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UNIFIED FORMALISM FOR EXCESS CURRENT NOISE IN RANDOM WALK
MODELS

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by: Jeppe Dyre

ABSTRACT

Excess current noise in random walk models with a frequency
independent conductivity is studied from a general point of view.
By introducing a "dynamical" diffusion constant, it is shown that
the current autocorrelation function in an external field probes
the equilibrium dynamical diffusion constant autocorrelation
function. From this a number of results, previously shown for
particular models, are derived . Also, it is shown that the
external field current autocorrelation function is proportional
to the equilibrium autocorrelation function for the absolute
value of the current. Thus, the excess noise spectrum probes the
equilibrium speed autocorrelation function. In the treatment
advanced here, the study of excess current noise in random walk
models reduces to a study of the stochastic point process
constituted by the particle jump times. This point process
contains all information about the noise. As an illustration of
the general theory, the CTRW model is briefly reviewed and a
simple derivation of the excess noise in the model is given.
Finally, the role of fermistatistics in models for 1/f noise is
discussed. It is argued that number fluctuation models, i.e.
models with long trapping times, are incompatible with
fermistatistics. On the other hand, it is shown there is a
peculiar "single particle" 1/f noise which is <u>due</u> to
fermistatistics but has nothing to do with the observed 1/f
current noise.

1. INTRODUCTION

Electrical 1/f noise has been a major puzzle in solid state physics for many years and is still far from understood [1-11]. This noise is found at low frequencies in apparently any conducting solid in an external electric field. 1/f noise is always observed together with white noise, the origin of which is well understood, and therefore 1/f noise is often referred to as excess noise. One may speak of excess current noise in a constant voltage circuit or excess voltage noise in a constant current circuit. The spectra of the two are always identical and only excess current noise will be discussed here.

Experimentally, the spectrum of excess current noise is given by

$$S_{exc,J}(\omega) = K \frac{\langle J \rangle_E^2}{V} \omega^{-\alpha}$$

(1)

where $\langle J \rangle_E$ is the average current in the electric field E, V is the volume of the sample, K is a constant, and the exponent α is close to one. The case $\alpha=1$ has given name to the subject: 1/f noise, where f is the frequency. The fact that the noise is proportional to $\langle J \rangle_E^2$ suggests it is the resistance that fluctuates and consequently one often speaks about 1/f resistance fluctuations. If really the resistance fluctuates, however, there should be 1/f fluctuations in the magnitude of the Nyquist noise in zero external field. This was shown actually to be the case by Voss and Clarke in 1976 [12]. Their work was a major breakthrough because it showed 1/f noise is an equilibrium phenomenon and is not created by the rather strong electric fields usually applied when measuring 1/f excess noise. The Voss

and Clarke experiment raised the obvious question: How can noise

fluctuate, being itself due to fluctuations? It was soon shown

that fluctuations in the magnitude of the Nyquist noise is due to

non-trivial fourth order correlations in the equilibrium current

or voltage fluctuations [13,14], implying these fluctuations are

non-gaussian.

During the eighties there has been considerable interest in

random walk models for 1/f noise. These models are probably the

simplest one can think of as a means of getting a better

understanding of the purely statistical properties of the noise.

In particular, the appearance of non-gaussian equilibrium current

fluctuations can be studied in detail. In sec. 2 of the present

paper we study general features of the current noise in random

walk models. The treatment is centred around the concept of a

"dynamical" diffusion constant. In sec. 3 the continuous time

random walk (CTRW) model for 1/f noise is briefly reviewed as an

illustration of the general theory. Finally, section 4 contains

a discussion where the role of fermistatistics for the

application of random walk models is emphasized. It is argued

that any model for 1/f noise based on long trapping times,

including the CTRW model in its multiple trapping realisation, is

incompatible with fermistatistics and is therefore unrealistic.

2. EXCESS CURRENT NOISE IN RANDOM WALK MODELS

To simplify the discussion we consider just one particle which performs a random walk in one dimension on a lattice with lattice constant a . No assumption is made about the underlying dynamics which does not have to be markovian. It is assumed the direction of any single jump is random; via the fluctuation-dissipation theorem this ensures a frequency-independent conductivity. If the particle jumps at times τ_i we assign to it a "dynamical" diffusion constant, D(t), defined by

$$D(t) = \frac{a^2}{2} \sum_i \delta(t-\tau_i) .$$

(2)

D(t) is characterised by

$$\langle v(t) v(t') \rangle_{re} = 2 D(t) \delta(t-t')$$

(3)

where v is the velocity, or

$$D(t) = \int_0^\infty \langle v(t) v(t+\tau) \rangle_{re} \, d\tau .$$

(4)

Here, $\langle \; \rangle_{re}$ denotes a "restricted ensemble average" by which is meant an average over the trajectories of all particles with the same D(t) , i.e. with the same jump times. The time average of D(t) is the ordinary diffusion constant D :

$$D = \langle D(t) \rangle = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_0^{T_0} D(t) \, dt .$$

