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socio-economic demands and influences

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Mathematical Education in Iceland in Historical Context

Socio-Economic Demands and Influences

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Ph.D. Dissertation

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Mathematical Education in Iceland in Historical Context
Socio-economic Demands and Influences

By: Kristín Bjarnadóttir, December 2006

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This study examines the history of mathematical education in Iceland and its position in comparison to its neighbouring countries. The Icelandic educational system and the socio-economic implications on mathematical education are subjected to a historically based analysis. The study is divided into three parts: The first section deals with the period from the settlement of Iceland in 874 A.D. into the modern period. The second section commences in 1800 and focuses on the late 1870s. It explores the effects of legislation on public education in arithmetic and regulations introducing a language stream in the Learned School, which had socio-economic consequences in that it yielded a long-term shortage of mathematics teachers, and delayed independent Icelandic initiatives in technical affairs. Meanwhile, non-mathematicians spearheaded public education in arithmetic. The third section focuses on the years 1965–1975, when the educational system was reformed with the introduction of “modern” mathematics.

After World War II, a modern egalitarian educational system was built up. It suffered from lack of facilities, and trained teachers, textbooks and curricula in mathematics. These factors in addition to population growth and demands for “education for all” had stretched the system to a breaking point by the 1960s. Theories were introduced then, initiated by the OECD, arguing that education, especially in mathematical subjects, was central to social and economic progress. A “modern” mathematics reform movement stimulated by the OECD, introducing logic and set theory into school mathematics, was part of a post-war awakening in science education. Iceland joined the reform movement with official backing and high expectations of economic progress.

The study compares developments in Iceland and neighbouring countries, Norway, Denmark and England. Mathematics education in Iceland differs primarily from its neighbours in the lengthy absence of demand for furthering higher mathematical education, nearly total dominance of a few institutions, and initiatives of individuals. During the 1960s reform period traits in common emerged, for instance attempts to dissolve stratified systems of education, and hopes that educational reform would lead to economic progress and improved understanding of mathematics by the pupils in schools.

The introduction of “modern” mathematics in Iceland proved to be strenuous. However, the challenge of coping with “modern” mathematics stimulated the creativity and initiative of a generation of mathematics teachers, and became a long-needed opportunity for their continuing education.

Kristín Bjarnadóttir, December 2006

Resumé

Denne afhandling handler om matematikuddannelsens historie i Island og dens stilling set i forhold til Islands nabolande. Der benyttes et historisk baseret ræsonnement for at undersøge det islandske uddannelsessystem og de sociale og økonomiske implikationer, som har haft indflydelse på matematikuddannelsen.

Afhandlingen er delt op i tre hovedafsnit: Det første afsnit handler om perioden fra landnam i det niende århundrede og op til den første del af den moderne tid. Det andet afsnit beskæftiger sig med perioden fra i begyndelsen af det nittende århundrede og fokuserer på begivenheder sidst i 1870erne. Her træder en lov i kraft om børns undervisning i regning og bekendtgørelser, som medførte, at der udelukkende blev oprettet en sproglig linie i den lærde skole. Udviklingen herefter op til midten af det tyvende århundrede undersøges. I det tredje afsnit fokuserer afhandlingen særligt på vigtige begivenheder i perioden 1965 til 1975, hvor undervisningssystemet blev reformeret og den “moderne” matematik blev indført.

Den islandske historie har sin oprindelse i den gamle middelalderkultur, hvorfra der bl.a. er bevaret nogle beretninger, som vedrør matematiske emner. Af særlig interesse er nogle tidlige oversættelser af tekster om hindu-arabisk aritmetik og korte fra begyndelsen af den moderne tid. De ændringer, der blev foretaget i 1740erne, skabte grundlaget for den lærde skole og en stærk tradition for hjemmeundervisning, som varede helt til 1970erne.

I den sidste halvdel af det nittende århundrede var der fremskridt i børne- og ungdomsuddannelserne. Loven af 1880 foreskrev undervisning for børn i regning uden at der eksisterede en egentlig skolelov. Modsat blev matematikundervisningen i den lærde skole reduceret ved, at man kun indførte den sproglige linie fra den nyligt reviderede danske lov om de lærde skoler. Det havde socio-økonomiske konsekvenser og resulterede bl.a. i en langvarig mangel på matematiklærere. Det var også med til at forsinke islændingenes egne initiativer på det tekniske område. De mest avancerede uddannelsesinstitutioner, Den Lærde Skole i Reykjavík og Islands Universitet, deltog ikke i den forrygende tekniske udvikling, som foregik i Island fra 1890erne til 1920erne, da der ikke eksisterede nogen form for undervisning i højere matematik. Derimod var der visse fremskridt i regning i den almene skoleuddannelse, ikke mindst på initiativ af ikke-matematikere.

Efter Anden Verdenskrig blev der etableret et moderne undervisningssystem baseret på ligeberettigelse, men manglende skolebygninger, mangel på uddannede matematiklærere, forældede lærebøger og usamtidige læseplaner hæmmede udviklingen. Et stigende pres fra en voksende befolkning og et øget krav om ”uddannelse for alle” havde fået systemet til at nå sine yderste grænser i 1960erne. På det tidspunkt fremkom der teorier, som blev udarbejdet af OECD, om at uddannelse, herunder matematik i særdeleshed, ydede et bidrag til sociale og økonomiske fremskridt. En ny type skolematematik, den ”moderne” matematik, blev indført globalt med støtte fra OECD. En generel reform af det islandske skolesystem blev påbegyndt med massiv økonomisk støtte fra det offentlige, specielt set i lyset af de tidligere tilskud til uddannelse. Island blev en del af den ”moderne” reformbevægelse i et miljø af offentlig interesse og forventninger om økonomiske fremskridt.

Afhandlingen er et komparativt studie, hvor der inddrages data fra nabolandene Norge, Danmark og England. I hovedtræk afviger matematikuddannelsens historie i

Island fra nabolandenes historie ved, at der i lange perioder er en mangel på krav om at fremme den højere matematikuddannelse, at enkelte institutioner besidder en position, der ligner et monopol på matematikundervisningen, og ved at enkeltindivider har taget særlige initiativer på egen hånd. Indbyggerne i de lande der sammenlignes med har på samme måde som islændingene baggrund i hele den europæiske kulturarv, herunder de varierende reformbevægelser, som Humanismen og Oplysningen. I løbet af reformen i 1960erne udgjorde de fælles træk nogle skridt i retning af en opløsning af de lagdelte uddannelsessystemer og forventninger om økonomisk fortjeneste baseret på pædagogisk reform og forbedret forståelse af matematikken blandt eleverne.

Indførelsen af den ”moderne” matematik i Island viste sig at blive en besværlig proces. Alligevel blev arbejdet en stimulerende og kreativ udfordring for en hel generation af matematiklærere, og det bød på en længe ventet mulighed for videreuddannelse. En forbedret almen skoleuddannelse, en stigende fokusering på matematikuddannelsen og en mere alsidig økonomi har præget det islandske samfund siden 1970erne. Det kan man delvis takke skolereformen i 1960erne og 1970erne for. Indførelsen af den ”moderne” matematik var en vigtig begivenhed i denne proces. Den henledte islændingenes opmærksomhed på matematikkens eksistens, og hvor vigtig den er for de tekniske og økonomiske fremskridt i landet samt på matematikkens kulturelle værdier.

Ágrip

Rannsóknin, sem greint er frá í þessari bók, fjallar um stærðfræðimenntun á Íslandi og stöðu hennar í samanburði við nágrannaþjóðirnar. Íslenskt menntakerfi og félagsleg og efnahagsleg áhrif á þróun stærðfræðimenntunar eru greind með aðferðum sagnfræðinnar.

Rannsókninni má skipta í þrennt: Í fyrsta hluta hennar er fjallað um tímabilið frá landnámi fram um 1800. Í öðrum hluta hennar er fjallað um stærðfræðimenntun frá upphafi 19. aldar og er sjónum beint sérstaklega að áttunda áratug aldarinnar. Rannsökuð eru áhrif laga um uppfræðslu barna í skrift og reikningi og reglugerðar um Lærða skólann í Reykjavík. Þróun fræðslumála í framhaldi af hvoru tveggja fram á miðja 20. öld er rakin. Í þriðja hluta rannsóknarinnar er megináhersla lögð á áratuginn 1965–1975, þegar umbætur voru gerðar á menntakerfinu ásamt því að innleidd var svonefnd nýstærðfræði.

Nokkur stærðfræðileg rit á íslensku er að finna í handritum frá miðöldum, meðal annars um indó-arabíska talnaritun og viðeigandi reikniaðferðir. Í upphafi nýaldar fylgdust Íslendingar með framförum þess tíma í landmælingum og mældu stöðu Ísland á heimskortinu. Reglugerðir um fræðslumál frá fimmta áratug átjándu aldar lögðu grunn að starfi lærðu skólanna og sterkri hefð fyrir heimafræðslu og sjálfsnámi sem eimdi af í landinu fram yfir 1970.

Á síðari hluta 19. aldar uxu barnafræðslunni og menntun ungmenna ásmegin. Mælt var fyrir um reikningsnám barna innan heimafræðslunnar með lögum frá 1880 og fyrstu gagnfræðaskólarnir voru settir á stofn um sama leyti. Jafnframt var stærðfræðikennsla í Lærða skólanum í Reykjavík skorin niður árið 1877 þegar einungis var tekin upp máladeild eftir að fyrirkomulag lærðra skóla í Danmörku hafði verið endurskoðað og deildaskiptingu komið á. Ákvörðunin hafði langvinn félagsleg og efnahagsleg áhrif sem birtist í viðvarandi skorti á stærðfræðikennurum og tafði frumkvæði Íslendinga í tæknilegum efnum. Í helstu menntastofnunum á Íslandi, Reykjavíkurskóla og Háskóla Íslands, var mönnum að mestu ókunnugt um vöxt stærðfræðinnar sem fræðigreinar og þessar skólar áttu lítinn þátt í örum tækniframförum í landinu á síðasta áratug nítjándu aldar og í upphafi tuttugustu aldar. Á sama tíma beittu ýmsir menntamenn, sem fæstir voru stærðfræðingar, sér fyrir því að auka og bæta reikningskennslu fyrir almenning.

Markmið fræðslulaganna 1946 var að stuðla að jafnrétti til menntunar. Skortur á skólahúsnæði, menntuðum kennurum í einstökum námsgreinum, kennslubókum og námskrám hamlaði framkvæmd laganna. Þessi vöntun, ásamt örri fólksfjölgun og kröfu um menntun fyrir alla, varð til þess að menntakerfið var komið í öngstræti upp úr 1960. Þá bárust til Íslands kenningar, studdar af OECD, Efnahags- og framfarastofnun Evrópu, um að menntun, sér í lagi á sviði stærðfræði og raungreina, væri mikilvæg forsenda félagslegra og efnahagslegra framfara. Allsherjar endurskoðun á íslenska skólakerfinu var hrundið af stað með meiri fjárframlögum til menntamála en áður hafði þekkst. Nýstærðfræði, þar sem rökfræði og mengjafræði var kennd allt niður á barnaskólastig, var þáttur í endurnýjun skólastarfs á Vesturlöndum og studd af OECD. Ísland slóst í för með alþjóðlegri hreyfingu um endurskoðun á stærðfræðikennslu með fullum stuðningi opinberra aðila og miklum væntingum um efnahagslegar framfarir.

Í rannsókninni er þróunin á Íslandi borin saman við það sem átti sér stað í nágrannalöndunum Noregi, Danmörku og Englandi. Stærðfræðimenntun á Íslandi var

frábrugðin því sem gerðist í hinum löndunum að því leyti hve lengi voru litlar kröfur gerðar um æðri stærðfræðimenntun, hvernig fáeinar stofnanir höfðu nánast einokunaraðstöðu og hve einstaklingar gátu haft mikil áhrif. Allar þjóðirnar fjórar deildu sameiginlegum arfi evrópskra menningarstrauma, svo sem áhrifum fornmenntastefnunnar og upplýsingastefnunnar. Á meðan endurnýjun stærðfræðimenntunarinnar átti sér stað komu fram nokkrir sameiginlegir drættir, svo sem viðleitni til að leysa upp stéttaskiptingu innan menntakerfisins og væntingar um efnahagslegan ávinning og dýpri skilning nemenda á stærðfræðinni en við notkun hefðbundins námsefnis.

Innleiðing nýstærðfræðinnar reyndist erfiðleikum háð á Íslandi, og í meira mæli en í samanburðarlöndunum vegna þess hve víðtæk hún varð og skólakerfið vanbúið. Á hinn bóginn urðu kynnin við hið nýja námsefni til að leysa úr læðingi sköpunarkraft og frumkvæði heillar kynslóðar stærðfræðikennara, og þau urðu kennurunum langþráð tækifæri til endurmenntunar. Bætt almenn menntun, aukin meðvitund um gildi stærðfræðimenntunar og sívaxandi fjölbreytni í efnahagslífinu og leiðum til mennta hefur einkennt íslenskt samfélag á síðustu áratugum. Endurnýjun skólakerfisins um 1970, þar sem nýstærðfræðin gegndi lykilhlutverki, á stóran þátt í því. Íslendingar lærðu að meta hlutverk stærðfræðinnar í tæknilegri og efnahagslegri þróun og þátt hennar í heimsmenningunni.

Contents

Lists

List of Figures

List of Tables

Notes on Conventions

This study concerns Icelandic affairs. In Iceland, family names are an exception rather than the rule. A person’s name is his/her first name. The last name is a patronymic, or more rarely a matronymic, which serves exclusively to distinguish the person from others bearing the same first name. Icelanders are ordinarily referred to by their first name or both in official discussion in Iceland. This custom has been retained in this account. In the list of sources, Icelandic authors have been listed in alphabetic order of their first names, while other authors are arranged by surname.

Furthermore, after the 1960s, the use of titles in official discussion diminished. Persons who completed their doctoral degree before or during the World War II are referred to as Dr. as was customary in their times, while the titles of younger persons will be dropped or only mentioned the first time a person enters the scene.

A few words on the intricacies of Icelandic: The Icelandic alphabet contains letters which are unfamiliar to the international reader. The first one, “ð”, is pronounced as “th” in the English “bathing”, while the letter “þ” is pronounced as “th” in “thunder”. Names beginning with “Þ” are arranged as “Th” within T in the list of sources and in the index.

The vowels have variations with or without accents, e.g., a, á, e, é, i, í, o, ó, ö, u, ú, which alter their pronunciation. Finally, there is the vowel æ, which may be taken as “ae” and the letters å (aa) and ø (oe) appearing in Danish texts.

Glossary

Institutes, Boards and Societies

Alþingi: An assembly to pass legislation and put it into effect, originally established in 930. It had mainly judicial authority in 1662–1805. It was laid down in 1805, and restored in 1845 with a consultative status. It was granted legislative power in 1874 and acquired parliamentary status in 1904.

The Association of Chartered Engineers in Iceland: *Verkfræðingafélag Íslands.*

Directorate of Educational Affairs: *Fræðslumálastjórn.*

Director of Educational Affairs: *Fræðslumálastjóri*. Directors were Jón Þórarinsson, 1908–1926, Cand. Theol. Ásgeir Ásgeirsson 1926–1931 and 1934–1939, the Rev. Jakob Kristinsson 1939–1944 and Helgi Elíasson 1944–1974.

The Icelandic Literary Society: *Hið íslenzka bókmenntafélag*, HÍB

The Icelandic Mathematical Society: *Íslenzka stærðfræðafélagið.*

Ministry of Education: *Menntamálaráðuneyti.*

National Centre for Educational Materials: *Námsgagnastofnun.*

Reykjavík Education Office: *Fræðsluskrifstofa Reykjavíkur.*

Reykjavík Director of Education: *Fræðslustjórinn í Reykjavík.* Directors were: Jónas B. Jónsson (1943–1973), Kristján J. Gunnarsson (1973–1982) and Áslaug Brynjólfsdóttir (1982–1997).

School Affairs Board: *Skólamálanefnd.*

State Textbook Imprint: *Ríkisútgáfa námsbóka.*

Structure

Governor: *stiftamtmaður* (before 1873), *landshöfðingi* (after 1873).

High school (matriculation) examination: *stúdentspróf.*

Home Rule: *heimastjórn.*

National examination of the middle school: *landspróf miðskóla.*

Benefice: *prestakall.*

Parish: *sókn.*

(Episcopal) see: *biskupsstóll.*

Schools, Colleges and Universities

Primary school: *barnaskóli*.

Lower secondary school: *gagnfræðaskóli*.

Youth school: *unglingaskóli*.

High school: *menntaskóli*.

The Learned School: *Lærði skólinn.*

Continuation department: *framhaldsdeild (gagnfræðaskóla)*.

Comprehensive multi-stream school: *fjölbrautaskóli*.

Reykjavík Technical School: *Iðnskólinn í Reykjavík.*

Technical College of Iceland: *Tækniskóli Íslands.*

Commercial School of Iceland: *Verzlunarskóli Íslands.*

The Co-operative Commercial College: *Samvinnuskólinn.*

Theological Seminary: *Prestaskólinn*.

Iceland Teacher Training College: *Kennaraskóli Íslands.*

Royal Danish School of Educational Studies: *Danmarks Lærerhøjskole*.
Denmark University of Education: *Danmarks Pædagogiske Universitet*.
Iceland University of Education: *Kennaraháskóli Íslands*.
The Polytechnic College: *Det Polytekniske Læreanstalt*.
Technical University of Denmark: *Danmarks Tekniske Universitet*.
University of Copenhagen: *Københavns Universitet*.
University of Iceland: *Háskóli Íslands*.

Political Parties

Independence Party: *Sjálfstæðisflokkur.*
Labour Party, Social Democrats: *Alþýðuflokkur.*
People's Alliance: *Alþýðubandalag.*
Progressive Party: *Framsóknarflokkur.*

Periods

874 Settlement of Iceland commences.
1262 End of Commonwealth.
1397 Kalmar Union.
1874 *Alþingi* became a legislative body.
1904 Home Rule.
1918 Sovereignty.
1944 Republic of Iceland.

Abbreviations

FHK *Félag Háskólamenntaðra kennara* : The Union of University Educated Teachers.
HÍB *Hið íslenzka bókmenntafélag* : The Icelandic Literary Society.
MUNK–cooperation, *Matematikk-undervisningens Nordiske Komité* : The Nordic Committee for Modernizing Mathematics Teaching, active from 1967.
NCEM National Centre for Educational Materials : *Námsgagnastofnun*.
NKMM *Nordiska kommittén for modernisering af matematikundervisningen* : The Nordic Committee for Modernizing Mathematics Teaching. Active 1960–1967.
NMT *Nordisk Matematisk Tidskrift* : Nordic Mathematical Journal.
OECD Organisation for Economic Co-operation and Development. Active 1961– .
OEEC The Organisation for European Economic Co-operation. Active 1948–1961.
PISA Programme for International Student Assessment, an OECD project, run in 2000 and 2003.
SMP School Mathematics Project. A project in England from 1960s.
SMSG School Mathematics Study Group. A project in the USA in 1950s and 1960s.
SRD School Research Department : *Skólarannsóknadeild Menntamálaráðuneytisins.*
TIMSS Third International Mathematics and Science Study. An OECD-study in 1995.
UICSM University of Illinois Committee on School Mathematics. A project in the USA, 1950s and 1960s.

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That this study has been completed is due to the support and encouragement from a number of people.

First I thank my adviser Prof. Mogens Niss for trusting me to undertake such an extensive piece of work. He not only encouraged me to decide to write it. He also pointed out a number of sources, taught me to appreciate what I had already achieved at times when I found the task overwhelming, and offered encouraging remarks to improve deficient parts. Without his support and generous advice this work would never have reached this stage.

Secondly, I thank my family. All the family members have wholeheartedly participated in this work. The drafts to this book were originally written in Danish. My daughter, Valgerður Halldórsdóttir, helped me to turn the draft into proper Danish. My elder son, Magnús M. Halldórsson, has read the work at various stages of its development and shared with me valuable comments and opinions. My younger son, Bjarni V. Halldórsson, read the manuscript in the final version, and offered useful suggestions on its arrangement. Their father and my husband, Halldór S. Magnússon, read the text at various stages and offered good advice regarding content and vocabulary. Still more precious, he has assumed increasing domestic responsibility after I decided to accept the challenge of this study. My mother, Kristín Eiríksdóttir, both encouraged and assured me regularly that my father, Bjarni Vilhjálmsson, would have appreciated this piece of work. My gratitude to all of them is immeasurable.

The most enduring influences on my attitude to research have come from my father. Firstly, by his model as a researcher who performed his tasks in his earlier years in the difficult conditions of a freelance worker without an official position, and who never compromised scientifically. Secondly, from all the tasks that he entrusted to me at a young age. They were as much a school to me in methodology as my later studies.

Furthermore I thank Prof. Helgi Skúli Kjartansson who read the first half of the work in manuscript and the National Archivist Ólafur Ásgeirsson, who read the whole manuscript in its final version. Both shared with me their extensive historical knowledge and made invaluable comments. I also thank science historians Prof. Einar H. Guðmundsson for reading and commenting on section 3, the early modern age, and Dr. Skúli Sigurðsson, who has read a large part of the work and made valuable comments on writing style. My lifelong friend, María Jóhannsdóttir, read the last proofs of the whole book. She made many clarifying comments on its writing style, use of the English language and my interpretation of various events in history. I thank her cordially for her warm support.

I thank the Iceland Ministry of Education for allotting me a half-year's sabbatical leave while working at *Fjölbrautaskólinn í Garðabæ* (upper secondary school) as Assistant Principal. In 2003 I was appointed Assistant Professor at the Iceland University of Education. I fulfilled the research requirements there by completing the study.

I thank all those who allowed me to interview them about the remarkable times in the realm of education in Iceland since the establishment of the Republic in 1944. I wish in particular to mention Director Jónas B. Jónsson, Minister Dr. Gylfi Þ. Gíslason and Director Andri Ísaksson, who were central actors in the reformation of mathematics education in the 1960s, and mathematics teachers Haraldur Steinþórsson and Guðmundur Pétur Sigmundsson. Although they have now passed away their legacy bears witness to their strength and prescience.

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Not the least, I thank those who have explained to me their painful experience of learning mathematics and left the unanswered question in my mind: Why cannot learning mathematics become a similar experience to learning any other subject, such as Icelandic or English?

Preface

Born in 1943, I grew up in the capital, Reykjavík, in the early years of the newborn Republic of Iceland, under new education legislation passed in 1946. The schools were crowded and there were housing problems in the capital area, but optimism prevailed. Some people still lived in Nissen huts left by the British and US military after World War II. My parents' family of six was lucky to have a small apartment in a new building on the outskirts of the town. The living room served as a common working space, where my father had his office, my mother sewed and we children did our homework.

My father, Bjarni Vilhjálmsson, studied philology, Icelandic language and history, at the University of Iceland 1936–1942. He worked part-time as teacher at the Iceland Teacher Training College, and in a one-year lower secondary school for those who were attempting the national entrance examination to the high schools in Iceland. As my father worked mainly freelance, he had many temporary tasks. One of them was to be the chairman of the national examination board. In the period 1947–1964 this event was organized from our living room. That meant contact with the headmasters from all around the country, and computation of the averages and the simple statistics that were to be included in the board's report.

From the age of eleven I served as my father's assistant clerk in administrating the national examination. At the age of twenty, I was appointed to be the board's special representative in one of the lower secondary schools where the examination was held. At that time I had graduated from the *Menntaskólinn í Reykjavík*, Reykjavík High School, and begun studying physics and mathematics at the University of Iceland. My teachers were Björn Bjarnason and Guðmundur Arnlaugsson, the main proponents of the introduction of "modern" mathematics in Iceland. They had also been my teachers at the Reykjavík High School. Even if the contact was mainly one-way communication from teacher to student, their views and attitudes had immense influence, and I identified myself with their progressive intentions to improve mathematics education in the country.

In 1967, I became a part-time teacher at the Iceland Teacher Training College. I had the support of my mentors, Guðmundur Arnlaugsson and Björn Bjarnason, in selecting a "modern" mathematics textbook, and soon I was busy introducing the set theoretical concepts to my student-teachers, preparing them for the work that would await them in their prospective profession. Later I realized that many of them were only using the Teacher Training College as an alternative route to high school, on their way to university. The upper secondary school level was at that time on the point of bursting, as educational opportunities for young people were scarce, in this period of growing national prosperity.

From 1969 to 1981 I served as mathematics teacher at lower secondary schools, where I introduced "modern" mathematics, first at *Kvennaskólinn*, a selective school for girls, and later at an ordinary rural school in a small town. I started to wonder why the pupils were more at ease with the conventional textbooks than with exotic set theoretical material. Could it be the crudely copied books on bad paper, or was it because the content did not motivate them?

In 1978 I joined a group which had the goal of creating a second wave of "modern" mathematics material for the School Research Department of the Ministry of Education. The material was not set-theoretical, but structuralistic or formative, led by

Jerome Bruner's theories about the spiral curriculum, with investigations and open-ended problems. The work of the group carried on, with intervals, into the 1990s. My pupils were more comfortable with this new approach, so my attitudes towards the "right" trends in mathematics education altered.

While continuing to develop educational material I decided to explore mathematics education further, in order to gain a deeper insight into the nature of mathematics learning. However, I believed that this was not possible without a better understanding of mathematics itself. As this was not available in Iceland, I went to the United States, where I obtained a master's degree at the University of Oregon in 1981–1983. Upon my arrival back to Iceland, I was offered the position of head of the mathematics department in a new upper secondary level school, *Fjölbrautaskólinn í Garðabæ*. The early 1980s was the last phase of the upper secondary level expansion period. I followed the debate about its legislation, passed in 1988, and led a work-group on the mathematics part of its national curriculum. I was soon drawn into the organisational work as assistant principal of my school, which was my main occupation for 18 years until 2003. From this period I retain an interest in the development and evolution of the upper secondary school level in general.

In the early 1970s, I began to study the history of mathematics education, with the encouragement of my father, who had by then become the National Archivist. I went through documents from the early years of the Bessastaðir Learned School in the 1820s and I learnt about mathematician Björn Gunnlaugsson and his mathematics teaching. These studies only resulted in notes in a little notebook, which have now become useful in the work on this book. My father also pointed out to me the mathematical treatise *Algorismus* in *Hauksbók*, a collection of medieval manuscripts. I did not give myself time to study *Algorismus* while my father was alive. It was through his successor Ólafur Ásgeirsson's encouragement that I started to explore *Algorismus* in 1995. I used a week in a summer cottage to read through the ancient descriptions of algorithms, and discover that they were indeed absolutely correct, even the extraction of a cubic root.

In 1995–1999, I chaired two groups writing national curriculum documents for the public school system, i.e. for the compulsory and upper secondary school level. There I could draw upon my earlier work and experience and learn about development in other countries. When that task was completed, several tasks that I had begun earlier waited. Would I ever be able to elaborate on them? There was Björn Gunnlaugsson's history and *Algorismus*.

In spare time during a long teachers' strike toward the end of the year 2000, I was able to collect my thoughts. Attending the ICME-9, the Ninth International Congress on Mathematical Education, in the year 2000 led me to visit Roskilde University in the spring of 2001. A meeting with two professors in IMFUFA, Department II of Studies in Mathematics and Physics and their Functions in Education, Research and Applications, Bent C. Jørgensen and Mogens Niss, convinced me to write a piece of work, which could contribute to the combined history of education, especially mathematical education.

I have been a spectator to and a participant in a series of important tasks and events, such as the national examination, the implementation of "modern" mathematics in Iceland, and the development of upper secondary education for all. The main proponents of "modern" mathematics, Guðmundur Arnlaugsson and Björn Bjarnason, were my teachers and mentors, and Guðmundur Arnlaugsson later a collaborator. I studied education with Dr. Matthías Jónasson, who put me in touch with the primary school experiments and introduced to me the pedagogical background of the educational currents of that time. I worked with mathematics consultant Anna Kristjánsdóttir in the 1970s and 1980s, and I know personally and have worked with many other central actors in the story recounted in this book. It was therefore tempting to bring together facts and ideas in an organized piece of work. The result is this study.

I hope that this work will contribute to a deeper understanding of the history of mathematical education within the Icelandic educational system, both on behalf of the international community of education and of Icelanders themselves. As Iceland is a small and well defined society it lends itself easily to be an international sample for research in many areas of sciences such as mathematical education. Such research can lead to results applicable to larger societies. For Icelanders the account may throw light on earlier progresses and mistakes and be of use in designing ways to improve the present educational system with respect to mathematical education.

This work served as a doctoral thesis, which was defended at Roskilde University, Denmark on February 24, 2006. It has been slightly amended, mainly to correct minor inaccuracies and discrepancies.

Garðabær, Iceland, December 1, 2006

Kristín Bjarnadóttir

The Research Question

In the interval between two landmark events, i.e. legislation and regulations in 1904–1907 and legislation and national curricula in 1995, 1996 and 1999, increased attention was paid to mathematics education in Iceland. Its importance was discussed with respect to its role in society and its technological development, for each individual in private, professional and public life, and for the maintenance of the cultural heritage, no longer confined to the Icelandic ancient tradition but also including mathematics education.

During World War II, Iceland was occupied by Allied military forces. In the midst of those events, Iceland became a republic in 1944, after being in relation to Denmark since the 14th century. One of the first acts of the new republic was to adopt new legislation in education. Its primary aim was to establish a new egalitarian and technological society. Less than two decades later waves from an international reform movement in mathematics education reached the Icelandic educational system. The reform movement hit the system in a crisis. The time which had passed since the adoption of the legislation was not adequate to build the required school facilities for the rapidly growing population, which was increasingly migrating to urban areas. Development of the necessary textbooks and in general of the content of the schoolwork had not been devoted attention. International reform movements in mathematics and natural sciences, and general didactics debate, stirred up the system for a decade and a half and altered it permanently. The question arises, whether it was for better or for worse. How did the alterations function relative to those in other countries meeting similar challenges? What was the situation in Iceland when the turbulent times of the international reform movement were over? These matters are the objects of this research. The research question is formulated thus:

To what extent has mathematics education developed similarly or differently in Iceland from that in other northern European countries, and what explanations can be offered for this?

This question cannot be answered without explaining the political scene in Iceland in general and its educational scene in particular. In the following the main facts concerning Icelandic history, politics, education and mathematics teaching will be traced. To illustrate the situation in Iceland some comparative examples from the neighbouring countries, Norway, Denmark and England, will be studied, and influences from these countries and from the United States will be discussed.

In 1965 Dr. Klaus Bahr, director of the Educational Investment and Planning Programme of the Organisation for Economic Cooperation and Development (OECD), gave a talk in Reykjavík to Iceland's most prominent educators.[1] In his presentation Dr. Bahr explained that the structure and content of formal education in most OECD countries were based on traditional concepts dating back to the late 19th century; and that education had primarily been regarded as serving only cultural purposes. Dr. Bahr introduced that

[1] Efnahagsstofnunin (July 1965)

...

2. New concepts of the role of education are recently being developed:
   a) education contributes substantially to economic and social progress and stability;
   b) education is as much a sector of society and national economy as traditional sectors;
   c) education competes with other sectors for scarce financial, natural and human resources.

...

3. Education is a system governed by its own logistic rules. Some of the main characteristi[c]s of this system:
   a) time-lags (teacher supply and demand; demographic development; student input and output);
   b) freedom of choice of the individual;
   c) efficiency of the system;
   d) co-ordination of educational decision-making;
   e) long-term financial planning;
   f) needs for a continuous flow of information (statistical and other -).

4. Education is an integrated socio-economic sector of society.
   a) Main problem areas: monopolistic position of the state; ad-hoc adjustment processes leading to cycles and bottlenecks; technical progress and eco[no]mic policy targets demand continuous adjustments; social objectives such as equal educational opportunities.
   b) The educational system supplies, the labor market transmits demand for human capital.[2]

The views of the OECD, presented in Dr. Bahr's talk, had a strong influence on thinking about education in Iceland during the following decade. The influences of OECD on education in Iceland will discussed further in chapters 6 and 7.

Dr. Bahr's remark about time-lags draws attention to the importance of studying historical development. The crucial events in mathematics education in the 1960s originated earlier. The events of the 1960s are rooted in the Education Act of 1946, which again has its background in the 1907 Act and regulations of 1904 and 1877, discussions in the 19th century and important regulations dating from the mid-18th century. Why was society and education as it was in the 18th century? All history in Iceland has its roots in medieval times, when the settlers established their own domestic culture.

For this reason the answer to the research question will be sought by investigating the history of education, and mathematics education in particular, through the eleven centuries of habitation in Iceland. The answer will be more detailed than is strictly necessary, mainly because much of this history has never been written, except about individual details, and because various materials were found during the investigation, which partly answer the research question.

The personal histories of several individuals will characterize the account to some degree. Due to the extremely sparse population in Iceland compared to neighbouring countries during most of its history, individuals can have, and have had, a great influence on the development of important events and matters. Their story deserves therefore special attention.

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[2] Efnahagsstofnunin (July 1965): 8–9

1. Introduction[3]

1.1. Iceland, Its Geography, History and People

Icelandic society is well defined in time and space. The habitation of Iceland has a history of 1130 years, on an isolated island in the Northern Atlantic Ocean. In order to put mathematics education in context, some introduction to its geography and the history of the country and its people is useful in order to understand the environment of the cultural life which has developed there throughout the centuries.

Iceland is situated about 1,000 km west of Norway and 800 km north-west of Scotland. It is the second-largest island in Europe; it has a total area of 103,000 km^2 and is 500 km long from east to west and 313 km across from north to south. Two main ocean currents, the Gulf Stream and the Polar Current meet around Iceland. Together they create rich fishing grounds and the Gulf Stream contributes to richer vegetation than otherwise could be expected at latitudes between 63° and 67° north.

Geologically, Iceland is a young country. Its oldest parts were formed in volcanic eruptions in the Tertiary era, about 14 million years ago. It lies astride the Mid-Atlantic Ridge and is the largest supramarine part of the mid-oceanic ridge system. The country remains volcanically active. Earthquakes and volcanic eruptions have had considerable influence on the history of its inhabitants. Currently glaciers cover 11% of its area.

