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What is physics problem solving competency? The views of Arnold Sommerfeld and Enrico Fermi

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Abstract

A central goal of physics education is to teach problem-solving competency, but the nature of this competency is not well-described in the literature. The present article uses recent historical scholarship on Arnold Sommerfeld and Enrico Fermi to identify and characterize two positions on the nature of physics problem-solving competency. The first, Sommerfeld’s, is a “theory first, phenomenon second” approach. Here the relevant problems originate in one of the theories of physics and the goal of the problem-solver is to make a mathematical analysis of the suitable equation(s) and then give a qualitative analysis of the phenomenon that arise from these mathematical results. Fermi’s position is a “phenomenon first, theory second” approach, where the starting point is a physical phenomenon that is analyzed and then brought into the realm of a physics theory. The two positions are illustrated with solutions to two problems and it is shown that the two positions are reflected in problem collections of university educations in physics.

Keywords: Problem Solving Competency, Physics Education, History of Physics, Philosophy of Physics

1. Introduction

Problem-solving plays an extensive role in the physics curriculum on most educational levels. Two types of justification are given for this situation. The first type focuses on how problem-solving may facilitate students' learning of physics, e.g. of concepts. The other type sees the development of students' problem-solving competency in itself as a goal of physics education (e.g., Rigden 1987; Mestre et al. 1993; Maloney 1994; Hsu et al. 2004, Gerace and Beaty 2005; Walsh et al. 2008; Maloney 2011). An argument for the latter view is that physics education (at least at the higher educational levels) ought to reflect the nature of physics as a scientific discipline and problem-solving is an important activity of scientific inquiry in general (Laudan 1977), and in physics in particular. The proponents of this view also argue that physics is in a unique position to help students develop the required skills for solving genuine problems of the real world (Rigden 1987), so physics have something to offer in this respect. While the two types of justification for problem-solving do not necessarily collide, they do represent two different orientations regarding what are the goals and what are the means. Moreover, due to their different aims, the two types often have different views on which problems are relevant for physics education.

In the present article, we focus on the development of problem-solving competency as an instructional goal in itself at the university/college level. The question we ask is: what is the nature of physics problem-solving competency? The answer to this question is important because students and teachers alike need to be aware of this nature as their beliefs impact how they deal with problem-solving in teaching situations: Mason & Singh (2010) pointed out that students' attitudes and approaches to problem-solving in physics can profoundly affect their development of physics expertise, including problem-solving competency. Ding (2014) argued that physics teachers' views of problem-solving influence their teaching: "The interplay between faculty's views of problem-solving and their choice of related activities can influence the conceptualization, design, and implementation of these introductory courses, thus having far-reaching implications for higher education." (Ding, 2014, p. 137) More generally, being an educational goal in itself, the specific nature of this competency should influence the choice of tasks, pedagogical methods, assessment strategies etc. that are employed in educational settings. Consequently, physics teachers and students should understand the nature of physics problem-solving competency and therefore we need to have an understanding of this nature.

Unfortunately, the description of the nature of physics problem-solving competency is left either implicit or somewhat fragmentary in the physics education literature. Policy documents on physics education, such as Beneitone et al. (2007), Tuning Project (2007), The Quality Assurance Agency for Higher Education (2017), state that university/college students should learn physics problem-solving competency, which the Latin-American Tuning Project characterized as “The capacity to pose, analyse and solve physical problems, both theoretical and experimental, through the use of numerical, analytical or experimental methods.” (Beneitone et al., 2007, p. 155). Physics education researchers and physics educators, e.g., Walsh, Howard, Bowe, (2007), Gerace and Beatty (2005), Heron & Meltzer (2005), Knight (2004), embrace this goal: “one of the principal goals of a physics course is to produce adept problem solvers who can transfer their knowledge and understanding to real world situations” (Walsh et al. 2007, p. 020108-1). So, these documents take a somewhat holistic approach and state that physics students should be able to tackle problems in physics. In effect, they take competent problem-solving to be the ability to solve physics problems, so the characterization of problem-solving competency is implicit and is displaced to identifying what is meant by a proper physics problem. However, no definition of such problems is given neither in these texts nor in the reviews of problem-solving in physics (Hsu et al., 2004; Maloney, 1994; Maloney, 2011). There is agreement among physics educators that proper physics problems are closer to the real world than the traditional and somewhat contrived problems found at the end of the chapter in textbooks (such as inclined plane problems) (Ding, 2012), but otherwise no consensus seems to have emerged about the nature of a physics problem: are they well-structured, ill-structured (e.g, Shekoyan and Etkina 2007) or multifaceted (Ogilvie, 2009), context-rich (Heller et al. 1992) or context-poor, do they require the making of assumptions about the real world (Fortus, 2005) etc.?