(5)

The relevance of D(t) to the excess current noise becomes clear when the current in an external electric field is evaluated. In

a weak field it is slightly more probable for the particle to jump to one side than to the other. To lowest order in E the total jump probability does not change, however, and the particle still jumps at times τ_i. The restricted ensemble average current is thus proportional to the equilibrium D(t):

$$\left\langle J(t) \right\rangle_{E,re} = \frac{q^2 E}{kT} D(t) \tag{6}$$

where q is the particle charge, and k and T have their usual meaning. The constant of proportionality follows from requiring the time average of eq. (6) to obey the Nernst-Einstein relation. The measured current noise is the cosine transform of the current autocorrelation function. Since the currents at different times within the restricted ensemble are uncorrelated, one has

$$\left\langle J(t) J(0) \right\rangle_{E,re} = \left\langle J(t) \right\rangle_{E,re} \left\langle J(0) \right\rangle_{E,re}$$

$$= \left(\frac{q^2 E}{kT} \right)^2 D(t) D(0) \qquad (t>0). \tag{7}$$

When averaged over the whole ensemble of possible jump times this leads to

$$\left\langle J(t) J(0) \right\rangle_{E} = \left(\frac{q^2 E}{kT} \right)^2 \left\langle D(t) D(0) \right\rangle \qquad (t>0) \tag{8}$$

where $\langle \ \rangle$ on the right hand side denotes an equilibrium average. To obtain the total current autocorrelation function one should add to this expression a white noise term proportional to $\delta(t)$. This term is not of interest here. According to eq. (8) the excess noise measures the spectrum of dynamical diffusion

constant fluctuations in equilibrium.

The autocorrelation function $\langle D(t)D(0)\rangle$ has a simple physical interpretation. From eq. (2) it follows immediately that

$$\langle D(t)D(0)\rangle \simeq \frac{D\,a^2}{2}\,p(t|0) \qquad\qquad (t>0) \qquad (9)$$

where $p(t|o)$ denotes the probability density for a jump at time t given the particle jumped at $t=0$. Except for a numerical constant, low frequency excess noise is thus the cosine transform of $p(t|o)$ [15]. $1/f$ noise implies long time correlations in this probability. The particle or the medium in which it jumps somehow has a long term memory. For an ordinary random walk, on the other hand, $p(t|o)$ is constant and there is no excess noise.

Generalising the above results to more than one particle is straightforward. Assuming the particles are independent and non-interacting, one just lets $\{\tau_i\}$ denote the collection of jump times for all the particles. In generalising to d dimensions the factor 2 in eqs. (2) etc should be replaced by 2d. Equations like (6) and (8) apply unchanged where J is now, of course, the component of the current in the direction of the field. No new features appear and in the rest of the paper only the one-dimensional case will be considered. The important thing, which is independent of the dimension and number of particles, is that all information about the noise lies in the statistical properties of the collections of jump times τ_i. In statistics a stochastic collection of times is referred to as a "point process". The study of point processes is a mature branch of the theory of stochastic processes [16-18]. Point processes have been applied in the study of photoelectron statistics, cosmic ray

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showers, kinetic theory, population growth, telephone traffic, etc. An important class of point processes is the class of so-called doubly stochastic Poisson processes. An example is the jump times τ_i for a particle performing a random walk where the jump probability at time t, $\gamma(t)$, is itself a stochastic process. Here one may define a second kind of time dependant diffusion constant, $\widetilde{D}(t)$, by

$$\dot{\widetilde{D}}(t) = \frac{a^2}{2}\, \gamma(t) .$$

(10)

While $\widetilde{D}(t)$ is of course different from $D(t)$, their statistical properties are identical: For the average over a particular realisation of $\gamma(t)$ it is easy to see that $\langle D(t_1) \cdots D(t_n) \rangle_{\gamma(t)} = \widetilde{D}(t_1) \cdots \widetilde{D}(t_n)$. Averaging this expression over all possible $\gamma(t)$'s leads to

$$\langle D(t_1) \cdots D(t_n) \rangle = \langle \widetilde{D}(t_1) \cdots \widetilde{D}(t_n) \rangle .$$

(11)

Thus, our definition of $D(t)$ in eq. (2) is consistent in the case where the ordinary diffusion constant really does fluctuate in time via a time dependent jump probability.