The Icelandic nation has a recorded history, defining the origin of its inhabitants and the specific point of time of their settlement. In the Viking Age, 800–1066, Nordic seamen had achieved control over all sailing routes in Northern and Western Europe. Around the year 900 Norwegians, together with their Celtic slaves, were rapidly settling in Iceland, the year of the origin of the settlement traditionally being counted as 874. The main attraction of Iceland was freely available land, suitable for animal husbandry. Before the settlement, more than 60% of the country is believed to have been covered with more-or-less continuous vegetation.

By the year 930 Iceland is believed to have been fully settled. The population at that time is estimated to have been a minimum of 10,000 and possibly up to 30,000. That same year, the Icelanders assembled to establish a common state and a general assembly for their land, *Alþingi*. Its sessions lasted two weeks every summer. The Icelandic laws were based on the Norwegian ones, but soon they were amended and supplemented. The Icelandic Commonwealth lasted for the next three-and-a-half centuries and under its aegis a rich culture blossomed. In 1262, Iceland lost its independence by submitting to the Norwegian King. At that time the population is estimated at 40–70,000. Iceland was brought into the Kalmar Union, formed by the unification of the crowns of Norway, Denmark and Sweden, in 1397. Iceland and the Faroe Islands were considered as Norway's tributaries in this union until 1814, when Norway came under the Swedish throne, while the tributaries remained under Danish rule.

Gradually living conditions became more difficult, and agriculture was characterized more by decline than by progress. A gradual deterioration of climate began as early as the 12th century, reaching its worst point in the 17th to 19th centuries.

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[3] The general facts about Iceland in the introduction are drawn from Helgi Skúli Kjartansson (1996), Þorleifur Einarsson (1996), Gunnar Karlsson (2000) and Jónas Kristjánsson (1980). See further Sigurður Nordal (1990) for Icelandic culture.

That period is known as the "Little Ice Age", when cold oceanic currents often brought sea ice to Iceland's shores and caused the temperature on land to drop, resulting in damage to vegetation, and famine. Volcanic eruptions and epidemics caused further disasters, so that the population is estimated to have been below 40,000 after catastrophic events in the 1780s.

The development of the Nordic language, *Norse*, further contributed to Iceland's isolation but also to its cultural independence. In the 14th century the languages spoken in the western part of the Nordic countries had begun to develop differently, and by the 16th century Icelanders found it impossible to call their language *Norse* any more, and coined the term *Icelandic*.

The first half of the 19th century saw the general ascendancy of nationalism in Europe. A handful of young Icelandic intellectuals advocated a distinct nationalism and demanded autonomy. The first step towards constitutionalism was a separate assembly for Iceland with a consultative role. The assembly, named *Alþingi* after its ancient predecessor, was inaugurated in 1845 in Reykjavík. The *Alþingi* was the first landmark towards a democratic government in Iceland. The period 1845 to 1944 was characterized by a movement for independence. The next important step was the proclamation of a Constitution in 1874, serving as a legal foundation for control of the relationship between Iceland and Denmark, and ensuring basic human rights in accordance with the liberal ideas of that time.

The next stage in the independence process was reached in 1904 with Home Rule, in the form of a native Icelandic Minister of Icelandic Affairs, residing in Iceland and responsible to *Alþingi*. In 1918 Iceland became a free and sovereign state in personal union with Denmark. After that the influence of the Danish government was largely confined to foreign affairs, although cultural influences on the educational system persisted for decades, even up to the 1960s. The final step was the declaration of full independence from Denmark and the establishment of the Icelandic Republic on June 17, 1944.

1.2. Culture

The geography, geology, meteorology and history of Iceland constitute the background to the inhabitants developing their own special culture. Christianity brought literacy to Iceland and thereby it entered the orbit of Western civilisation. In the 12th century, learned men began to use the Latin alphabet for codification of vernacular texts. A lay culture was established, and an educational elite which kept in touch with contemporary scholarship, including the latest mathematics education of that time.

The Icelanders began to lag behind in scholarly respects around 1500, even though the cultural heritage was never forgotten. Worsening living conditions in Iceland, the establishment of universities in Europe, and the dissolution of monasteries and convents in Iceland following the Reformation, which introduced the Evangelical Lutheran faith in 1550, were important factors in this development. The Enlightenment and rising nationalism contributed to a break from an essentially still a medieval society by the end of the 18th century, to the formation of towns and an increased emphasis on learning and modern culture, a development that has continued until the present day.

At the end of the 19th century Iceland had hardly any infrastructure, no roads, bridges, motor-powered boats or ships, only a few durable buildings and very few schools and hospitals. Since then Iceland has developed into a modern prosperous technological society. However, Icelanders of a certain age still remember other times, and a question burns on their lips: How will we be doing in the future? They try to compare their situation and capacity to those of other nations – the neighbouring Nordic nations, Europeans and the rest of the world – in order to evaluate themselves and decide how to improve, and where to direct the course for the future.

This study is an examination of a small part of Icelandic culture, mathematics education and its development in comparison with other countries. To what extent did international currents reach Iceland, how did Icelanders cope with them, and were they able to integrate them into their own culture?

1.3. Education

For more than one-and-a-half centuries, from the 1740s, public education in Iceland was based on regulations on the responsibility of the homes for children's literacy and knowledge in Christendom. It was only in 1907 that legislation was passed in Iceland about public schooling for all children, 10–14 years old. It was followed up with the establishment of a teacher training college in 1908. The legislation was a crucial event on the road towards a fully democratic society, even if at that time the neighbouring countries offered their subjects schools from the age of seven. Lack of infrastructure in the large country, where the population was predominantly rural until the 1920s, precluded for 70 more years the provision of such privileges to all Icelandic children, even though legislation passed in 1946 so prescribed.

In cultural respects, the focus was on preserving the national cultural heritage. When the schools of theology, medicine and judicial studies were combined into the University of Iceland in 1911, the first subject within the humanities was Icelandic studies: the Icelandic language and history. The only high school in the country offered a language stream exclusively, and a mathematics stream was not established until 1919. Only much later, in 1970, was Faculty of Engineering and Science established at the University. Prior to that time mathematics and physics had been taught to engineering students, including some student teachers, since 1940.

The 20th century was characterized by progress in all respects of national life, due to increased technological knowledge. Roads, bridges and buildings were constructed. Transport and communications improved on land, sea and in air. Increased exchanges with other countries brought cultural influences, and gradually traditional Icelandic culture amalgamated with western cultural life. Due to the sparse population, currents in culture and education either passed the country by, or spread throughout it through personal acquaintanceships and contacts between people in the same trade, based on a common education at the same institution.

The Public Education Act of 1907 devoted a few lines to explaining what knowledge in arithmetic was expected, i.e. the four operations in whole numbers and fractions, the computation of area and volume and mental arithmetic. There was no direct route from public schools to higher and further education. The requirements for the upper secondary school level, the high school, preparing for university studies, were adjusted to the Danish educational system by a new regulation published in 1904, without adopting the democratic advantages of the Danish system.

In the mid-1960s mathematics reform movements, promoted by the OECD, began to exert strong effects on the Icelandic education system. As a result, a series of curriculum documents, at least in preliminary form, and new textbooks in revolutionary form, were introduced, and had lasting effects.

By the end of the 20th century, a 126-page booklet was published, containing a detailed description of the expectations of the output of the educational system in mathematics, after ten years' study in the age range six to sixteen years in compulsory school. The compulsory school has a defined continuation to the upper secondary level, which comprises nearly 30 schools. An 80-page booklet describes the mathematics requirements at its end.

1.4. Research Method

The answer to the research question will be approached by a historical method, i.e. a careful analysis of a range of documents and interviews. The purpose of using a historical approach is to illustrate how trains of events and official decisions may lead to partly inevitable consequences for educational matters in general and mathematics education in particular.

The history of mathematics education will be told within the framework of the general history of education and schools. This story can only be told as a part of the history of Iceland. The history was traced by referring to scholars' published works, legislation, regulations, reports and documents preserved in official archives. Where applicable, events were explored by referring to contemporary articles in newspapers and journals. Supplementary knowledge was acquired through interviews with persons involved or knowledgeable observers, and from published memoirs and biographies. For the purpose of finding out how the actual teaching proceeded at different times, mainly textbooks have been consulted, in addition to some personal experiences and a few memoirs of contemporaries. Where possible, evidence is supported by tables and graphs of quantitative elaboration on available data.

The history is for the most part told in chronological order, with slight deviations when several parallel topics are treated separately.

Throughout the study, the development will be measured by the fundamental reasons for mathematics education as identified by Prof. Mogens Niss:

> Analyses of mathematics education from historical and contemporary perspectives show that in essence there are just a few types of fundamental reasons for mathematics education. They include the following:
>
> - contributing to the *technological and socio-economic development* of society at large, either as such or in competition with other societies/countries;
> - contributing to *society's political, ideological and cultural maintenance and development*, again either as such or in competition with other societies/countries;
> - providing *individuals with prerequisites which may help them to cope with life* in various spheres in which they live: education or occupation; private life; social life; life as a citizen.[4]

Did Icelandic society need mathematics education for its economical or cultural development, did it cultivate mathematics for its own sake, or did individuals need to

[4] Niss, M. (1996): 13

be provided with mathematical prerequisites to cope with their private or professional lives? The fundamental reasons for mathematics education or its absence in Icelandic society at each particular period of time will be analysed.

There are some provisos. It may be difficult, if not impossible, to access the ultimate truth concerning the various issues. Contemporary sources often only account for part of the progress on an issue, while they may say nothing of other aspects, which at that time may have been self-evident but no longer are.

Furthermore, written sources are limited. Many decisions may have been taken at informal meetings where no minutes were taken, or they are lost. Compared e.g. to Norway, where there was a structured decision-making process concerning the 1960s mathematics reform, this is especially true in Iceland, where only a few persons were involved in decisions concerning similar matters.

Other important sources may still be undiscovered. Retrospective sources, such as memoirs and interviews with persons about events in long gone times, are later interpretations which may have to be taken with a pinch of salt. The interpreter may never have had the overview of the whole course of events, or he/she may interpret it in his/her own or some other person's favour, or may simply have forgotten important items or aspects of the matter. The history to be told is therefore limited to what is available, what the author discovered, and what she found relevant.

However, the author has found and considered relevant a number of sources which have been drawn into an account intended to become the basis to the answer to the research question. Some items mentioned and included in the discussions may deserve further investigation and research, while all effort will be made to answer the research question as honestly as possible by interpreting the collected data.

Many items will not be included in this educational account. Schools such as *Kvennaskólinn í Reykjavík*, a school for girls established in 1874, and *Verzlunarskóli Íslands*, a school of commerce established in 1904, are certainly proud representatives of rising interest in public education at the time of their establishment, and have proved their value. However, they will only be mentioned marginally, because their mathematical education is not known to have had any marked influence on the country's history in that field. Similar reasons apply to agriculture and navigation colleges. A strong home- and self-education tradition prevailed up to the turn of the 20th century, while it naturally dwindled with the increased number of schools and easier transport. One remarkable offspring of this tradition was Vilhjálmur Ögmundsson (1897–1965), who will not be mentioned either. Vilhjálmur Ögmundsson returned, after his studies at *Verzlunarskólinn*, to his family's farm and pursued his mathematical interests to such a high standard that his mathematical articles were published in distinguished journals. His work did not, however, contribute to the general mathematics education and will not be reported here. Other institutes and persons, such as a number of 20th century mathematicians, have been deemed to fall into that category. In that choice some mistakes may be made. In those cases, new research projects might emerge.

Throughout the work, the motto of the ancient scholar Ari the Learned (1068–1148) will be kept in mind: "Hvatki es missagt er í fræðum þessum skal hafa það er sannara reynist." ("Whatever may be wrongly said in these studies, one shall adhere to the more truthful").

2. Medieval Times

2.1. Introduction

For the first several centuries after settlement began in Iceland, the Icelanders developed a dynamic culture, creating new knowledge based on their own experiences, while exploring and exploiting their new country. They were curious observers of their new environment. They wrote down their observations and gradually merged these with imported knowledge from Europe. However, the originality which characterized the very first centuries of settlement disappeared over time. The next step in the cultural development was to adjust the imported knowledge to domestic culture by translation, which was almost unique in European culture. This knowledge was in many cases recent when it was implemented into Icelandic culture. An example of this is the translation of the arithmetic treatise *Algorismus*, preserved in several ancient manuscripts. By the late 14th century this stage was over as well. The heritage of written knowledge and stories was preserved by copying it during the following centuries, while little of value was added to it.[5]

Cultural exchanges and other contacts with Europe, such as trade, depended on sailing. Sailing from Iceland to Europe was very common in the 11th and 12th century, so common that in fact it was necessary for the bishop of the Skálholt See to limit the travels of priests, so that there would be enough priests left in the country to serve in churches.[6] When the Icelanders submitted to the rule of the Norwegian King in 1262, a part of their agreement with the King was that he would ensure that six ships sailed to Iceland each year. This indicates that the Icelanders themselves may not have had many ships at that time. However, they were allowed build and operate their own ships. This changed in the second half of the 14th century, when merchants in Bergen acquired a monopoly on trade with Iceland.[7]

Sees and cathedral schools established in the 11th and early 12th century remained relatively unchanged until the beginning of the 19th century. The sees and the great estates were situated far inland, surrounded by rivers and mountains. The end of the middle ages, in the mid-16th century, marked a change in Iceland of an opposite kind to that which characterized most European countries. In Europe it generally meant the beginning of greatly increased trade. In the modern age, Iceland remained outside the mainstream of trade, under the trade monopoly of the Danish King.[8] In the late 14th century there were six main harbours, where markets for import and export goods were located, but no towns grew up at these ports. Trade within the country was so small that no infrastructure existed; neither roads nor bridges were needed for any major transport.[9] Not until the late 18th century would the first town begin to form, at Reykjavík. The structure of society thus remained rural and somewhat medieval for centuries after Europe had changed towards a more modern structure.

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[5] Þorsteinn Vilhjálmsson (1990): 3–50
[6] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 80
[7] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 136
[8] Gunnar Karlsson (2000): 127, 138–142
[9] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 136–137

The fact that the tithe, introduced by *Alþingi* in 1096, was 1% property tax[10], while in other countries it was 10% income tax, may be an example of the homogenous structure of Icelandic society and the absence of internal trade. One thus assumed 10% income of all properties.[11]

The object of this chapter, mathematical education in the medieval period, will be viewed from a broad perspective, in the absence of schools in the modern sense. Accounts will be given of scientific observations, originally preserved in oral forms, followed by the influence of Christianity, literacy, schools and recording on parchment.

2.2. Early Scientific Observations[12]

Þorsteinn Surtur and His Calendar[13]

The first settlers in Iceland were pagan and illiterate. They brought with them oral wisdom. Their primary field of scientific knowledge was chronology and navigation. Probably the initial way of keeping track of time was by counting the weeks by the change of lunar quarters. A primitive calendar, counting 52 weeks or 364 days in the year, may have been a temporary trial, from which the need for a more reliable system of time-computing was quickly discovered.[14]

At the establishment of *Alþingi* in 930, the date of assembly was agreed upon in the 10th, later the 11th, week of summer. Only 20 years later, it had become clear that the summer "went back to the spring", i.e. the summer began earlier and earlier according to computations by this calendar. This was inconvenient, as *Alþingi* had to assemble at a time during the short Icelandic summer when certain duties of farmers were over and others were not yet due. It is believed that Þorsteinn Surtur may have realized the error around the year 955, by observing the location of the sunset, which in northern areas moves rapidly clockwise along the horizon before the summer solstice, and subsequently anti-clockwise. This method would not be as suitable at lower latitudes, and is most likely an Icelandic invention.

To correct the error a week called "sumarauki", summer's extra week, was inserted at regular intervals in the Icelandic week-calendar, which remained in use for many centuries. This correction was first developed by Þorsteinn Surtur, by inserting a week every seventh year, making the year 365 days. Later it was adapted to the Julian calendar of the church, by inserting a week every sixth year, and every fifth year if there were two leap years in between. Þorsteinn Surtur's method is explained in *Íslendingabók*[15] / *Book of the Icelanders* by Ari the Learned, one of the most reliable sources in Icelandic medieval literature, written in the period 1122–1133.

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[10] Gunnar Karlsson (2000): 38–40
[11] Magnús Stefánsson (1974): 287–291
[12] The section on early scientific observations attributed to Þorsteinn Surtur and Stjörnu–Oddi is drawn upon the work by Þorsteinn Vilhjálmsson, professor of physics and the history of science at the University of Iceland.
[13] Þorsteinn Vilhjálmsson (1990): 16–24
[14] Helgi Skúli Kjartansson, e-mail, November 9, 2004
[15] Jakob Benediktsson (1968)

The week-calendar was later adjusted to the Gregorian calendar. Remnants of the old system have survived to the present day, mainly in celebrations on special occasions, such as of the beginning of summer around April 20, and mid-winter feasts.

2.3. Christian Era

Literary Activities

In the year 1000, a political agreement to adopt Christianity was made at *Alþingi*, upon the initiative of the Norwegian King Ólafur Tryggvason. Icelanders adopted the Christian faith, yet not the Christian Church, which took decades to establish. The church in Iceland was a private church, and a source of profit for the chieftains' dynasties which ruled the two Icelandic bishoprics, the monasteries and convents, and the churches. Many chieftains acquired ecclesiastical education while retaining the ruling power. The owners of the churches were interested in education, and some of them ran schools on their estates, where famous scholars, such as Ari the Learned and Snorri Sturluson (1179–1241), were educated. The priests were employed by the chieftains who were in charge of their churches. Some of the chieftains themselves were ordained as priests, and both ruled their own churches and belonged to the innermost circle of power in the country.[16]

Toward the end of the 12th century the archbishop in Niðaróss, Norway, forbade those in higher orders to occupy important secular positions, and later celibacy was demanded of these orders. The chieftains were, however, reluctant to submit to these conditions.[17]

Gradually several dynasties came to dominate in the country in the late 12th and early 13th centuries. The chieftains wanted to maintain their dignity and the respect of their people. They did not build castles or palaces, but kept up with foreign fashion in the field of education and book writing.[18]

Books were brought to Iceland with Christianity: books in Latin, the sacred writings of the Catholic Church, written on parchment. The earliest work known to have been recorded in writing in the Icelandic language was the laws of the ancient Commonwealth. The recording of the laws began about 1100.[19]

Literature in Iceland was created in the vernacular, Icelandic or Old Norse, in contrast to other Christian countries where the literary language was Latin. This may be explained by the following facts:

- The Church in Iceland was from the start subject to lay control, so the church acquired a national character and the lay chieftains became literate.
- From the time of the settlement the Icelanders possessed an immense store of orally preserved poems and stories.
- The parliament, *Alþingi*, passed complicated laws full of detailed provisions, which received a priority in being written down.

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[16] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 71–74
[17] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 90
[18] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 73
[19] Jónas Kristjánsson (1980): 29

Widespread writing activities meant that religious works were also translated and composed in Icelandic. Other foreign writings were accorded the same treatment. Native literature of considerable volume and quality was thus created. This especially applied to the period 1100–1350. Part of the native knowledge was encyclopaedic. Extant manuscripts from the 12th to 14th century contain foreign accounts of mathematics, astronomy and chronology, but also independent Icelandic studies.

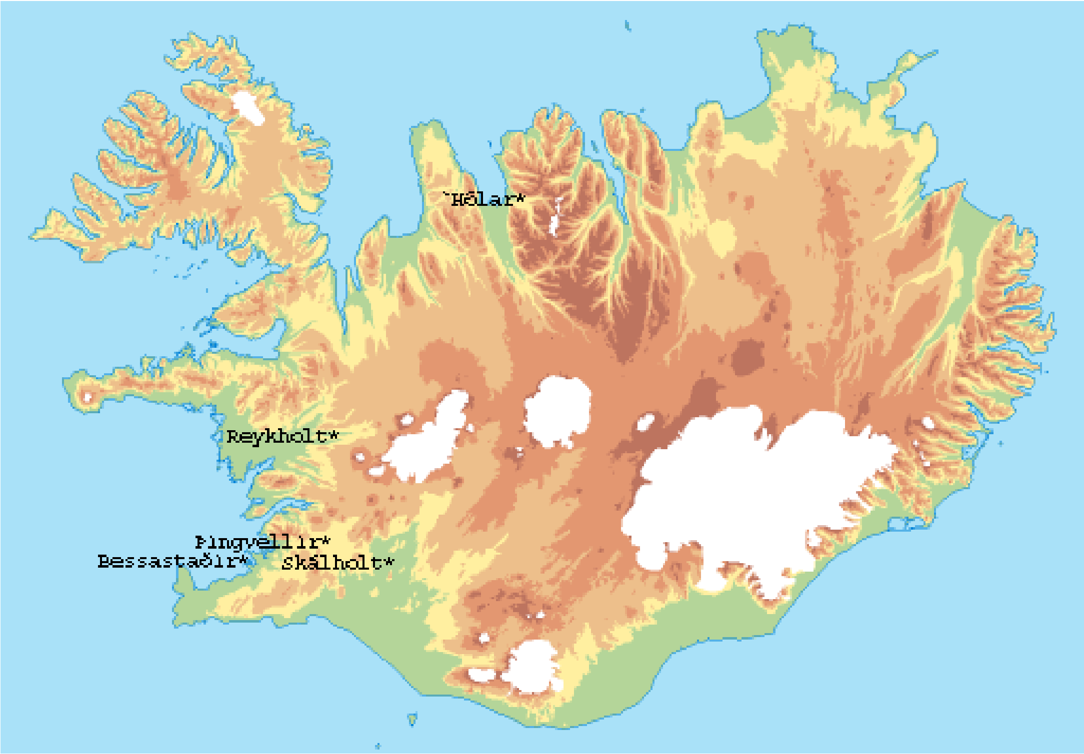

Figure 2.1: Location of the main historical sites in Iceland up to 1800.[20]

Schools

There were three kinds of educational seats in Iceland in the middle ages: monasteries, cathedral schools and private schools. Schools in medieval times share, however, only their name with modern educational institutions. Their main purpose was to train young boys for the priesthood. The training lasted about six years, more or less.[21]

The first bishop at the See of Hólar in Northern Iceland, Jón Ögmundarson (bishop 1106–1121) studied in a private school of the bishop of Skálholt and was ordained in Lund in Sweden. Bishop Jón Ögmundarson directed a cathedral school at Hólar of high quality, with a French or Franconian teacher, Rikini, to teach singing. Bjarni Bergþórsson (d. 1173), known as Bjarni the Reckoner and believed to have written the chronological treatise *Rímbegla*, probably taught arithmetic, which points to mathematical activities at the Hólar Cathedral School. Sources next tell of a school at Hólar in 1218, thereafter of a regular school in the first half of the 14th century, and irregularly after that until the Reformation in 1550.[22]

[20] National Land Survey of Iceland: website, accessed October 2004
[21] Sverrir Tómasson (1988): 15–22
[22] Janus Jónsson (1893): 19–26

Priests were also taught at the See of Skálholt. No schools are explicitly mentioned in monasteries in the sources, but various comments point to instruction of youngsters. For example, boys were admitted for learning at the monastery at Helgafell, and an inventory list of Viðey Monastery from 1397 mentions a good library with several school books. School books are also mentioned in an inventory list of Hólar Cathedral in 1396.[23] School books were of two kinds, i.e. on grammar and etymology and on Biblical exegesis.[24]

While girls might go to convents, and there is an account of one Icelandic girl, Ingunn, studying at Hólar Cathedral School in northern Iceland,[25] we may assume that to be an exception and that learning was mainly confined to boys.

The Curriculum

Not much is known about what was taught in the medieval schools and consequently little is known of mathematical education at that time. One should, however, not conclude from the modest amount of sources available that the training for the priesthood was more primitive than in other Catholic countries.[26]

Traditionally learning was divided into seven arts, *septem artes liberales*. The scholar Boëthius (480–520) counted four of them in one, the *quadrivium*, the combined subject of arithmetic, geometry, astronomy and music. In the 9th century, the three subjects, rhetoric, grammar and logic, were counted together in the *trivium*. Latin grammar and rhetoric were taught in Iceland as part of the *trivium*. This is confirmed by the available sources on school work and books in the inventory lists of the educational institutions of the time. For evidence about the *quadrivium*, other sources must be cited, such as treatises preserved in ancient manuscripts.

Stjörnu-Oddi's Astronomical Observations[27]

In the 12th century, probably in the first half, Oddi Helgason, called Stjörnu-Oddi (Star–Oddi), a farm-worker, is believed to have made remarkable observations about the annual motion of the sun. His observations are found in the treatise *Odda-tala / Oddi's Tale*. It comprises three sections, treating different aspects of the sun's motion.

Firstly, Stjörnu-Oddi observed the summer solstice and the winter solstice to be a week earlier than the official date, i.e. on June 15 and December 15 instead of June 21 and December 21. Later, another Icelandic author in the 13th century explained this discrepancy by the northern location of Iceland compared to "the middle of the world", which shows independent observations of the errors of the Julian calendar.

Secondly, Stjörnu-Oddi explained the curve of the height of the sun during the year by counting the weekly increase in the first half of the year and decrease in the second half. The approximating curve is a kind of a spline-function of second degree curves, probably of European origin. As a measuring scale, Stjörnu-Oddi used the diameter of the sun, the sun rising a total of 91 diameters.

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[23] *Diplomatarium Islandicum* III (1896): 613
[24] Sverrir Tómasson (1988): 28–32
[25] Sigurður Líndal (1974): 261
[26] Sverrir Tómasson (1988): 34
[27] Þorsteinn Vilhjálmsson (1991): 69–76. Þorsteinn Vilhjálmsson (1990): 272

As the difference is 47 degrees, Oddi measured the diameter as a little more than half a degree, which is a fairly good approximation. This implies that Oddi placed the equinoxes at equal time intervals from the solstices, thus dividing the year into four equal parts. This assumption for the timing of the equinoxes would have been several days off in the twelfth century. It seems that on issues like this, Oddi was led more by a concept of mathematical symmetry and simplicity than by exact observation.

The third part of *Oddi's Tale* concerns the time of dawn.

In summary, Stjörnu-Oddi's results may quite well be some kind of crystallisation of knowledge and motivation gained in the Viking Age. Besides this, Oddi was living at a time and place close to flourishing literary and cultural interests at the Hólar See in Northern Iceland. There are many complicated reasons for *Oddi's Tale* being known to us. One of the most important ones is that other people have found this kind of knowledge useful, and considered it worthwhile to copy the text onto precious vellum again and again. In all likelihood, the knowledge in *Oddi's Tale* was not gained through foreign books, but through independent observations, maybe spanning several generations, and perhaps with a little help from oral (e.g. mathematical) information from the continent.

In the 11th and 12th centuries, pilgrimages to Rome or Jerusalem were common and there was still a connection to Greenland.[28] Sailors, who sailed on the ocean for weeks and months, found their way from the location of the sun and the stars. It was useful for them to know how far north of east and north of west the sun rose and set in each week of the summer. This was measured by Stjörnu–Oddi with great accuracy. Icelanders' interests in chronology and navigation points to the needs of people concerned with sailing the high seas and discovering new countries.

2.4. Scientific Treatises and Manuscripts

Manuscripts originating in the 12th and 13th century contain material revealing knowledge of well-known school-book authors, such as Alexander de Villa Dei and Sacrobosco, and indications of knowledge of Euclid's work are also found. In the 12th and 13th century ancient Greek writings were translated into Latin and worked into textbooks. In some cases these writings were used as a basis for Icelandic treatises only one or two decades after they had been composed in mainland Europe.[29]

Treatises

Three treatises on chronology, *Rím I* or *Rímbegla*, *Rím II* and *Rím III*, written in Icelandic, are found in ancient manuscripts. These treatises were researched by N. Beckman and Kr. Kålund in *Alfræði Íslands* in 1914–1916, and *Rímbegla* by Stefán Björnsson in 1780. Included in *Rímbegla,* and also as a special treatise, is *Oddi's Tale*.

Furthermore there is the arithmetic treatise *Algorismus*, which has been researched by several scholars. P.A. Munch published it in 1847 and Finnur Jónsson et al. in 1892–1896.[30] Suzan Rose Benedict wrote about it in 1914 in her Ph.D. thesis on early treatises introducing into Europe the Hindu art of reckoning;[31] Jón Helgason

[28] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 80
[29] Beckman, N. et al. (1914–1916): xxxviii
[30] Finnur Jónsson (1892–1896)
[31] Benedict, S.R. (1914)

published a facsimile with scientific comments in 1960,[32] and Otto B. Bekken et al. wrote about its mathematical content in 1985 and 1986.[33]

In addition, there are scattered pieces of mathematical material in encyclopaedic collections.

The treatise *Rím I* or *Rímbegla*, the first Icelandic chronological treatise, is believed to have been written in the middle of the 12th century, possibly by Bjarni Bergþórsson the Reckoner.[34] Its main manuscript is contained in AM 625 4to, written around 1470, while its oldest one is in GKS 1812 4to, written around 1192.[35] Some of its content is of Icelandic origin, such as *Oddi's Tale*. However, the majority is of foreign origin. Its most famous models are the writings of Bede, Isidorus and Honorius.[36] In spite of the partly foreign content of *Rímbegla*, it is an independent treatise with its own structure, written in domestic Icelandic style.[37] Only Roman numerals are used in *Rímbegla*.

The treatise *Rím II* is believed to be written in 1275–1300[38], partly under the influence of, or as a free translation of, *Compotus* by well-known textbook author Johannes de Sacrobosco.[39] Its main manuscript, AM 624, 4to, p. 148–234, is dated around 1500.[40] However, fragments of the treatise are found in older manuscripts, such as AM 415, 4to, from around 1310, GKS 1812, 4to, (1300–1400), AM 732, 4to, (1300–1325), AM 736 I, 4to, from the same period, and a couple of others, also from the same period, in addition to AM 727, 4to, written in 1594.[41]

Rím II contains e.g. the following mathematical content:[42]

- reference to Euclid's third common notion, and to two sentences about proportions in the *Elements* from 300 BC,
- the approximation 22/7 to the ratio of the circumference of a circle to its diameter,
- chronology, with reference to *Oddi's Tale*,[43]
- tidal calculations, which point to south and southwest Iceland, most likely the island Viðey off Reykjavík, as the location of the writing,[44]
- interesting attempts to explain spring tides,
- Hindu-Arabic numerals.

Rím II contains an interesting sentence: "Þetta prófar reglu sú, er geometrici hafa (This proves the rule that the geometers have): Ef þú tekur jafn mikið af jafn miklum hlutum, þá verður jafnmikið eftir í hverjum stað.[45] (If equals are subtracted from equals, the remainders are equal)" This is identical to the third common notion in

[32] Jón Helgason [Dr. Phil.] (Ed.) (1960)
[33] Bekken, O.B. et al. (1985). Bekken, O.B. (1986)
[34] Beckman, N. et al. (1914–1916): xcvi-xcviii, 93
[35] *A Dictionary of Old Norse Prose – Indices* (1989): 471. Beckman, N. et al. (1914–1916): lxxx–xc
[36] Beckman, N. et al. (1914–1916): xix
[37] Beckman, N. et al. (1914–1916): xxxi–xxxvii
[38] Beckman, N. et al. (1914–1916): xliv–xlv, xcviii
[39] Pedersen, O. (1966): 494
[40] *A Dictionary of Old Norse Prose – Indices* (1989): 457
[41] Beckman et al. (1914–1916): xc–cii, cxcvi–ccxxviii
[42] Þorsteinn Vilhjálmsson (1990): 43–45, 48. Pedersen, O. (1966): 495
[43] Beckman, N. et al. (1914–1916): 91
[44] Beckman, N. et al. (1914–1916): l (Roman 50)
[45] Beckman, N. et al. (1914–1916): 111

Euclid's *Elements*.[46] This sentence is again found in an early 18th century textbook in manuscript, *Arithmetica,* ÍB 217, 4to. A continuation of this reference concerns proportions, that they are preserved in multiplication and division, contained in book 7, proposition 17 in Euclid's *Elements*.[47]

According to Beckman, *Rím II* is not a better treatise than *Rímbegla*, except in the sense than that it is based on later and better sources. It contains hardly as many original observations and thoughts as *Rímbegla*.[48] However, the domestic observations about chronology and tidal calculations are of considerable interest.[49] Actually, the tidal calculations could point to a large part of the coasts of south-west Iceland, but the only educational seat where these computations could have been performed, was on the island Viðey.[50]

The third chronological treatise, *Rím III*, contained in AM 624, 4to (c. 1500), and AM 727, 4to, (1594),[51] is less independent vis-à-vis foreign sources and contains mainly memory rules, some of them by Sacrobosco.[52]

If *Oddi's Tale* is considered as a separate treatise, it is among the earliest Icelandic chronological treatises. It is found within the treatise *Rímbegla*, as an independent treatise in the earliest part of the manuscript GKS 1812, 4to from around 1192, and its two first parts in *Hauksbók's* cosmological treatise *Heimslýsing og helgifræði*.[53]

The arithmetic treatise *Algorismus* is found in the manuscript AM 544, 4to in the manuscript collection *Hauksbók,* the *Algorismus* part dated in 1302–1310, most likely in 1306–1308. It also exists in the manuscripts GKS 1812, 4to (c.1300–1400), in AM 685, 4to (c.1400–1500), and a fragment in AM 736 III, 4to, dated c. 1550.[54] All these texts are in most respects identical.[55] The content of *Algorismus* will be treated in a special section.

Manuscript GKS 1812, 4to

The manuscript GKS 1812, 4to contains mathematical material. A connection has been postulated with Viðey Monastery, as it was handed down to Bishop Brynjólfur Sveinsson (1605–1675) by an heir of Alexíus Pálsson (1485–1568), the last abbot in Viðey.[56] It is in four parts.

The first one has handwriting dated in 1192 and contains a Latin glossary, ecclesiastical calculations and chronology, including *Rímbegla*, the story of Þorsteinn Surtur and *Oddi's tale*.