While the characterization of physics problem-solving competency as the ability to tackle physics problems has much to commend it, including its simplicity and holistic nature, it is too implicit to inform an understanding of the nature of problem-solving in physics.

Some attempts to specify the nature of the problem-solving competency have been offered. Some of these describe the steps required when solving a problem. Fortus (2007) used the IDEAL-approach for general problem solving given by Bransden and Stein (1984) where problem solving is seen as involving the steps 1) Identify the problem, 2) Define and represent the problem, 3) Explore possible strategies or solutions, 4) Act on a selected solution, 5) Look back and evaluate. Reif (2008) devised his own general problem-solving strategy (1. Describe problem; 2. Analyze problem; 3. Construct solution; 4. Assess solution; 5 Exploit the solution). However, these approaches do not capture the essence of physics problem-solving competency since they use models for general problem-solving. In contrast to these general approaches, The Quality Assurance Agency for Higher Education (2017) and Bolton and Ross (1997) focused on *physics* problem-solving and listed a number of skills that this competency consists of: “For example, students learn how to identify the appropriate physical principles, how and when to use special and limiting cases and order-of-magnitude estimates to guide their thinking about a problem and how to present the solution, making their assumptions and approximations explicit.” (Quality Assurance Agency for Higher Education, p. 10). Bolton and Ross (1997) added the skill to be able to identify and label variables. However, while each of these skills may be relevant, simply listing such individual skills does not constitute an integral approach that is required for the description of a competency, which is a cluster of related knowledge, skills and attitudes. As the same ingredients can lead to either a brownie or a chocolate soufflé depending on the sequence of actions, in order to characterize problem-solving competency we not only need to specify the skills ingredients but also how they appear in the overall scheme.

What is needed is a description of the nature of physics problem-solving that gives a holistic, explicit characterization. In this article, we use recent historical scholarship to show that two distinct views on the nature of problem-solving in physics can be discerned among research physicists in the 20th century. The views are epitomized by Arnold Sommerfeld (1868-1951) and Enrico Fermi (1901-1954), two outstanding and famous physicists of the 20th century. The purpose of the present article is to use their views to characterize and illustrate two positions on the nature of physics problem-solving competency. The two positions will be called the "theory first, phenomenon second" position (or "theory first", for short) and the "phenomenon first, theory second" positions (or "phenomenon first", for short), respectively.

There are three reasons why we focus on Sommerfeld and Fermi. First, their positions seem to be characteristic of physicists at various times and places. Among Physics Nobel Laurates, Lev Landau (1908-1968), Pyotr Kapitza (1894-1984) and Pierre Gilles de Gennes (1932-2007) (see Livanova, 1978, Kapitza, 1980, Plévert 2011) adhered to something like Fermi's approach, while Werner Heisenberg (1901-1976) and Felix Bloch (1905-1983) subscribed to Sommerfeld's approach (see Hahn, 1990). Hans Bethe (1906-2005) and John Bardeen (1908-1991) used both approaches depending on the situation (see Schweber, 2012, and Hoddeson and Daitch, 2002). The second reason is that while one can find proponents of the two approaches that are closer to our time, the approaches are in fact most elaborately described in the literature on Sommerfeld and Fermi. The final reason is that Sommerfeld and Fermi not only used their particular problem-solving approach in their research, but also subscribed to it in their teaching, for instance when they assigned problems to their students. Hence, for Sommerfeld and Fermi, their teaching reflected their research practice.[1]