We now turn to the problem of expressing the current autocorrelation function in an external field in terms of equilibrium current fluctuations. Focusing attention on diffusion constant fluctuations according to eq. (8), we note that the integrated dynamical diffusion constant counts the number of jumps, N, in time t :

$$N = \frac{2}{a^2} \int_0^t D(t')dt' .$$

(12)

For the variance of N one has

$$\langle (\Delta N)^2 \rangle_t = \frac{4}{a^4} \left[\int_0^t \int_0^t \langle D(t') D(t'') \rangle \, dt' dt'' - \left(\int_0^t \langle D(t') \rangle \, dt' \right)^2 \right]$$

$$= \frac{8}{a^4} \int_0^t (t - \tau) \, C_D(\tau) \, d\tau \qquad\qquad (13)$$

where $C_D(\tau)$ is the autocorrelation function for the dynamical diffusion constant,

$$C_D(\tau) \equiv \langle D(\tau) D(0) \rangle - D^2. \qquad\qquad (14)$$

Equation (13) leads to

$$C_D(t) = \frac{a^4}{8} \frac{d^2}{dt^2} \langle (\Delta N)^2 \rangle_t. \qquad\qquad (15)$$

The next step is to relate the right hand side to averages of the displacement in time t, $\Delta X(t)$. The quantity $\langle e^{ik\Delta X(t)} \rangle$ is an average of a product of independent factors $e^{\pm ika}$ and thus

$$\langle e^{ik\Delta X(t)} \rangle = \langle \cos^N(ka) \rangle_t. \qquad\qquad (16)$$

From this equation averages of all powers of $\Delta X(t)$ can be found. For the first two non-zero averages one finds

$$\langle (\Delta X)^2(t) \rangle = - \frac{d^2}{dk^2} \langle \cos^N(ka) \rangle_t \Big|_{k=0} = a^2 \langle N \rangle_t$$

$$\langle (\Delta X)^4(t) \rangle = \frac{d^4}{dk^4} \langle \cos^N(ka) \rangle_t \Big|_{k=0} = a^4 \left(3\langle N^2 \rangle_t - 2\langle N \rangle_t \right). \qquad (17)$$

In particular, the fourth order cumulant of $\Delta X(t)$,

$$\langle \Delta X^{(4)}(t) \rangle \equiv \langle (\Delta X)^4(t) \rangle - 3 \langle (\Delta X)^2(t) \rangle^2, \text{ is given by}$$

$$\langle \Delta X^{(4)}(t) \rangle = a^4 \left(3 \langle (\Delta N)^2 \rangle_t - 2 \langle N \rangle_t \right). \tag{18}$$

Since $\langle N \rangle_t$ is always proportional to t, eq. (18) in conjunction with eq. (15) implies [19-22]

$$C_D(t) = \frac{1}{24} \frac{d^2}{dt^2} \langle \Delta X^{(4)}(t) \rangle. \tag{19}$$

Equation (19) was first derived by Kuzovlev and Bochkov as a consequence of

$$\left\langle e^{ik\Delta X(t)} \right\rangle = \left\langle e^{-k^2 \int_0^t D(t')dt'} \right\rangle \tag{20}$$

derived for "slowly fluctuating" $D(t)$ [20]. Their definition of $D(t)$ is not obvious, but eq. (20) is not valid for any reasonable definition of $D(t)$ as long as $a > 0$, since it does not apply even in the case of an ordinary random walk with a constant $D(t)$: For $D(t) = D$ eq. (20) reduces to $\ln \left\langle e^{ik\Delta X(t)} \right\rangle = -k^2 D t$ which implies $\langle \Delta X^{(4)}(t) \rangle = 0$. In reality $\langle \Delta X^{(4)}(t) \rangle \propto t$ in this case since cumulants are additive and $\Delta X(t)$ is a sum of independent increments. Instead, eq. (20) must be replaced by

$$\left\langle e^{ik\Delta X(t)} \right\rangle = \left\langle e^{\frac{2\ln(\cos(ka))}{a^2} \int_0^t D(t')dt'} \right\rangle \tag{21}$$

which just combines eqs. (12) and (16). For $a \to 0$ eq. (21) reduces to eq. (20). This limit, however, is only permissible in certain models. From eq. (21) it is possible to derive eq. (19) directly by expanding the logarithm of the equation to fourth

order in k , but we found it more instructive and also useful below to arrive at eq. (19) via averages of N and its fluctuations.

Defining as usual the fourth order cumulant of the velocity by

$$\langle v(t_1), v(t_2), v(t_3), v(t_4) \rangle \equiv \langle v(t_1) v(t_2) v(t_3) v(t_4) \rangle - \langle v(t_1) v(t_2) \rangle \langle v(t_3) v(t_4) \rangle$$
$$- \langle v(t_1) v(t_3) \rangle \langle v(t_2) v(t_4) \rangle \tag{22}$$
$$- \langle v(t_1) v(t_4) \rangle \langle v(t_2) v(t_3) \rangle$$

equation (19) can be rewritten as

$$C_D(t) = \int_0^t d\tau' \int_0^{\tau'} d\tau'' \langle v(t), v(\tau'), v(\tau''), v(0) \rangle. \tag{23}$$

The right hand side is the Burnett coefficient [21,22]. Equation (23) was derived by Kuzovlev and Bochkov [20] from eq. (20), and also by Nieuwenhuizen and Ernst from a different point of view [22]. The latter authors define a "fluctuating diffusion coefficient", $D(t,\tau)$, by

$$D(t, \tau) = \frac{1}{2} \frac{\partial}{\partial t} \left[(x(t) - x(\tau))^2 \right]. \tag{24}$$

They then proceed to show for the markovian random walk model under study that $C_D \equiv \langle D(t, \tau) D(t', \tau') \rangle - D^2$ is actually a function of t-t' only which obeys eq. (23).