[46] Heath, T. L. (Ed.) (1956): Vol. I, 155
[47] Beckman, N. et al. (1914 – 1916): 100–101
[48] Beckman, N. et al. (1914–1916): lvi–lvii
[49] Þorsteinn Vilhjálmsson (1990): 45
[50] Þórir Stephensen (1992)
[51] Beckman, N. et al. (1914–1916): cii
[52] Þorsteinn Vilhjálmsson (1990): 45–46
[53] Þorsteinn Vilhjálmsson (1990): 25–37
[54] *A Dictionary of Old Norse Prose – Indices* (1989): 26
[55] Finnur Jónsson (1892–1896): cxxxi
[56] *Diplomatarium Islandicum* I (1857–1876): 183–184

The second part is dated in the period 1225–1250 and also contains chronology, calendars etc. The handwriting of Styrmir Kárason, prior of Viðey Monastery in 1235–1245, is believed to be possibly one of six handwritings on one piece of vellum from this part.[57]

The third and fourth parts of GKS 1812, 4to are dated in the period 1300–1400.[58] They contain, among other things, a manuscript of *Algorismus* in the Norwegian language, which is of the same origin as the manuscript in *Hauksbók*, and also fragments of *Rím II*.[59] The manuscript GKS 1812, 4to thus largely treats subjects related to arithmetic, calendars and astronomy. One might even conjecture that it previously contained the original translation of *Algorismus*.

2.5. *Algorismus*[60]

Introduction

The treatise *Algorismus* is found in several Icelandic manuscripts dated from the 14th, 15th and 16th centuries. They are copied from an older original, probably from the latter half of the 13th century.[61] The treatise explains the Hindu-Arabic number system. The main bulk of *Algorismus* is an Icelandic translation of a well-known poem, written in Latin hexameter, *Carmen de Algorismo*,[62] by Alexander de Villa Dei (1170~1250), a French priest. Yet it also contains references to other sources, some of which seem to be of Neo-Pythagorean origin. Possibly the author of *Algorismus* knew a related work, *Algorismus Vulgaris* by Johannes de Sacrobosco. That work was elaborated upon in Latin by the Dane, Peder Nattegal, Petrus Philomeni de Dacia, in 1291 in his *Commentary to Sacrobosco's Algorismus.* [63] Nattegal's commentary was probably not known to the translator of *Carmen*, as we will conjecture later in this section that *Algorismus* was written earlier than the commentary, even before 1245.

Content

In *Algorismus* the Hindu-Arabic number system is introduced to the reader by explaining the decimal place-value system and its computation algorithms. The content can be divided into the following sections:

- The ten characters to designate the numbers, including the zero.
- The place-value number system.
- Even numbers and odd numbers.
- The seven arts of computing: Addition, subtraction, doubling, halving, multiplication, division and extraction of roots, the square root and the cubic root.
- The elements: fire, air, water and earth, and numbers designated to them.

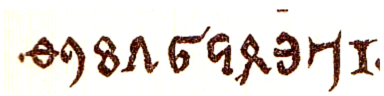

Figure 2.2. The ten digits as presented in the *Hauksbók* edition 1892–1896, p. 417

[57] Þórir Stephensen (1992)
[58] *A Dictionary of Old Norse Prose – Indices* (1989): 26
[59] Beckman, N. et al. (1914–1916): ccx–ccxvii
[60] Kristín Bjarnadóttir (2003): 99–108. Kristín Bjarnadóttir (2004): *netla.khi.is* website
[61] Finnur Jónsson (Ed.) (1892–1896): 417–424, cxxxii
[62] Steele, R. (Ed.) (1988): 72–80
[63] Pedersen, O. (1966): 498

Many paragraphs of *Algorismus* are recognisable as a direct translation of *Carmen de Algorismo*.[64] In the following, *Algorismus* will be compared to *Carmen* in Robert Steele's edition published by Early English Text Society in 1922, reprinted in 1988.

Algorismus has taken from *Carmen* the reference to ten Indian number figures, including the cipher or zero, an idea which first appears in *Carmen* in European literature.[65] However the cipher, called *siffra*, is given special attention (citations from *Algorismus* have modern spelling):

> Siffra merkir ekki fyrir sig en hún gerir stað og gefur öðrum fígúrum merking.
>
> Siffra has no meaning in itself but it marks place and adds meaning to other figures.[66]

It seems, though, that the Icelandic writer knew other sources than *Carmen de Algorismo*. He has for example added to the text:

> Jafnir fingur eru fjórir; 2, 4, 6, 8 en ójafnir aðrir fjórir; 3, 5, 7, 9. En einn er hvorki því að hann er eigi tala heldur upphaf allrar tölu.
>
> Even digits are four; 2, 4, 6, 8, and uneven another four; 3, 5, 7, 9. One is neither as it is not a number but the origin of all numbers. [67]

Bekken et al. have pointed out likeness to the statement that one is not a number, in al-Kwarizmi's *Arithmetic*, which again refers to another book on arithmetic, most likely either Euclid's *Elements*, book VII, or *Arithmetica* by the Neo-Pythagorean Nicomachus.[68] The citation referred to is the following from the Latin translation *Dixit Algorizmi* of al-Kwarizmi's work:

> Et iam patefeci in libro algebr et almucabalah, idest restaurationis et oppositionis, quod uniuersus numerus sit compositus et quod uniuersus numerus componatur super unum. Unum ergo inuenitur in uniuerso numero. Et hoc est quod in alio libro arithmetice dicitur quia unum est radix uniuersi numeri et est extra numerum :
>
> And I have already explained in the book on algebra and almucabalah, that is on restoring and comparing, that every number is composite and every number is composed of the unit. The unit is therefore to be found in every number. And this is what is said in another book on arithmetic that the unit is the origin of all numbers and is outside numbers :[69]

The idea that one is not a number was still referred to in an Icelandic 18th century manuscript, as will be seen.

Computation Methods

All the computation methods in *Algorismus* are explained rhetorically. No numerical examples are shown, except for multiplying two one digit numbers, "vii" and "niu (nine)", (so written, not with Hindu-Arabic numerals), an example which is not found in *Carmen de Algorismo*. This example is done by the method

$7\cdot9 = 10\cdot7 - 7$ or

$a\cdot b = 10b - (10 - a)b, \ (0 < a, b < 10)$.

---

[64] Finnur Jónsson (1892–1896): cxxxii
[65] Benedict, S. R. (1914): 122
[66] Finnur Jónsson (Ed.) (1892–96): 417
[67] Finnur Jónsson (Ed.) (1892–96): 418
[68] Bekken, O. B., and Christoffersen, M. (1985): 27
[69] Allard, A. (1992): 1

In the following, examples have been made up to illustrate the computing methods presented rhetorically in *Algorismus*. Bekken et al. constructed similar examples.[70]

Addition and subtraction of multi-digit numbers are performed from the right hand side, as is still customary. Addition is only done with two numbers at the time, the larger one written above and the smaller one below, right justified. Gradually, the smaller number adds to the larger one, as the sum seems to be meant to be written over it, starting at the unit-place and continuing to the left.

An example of 4073 added to 536, shown in four steps:

4073	407**9**	4**109**	**4609**
536	536	536	536

Subtraction is performed similarly. The number 463 subtracted from 1427:

1427	142**4**	13^{1}2**4**	13**64**	**964**
463	463	4 63	463	463

Doubling is considered to be an operation of its own, performed from the left side. This operation is rather inaccurately explained in *Algorismus* compared to *Carmen de Algorismo*, i.e. *Algorismus* is not an accurate translation in this chapter. It is specially noted that if the character "semiss" (one half) is found "yfir uppi í ysta stað" (up above in the farthest place) then one should be added as then there was previously an even number divided into halves. (This should be uneven / odd number). Possibly the translator (or a copyist) did not quite understand what is going on here.

An example of doubling 653½ (Instead of the semiss-character above 3, the fraction ½ is written at the end):

653½	**12**53½	**130**3½	**1307**

Halving, which also is a separate operation, is performed from the right. An example of halving 1787:

1787	178**3½**	17**43½**	1**393½**	**893½**

Multiplication of multi-digit numbers is done from the left as is the doubling. The advantage of multiplying from the left is that the product of the digits can be added to the previous product as they are found and it is not necessary to carry.

Example: Multiply 523 by 217:

First 523 are multiplied by 2 and 2 disappears under the product:

217	**1046**17
523	523

Then the first digit on the right of the lower number is moved one place to the right below the second digit from the left of the upper number which now is the multiplying digit. In this example the digit 3 is moved to be placed below 1 and other digits correspondingly. Now 523 are multiplied by 1. It is not quite clear if the digits of the product should be added as they are calculated or afterwards all at the same time:

[70] Bekken, O. and M. Christoffersen (1985): 49–57

52	
104637	**10983**7
523	523

In the next step the digit 3 of the multiplicand is moved below 7 and the other digits similarly. Then 523 is multiplied by 7:

	35		**14**	**2**	
109837	1**0983**7	**11333**7	**11333**7	**113471**	**113491**
523	523	523	523	523	523

The order of the multiplications in a product of two three-digit numbers is as follows:

$$(100a + 10b + c) \quad \text{multiplied by}$$
$$(100d + 10e + f)$$
$$= 100a\cdot 100d + 100a\cdot 10e + 100a\cdot f + 10b\cdot 100d + 10b\cdot 10e + 10b\cdot f + c\cdot 100d + c\cdot 10e + c\cdot f$$

<u>Division</u> is essentially performed similarly to modern algorithms.

<u>Extracting the square root</u> of $(100a + 10b + c)^2$ in *Algorismus* is done by using the following relation, guessing the digits a, b and c, and subtracting term by term:

$$(100a + 10b + c)^2 - (100a)^2 - 2\cdot 100a\cdot 10b - (10b)^2 - (2\cdot 100a + 2\cdot 10b)\cdot c - c^2 = 0$$

while a common method in Icelandic schools in the 20th century was more compact, subtracting the following terms:

$$(100a + 10b + c)^2 - (100a)^2 - (2\cdot 100a + 10b)\cdot 10b - (2\cdot 100a + 2\cdot 10b + c)\cdot c = 0$$

The older method can lead to guessing too high a value of *b*. However, it has the advantage of treating each place in the number, while the more modern method needs jumping over every other place.

<u>Extracting a cubic root</u> is done in a compact way in *Algorismus*:

$$(100a + 10b + c)^3$$
$$- (100a)^3 - (3\cdot 100a + 3\cdot 10b)\cdot 100a\cdot 10b - (10b)^3$$
$$- (3\cdot 100a + 3\cdot 10b)\cdot(100a + 10b + c)\cdot c - c^3 = 0$$

while the method of Āryabhaṭa (born 476) in his work *Āryabhaṭīya* is[71]

$$(100a + 10b + c)^3$$
$$- (100a)^3 - 3\cdot(100a)^2\cdot 10b - 3\cdot 100a\cdot(10b)^2 - (10b)^3$$
$$- \quad 3\cdot(100a + 10b)^2\cdot c - 3\cdot(100a + 10b)\cdot c^2 - c^3 = 0$$

The chapter on the cubic root in *Carmen de Algorismo* is said to be the first one where extracting a cubic root is introduced in Latin.[72] It is not found in *Dixit Algorizmi* or other translations of al-Kwarizmi's *Arithmetic.*[73]

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[71] Katz, V.J. (1993): 202
[72] Beaujouan, G. (1954): 106
[73] Allard, A. (1992): xxxi

Elements

This chapter is not to be found in *Carmen*, and its content is unrelated to the rest of *Algorismus* in modern understanding. In short, cubic numbers are assigned to the elements earth and fire, 2^3 to the earth and 3^3 to the fire. It is found necessary to put something in between them which will unite them in their disagreement. So the numbers $2^2 \cdot 3 = 12$ are selected for water, which then has two factors from earth and one from fire, and $2 \cdot 3^2 = 18$ for air, gaining one factor from earth and two from fire. This puts the elements in the correct order by lightness: Earth (8), water (12), air (18) and fire (27). This is probably the only evidence that these cosmological discussions from *Timaios* by Plato were known in the Nordic countries.[74] The arithmetic ideas are, however, not made as explicit in *Timaios* as in *Algorismus*. The Greek Neo-Pythagorean Nicomachus, his Latin follower Boëthius, Sacrobosco and his commentator Petrus Philomena de Dacia have elaborated on these arithmetic ideas of the numbers 8, 12, 18 and 27 but without reference to the elements.[75]

Hauksbók

The Lawman Haukur Erlendsson (1265-1334) collected knowledge of various origins in *Hauksbók*, a parchment (vellum) written by himself and scribes under his supervision. It also contains manuscripts of well-known sagas. *Hauksbók* is one of the main sources of *Algorismus*.[76]

The part of *Hauksbók* containing *Algorismus* is assumed to have been written in the period 1302–1310, most likely in 1306–1308.[77] The calligraphy of the Hindu–Arabic numerals in *Algorismus* has been assumed to be from before 1270, while the main manuscripts of *Algorismus* are dated from the 14th century. This, including several errors common to the manuscripts, suggests that the *Algorismus* manuscript in *Hauksbók* was copied from an older manuscript.[78]

Haukur Erlendsson wrote a large part of *Hauksbók* himself. He was an industrious man with multiple interests, but he was neither the author nor the translator of *Algorismus*, nor did he copy *Algorismus* himself. The copyist is believed to have been his scribe, an Icelander, writing under Haukur Erlendsson's supervision.[79]

Haukur Erlendsson

Very little is known about Haukur Erlendsson's childhood, upbringing and education. Haukur Erlendsson is first mentioned in sources in 1294, when he was Lawman for the Southern and Eastern quarters of Iceland. He kept the position only for one year.[80] It is likely that he was about thirty years old when he first became Lawman, so he could have been born around 1265.[81]

[74] Pedersen, O. (1966): 498
[75] Bekken, O. B. (1986): 12–13
[76] Finnur Jónsson (1892–1896): iv
[77] Stefán Karlsson (1964): 119
[78] Finnur Jónsson (1892–1896): cxxxi
[79] Finnur Jónsson (1892–1896): xiv
[80] Stefán Karlsson (1964): 114
[81] Jón Helgason (1960): vi–vii

In 1302 it is known that Haukur Erlendsson was in Norway, where he calls himself Lawman in Oslo. In 1304 he stayed in Iceland, perhaps as a representative of the Norwegian King. In 1305 he was in Norway and he came to Iceland in 1306 and stayed there until 1308. His title is Lawman until 1322, and sources reveal that his district was Gulaþing in Western Norway. Haukur Erlendsson died in Norway in 1334.

Haukur Erlendsson's residence in Iceland 1306–1308 is unknown. One of the main treatises in *Hauksbók* is the *Book of Settlements,* which was written by Haukur Erlendsson himself in this period. It is suggested from the additions he made to the *Book of Settlements*, compared to its earlier versions, that it could have been written in the vicinity of the island of Viðey off Reykjavík and that he acquired *Algorismus* there.[82] It is very likely that Haukur Erlendsson was acquainted with the library of the monastery on Viðey. Furthermore, mathematics was among his interests and it can be seen that he knew for example how to calculate the circumference of a circle.[83]

Who wrote the first version of *Algorismus*? We turn our attention to Styrmir Kárason.

Styrmir Kárason

Styrmir Kárason the Learned is known for his *Styrmisbók*, a lost version of the *Book of Settlements*, a source that Haukur Erlendsson informs the reader that he used for his own *Book of Settlements*. Styrmir Kárason was Law Speaker from 1210 to 1214, when Snorri Sturluson succeeded him. Styrmir Kárason and Snorri Sturluson were companions, and Styrmir stayed for long periods at Snorri's home.[84] For the Law Speaker

> ... it [was] imperative that an articulate man with administrative abilities and profound knowledge of both law and chronology be elected ...[85]

so it is clear that Styrmir Kárason mastered chronology. Styrmir became Law Speaker again in 1232–1235, after which he entered Viðey Monastery, where he was prior until his death in 1245.

Viðey Monastery

The Augustinian monastery on Viðey was established in 1225. Snorri Sturluson was one of the initiators of its foundation.[86] From an inventory of books in the monastery, dated 1397, it can be concluded that no place in Iceland had a better collection of schoolbooks and history books. Amongst them was the grammar textbook *Doctrinale Puerorum* by Alexander de Villa-Dei, and nine other books of verses.[87] Could other books by Alexander have been amongst them, e.g. *Carmen de Algorismo*?

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82 Jakob Benediktsson (1968): lxxix. Helgi Guðmundsson (1967): 68
83 Helgi Guðmundsson (1967): 82
84 Jón Jóhannesson (1941): 137
85 Jón Jóhannesson (1974): 48
86 Jón Jóhannesson et al. (1946): 302
87 *Diplomatarium Islandicum* IV (1897): 110–111

At least the Viðey Monastery had financial resources to buy foreign scholarly writings. Another work in the inventory is an etymology by Isidorus of Seville, *Ysidorus ethimologiarum*, containing a collection of knowledge on grammar, rhetoric, arithmetic, geometry, music, astronomy, medicine etc. This book is only known to have been owned by one other establishment in the country, Hólar Cathedral.[88]

Studies of ecclesiastical and tidal calculations and other mathematical subjects seem to have been practised in the monastery, even at the time of Styrmir Kárason. Old writings connected to Viðey, i.e. the treatise *Rím II* for its tidal calculations[89] and the manuscript GKS 1812, 4to, mentioned earlier,[90] suggest this conjecture. Recent archaeological excavations carried out on Viðey by Reykjavík Museum in 1987–1995 have revealed wax tablets, similar to those which are believed to have been used in medieval schools for writing and computing.[91]

Figure 2.3: A wax tablet found on Viðey with a religious text from the 15th century[92]

Was *Algorismus* written in Viðey Monastery?

Texts like *Algorismus* cannot be translated or even copied unless they are thoroughly understood. This knowledge was available at Viðey Monastery. It seems that the text has come through un-garbled, even if there are several items in the manuscripts which may point to misunderstanding.

As mentioned before, the calligraphy of the Hindu–Arabic numerals in *Algorismus*' manuscript, being from before 1270, suggests that the first manuscript of *Algorismus* might have already existed before that time. *Carmen* was written around 1200[93] and might easily have been brought to Iceland before the middle of the thirteenth century, even as early as in 1235.

Alexander de Villa Dei and Styrmir Kárason were contemporaries. Styrmir lived for more than four decades after *Carmen* was written, so he could have acquired it and translated it.

[88] *Diplomatarium Islandicum* III (1896): 613
[89] Beckman et al. (1914-1916): xlv-l
[90] *Diplomatarium Islandicum* I (1857–1876): 184
[91] Árbæjarsafn: website, accessed November 4, 2004
[92] Reykjavík City Museum: website, accessed November 4, 2004
[93] Beaujouan, G. (1954): 106

Many Icelanders travelled to Europe in the thirteenth century, for example to study abroad or undertake pilgrimages. In *Hauksbók* one can find an itinerary of a journey to Rome.[94] Education improved greatly in Europe in this period and Icelanders watched closely the development there. It is not unlikely that some pilgrims brought back with them the latest studies.

A detail that points to Viðey is the conjecture that Haukur Erlendsson stayed in the vicinity of Viðey or had free admission to its library during the time of the writing of *Algorismus* in *Hauksbók*.

Algorismus may have been written before *Rím II*. In *Rím II* Hindu-Arabic numerals are used in the text, on almost every page. In *Algorismus* the Hindu-Arabic numerals are introduced and used only in the first few sections. After that Roman numerals or words are used when numerals occur, until the final section about the elements, which is not found in all manuscripts of *Algorismus* and originates from a source different from *Carmen de Algorismo*. In both treatises, both types of numerals are used alternatively, which was the case for a long time after the introduction of Hindu-Arabic numerals.

A Conjecture

Sturlunga saga, written by Sturla Þórðarson, one of Snorri Sturluson's nephews, says of Snorri Sturluson and Sturla Sighvatsson, another of Snorri's nephews, in 1230:

> Snorri reið eigi til þings, en lét Styrmi prest inn fróða ríða til þings með lögsögn. Nú tók at batna með þeim Snorra ok Sturlu, ok var Sturla löngum þá í Reykjaholti ok lagði mikinn hug á at láta rita sögubækr eftir bókum þeim, er Snorri setti saman.
>
> Snorri did not ride for Althing but ordered Styrmir the Learned to ride for Althing to speak the law. Now Snorri's and Sturla's friendship increased and Sturla stayed for long periods in Reykjaholt and he expressed a strong desire to have stories written on the basis of the books Snorri was composing.[95]

In the following couple of years friendship still existed between the two men, who were otherwise rivals in the battle for power in the Sturlung Era, a period of civil strife in which the Sturlung clan played a major role. Sturla Sighvatsson went on a pilgrimage to Rome in 1233 and returned in 1235. It is not unthinkable that Sturla brought some manuscripts with him which were sold or given to Viðey. A modern historical novel by Thor Vilhjálmsson, *Morgunþula í stráum*,[96] pictures what could possibly have happened. In the novel, Sturla searches for manuscripts on his journey, encouraged by Snorri Sturluson, in fact for the books of Sæmundur the Learned who studied in Franconia in the 11th century. It is tempting to conjecture the following course of events:

- Sturla Sighvatsson may have acquired manuscripts on his journey to Rome in 1233–1235, among them *Carmen de Algorismo,* encouraged by Snorri Sturluson and Styrmir the Learned, who knew Sturla as is implied in the above quotation.
- A copy of *Carmen* was brought to Viðey Monastery where Styrmir Kárason became prior in 1235.

[94] Finnur Jónsson (Ed.) (1892–1896): 502
[95] Jón Jóhannesson et al. (1946): 342
[96] Thor Vilhjálmsson (1998)

- *Carmen* was translated by Styrmir or under his supervision and *Algorismus* was composed with some omissions and additions to the translation.
- Haukur Erlendsson knew about Styrmir Kárason's writings, such as the *Book of Settlements* and *Algorismus*. He copied them or had them copied while he stayed on or near Viðey.

Of course Thor Vilhjálmsson's book is a novel, not a historical research treatise, and Sturla Sighvatsson had the reputation of a warrior, not a scholar. To be realistic, one should also consider the alternative that a clergyman brought the treatise to Iceland, possibly one of the foreign bishops reigning at that time, or Icelandic candidates for the episcopates.

2.6. The End of the Commonwealth

Society

Conflicts over wealth and power between the main dynasties increased during the 13th century. Moreover, the Icelanders were dependent on the Norwegians for trade. In the Sturlung Era, in the middle of the 13th century, when the chieftains could not agree on adhering to the law, their solution was to submit their affairs to the decision of the Norwegian King. Finally in 1262 the Icelanders, the land and the subjects, swore allegiance to the Norwegian King. The Icelanders agreed to pay tax, in return for the promise that six ships would sail each year from Norway to Iceland, and various rights for Icelanders in Norway.[97]

The next decades were characterized by a long-needed peace. Farmers, i.e. landowners, acquired increased influence. The power of the church increased strongly. There were three upper classes of people: the nobility of the King's men, the landowners and the clergy. There was no middle class, no merchants or skilled workers. Farmers could be landowners or tenants. Servants and farm workers were mostly unmarried people, and the majority probably did not have service as a lifelong occupation.[98] The beginning of the 14th century was characterized by the consequences of a volcanic eruption, disease and cold periods with pack ice on the coast in winter. However, fishing became more profitable than before, leading to improved living conditions. All fishing was the property of the landowners, and the fishermen were their tenants. Previously woollen textiles had been the main export, but they were superseded by stockfish (dried fish)[99] around the mid-14th century.

More than a third of the population died in the Black Death in 1402–1404, so that no more than 30–40 thousand people remained. Only few ships came to Iceland for many years. Seafarers avoided the plague. Icelanders had few or no ships at that time, so that they were isolated for some period. During the 15th century Englishmen began to seek Icelandic seas. The rich fishing grounds attracted European fishermen through the centuries until 1975, when the Icelandic fishing limits were expanded to 200 miles.[100]

[97] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 119–120
[98] Gunnar Karlsson (2000): 166
[99] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 131–137
[100] Björn Þorsteinsson and Bergsteinn Jónsson (1991): 149–150

Language

When Iceland was settled, the settlers spoke the Old Norse language. While in earlier times the language is assumed to have been the same in Denmark, Sweden and Norway, it had already split into the East-Nordic languages, Danish and Swedish, and the West-Nordic language or Old Norse, spoken in Norway and in Norway's colonies, Iceland, the Faroe Islands and Greenland. When the first Icelandic written sources entered the scene in the second half of the 12th century, there was only a small difference between Icelandic and Norwegian Norse, or at least the Norwegian spoken in West-Norway. This difference increased in the 13th and especially in the 14th century: changes took place in the Icelandic vowels, and the system of declensions of nouns and conjugations of verbs was simplified in the Norwegian language.[101] For example, the version of *Algorismus* in GKS 1812, 4to is written in the Norwegian language and it is known that in the period 1300–1400, when this part of GKS 1812 is believed to have been written, there were differences in the written languages.

In the middle of the 12th century, an unknown Icelandic author created complete writing rules for the language, which were influential in the following centuries, even though no one followed them completely.[102] Thus by the 15th century the Icelanders had their own language. Many changes in pronunciation occurred during the first centuries, while all the main changes were completed by the early 17th century. Reading books and recitals of the medieval literature century after century has without doubt contributed to the maintenance of the ancient vocabulary and system of declensions and conjugations.[103]

The three Nordic kingdoms, Denmark, Norway and Sweden, were united in the Kalmar Union in 1397. The union was dominated by Denmark, where the kings resided, so that the Icelanders' contact with Norway diminished.

During the early 15th century there were strong English influences in Iceland, through trade with English fishing fleets. The Low-German Hanseatic League, mainly from Hamburg, had considerable influence and trade in Iceland from 1470 to the middle of the 16th century. Their main base was Hafnarfjörður, where they even built permanent wooden houses. In 1542 King Christian III renewed a prohibition on foreigners dwelling in Iceland.[104] During this period Icelandic, like other Nordic languages, absorbed many Low-German words. They were deliberately rejected from the Icelandic language when the first theological works were printed on the initiative of Bishop Guðbrandur Þorláksson in the late 16th and early 17th century.[105]

Thus in the culturally important field of language the distance between Iceland and its partners in the union was increasing. Instead of being predominantly connected through a common language, Icelandic society became, together with its geographical remoteness, separated mainly by its language.[106]

[101] Stefán Karlsson (2000): 20–22
[102] Stefán Karlsson (2000): 19
[103] Stefán Karlsson (2000): 65–67
[104] Gunnar Karlsson (2000): 126
[105] Baldur Sigurðsson, an email September 16, 2004
[106] Gunnar Karlsson (2000): 102–105

A colder climate contributed to worse conditions for travel between the countries than during the 12th and 13th century. These factors – language, climate and the transfer of administration to Copenhagen – contributed to the isolation of Iceland and the Icelanders from the other Nordic countries, which was to persist into the 20th century.

Learning

Sources about schools are scarce. However, it is known that the education of priests was at times inadequate. In 1307 several clergymen were suspended due to lack of knowledge.[107] The Black Death of the early 15th century killed the majority of learned men at both sees, at Skálholt and Hólar. Another destructive plague broke out in 1494. The sparse population in that country on the border of the habitable world was sensitive to calamities such as epidemics, which were sometimes accompanied by volcanic eruptions and other natural disasters. Therefore, for long periods the learned class might become severely disabled from carrying on a tradition of acquiring more than just the most vital knowledge.

In the 15th century, in 1479, a university was established in Copenhagen, and another at Uppsala in Sweden in 1477. As a consequence the centres of learning and scholarly activities in the Nordic countries began to move from the medieval Catholic centres of learning at the cathedrals and monasteries to the universities,[108] which for Iceland meant that its centre of learning gradually moved to Copenhagen. After that time there are no sources indicating mathematical initiative in Iceland for several centuries.

Medieval Mathematical Heritage

What role does the medieval mathematical heritage play in Icelandic mathematics education? The answer is that its influence may be traced from time to time through history. It appeared at times of cultural resurgence. We know that *Hauksbók* with *Algorismus,* and GKS 1812, 4to with *Oddi's Tale*, *Algorismus, Rímbegla* and fragments of *Rím II*, were among the manuscripts in the possession of Bishop Brynjólfur Sveinsson, who restored mathematics learning in his cathedral school in the 17th century. *Rímbegla* and *Oddi's Tale* were among the first treatises introduced to the European learned world when translated into Latin in the late 18th century. All these treatises were published in scientific editions at the turn of the 20th century, when Iceland was acquiring Home Rule. A reference to algorithms in *Algorismus* appeared in one of the very few articles discussing mathematics education in the early 1960s.[109] These treatises thus had their roles in Icelandic education and self esteem, even if minimal and probably mainly confined to the educated elite.

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[107] Janus Jónsson (1893): 8–26
[108] Pedersen, O. (1966): 497
[109] Gestur O. Gestsson (1962): 114–137

3. Early Modern Times

3. 1. Education after the Reformation

In 1550 the Catholic faith was abandoned in Iceland and the Evangelical Lutheran faith adopted by the order of the King Christian III. Politically this meant that the monasteries and the convents became the property of the Danish kingdom, and were closed down as seats of learning. For the next two and a half century, two cathedral schools were run in Iceland, at the sees of Skálholt and Hólar. Regulations were set for the schools, such as the salaries of the schoolmaster and his assistant, the "hearer". The number of pupils was to be 24 at each see. In 1743 the numbers were 24 at Skálholt and 16 at Hólar. Initially, the school term only lasted about 26 weeks, and was even shorter if there was not enough food for the pupils.[110]

Bishops Guðbrandur Þorláksson and Oddur Einarsson

Two bishops, Guðbrandur Þorláksson (1541/42–1627) at Hólar and Oddur Einarsson (1559–1630) at Skálholt, were known to have studied mathematical sciences, such as astronomy, in Copenhagen. Guðbrandur Þorláksson is believed to have corresponded with Tycho Brahe, and Oddur Einarsson was a guest of Tycho Brahe at the astronomical institute at Hven in Øresund. Guðbrandur Þorláksson was headmaster at Skálholt School in 1564–1567, and later at Hólar School before his appointment in 1571 as bishop of Hólar. Oddur Einarsson was headmaster with Bishop Guðbrandur Þorláksson at the Hólar See in 1586–1588, before he became bishop of Skálholt in 1589.[111] One may suppose that both of them supported mathematical studies at their schools during the last decades of the 16th century and during their periods of office as bishops in the early 17th century.

Printing Technology

One of the European innovations that were brought to Iceland at an early time was printing technology. A printing press was established at the see of Hólar around 1530.[112] The publications printed for Bishop Guðbrandur Þorláksson, such as the first translation of the entire *Bible* into Icelandic in 1584, became a basis for public education, and they reflect his efforts to return to the language of early Iceland. Danish never became the official language in Icelandic churches, and consequently not in the two cathedral schools either. The Icelandic language had already become so different from Danish that it was regarded as necessary to translate the *Bible* into Icelandic for church use.

Bishop Guðbrandur Þorláksson published mainly religious books. No mathematical writing is attributed to him or contained in his publications, in spite of his mathematical learning, except *Calendarium* (1597). The *Calendarium* is the first almanac adjusted to Iceland, written by the bishop or by his friend Arngrímur Jónsson (1568–1648) the Learned in cooperation with the bishop.[113]

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[110] Janus Jónsson (1893): 35–39
[111] Janus Jónsson (1893): 46, 67
[112] Gunnar Karlsson (2000): 136. Hólarannsóknin: website, accessed July 23, 2005
[113] Einar H. Guðmundsson (1996)

Bishop Guðbrandur Þorláksson measured the latitude of Hólar, 65° 44’, a measurement which was very close to being correct. Prof. Einar H. Guðmundsson has suggested that the quadrant used for this measurement was made in Tycho Brahe’s workshop at Hven and that Oddur Einarsson brought it to Iceland in 1589.[114] The measurement is used in the 1597 *Calendarium*[115], while less accurate latitude for Hólar, 66° 55’, is used in a map of Iceland made by Bishop Guðbrandur Þorláksson in 1585. The bishop also computed the solar eclipse in Northern Iceland and published it in the *Calendarium*.

Bishop Guðbrandur Þorláksson’s Map

Bishop Guðbrandur Þorláksson’s map of Iceland was published in the collection of the Dutch cartographer Abraham Ortelius in 1590.