The existence and characterization of the two positions are relevant for actors at various levels. From a policy point of view, the two positions offer quite different perspectives on what

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[1] Lev Landau often used a "phenomenon first" approach in his research, but the problems he assigned in his famous theoretical minimum were oriented towards a "theory-first" approach. Ioffe (2013) gave an example of problem and the famous Landau and Lifschitz textbooks give other problems, which according to Hall (2006), stem from the theoretical minimum.

physics can offer for the development of students' problem-solving skills, for instance in relation to 21st century skills with its heavy focus on problem-solving (see, e.g., McComas, 2014). From a learning perspective, as noted above, it is, first, important that the teacher as well as the students are aware of the nature of the enterprise they are engaged in and hence they should know the nature of the problem-solving competency they strive to develop; second, the learning environments in which the two competencies can be developed are most likely different as they require different problems, scaffolding activities etc. Finally, from a research point of view, the distinction may help to inform discussions on problem-solving in physics, for instance, about whether a given problem contributes to one or the other competency.

The distinction between the two positions on problem-solving is meant to offer a distinction between two holistic approaches to the issue of what is physics problem-solving competency in terms of the relation between theory and phenomenon. We do acknowledge, however, that problem-solving requires more than identifying the relevant theory and pertinent aspects of the phenomenon. As noted above, it involves identifying the appropriate physical principles, how and when to use special and limiting cases and order-of-magnitude estimates, as well as the identification and labelling of variables and the use of approximations and idealizations. Moreover, it involves extensive use of mathematics (see Niss (2017)). It is important to keep in mind that these aspects are part of both approaches.

2. Two positions on physics problem solving competency

Both Sommerfeld and Fermi had to invent their own approach to the teaching of theoretical physics. Sommerfeld because he was a maverick of the budding discipline of theoretical physics, and Fermi because he was the first professor of theoretical physics in a relatively scientifically isolated Italy. Sommerfeld's Institute of Theoretical Physics at the University of Munich, which he liked to call "a nursery of theoretical physics," was very influential and successful and he educated a generation of German physicists including the Nobel Laurates Wolfgang Pauli, Werner Heisenberg, and Hans Bethe. Some of Sommerfeld's students set up branches of the schoolin Leipzig, Zürich, Stuttgart, and Hamburg (Eckert 2013). Fermi developed his approach in Rome, then took it to a larger scale in Chicago, where he trained several outstanding physicists, including the Nobel Laurates Chen-Ning Yang, T.D. Lee and Jack Steinberger.

2.1 Sommerfeld's view – The theory-first position

The historian Suman Seth (2010) has characterized the approach of Arnold Sommerfeld (1868-1951) as "a physics of problems" in contrast with the "physics of principles" advocated by Max Planck, Albert Einstein and Niels Bohr. The latter searches for general principles, while the former aims at solving concrete problems. Sommerfeld focused on specific questions and their specific solutions and searched for a mechanism or a process rather than a generalizing postulate. The British physicist Frederick Lindemann characterized Sommerfeld's students in a 1933 letter to Einstein: "I have the impression that anyone trained by Sommerfeld is the sort of man who can work out a problem and get an answer, which is what we really need at Oxford, rather than the more abstract type who would spend his time disputing the philosophers." (Quoted in Seth 2010, p. 3) Werner Heisenberg also stressed this focus on problem-solving in Sommerfeld's teaching: "In his pedagogy, he was not satisfied with presenting fundamental theoretical relations; rather, he showed students 'how it is done,' how one actually treats a physical problem mathematically through to its conclusion." (Heisenberg, 1948), quoted in Eckert 2013, p. 411). In fact, attacking a wide range of problems mathematically was a characterizing feature of the Sommerfeld's school (Eckert, 2013).