Combining eqs. (8), (14) and (23) we finally arrive at

$$\langle J(t) J(0) \rangle_E = \langle J \rangle_E^2 + \left(\frac{E}{kT} \right)^2 \int_0^t d\tau' \int_0^{\tau'} d\tau'' \langle J(t), J(\tau'), J(\tau''), J(0) \rangle. \tag{25}$$

Since cumulants are additive, this equation implies the excess current noise is proportional to $\langle J \rangle_E^2 / V$ as in experiments (eq. (1)). Actually, this follows also directly from eq. (8). It

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was first shown for a random walk model by Tunaley [23].
Equations like eq. (25) has been derived by a number of authors
[13,14,20,21,22,24], thereby greatly clarifying and simplifying
the subject by reducing the current noise in an external field to
equilibrium current fluctuations. We note here that it is
actually possible to express $\langle J(t)J(0)\rangle_E$ in terms of
equilibrium two point correlation functions. Since

$$|J(t)| = q\alpha \sum_i \delta(t-\tau_i) = \frac{2q}{\alpha} D(t),$$

(26)

equation (8) can be rewritten as

$$\left\langle J(t)J(0)\right\rangle_E = \left(\frac{q\alpha E}{2 kT}\right)^2 \left\langle |J(t)| |J(0)|\right\rangle \qquad (t>0)$$

(27)

where the right hand side is, as usual, an equilibrium average.
This result is simpler than eq.(25) and, perhaps, more
aesthetically pleasing. But it is less general than eq.(25)
since it explicitly involves the lattice constant and thus
depends on the existence of a lattice.

For the generation of 1/f noise one needs long time
correlations in the fluctuations of D(t). Actually, noise that
varies approximately as 1/f is obtained only if $C_D(t)$ is almost
constant, typically varying as log(t) to some negative power. One may
imagine two different ways of generating 1/f noise. The one case
is that of "genuine" mobility fluctuations, i.e. when the random
walk is a doubly stochastic Poisson point process with 1/f noise
in the $\widetilde{D}(t)$ fluctuations of eq. (10). The other case is when
long time correlations in D(t) arise because of occasional long
trapping times of the particles. Here one must assume the
existence of a broad spectrum of trapping times exceeding the

longest experimental times. This may be regarded as the case of number fluctuations since a charge carrier trapped for the whole period of observation for all practical purposes is non-existent.

We close this section by showing that only 1/f noise with strong fluctuations in the number of jumps is observable. By strong noise is here meant noise obeying

$$\langle (\Delta N)^2 \rangle_t \gg \langle N \rangle_t \qquad (28)$$

on the relevant time scale. This is the criterion for N fluctuations much larger than for an ordinary random walk where there is equality in eq. (28) on account of the Poisson statistics. Both the mobility and the trapping mechanism may satisfy eq. (28) which rules out only the case of very weak mobility fluctuations. To show eq. (28) we calculate first the white noise in equilibrium. From $J = qa \sum_i \pm \delta(t-\tau_i)$ we get immediately

$$S_{white,J}(\omega) = 2q^2 a^2 \frac{\langle N \rangle_t}{t} \quad , \quad t = \omega^{-1}, \qquad (29)$$

where the number of jumps per unit time, $\langle N \rangle_t / t$, is of course independent of t . In a weak external field there is a slight increase in the white noise which, however, is insignificant for the present calculation.[22,23] Since $C_D(t)$ varies only very slowly one has $S_D(\omega) \lesssim 4t C_D(t)$ and eq. (13) implies $C_D(t) \simeq \frac{a^4}{4} \frac{\langle (\Delta N)^2 \rangle_t}{t^2}$ where $t = \omega^{-1}$. Combining these results we get for the excess current noise

$$S_{exc,J}(\omega) \lesssim q^2 a^2 \left(\frac{qaE}{kT} \right)^2 \frac{\langle (\Delta N)^2 \rangle_t}{t} , \quad t = \omega^{-1} \qquad (30)$$

In linear response theory $qaE/kT \ll 1$ and the criterion for

measurable excess noise reduces to eq. (28). This condition is independent of the size of the system since both sides of eq. (28) are additive.

