of Chinese Studies* 1, 1987: 301 - 316.

① 一个定理显然要有证明,如果没有证明,我们通常只称猜想,除非发明者是费马。

② 明希豪森男爵(Baron von Münchhausen)是 18 世纪小说中的虚构人物,小说中他骑着炮弹和巨人搏斗并旅行到月球—译者注。

[6] Chemla, Karine. Theoretical Aspects of the Chinese Algorithmic Tradition (First to Third Centuries)[J]. *Historia Scientiarum* 42, 1991: 75 - 98.

[7] Høyrup, Jens. *Algebra in Cuneiform: Introduction to an Old Babylonian Geometrical Technique*. Berlin: Edition Open Access, 2017.

[8] Høyrup, Jens. Was Babylonian Mathematics Algorithmic? [J]. 2018: 297 - 312 *in* Kristin Kleber, Georg Neumann & Susanne Paulus (eds), *Grenzüberschreitungen: Studien zur Kulturgeschichte des Alten Orients*. Festschrift für Hans Neumann zum 65. Geburtstag am 9. Mai 2018. Münster: Zaphon, 2018.

[9] Høyrup, Jens. From the Practice of Explanation to the Ideology of Demonstration: an Informal Essay[J]. Forthcoming in Gert Schubring (ed.), *Interfaces between Mathematical Practices and Mathematical Education*. New York: Springer, 2019.

[10] Hudecek, Jiri. Ancient Chinese Mathematics in Action: Wu Wen-Tsun's Nationalist Historicism after the Cultural Revolution[J]. *East Asian Science, Technology and Society* 6, 2012: 41 - 64.

[11] Hudecek, Jiri. *Reviving Ancient Chinese Mathematics: Mathematics, History and Politics in the Work of Wu Wen-Tsun* [M]. London & New York: Routledge, 2014.

[12] Imhausen, Annette. *Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten* [M]. Wiesbaden: Harrassowitz, 2003.

[13] Knuth, Donald E.. "Ancient Babylonian Algorithms". *Communications of the Association of Computing Machinery* 15, 1972: 671 - 677, with correction of an *erratum* in 19, 1976: 108.

[14] MCT: Otto Neugebauer & Abraham Sachs. *Mathematical Cuneiform Texts* [M]. New Haven, Connecticut: American Oriental Society.

[15] MKT: Otto Neugebauer, *Mathematische Keilschrift-Texte* [M]. 3 vols. Berlin: Julius Springer, 1935, 1935, 1937.

[16] Neugebauer, Otto, 1957. *The Exact Sciences in Antiquity* [M]. Second edition. Providence, Rh.I.: Brown University Press.

[17] Peet, T. Eric. *The Rhind Mathematical Papyrus, British Museum 10057 and 10058*[M]. Introduction, Transcription, Translation and Commentary. London: University Press of Liverpool, 1923.

[18] Ritter, Jim. Reading Strasbourg 368: A Thrice-Told Tale[J]. Karine Chemla (ed.), *History of Science, History of Text*. Dordrecht: Kluwer, 2004: 177 - 200.

[19] Siu, Man-Keung, & Alexeï Volkov. Official Curriculum in Traditional Chinese Mathematics: How Did Candidates Pass the Examinations? [J]. *Historia Scientiarum* 9, 1999: 85 - 99.

[20] Struik, Dirk J. *A Concise History of Mathematics*[M]. 2 vols. New York: Dover, 1948. TMB: François Thureau-Dangin, *Textes mathématiques babyloniens*[M]. Leiden: Brill. van der Waerden, B. L. *Science Awakening*. Groningen: Noordhoff, 1954.

[22] Wu, Wenjun, 2017. Autobiography of Wentsun Wu (1919 - 2017)[J]. written 2006. *Notices of the AMS* 64, 2017: 11, 1319 - 1320.

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On Being First, Being Wrong and Being Right
Knuth, "Knuth", Wu Wenjun, and Algorithms

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Before approaching my topic I need to make clear that I do not read Chinese. What I deal with is thus a topic that can be approached through languages I am familiar with – and indeed, through English and French.