Hans Bethe (1906-2005), who did his PhD with Sommerfeld, has described Sommerfeld's approach in more detail: "The method to follow was to set up the differential equation for the problem (usually the Schrödinger equation), to use your mathematical skill in finding a solution as accurate and elegant as possible, and then discuss this solution. In the discussion finally, you would find out the qualitative features of the problem. Sommerfeld's way was a good one for many problems where the fundamental physics was already understood, but it was extremely laborious. It would usually take several months before you knew the answer to the question." (Segrè 1970, p. 59) The differential equation for the problem did need not to be the Schrödinger equation, but could come from other areas of physics, including electromagnetism or classical mechanics.

So, Sommerfeld advocated an approach that was mathematically as well as physically rigorous. The starting point is a problem that is defined within the framework of a theory of physics. Hence, the first major task for the problem-solver is to adapt the equation or principle correctly to the situation; to do this requires introducing approximations and idealizations, identifying the relevant variables, boundary conditions and constraints. The next step is to set up the equation within that framework. Then the equation is solved and finally the solution is discussed in terms of the relevant physical theory. In short, it was a "theory first, phenomenon second" approach in the sense that the problems concern whatever phenomena that arise within the theory. It is clear from the theory what equation(s) is (are) relevant and the task of the problem-solver is mainly to solve the mathematical problems involved.

Sommerfeld's teaching reflects these research ideals, e.g., for the problems he suggested to his students. One student, Paul Ewald, recalled that Sommerfeld had a list of a doctoral thesis topics, such as the calculation of self-inductances of solenoids for alternating currents, the propagation of radio waves over a surface of finite conductivity or an unsolved problem of gyroscopic theory (Eckert 2013). Each subject had its own merit and its own type of mathematical technique that were pointed out by Sommerfeld.

Sommerfeld's position can be described as the "theory-first" position on physics problem-solving competency. The problems to be solved originate in and are defined by one of the theories of physics, be it quantum mechanics, classical mechanics, electromagnetism etc. The problems concern physical phenomena, but these phenomena arise within the theory rather than the other way around, that is observe some phenomenon that we try to understanding using whatever available theory. This implies that not only the physical theory is given, but typically the relevant equation or principle of that theory is also given, so the identification of the theory and the equation/principle play a minor role in the problem-solving process. Now the skills mentioned in the introduction come into play. The next step is to make some idealizations and abstractions concerning the physical system in question, such as assuming perfectly spherical objects and neglect air resistance. Then the problem-solver needs to identify the relevant variables, boundary conditions and constraints. Next the problem-solver has to adapt the equation or principle correctly to the situation. This may be quite demanding on the part of the problem-solver. Next, the problem-solver should apply her mathematical skills and solve the mathematical problem "as accurately and elegantly" as possible. Of course, this could be quite demanding. Finally, the problem-solver should interpret and discuss the solution in terms of the original problem situation, as well as consider the qualitative features of the solution.