3. EXAMPLE: EXCESS NOISE IN THE CONTINUOUS TIME RANDOM WALK MODEL

One way or the other 1/f noise arises from long time correlations in the diffusion constant fluctuations. This may occur for instance via occasional very long trapping times for the change carriers. The simplest example of this is the continuous time random walk (CTRW) model of Montroll and Weiss [25]. Here the jump probability at any time is a function only of the time elapsed since the preceding jump. The CTRW model was first applied to the 1/f noise problem by Tunaley [23] and later by Nelkin and co-workers [24,26]. The central quantity in this model is the waiting time distribution function, $\psi(t)$, which is the probability density for jumps the time t after the latest jump. In the language of stochastic point processes, the sequence of waiting times is a so-called renewal process [16,27]. In this section we calculate the excess noise from $\psi(t)$. This was first done by Tunaley [23] but is repeated here as an illustration of the general theory of sec. 2 and also because the below derivation is simpler than that of Tunaley.

If the particle jumps at $t=0$ we let $\psi_n(t)$ denote the probability density for the n'th jump hereafter occurring at time t. Obviously one has

$$\psi_{n+1}(t) = \int_0^t \psi_n(t-\tau)\,\psi(\tau)\,d\tau \tag{31}$$

For the function $f(t)$ defined by

$$f(t) = \sum_{n=1}^{\infty} \psi_n(t) \tag{32}$$

equation (31) implies

$$\int(t) - \psi(t) = \int_0^t \int(t-\tau)\,\psi(\tau)\,d\tau \qquad (33)$$

Taking the Laplace transform of this equation one gets

$$\tilde{f}(s) - \tilde{\psi}(s) = \tilde{f}(s)\,\tilde{\psi}(s) \qquad , \text{ or}$$

$$\tilde{f}(s) = \frac{\tilde{\psi}(s)}{1 - \tilde{\psi}(s)} \qquad . \qquad (34)$$

The quantity $p(t|o)$ occurring in eq. (9) is just $f(t)$ and thus

$$S_{exc}(\omega) = \frac{S_{exc,0}(\omega)}{D^2} = \frac{a^2}{2D}\,4\int_0^\infty \int(t)\cos(\omega t)\,dt$$

$$= \frac{2a^2}{D}\,Re\,\tilde{f}(i\omega) = \frac{2a^2}{D}\left[Re\,\frac{1}{1-\tilde{\psi}(i\omega)} - 1\right] . \qquad (35)$$

This is Tunaley's result for the excess noise [23]. It is convenient to write $\psi(t)$ as a sum of exponential decays, $\psi(t) = \langle \gamma e^{-\gamma t}\rangle$, where the average is over a distribution of jump rates [24,26]. From this $\tilde{\psi}(s) = \langle\frac{\gamma}{\gamma+s}\rangle$ and $D = \frac{a^2}{2}\langle\gamma^{-1}\rangle^{-1}$ which substituted into eq. (35) yields

$$S_{exc}(\omega) = \frac{2a^2}{D}\left[Re\,\frac{1}{i\omega\langle\frac{1}{\gamma+i\omega}\rangle} - 1\right] = \frac{2a^2}{D\omega}\left[Im\,\langle\frac{1}{\gamma+i\omega}\rangle^{-1} - \omega\right], \qquad (36)$$

i. e.

$$S_{exc}(\omega) = \frac{4\langle\gamma^{-1}\rangle}{\omega}\left[Im\,\langle\frac{1}{\gamma+i\omega}\rangle^{-1} - \omega\right] . \qquad (37)$$

A simple example yielding 1/f noise is the case of all γ's equally likely:

$$P(\gamma) = \frac{1}{\gamma_0} , \quad \gamma_{min} < \gamma < \gamma_0 , \tag{38}$$

where $\gamma_{min} \ll \gamma_0$ is assumed. One may think of γ_0 as a phonon frequency and γ_{min} as corresponding to long waiting times, e.g. one day or one year. For the whole range of intermediate frequencies $\langle (\gamma + i\omega)^{-1} \rangle \cong \gamma_0^{-1} \ln(\gamma_0/i\omega)$ and

$$S_{exc}(\omega) \cong \frac{4}{\omega} \ln\lambda \, \text{Im} \, \frac{1}{\ln(\gamma_0/i\omega)} \cong \frac{2\pi \ln\lambda}{\ln^2(\gamma_0/\omega)} \frac{1}{\omega} \tag{39}$$

where $\lambda = \gamma_0/\gamma_{min}$. We thus find $S_{exc}(\omega) \propto \omega^{-\alpha}$ where

$$\alpha = 1 - \frac{2}{\ln(\gamma_0/\omega)} . \tag{40}$$

At ordinary laboratory frequencies one has $\alpha \cong 0.9$. This model may be termed the "standard model of 1/f noise" since it is probably the simplest exactly soluble random walk model giving 1/f excess noise. Of course 1/f noise is built into the model via eq.(38). This is equivalent to a waiting time distribution function $\psi(t) \propto t^{-2}$ [28] above γ_{min} , which implies a logarithmically divergent average waiting time. The excess noise of the standard model is identical in functional form to the expression derived by Kuzovlev and Bochkov for a scale-invariant random walk [10,20]. Their derivation is based on eq. (20) which has been criticised above. It is not quite transparent, also the factor $\ln\lambda$ in eq. (39) is absent in their calculation of the excess noise.