My starting point is a passage in Jiri Hudecek's political biography of Wu Wenjun [Hudecek 2014: 117*f*]:

> although Greek geometry was axiomatized, axiomatization was not its working method in the same way as in modern mathematics. Perhaps because of these problems, in his later works Wu introduced an opposition between proofs and algorithms as a replacement.[1] A crucial influence in this shift was the work of the computer scientist Donald Knuth. Wu studied Knuth's textbook *The Art of Computer Programming* (first volume 1968, second 1969, third 1973), which consists of commented algorithms, just like ancient Chinese mathematical classics consisting of problem-solving methods. Knuth also wrote an article on ancient Babylonian algorithms (Knuth 1972). Although never cited by Wu himself, it was mentioned by his younger colleagues Li Wenlin and Yuan Xiandong (1982) in the same volume as Wu Wen-Tsun (1982c). Both Li Wenlin and Karine Chemla, who studied ancient Chinese mathematics in Beijing in the early 1980s, confirmed the influence of this article on Wu Wen-Tsun's thought (personal communication).
>
> Wu might as well have drawn inspiration from Knuth's opening sentence: "One of the ways to help make computer science respectable is to show that it is deeply rooted in history, not just a short-lived phenomenon." Wu Wen-Tsun had a similar motivation for his own turn to history, although it could also be said that he proceeded in the opposite direction, making ancient Chinese mathematics respectable by showing what can be rooted in it.
>
> Knuth drew a series of analogies between ancient Babylonian mathematics and computer science: Babylonian sexagesimal notation was actually the first floating-point notation; their algebraic algorithms were "machine language" as opposed to the "symbolic language" of our modern algebra; they used numerical algebra disregarding physical and geometrical significance. Knuth also compared particular algorithms to a "stack machine" or to a "macro expansion". His article was not a serious history of algorithms, but rather a reminder of the venerable ancestry of the basic techniques of computer science. But Knuth's last paragraph must have been very suggestive to Wu:
>
>> What about other developments? The Egyptians were not bad at mathematics, and archaeologists have dug up some old papyri that are almost as old as the Babylonian tablets. The Egyptian method of multiplication, based essentially in the binary number system (...) is especially interesting. Then came the Greeks, with an emphasis on geometry but also such things as Euclid's algorithm; the latter is the oldest nontrivial algorithm which is still important to computer programmers. (...) And then there are the Indians, and the Chinese; it is clear that much more can be told.
>>
>> (Knuth 1972: 676).

[1] [Namely of the dichotomy axiomatization–mechanization, which Hudecek and perhaps Wu sees/saw as "not quite satisfactory from the historical perspective"./JH]

> In comparison to Knuth's article, Wu's sole emphasis on Chinese mathematics appears narrow-minded. It might even be suggested that Wu tried to take away some credit from "rival" ancient civilizations in his later attempts to demonstrate the computational superiority of specific Chinese algorithms over Western ones.

I have no opinion whether Knuth's *Art of Computer Programming* was a crucial influence, and it is irrelevant to my topic; in any case it seems to regard only the terminological shift from "mechanization" to "algorithms".[2] The confirmation of "the influence [of Knuth's algorithmic interpretation of Babylonian mathematics] on Wu's thought arouses my doubts. Firstly, according to a number of quotations in [Hudecek 2014: 116–119], Wu was no miser when it came to recognizing his debts. Secondly, he had strong doubts concerning the attribution of, for instance, "algebra" to the ancient Babylonians,[3] and also concerning Dirk Struik's belief in the existence of an undifferentiated "Oriental" mathematics [Struik 1948: I, xii and *passim*]. Knuth, on the other hand, had no doubts as to the authenticity of Babylonian algebra, and also suggest is in the passage quoted by Hudecek that ancient Babylonian, Egyptian, Indian and Chinese mathematics belonged to a shared genre. Wu may well have known [Knuth 1972] at some moment, but in that case he seems to have followed the principle formulated by my old friend Marinus Taisbak in a private letter, "in the interest of peace on earth not to cite those with whom I disagree". In any case, his autobiographical note [Wu 2017] suggests that his insight in the algorithmic (at first named "mechanical") nature of ancient Chinese mathematics must antedate his supposed encounter with Knuth's article:

> During the cultural revolution I was sent to a factory manufacturing computers. I was initially struck by the power of the computer. I was also devoted to the study of Chinese ancient mathematics and began to understand what Chinese ancient mathematics really was. I was greatly struck by the depth and powerfulness of its thought and its methods. It was under such influence that I investigated the possibility of proving geometry theorems in a mechanical way.

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[2] Capriciously, I come to think of Charles Darwin's words [1872: 49] from the sixth edition of the *Origin of Species* concerning Herbert Spencer's "survival of the fittest" – namely that the expression "is more accurate, and is sometimes equally convenient". Of course, this ambiguous praise had no impact on Darwin's own theory, and is not even found worthy a reference in the index.