Fig. 3.1. Bishop Guðbrandur Þorláksson’s map of Iceland published in 1590.[116]

The website of the National and University Library of Iceland contains the following:

> In 1590 Abraham Ortelius published a new supplement, *Additamentum IV*, to his collection of maps, *Theatri orbis terrarum*. Amongst the new maps is a map of Iceland (*Islandia*). The author is not mentioned but on it says that it was engraved in the year 1585. On it we can also find a dedication to Fridrik II of Denmark by Andreas Velleius (Andreas Sørensen Vedel 1542-1616); a well-known Danish historian of the period. It has been known for a long time that Vedel is not the author of the map and he could not have made it. The map is so superior to all earlier maps of Iceland in content and execution that an Icelander must be its originator. All clues point in the direction of Gudbrandur Thorláksson, bishop of Hólar. In his school days

[114] Einar Guðmundsson (1996)
[115] Þorsteinn Sæmundsson (Ed.) (1968): 15
[116] National and University Library of Iceland: website, maps. Accessed June 1, 2005

in Copenhagen he had studied mathematics and astronomy alongside theology. He had calculated the position of Hólar and arrived at an amazingly accurate result. He made a map of the North in 1606. The map of Iceland by bishop Gudbrandur does no longer exist and it is not known when he made it or what resources he had at his disposal. A list of churches and fiords has been found which he seems to have used, especially the latter, which he follows very closely for a large part of the coast.[117]

Bishop Guðbrandur Þorláksson's map did not become widespread, but it was the basis for other maps of Iceland published in the 17th century. The Danish authorities had made new surveys in the early 18th century, first by Magnús Arason and later under the leadership of Thomas Hans Henrik Knoff to continue and complete the work, but did not publish a complete map, as it was considered a military secret. Maps based on these measurements were published in Copenhagen in 1752 in a book about Iceland by Niels Horrebow[118], and by Homanns of Nürnberg, Germany in 1761. These maps became models for most maps of Iceland until the 19th century.[119]

Learning in the 17th Century

Natural sciences were highly regarded in Denmark in the 17th century. Many Icelanders studied these subjects at the University of Copenhagen.[120] Gísli Einarsson taught mathematics at Skálholt. Runólfur Jónsson (–1654), headmaster at Hólar in 1645–1649, seems to have run a private school in natural sciences in Copenhagen in 1649–1651. He presided at a symposium when ten Icelanders, his students, debated on physics and philosophy in Copenhagen in 1652. At the same time, young men made astronomical observations. However, when influences from the great interests in these subjects in Denmark had arrived in Iceland, witch-hunts had preoccupied even those same individuals, and others among Iceland's best-educated and most influential people.[121]

Bishop Brynjólfur Sveinsson

Brynjólfur Sveinsson (1605–1675) of Skálholt was the most renowned bishop of the 17th century. Brynjólfur sailed to Copenhagen in 1624 and studied theology and philosophy, and also medicine, astronomy, Greek and Hebrew. On his return home he tried to become a teacher at Skálholt or Hólar, but in vain. So he continued his Greek studies, first at home and then in Copenhagen. Thereafter he was appointed deputy headmaster (*conrector*) at a Latin school in Roskilde in Denmark. He remained there for seven years (1632–1639), and was awarded a master's degree in philosophy in 1633. In 1638 Brynjólfur Sveinsson was elected bishop, although his expertise was in the field of philosophy rather than theology. When he objected on the grounds that he was primarily a teacher and an educator, he was told that this was exactly what was needed in Skálholt.[122]

Fig. 3.2. Bishop Brynjólfur Sveinsson

[117] National and University Library of Iceland: website, text on maps. Accessed February 2 2005
[118] Horrebow, N. (1966)
[119] National and University Library of Iceland: website, text on maps. Accessed September 16, 2005
[120] Einar H. Guðmundsson (1998): 188–191
[121] Helgi Þorláksson (2003): 380–384
[122] Gunnar Harðarson (1988): 91–92

Bishop Brynjólfur Sveinsson adhered to Humanism and was a strong supporter of learning. At his time the cathedral school in Skálholt was larger than required, and he favoured the *quadrivium.* At least two of his headmasters had studied sciences and mathematics in Copenhagen: Þorleifur Jónsson, who debated in Copenhagen on the world, *Dissertatio physica de mundo*, in 1644, and Gísli Einarsson, who became the first person in Iceland to be appointed a mathematics teacher.[123]

Professor Jón Helgason compiled a list of Bishop Brynjólfur Sveinsson's library. The library included a selection of books on astronomy, mathematics and natural philosophy.[124] Among them was *De Sphæra* by Johannes de Sacrobosco, and many other works about astronomy, which indicate that the bishop must have been very knowledgeable in astronomy if he ever read these books. His books about mathematics were fewer. However, Euclid's *Elements* are found in commentaries by Campanus from Novara (d. 1296) and Bartolmeos Zambert (b. 1473). Furthermore, Bishop Brynjólfur Sveinsson had a book on geometry, *De veritatibus geometricis* by Villum Lange, a work about the life of Pythagoras, *De vita Pythagoræ* by Jamblichus, *Qvadratura circuli* by Christen Longomontanus, and *Rami Arithmetica cum Shoneri Arithmeticis*, (probably *Arithmeticæ libri duo et algebræ todidem* from 1586 with additions and explanations by Lasarus Schöner) by Pierre de la Ramée. Bishop Brynjólfur Sveinsson was a supporter of Ramée's philosophy, as Oddur Einarsson had been, and Brynjólfur Sveinsson wrote commentaries on Ramée's book on *Dialecticarum*. The majority of Bishop Brynjólfur Sveinsson's library seems to have been acquired before he left Denmark, which reflects the difficulty of maintaining an up-to-date library in Iceland.[125]

It is interesting to learn about the bishop's interest in Ramée's philosophy. In Boyer and Merzbach's *History of Mathematics*, this account of Pierre de la Ramée is found:

> … Pierre de la Ramée, or Ramus (1515–1572), [was] a man who contributed to mathematics in a pedagogical sense. … Ramus was at odds with his age in many ways, and while his Humanist contemporaries had little use for mathematics, he had almost a blind faith in the subject. He proposed revisions in the university curricula so that logic and mathematics should receive more attention; his logic enjoyed considerable popularity in Protestant countries … Not satisfied even with the *Elements* of Euclid, Ramus edited this with revisions. However, his competence in geometry was very limited, and his suggested changes in mathematics were in the opposite direction from those in our day. Ramus had more confidence in practical elementary mathematics than in speculative higher algebra and geometry. Looking back on his age we see that the mathematics of that time seems already to have been excessively concerned with practical problems in arithmetic, while weakness in geometry was quite conspicuous.[126]

It is fair to conclude from the library of Bishop Brynjólfur Sveinsson and from his adherence to Ramée's philosophy that his interest in mathematical education was genuine. Considering that he had already been deputy headmaster in Roskilde, he may be thought of as a person who brought modern educational ideas of that time into the Icelandic school. The mathematical interests can also be concluded from his manuscript collection, and the fact that he appointed the first and only formally-

[123] Einar H. Guðmundsson (1998): 203–205
[124] Jón Helgason [Dr. Phil.] (1948): 116–127
[125] Einar H. Guðmundsson (1998): 207–210. Jón Helgason [Dr. Phil.] (1948): 126–127
[126] Boyer, C.B. and Merzbach, U.C. (1989): 293

appointed mathematics teacher of the Protestant cathedral schools, Gísli Einarsson, as we will learn about.

Professor Jón Helgason wrote an account of the existence of a story confirming Bishop Brynjólfur Sveinsson's learnedness, saying that the bishop was asked to decide who was right in a debate between Prof. Villum Lange and Marcus Meibom about a mathematical problem. Professor Jón Helgason considered it obvious that there was no likelihood that Bishop Brynjólfur Sveinsson had been capable of making any judgement in mathematical disputes, and no one would have thought of requesting that from him. In his letters the bishop himself had said that he only understood half the content of the books.[127] This modest answer, that he only understood half the content, could, however, equally well be conceived so that he had at least attempted to read the books and understood some of their content, which might indicate more than average knowledge in mathematics.[128]

Bishop Brynjólfur Sveinsson was an avid collector of old manuscripts, which witnesses his humanistic interests. It is known that he possessed the mathematical manuscript collection GKS 1812, 4to. Brynjólfur Sveinsson's household manager and friend, Hákon Ormsson, whose name is written on the book, was a great-great-nephew of Abbot Alexíus Pálsson, the last abbot at Viðey Monastery, and thus a link between the manuscript GKS 1812, 4to and Viðey Monastery. Bishop Brynjólfur Sveinsson sent the manuscript to the library of King Frederik III around 1656.[129]

Another manuscript collection that is known to have come into Brynjólfur Sveinsson's possession is *Hauksbók*, probably in the period 1652–1660. From Brynjólfur Sveinsson's heirs the manuscript came into keeping of the well-known collector of manuscripts and documents Prof. Árni Magnússon, probably in 1691, to remain in his collection, the Arnamagnean Institute in Copenhagen, to the present day.[130]

Thus two of the most important Icelandic mathematical manuscript collections, GKS 1812 4to and *Hauksbók*, containing the important mathematical treatises *Algorismus*, *Rímbegla*, *Oddi's Tale* and fragments of *Rím II*, were in the possession of Bishop Brynjólfur Sveinsson. It is very likely that Bishop Brynjólfur studied these treatises. He had copyists working for him, and he probably kept a copy of those manuscripts he sent abroad. The bishop's attempt to restore learning was thus based on the Icelandic manuscript heritage, also in mathematical subjects.

Fig. 3.3. Skálholt Cathedral at Bishop Brynjólfur Sveinsson's time.

[127] Jón Helgason [Dr. Phil.] (1948): 133–134
[128] Helgi Skúli Kjartansson, an email, November 10, 2004
[129] *Diplomatarium Islandicum* I (1857–1876): 183–184
[130] Jón Helgason [Dr. Phil.] (1960): xxvii–xxviii

Bishop Brynjólfur Sveinsson's education was within the learned Latin and Greek world. However, precisely through this education Brynjólfur Sveinsson was able to understand the Icelandic culture to which he belonged. One could say that he worked at bridging the gap between European Latin education and the ancient Nordic culture, at the same time as he worked at drawing his contemporaries' attention to the value of the ancient Icelandic culture.[131] For this reason he worked hard to have the ancient manuscripts published in print. He could not acquire a licence to establish a printing press in Skálholt, so he hoped that the manuscripts would be published in Copenhagen. Brynjólfur Sveinsson's successor, Þórður Þorláksson (bishop 1674–1697) had the Hólar printing press moved to Skálholt during 1686–1700, where the first Icelandic publications of ancient Icelandic manuscripts were printed.[132] Bishop Þórður Þorláksson was Bishop Guðbrandur Þorláksson's great-grandson, educated in Copenhagen and Germany. Bishop Þórður Þorláksson had wide interests, not the least in mathematical subjects, geography, astronomy, chronology and map-making. He improved on his great-grandfather's surveying and maps, while his measurements unfortunately were not published until more than two centuries later, and became of no use.[133]

Gísli Einarsson[134]

Gísli Einarsson (c. 1621–1688) studied at the Skálholt Cathedral School in Bishop Brynjólfur Sveinsson´s time, and thus he enjoyed the bishop's guidance. Gísli probably graduated in 1644. Thereafter he studied mathematics and astronomy for five years at the University of Copenhagen with the learned Jørgen From (1605–1651) as private teacher. At that time Icelandic students mainly studied theology and law, preparing them for official positions back home in Iceland. A letter in the Danish State Archives is dated in April 1649 from Gísli Einarsson to Otte Kragh, secretary of King Frederik III. In the letter Gísli Einarsson asked for the King's grant to continue his studies in arithmetic, geometry and *sphærica*, which up to that time had not been taught at Skálholt Cathedral School. As an answer to this request Gísli Einarsson was appointed to the teacher's position four days later. Gísli Einarsson was the first person to be formally appointed as teacher of mathematics and astronomy in Iceland. He stayed for 12 years at the cathedral school in Skálholt, first as "hearer" and later as headmaster.

According to contemporary accounts, Gísli Einarsson had considerable talents as a mathematician. Just before he left Copenhagen for Skálholt he was chosen to compute the annual astronomical almanac for Denmark for the year 1650, a task allotted only to professors of mathematics and astronomy at the University and their best students. This almanac, which was published in two slightly different editions, is the only work by Gísli Einarsson which has survived intact through the centuries. Gísli Einarsson was the second Icelander to publish a yearly almanac with astronomical computations, as Bishop Guðbrandur Þorláksson at Hólar had published a table on the course of the sun in his *Calendarium* in 1597.

[131] Gunnar Harðarson (1988): 89–90
[132] Gunnar Harðarson (1988): 98–99. Hólarannsóknin: website, accessed July 23, 2005
[133] Haraldur Sigurðsson (1982b): xv
[134] Einar H. Guðmundsson (1998): 185–231. Astrophysicist Prof. Einar H. Guðmundsson has studied the life of Gísli Einarsson. This section draws on his article.

As an example of how unusual this was, thirty years were to pass until the next almanac was published by Icelanders. That one was not computed by an Icelander, but was a translation of a Danish almanac for the year 1684 by the astronomer Bagge Wandel (1622–1684). Only after yet another 153 years, in 1837, did regular publication of Icelandic almanacs begin; this has continued uninterrupted until this day. The almanac was computed by Danish astronomers until 1923, when mathematician Dr. Ólafur Dan Daníelsson and physicist Þorkell Þorkelsson took over the computations, 273 years after Gísli Einarsson. However, through the centuries, the general public had studied chronology from handwritten calendars called *rím*, both in prose and rhyme. Some of them were translated and adjusted to the Icelandic environment, while others were home-made.

Gísli Einarsson determined the latitude of Skálholt fairly precisely, which indicates that he must have had access to a good angle-measuring device, such as a quadrant. His measurement is not as precise as the earlier determination of the latitude of the episcopal see at Hólar by Bishop Guðbrandur Þorláksson. Further refinement of the latitude of Skálholt was later made by the Bishop Þórður Þorláksson.

The best Icelandic description of the 1652 comet was written by Gísli Einarsson. Unfortunately the letter containing the description is no longer available, while it exists in an account by the lawyer and historian P. H. Resen (1625–1699). Resen possessed the letter, which reflects that Gísli Einarsson was knowledgeable in astronomy, and that his description surpasses all Icelandic accounts of comets until Björn Gunnlaugsson's accounts of comets in 1826 and 1858. However it can be seen from Gísli Einarsson's account that he believed the comet to be closer to the earth than the moon, a theory that Tycho Brahe had rejected in 1577.

No sources give any account of what Gísli Einarsson taught or how he did it, while they do recount that he was popular amongst his pupils. An inventory of Skálholt Cathedral from 1744 reveals a textbook, *Frommii Arithmetica*, i.e. *Arithmetica Danica* by Jørgen From, Georgius Frommius, Gísli Einarsson's teacher in Copenhagen.[135] The book was published in Copenhagen 1649, the year when he left Copenhagen for Iceland. It is possible that he brought the book to Iceland and used it at the cathedral school. The book is not mentioned in the inventories of Skálholt Cathedral in 1674, 1704 or 1722. One reason could be that not all these lists were preserved complete, or that they did not include schoolbooks. Furthermore, Gísli Einarsson was still alive in 1674 serving as priest at Helgafell on Snæfellsnes. The book may have been his private property, and only sold later to the cathedral school, even after 1704, when manuscript collector Prof. Árni Magnússon made an invaluable list of books at Skálholt at that time.

Frommii Arithmetica[136]

Frommii Arithmetica exists in the Royal Library in Copenhagen, named *Arithmetica Danica Seu Brevis Ac Perspicua, Institutio Arithmeticæ Vulgaris, Astronomicæ, Geodætice, In Usum Gymnasiorum Et Scholarum Daniæ Ac Norvegiæ*, Fussu Regio Adornata À Georgio Frommio, In Academiâ Hafniensi Mathematum, Professore Ordinario, published in 1649, written in Latin.

[135] Hörður Ágústsson and Kristján Eldjárn (1992): 347
[136] Frommius, G. (1649)

According to its title *Frommii Arithmetica* contains general, astronomical and geodetic arithmetic to use in Danish and Norwegian (learned) schools. This is a typical textbook in arithmetic, and in fact a rather good one. It is 164 pages long, comprising four books.

Book I treats simple arithmetic (*De Arithmetica Simplici*), i.e. the four operations in positive integers and vulgar fractions in addition to extraction of a square root and a cubic root. A number is defined by referring to Euclid's *Elements*, book VII, def. 2: A number is a multitude composed of units. Hindu-Arabic number notation is explained, as well as Roman number notation.

Whole number arithmetic is well explained, e.g. multiplication with a picture of the area of a rectangle with sides of 2 and 3 units. Multiplication by powers of ten is treated specifically with examples, as is multiplication by 6 as being first by 2 and then by 3, and multiplication by 8 as doubling three times. Finally, a shorthand multiplication of two 5-digit numbers is shown. The answer has only 5 digits, but it is said to have been divided by 10,000. Division is still somewhat messy, and is not explained further than with several examples. Testing of all four operations is shown, and of addition and subtraction in more than one way.

Fractions are explained fairly thoroughly concerning size, comparison, reduction, addition and subtraction. Multiplication and division are only shown with examples, without explanations.

Extractions of square roots and cubic roots are basically made by the same method as shown in *Algorismus*. However, the explanations are much clearer, with a number of numerical examples for demonstration. The meaning of a square root is demonstrated by a drawing of a 9-unit square with sides of 3 units.

Book II concerns comparisons (*De arithmetica comparata*). First it treats proportions, and then turns to arithmetic and geometric progressions. No algebraic notation is used, which is natural as it was rather undeveloped before the middle of the 17th century.

In arithmetic progressions the sum is computed according to $S_n = (a_1 + a_n)\cdot n/2$ where a_1 and a_n are the first and the last terms in a progression with n terms. The number of terms is shown computed from the first term, the last term and the difference between every two terms. The first term is shown computed from the last term, the difference and the number of terms. A practical example is given of computing the number of seats in a theatre, given a constant difference between the number of seats in a row, the number in the first row and the number of rows.

The geometric progression is only treated with quotient 2. Elaborate methods are shown for computing terms in the progression.

The second part of the Book II treats *Regula Trium*, simple, inverse and composite, *Regula Societatum* and *Regula Falsi*. The remaining sections of the book concern calculations in the sexagesimal system and geodetic computations.

Frommii Arithmetica seems to be a flexible textbook, with its explanation of the decimal system and attempts to support explanations with simple pictures, methods of testing and shorthand methods. Pupils who managed these methods must have been proficient in all general arithmetic in whole numbers and fractions. The first printed books in Icelandic that reach that level were published in the 1780s.

Gandreið

The Rev. Jón Daðason (1606–1676), for a period a cathedral priest in Skálholt, wrote in 1660 a kind of encyclopaedia in Icelandic, called *Gandreið* (literally Witch-ride), in which he collected knowledge appropriate for an educated man of the 17th century.[137] In this manuscript he explains briefly the basic features and spelling of the decimal system and the four operations. It seems that the Rev. Jón Daðason enjoyed mathematical patterns. His "practical" examples contain the addition of the first 99 whole numbers to the sum 4950, under the disguise of being number of units of butter a cook could eat. Another example is of 12 to the power 6, hidden in a story of 12 boats, with 12 men on each boat, each man having 12 harpoons, each hitting 12 seals, etc. One is tempted to conclude that, at the time of Bishop Brynjólfur Sveinsson and Headmaster Gísli Einarsson in Skálholt, solving arithmetic problems was a leisure-time activity.

Remarks

Several examples have been cited above of Icelanders in the 17th century who were engaged in mathematical activities in their early years while in Copenhagen. These persons have in common interests and talents in the field of mathematical sciences. There are only sparse accounts of their mathematical activities after they returned to Iceland. There may be various explanations for this. There were no positions for scholars at that time. The only positions for learned men were at the bishoprics and schools, where they had other obligations. After returning to Iceland, most university graduates became government officials, priests or teachers. Such positions left little time for further study or practice of the sciences. Connections with European scholars were scarce and it was difficult to acquire books and manuscripts.

Their main efforts of which we have records in the mathematical field concerned astronomy, a semi-practical activity. In a few cases, learned men are known to have measured the latitude of sites of significance in Iceland, and even have tried to find the corresponding longitude, a much more difficult task. Some became cartographers and published perpetual calendars.[138]

Moreover, other learned men, who wrote accounts of these scholars' life and work, did not have insight into their mathematical activities, and wrote instead about their personal circumstances and behaviour, such as their inclination for alcohol or their family problems. The same was the case far into the 19th century. Knowledge of their scientific activities may thus have been forgotten.

3.2. Early 18th Century Arithmetic Manuscripts

Three manuscripts in Icelandic called *Arithmetica* exist in the National and University Library of Iceland from the early 18th century. The manuscripts are not related, and the authors of two of them are not known. The first manuscript, Lbs. 1694, 8vo, *Arithmetica Islandica*, explicitly dated in 1716, is torn, while mended, a total of 160 pages, ten handwritten quires. It contains numeration, money, weight, the four operations, progressions, fractions, *Regula Trium, Regula Duples Reciproca,* etc.

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[137] Lýður Björnsson (1993): 13–17

[138] Einar H. Guðmundsson (1998): 230

Arithmetica. Þad er Reikningslist

The second one, ÍB 217, 4to, *Arithmetica. Þad er Reikningslist*, is not dated, but from an addition problem it can be concluded that the "present time" is 1721.[139] This copy may be younger, and is dated to 1750 by the manuscript department of the National and University Library of Iceland. The manuscript is 130 pages. It is not worn at all and is well readable. It may be a translation, possibly a free translation, of a Latin textbook about arithmetic. It is not related to the other contemporary manuscript, Lbs. 1694, 8vo. The textbook contains all the traditional topics of arithmetic, the four operations in whole numbers and fractions and extraction of a square root, but neither currency nor measuring units. Furthermore it contains arithmetic and geometric progressions and formulas for their sums, and several variations of the *regula de tri.* The formula for the sum of a geometric progression goes for any quotient, at least a whole number greater than 1, and thus goes a little further than *Frommii Arithmetica's* formula, which only works for the quotient 2.

The first three pages of introduction to numeration are interesting reading, revealing the cognitive world of the author/translator. He counts the ten digits and then says:

> Af þessum merkia niju vissa Taulu edur Fiöllda fyrir sig siálfa. Sá tíjundi sem er zyphra merkir ekkert fyrir sig helldur eykur einasta Merking þeirrar Tölu sem fyrir framan hana er til vinstri Handar.
>
> Margir af þeim Lærdu Mathematicis hafa viliað halda að 1. / unitas / væri ej Tala /numerus/ helldur væri Talann Fiölldi af 1. /:ex unitatibus / til samans lagdur við Evcl: Elementa Lib. 7 Def. 2. hvað um 1 kyni ej seigiast, einasta álitid unitatem, sem upphaf og undirrót til allrar Tölu.
>
> Of those nine mean a certain number or magnitude for itself. The tenth one, which is cipher (zyphra) means nothing for itself while it only increases the meaning of the number in front of it to the left.
>
> Many of the learned mathematicians have meant that 1 /unitas/ was not a number /numerus/, rather the number was a multitude of 1 /:ex unitatibus/ added together, Evcl: Elementa Lib. 7. Def. 2. what could not be said about 1, only supposed to be unitatem, as the origin and basis of all number.[140]

This reference reminds the reader of the comments on the numbers cipher and one in the 13th century treatise *Algorismus*, cited earlier, where Algorismus says: "One is ... not a number but the origin of all numbers" (see section 2.5.). It is tempting to believe that the writer / translator knew *Algorismus*, which was in the hands of Bishop Brynjólfur Sveinsson a few decades earlier in two different vellum copies, in *Hauksbók* and in GKS 1812, 4to. It also refers directly to Euclid's *Elements*, Book 7, Def. 2 which says:

> A number is a multitude composed of units.[141]

Roman numeration is introduced with an amusing commentary of how the Romans could have used ten letters to write any number in the present Hindu-Arabic way, explaining that this is the way many merchants express their prices.

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[139] National and University Library of Iceland: ÍB 217, 4to: 10
[140] National and University Library of Iceland: ÍB 217, 4to: 1
[141] Heath, T. L. (Ed.) (1956): Vol. 2, 277

In an example on page 6, a reference is found to Archimedes' sand-calculations to demonstrate huge numbers. On page 7, as an introduction to arithmetic, common notions are listed, some of which are a direct translation of Euclid's common notions: [142]

1. Heillt er meira sijnum Parti. (The whole is more than its part. Common notion 5).
2. Heillt er jafnt sijnum Pörtum öllum til samans. (The whole equals all its parts).
3. Hvers Parta allir jafnast, þad er sijn i milli jafnt. (That which parts all are equal, are equals).
4. Ef jafnt tilleggst Jöfnu, verdur sijn i millum jafnt. (If equals be added to equals, the wholes are equal. Common notion 2).
5. Ef jafnt er Jöfnu frátekid, verdur Jafnt eftirskilid. (If equals be subtracted from equals, the remainders are equal. Common notion 3).
6. Ef jafnt er med Jöfnu multiplicerad, framkiemur þad Jafnt. (If equals be multiplied by equals, the wholes are equal).
7. Þess sem Jafnt er, þess Helmingar, Þridiungr, Fiordungr etc. eru Jafnir. (Of equals, their halves, thirds, fourths etc. are equal).
8. Jafnrar Tölu er ferköntud Tala samjöfn. (The squares of equals are equal).
9. Ef ferköntud og cúbisk Tala eru sijn í millum jafnar, þá eru og þeirra rhætur Jafnar. (If square numbers and cubic numbers are respectively equal, then also their roots are equal).
10. So jafnast og þeirra Helmingun og tvöfölld Tala. (Thus also their halves and doubles are equal).
11. Einginn kann fyrir sig alla Tölu mæla. (No one can for himself measure all numbers).
12. Öll Tala hefur sijna Mæling fyrir Einingar. (All numbers have their measure of units).
13. Eininginn hvorki multiplicerar nie dividerar. (The unit neither multiplies nor divides).

Items 6 and 7 also have their parallels in some editions of Euclid's *Elements*.[143]

The common notions are called "Óbrigdanlegar Riettarbætur, þad er so augliósar reglur, ad hvers og eins Skilningur fær þær ad medkienna." / "Incontestable procedures, that is, so obvious rules that each one's understanding must acknowledge them."

Finally, when it comes to extraction of a square root one finds another reference to Euclid's *Elements*, i.e. Book 1, Cap. 47 [proposition 47], the Pythagorean Theorem. The theorem is applied to a delightful drawing of a castle tower, a building otherwise unknown in Iceland, surrounded by moats, so that a ladder has to be raised.

The authors and the origins of those two manuscripts are subject to the silence of long gone times, but one could make a suggestion. Magnús Arason Thorkelin, or Magnus Arethæ Torchilius, was matriculated in the University of Copenhagen in 1705. He studied mathematics and geodesy and was sent to Iceland in 1721 to do geodetic measurements after serving in the Danish and later Norwegian navy.[144] He drowned in 1728. The 18th century maps of Iceland are partly based on his measurements (see section 3.1.). Magnús Thorkelin could be the writer or the translator of *Arithmetica. Þad er Reikningslist.* Another suggestion is the Rev. Magnús Einarsson, who according to his son, Skúli Magnússon, the Treasurer of Iceland, composed an Icelandic arithmetic textbook after *Frommii Arithmetica*.[145] There are some similarities between these two books, but ÍB 217 4to is definitely not a translation of *Frommii Arithmetica*.

142 National and University Library of Iceland: ÍB 217, 4to: 7. Heath, T. (Ed.) (1956): Vol. 1, 155
143 Heath, T.L. (Ed.) (1956): Vol. 1, 223
144 Bjarni Jónsson (1949): 61
145 Skúli Magnússon (1947): 45

The manuscript ÍB 217, 4to, reveals a certain connection to the European cultural heritage, with the references to Euclid's *Elements*, a little knowledge of Archimedes' arithmetic, and an effort to bring them into Icelandic culture by translating or recomposing the text. The manuscript is clean and unworn, but not untouched by hands, at least not the *Abacus Pytagora*, the multiplication table, which clearly has been used.

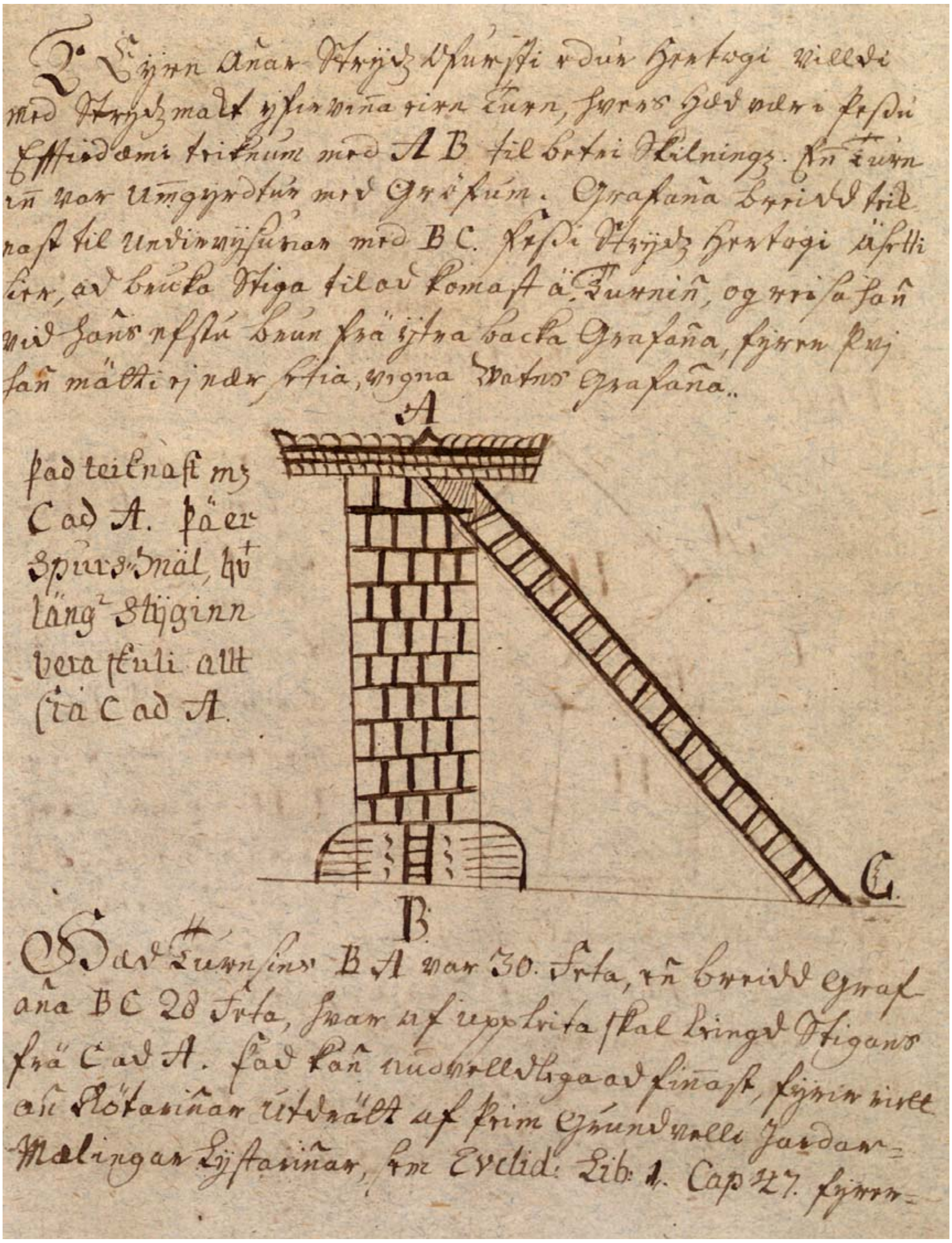

Fig. 3.4: A page from the manuscript ÍB217. 4to, *Arithmetica. Þad er Reikningslist.*

Limen Arithmeticum

The third manuscript is Lbs. 1318, 8vo, *Limen Arithmeticum edur Eynfaldlegur Inngangur til Rettelegs Nams og Brukunar Þeirrar nafn frægu Reiknings Listar / Limen Arithmeticum or Simple Introduction to Correct Learning or Use of the Renowned Computing Art.* The manuscript is 175 pages, written in 1777 as a copy of a (handwritten) book, dated 1736 and attributed to the Reverend Stefán Einarsson (1698–1754) at Laufás in Eyjafjörður. Similar to Lbs. 1694, 8vo, *Arithmetica Islandica*, it contains the traditional topics of arithmetic, the four operations in whole numbers and common fractions, units for weight and measure and monetary units. Furthermore it contains extraction of a square root and cubic root and arithmetic progressions. There is the *Regula de Tri* and its variations, including simple and compound interests, and there are tables to compute the tithe.

The Manuscripts

All the above-mentioned manuscripts deserve further investigation. Their main content is arithmetic, an effort to explain the Hindu-Arabic numeration and algorithms, while there is hardly any reference to Euclidian geometry in this 18th century heritage. The ÍB 217, 4to, *Arithmetica* (1721) seems the most theoretical one, and contains references to Euclid's *Elements* a couple of times. It does not contain any information about money and measuring units. Lbs. 1694, 8vo, *Arithmetica Islandica* (1716) and the Lbs. 1318, 8vo, *Limen Arithmeticum* (1736) are more practically oriented with measuring and monetary units and *Limen Arithmeticum* with interests and the tithe. However, they do not seem to be related. Their vocabulary is different. The arithmetic concepts addition, subtraction, multiplication and division have been translated from Latin into Icelandic, but the translations are different as is the spelling. Mathematical vocabulary was not yet standardized. The spelling and vocabulary in the 1780s were more similar to the contemporary Icelandic. In the early 20th century it reached the standards of present times.

3.3. Rise of Enlightenment

Society

The 18th century was the most difficult period in Icelandic history, while at the same time it brought a new dawn to education. In 1707 a smallpox epidemic killed a large number of people, and by the middle of the century a series of cold years caused a famine. The situation reached its worst during the so called Haze Famine, following the massive volcanic eruption of Lakagígar in 1783–1785, when toxic volcanic gases and ash poisoned the grass and killed the majority of all livestock. Even Iceland's continued existence as an inhabited country was in doubt.[146] Tens of thousands of sheep, cattle, and horses died, mostly from fluorosis.[147] From the end of 1783 to the end of 1786 the population decreased from 49,753 to 39,190 or just over 10,500, one fifth of the whole population.[148]

The Lakagígar crater poured out the greatest lava flow witnessed on Earth, at least in the last millennium; it covered 580 square kilometres and its volume totalled 12

[146] Helgi Skúli Kjartansson (1996): 81
[147] Lýður Björnsson (2006): 208
[148] Gunnar Karlsson (2000): 181

cubic kilometres.[149] In the midst of these events, earthquakes destroyed all the buildings at Skálholt except the cathedral.