4. DISCUSSION

Random walk models provide a simple framework for understanding the fluctuation-dissipation theorem. It is therefore an obvious idea also to use these models for getting a better understanding of low frequency resistance fluctuations. Various random walk models have been studied with this purpose [19-24,26,28-30]. In this paper a general framework for discussing excess noise in random walk models has been proposed. Following previous work, in sec. 2 we focused on the concept of a fluctuating diffusion constant, the "dynamical" diffusion constant $D(t)$. While the exact definition of $D(t)$ hitherto has not been clear, we here use a definition of $D(t)$ which is simpler than previous definitions. For all practical purposes, however, it is identical to these since the expressions for $\langle D(t)D(0)\rangle$ in eqs. (19) and (23) are identical to those given by Kuzovlev and Bochkov [19,20], and Machta, Nelkin, Nieuwenhuizen and Ernst [21,22]. The relevance of $D(t)$ to excess current noise is shown by eq. (8) according to which the current fluctuations in an external field directly probes the equilibrium $\langle D(t)D(0)\rangle$. An important general property of random walk models is the fact that the noise is proportional to $\langle J\rangle_E^2$ and inversely proportional to the volume or any other extensive property like the number of charge carriers. This follows immediately from eq. (8) since $D(t)$ and autocorrelation functions are additive.

As an example of an application of the general formalism, note that eq. (16) is valid also in a time-dependant external field. From this it is straightforward to show that in a sinusodial field one finds the so-called $1/\Delta f$ noise [31-33] which is directly proportional to the magnitude of the 1/f noise in a

constant field. A weakness of the formalism of sec. 2 should be mentioned, namely that it apparently does not easily allow for an exact calculation of the white noise in an external field. Though insignificant in the linear regime, there is an interesting small increase in the white noise when the field is turned on [22,23].

The dynamical diffusion constant is proportional to the absolute value of the current. This leads to a simple expression for the current fluctuations in an external field in terms of equilibrium fluctuations (eq. (27)), a result which can be rewritten in terms of autocorrelation functions as

$$\left\langle \left(J(t)-\langle J\rangle_E\right)\left(J(0)-\langle J\rangle_E\right)\right\rangle_E = \left(\frac{qaE}{2kT}\right)^2 \left\langle \left(|J(t)|-\langle|J|\rangle\right)\left(|J(0)|-\langle|J|\rangle\right)\right\rangle . \tag{41}$$

Analogous to the frequency-dependent conductivity which probes the equilibrium _velocity_ autocorrelation function, the excess noise spectrum thus probes the equilibrium _speed_ autocorrelation function. But it should be remembered that a frequency independent conductivity must be assumed to derive eq. (41), and also that the random walk takes place on a lattice. Equation (41) cannot be expected to apply more generally. For this to be the case, one should be able to define a characteristic length to play the role of the lattice constant a in eq. (41). The only possibility for this seems to be to let a be the length for which $qaE \simeq kT$ at fields marking the onset of non-linearities. Since, however, non-linearities involve new physics which is in general uncorrelated to linear response phenomena, there is little hope this approach can be generally valid. Thus, eq. (41) must be limited to lattice models.

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models developed above provides a convenient starting point for a discussion of general properties of 1/f noise. In particular, the point process approach makes it possible to throw some light on the old controversy as to whether 1/f noise is due to mobility or to number fluctuations [6,9]. We end this paper by giving a general argument to the effect that fermistatistics rules out the number fluctuation mechanism. Mobility fluctuations correspond to noise _in_ the effective charge carrier Hamiltonian, whereas number fluctuations due to occasional deep trapping is noise generated _by_ the Hamiltonian itself. In the language of point processes, the standard example of mobility fluctuations is the case when the jump times τ_i constitute a doubly stochastic Poisson process, while the standard example of trapping noise is the case when τ_i is a renewal process, i.e. when we have a continuous time random walk. This case is non-markovian and therefore in a sense unphysical, admittedly, but the CTRW model is equivalent to a markovian multistate trapping model where the noise then is indeed generated _by_ the Hamiltonian [26].

In sec. 3 the excess noise in the CTRW model was calculated. A simple example termed the standard model was worked out in detail. This model is one out of the class of CTRW models discussed by Nelkin and Harrison [26]. In the multistate trapping realisation of the CTRW, the standard model corresponds to a density of energies, ε , given by

$$ p(\varepsilon) \propto \exp\left(\frac{\varepsilon}{kT}\right). \tag{42} $$

Though this implies only very few deep trapping states, in the course of time all energies are equally likely occupied because of the Boltzmann factor to be multiplied to eq. (42) to get the

probability. Thus, the standard model is just another example of the old idea of 1/f noise arising when all activation energies are equally likely. At the same time the standard model satisfies the requirement for 1/f noise given by Nieuwenhuizen and Ernst, namely an exponential density of states [22,28].