[Hudecek 2012: 55], speaking of Wu's endeavour to make use of mechanization in contemporary production of mathematical knowledge, only states that "Wu perhaps saw a positive example in the works of the computer scientist Donald Knuth". No claim of crucial influence here.

[3] Had he had access to Neugebauer's or Thureau Dangin's text editions he might *perhaps* have been less sanguine on this account. But this is hypothetical history, only pertinent by emphasizing that he did not have this access.

Babylonia

Let us go on with a closer look at what Knuth says about ancient algorithms, firstly in order to judge the quality of his arguments in their chronological context, secondly and briefly (and somewhat anachronistically) from an actual point of view.

As formulated by Hudecek, Knuth's paper was no piece of serious history writing (neither of algorithms nor otherwise, one may add); nor was it meant to be. In Knuth's own opening words [1972: 671],

> One of the ways to help make computer science respectable is to show that it is deeply rooted in history, not just a short-lived phenomenon. Therefore it is natural to turn to the earliest surviving documents which deal with computation, and to study how people approached the subject nearly 4000 years ago.

Knuth's central argument consists in the presentation of some stepwise calculations in Neugebauer's translation (the best available at the time – somewhat straightened by Knuth in the interest of readability), in which the texts seem to prescribe a sequence of purely arithmetical steps. These are taken to illustrate (p. 672) that

> The Babylonian mathematicians were [...] adept at solving many types of algebraic equations. But they did not have an algebraic notation that is quite as transparent as ours; they represented each formula by a step-by-step list of rules for its evaluation, i.e. by an algorithm for computing that formula. In effect, they worked with a "machine language" representation of formulas instead of a symbolic language.

At the same time Knuth complains (p. 674) that his examples represent

> only "straight-line" calculations, without any branching or decision-making involved. In order to construct algorithms that are really nontrivial from a computer scientist's point of view, we need to have some operations that affect the flow of control.

Knuth overlooks that this linearity is exactly what allows him to conflate the single calculation – by necessity unbranched – with an algorithm which it is supposed to represent.[4] In order to get something which smacks of a loop with a criterion for when to stop Knuth then points to problems about composite interest, where (thus Knuth, p. 674) a "longwinded and rather clumsy procedure reads almost like a macro expansion" – which it evidently only does in the eyes of somebody familiar with macros. The Babylonian calculator simply repeats the calculation.

Knuth also observes (p. 674) that

[4] *If* the calculations corresponded to an actual branched algorithm, they might of course justify the road taken by a reference to the criterion deciding what to do at the branching point. The language for that was at hand – regularly, texts justify a step with the words "because he [the master formulating the problem] has said; but no such justification ever corresponds to a branching.

> We don't find tests like "Go to step 4 if x < 0", because the Babylonians didn't have negative numbers; we don't even find conditional tests like "Go to step 5 if x = 0", because they didn't treat zero as a number either! Instead of having such tests, there would effectively be separate algorithms for the different cases. (For example, see [MKT I, 312-314] for a case in which one algorithm is step-by-step the same as another, but simplified since one of the parameters is zero.)

Nor do we find anything like "Go to step 4 if x > 10" – the absence of zero and negative numbers are no reason that the Babylonian texts contain no decisions or branchings. They simply do not. That there are "separate algorithms for the different cases" illustrates instead that Knuth's algorithm concept is empty when applied to the Babylonian record, and hides the genuine character of the texts. The Babylonian texts contain examples meant to be paradigmatic but also meant to be applied with the necessary flexibility.[5] Some texts even show that different approaches are possible – thus the Old Babylonian text AO 8862 [MKT I, 108–117], whose first three problems *could* be solved by application of the same method; instead of doing that, the text applies three different tricks.