2.2 Fermi's view – The phenomenon-first position

The physics approach of Enrico Fermi (1901-1954) can, like Sommerfeld's, be characterized as a physics of problems, as he preferred to work on concrete problems rather than study abstract and general principles. C. N. Yang recalled about Fermi's teaching that "We learned that abstractions come *after* detailed foundation work, not before." (Segrè 1970, p. 170). However, while Sommerfeld favored a very theoretical approach, Fermi tried to make the physics of a problem clear and he often gave beautiful, simple and clear explanations of puzzling phenomena (Segrè 1970). Hans Bethe, who worked with both Sommerfeld and Fermi, agreed and called Fermi's approach "pragmatic" (Segré, 1970, p. 60). Fermi would solve a problem by thinking about it in a general way, making an analysis of the essentials and providing a few order-of-magnitude estimates: "He was able to analyze into its essentials every problem, however complicated it seemed to be. He stripped it of mathematical complications and of unnecessary formalism. In this way, often in half an hour or less, he could solve the essential physical problem involved. Of course, there was not yet a mathematically complete solution, but when you left Fermi after one of these discussions, it was clear how the mathematical solution should proceed.” (Segrè 1970, p. 59). Fermi’s was a “phenomenon first, theory second” approach, in the sense that it first involved an analysis of the phenomenon, then a description of it within a theory of physics. Here, analysis of the phenomenon means an explanation of the phenomenon. “Often, when just talking to [Fermi], one heard a beautiful explanation develop, simple and clear, which would resolve a puzzling phenomenon.” (Segré, 1970, p. 54). Such an explanation is based on physical concepts and principles and gives a physical narrative of the phenomenon, and hence “resolves” the phenomenon in the physics and the essential physics had been found. An important check that the explanation resolves the phenomenon is the use of order-of-magnitude estimates based on the explanation to see whether the explanation gives roughly the right numbers.

The mathematics was always subordinate to the physics for Fermi and he chose the mathematical tools for the occasion. Personally, Fermi was not intimidated by mathematical difficulty, but he did not seek elegant mathematics for its own sake: “the important point is whether it illuminates the essential physical content of the situation.” (Segrè 1970, p. 18). Bethe has given this account of the difference between Sommerfeld and Fermi: “Sommerfeld said ‘Well here is the title of your problem, now you do it and then you had to put in differential equations and if possible Bessel functions. For Fermi that didn’t matter. You just did the mathematics the best way that came to your mind, and the physics was clear by the time you started.” (Schweber 2012, p. 195)

Like Sommerfeld, Fermi practiced what he preached, that is he tried to develop the same problem-solving skills in his students that he also used in his research as testified in the collection of problems that he used at University of Chicago and which led to (Cronin, Greenberg and Telegdi, 1979).

Fermi’s position can be described as the “phenomenon-first” position in problem-solving in physics.[2] The starting point of problem-solving is some phenomenon in the physical world. The first task of the problem-solver is to make a qualitative analysis of the phenomenon and describe the

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[2] Fermi is the inventor of Fermi problems. However, one should note that while the above approach in general accords with Fermi problems, the latter, such as the famous number of Piano Tuners in Chicago, often do not require physical reasoning, in contrast to the crucial steps involving physics of the above approach.

essentials of the situation by giving an explanation of the phenomenon in terms of physical concepts and principles. This should lead to an identification of the relevant physical theory and the relevant equations. This step could be a quite substantial part of the solution process. Then the problem-solver uses this identification of the physics to describe the situation and convert the problem into a model susceptible to quantitative analysis by extracting essential elements and idealizing these elements if necessary; this includes comparing effects by making rough order-of-magnitude calculations. Next, the problem-solver should use whatever mathematics might be deemed relevant for analyzing the model. The problem-solver then conducts a mathematical analysis, by making the necessary approximations and using the relevant information. Finally, the mathematical results obtained are interpreted in terms of the real-world situation and an answer to the specific question is given.

The two positions differ substantially on a number of issues. First, there is a difference when it comes to the required physical analysis. In the phenomenon-first approach, a major task is to give a qualitative analysis of the situation, whereas this grows out of the solution for the theory-first approach. Second, the role of theory is different. In the theory-first approach, the problem originates in the theory and is defined by it, whereas according to the phenomenon-first approach, the theory is a framework that delivers tools that can be used for a particular problem. Finally, the interpretation of the mathematical result in terms of the situation differs. For the phenomenon-first approach, the result answers a specific question, whereas for the other approach the result gives general insight into the behavior of the problem.

3. Illustration of the two positions

The two positions can be illustrated by looking at the solutions to the following two problems that come from problem collections used in physics education.