Society of the Learned Arts

The Enlightenment movement had considerable influence in Iceland, although the calamities which befell the nation in the 18th century worked against the spirit of optimism which was a part of the movement. Enlightenment influence was first felt among the Icelanders near the middle of the 18th century, and strongly from the 1770s onwards. Until that decade, the Enlightenment movement was largely determined by the Danish authorities. The period from the late 1770s to the early 1790s was characterized by the activities of the Society of the Learned Arts (*Lærdómslistafélag*), established in 1779. This society of the adherents of the Enlightenment movement published a journal, called *Rit þess konunglega íslenzka Lærdómslistafélags / Writings of the Royal Icelandic Society of Arts* or *Félagsrit*, in which emphasis was placed on information and education in various practical matters.[150]

The aims of the Society of the Learned Arts were to educate the Icelanders, primarily in practical matters and secondly in sciences. A number of Icelanders and others interested in progress in Iceland – those who worked as officials abroad, Icelandic students and governmental officials in Iceland – supported the society by participating in its publishing activities, writing articles and books on various "useful" matters and by running the society itself. Among its members were Professor Thomas Bugge, Ólafur Stefánsson, later Governor of Iceland, Royal Customs Official in Skagen Ólafur Ólafsson Olavius, Stefán Björnsson, the calculator, Hannes Finnsson, later bishop and Geir Vídalín, later bishop.[151]

Public Education

The Danish priest Ludvig Harboe and Jón Þorkelsson, former headmaster at Skálholt, (1697–1759, headmaster 1728–1737), were sent by Danish authorities to Iceland in 1741. Their activities in 1741–1745 resulted in "a literacy drive, spurred by the realization that the religious education of Icelandic youth, growing up at widely scattered farms, was hard to achieve without the use of printed textbooks."[152] As a result of that effort, by the end of the century almost every boy and girl in the country was able to read. Yet there were no elementary schools and no professional teachers, only the parish priests who monitored the education provided by the home. At the same time the educational standards of the clergy were raised, an increasing number of Icelandic students attended the University of Copenhagen, and well-educated lawyers took over most of the higher administrative posts in the country.[153] Icelandic students had priority for grants at Regensen student residence in Copenhagen after it was built in 1623–1628.[154]

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[149] Þorleifur Einarsson (1996): 27
[150] Ingi Sigurðsson (1990): 293–294
[151] Ottó Björnsson (1990): 12
[152] Helgi Skúli Kjartansson (1996): 80
[153] Helgi Skúli Kjartansson (1996): 80
[154] Aðalgeir Kristjánsson (1999): 12. Helgi Þorláksson (2003): 382

Ludvig Harboe was a priest at the Castle Church in Copenhagen when he was sent to Iceland as a Visitator of the Icelandic Church. He had been given the mandate of a bishop, and he moved immediately at his arrival in 1741 to the see of Hólar, which was without a bishop at that time. He left the country in 1745 to become bishop of Trondheim, Norway, after which he became bishop of Sjælland, Denmark.[155] During his time in Iceland a series of ordinances streamed from the government about the schools, education of children, house discipline etc. In 1743 an ordinance on the Latin schools, *Tilskipun 3. maí 1743 um latínuskólana á Íslandi / Forordning om de latinske Skoler paa Island / Directive on the Latin Schools in Iceland May 3 1743*, initiated by Harboe and his assistant Jón Þorkelsson, reorganised the schools. There, for the first time, the requirements for the pupils in all subjects were written down. In arithmetic they were to know at least the four operations in whole numbers and fractions.[156] Geometry was not mentioned.

The First Printed Arithmetic Textbook in Icelandic

An ordinance on trade and prices was published in 1702. In 1746 it was printed with an addendum containing a compendium of the four operations, probably to assist the general public's understanding of their trade with the foreign merchants.

This first arithmetic textbook in Icelandic is short, 12 pages, with a long name:

Lijted Agrip Vmm þær Fioorar Species I Reiknings Konstenne, Þa undann eru geingenn Numeratio edur Talann. 1. Additio edur Tillags Talann, 2. Subtractio edur Afdraattar Talann. 3. Multiplicatio Margfiølgande Tala. 4. Divisio Skipta edur Sundurdeilingar Talann. Handa Bændum og Børnum ad komast fyrst i þa Støfun, og til mikillrar Nitsemdar ef ydka sig i því sama, sierdeilis i Kaupum og Sølum, i hvørjum Additio og Subtractio hellst brwkast. Innrettud Þad næst hefur orded komest Epter E. Hatton Reiknings Konst Edur Arithmetica. / A Little Compendium of the Four Species in the Calculation Art when Preceded by the Numeration. 1. Addition. 2. Subtraction. 3. Multiplication. 4. Division. Intended for Farmers and Children to Enter as Early as Possible that Lettering and for Great Usefulness if Training in the Same, Especially in Buying and Selling, in Which Addition and Subtraction is Most Often Used. Designed as far as has become known from E. Hatton's Computation Art or Arithmetica.[157]

As appears in the title, the author of the book is supposed to be a certain E. Hatton. It was translated by Bishop Halldór Brynjólfsson[158] of Hólar, who served in 1746–1752. Bishop Halldór Brynjólfsson was nominated by Ludvig Harboe in 1745, so the book could have been published on Harboe's initiative. The bishop's mathematical activity is not mentioned in his section in the biographical lexicon of bishops of Hólar.

In the booklet's introductory section about numeration, the place-value system is presented. "123456789 and 0 are the basis of all larger numbers". A million and larger numbers are explained.

[155] Jón Halldórsson (1911–1915): 185
[156] Janus Jónsson (1893): 38–41
[157] Hatton, E. (1746)
[158] Einar H. Guðmundsson (1995): 9. Þorvaldur Thoroddsen (1892–1904): Vol II, 276

In the section about addition, keeping tens mentally is explained. Thereafter the main bulk of the text concerns measuring units, like fishes. Five fishes = 1 farthing/quarter, 8 farthings/quarters = 1 *vætt*. The relation between fishes, skildings, marks and *ríxdalir* (rix-dollars), etc. is followed by computation examples.

In multiplication, the main topic is the distinction between the ordinary "hundred" (100) and the "large hundred", i.e. 120. The multiplication table is introduced.

In the division section, the main concepts are introduced by an example about 12: *Dividendus* 12, *Divisor* 4 and *Quotus* 3, *Remanens* 0. Thereafter, an example of the division of 37642 by 7 is explained.

The booklet is a handbook, rather than a textbook to learn from. The text and its spelling are not fluently written. It is noticeable that it is intended for farmers, presumably so that they will not be cheated in their trade with the monopoly merchants, at a time when most trade was pure barter.[159] Its main purpose was to orient people to decimal notation and the four operations in whole numbers, in addition to conversion tables of various units. The stockfish (dried fish), was still the typical export commodity at the time when the booklet was published.

The fact that this first textbook was printed as an addendum to an ordinance about trade and prices: *Tilskipan umm þann Islendska Taxta og Kauphöndlan*, suggests that the barter trade was the general public's main use for mathematics.

Mathematician Stefán Björnsson[160]

Stefán Björnsson (1721–1798) enrolled at Hólar Cathedral School in 1736, and graduated in 1744. Harboe, who at that time served as Bishop of Hólar, declared Stefán Björnsson in 1744 to be by far the best student in mathematics. Stefán Björnsson went to the University of Copenhagen in 1746 to study theology, and graduated after five months, an exceptionally short time. He had served as a deacon for some period earlier. Back in Iceland he could not immediately find employment, although Bishop Halldór Brynjólfsson was his friend. In 1750 the bishop said of Stefán Björnsson that nobody in the whole Hólar diocese was his equal at gifts and learnedness, and he doubted that in Copenhagen any of his fellow countrymen exceeded him in gifts, industriousness and courtesy.

Unfortunately Bishop Halldór Brynjólfsson had passed away when Stefán Björnsson was at last appointed headmaster at Hólar School, and Stefán Björnsson found himself at odds with his temporary successor. In 1755 Stefán Björnsson left for Copenhagen, where he lived for the following 43 years. He studied mathematics and physics, and soon started to work under the supervision of Thomas Bugge at the geodetic department of the Royal Danish Science Society, where Stefán Björnsson was appointed as calculator. It seems that he suffered from a difficult temper, in spite of the bishop's recommendations.

Stefán Björnsson wrote several scientific pieces of value in the period 1757–1794. They can be divided into the following categories:

[159] Gunnar Karlsson (2000): 138–142, 244

[160] Einar H. Guðmundsson (1995): 8–27. The content of this section draws on writings about Stefán Björnsson by Prof. Einar H. Guðmundsson.

In 1757-1760 Stefán Björnsson published several treatises or dissertations on astronomy and philosophy.

In 1780 his main work was published: *Introductio in tetragonometriam ad mentem V.C. Lambert analytice conscripta a Stephano Biörnsen mathem. et philosoph. cultore*, Havniae 1780, 454 pages, 8vo. The book concerns tetragonometry, a mathematical treatment of the geometric qualities of plane quadrilaterals, similar to how trigonometry treats plane triangles. The book seems to have been of some value. While there is no reason to infer that this work had any influence on Icelandic education, Stefán Björnsson was the first Icelander to write a book on higher mathematics and have it published.

That same year, 1780, Stefán Björnsson published in print the Icelandic arithmetic treatise *Rímbegla* in Latin translation, in addition to *Oddi's Tale* and a couple of other smaller treatises. In 1785 he published an Icelandic saga, and several more of his Latin translations of sagas exist in manuscript. Thus Stefán realized Bishop Brynjólfur Sveinsson's dream of publishing the Icelandic cultural heritage in Latin, the *lingua franca* of the learned world.

In the period 1782–1794, Stefán Björnsson, in his sixties and seventies, published six treatises on science for his fellow countrymen, in the journal of the Learned Arts Society. These articles were intended for farmers and took account of their daily environment. No evidence exists, however, for whether they had any influence on Icelandic farming. The treatises total 103 pages in 8vo, explaining basic statics and mechanics through machines, such as wheels, pulleys, wedges and scales. This material was probably taught in higher European schools at this time. However, it seems that the text is not translated from another language, i.e. that its primary version was written in Icelandic. It is difficult to assess the influence of the treatises on Icelandic education. Yet it is likely that Björn Gunnlaugsson (1788–1876), the prime mathematician of the 19th century, who was self-educated to a large part, studied these texts.

Finally, in 1792 at the age of 71, Stefán Björnsson earned *Accessit* for his treatise, when C.F. Degen earned the first gold medal in mathematics at the University of Copenhagen, and in 1793 Stefán Björnsson himself earned the gold medal when the competition was held for the second time.

While Stefán Björnsson is not known to have taught mathematics, his importance in the history of mathematics education is founded upon his being the first Icelandic mathematical scientist, who also made a serious attempt to introduce the mathematics of modern physics to his fellow countrymen. If he did influence Björn Gunnlaugsson, he may be seen as an important link in education history.

Mathematics at Hólar School

When Stefán Björnsson left Hólar School in 1755, Hálfdan Einarsson (1732–1785) took over as headmaster. Hálfdan Einarsson's biography was written by a 20th century bishop, Jón Helgason. Hálfdan Einarsson was a young man when he became headmaster, and he remained in that office until his death. His biography describes the conditions at Hólar School in the second part of the 18th century. There were two teachers, the headmaster and his assistant, the "hearer."

The school was supposed to operate from September 29 to July 1. In reality the school year was never longer than to the middle of May, and sometimes only to the end of April. For two academic years the school was closed, when there was no way to provide enough food: in 1756–1757, after pack-ice closed the nearby harbour of Hofsós for the whole summer of 1756, and in 1784–1785, when neither of the two schools was open after the first year of the Haze Famine.[161]

The studies were of three kinds: grammatical subjects, general educational subjects and specialized subjects. The grammatical subjects were mainly concerned with perfecting the Latin knowledge of the pupils. The general educational subjects were meagre; some history and geography, studied without maps, and imperfect mathematics: only the four main operations in whole numbers, and *regula de tri*. The pupils did not have to struggle with fractions unless they particularly wished to do so, and those who did wish had to do so outside class, without any instruction.[162] One would like to imagine that more mathematical studies were provided during the times of Bishop Halldór Brynjólfsson and Stefán Björnsson. This description does not cover the minimum requirements set by Harboe in the directive from 1743.[163] Probably neither of the two teachers felt inclined to teach the subject, possibly because their own mastery of it was limited, or they did not see it as important. After all, what future officials and priests needed to compute were monetary sums.

Hálfdan Einarsson was still headmaster at Hólar School when Stefán Björnsson's predecessor as headmaster, the Reverend Gunnar Pálsson (1714–1791), wrote two letters in 1780–1782 to Hálfdan Einarsson and a letter to his assistant too, about an arithmetic textbook by Ólafur Olavius.[164] That book, which could have added considerably to the mathematical education of the Hólar pupils, will be referred to later in this chapter.

Scientific Activities[165]

In the second half of the 18th century, the Danish government made an attempt to set up an astronomical observation tower in Iceland. While the attempt was only partially successful, it bears witness to various good intentions which were later realized. In 1749 Niels Horrebow was sent to Iceland to make astronomical and meteorological observations. He had an astrological tower built at Bessastaðir and did some measurements, e.g. the geographical position of Bessastaðir. Soon Horrebow turned his attention to social conditions and wrote a remarkable book, *Tilforladelige Efterretninger om Island*, published in 1752. Horrebow left in 1751.

Horrebow's book has been translated into Icelandic. It is full of sympathy for Iceland and Icelanders. The author tries his best to reject defamation that had appeared in a book on Iceland by the German Johann Anderson, mayor in Hamburg, who had never been there but collected his information from persons who had stayed there temporarily. Horrebow says about learning:

> Allur þorri þeirra, sem ekki er skólagenginn, hefur gert sér far um að læra að skrifa, svo að ég fullyrði, að bæði meðal klerkastéttarinnar og leikmanna eru fleiri skrifandi

[161] Jón Helgason [bishop] (1935): 59–60
[162] Jón Helgason [bishop] (1935): 62–63
[163] Janus Jónsson (1893): 38–41
[164] Gunnar Sveinsson (Ed.) (1984): 372–373, 387, 394–395
[165] Einar H. Guðmundsson (1988): 110–125. This section is drawn on Prof. Einar Guðmundsson's article about astronomical research in the latter part of the 18th century.

menn á Íslandi en hjá oss. … Enda þótt góðir reikningsmenn séu færri en góðir skrifarar, eru þar þó margir ósigldir menn vel færir í reikningslist, svo að vér skulum viðurkenna, að Íslendingar almennt skrifa góða rithönd og eru liprir reikningsmenn.

The majority of those who have not attended schools, have done their best to learn how to write, so I maintain that both among the clergy and laymen there are more people able to write than among us … Even though good calculators are fewer than good writers, out there though exist many people, who have not been abroad, well competent in the computing art, so that we should admit that Icelanders in general write a good hand and are agile calculators.[166]

In continuation, Horrebow maintains that it is not subject to criticism that the Icelanders use the sun and the tides to know the time. This, he says, is common in all places. Icelanders do not know the 24-hour division of the day; they have time-marks at one-and-a-half hour intervals, such as midday, midnight, mid-evening, high day, etc.[167]

In 1779 Thomas Bugge, then recently appointed professor in astronomy at the University of Copenhagen, proposed a Norwegian student, Rasmus Lievog, to be an astronomer in Iceland. Lievog was to make his observations at Lambhús, Álftanes, close to Bessastaðir, the residence of the most powerful man in the country, Governor Thodal. In 1780, Governor Thodal had the idea of building the observation tower in connection with a public school "for the common support to both institutions" as he wrote to Bishop Ludvig Harboe in Sjælland. The star-master was supposed to be schoolmaster as well, and he was to be assisted in both tasks by an Icelandic student.

Nothing came of these intentions, as the government in Denmark did not support the plan. Lievog had his tower and continued his astronomical and meteorological observations, for which he has been reported as "one of the best meteorological observers of the eighteenth century" in an article by J. A. Kington, published in the British journal *Weather* in 1972. Professor Bugge used Lievog's measurements, as well as measurements from Godthåb in Greenland, Vardø in Norway and Trankebar in India, in his articles in the writings "*Skrifterne*" of the Royal Danish Science Society. Lievog was favoured by Professor Bugge and Governor Thodal, although later in his stay in Iceland his star seems to have fallen.

Meanwhile the country suffered from the great disasters of the Haze Famine. The Skálholt School, whose buildings had been destroyed by earthquakes, was moved to Reykjavík in 1785, and located at Hólavellir. A new governor, Levetzow, suggested that the observation tower be moved to Reykjavík. There was some discussion of the possibility of appointing Lievog as teacher of mathematics and physics at Hólavellir School at the same time. Levetzow expressed his view that even if Lievog was knowledgeable, he doubted that he could teach other people properly, as he was slow and rather verbose when expected to explain something. This plan was never realized, and Lievog continued to work at Lambhús. But at the beginning of the 19th century there were still plans about appointing Lievog as teacher. As before, the plans came to nothing, and Lievog left Iceland in 1805. Hólavellir School had a poor reputation for mathematics teaching, as we shall discuss later.

166 Horrebow, N. (1966): 221
167 Horrebow, N. (1966): 222

A Primary School at Hausastaðir

The above account reveals concerns about public education as well as scientific activities. Indeed, one of the first public schools was established in this period at Hausastaðir on Álftanes in 1791. (A public school had been established in the Westman Islands in 1745 but was laid down in 1760). Headmaster Jón Þorkelsson, Ludvig Harboe's collaborator, is considered to have been among the first to bring the ideas of the Enlightenment movement to Iceland.[168] In accordance with his idealistic views on education of the common people, he bequeathed all his belongings to establish a school for poor children in Kjalarnes Deanery, the far southwest area of Iceland, including Reykjavík and Álftanes. The Thorkillii-fund was established for this endowment. The foundation became the basis for many experiments with schools during the subsequent century. The prime proponents of the Enlightenment movement at this time, Bishop Hannes Finnsson and Governor Ólafur Stefánsson, reasoned in 1791 that it would be safe to establish a school supported by the fund. For the first 13 years the school was well run by schoolmaster the Reverend Þorvaldur Böðvarsson. After 1804 troubles began. To find a schoolmaster was not easy and other difficulties, such as inflation in Denmark resulting from the Napoleonic wars, led to the closure of the school in 1812.[169] Other sources say that the establishment was more an orphanage than a school.[170]

As this school experiment was going on almost next door to Lievog in Lambhús, Álftanes, it must have been thought natural to have Lievog in mind when seeking an appropriate schoolmaster. However, it reflects some underestimation to believe that these two tasks of teaching and running the scientific measurements could be completed by the same person, who probably did not speak Icelandic.

3.4. Three Mathematics Textbooks in the 1780s

Clear Guidance by Ólafur Olavius

One of the most prolific writers of the Society of the Learned Arts was Ólafur Ólafsson Olavius (c. 1741–1788), who wrote a number of books, among them books about practical matters of agriculture, and one about arithmetic, called *Greinilig Vegleidsla til Talnalistarinnar med fiórum høfudgreinum hennar og þriggja lida Reglu skipud eptir Landsvísu og Kaupløgum Íslendínga / A Clear Guidance to the Number Art with its Four Main Operations and Regula de Tri arranged by the Country's Customs and the Icelanders' Trade Arrangements*[171], published in Copenhagen in 1780 (374 pages + foreword, xxviii pages) .

In short, the *Greinilig Vegleidsla / Clear Guidance* is a thorough textbook in arithmetic. The book is dedicated to a Mr. Schach Rathlau with a foreword, containing:

> Om Regnekunsten, denne vigtige og for Mennisket uundværlige Kundkab har man hidindtil ikke havt nogen betydelig Underretning i Landets Sprog. ... Men hvo indseer vel ikke deraf flydende skadelige Fölger, især naar der handles med fremmede Kiöbmænd og erfarne Regnemestere, som maaskee undertiden kunde og vilde vinde meer end de burde? Ja hvo skulde troe at Landet, blottet for denne Kundskab iblandt

[168] Haraldur Sigurðsson (1990): 285
[169] Sverrir Jakobsson (1994): 73
[170] Gunnar Karlsson (2000): 171
[171] Ólafur Olavius (1780)

andre, skulde kunde bestaaet saa længe, som det dog har, skiönt derhos bragt i den Form, som det nu befindes i, og hvorover man ej maae forundre sig? Jeg har da stræbt at afhielpe denne Kundskabs Mangel ved Udgaven af nærværende *Veileedning til Regnekunsten*, indrettet saavel til de latinske Skolers Brug i Island, som andres af Landets Börn, der maatte finde Lyst til at öve sig i Regning.

About the computing art, this important and for the human being indispensable knowledge one has up to now not had any instruction in the country's language. … But who cannot realize the harmful consequences thereof, especially when trade is done with foreign merchants and experienced calculators, who perhaps sometimes could and wanted to earn more than they should? Yes, who should believe that the country, deprived of this knowledge amongst others, should be able to stand up so long, as it nonetheless has, even if brought into the shape it presently has, and over which one cannot be surprised? I have tried to help out of this knowledge's shortage by the publication of the present *Guidance to the Computing Art*, arranged as well to the use of the Latin schools in Iceland as of other children of the country, who might find an urge to exercise in computing.[172]

While one can sense a sincere wish to improve education in Iceland with this book, a sentence like "... who should believe that the country, deprived of this knowledge amongst others, should be able to stand up so long, as it nonetheless has, even if brought into the shape it presently has, and over which one cannot be surprised" disturbed many a good inhabitant of Iceland. The writer next addresses the benevolent reader for 20 pages, where he informs the reader that among his models were textbooks by the Dane Chr. Cramer, adherent to the Enlightenment, and the German Christlieb von Clausberg, the author of *Demonstrative Rechenkunst* to which it bears resemblance.[173] Then this:

... þá var áhugi siá med verklíngi þessum, at sem flestir mættu hafa nokkur not af út á Islandi, og fyri því hefir eg iafnan stund á lagit, at skýra svo greiniliga frá øllu sem máttag og nauzynligt þótti edr þó nytsamligt at vita, og søkum þess verinn sva fiølmælltr vída, at meira hefir úr vordit, og þó minna tilheyriligs efnis, enn til var hugat í fyrstu, og man því rit þetta lítt skapfelligt verda þeim, er líta á stuttleikinn einn, og rita kunnu svo samslegit næsta, er eigi vyrdi hálft úr. En eg veit ekki af, er sá se nokkurr úti þar, sem kenni almenníngi nokkvat í Talnalist (þat er sem flest ønnur skipan á ólandinu!), eg fráqved lítinn hlut í skólunum, og mun þá siálfmællt, at hverr almenníngsmadr skal vera kennandi sinn siálfr, er læra vill nokkut. At skrifa þeim þá bók slíka, er á litlu edr aungu veri grein giør, og synt at eins hve atfara skyli med fám dæmum (sem bækur margar eru í útløndum, og hælast þó af nafni vel grundvalladrar Talnalistar, iá, algerrar) þat gat eg engi at mer.
... then the interest brought by this piece of work was that as many as possible could have some use of it out in Iceland, and for that reason I have always tried to explain so clearly everything as I could and is necessary or though useful to know, and for that reason I have become so verbose in many places that it has become larger, though with still fewer topics, than I planned in the beginning, so this script will be little likeable to those, who look at the shortness alone, and can write so compactly that the result is not half. But I know not of anyone out there, who teaches the general public something in the computing art (that is as most other arrangements in the non-country!), I count off a little part in the schools, and it therefore is self-said that every common person, who wants to learn something, must be his teacher himself. To write for them such a book where little or nothing is explained, and only shown how to do with few examples (such as many books are abroad and pride

[172] Ólafur Olavius (1780): vi–ix
[173] Clausberg, C. v. (1732, 1748, 1762)

themselves with a name of well founded computing art, yes complete) that I could not do.[174]

Again, the good intentions are obvious, while the phrases about the "non-country" and that every person must be his teacher himself seem to have been taken as arrogance. It has entered historical research[175] through two letters, written by ex-headmaster the Rev. Gunnar Pálsson, who does not seem to have read more than the foreword. In his letter to Headmaster Hálfdan Einarsson at Hólar Cathedral School, dated October 5, 1780 to February 20, 1781, he says that he read the foreword two or three times and is not impressed: "... *affecteran apluda, floces, tilica, colloqvium cum matre Evandri etc.* og hvilik [and what] *prudentia didactica* her sig sýnir [here shows itself] …"[176] while the main obstacle is the phrase *óland* / non-country.[177] In a later letter to the headmaster, dated April 24 – May 31, 1782, the Rev. Gunnar Pálsson recounts that he has seen the cover of the book in the house of some other person, who complained about the style and then he comments: "Þad er eingi [it is no] *prudentia* né [nor] *dexteritas didactica*, ad [to] *prostituera* sig þannig [oneself thus] *affectate*."[178] The letters are full of Latin and Danish flourishes, and as such they are in the eyes of modern Icelanders much more affected than the *Greinilig Vegleidsla / Clear Guidance,* which is written in a pure Icelandic language, still entirely readable, without any learned Latin jargon. In the foreword the author emphasized that he used only Icelandic and not Latin, as

> ... almúgalidi myndu jafnhægt í munni falla íslendsk ord sem látínsk, er þat kann varla at lesa rett, og þá miklúngi minnr beygia heyrilega.
>
> ... for the common people, Icelandic words are as easy in the mouth as Latin, which they hardly can read correctly, and then much less do the declensions audibly.[179]

This clause may have offended the Latin-learned as well as those who were not.

In the second letter to Headmaster Hálfdan Einarsson and in a letter to his assistant, both written in 1782, the Rev. Gunnar Pálsson mentioned that unfortunately *Arithmetica* by the late Rev. Stefán Einarsson had not achieved publication. This *Limen Arithmeticum* was written in 1736. The manuscript now extant, Lbs. 1318 8vo, is dated 1777, so this indicates that mathematical manuscripts were passing among people at that time.[180]

The fact that the Rev. Gunnar Pálsson's unfavourable review of the *Greinilig Vegleidsla / Clear Guidance* reached the headmaster of the Hólar School may have had a serious effect on the book's distribution. The author had moved to Skagen in Denmark at the time of its publication, so he had to rely on others for promoting his book. Olavius died in 1788 in Skagen, and may never have visited Iceland after the book's publication.

The author had pedagogical ideas on how to study mathematics:

> Eg segir of mig, at eg læri meirr af einu dæmi leystu, enn 10 óláustum, þvíat ávallt er hvar nockvat afbrugdit, og má þá betr læra af 3 edr 4 enn einu, hve fara skal annarstadar. Þá er vidvaníngum verda fengin dæmi þessi til lausnar, edr taka þeir

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[174] Ólafur Olavius (1780): xiii–xiv
[175] Loftur Guttormsson (1990): 177
[176] Gunnar Sveinsson (Ed.) (1984): 372
[177] Gunnar Sveinsson (Ed.) (1984): 372–373
[178] Gunnar Sveinsson (Ed.) (1984): 394–395
[179] Ólafur Olavius (1780): xxvi
[180] Gunnar Sveinsson (Ed.) (1984): 387, 396

seálfir id sama, og leysa rángt or, þá megu þeir siá af verknadum bókarinnar, hvar í se fólgin villa sín, og koma þá svo á retta leid tølu sinni, en þat ser æva á spurníng og svari einu. At þessu leiti kallar eg verklíng minn greiniliga Vegleidslu til Talnalistarinnar.

I say of myself, that I learn more from one solved problem than 10 unsolved ones, as there are always some differences, and then more can be learnt from 3 or 4 than from one, how elsewhere shall be done. When beginners will be handed these problems for solving, or they take themselves the same and solve wrongly, then they may see from the doings of the book, wherein their error is hidden, and thus bring their number to the right track, but it must be exercised on a question and an answer. For this reason I call my little work clear Guidance to the Number Art.[181]

Concerning the content, the book treats whole number arithmetic in 169 pages. In addition to explaining standard procedures for these operations, the author gives many examples of useful calculation tricks, such as multiplying by 100 and dividing by 4 instead of multiplying by 25, or multiplying by 8 and dividing by 1000, instead of dividing by 125. He often uses extremely large numbers which he explains as having greater effect than small ones. In fact he knows many neat examples revealed by large numbers, such as in multiplication:

998877665544332211 times 9 equals 8989898989898989899

There is a section about ratio and proportions, and arithmetic and geometric progressions are mentioned, although due to constraints of space they are not treated fully. In the section about fractions, numbering 112 pages, the greatest common divisor is found by Euclid's algorithm in a somewhat advanced version. The final chapter on *regula de tri* numbers 84 pages.

On the whole, the book is a good introduction to arithmetic, and much of its content could still be of use today as an aid to mental arithmetic. The good intentions to explain and be of assistance seem to shine from the pages, even if the author may have managed to offend his intended readers.

Vasa-qver **1782**

In 1782 a learned man, Jón Jónsson Johnsonius (1749–1826), published a pocket-book with tables, handy for the exchange of goods. It was called *Vasa-qver fyrir bændur og einfalldlínga á Islandi : edr ein audvelld Reiknings-List, hvarí finzt Allskonar Utreikningr á upphæd og verdaurum í kaupum og sølum, bædi eptir innlenzku og útlenzku verdlagi : Einnig Utdráttr af hinni Konúngl. Islenzku Kaup-Taxta og Brefburdar-Tilskipun* [182] / *A Pocket-Book for Farmers and Simpleminded People or an Easy Computing Art, Wherein is Found All Kinds of Computations of Sums and Prices in Purchases and Sales , Both by Domestic and Foreign Price Levels : Furthermore, an Extract of the Royal Icelandic Ordinance on Buying Rates and Mailings* (249 pages).

In the foreword of the *Vasa-qver* the author says that even if farmers and the bourgeoisie in Denmark are much better informed in the computing art than the common people in Iceland, a number-booklet such as this one has been published in Copenhagen long ago, and many times. They are better informed, as there they at least have some experience of it in the primary schools. And even if Iceland has already had printed *Number Art* [*The Clear Guidance to the Computing Art*], this present

[181] Ólafur Olavius (1780): xv
[182] Jón Jónsson (Johnsonius) (1782)

booklet might nonetheless be of good use to many, as the other, even if good and well composed, might not be of good use for the common people. It was mainly intended for young people who would have the will and opportunity to study such things.[183]

Content of the *Vasa-qver*: Firstly, there are multiplication tables up to 100 times 100 on 25 pages, then multiplication of the present currency on 102 pages, and interest-tables on 25 pages. The remainder of the book contains mainly exchange tables between different currencies and scales, and an extract of the charge-table published as a royal ordinance, dated in 1776, about the Danish trade monopoly which had been in force since 1602, and was abolished in 1786–1788. These were basically Danish measures, in addition to the ancient *landaurar*, the *hundrað*/hundred equivalent to 120 *alin*/ells (an ell was approx. 60 cm of woollen cloth), 240 fishes or one cow.[184]

The *Vasa-qver* can therefore not be considered as a mathematics textbook but a collection of tables. Its contents are in most respects similar to the first textbook, published in 1746, *Lijted Agrip*. Again the "fish" trading unit is related to other units, such as rix-dollars and skildings, called the *Cúrant*. The *Cúrant*-coins comprised *Cronas* and *Specias*. Also included is a table of the *tíund* or tithe, which was computed as a 1% property tax, 1 sheep of every 100 sheep, and 1 cow of every 100 cows.

Possibly this book had more influence on the mathematics education of the public than the two good textbooks published by the proponents of the Enlightenment in that same decade.

Society

The foreword of the *Vasa-qver* reveals that Icelanders were becoming aware that they were lagging behind, especially behind the Danes, in public education. As stated before, the school at Hausastaðir, Álftanes, was to be established a decade later, in 1791. The bourgeoisie was also about to be born. Reykjavík was granted its town charter in 1786, and by 1791 twenty persons had formally entered the class of bourgeoisie, which had right to pursue commerce and (small scale) industry. This modernization was connected to the abolition of the trade monopoly in 1786–1788. The Danish King had run the Iceland trade on his own account in the period 1774–1787. Because of the Haze Famine the trade was run at a great deficit, a fact that contributed to the abolition of the monopoly trade, but also a plan on behalf of the government of organized urbanization of the country.[185]

[183] Jón Jónsson (Johnsonius) (1782): 5–7

[184] *Hagskinna* (1997): Measures, weights and currency: A hundred = 20 *aurar* (pennies) = 120 *álnir* [ells, originally a measure of woollen cloth]. In terms of fish-value, one hundred = 6 *vættir* = 240 (valid) fishes. This Icelandic currency system existed from medieval times up to the 20th century, called *landaurar* (land-pennies). A hundred was the equivalent of a cow, i.e. a middle-aged, faultless cow in spring, or six sheep, woolly and carrying lambs, in spring. The monetary value of *landaurar* was variable, and up to and beyond the 18th century there were differences of opinion on how to compute it. … Farms were also measured by hundreds. An average farm was valued at 20 hundreds, and it was supposed to support livestock of 20 cows or 120 sheep.

[185] Gunnar Karlsson (2000): 182

Ólafs Arithmetík

In 1785, only five years after the publication of the *Greinilig Vegleidsla* / *Clear Guidance*, Ólafur Stefánsson (Stephensen) (1731–1812), later governor, published a textbook, called *Stutt Undirvísun í Reikníngslistinni og Algebra. Samantekin og útgefin handa Skóla-lærisveinum og ødrum ýnglíngum á Íslandi*[186] / *A Short Teaching on the Computing Art and Algebra. Collected and Published for School-Pupils and Other Youths in Iceland* (248 pages), often called *Ólafs Arithmetík*. This book had some influence on Icelandic mathematics education, as it was required reading in the Latin schools.[187] Unfortunately, the schools were at their nadir at that time, as we shall later see.

Before the foreword the author addresses the Direction for the Royal Greenlandic, Icelandic, Finnmarkish and Faroese Trade thus:

> Om Indbyggerne selv havde været Allernaadigst overladt at vælge, skulde de intet heller have tilønsket sig, i denne Henseende: end at Landets Handel maatte betroes til saa oplyste, saa retsindige, og for dets Velstand og Opkomst saa omhyggelige Herrers Bestyrelse.
>
> If the inhabitants themselves had most graciously been left to choose, they would have wished themselves nothing else in this respect: that the country's trade would be entrusted to such enlightened, so righteous and so much for its affluence and prosperity meticulous gentlemen's management.[188]

It is hard to credit the sincerity of these words, considering the situation of trade monopoly, which was abolished in the following year, but this was probably the style of the period. It seems that the author was aware of the forthcoming abolition, and a little later he says:

> Alt hvad jeg haver troet at kunne være mine Landsmænd, og især den opvoxende Ungdom, til Tieneste med, der kunde hielpe til at give dem noget Slags Kundskab og Beqvemhed, enten til selv, i sin Tid, at kunne tage nogen Deel i Handelen, eller som Betiente at befordres ved samme; det har været at overgive dem denne af mig sammenskrevne liden Regnebog, som jeg dog haaber vil medföre nogen Nytte.
>
> Everything that I have believed can be of service to my fellow countrymen and in particular the up-growing youth; which could help to give them some kind of knowledge and convenience, either to be able to themselves, in due time, to participate to some degree in the trade, or as servants to further the same; it has been to hand over to them this by myself composed little arithmetic book, which I however hope will bring some use.[189]

For this purpose he dedicated the book to these honourable Excellencies and good gentlemen.