What Knuth could not know is that the Babylonian texts do *not* "prescribe a sequence of purely arithmetical steps", as they do in Neugebauer's and Thureau-Dangin's translations; the so-called "algebra" texts prescribe cut-and-paste manipulations of areas within a square-grid geometry, in which the correctness of procedures is as intuitively obvious as in simple symbol-based equation algebra – see, for instance, [Høyrup 2017]. But this is a different matter which does not concern us here. What Knuth *could* have known in 1972 is that the "zigzag functions" describing planetary motion in the Babylonian mathematical astronomy of the later first millennium BCE must have been calculated according to fixed algorithms (with branchings) – see [ACT I, 30–32]. The planetary tables do not explain these algorithms, it is true, they only state what comes out of them. However, the astronomical "procedure texts" explain indubitable algorithms, often with decision of the type DO ... WHILE; a number of (mostly fragmentary) examples are found in [ACT I, 186–276]. A fairly well-conserved complete specimen was published by Lis Brack-Bernsen and Hermann Hunger in [2008]. In the late period, the Babylonian astronomer-priest who prepared these texts were thus fully able to think algorithmically – but that does not entail that the scribe-school teachers did so 1200 to 1700 years earlier.[6]

Nor can the passage [Knuth 1972: 676]

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[5] Whoever is tempted to introduce the notion of a "flexible algorithm" (Knuth did not!) should try to write a "flexible" computer program without making use of explicit branchings etc. Algorithms are, by definition, *not flexible* – they are, in Wu's original formulation, *mechanical* [Hudecek 2014: 117*f*].

[6] The question of algorithms in Babylonian mathematics as it can be judged today is dealt with in more detail in [Høyrup 2018].

> What about other ancient developments? The Egyptians were not bad at mathematics, and archeologists have dug up some old papyri that are almost as old as the Babylonian tablets we have discussed. The Egyptian method of multiplication, based essentially in the binary number system (although their calculations were decimal, using something like Roman numerals) is especially interesting; but in other respects, their use of awkward "unit fractions" left them far behind the Babylonians [...]. And then there are the Indians, and the Chinese; it is clear that much more can be told

be counted as arguments that Ancient Egyptian, Indian and Chinese mathematics contained algorithms. Nor can it be supposed that Knuth had more information which he chose not to present in detail. Apart from the Babylonian text collections [MCT, MKT, TMB], the only items in his bibliography that speak of history are Asger Aaboe's *Episodes from the Early History of Mathematics* [1964], a high-school book; and two popularizations, Neugebauer's *Exact Sciences in Antiquity* [1957] and B. L. van der Waerden's *Science Awakening* [1954]; none of these would have assisted him.[7]

Actually, Knuth was too wise to make the claim. This closing passage points back to the beginning of the article, with the exhortation to "turn to the earliest surviving documents which deal with computation, and to study how people approached the subject nearly 4000 years ago". The "Knuth" compared to whom Wu is found by Hudecek to be narrow-minded is a product of superficial reading.

Ancient China

Let us look now at ancient Chinese mathematics, Wu's case. Evidently, a text like the *Nine Chapters* prescribes calculations and can thus be claimed to "contain algorithms" if read as Knuth reads his Mesopotamian material. That seems to be the level at which Hudecek seems to understand him – at least, what [Hudecek 2014: 130] explains suggests nothing else. However, Wu's reference to his "being struck by the depth and powerfulness" of ancient Chinese mathematics shows – even to the one who cannot read the detailed arguments of his Chinese publications – that Wu moves at a different level. Knuth, indeed, when he tried to legitimize modern computer science, had complain about the shallowness of the Babylonian straight-line calculations; that leaves space for neither depth nor particular powerfulmess.

Firstly, there is the way numerical procedures such as the extraction of square and cube roots are presented in the *Nine Chapters*. As formulated by Karine Chemla [1991: 75], (summarizing earlier work from her hand), these sometimes make "use of iteration, conditionals and assignments of variables: three resources listed as basic concepts in D. Knuth's *Fundamental Algorithms. The Art of Computer Programming*". And, in contrast to the conditionals etc. which Knuth imposed on the Mesopotamian material, these

[7] A different question is whether algorithmic analysis can be applied to Mesopotamian and Pharaonic mathematics. Jim Ritter [2004] and Annette Imhausen [2003] have shown convincingly that it can. But none of them claim that the material they deal with consisted *in itself* of algorithms.

are really in the texts – see the appendix in [Chemla 1987], offering text translations and a flow chart corresponding to a critical passage.

I shall not go in detail with this – Chemla has treated the topic under various perspectives immensely better than I would be able to, and not only because she has access to the original texts. Instead, I shall move from the question of *algorithms* to that of *algorithmic culture*, which is rather what Wu speaks about though in different terms.