- **Problem 1: The spiral orbit problem.** A particle moves in two dimensions under the influence of a central force determined by the potential $V(r) = \alpha\, r^p + \beta\, r^q$. Find the powers p and q which make it possible to achieve a spiral orbit of the form $r = c\theta^2$, with c a constant. (Cahn et al. 1994, p. 10)
- **Problem 2: The pole vaulting problem.** When told that the world record for the pole vault was about 18 feet, the fast-rising athlete Rod Fibreglass told the press, “Give me a pole long enough, and I will raise the record to 30 feet”. Could he manage it? How high might he get if he tried hard? (Thompson 1987, p. 3)

The first problem is couched in technical language and may look daunting to an outsider with its reference to central forces and mathematical equation for the potential. It is clear that this technical information is required to solve the problem and the initiated reader will immediately understand that this is a problem in classical mechanics, but perhaps not know how to solve it within that theoretical framework. The other problem, on the other hand, can be immediately understood by the non-initiated reader. Here it is not clear, however, what physics is relevant to the problem or how to apply the theories of physics to it, perhaps even for a reader well-versed in physics.

3.1 The solution to the spiral orbit problem and the theory-first position

The following solution to problem 1 is a sketch of the solution given in the textbook Cahn & Nadgorny (1994). They start by pointing out that “The solution may be obtained most quickly by employing the differential equation for the orbit (see Goldstein, *Classical Mechanics,* §3-5)” (Cahn et al. 1994, p.100). Section §3-5 of this classical textbook is entitled “The differential Equation for the Orbit, and Integrable Power-Law Potentials” and is found in the chapter on the two-body central force problem, which applies to systems where two bodies interact with a force that acts only in the direction of the line connecting the two bodies. In the beginning of the chapter, Goldstein shows that for such systems the so-called angular momentum vector is conserved and he sets up the fundamental differential equations equivalent to Newton's second law for the situations in question. The relevant equation for our purposes, which the reader may wish to simply accept, is

$$m\frac{d^2r}{dt^2} - \frac{l^2}{mr^3} = -\frac{\partial V}{\partial r}. \qquad (1)$$

Here *r* is the distance, *m* is the mass, *l* is the magnitude of the angular momentum. $\frac{\partial v}{\partial r}$ is the derivative of the potential $V(r)$, which is a measure of the force between the particles.

Cahn et al. (1994) now states that problem 1 can be solved by applying Goldstein's line of reasoning in section §3-5 where he eliminates the time dependence from the above equation using the conservation of angular momentum. This leads to following differential equation

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2}\frac{d}{du}V\left(\frac{1}{u}\right). \qquad (2)$$

Here *u* is defined by $u = \frac{1}{r}$.

We can now substitute the problem's orbit equation $r = c\theta^2$ into the definition of *u* to get

$$u = \frac{1}{r} = \frac{1}{c\,\theta^2} \text{ so } \frac{d^2u}{d\theta^2} = \frac{6}{c\,\theta^4}.$$

Substituting this result and the problem's potential equation $\left(V(r) = \alpha\, r^p + \beta\, r^q = \alpha\left(\frac{1}{u}\right)^p + \beta\left(\frac{1}{u}\right)^q\right)$ into equation 2 yields

$$\frac{6}{c\theta^4} + \frac{1}{c\theta^2} = \frac{m}{l^2}\left(p\alpha\, c^{p+1}\theta^{2(p+1)} + q\beta c^{q+1}\theta^{2(q+1)}\right). \qquad (3)$$

Using that so far p and q are interchangeable, we can identify powers of θ on the two sides of equation 3 to obtain

$$-4 = 2(p+1)$$

$$-2 = 2(q+1)$$

and therefore we get the necessary condition

$$p = -3$$

$$q = -2$$

(Cahn et al. 1994, p.100-101).