In his address to the reader the author wrote in Icelandic. He informed the reader that he began to write the book in 1758, soon after his years in Copenhagen, where he had gathered some knowledge of mathematics or measuring.

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[186] Ólafur Stefánsson (1785)
[187] *Lovsamling for Island* 5. 1784–1791 (1855): 244. *Alþingisbækur Íslands* 1781–1790 (1986): 327
[188] Ólafur Stefánsson (1785): *3
[189] Ólafur Stefánsson (1785): *4

The main part of the *Stutt Undirvísun / Short Teaching* treats the same content as does the *Greinilig Vegleidsla / Clear Guidance,* in fewer pages. Whole number arithmetic takes 82 pages and fractions 24 pages. Decimal fractions are new vis-à-vis the *Clear Guidance* and progressions / sequences are treated more fully. *Regula de tri* has its place, as have extractions of square root and cubic root. The last forty pages are devoted to an introduction to algebra and first and second degree equations, which the author intends for the school pupils.

In his introduction the author says:

> ... ei er ritlíngr þessi samantekinn í þeirri meiníngu, at ei seu þeir margir her á landi, er vel kunni at reikna, og geti án hanns kennt þat ödrum út af ser, einkum af embættis-mönnum andligrar og veralldligrar stettar, helldr er hann ætladr þeim úngu og uppvaxandi til nota ...
>
> ... this booklet is not composed in the understanding that there are not many in this country that well can compute, and can, without it, teach it to others, especially officials of the religious and secular classes; rather it is intended for the young and up-growing to use ...[190]

This may be a sentence guarding the author against the kind of criticism that Ólafur Olavius probably received for his comment that he didn't "know anyone out there, who teaches the general public something in the computing art". Whatever the discussions were, the fact that the book was introduced as a required textbook for the Latin schools ensured some distribution of the book. As Assistant Governor, Ólafur Stefánsson was in better position to have his book adopted for use than his rival, who lived in Skagen, Denmark.

All things considered, the book *Short Teaching* is clear, written in proper Icelandic, and somehow the layout is more airy than that of the *Clear Guidance*. However, as emerges from this short overview, the *Short Teaching* is more compact than the *Clear Guidance*, and the *Clear Guidance* offers more variety of explanations and illustrative examples.

Distribution of the 18th Century Textbooks

Remarkable as it was to have a choice of two good printed arithmetic textbooks in the last decades of the 18th century, no proper textbook was published again until 1841. The two books and the *Vasa-qver* were therefore the basis for mathematical knowledge for over half a century, at least for those who did not attend a learned school.

A survey exists of inventories of books in seven out of nine benefices in the county of Austur–Húnavatnssýsla in northwest Iceland from the first three decades of the 19th century.[191] The sources were on one hand annual church censuses, and on the other hand probate records and records of administration of estates at death, available at the National Archives of Iceland. The inventories were not all compiled in the same year; it differs from place to place in which year the best list was taken. The first one was made in 1809, another in 1823, while the other five were made in 1826–1830. Books were counted on 159 farms, a total of 2,490 religious books; the average number on each farm being 16, not counting four larger libraries. Secular books were not counted on the regular farms, only when the estate of a deceased person was evaluated. For the

[190] Ólafur Stefánsson (1785): Til Lesarans
[191] Sólrún B. Jensdóttir (1969)

larger libraries complete, but not all equally accurate, lists exist. From them and from estates of deceased persons one can deduce that secular books comprised about one-eighth of the total number of books, approximately 350 books, which gives an estimated total of 2,840 in addition to 562 books in private libraries, a grand total of about 3,400 books.[192]

In 129 estates of deceased persons in the period 1800–1830, there were a total of 8 arithmetic textbooks out of 189 secular books. In the four private libraries there were 6 arithmetic books out of 562 total books and an estimated 70 secular books.

Out of the approximately 260 secular books found in estates of deceased persons and in private libraries, four copies were found of the *Vasa-qver*, three copies of the *Clear Guidance* and seven copies of the *Short Teaching*. All these books were at that time between 25 and 50 years old. The *Short Teaching* was found in three out of the four private collections, all owned by learned persons. Of the three copies of the *Clear Guidance* two existed in private collections.

One may estimate approximately 160 secular books not counted at the regular homes and not included in the estates of deceased persons, so one could expect 5–7 arithmetic books not counted.

Considering that the *Short Teaching* had been a required textbook for the schools it had remarkably little more distribution than the *Clear Guidance*. The *Short Teaching* might though have worn out early, as each copy may have had many users in the schools. Another aspect to consider is that, while people treated their books carefully, Icelandic farmhouses were not well suited for conserving book collections for many decades, in the badly heated buildings made of turf and stone, where things quickly moulded and rotted.

3.5. Aspects of Mathematics Education

This section concludes the history of mathematics education in the times when schools were located at the two sees at Hólar in northern Iceland and Skálholt in the south. Reflecting on the status and role of mathematics in Iceland during the first eight centuries of Christianity and European culture, it is useful to recall the fundamental reasons for mathematics education as identified by M. Niss, and consider whether it contributed to the technological and socio-economic development of society or its political, ideological and cultural maintenance and development, or provided individuals with prerequisites to help them to cope with life in various spheres in which they lived: education or occupation; private life; social life; life as a citizen.[193]

During the first few centuries it is fair to conclude that interest in mathematics was rooted in the need for calendar and ecclesiastical calculations, in addition to computing the tithe, and its use in the barter trade. The work of Þorsteinn Surtur and Stjörnu–Oddi, containing a certain degree of originality, may be thought of as contributing to the socio-economic development of society, since for example Þorsteinn Surtur worked on specifying the accurate dates of the summer solstice, in order to date the meetings of the *Alþingi* correctly in the vulnerable period of a summer so short for agriculture. Stjörnu-Oddi's work was e.g. of assistance to seafarers, while it also witnessed some inclination to value mathematical patterns for its own merits, thus contributing to cultural maintenance and development.

[192] Sólrún B. Jensdóttir (1969): 164
[193] Niss, M. (1996): 13

The ecclesiastical calculations may be understood as contributing to society's cultural maintenance, not of mathematics itself as a cultural value, but of the cultural values of the Church. As a result, individuals preparing for the religious profession had to be provided with certain required mathematical prerequisites.

The cultural values of mathematics itself do not stand out in Icelandic cultural history. Several encyclopaedic manuscripts contained fragments of classical mathematical knowledge, which the owners may have collected for the sake of search for knowledge only. There is not much evidence that for example Euclid's *Elements* were known or read in Iceland during this period. The exceptions are a book in the possession of Bishop Brynjólfur Sveinsson, who had been a schoolmaster in Denmark, and a few references in an undated manuscript, ÍB 217, 4to, by an unknown author. These references are confined to the arithmetic part of the *Elements* and the Pythagorean Theorem. People seem to have been interested mainly in arithmetic, probably for its practical purposes. This is e.g. reflected in the other two early 18th century manuscripts and the textbooks of the 1780s.

Another practical aspect was land-surveying and map-making, which could be measured by European standards in the late 16th century, thanks to Bishop Guðbrandur Þorláksson.

The interests of the proponents of the Enlightenment movement, Ólafur Olavius and Ólafur Stefánsson, can be read in their forewords to their arithmetic books. They both state clearly that their intentions are to provide individuals with prerequisites to cope with trade and skilled merchants in addition to school practices, while their main aim as members of the Society of the Learned Arts was to contribute to the technological and socio-economic development of Icelandic society.

As there were no mathematical requirements for students entering the University of Copenhagen, the learned schools did not have to take that aspect into consideration until the 19th century. There are not many accounts on which to base any conclusions about mathematics teaching at the learned schools at Hólar, Skálholt and later Hólavellir. The meagre evidence to be found, reflected in the biography of Headmaster Hálfdan Einarsson, and later in the memoirs of the Reverend Árni Helgason from Hólavellir School, supports the conclusion that there was little emphasis on mathematical education and skills in order to provide individuals with prerequisites to cope with further education and occupation.

The early printing of handbooks for farmers with conversion tables of trading units, the *Lijted Agrip* / *Little Compendium Intended for Farmers* (1746) and *Vasa-qver / A Pocket-Book for Farmers and Simpleminded People or an Easy Computing Art* (1782) reveal a need for handbooks as an aid in trade. These books may therefore be taken as a contribution to the technological and socio-economic development of society. They may have served more people than the comprehensive arithmetic textbooks, good though they were in themselves.

4. The 19th Century

4.1. Learned Schools

Society

By the beginning of the 19th century more modern judicial and ecclesiastical institutions in Reykjavík had replaced both the Skálholt and Hólar episcopal sees and the *Alþingi*, which by that time served only as a court of law.[194] Skálholt See was transferred to Reykjavík after the earthquakes in 1784. In 1805 the Governor's residence was moved to Reykjavík from Bessastaðir on Álftanes. Reykjavík thus became a centre of government at the beginning of the 19th century and acquired the position of a capital town. At the end of the 18th century the population of Iceland numbered only 47,000, and was almost exclusively rural. The number of inhabitants in Reykjavík was 307 in 1801.[195] In the first half of the 19th century population growth was 0.5% per year.

Fig. 4.1. Reykjavík 1820.

A Learned School at Hólavellir

The Skálholt School was moved to Hólavellir in Reykjavík in 1786, two years after the earthquakes. The Hólar School was closed down in 1802 and united with the Hólavellir School. The idea of the Hólavellir School was to establish a school in a modern style where the teachers would receive their salaries in cash, and the pupils would receive their alms in cash from the King's funds as well.

[194] Helgi Skúli Kjartansson (1996): 81

[195] *Hagskinna* (1997): Table 2.3. Population of parishes, towns and counties 1769–1990

The experiment proved a disaster. The Hólavellir school building was poor, as was the teaching. When the sees at Skálholt and Hólar were abolished, an agreement was made that a certain amount of money should be allocated to cover the costs of the new see and a school for 40 pupils, 24 from Skálholt and 16 from Hólar. The money declined in value during the Danish inflation years in the early 19th century. The see, which had previously had huge assets, went bankrupt and in 1804 the pupils of the Hólavellir School could not even be adequately fed. They were sent home and the school was closed down. For a year there was no learned school in the country, while in 1805 it was re-established at Bessastaðir on Álftanes, the former residence of the governor. The number of pupils fell to 24 in 1805.[196]

Governor Ólafur Stefánsson's book *Stutt undirvísun / Short Teaching* was used as a textbook in mathematics at Hólavellir School. In his memoirs from his school years, the Reverend Árni Helgason said:

> … kennsla var þar [í Skálholti, þegar faðir hans gekk í skóla kringum 1770] rígbundin og allt miðaði til þess, að gera lærisveina vel að sér í latínu og biblíufróða.
>
> … Í Reykjavíkurskóla var engu breytt viðvíkjandi því, sem átti að lesa í latínu og grísku. … Boðið var að piltar skyldu læra Geographia og Arithmetica, en hvorugt var þar kennt í minni tíð, engin danska, engin íslenzka, en okkur bara sagt, að við ættum að læra þetta, og það gekk þá upp og niður. … öllum sem komust í efra bekk var gefin stiptamtmanns Ólafs Arithmetík, en það var í sjálfra piltanna valdi, hvort þeir luku upp bókinni nokkurntíma eða aldrei.
>
> Teaching there [at Skálholt, when his father attended school around 1770] was rigid and everything aimed at educating the pupils in Latin and the Bible.
>
> … In the Reykjavík School nothing was altered concerning what was prescribed in Latin and Greek. … It was required that the pupils should study geography and arithmetic, but none of those were taught at my time, no Danish, no Icelandic, we were told that we should just learn this, and the results were variable. … Everyone who reached the upper grade was given Governor Ólafur's Arithmetic, but it was up to the pupils whether they ever opened the book or not.[197]

The new school at Hólavellir, like the old cathedral schools, had the primary aim of preparing the pupils for priesthood, so Latin and Greek were the most important subjects. Probably not many teachers knew mathematics either. As mentioned before, there had been plans to appoint star-master Lievog as mathematics teacher, plans which came to nothing.

The Bessastaðir School 1805–1846

By the implementation of the Royal Directorate of the University and the Learned Schools / *Den Kongelige Direction for Universitetet og de lærde Skoler* the learned schools in Denmark were transformed in 1809 from theological seminaries to schools offering general preparatory education for officials. This was not entirely the case with the Icelandic school, as no further education was available in the country, so a final examination from the school was sufficient qualification for the priesthood.[198] Certainly a number of the school's graduates went abroad to study at the University of Copenhagen, where Icelandic students had priority for grants at Regensen student residence.

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[196] Nanna Ólafsdóttir (1961): 67–93
[197] Árni Helgason (1907–1915): 85–86
[198] Hjalti Hugason (1983)

However, the majority of clergymen in Iceland were only learned-school graduates, or even graduates from private instruction without further education.[199] It was only after the Learned School was transferred to Reykjavík in 1846 that a theological seminary was established as a special institution in 1847.

The two learned schools, which in 1802 had become one, were arranged so that the pupils stayed there generally for six years, three years in the lower grade and three years in the upper grade. During the Haze Famine and its aftermath there had been no school in Iceland in 1784–1785, no school in southern Iceland in 1785–1786 (as the Skálholt School was destroyed by earthquakes), and the Hólavellir School not yet established, and again no school in the country in 1804–1805. It was therefore not at all unusual for pupils to be privately educated at home or by clergymen for part of or all their training time.

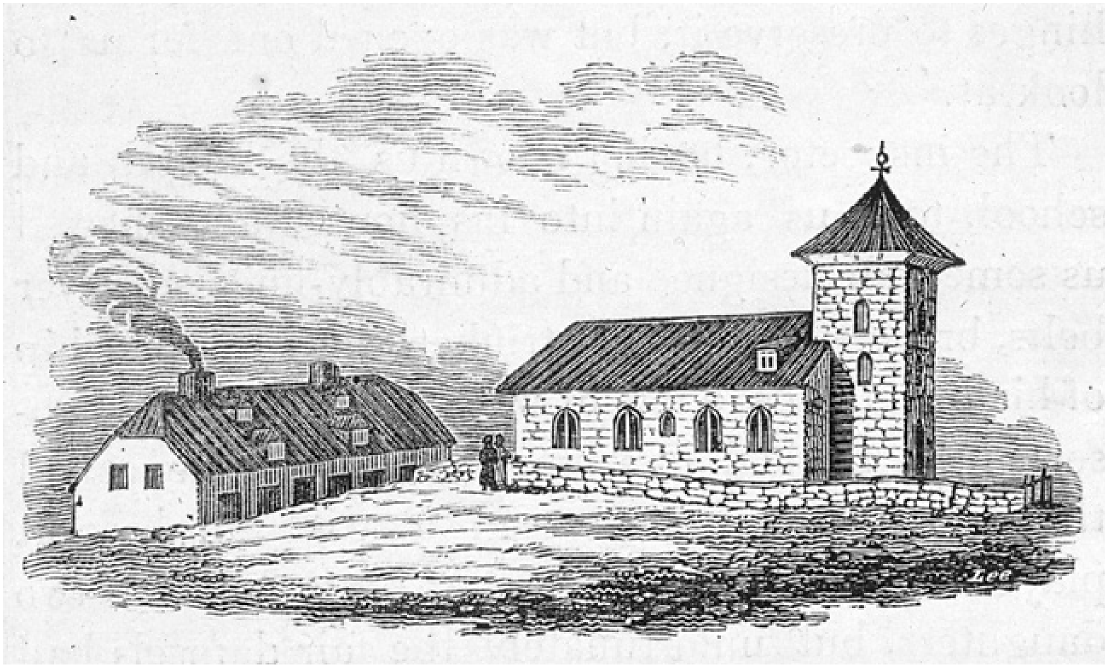

Fig. 4.2. Bessastaðir in 1834. The school building (background) dates from 1761-6 and is built of stone; the church, also of stone, was begun before 1780 and taken into use in 1795, and the tower was completed in 1822–3.

Complaints about Mathematics Teaching in Bessastaðir School

On November 7th 1822 two professors at the University of Copenhagen, E. G. F. Thune and F. Petersen, wrote to the Royal Directorate of the University and the Learned Schools and complained that the dimittends from Iceland did not fulfil the requirements in mathematics for the *artium* examination and that they had also had less Greek and Latin than prescribed by regulations dated August 10, 1818.[200]

Headmaster Jón Jónsson at Bessastaðir School was asked to respond to these complaints. He recalled that the Icelandic students had been exempted from mathematics before the academic year 1822–23, when no algebra and no geometry had been taught at the school. However, the dimittends after 1823 would have no valid reason for exemption from examination in arithmetic and geometry. Bishop

[199] Hjalti Hugason (1983): 10
[200] National Archives of Iceland: Skjalasafn kirkjustjórnarráðsins SK/4 (örk 23)

Steingrímur Jónsson supported the headmaster's case. The two letters are kept in the National Archives in Reykjavík.

One can imagine the problems in teaching, both in mathematics and the ancient languages, when each of the two grades were repeated for three years, in addition to the fact that a number of pupils came into the grades with home-education, which could be on different levels in the various subjects. In 1822 a new mathematics teacher was appointed at Bessastaðir School: Björn Gunnlaugsson, who had studied mathematics at the University of Copenhagen.

4.2. Björn Gunnlaugsson

Björn Gunnlaugsson's Life and Work

Björn Gunnlaugsson was born in 1788. His father had taught him mathematics and geodesy. Bishop Geir Vídalín wrote about the father, Gunnlaugur Magnússon (1747–1821), that he

> audveldlega forþenadi ad nefnast Eyiu vorrar Archimedes, ef hann hefdi ei vantad Efni og Konstar sinnar rettu reglur;
>
> easily deserved to be called our island's Archimedes, if he had not been short of means and the correct rules of his art;[201]

Gunnlaugur Magnússon earned several prizes from the King for invention of utilities, such as a loom and a harpoon, presumably on the initiative of the Society of the Learned Arts.[202] This suggests that Gunnlaugur Magnússon was an avid reader of the publications of the Learned Arts Society. A list is in existence of books in his home in 1796, registered by the parish priest. A total of 28 religious books are registered, and the 29th is *Bernhardi Indledning til Klogskaben*, a Danish book, in addition to "several other useful books", of which most were in Danish.[203] The priests usually did not register secular books, so books and journals from the Society of the Learned Arts could be among the "other useful" books. At least it is clear that Gunnlaugur Magnússon read Danish, so he was able to gather knowledge from European sources, and that his home had more books than the general public of his class, a poor tenant as he was.

When Björn Gunnlaugsson was supposed to go to school in 1804, the Hólavellir School had been closed down. When he applied for entrance to the Bessastaðir Learned School in 1805 his application came too late, and finally, when he applied in 1806 and 1807, he was too old. Other sources say that he was too poor. He therefore acquired some private instruction and was graduated in 1808 by Bishop Geir Vídalín, who had studied some mathematics at the University of Copenhagen.[204]

[201] P. + B. [Páll Melsteð and Björn Jónsson] (1947): 77
[202] Ottó J. Björnsson (1990): 8–13
[203] Ottó J. Björnsson (1990): 42–47
[204] Ottó J. Björnsson (1997): 3–4

In his testimonial about Björn Gunnlaugsson's final examination the bishop said that he had, by independent study, not only been through both the computing arts (probably arithmetic and algebra) but also geometry, trigonometry, stereometry, finite and infinite calculus, statics, mechanics and more parts of the natural measuring art, and for this he deserved the strongest praise (*Egregia Laude*). By this well-based knowledge he was able to relieve some of his father's labour.[205]

Björn Gunnlaugsson's activities in 1808–1817 are not known, except that he wanted to become a priest but was advised against it, as he was considered absent-minded about everything except geodesy. He is known to have become acquainted with lieutenants Scheel and Frisach, who were working on a survey of Iceland's coast in the service of the Danish military. They instructed him and gave him books on that subject.[206]

Finally in 1817, at the age of 29 years, Björn Gunnlaugsson went to Copenhagen to study at the University. He arrived too late for the autumn semester, so he used his time to solve the University's prize problem in mathematics, for which he received the gold medal in February 1818. The problem concerned deciding the attraction of the mass of a cylinder on a point on its axis or outside it. Björn Gunnlaugsson completed the Second Learning Examination (*Den anden lærdomsprøve*) in 1818, and continued to study higher mathematics and geodesy. In 1819 he again attempted the University's prize problem, to win the gold medal for the second time in 1820. Björn Gunnlaugsson's accomplishments are probably the finest examples of the results of the 18th century's restoration of education, in particular the efforts of the Society of the Learned Arts.

It is not known what Björn Gunnlaugsson studied for the next couple of years, except that he studied geodesy with the astronomer H.C. Schumacher and worked with him on surveying in Holstein in 1820 and 1821. At this time it was not possible to graduate in mathematics from the University of Copenhagen, so he would have had to graduate in an unrelated subject such as law or theology. He therefore chose to write to the education authorities in Denmark in April 1822. In his letter Björn Gunnlaugsson explained the importance of mathematical studies for school pupils, and the lack of mathematical tradition and education in Iceland. He suggested that a teaching position in mathematics be established at Bessastaðir School, and offered to take over the task.[207] Björn Gunnlaugsson was appointed to the position, as soon as May 14, 1822.[208] He sailed to Iceland shortly thereafter and began to work in the autumn of 1822, at the school where his applications for entrance had been rejected three times.[209]

On the first day of school in the autumn 1822 or 1823 Björn Gunnlaugsson gave a speech declaring his view of the role of mathematics education,[210] to which he most likely adhered throughout his teaching career. There he said:

> Til þess að geta lifað, og lifað þægilegu lífi, verðum vér að nota þau gæði sem guð hefur oss í náttúrunni fyrirbúið, til að nota náttúrunnar gæði verðum vér að þekkja hennar gang; til að geta þekkt hennar gang verðum vér eða að minnsta kosti nokkrir

[205] P. + B. [Páll Melsteð and Björn Jónsson] (1947): 77–78
[206] P. + B. [Páll Melsteð and Björn Jónsson] (1947): 66
[207] Ottó J. Björnsson (1997): 3–15, 18–19
[208] National and University Library of Iceland: Lbs. 609, fol
[209] Einar H. Guðmundsson (2003): 4–5
[210] Reynir Axelsson (1993): 52–53

af oss að rannsaka hann; til að rannsaka hann verðum vér að reikna hann út oft og tíðum með *mathesi applicata*; til að reikna með *mathesi applicata* verðum vér að þekkja *mathesin puram* og það til hlítar; og til þess að þekkja hana að gagni verðum vér að kynna oss öll veltingabrögð hennar að so miklu leyti sem oss er mögulegt; og höfum vér ekki allir tækifæri og tómstundir til þess, þá verðum vér að senda nokkra njósnarmenn út sem gjöri það fyrir oss. Sérhvör þjóð ætti því að hafa sína *mathematicos* til að senda þá út í náttúruna sem njósnarmenn á undan sér til að rannsaka hennar leyndardóma og sem vísi síðan þjóðinni á eftir hvört hún leita skuli til að finna þau gæði sem í henni eru fólgin.

In order to be able to live, and live comfortably, we have to utilize the resources which God has in nature prepared for us, in order to use the resources of nature we have to know its evolution; in order to know its evolution we, or least some of us, have to research it, in order to research it we have to calculate it, often with *mathesi applicata*; to calculate with *mathesi applicata* we have to know *mathesin puram* and that thoroughly; and in order to know it properly we have to investigate all its tricks to the degree that we possibly can; and if not all of us have the opportunity and leisure time for that, then we have to send out some scouts who do that for us. Every nation should therefore have its *mathematicos* to send them out into nature to research its mysteries and who then point out to the nation where it should search to find the resources which are hidden in it.[211]

Björn Gunnlaugsson continued to explain the use Archimedes had made of his knowledge in wartime, and for those who were not interested in warfare he mentioned a number of machines based on mathematical knowledge which could be useful for farmers, in addition to mechanics, optics, astronomy and architecture where physics and mathematics assisted each other. Then Björn Gunnlaugsson counted the various amusements people could gain from mathematics and its elegance. Finally he argued how mathematics could train people in logical thinking, as nowhere else was the truth as easy to research and easily distinguished from falsehood, even if it was not always obvious, and because its knowledge existed in abundance, as shallow or as deep as one wished to dive into it.[212]

In spite of this elegant and motivational speech, 19th century would not see any one of Björn Gunnlaugsson's caliber in mathematical learning in Iceland.

Björn Gunnlaugsson's address indicates that he considered it the goal of his teaching that the nation would be able to harness nature's resources, in addition to the official reason given for teaching mathematics, which was to ensure the admittance of Icelandic students to the University of Copenhagen. One can therefore identify, in early 19th-century Iceland, two of the fundamental reasons for mathematics education, stated by M. Niss, i.e. to provide the students with prerequisites for further studies and their private life, and to contribute to the technological and socio-economic development of society.

Björn Gunnlaugsson continued to teach at the learned schools at Bessastaðir and later Reykjavík for 40 years, until 1862. In the summers of 1831–1843, apart from 1836, he travelled throughout the country to make geodetic surveys as a basis for a new, scientifically drawn map of the country. A map of Iceland, drawn according to Björn Gunnlaugsson's survey, was printed and published by the Icelandic Literary Society in 1844 and 1849. This was a great feat, which alone suffices to uphold his name in Icelandic history.

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[211] Björn Gunnlaugsson (1993): 57
[212] Björn Gunnlaugsson (1993): 57–66

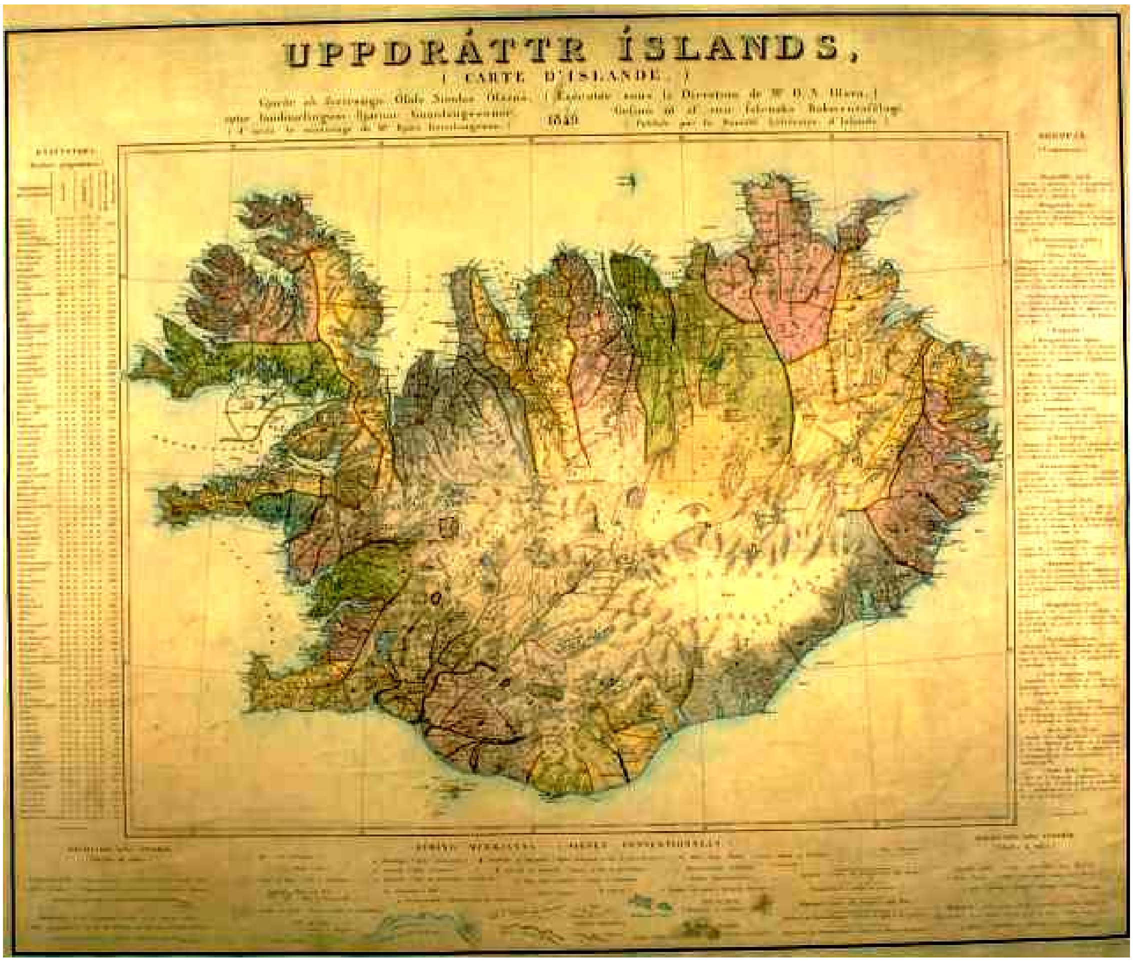

Fig. 4.3. Björn Gunnlaugsson's map of Iceland, published in 1844.[213]

For the 50 years from 1844 most maps of Iceland were more-or-less based on Björn Gunnlaugsson's map. It was not abandoned completely until after World War II, when new maps based on new surveys were published.[214]

Various writings of his exist, many of them published in connection with school reports and school invitations, concerning astronomy and geodesy, in addition to a number of articles in the current journals on the same subjects and the geography of Iceland. Björn Gunnlaugsson was best known in his time for the map and for his great philosophical poem *Njóla* where he combined his sincere faith in God, built on his thorough knowledge of the *Bible* and theological writings, and his philosophical thoughts, based on Kant's theories and on his astronomical knowledge.[215]

After Björn Gunnlaugsson retired, the Icelandic Literary Society published his book, *Tölvísi / Number Wisdom*, an extensive piece of work in Icelandic. The book was written on the encouragement and funding of the Literary Society, which published the first volume, but the second volume was never published. A contemporary source says that *Tölvísi* is a book that everyone praises but no one reads.[216]

213 National and University Library of Iceland: website
214 Haraldur Sigurðsson (1982a): 13
215 Einar H. Guðmundsson (2003): 9–78
216 P. + B. [Páll Melsteð and Björn Jónsson] (1947): 70

Björn Gunnlaugsson's Mathematics Teaching

In an account of the school year 1820–1821 Headmaster Jón Jónsson tells that he gave exercises in counting and the four operations in whole numbers, called "species", to the pupils in both grades. Some pupils also had exercises in common fractions, decimal fractions and *regula de tri.* His account in 1822 is similar.

In 1823 Björn Gunnlaugsson gave a report of his first school year. He divided the lower grade into two groups, *novis* and *veteranis*. The *novis* had the four species in whole numbers, while the *veteranis* had fractions and *regula de tri*. In the second semester he moved to fractions with the *novis* and algebra with the *veteranis*. In the upper grade Björn Gunnlaugsson repeated arithmetic and moved on to quadratic equations in the first semester and taught geometry in the second semester, to introduce finally, after Easter, some interesting geometrical problems in practical geodesy "in order to arouse the pupils' interest in the unknown mathematics". Next year Björn Gunnlaugsson taught only geometry in the upper grade, using a geometry textbook by the Danish author Hans Outzen Bjørn, and arithmetic in the lower grade.

According to the account for 1824–1825, Björn Gunnlaugsson taught the four species in whole numbers and fractions, decimal fractions, algebra and extraction of quadratic and cubic roots in the lower grade, following the arithmetic textbook by Bjørn. In the upper grade Björn Gunnlaugsson had to teach decimal fractions, algebra and extraction of roots.

> Dette var jeg nødsaget til at læse, uagted det også læses i nederste Classe, da nogle nykomne Discipler ikke have hørt dette, fordi Mathematik ikke er endnu blevet almindelig i Privatskoler, men disse Discipler havde lært saa megen Latin, Græsk og Dansk at de maatte sættes enddog øverst i den øverste Classe.
>
> This I was obliged to teach, even if it also was taught in the lower grade, as some newly arrived pupils had not heard of this as mathematics has not yet become common in the private schools, but these pupils had learnt so much Latin, Greek and Danish that they even had to be placed in the uppermost seats in the upper grade.[217]

In the second semester they studied exponents, logarithms and equations. That year, geometry was not studied as it had been covered for the whole of the previous academic year. Thus Björn Gunnlaugsson continued his programme through the years, while there were only two grades. In the lower grade he taught arithmetic, elementary arithmetic in the autumn semester and algebra in the second semester. In the upper grade he taught geometry every other year and as advanced arithmetic and algebra as possible during the other academic year. The two textbooks by H. O. Bjørn were studied alternately: *Lærebog i Geometrien* / *Geometry* (in 1826 and presumably even years) and *Mathematik* / *Mathematics* (in 1827 and odd years).[218]

Páll Melsteð was a pupil at Bessastaðir School in 1828–1834. Páll Melsteð was the P. in the P. + B., the authors of an obituary of Björn Gunnlaugsson, the main source about his life. Páll Melsteð said about Björn Gunnlaugsson in his memoirs, written in the 1892:

[217] National Archives of Iceland: Bps. C VII, 3a. The above mentioned reports are preserved in this packet.
[218] National Archives of Iceland: Bps. C VII, 3a

> Björn Gunnlögsen kenndi í neðri bekk reikníng, dönsku, íslenzku og landafræði, í efri bekk geometriu og landafræði. Meiri stjörnu- og stærð-fræðíngur mun vart hafa verið á þessu landi en Björn Gunnlaugsson, og gat kennt stærðfræðina vel, en hann gekk ekki eftir því, að við lærðum hana. Hann spurði okkur einatt: skiljið þið þetta? Við sögðum já, en stundum sögðum við ósatt; þá vantaði að láta okkur sýna, að við kynnum að reikna dæmið rétt.
>
> Björn Gunnlögsen taught in the lower class arithmetic, Danish, Icelandic and geography, in the upper class geometry and geography. A greater astronomer and mathematician has hardly existed in this country than Björn Gunnlaugsson, and he could teach the mathematics well, but he did not check if we learnt it. He often asked us: Do you understand this? We said yes, but sometimes we did not tell the truth; what lacked then was to have us show that we knew how to compute the problem correctly.[219]

Páll Melsteð also told a story about Björn Gunnlaugsson's absentmindedness, but said that no one smiled at him, as the pupils were used to hearing him talk to himself, and they could not but admire him; he was a true sage in their eyes.[220]

Mathematics in Bessastaðir School's Last Years

According to the last Bessastaðir School reports of 1840–1846, Björn Gunnlaugsson's mathematics teaching programme continued. In the lower grade there were the four species in whole numbers and fractions, proportions, algebra, quadratic roots and powers together with *regula de tri*, though possibly with different emphasis each year, as the pupils repeated the grade three times. In the upper grade there was geometry one year and advanced arithmetic and algebra the second year, both taught from textbooks by the Danish author G. F. Ursin.