As Hudecek paraphrases or interprets Wu, "although Greek geometry was axiomatized, axiomatization was not its working method". This "working method" refers to the way axiomatization unfolded from the outgoing 19th century onward and symbolized by Bourbaki ("whom" Wu knew well from proper professional experience). In a different sense, a culture of axiomatization developed in Greek geometry during the fourth century BCE, becoming hegemonic ideology from Euclid's time onward [Høyrup 2018]. That was preceded by a mathematical culture which still saw argument as an ideal, but where arguments would draw on the "locally obvious", with no reference to absolute first principles.[8]

Babylonian mathematical culture was different. It might also appeal to the locally obvious in didactical explanations, but on the whole implicitly (however, oral didactical expositions may have been more explicit). The main teaching aim, as mentioned, was to train through paradigmatic examples, meant to be followed with as much flexibility as needed – what Knuth sees as "separate algorithms for the different cases".

And ancient Chinese mathematical culture was different from all of these – its ideal, even when meant to convey understanding, was *algorithmic*. This may be illustrated by a look at Chapter 3 of the *Nine Chapters* [ed., trans. Chemla & Guo 2004: 280–311]. The chapter deals with distribution according to "degrees" – as explained by Liu Hui, according to *rank*.

Such distributions were well known in Pharaonic Egypt – one example is in Rhind Mathematical Papyrus no. 63 [trans. Peet 1923: 107]. The rule is also the same. Further, the problem type is very common in late medieval and Early Modern European commercial arithmetic, where it goes under the name of the "rule of company" (and similarly); even here the technique is the same. There is a difference, however, and that reflects the difference between mathematical cultures. The *Nine Chapters* at first sets out a rule in abstract terms – an (unbranched) *algorithm*. The follow a number of examples, which are really *applications* of the rule. This is a first hint that the mathematical culture of the *Nine Chapters* is *algorithmic*.

In this sense, even Sanskrit mathematics between Brahmagupta and Bhaskara is often (not always) algorithmic. But there is more to the *Nine Chapters*, Chapter 3. In

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[8] Hippocrates of Chios thus bases his investigation of lunules on two principles – the "Pythagorean rule" and the proportionality of areas to the square of a characteristic linear dimension. Both principles had been used in practical mensurational geometry at least since the earliest second millennium BCE. See [Høyrup 2019].

problems no. 1, 3 and 5, the weights are immediately given, and the rule can be applied as it is.[9] In no. 2, however, the three weights are not given directly, but they are told to be in ratios 1:2 and 1:2 – and in no. 4, 5 weights are in ratios 1:2, 1:2, 1:2, 1:2 and 1:2. Obviously, the weights have to be calculated first, as 1–2–4 and 1–2–4–8–16, respectively. However, this preliminary calculation is not explained in either case, its result is stated directly. That is, *the text only explains that part which is covered by the algorithm* as explained in the beginning of the chapter; what falls outside the algorithms also falls outside explanation. *Teaching the algorithm* is thus what the text is about – the higher-level aim of teaching how to make calculations is not directly visible but mediated through the algorithms.

This centrality of the algorithm does not characterize the *Nine Chapters* alone. This work, though a cardinal classic, after all did not constitute a mathematical culture. Ancient Chinese mathematical culture was *a practice,* in which use of the classic was important. But the two should not be conflated. However, a description of state examinations in mathematics written almost a millennium after the *Nine Chapters* (in the *Xin Tang shu*, compiled in 1060) specifies that one of the tasks the students have to perform is to *construct algorithms* (quotations in [Siu & Volkov 1990: 92*f*]. Commentaries (like that of Liu Hui) also explain why algorithms work, and the *creation of new algorithms* (not unspecifally of new mathematical knowledge) is something of which several authors boast (quotations *ibid.* p. 94). So, just as Greek-style mathematics from Euclid to modern times has tended to see the discovery of theorems[10] as the gist of the undertaking, ancient Chinese mathematics saw its task as constructing algorithms – obviously, again, demonstrably correct and coherent algorithms.

Even in this sense, ancient Chinese mathematics, from Han to Tang, was thus *algorithmic*. And in this sense, neither Sanskrit nor Mesopotamian nor Pharaonic (nor medieval Arabic and European) mathematics was.

In consequence, the attentive outsider must conclude that Wu was right – not only in his characterization of ancient Chinese mathematics but also when taking this as the *specific* character of ancient Chinese mathematics.

The same outsider (but now an insider) must conclude that Knuth was mistaken – and even more mistaken the reproach that Wu saw matters more narrowly than "Knuth".

Under these conditions, it is rather futile to discuss who was first. No doubt Baron von Münchhausen was first when it came to riding through the air (on a cannon ball), and the Wright Brothers second.