This solution illustrates the theory-first position. At first, the problem-solver should identify the relevant governing equation from the available ones; in this case it is the equations of classical mechanics. The problem-solver can make this identification by knowing equation 2 from the literature, say the Goldstein reference, or by deriving it herself using the outlined procedure.[3] Next, the problem-solver adapts this equation to the present situation, namely the given potential and the given equation for a spiral orbit in equation 2. Then the job is mainly mathematical, which is to rearrange this equation and identify powers on the two sides. One could then (but this is not done in the solution given in the book), give answers to qualitative questions, such as what physical system corresponds to the potential obtained, i.e.

$$V(r) = \alpha\, r^{-3} + \beta\, r^{-2}.$$

or how do *a*, *b,* and *c* relate to conserved angular momentum in a particular motion?

In the case of the spiral orbit problem, the initial physical analysis consists in saying that since we have central force motion (as described in the problem text), angular momentum is conserved, so we can use the equation from Goldstein. The fact that angular momentum is

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[3] The procedure is to use the definition of angular momentum *l*, which is conserved in central force motion, and substitute $\left(\frac{l}{\mathrm{mr}^2}\right)\left(\frac{\mathrm{d}}{\mathrm{d\theta}}\right)$ for $\frac{\mathrm{d}}{\mathrm{dt}}$.

conserved is standard reasoning in classical mechanics. The problem provides the potential as well as the orbit equation, so it neither requires that the problem-solver contemplates realistic potentials nor that she sets up the condition for spiral orbits. Basically, the analysis of the physics of the situation comes after the problem has been solved, namely answering qualitative answers such as the two questions mentioned above.

3.2 The solution to problem 2 and the phenomenon-first position

Turning now to problem 2, Thompson (1987) solved the problem in the following way. First, it should be realized that the dominant consideration is that the kinetic energy of the running man just before take-off is converted into gravitational potential energy during the jump at something less than 100% efficiency. This reasoning gives the equation

$$½\, mv^2 = mgh.$$

Here v is the speed of the running man, h is the height the man can reach, m is his mass and g is the gravitational acceleration. We have to isolate h, and get

$$h = \frac{1}{2}\frac{v^2}{g}.$$

We have to somehow estimate the speed of the pole vaulter during the run. The world record in 100 meters race is about 10 s, giving an average speed of 10 m s^{-1}. If the pole vaulter attains a speed of 10 m s^{-1} during the run, the corresponding height is

$$h = \frac{1}{2}\frac{\left(\frac{10m}{s}\right)^2}{9.82m/s^2} \approx 5m$$

To this number, we should add smaller terms arising from: (i) the fact that his center of mass is already about 1 m above the ground when he starts; (ii) the work done by his legs on take-off, and by his arms in climbing up the pole, which can be estimate to give an extra 0.5m; (iii) the fact that his center of mass actually passes *below* the bar, an extra 10cm. Adding these items together gives an answer of approximately:

$$H = \frac{1}{2}\frac{v^2}{g} + 1m\ + 0.5m\ + 0.1\ m = 6.6\ \text{m} = 21\ \text{feet}.$$

The difference between this and the observed 18 feet is due to an efficiency of less than 100% – or to errors in the estimated quantities. In any case, there is clearly no hope of Mr Fibreglass making good his boast that he could raise the record to 30 feet as stated in the problem formulation.

This problem illustrates Fermi's approach. First, the problem-solver has to make a physical analysis of the situation. She should realize that the dominant consideration is that the problem can be solved using the principle of the conservation of energy and that she then has to equate the kinetic energy during the running phase of the pole vaulting with the potential energy during the jump, as this will give an upper bound on the height the pole vaulter can reach. So, she has to recognize that the problem doesn't require that the specifics of the situation be taken into account, such as the elastic properties of the rod and also that the mentioned terms (the center-of-mass of the body is above the ground, the work of the feet, the center of mass of the body is going under the stick) are in fact not the dominant ones. With these considerations in place, the mathematical solution is not very complicated involved and the problem solver only has to isolate *h*. Next, the problem-solver has to realize the presence of the other terms and estimate their values – based on her physical understanding of the situation. Finally, the result is interpreted in terms of the original question of whether it is possible to jump 30 feet up.

4. Remarks on the two approaches

The theory-first and the phenomenon-first approaches emphasize different sides of the problem-solving competency: the initial physical analysis, as well as the theory and the interpretation of the mathematical results play different roles in these two understandings.