In the report from 1840–1841, Headmaster Jón Jónsson wrote:

> Undirkennari (Adjunctus) B. Gunnlaugsson hefir … kénnt efribekkíngum alla jarðmælíngarfræðina (Geometri) eptir Ursin. Þar les hann yfir hverja setníngu og útfærir sannanirnar á tøblunni, bætir og stundum við nýum sønnunum, svo lærisveinarnir sjái sømu setninguna sannaða á ymislegann hátt. Ef tíminn er nógur þá reynir hann lærisveinana, annars ekki. … Í neðrabekk kénnir hann Reikníngslist og fylgir þar í Byrni. Hann hefir yfirfarið ... hvernig talið sé (Numeratio), 4 reikningsgreinir (species) í heilum, margskonar og brotnum tølum, sømuleiðis Proportionir, tugabrot, bókstafa reikníng, kvaðratrætur og veldi; þetta hefir hann ként meira practice, enn theoretice; hann lætur því duga að kénna lærisveinunum í neðrabekk að reikna, og sýna til hvers aðferðirnar séu. Þegar bókstafareikníngurinn byrjar, útskírir hann hvers vegna bókstafir séu hentugri enn tölur, og sýnir ýmisleg dæmi uppá positiv og negativ stærðir, o.s.fr.
>
> Adjunct B. Gunnlaugsson has … taught the upper grade pupils all of Ursin's Geometry. There he reads every sentence and demonstrates the proofs on the board, adds sometimes new proofs, so that the pupils see the same sentence proved in various ways. If time allows, then he tests the pupils, otherwise not. … In the lower grade he teaches computing art and follows Bjørn in that. He has covered … counting (Numeratio), 4 operations (species) in whole, manifold and fractional numbers, furthermore proportions, decimal fractions, letter computing, quadratic roots and powers; this he has taught more practically than theoretically; he lets it suffice to teach the lower grade pupils to compute and show what the methods are used for.

[219] Páll Melsteð (1912): 28
[220] Páll Melsteð (1912): 34

When the letter computing begins, he explains why letters are more convenient than numbers, and shows various examples upon positive and negative quantities, etc.[221]

In the report from 1841–1842 Björn Gunnlaugsson writes:

Hjá efrabekkíngum yfirfór ég reikníngsfræðina (þ.e. bæði talna- og bókstafareikning ásamt með Algebra) eptir Ursins lærdómsbók, þannig: að eg fyrst las upp af bókinni hvørja grein, þó á íslendsku, sleppti síðan bókinni, og sýndi hið sama og sannaði á tøblunni, og útskírði munnlega eptir sem verða vildi og tímin leyfði. En þætti mér það ekki verða nógu greinilegt eða reglulegt, sem skyldi, en vildi ekki eyða leingri tíma þartil, eða vildi bæta þar við einhvørju, eða sanna øðruvísi, þá skrifaði eg það upp heima hjá mér og afhendti síðan nærsta dag í skólanum. … Í neðrabekk yfirfór eg reikníngslist eptir Bjørns lærdómsbók eins og í fyrra, en hvað Þríliðuna snertir fór eg eptir þeirri nýútkomnu íslendsku reikningslist Klausturhaldara Jóns Guðmundssonar, hvar greinilega er skírt frá þeim velska máta í Þríliðunni, hvør og stundum hefur brúkaður verið hér við skólann.

In the upper grade I went through the arithmetic (i.e. both numerical and letter arithmetic together with algebra) according to Ursin's textbook, such that I first read aloud from the textbook each paragraph, though in Icelandic, then put the book aside, and showed the same and introduced proofs on the board, and explained orally as needed according to the time available. If I did not think it was clear or tidy enough, as it should be, and I did not want to use more time on it, or wanted to add something or prove in another way, then I wrote it up at home and handed it out in school the following day. … In the lower grade I went through Bjørn's textbook like last year, while concerning the *regula de tri* I followed the newly published Icelandic *Computing Art* by convent steward Jón Guðmundsson, where is clearly explained the foreign way of the *regula de tri*, which also from time to times has been used here at this school. [222]

In this report there is an account of books sent as a gift from the Royal Directorate of the University and the Learned Schools. Among the books is *Algebra og Functioner* by the author Ramus. Books were also sent for sale to the pupils at a one-third discount from the library price.[223]

In 1842–1843 the Directorate sent a number of books as before, since 1819, ordered by the school to sell to the pupils with the above-mentioned discount, and a number of other books, amongst them the *Lalandes Logarithmer*, *Trigonometri* by Ramus, *Kellners descriptive Geometri*, Madvig's *Bemærkninger* to his *Grammatik* and Svenningsen's *Geometri* for the library.[224] It is noteworthy that Björn Gunnlaugsson often changed textbooks and usually had the latest version in hand, which may be explained by these books being sent by the Directorate.

Björn Gunnlaugsson wrote in the report 1842–1843:

Hjá Efrabekkíngum yfirfór eg Rúmmálsfræðina (Geometria), sem eptir griska nafni sínu heitir Jarðmæling. Þar í fylgdi eg Ursins lærdómsbók og brúkaði sömu aðferð sem undanfarin ár.

... Í neðrabekk kénndi eg talnalist, og fylgdi þar í Ursins reiknings bók. Það gérði ég með því, að lærisveinarnir æfðu sig í að reikna út dæmin sem í bókinni standa, sumir á töblunni til skipta, en hinir í sætum sínum. Þannig yfirfórum við þær 4 reikningstegundir í heilu og brotnu, viðkénndum og óviðkénndum tölum, samt þriggjaliðareglu í heilum tölum og brotnum, einnin öfuga þriggjaliðareglu og rétta samsetta þríliðu.

[221] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1840–1841*: 22
[222] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1841–1842*: 14–15
[223] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1841–1842*: 9–10
[224] *Skírsla um Bessastaða-skóla fyrir skóla-árið 1842–1843*: 4–8

> With the upper grade pupils I went through geometry which by its Greek name is called Earth Measure. In that I followed Ursin's textbook and used the same method as in previous years.
>
> In the lower grade I taught arithmetic, and followed therein Ursin's arithmetic. That I did by having the pupils compute the problems from the book, some of them taking turns at the board, and the others in their seats. Thus we went through the 4 operations in whole numbers and fractions, named numbers and un-named, in addition to the *regula de tri*, also inverted *regula de tri* and composite *regula de tri*.[225]

There is no indication in the reports that the Icelandic textbook *Reikningslist* / *Computing Art* by Jón Guðmundsson was used any more.

In the following year, 1843–1844, Björn Gunnlaugsson used Ursin's geometry textbook with the same method as the last year but one in the upper grade, and in the lower grade Ursin's arithmetic textbook, using the same method as the previous year. The most advanced pupils computed exercises from *Nyttige og curiøse Opgaver til Øvelse* / *Useful and Curious Problems for Practice*.[226]

In 1844–1845 Björn Gunnlaugsson accounted that he had introduced *Geometri* by Svenningsen, six chapters out of eight. The book could not be completed, as the number of theorems was nearly twice as many as in Ursin's geometry. Some of the theorems also needed long explanations. Björn Gunnlaugsson concluded that Svenningsen's book would be more suitable where there were more grades in the school. Svenningsen's geometry had arrived in the Directorate's package in 1843. The lower grade was taught in the same way as in previous years. Those more advanced had partition of inheritance and decimal fractions.[227] The next year, 1845–1846, the upper grade read arithmetic and algebra using Ursin's textbook, while the lower grade was taught the same way as previously. This was the last year of Bessastaðir Learned School.

Descriptions by Headmaster Jón Jónsson and Björn Gunnlaugsson of the latter's teaching in the reports are unusual sources about how the teaching proceeded. They show his sincere concern for the subject and his pupils, and indicate the teaching methods of those times. One notices that Headmaster Jón Jónsson said about Björn Gunnlaugsson's instruction in 1841 "If time allows, then he tests the pupils, otherwise not" while Páll Melsteð said ten years earlier that "what lacked was to have us show that we knew how to compute the problem correctly". However, Björn Gunnlaugsson reported in 1843 that "That I did [taught arithmetic] by having the pupils compute the problems from the book, some of them taking turns at the board, and the others in their seats." This indicates that he did test the pupils, but was not a strict teacher.

Concerns Regarding the Icelandic School

Romanticism and nationalist movements in Europe in the first half of the 19th century influenced Icelandic intellectuals. The cradle of an Icelandic movement for independence was in the Bessastaðir School, and amongst Icelandic university students in Copenhagen.

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[225] *Skírsla um Bessastaða-skóla fyrir skóla-árið 1842–1843*: 13–14
[226] *Skírsla um Bessastaða-skóla fyrir skóla-árið 1843–1844*: 7
[227] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1844–1845*: 10–11

One of them was Baldvin Einarsson (1801–1833). Troubled by the professors' complaints about the poor knowledge of the Icelandic students, he wrote two articles, *Tanker om det lærde Skolevæsen i Island* / *Thoughts about the Learned School System in Iceland*, in 1829 and *Tillæg til Tanker om det lærde Skolevæsen i Island* / *A Supplement to Thoughts about the Learned School System in Iceland*, in which he blamed the circumstances of the school, e.g. its buildings, for its situation. Baldvin Einarsson took these documents to an audience with the King himself. He explained the problems of the school; the over-crowded buildings and inadequate financial means. He referred to the agreement made when the assets of the two episcopal sees and two schools were sold, that in their place there should be a fixed sum to cover the expenses of the see and a school for 40 pupils. The sole see, located in Reykjavík, had gone bankrupt, and the number of school pupils had dropped to 24 in 1805.[228]

While Baldvin Einarsson died sadly in an accident, his articles were given thought by the authorities, and they were discussed in the Danish administration for some years without any further suggestion from officials in Iceland. Jón Sigurðsson (1811–1879), who was to become leader of the campaign for Iceland's independence, took up Baldvin Einarsson's cause, which resulted in the establishment of the Reykjavík Learned School in 1846 and a new building for it, which continues to serve its purpose to the present day.[229]

The material situation of Bessastaðir School was poor. In spite of that, it enjoyed high prestige. It managed to have excellent teachers who dedicated their lives to teaching, and were not simply theologians teaching temporarily while waiting for a good parish. There were especially two outstanding teachers, Sveinbjörn Egilsson, who taught Greek, with a special talent for training his pupils in Icelandic through their translations from Greek classics, and the mathematician Björn Gunnlaugsson.

Printed Reports on Bessastaðir School

During 1828–1840 the Bessastaðir School published a periodical, *Boðsrit* / *Invitation Writings*, on the occasions of the birthday of the King, first King Frederik VI and later Christian VIII. In the *Invitation Writings* learned articles by the teachers of the school were published. In the first of the publications Björn Gunnlaugsson wrote about the trajectory of the moon, in 1834 he wrote in Latin about the survey and the map of inland Iceland, and in 1836 tables of sunrise, sunset, dawn and dusk, while Sveinbjörn Egilsson translated Homer's *Odyssey* and published it in parts in 1829–1840.

In 1839 a circular, dated September 14, came from the Directorate of the University and the Learned Schools to all the schools in Denmark, announcing that every headmaster should yearly publish in print a report about the internal situation of his school. The report was to be published at the time of the official learning examination. The government had, however, not decided upon the format of the report, as reports from all the schools were to have the same format and be bound together in one volume. This decision was not published until in November, the same year and could not be brought to Iceland until the spring ships arrived in 1840. The headmaster asked in a letter dated July 29, 1840, in which language the report should be written.

[228] Nanna Ólafsdóttir (1961): 67–81
[229] Sverrir Jakobsson (1994): 75

The report for the school year 1839–1840 was therefore not published in print, as no decision had been made on the language. In May 1841 a message came, confirming that it could be in Icelandic.[230] This anecdote illustrates Iceland's geographical isolation from its government in Denmark. Jón Sigurðsson took this matter up in his journal *Ný félagsrit*, as an example of the inconvenience for Icelanders of not being able to govern their own school affairs.[231]

The first report, for 1840–1841, was published together with *Invitation Writings*, now to the official examination hearing in Bessastaðir School. In this report, the headmaster explained the conditions of the school building, which he said was unsuitable for a school. After the number of pupils was increased, the library, for example, had to be located in the tower of the church, which leaked, so the books suffered from dampness. From this report it is clear that the number of pupils had increased to 40, and had been at this level for quite a number of years. The number had previously been 27 to 32, later 36 pupils. In addition, in recent years there had been 2–4 extra pupils, who were to provide their own food and shelter, probably on farms in the neighbourhood. As a result the pupils were like lambs in a crib, the schoolrooms were so small.[232] In 1841–1842 there were 40 pupils at the school, 23 sons of officials, 15 sons of farmers and craftsmen and two sons of merchants. In addition to these, there were four extra pupils.[233] The fact that sons of farmers were 35–40 percent of the pupils indicates that there was some flow between the classes of officials and farmers, or at least landowners.

The report was a part of a compilation of reports from all the learned schools in Denmark and Iceland, so the authorities may not have been waiting specially for it for decisions upon the Icelandic school. At least a resolution came from the King, dated June 7, 1841, saying that the Icelandic school had to be re-planned and a new institution for higher education had to be established, so that the learned school could be made equivalent to the Danish learned schools.[234] However it cannot be read from the headmaster's report that any decision was made about a new building until after the report for 1842–1843.[235] From the report 1843–1844 it is clear that only the most urgent repairs would be made, as the school was to be transferred to Reykjavík.[236] *Alþingi*, the parliament, was reinstated in 1845 and it met in a new building, intended for the Learned School, which moved into it in 1846.

The *Invitation Writings* continued to include learned articles, written by the teachers. In 1842 Björn Gunnlaugsson published his great philosophical poem *Njóla*. Dr. Hallgrímur Scheving published in 1843 his collection of proverbs, which is one of the bases for the current collection of Icelandic proverbs, and in 1845 and 1846 Björn Gunnlaugsson published a comprehensive guide to recognizing the stars.

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[230] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1840–1841*: 7–10

[231] Jón Sigurðsson (1994): 124–125

[232] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1840–1841*: 16

[233] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1841–1842*: 3–4

[234] Nanna Ólafsdóttir (1961): 89

[235] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1842–1843*: 3–4

[236] *Skírsla um Bessastaða-Skóla fyrir skóla-árið 1843–1844*: 1

4.3. Public Education

Reikningslist by Jón Guðmundsson

In 1841, more than half a century after the two Enlightenment arithmetic textbooks were published, the *Greinilig vegleiðsla / Clear Guidance* by Ólafur Olavius and the *Stutt undirvísun / Short Teaching* by Governor Ólafur Stefánsson, a 260-page textbook, *Reikníngslist, einkum handa leikmönnum / Computing Art, Mainly for Laymen*, was written by "convent steward" Jón Guðmundsson and published at the expense of O. M. Stephensen, one of Governor Ólafur Stefánsson´s descendants.

Jón Guðmundsson (1807–1875) was a pupil of Björn Gunnlaugsson at Bessastaðir in 1824–1832. He qualified as a lawyer in 1851 from the University of Copenhagen and was temporarily appointed a county magistrate in 1849, but was dismissed in 1851 for high treason due to his political activities as Jón Sigurðsson's ally in the independence campaign. Jón Guðmundsson became editor of *Þjóðólfur*, one of the leading journals of the second half of the 19th century. The "convent" where he was steward, at the time when the *Reikníngslist* was published, was Kirkjubæjarklaustur in East Iceland, where he was an overseer of royal properties. The convent itself, of course, was long gone.

Björn Gunnlaugsson only used the book for one year. The book was intended for laymen, so possibly it was not deemed theoretical enough for the school. In the introduction the author said:

> ... þó eg sé þess fullviss, ad hvörr sæmilega gáfadur madur sem er, géti lært af kveri þessu þann reikníng sem þad inniheldur, tilsagnarlaust edur tilsagnarlítid, þá má samt einginn sá hugsa sér ad læra neitt af því – nema með góðri tilsøgn – sem ekki kann vel ad lesa, þ.e. sem gétur gjört sér greinileg meiníngaskil eptir lestrarteiknum og lestrarreglum; hvörr sá sem ekki er fær um þad, þarf hvörki ad ætla sér ad læra neitt af bókinni, né heldur ad dæma hana óvægilega fyri þad, þótt ekkért skilji.
>
> ... even if I am certain that every reasonably gifted man can learn from this booklet the arithmetic it contains, without or with minimum instruction, then no one should expect to learn anything from it – without good instruction – who cannot read, that is can make meaning of the reading symbols and reading rules; everyone who is not capable of that should not expect to learn anything from the book, nor judge it harshly for that, even if he does not understand anything.

Obviously the author feels necessary to safeguard himself against criticism, not only from those who cannot read, but also from those who know more than the book presents:

> Þessum þarf eg ekki ad segja, ad bókin er ekki samin handa þeim, heldur einkum handa leikmønnum; ad eg þessvegna vard ad fara fleiri ordum um margar reikníngsreglur, en þeim kann að þikja vidþurfa, sem kunna þær.
>
> Those I do not have to tell that the book is not composed for them but for laymen; that I therefore had to use more words on many computing rules than those who know them might find necessary.[237]

These quotations reflect the expected attitude towards publications of that kind in the mid-19th century Iceland and probably much longer. The attitude seems to have been: "Who does the author think he is?", both on the part of those who were ignorant of the matter and those who knew better.

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[237] Jón Guðmundsson (1841): Til lesarans (To the Reader)

Official criticism was not uncommon in these days. When Jón Guðmundsson later became editor of *Þjóðólfur* he criticized Headmaster Bjarni Jónsson harshly for being too fond of the Danes and the Danish language. During Bjarni Jónsson's time in office, the school reports were written both in Icelandic and Danish.

Headmaster Bjarni Jónsson used page after page in the school reports to defend himself. The school report 1862–63 is nearly three times as long as earlier. The headmaster explained that professionally he had studied and worked in English and French in addition to the classical languages, while his knowledge of Danish, German, and the Germanic languages was not on professional level. However, by the arrangement of fate he had stayed in Denmark for 22 years, and he declared once and for all that he loved Denmark to which he was attached by many bonds of gratitude. And to confirm his learnedness in Latin and not in Danish he printed his address in Latin to his stepfather, Björn Gunnlaugsson, on Björn Gunnlaugsson's retirement.[238]

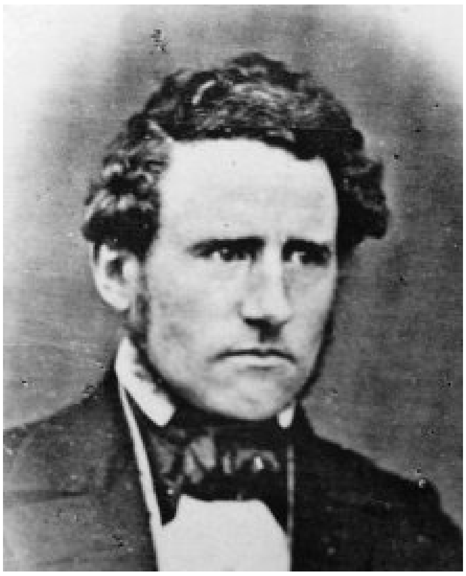
Fig. 4.4. Editor Jón Guðmundsson.

The *Computing Art* / *Reikningslist* textbook is conventional. It explains the four operations on whole numbers, named numbers, such as money, weight and time, fractions, *regula de tri*, percentages and partition of inheritance. It contains topics such as Euclid's algorithm and testing of the operations, while it is mainly written in the style of how to do, rather than why, and there are no algebraic explanations.

It is not easy to decide if the *Computing Art* / *Reikningslist* had any influence. No evidence has yet been found about its use except that it was used in the Bessastaðir School for one year.

A Book on Arithmetic for Farmers

Strangely enough, another mathematical book was published in 1841. The book in question was *Stuttur Leiðarvísir í Reiknings-list handa Bændafólki* / *A Short Guide in Computing-Art for Farming Folk*, composed by the Rev. S. Br. Sivertsen of the Útskálar and Hvalsnes parishes in the far southwest of Iceland. The book is an unpretentious guide to counting and computing in 16 pages, followed by conversion tables for the various currencies, weight and measuring units in 5 pages and computations with them in 8 pages. Fraction computations are covered on 9 pages and the indispensable *regula de tri* in 7 pages. The book concludes with interest–tables.

Nothing is known about the distribution of this booklet. However, like the two 18th-century books, the *Lijted Agrip* / *Little Compendium Intended for Farmers* (1746) and *Vasa-qver* / *A Pocket-Book for Farmers and Simpleminded People or an Easy Computing Art* (1782), it meets the need of the common farmer for conversion between units, probably the main mathematical activities of the general public. It contains the following relations:

[238] *Skýrsla um Hinn lærða skóla í Reykjavík 1862–63*: 28–34

1 lest er 12 tunnur eða 30 vættir (1 *lest* equals 12 barrels or 30 *vættir*)
1 vætt er 40 fiskar eða 8 fjórðungar (1 *vætt* equals 40 fishes or 8 farthings)
1 fjórðungur er 20 merkur eða 5 fiskar (1 farthing equals 20 marks or 5 fishes)
1 hundrað er 20 landaurar eða 120 álnir (1 hundred equals 20 *landaurar* or 120 ells (of woollen cloth))

The rates are the same as were introduced in the first printed book, the *Little Compendium*. Society had not changed much in a hundred years. However, several more modern units were introduced, such as pounds, miles, feet and fathoms.

Reasons and Argumentations for Mathematics Teaching

Certainly the reason for teaching mathematics at the learned schools in Bessastaðir and later in Reykjavík was primarily to ensure the entrance of Icelandic students into the University of Copenhagen. Few had realized that mathematical studies would have anything to do with economic and social progress in Iceland. Baldvin Einarsson seems to have done so, as at the time when he was becoming a lawyer in 1832, he enrolled into the newly established Polytechnic College in Copenhagen in order to complete an examination in practical sciences, although he already had a family to support. A fatal accident in 1833 brought his plans to nothing.[239]

As in the 18th century, books were published for the general public, such as the farmers, with the aim of helping them in their trade, and thus contributing to the socio-economic development of the society.

Jón Sigurðsson's Vision for Public Education

Jón Sigurðsson, the 19th century leader of the movement for independence from Denmark, did not attend the Bessastaðir School himself, but graduated after private instruction by his father, in which he did not learn much mathematics. His first school was the University of Copenhagen, where he studied philology and history (*philologicum magnum*), an uncommon subject for Icelanders at that time. In the 1840s he had become an undisputed leader of the independence movement. He published a journal, *Ný félagsrit* / *New Society Journal*, referring to the journal of the earlier Learned Arts Society. Jón Sigurðsson led the campaign by arranging petitions to the King, and by extensive political writing in Scandinavian newspapers, for Icelanders in *Ný félagsrit*, and for the Danes in newspapers such as the *Berlingske Tidende*.

Fig. 4.5. Jón Sigurðsson, leader of the Independence Movement.

[239] Nanna Ólafsdóttir (1961): 23–24

In the 1840s Jón Sigurðsson wrote, for example, a series of articles about education and schools in *Ný félagsrit*. His vision was compulsory instruction for all people in the country. According to the theories of the French Revolution about the fundaments for a people's state, schools were needed if the people were to speak the same language and adhere to the same legislation. Jón Sigurðsson's ideas about schools were drawn from Prussia. He wanted six hours' instruction a day for all children aged 7–14.[240] The establishment of such schooling in Iceland would not be a reality for more than a century.

Jón Sigurðsson wrote detailed proposals about schools for farmers and about a lower secondary school for the middle class, which hardly existed in Iceland and was to be promoted. Theoretical mathematics was not at the top of his lists, while he did provide for arithmetic and geometry for those heading for a learned school, probably with university entrance in mind. For farmers he proposed practical measuring skills and geodesy.[241]

Progress in the First Half of the 19th Century

The time was not yet ripe for progress in public education in Iceland. The home-education tradition was still strong, and was seen by the Icelandic authorities as the only method available in the rural areas, even if Denmark had legislation on public schools in 1814. Legislation on teaching children writing and arithmetic had yet to be seen. A primary school was established in Reykjavík in 1832, funded by the Thorkillii-fund, which had previously supported the first school for children at Hausastaðir on Álftanes. A timetable exists from the year 1840. Arithmetic was taught for six hours a week. Every other day there was computation on the board, and the other day mental arithmetic. This school closed down in 1849 when the grant from Thorkillii-fund was no longer available.[242] A school was re-established in Reykjavík in 1862.

However, there was considerable progress in the campaign for independence. After a series of petitions to King Christian VIII, he decided, in opposition to his advisers, to surrender to the wishes of the Icelanders and let his officials investigate the possibility of establishing an elected body with a consultative role, to be named *Alþingi* like the ancient establishment at Þingvellir.[243] A royal declaration of this arrangement was published in 1843. The restored *Alþingi* was inaugurated in 1845, in the new building of the Learned School.

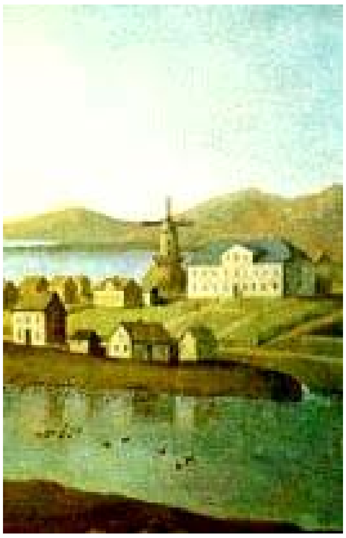
Fig. 4.6. The Learned School.

Why? In the 19th century's romantic wave of nationalism many Danes had begun to think of Iceland and Icelanders as precious antiques, due to their ancient manuscripts and archaic language. They wanted to keep the Icelanders happy, and in particular to reinstate such a historical establishment as *Alþingi* was.[244]

[240] Jón Sigurðsson (1994): 84–85
[241] Jón Sigurðsson (1994): 85–87, 142
[242] Ármann Halldórsson (2001): 17–32
[243] Bragi Guðmundsson and Gunnar Karlsson (1997): 104–105
[244] Bragi Guðmundsson and Gunnar Karlsson (1997): 124

4.4. The Reykjavík Learned School

Society in the Second Half of the 19th Century

The frame of this period is the Reykjavík Learned School, which operated in 1846–1904. The period is also marked by two outstanding events in the history of Iceland: the restoration of the *Alþingi* in 1845, and the establishment of Home Rule in 1904. In the 1870s, important events occurred in the fields of both independence and education. Iceland was granted its own Constitution, under which the *Alþingi* became a legislative body, by King Christian IX in 1874. New regulations on the learned school were issued in 1877, which made it equivalent to a language-history stream in the Danish learned schools. Finally, new legislation on public education was passed in the *Alþingi* in 1879 and approved by the King in 1880, mandating instruction for children in writing and arithmetic. Instruction was still the responsibility of the home, under the supervision of the church. These tasks were beyond the capacity of many households. No teacher training was yet available, and legislation on schools was first adopted in 1907.

In spite of this arrangement, primary schools began to be established, some only surviving for a short period. The first school to operate uninterruptedly to the present day is Eyrarbakki Primary School, established in 1852. A primary school has operated in Reykjavík since 1862, and in Akureyri since 1871. From 1878 primary instruction was included in the National Budget.[245]

The population grew in the 19th century, although more slowly than in other European countries. Reykjavík, which had been granted its town charter in 1786, grew still faster.[246]

Year	Iceland	Reykjavík	% of population
1801	47,240	307	0.6%
1835	56,035	639	1.1%
1860	66,987	1,444	2.2%
1880	72,445	2,567	3.5%
1901	78,470	6,682	8.5%

Table 4.1. Population in the 19th century

Means were limited in Iceland in the mid-19th century. In the fiscal year 1850–1851 the income of Iceland was 28,320 rix-dollars. Expenses were 51,764 rd., of which the cost of the Learned School was 8,350 rd. Obviously the school's expenses weighed heavily in the National Budget. Jón Sigurðsson's tactic was to claim that the Danes owed Iceland for sold properties, e.g. the episcopal sees, and for profit from the monopoly trade. He calculated the debt as 99,725 rd. a year for the indefinite future. While the Danes would never admit to such a debt, it comforted the Icelanders not to have to regard as charity the necessary funding to cover the yearly deficit.[247]

[245] Sverrir Jakobsson (1994): 77–78
[246] *Hagskinna* (1997): Table 2.3. Population of parishes, towns and counties 1769–1990
[247] Bragi Guðmundsson and Gunnar Karlsson (1997): 126–127

Provisional Regulations in 1846

At its establishment in 1846 the Reykjavík Learned School was given provisional regulations by its Danish authorities. The prospective headmaster Sveinbjörn Egilsson participated in their composition. However, he obtained less than he wished, and other things were organized differently from his wishes.[248] The school's mission statement says:

> … bæði á hann að veita lærisveinum svo mikla þekkingu og svo mikla mentun, sem þeir menn þurfa við, er ætla sér að yðka sérstaklegar vísindagreinir í Kaupmannahafnar háskóla, svo á hann og að undirbúa þá, sem æskja að komast í prestaskóla þann, er setja skal á Íslandi í sambandi við latínuskólann, handa þeim mönnum, sem vilja verða prestar þar í landi.
>
> … it shall both offer as much knowledge and education as needed for the aim to study special research subjects at the University in Copenhagen, and then the school shall also prepare those who wish to enter the Theological Seminary which is to be established in Iceland in connection to the Latin School, for those who want to become clergymen in that country.[249]

The main goal concerns the prerequisites for further studies, now at the University of Copenhagen and at the Theological Seminary to be established in Reykjavík. Other societal or personal needs are not mentioned.

The Theological Seminary was established in 1847. Thus the Reykjavík Learned School was a genuine learned school, more similar to other learned schools in the Danish school system than the Bessastaðir School. By the terms of the 1846 regulation, the school was to be divided into four grades. The pupils were to attend the second grade for one year, while the other three grades were to be repeated. This seven-year school never became a reality, and initially the grades were three. During the period 1846–1851 the total number of pupils was approximately 50–60. Those who intended to attend the Theological Seminary were to study Hebrew, while the others were to study English or French, as Jón Sigurðsson had suggested.[250] There were nine graduates in 1847, nine in 1848, eight in 1849, five in 1850 and thirteen in 1851.[251]

Icelandic, Danish, Latin, Greek, Hebrew, mathematics and physics were to be studied for six years. The school had a total of 106 teaching hours a week, of which Latin was allocated 26 hours, mathematics 20, Greek 14, Icelandic 3, Danish 9 and German 7 hours.[252] Mathematics thus accounted for 18.5% of the teaching hours. Pupils studying in the school for six years had therefore a total of 40 weekly hours of mathematics.

Soon the school was divided into four grades. Icelandic was allocated more hours, as were natural history, i.e. biology and botany, writing and drawing, so mathematics accounted for 13.2% of the total number of hours. This ratio continued until 1850.

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[248] Heimir Þorleifsson (1975): 15
[249] Heimir Þorleifsson (1975): 15–16
[250] Heimir Þorleifsson (1975): 16–18
[251] Heimir Þorleifsson (1975): 18
[252] Heimir Þorleifsson (1975): 18

Regulations of 1850

On May 13, 1850 regulations on instruction and examinations for the learned schools in Denmark were published on the initiative of Professor J. N. Madvig, Minister of Education at that time. The regulations were published in Iceland on July 30, 1850, with the necessary adjustments to Icelandic circumstances. The regulations contained the following mission statement:

> Það er ætlunarverk hins lærða skóla, að veita lærisveinum þeim, sem honum eru á hendur faldir, þá tilsögn, er leiða megi til sannrar og röksamlegrar frummenntunar, og jafnframt með fróðleiksauka og eflingu sálargáfnanna búa þá á þann hátt, sem bezt má verða, undir enn fremri menntunarframfarir í prestaskólanum eða bókiðnir við háskólann ….
>
> It is the Learned School's goal to supply the pupils, for whom the school will be entrusted, the guidance that may lead to a true and logical education and furthermore, by increase of knowledge and support of the soul's gifts, prepare them as best is possible, for still further educational progress in the Theological Seminary or theoretical activities in the University … [253]

Compared to the 1846 regulations, the novelty in the new main goal, compared to the earlier one, is to offer a "true" general education, while preparation for university studies comes second, in agreement with Professor Madvig's ideas.[254]

The length of the school was to be six years in four grades, one year in each of the two first grades and two years in third and fourth grades. Later these two grades were also split in two. This arrangement continued until 1948. As before, the mathematical subjects – arithmetic, which included algebra, and geometry, which included plane geometry, stereometry, trigonometry and astronomy – were taught throughout the school, four or five hours a week each year. The language curriculum seems to have become more traditional than before. Hebrew and Greek, however, were reduced slightly, while English and French were optional, and Latin had an increasing share of the hours, 8–10 hours a week. Mathematics accounted for about 11.4% of total teaching hours.