A temperature-dependent density of states as in eq.(42) arises in a system of fermions. Here a single particle feels, in a mean-field approximation, a density of available states below the fermienergy, $\rho_{eff}(\varepsilon)$, which is given by

$$\rho_{eff}(\varepsilon) \cong n(\varepsilon)\, e^{\frac{\varepsilon - \varepsilon_F}{kT}} \qquad\qquad (\varepsilon < \varepsilon_F) \qquad (43)$$

where $n(\varepsilon)$ is the ordinary density of states. For a constant $n(\varepsilon)$ eq.(42) is obtained. Equation (43) implies 1/f noise in the motion of the single particles, a point we will return to below. While interesting on its own, this has nothing to do with the observed 1/f resistance fluctuations, however. This is because the mean-field approximation does not apply due to strong interparticle correlations: Below the fermilevel the particle number fluctuations (in one energy level) are exponentially small, $\langle(\Delta n)^2\rangle = \langle n\rangle\langle 1-n\rangle \ll \langle n\rangle$ [34], while for independent particles one would have $\langle(\Delta n)^2\rangle \approx \langle n\rangle$ (because of the Poisson statistics), i.e. much larger fluctuations.

We now extend the above and argue that fermistatistics actually kills any 1/f noise model based on the trapping mechanism. Basically, one may imagine two different ways of producing long trapping times. The first case involves hopping between localised states of similar energy separated by various long tunnelling distances or large energy barriers. In this case, inevitably, there is a strong frequency-dependence of the

conductivity and models of this kind cannot explain the usual case of 1/f fluctuations in a frequency-<u>in</u>dependent conductivity. (Note that McWhorter's model based on tunnelling to surface states [9] is not ruled out by this argument which is only concerned with bulk and isotropic 1/f noise.) The second way of having long trapping times is that of trapping into deep energies. This only works for independent particles with the peculiar density of states of eq. (42). For fermions, the density of states above the fermienergy is not temperature-dependent, and below the fermilevel, where eq. (43) does apply in a mean-field sense, the particles are not independent as discussed already.

To summarise the effect of fermistatistics, it has been argued that for fermisystems 1/f resistance fluctuations cannot be due to occasional deep trapping of the charge carriers. The number fluctuation mechanism thus can be ruled out on general grounds, and e.g. the CTRW model in the multiple trapping realisation is not realistic. Thus, assuming the noise is markovian, the noise must be caused by mobility fluctuations. The central problem, which till today remains largely unsolved, is to identify the origin of the mobility fluctuations.

While fermistatistics rules out number fluctuations, it implies on the other hand a peculiar kind of "single particle" 1/f noise: The motion of a single localised fermion exhibits 1/f noise as a consequence of eq. (43). In principle this is observable, since, for a system of fermions described by a master equation (an implicit assumption in the argument), there are no quantum coherence effects and, in effect, the particles are classical and distinguishable. Similarly, atoms or ions diffusing in a disordered medium with a distribution of available potential minima also exhibit single particle 1/f noise, since

they behave as fermions because of their strong repulsion. This

should be observable by monitoring the motion of tracer atoms or

ions in time.

Except for the above arguments ruling out number fluctuations,

the cause of 1/f noise has not been discussed in the present

paper. We feel there is no simple and generally valid

mechanism explaining 1/f noise. Rather, 1/f noise may arise from

a number of different mechanisms which probably, one way or the

other, involve lattice defects overcoming or tunnelling through

barriers. Following this line of thought, 1/f noise and its

dependence on temperature, etc, provides important information

about the solid under study. Note that while $S_{exc}(\omega)$ probes

only $\langle D(t)D(0)\rangle$ there is an enormous amount of information

hidden in the higher order correlation of $D(t)$. This is

illustrated by the fact that the total amount of information in

the noise is contained in a function of two variables

$$P(u,t) = \langle e^{uN} \rangle_t$$

(44)

while $S_{exc}(\omega)$ is only a function of one variable. The fact that

there is more to the noise than just the spectrum has been emphasized already 10 years

ago by Voss in a discussion of linearity of the 1/f noise

mechanism [35]. The higher order correlations of J(t) and

thereby D(t) are accessible today by digital techniques and their

determination should provide a means of distinguishing between

various models for 1/f noise. Other promising lines of research

is the measurement of 1/f noise anisotropy [11], and the problems

of stationarity of the noise and the dependence of the noise on

the annealing state (in any system with long relaxation times the

possibility of "glass transitions" should not be forgotten). -

1/f noise is still mainly an experimental field and it seems a

lot of work remains to be done here before reliable theories can

be arrived at.

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PEACE RESEARCH SERIES NO. I
Af: Bent Sørensen
nr. 83 er p.t. udgået

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Af: Jens Højgaard Jensen, Mogens Niss m. fl.

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Specialerapport af: Per Hedegård Andersen, Carsten Holst-
Jensen, Else Marie Pedersen og Erling Møller Pedersen.
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Projektrapport af: Biger Lundgren, Henning Sten Hansen
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Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn
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　　　　GLASS TRANSITION".
　　　　Af: Tage Christensen.