The two approaches are not contradictory; in fact, the phenomenon-first approach is often seen as a sort of supplement to the theory-first approach. In the 1960s, the physics department at the

University of Bristol, for instance, introduced an exam using problems like the pole vault one in order to see whether the student could use the material previously learned in courses with more traditional exam problems such as the spiral orbit problem (Thompson 1987). Moreover, it was hoped that the exam could encourage the cultivation of a group of skills considered as an important constituent of the expertise of the professional physicist, including the ability to convert a real problem into a model. The book is entitled “Thinking like a physicist”, to stress the importance of these skills, which are not trained by solving the more traditional exam problems.

The Nobel Laurate Pyotr Kapitza (1894-1984) had a similar agenda and advocated using problems related to Bristol’s problems in his general physics course in the 1940s. Kapitza found that the problems could cultivate the creative scientific thinking of future scientists, as it “is well-known that fruitful scientific work requires not only knowledge and understanding but also a capacity for independent analytical and creative thinking. In effect, these problems were compiled as a useful means for the discovery, evaluation and cultivation of these qualities during the teaching process.” (Kapitza 1980, p. 198) Some physicists combined both approaches in their research; this includes Hans Bethe, whose craftsmanship as a physicist has been described by his biographer as “an amalgam of what he learned from these two great physicists and teachers [Sommerfeld and Fermi], combining the best of both: The thoroughness and rigor of Sommerfeld with the clarity and simplicity of Fermi. He had learned from them how to balance rigorous analysis with approximate methods.” (Schweber 2013, p. 196) This craftmanship meant that Bethe, in the words of Freeman Dyson, was “the supreme problem solver of the past century.” (Dyson 2005, p. 219) In short, the two understandings of physics problem-solving competency are two co-existing approaches rather than two mutually exclusive ones.

5. Concluding remarks

Given that physics students are to acquire problem-solving competency during their physics education, educators need to make decisions about the kind of problem-solving competency. In this article, we have presented two different views on physics problem solving, the theory-first and the phenomenon-first approaches. Historical scholarship has shown that these are characteristic for the university level teaching of two prominent physicists of the 20th century. As shown above, the two positions can be found in other parts of the university level physics education community as well, for instance when it comes to the selection of physics problems at college/university level. A further example is a problem-solving course in physics at Roskilde University (Niss & Højgaard Jensen, 2010). The course is based on solving the so-called unformalized problems in physics, where problems are formulated in everyday language and often concern real world phenomena (Højgaard Jensen, Niss & Jankvist, 2017). Consequently, a major aspect of the problem-solving process is to formalize the problem in physics terms. Hence, these problem sits squarely within the Fermi tradition and a similar view has informed the development of the course (Niss & Højgaard Jensen, 2010).

The two positions seem to have something to offer for educational discussions on problem-solving at college/university level physics. An example from Danish Highschool indicates that the two positions are also relevant for discussion for physics education at lower levels that presumably are farther removed from research physics. The problems at the written exam at the highest level of physics in the Danish high school have traditionally been of the theory-first kind. However, within the last ten years or so, the problem sets at the exams have begun to be supplemented by problems that follow more the phenomenon-first approach as the problem-solver now has to make a thorough physical analysis of the problem situation. So, today both views on problem-solving competency is at play in the exam sets in Danish Highschool. An awareness of the two positions might clarify for teachers and students alike that actually two fundamentally different views are at play in the exam problems.

Finally, it should be pointed out that the two versions of the problem-solving competency need to be taught differently. Different kinds of problems are needed, depending on which competency we are talking about, but it is reasonable to assume that the two competencies also require different scaffolding activities as the solution processes require different planning, monitoring and justifications. While some of the extensive research that has been done on problem solving in physics might be relevant, it seems that further works need to be done in this respect.

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Response to comments

The manuscript has been revised according to the comments given directly in the manuscript.