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[9] The weights are not always ranks, but Liu Hui has explained already in his commentary to the rule how to apply it if the "degrees" represent the number of members of families, where the distribution is meant to be equal between individuals.

[10] Obviously with appurtenant demonstrations, without which we usually speak of *conjectures* and not of theorems, unless the inventor is a Fermat.

References

Aaboe, Asger, 1964. *Episodes from the Early History of Mathematics*. New York: Random House.

ACT: Otto Neugebauer, *Astronomical Cuneiform Texts: Babylonian Ephemerides of the Seleucid Period for the Motion of the Sun, the Moon, and the Planets*. London: Lund Humphries, 1955.

Brack-Bernsen, Lis, & Hermann Hunger, 2008. "BM 42484+42294 and the Goal-Year method". *SCIAMUS* **9**, 3–23.

Chemla, Karine, & Guo Shuchun (eds), 2004. *Les neuf chapitres. Le Classique mathématique de la Chine ancienne et ses commentaires*. Paris: Dunod.

Chemla, Karine, 1987. "Should They Read Fortran As If It Were English?" *Bulletin of Chinese Studies* **1**, 301–316.

Chemla, Karine, 1991. "Theoretical Aspects of the Chinese Algorithmic Tradition (First to Third Centuries)". *Historia Scientiarum* **42**, 75–98.

Høyrup, Jens, 2017. *Algebra in Cuneiform: Introduction to an Old Babylonian Geometrical Technique*. Berlin: Edition Open Access.

Høyrup, Jens, 2018. "Was Babylonian Mathematics Algorithmic?", pp. 297–312 *in* Kristin Kleber, Georg Neumann & Susanne Paulus (eds), *Grenzüberschreitungen: Studien zur Kulturgeschichte des Alten Orients*. Festschrift für Hans Neumann zum 65. Geburtstag am 9. Mai 2018. Münster: Zaphon, 2018.

Høyrup, Jens, 2019. "From the Practice of Explanation to the Ideology of Demonstration: an Informal Essay". Forthcoming in Gert Schubring (ed.), *Interfaces between Mathematical Practices and Mathematical Education*. New York: Springer, 2019.

Hudecek, Jiri, 2012. "Ancient Chinese Mathematics in Action: Wu Wen-Tsun's Nationalist Historicism after the Cultural Revolution". *East Asian Science, Technology and Society* **6**, 41–64.

Hudecek, Jiri, 2014. *Reviving Ancient Chinese Mathematics: Mathematics, History and Politics in the Work of Wu Wen-Tsun*. London & New York: Routledge.

Imhausen, Annette, 2003. *Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten*. Wiesbaden: Harrassowitz.

Knuth, Donald E., 1972. "Ancient Babylonian Algorithms". *Communications of the Association of Computing Machinery* **15**, 671–677, with correction of an *erratum* in **19** (1976), 108.

MCT: Otto Neugebauer & Abraham Sachs, *Mathematical Cuneiform Texts*. New Haven, Connecticut: American Oriental Society.

MKT: Otto Neugebauer, *Mathematische Keilschrift-Texte*. 3 vols. Berlin: Julius Springer, 1935, 1935, 1937.

Neugebauer, Otto, 1957. *The Exact Sciences in Antiquity*. Second edition. Providence, Rh.I.: Brown University Press.

Peet, T. Eric, 1923. *The Rhind Mathematical Papyrus, British Museum 10057 and 10058*. Introduction, Transcription, Translation and Commentary. London: University Press of Liverpool.

Ritter, Jim, 2004. "Reading Strasbourg 368: A Thrice-Told Tale", pp. 177–200 *in* Karine Chemla (ed.), *History of Science, History of Text*. Dordrecht: Kluwer.

Siu, Man-Keung, & Alexeï Volkov, 1999. "Official Curriculum in Traditional Chinese Mathematics: How Did Candidates Pass the Examinations?" *Historia Scientiarum* **9**, 85–99.

Struik, Dirk J., 1948. *A Concise History of Mathematics*. 2 vols. New York: Dover.

TMB: François Thureau-Dangin, *Textes mathématiques babyloniens*. Leiden: Brill.

van der Waerden, B. L., 1954. *Science Awakening*. Groningen: Noordhoff.
Wu, Wenjun, 2017. “Autobiography of Wentsun Wu (1919–2017” [written 2006]. *Notices of the AMS* **64**:11, 1319–1320.