　　　　"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
　　　　Af: Jeppe C. Dyre.
　　　　Contributions to the Third International Conference
　　　　on the Structure of Non – Crystalline Materials held
　　　　in Grenoble July 1985.

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　　　　Af: Bent Sørensen.

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　　　　– flodblindhed som eksempel på matematisk modelle-
　　　　ring af et epidemiologisk problem.
　　　　Projektrapport af: Per Hedegård Andersen, Lars Boye,
　　　　CarstenHolst Jensen, Else Marie Pedersen og Erling
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　　　　Vejleder: Jesper Larsen.

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　　　　Projektrapport af: Mikael Wennerberg Johansen, Poul Kat-
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　　　　Af: Bent Sørensen

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　　　　og Jimmy Staal.
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112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION
　　　　FREM TIL 1950 – BELYST VED EKSEMPLER".
　　　　Projektrapport af: Erik Odgaard Gade, Hans Hedal,
　　　　Frank C. Ludvigsen, Annette Post Nielsen og Finn
　　　　Physant.
　　　　Vejleder: Claus Bryld og Bent C. Jørgensen.

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　　　　AF KONTIGENSTABELLER".
　　　　Projektrapport af: Lone Biilmann, Ole R. Jensen
　　　　og Anne-Lise von Moos.
　　　　Vejleder: Jørgen Larsen.

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　　　　Af: Mogens Niss.

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　　　　KUNNE NOK SKABE RØRE I ANDEDAMMEN".
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　　　　Vejleder: Mogens Niss.

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　　　　Af: Jeppe C. Dyre.

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　　　　Fysiklærerforeningen, IMFUFA, RUC.

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　　　　Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β – systemet – en effektiv fotometrisk spektral-
　　　　klassifikation af B-,A- og F-stjerner".
　　　　Projektrapport af: Birger Lundgren.

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　　　　Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
　　　　Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE
　　　　ALGEBRA".
　　　　Projektrapport af: Pernille Sand, Heine Larsen &
　　　　Lars Frandsen.
　　　　Vejleder: Mogens Niss.

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　　　　Projektrapport af: Niels Jørgensen & Mikael Klintorp.
　　　　Vejleder: Jeppe Dyre.

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　　　　Lecture Notes 1983 (1986)
　　　　Af: Bent Sørensen

130/86 "Studies in Wind Power"
　　　　Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" – Et integreret fysik/historie-
　　　　projekt om naturanskuelsens historiske udvikling
　　　　og dens samfundsmæssige betingethed.
　　　　Projektrapport af: Jakob Heckscher, Søren Brønd,
　　　　Andy Wierød.
　　　　Vejledere: Jens Høyrup, Jørgen Vogelius,
　　　　Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
　　　　Projektrapport af: Søren Brønd, Andy Wierød.
　　　　Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA.
　　　　ENERGY SERIES NO. 15.
　　　　AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
　　　　Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen,
　　　　　　　　Petr Viscör

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSES-
　　　　TEORETISKE FORUDSÆTNINGER"
　　　　MASTEMATIKSPECIALE: Claus Larsen
　　　　Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens
　　　　første og andet møde med græsk filosofi"
　　　　Projektrapport af Frank Colding Ludvigsen
　　　　Vejledere: Historie: Ib Thiersen
　　　　　　　　　　Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE
　　　　FASTE STOFFER" – Resume af licentiatafhandling
　　　　Af: Jeppe Dyre
　　　　Vejledere: Niels Boye Olsen og
　　　　　　　　　　Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International
Workshop on Teaching Nonlinear Phenomena
at Universities and Schools, "Chaos in
Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

13 9/87 "Machbarkeit nichtbeherrschbarer Technik
durch Fortschritte in der Erkennbarkeit
der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR -
ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With
Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircan Approach

Paper presented at the Symposium on the
Foundations of Modern Physics The Copenhagen
Interpretation 60 Years after the Como Lecture.
Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND
MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at
ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM"

- en ny frekvensbaseret målemetode.

Fysikspeciale af Jan Vedde

Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"

redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med
Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

149/87 "In-Situ Measurements of the density of amorphous
germanium prepared in ultra high vacuum"

by: Petr Viščor

150/87 "Structure and the Existence of the first sharp
diffraction peak in amorphous germanium
prepared in UHV and measured in-situ"

by: Petr Viščor

151/87 "DYNAMISK PROGRAMMERING"

Matematikprojekt af:
Birgit Andresen, Keld Nielsen og Jimmy Staal

Vejleder: Mogens Niss

152/87 "PSEUDO-DIFFERENTIAL PROJECTIONS AND THE TOPOLOGY
OF CERTAIN SPACES OF ELLIPTIC BOUNDARY VALUE
PROBLEMS"

by: Bernhelm Booss-Bavnbek
Krzysztof P. Wojciechowski