A table of the results of 19 Icelandic students in the First Learning Examination at the University of Copenhagen in 1847–1851 shows that their results in mathematics, both in arithmetic and geometry, were good vis-à-vis history and exegesis, and even Latin and Greek.[255] A total of eight students had *Laudabilis*, or first grade, in arithmetic, and nine students had *Haud Illaudabilis*, or second grade, while two had *N. cont.*, *Non Contemnendus*, not contemptible, or third grade, i.e. 8–9–2. The few third grades are notable. The results in geometry were slightly worse: 8–7–4. In history they were: 8–6–5. In Danish the grades were lower. In Latin the ratio was 13–5–1 while in Latin composition 4–10–5.

[253] Heimir Þorleifsson (1975): 19
[254] Heimir Þorleifsson (1975): 19
[255] *Skýrsla um hinn lærða skóla í Reykjavík skólaárið 1851–52*: Appendix

Björn Gunnlaugsson's Last Teaching Years

By the time the Learned School moved to Reykjavík in 1846, Björn Gunnlaugsson had become highly regarded. He was 58 years old and had taught at Bessastaðir School for 24 years. In a letter from the King dated May 14, 1822 he was appointed as adjunct to the Bessastaðir School. On June 16, 1851 he was appointed Assistant Headmaster (*Overlærer*). A letter dated September 21, 1846 announced that he had been decorated as Knight of Dannebrog, and another letter, dated November 7, 1857, that he had been decorated as Knight of the Honorary Legion (*Æreslegionen*).[256] Björn Gunnlaugsson thus earned both honour and distinction. He was released from his duties by a letter dated March 31, 1862.

Fig. 4.7. Björn Gunnlaugsson.

The reports of Reykjavík School do not reveal such personal comments on the teaching as the Bessastaðir School reports do. The teaching became easier than before, as there were more grades. In 1846–1847 Björn Gunnlaugsson taught mathematics in three grades. In the next year, 1847–1848, there were already four grades, 1, 2, 3a and 3b.

In 1850 the school had problems, a rebellion against the highly-regarded headmaster, Sveinbjörn Egilsson. The rebellion, called the *pereat*, was partly connected with the fact that the school was now located in a town, where it was more difficult to maintain discipline than in Bessastaðir School, and partly it was inspired by the 1848 revolutionary movement in Europe. Björn Gunnlaugsson, gentle as he was, thanked the rebellious pupils, when it suited them to attend class.[257] No report was published for 1849–1850.

In 1850–1851 Ursin's books were used in all grades. From 1851 there were four grades, with the third grade partly divided, so the grades were in practice five. The last grade was repeated once. Mathematics was taught jointly in the two third grades, and the syllabus varied slightly from year to year. During the following years Björn Gunnlaugsson tried out books by Fallesen and Ramus, and from 1856 to 1862, when Björn Gunnlaugsson retired, textbooks by Ursin, Ramus and Assen were used side by side. The topics were elementary and advanced arithmetic, algebra, roots, decimal fractions, equations, proportions, chain fractions, logarithms, elementary geometry, trigonometry and stereometry.[258]

[256] National and University Library of Iceland: Lbs. 609, fol
[257] Bragi Guðmundsson and Gunnar Karlsson (1997): 11–20
[258] See Appendix A

Bjarni Jónsson, Björn Gunnlaugsson's stepson from his first marriage, took over as headmaster in June 1851, and Björn Gunnlaugsson was appointed as his assistant at the same time. From that time on, the reports, which previously had been in Icelandic, were written both in Icelandic and Danish. They were more detailed than before, but there were no personal remarks from the teachers, and the syllabus was only a list of page-numbers in textbooks. From the same time, examination problems were printed in the reports. The reports give special accounts of changes of textbooks, and new books added to the library.[259] The school bought Steen's *Elementær Arithmetik* in 1856. In 1857–1858 the school bought 93 books, among them Ramus's *Elementær Algebra* (1855), Ramus's *Elementær Geometrie* (1850), G.F. Ursin's *Regnebog* (1856) and *Stereometrien* (1847) [1547 in text] and Assen's *Arithmetikkens Begyndelsesgrunde* (1852). Most of the books were for the teaching of classical languages, while there were also books on history and English, French and German literature. For the next four years, until Björn Gunnlaugsson retired, no mathematics books were bought. The school library seems only to have bought mathematics schoolbooks in his time, not scientific books, as far as can be seen from the school reports 1851–1862. This agrees with Prof. Einar H. Guðmundsson's opinion that Björn Gunnlaugsson did not study contemporary mathematics after he returned from Copenhagen, only philosophy.[260]

In 1864–1865 *Euclidis elementa graece et latine*, ed. J. Guilelmus Camerer, (Berolini 1824–25) Tom. I.–II., appears in the list of books purchased, two years after Björn Gunnlaugsson's retirement.

Björn Gunnlaugsson doubtlessly devoted much thought to mathematics teaching. A manuscript[261] exists in the National and University Library in which he has copied a draft of a teaching plan for the learned school, presumably in Denmark (*Udkast til en Underviisningsplan i de udvidede lærde Skoler*), both for mathematics, astronomy, natural sciences (physics and chemistry), and natural history (biology and botany), which were the subjects he taught. The draft contains a detailed description of teaching methods which might motivate the pupils for computing exercises. Attached to it is a copy of the mathematics part of a school report from the Danish Metropolitan School in July 1846. Also attached is a short discussion (four written pages) by Björn Gunnlaugsson about examination regulations published in 1852, where he argues about the contribution of the teachers to the examinations. He regards it as impossible that the Ministry should make up the examinations on its own; it must have some reports from the schools about what they have taught. This seems to be a kind of a defence for Björn Gunnlaugsson writing his own examinations:

> Í reglugjörðinni um lærdómsprófin 1852 §14 stendur so:
>
> "De skriftlige Opgaver tilstilles Skolen af Ephoratet, som lader dem sig foreslaae af Examenscommissarierne eller andre dertil egnede Mænd."
>
> Hér er því öldungis ekki bannað að kennarar seu meir eða minna spurðir til ráða; og þó so væri að það væri óleyfilegt í öllum öðrum fögum en Mathematik, þá verður það þó æfinlega nauðsynlegt í henni; so lengi sem Island er þar sem það er, og so lengi sem Mathematiken er sjálfri sér so ólík í sýnum pörtum sem hún er og þó "ubegrændset". Þetta mun einnig ega sér stað í Danmörk, því verkefnin í Mathematik i Programminu bera það með sér, að einginn fjallar um þau, nema sá sem hefur Mathematik fyrir daglegt Studium, og þessir hljóta að fá að vita hjá kénnurunum eða

[259] *Skýrsla um hinn lærða skóla í Reykjavík skólaárið 1855–56*: 22
[260] Einar H. Guðmundsson (2003): 23–24
[261] National and University Library of Iceland: Lbs. 2008, 4to

Ministerienu hvörjar bækur notaðar eru við Skolana og ekki einungis það, heldur og einnig að hvað miklu leiti hvorri bók hefur fylgt verið. Því ekki munu menn þrælbinda so kennarana að þeir skuli kenna þær heilar spjalda á milli og hvorgi frávíkja eða útfyrir þær fara hvorsu sem þeim kann ílla að líka við þær. Prógrömmin sýna það einnig að hlaupið er yfir sumt í einni bók og lesið kannske í annari; og þó ekki ætíð nákvæmlega tiltekið hvað það er, ("Med Udeladelse af nogle Stykker") hvað eð mér sýnist benda til þess að Ministeríið viti það ekki gjörla. Stundum er sagt að lesið hafi verið eptir Dictatis, og þá sýnist mér verði að ráðfæra sig við kennarana. Á móti þessu sannar það ekkert, þó sömu verkefni gangi yfir alla skóla í Danmörk, því áður en þau eru samin, géta Commissarii verið búnir að fá nákvæmar skírslur frá öllum kénnurum um það hvað og hvornin allt hefur lesið verið, og vinsað þar úr so aungvum verði gjört órétti, og er þá komið að því sem ég sagði, að kennararnir eiga nokkurn þátt í ráðum með commisariunum, eða skíri þeim frá aðferð sinni. Líki Commissörunum ekki aðferð þeirra, þá géta þeir fundið að henni, og varað við henni framvegis. Hér er þá komið sýnist mér allt nauðsynlegt Controle, því öðrumegin standa kennararnir en hinumegin Commissar og Ministerium.

The Regulations of the Learning Examinations §14 say:

"The written examination problems are provided to the school by the Ephorate, which has them suggested to itself by the examination commissars or other for that task suited persons."

Here it is therefore not at all forbidden that the teachers are more or less consulted; and even if it were not permitted in all other subjects than the mathematics then it will always be necessary in that subject, as long as Iceland stays where it is and as long as the mathematics is so different from itself in its parts as it is and still unlimited. This is also said to be the case in Denmark, as the problems in the programme reveal that no one handles them except the one who has mathematics as a daily exercise and those must be informed by the teachers or the Ministry about which books are used at the schools and not only that but also to which degree each book has been followed. As people are not to be supposed to tie the teachers down in the sense that they [the books] have to be taught from the beginning to the end and nowhere be parted with or extended, however badly they may like them. The programmes show also that some items are skipped and maybe read in another book; and still it is not exactly quoted what, ("by dropping some items") where it seems to me that the Ministry does not know it exactly. Sometimes it says that dictates have been used and then it seems to me that the teachers have to be consulted. Against this it does not prove anything although the same problems are used all over Denmark as the commissars can have had exact reports from all the teachers, about what and how everything has been read, and sorted out from them so that no one will be done any injustice, and then it is like I said that the commissars consult the teachers or the teachers explain to them their methods. If the commissars do not like it they can criticize it and warn about its future use. Here we have all the necessary control, as on one side there are the teachers and on the other side the commissars and the Ministry.[262]

Björn Gunnlaugsson was the only mathematician in Iceland all his life. He could not consult with anyone, and no one in the country could write examinations for him. Denmark was hundreds of miles away. Ships did not sail across the ocean in wintertime, so examinations could not be sent to Björn Gunnlaugsson or other teachers at the Reykjavík School. In these notes he seems anxious that examinations could be imposed upon him, containing problems for which he has not prepared his pupils. His examination problems were not always particularly difficult,[263] but his

[262] National and University Library of Iceland: Lbs. 2008, 4to
[263] A personal note from Skarphéðinn Pálmason, November 2004

students normally did well on the first examinations at the University of Copenhagen, as previously cited.

Björn Gunnlaugsson's pupils loved and respected him, although they were perhaps not carried away with him in his mathematical thoughts. His manners may have been too eccentric. When he left the school in 1862 the pupils brought him a poem, written by Matthías Jochumsson, later a laureate of Icelandic poetry.[264] Björn Gunnlaugsson died in 1876, at the age of 88.

The Icelandic Literary Society

The Icelandic Literary Society was active from 1816. It acted in two branches, in Copenhagen, where it was established on April 13, and in Reykjavík, established on August 1. It continued the activities of the Society of the Learned Arts, established in 1779, and the societies were formally united in 1818. The Literary Society published a number of books related to Icelandic research, and also books for increased knowledge presented in Icelandic, such as translations of G. F. Ursin's *Stjörnufræði* / *Astronomy* by Jónas Hallgrímsson in 1842 and J. G. Fischer's *Eðlisfræði* / *Physics* by Magnús Grímsson in 1852, and other original textbooks written in Icelandic.

Jón Sigurðsson was the president of the Icelandic Literary Society from 1851 to his death in 1879. His brother Jens Sigurðsson was adjunct at the Reykjavík School and the treasurer of the Reykjavík branch of the society from 1850. The two branches cooperated, although they had each their own publishing scheme and funds. Jens Sigurðsson was a colleague of Björn Gunnlaugsson at the Reykjavík School, and his son-in-law, as he married Björn Gunnlaugsson's only daughter, Ólöf Björnsdóttir, half-sister of Headmaster Bjarni Jónsson. Jens Sigurðsson was headmaster from 1869 until his death in 1872.

Tölvísi

In the 1850s Björn Gunnlaugsson had begun to prepare a textbook on mathematics, which he called *Tölvísi / Number Wisdom*. The minutes of the Reykjavík branch of the Literary Society reveal that on February 19, 1855 the members of the society assigned Björn Gunnlaugsson to write a theoretical arithmetic book, to which he agreed. There is no indication that this matter was discussed in the Copenhagen branch.[265] However, a letter, dated, August 12, 1861, exists from Björn Gunnlaugsson to Jón Sigurðsson about preparations of the book:

> Jeg hef verið í sumar að keppast við reikningsbókina, sem felagið so kallar, og stendur til að jeg sýni félaginu 40 arkir skrifaðar með minni hendi nú á félagsfundi. Það er hin theoretiska arithmetik, þó hinu practiska sé blandað þarí með, þá matti jeg það minna, þar þær íslenzku reikningsbækur hafa það. Jeg kalla þessa reikningsbók Tölvísi, eins og Konráð Gíslason hefur yfir Arithmetik, og sem er í Snorra eddu. Eptir því sem ég hef farið í þetta, hef jeg sett þar í margt, sem ekki er algengt, og ætlast til að þar verði allt sem í skolum er kénnt og jafnvel fleira.

[264] Matthías Jochumsson (1862)
[265] National and University Library of Iceland: *Deild hins íslenzka bókmenntafjelags í Kaupmannahöfn. Samkomubók 1816–1874*

> This summer, I have been working hard on the Arithmetic, so called by the Society, and it is planned that I shall show the Society 40 quires written by my hand now at a Society meeting. It is the theoretical arithmetic, even though the practical items are mixed in with it, then I valued them less as the Icelandic arithmetic textbooks contain them. I call this arithmetic textbook *Tölvísi,* as Konráð Gíslason uses on arithmetic, and likewise is in Snorra-Edda. As I have been working on this, I have included many items, which are not common, and had in mind that it will contain all that is taught at schools and even more.[266]

Regrettably, the book was never used in schools. At this time Björn Gunnlaugsson was surveying Þingvellir for Jón Sigurðsson, and the above remark was not the main subject of the letter. That same month, in August 1861, Björn Gunnlaugsson handed over the 40 handwritten quires, for which he received 100 rd. An agreement was made that he should receive 16 rd. for each printed quire. In April 1864 he handed over six printed quires of the book. The book was printed in Reykjavík by Prentsmiðja Íslands, E. Þórðarson, and published in 1865, 25 printed quires in total.[267] A letter of thanks from Björn Gunnlaugsson was read at the society meeting in April 1866.

In October 1867 Björn Gunnlaugsson brought the Society a manuscript for the second part of *Tölvísi* which he took back for revision, and handed in for final submission in May 1868.[268] The manuscript collection Lbs. 2397, 4to contains *Tölvísi*, both the printed work and the manuscript to the second part that was never printed.

The published part of *Tölvísi* contains an introduction, sections on counting and the four operations, where the author introduces algebraic operations, and fractions. This is followed by a section on the nature of the whole numbers, i.e. introduction to number theory, such as modular arithmetic, congruence, prime numbers, divisibility and Fermat's Little Theorem. The book continues with decimal fractions, periodic decimals, error bounds on all operations, powers, exponents and roots, quadratic roots, cubic roots, bi-quadratic roots and fifth roots, the binomial theorem and chain fractions. The book ceases abruptly on page 400 after 25 quires, in the middle of the chain fractions. The writing, in algebraic language, is of a high standard.

The second, unpublished, part of *Tölvísi* contains chain fractions, ratios and proportions, equations in one and more variables, of first and second degree and higher degrees by guessing, and introduction to imaginary numbers. Then the author moves over to differential and trigonometric equations, exponential equations and logarithms, arithmetic and geometric progressions, compound interests and at last permutations and combinations. In several places, the author indicates what he has borrowed from the Danish mathematics textbook author Ramus and how Ramus addresses some specific items.

The topics treated in *Tölvísi* are certainly not new, nor did Björn Gunnlaugsson invent them. However, he decided to introduce them to the Icelandic reader in Icelandic vocabulary and with references to situations understood by Icelanders. The text is mostly presented with high-level algebra, and as such only understood in Iceland by his pupils at most.

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[266] National and University Library of Iceland: Lbs. 2590, 4to. Bréfasafn Jóns Sigurðssonar forseta
[267] Björn Gunnlaugsson (1865)
[268] National and University Library of Iceland: *Fundabók hins íslenzka Bókmentafélags í Reykjavík*, begun 1816 (to 1879)

Possibly Björn Gunnlaugsson hoped that the book would be used as a textbook in the Reykjavík School, which never became the case. Björn Gunnlaugsson's successor, Halldór Guðmundsson (1826–1904), continued to use Ursin's and Ramus' textbooks and in 1864 introduced Steen's *Elementær Arithmetik* and Steen's *Mathematiske Opgaver i det indledende Cursus* for the second grade.

Other Works

In the same year as Björn Gunnlaugsson submitted his completed *Tölvísi* part two, in September 1868, he offered the Literary Society the option of buying a manuscript of his plane geometry that he had begun to write but had not finished. A meeting of the Society decided to consult with the Copenhagen branch.[269]

A letter to the Reykjavík branch was dated in June 28, 1869, written on the letterhead of the Icelandic Literary Society and signed by Jón Sigurðsson, referring to a letter dated September 20 the previous year, stating that as nothing definite could be said about the book yet, such as its size, nothing final could be decided and for that reason it had not been found possible to propose the matter in a meeting in the Copenhagen branch. He trusted the Reykjavík branch to decide upon the matter, according to circumstances, supposing that the members would hardly oppose the manuscript being bought, especially for the author's sake, while they might think it risky to use much money on printing it.[270]

And so it remained. Björn Gunnlaugsson was the author of the map of Iceland, which the Society in Copenhagen had published in the 1840s and was one of its gems. The year before, in 1868, the Society in Copenhagen had published his *Einföld landmæling / Simple Land-Surveying*. Furthermore Jón Sigurðsson and Björn Gunnlaugsson had family connections. Jón Sigurðsson therefore had reason to act kindly to the honourable 81-year-old gentleman, although as a realistic and practical man he probably saw that it would produce too much deficit to publish such a book.

Some manuscript collections, attributed to Björn Gunnlaugsson, deserve further investigation. The collection Lbs. 2007a, 4to contains parts of the manuscript to his *Tölvísi* and *Promemoria*, a worn draft of an introduction to plane geometry, 22 pages long. It also contains a handbook in geodesy and a manuscript on astronomy, not so thin. The collection Lbs. 2007b, 4to contains e.g. botany, notes on astronomy, a draft to an introduction to a classification of the mathematical sciences and two sets of drafts of arithmetic textbooks, *Calculus numericus eður Talnareikningur* and *Arithmetica eður Talnafræðinn*, which are less algebraic than *Tölvísi*.

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[269] National and University Library of Iceland: *Fundabók hins íslenzka Bókmentafélags í Reykjavík*, begun in 1816 (to 1879)

[270] National and University Library of Iceland: *Bréfasafn Hins íslenska bókmenntafélags, Reykjavíkurdeild*, 1860–1869

Björn Gunnlaugsson's Legacy

In spite of Björn Gunnlaugsson's efforts for 40 years, his mathematics teaching does not seem to have had as much influence as might be expected. Only one of his mathematics textbooks was printed; the first half of *Tölvísi*. It was never used as a textbook, and nothing came of his plane geometry manuscript. The time seems not to have been ripe for mathematics as a discipline in 19th-century Iceland. People may not have seen many applications for such higher mathematics, in a country where all transport was by horses and open boats, and there was no technical industry of any kind. People valued, however, Björn Gunnlaugsson's geodetic work, and his textbook on land surveying was taught in the Reykjavík School from 1872.[271]

Few of Björn Gunnlaugsson's pupils studied mathematical subjects. The natural scientist and poet Jónas Hallgrímsson, who was his pupil in the 1820s, enjoyed his mathematics teaching. Jónas Hallgrímsson later became Björn Gunnlaugsson's successor in Icelandic geographical research. He appreciated Björn Gunnlaugsson highly and dedicated to him his translation of Ursin's *Stjörnufræði* / *Astronomy* in 1842, even if they disagreed on poetry and aesthetics.[272]

Magnús Grímsson, who translated Fischer's *Eðlisfræði* / *Physics*, expressed in his foreword special thanks to Björn Gunnlaugsson, who had read the manuscript:

> ... þá þakka eg sérílagi herra yfirkennara Birni Gunnlaugssyni, R af D., fyrir alla þá alúð og umönnun, sem hann hefir borið fyrir útleggingunni, þar sem hann hefir – svo vandlega sem embættis annir hans leyfðu – lesið handritið allt og víða lagað það mikið; sumstaðar er og bókin auðguð með athugasemdum hans (B.G.).
>
> Then I especially thank Assistant Headmaster Björn Gunnlaugsson, Knight of Dannebrog, for all the meticulousness and care which he has shown for the translation, where he has – as carefully as his official duties have allowed – read through the whole manuscript and in many places amended it substantially; in some places the manuscript is enriched by his remarks (B.G.).[273]

Magnús Grímsson had been a pupil of Björn Gunnlaugsson at Bessastaðir in 1842–1848 and was married to Björn Gunnlaugsson's stepdaughter. He graduated from the Theological Seminary and had wide interests. He had been the guide of foreign natural scientists in their research travels in Iceland. He also invented technical tools, such as machines to mow, pump water, row, etc.[274]

Arithmetic textbook author Jón Guðmundsson was Björn Gunnlaugsson's pupil in 1824–1832. Furthermore, The Reverend Eiríkur Briem, who wrote a very influential arithmetic textbook, first published in 1869, must have been his pupil for some time, as he graduated from the Learned School in 1864, two years after Björn Gunnlaugsson's retirement.

Only three of Björn Gunnlaugsson's pupils enrolled at the Polytechnic College in Copenhagen. They were Baldvin Einarsson, mentioned earlier, Bjarni Thorlacius, who transferred to medical studies, and Halldór Guðmundsson, who studied there for two years in 1854–1856 and became Björn Gunnlaugsson's successor as mathematics teacher at the Reykjavík Learned School in 1862. The next student to enrol and complete the first half of the engineering examination, in 1878, was Björn Jensson,

[271] Heimir Þorleifsson (1975): 221
[272] Páll Valsson (1999): 26–27
[273] Magnús Grímsson (1852): Foreword
[274] Páll Eggert Ólason (1950): Vol. III, 423–424

Björn Gunnlaugsson's grandson and Jón Sigurðsson's nephew.[275] Jón Sigurðsson became Björn Jensson's closest relative after the untimely death of his parents, Headmaster Jens Sigurðsson in 1872 and Ólöf Björnsdóttir in 1874, and Björn Jensson was staying with Jón Sigurðsson in Copenhagen at his death in 1879.[276] Björn Gunnlaugsson, the son of a tenant farmer, thus became closely related to Iceland's educational and political elite. Björn Jensson became mathematics teacher at the Reykjavík School in 1883.

4.5. The Reykjavík Learned School after 1860

The Reykjavík Learned School before 1877

Bjarni Jónsson, headmaster from 1851, was previously a teacher in Aalborg, Denmark for ten years, and assistant headmaster in Horsens, Denmark for five years. He was concerned about the Icelandic pupils' knowledge of Latin and wrote long letters about the matter. We have earlier learnt about his defence against Jón Guðmundsson's attack on him for being too fond of the Danes and about his professional knowledge of English, French and the classical languages. He wrote e.g. the following in the school report in 1863:

> Men ved saaledes at fremhæve de gamle Sprogs Vigtighed i Skolen, baade i Almindelighed og i Særdeleshed for os, maa jeg forvare mig imod, at jeg derfor nedsætter de andre Fag og navnlig Mathematik og Naturvidenskaberne. Jeg erklærer da engang for alle, at jeg anseer disse Videnskaber af uhyre Vigtighed, og navnlig for os, som i den Henseende have bestandig staaet tilbage, og hvis Lands naturlige Hjelpekilder ere saa lidet bekjendte, men jeg paastaaer, at deres Plads ikke er i Skolen, idetmindste Naturvidenskabernes, paa et saa tidligt Stadium, som nu er; de maa dyrkes som Specialstudium, eller ikke för end i Skolens överste Classe. "Ne donnez pas à votre fils un seul livre de mathemathiques avant qu'il ait achevé ses études classiques", sagde La Place til en Ven, som raadspurgte ham, hvorledes han bedst kunde anbringe til mathematiske Studier en Sön, som han troede havde Anlæg derfor (dette læstes i „Journal des Debats" for et Par Aar siden). Erfaringen stadfæster dette hos os.[277]

> But by thus emphasizing the importance of the ancient languages in the school, both in general and in particular for us, I must guard myself against belittling other subjects and in particular the mathematics and the natural sciences. I declare once and for all that I consider these sciences of utter importance, and in particular for us, who in this respect all the time have been behind, and whose country's natural resources are so little known, but I claim that their place is not in the school, at least not the natural sciences', in such an early stage as they are now; they must be nurtured as a special study, or not until the school's uppermost class. "Ne donnez pas à votre fils un seul livre de mathématiques avant qu'il ait achevé ses études classiques", La Place said to a friend, who consulted him about how he best could introduce mathematical studies to a son, who he thought had an inclination for those (this was read in "Journal des Débats" a couple of years ago). Experience confirms this with us.

Considering the role of Laplace (1749–1827) in the French Academy, this is an interesting quotation.

[275] Sveinbjörn Björnsson (1981): ix
[276] Guðjón Friðriksson (2003): 533, 552–553
[277] *Skýrsla um hinn lærða skóla í Reykjavík skólaárið 1862–63* (1863): 103–105

Obviously time in school had to be shared between the various subjects. Discussions were going on about ancient versus modern languages. Clearly, if the modern languages were to have some space, the hours had to be taken from something else, and in the headmaster's opinion it could not be the ancient languages. There were also other problems which the school had to face. The number of pupils was falling. During 1851–1856 they were 45 in total. During the following years the number fell, to 30 in 1862–1863. In the view of the headmaster, the reason was inferior home-instruction. The pupils were expected to be prepared from home in Danish and Latin. A subsidy for translating textbooks into Icelandic was sought.[278] In the Minister Prof. Madvig's opinion, the reason for the fall in the number of pupils could be that the interest in ancient languages was diminishing. He did not think it would be wise to translate textbooks for Icelandic pupils as

> ... jo mindre Kredsen for dette Modersmaal er (for Islandsken noget over 60,000 mennesker), desto mindre kan den afslutte sig, desto mere bliver for dem, der skulle opnaa og repræsentere en höiere Dannelse, tidlig og fuldstændig Tilegnelse af et mere udbredt Meddelelsesmiddel nödvendigt.
>
> The less the circles of this vernacular is (for the Icelandic a little more than 60,000 persons), the less it can isolate itself, the more necessary is an early and perfect adoption of a more widespread communication medium for those who are to gain higher education.[279]

Prof. Madvig did not think it wise at all to recommend sole use of Icelandic textbooks, but was inclined to advise them for the ancient languages. However, he did not want the present version of his own Latin grammar textbook translated, as he intended to shorten it, and he would prefer an older shorter edition to be translated. Finally, he said that a possible subsidy would have to be limited to a suitable honorarium for a translation and eventually some processing, and a yearly grant of a certain amount was not advisable. But at least a Latin grammar textbook, *Latnesk orðmyndafræði*, written by the teachers in the Learned School, Headmaster Jón Þorkelsson et al., was published in Icelandic in 1868.[280]

The discussions continued with petitions to the *Alþingi* and from the *Alþingi* to the King, in which for example in 1861 *Alþingi* suggested that the main emphasis would be placed on Greek and Latin, and instruction in other subjects would be adjusted to the needs of the nation.[281] From 1862–1863 the number of pupils began to increase, rising to 87 in 1868–1869.[282] New regulations were finally issued in 1877.

[278] Heimir Þorleifsson (1975): 24–25
[279] *Lovsamling for Island* 18 (1884): 20–23
[280] Jón Þorkelsson et al. (1868)
[281] *Alþingistíðindi* 1861: 813
[282] Heimir Þorleifsson (1975): 28

The 1877 Regulations

In 1871, the Danish parliament passed new legislation, by which the Latin schools' uppermost grades were divided into a language-history stream and a mathematics-science stream.[283] In 1875, the *Alþingi*, which had acquired legislative powers in 1874, appointed a school commission, called the School Affairs Board, composed of Bishop Pétur Pétursson, Provost and Member of Parliament Þórarinn Böðvarsson, Learned School Headmaster Jón Þorkelsson, Member of Parliament, poet Dr. Phil. Grímur Thomsen, and Reykjavík Primary School Headmaster Helgi E. Helgesen. The School Affairs Board wrote a proposal for regulations for the Reykjavík Learned School, another for the Theological Seminary and the third for a new lower secondary school, which led to the establishment of a school at the farm of Möðruvellir in Hörgárdalur near Akureyri, northern Iceland, in 1880. The proposal was dated October 5, 1876.[284]

Fig. 4.8. Dr. Grímur Thomsen.

Dr. Grímur Thomsen was an influential person. He was the first doctor of philosophy in modern English and French literature at the University of Copenhagen and had worked for years in a high position at the Danish Ministry of Foreign Affairs.[285] He may therefore have had different views on education from those who had been involved in Icelandic school affairs. In addition he was a poet and became highly regarded as such later. His opinions may have had great weight, as the proposal suggests.

The proposed changes in the new regulations for the Reykjavík Learned School were far smaller than in the Danish legislation, and the proposal was in many respects similar to the 1850 regulations. [286] The goals were about the same as in 1850. There were to be five grades, of which the last was to be repeated. Danish was to be taught for four years as before, and mathematics in all grades as previously. The mathematics content was to be the same as in the 1850 regulations, while it was described more accurately according to the Danish regulations dated August 5, 1871. Physics was to be taught in the third and fourth grades, which offered the possibility of teaching it jointly with a proposed lower secondary school in Reykjavík. Latin and Greek were to keep their former position, though with a slight decrease in Latin composition. Hebrew was to be abolished, while French was to be taught in all grades, English for the first four years and German for the last two years as an optional subject.[287] Previously German had been a compulsory subject for the first four years. According to the board, the reasons why French was chosen as the main foreign language were that priests, physicians, county magistrates and other officials needed to communicate with French seamen on the Icelandic coasts.[288] Another reason may have been that Dr.

[283] Skovgaard-Petersen, V. (1976): 12
[284] Heimir Þorleifsson (1975): 38
[285] Páll Eggert Ólason (1949): Vol. II, 105
[286] Heimir Þorleifsson (1975): 38–50
[287] *Álitsskjal nefndarinnar í skólamálinu* (1877): 19–47
[288] *Álitsskjal nefndarinnar í skólamálinu* (1877): 33

Grímur Thomsen was a proponent of French and English literature. It is not possible to conjecture about who suggested running the two streams jointly. According to what happened later it was not Headmaster Jón Þorkelsson.

When the regulations were published, the following main alterations had been made to it: Danish and religious studies were to be taught in all grades, while mathematics was to be completed in the fourth grade.[289] Several documents from the archives of the Governor of Iceland and the Ministry of Icelandic Affairs, preserved in the National Archives of Iceland, reveal the lobbyism going on in 1876–1877.[290]

Governor Hilmar Finsen sent the proposals of the School Affairs Board to the Minister of Justice and of Icelandic Affairs, J. Nellemann in Copenhagen, together with a letter dated October 20, 1876, containing the Governor's own comments on them. In a 34-page letter about the three issues, of which 17 were about the Learned School, the Governor expressed his concern about the workload of the pupils, having to study mathematics and Latin at the same time. He suggested that mathematics terminate after the fourth year, after which German would become a compulsory subject for two years. The Reykjavík Learned School would then become similar to the Danish learned schools' language-history stream. The Governor did not mention Danish. In the letter he stated that:

> ... den sproglig-historiske Undervisning maa ansees for den, der efter de for Haanden værende Forhold, er bedst skikket til at forberede Skolens Disciple til den Fagdannelse, som de senere agte at erhverve sig, og som, efter de herværende Forhold, i Reglen vil söges opnaaet ved at tage de som Kvalificationer, for at opnaae Embede heri Landet foreskrevne Embedsexamina enten ved de herværende höiere Dannelses-Anstalter, nemlig Præsteskolen og Lægeskolen eller, for Juristers og Filologers Vedkommende, ved Universitetet i Kjöbenhavn; men for alle disse Fags vedkommende maa den sproglig-historiske Undervisning, saavidt jeg skjönner, betragtes som den hensigtsmæssigste Forberedelse. Det vil höre til de meget sjeldne Undtagelser, at en Student fra den herværende Skole vil söge en videre gaaende Uddannelse ved Universitetet i de Fag, for hvilke den mathematisk naturvidenskabelige Undervisning maa betragtes som den bedste Forberedelse, og her i Landet have vi ikke Læreanstalter, hvor saadan Uddannelse kan erhverves.

> ... the language-historical teaching must be considered as that which for the present situation is best suited to prepare the school's pupils for the professional education they later plan to acquire, and which, by the present situation, usually will be attempted to gain by seeking it as qualifications for office examinations prescribed to obtain an office in this country, either at one of the present higher education institutes, that is the Theological Seminary or the Medical School, or concerning the lawyers or philologists, at the University in Copenhagen; but concerning all these subjects the language-historical education must be considered the most suitable preparation. It is an extremely rare exception if a student from the present school will seek further education at the University in subjects for which instruction in the mathematics and natural sciences must be considered as the best preparation, and in this country we do not have learning institutions where such instruction can be acquired.[291]

Minister Nellemann forwarded the proposals to King Christian IX with a letter dated July 10, 1877, where he expressed his view that it was necessary to increase instruction in Danish in the Icelandic learned school, as that language was of the greatest importance to Icelandic officials as a business language. Furthermore,

[289] *Stjórnartíðindi* 1877 no. 8, July 12
[290] See Appendix B for transcripts of details of the documents and their translations
[291] National Archives of Iceland: Íslenska stjórnardeildin. S. VI, 5. Isl. Journal 15, nr. 680 *Skólamál*.

religious studies should be taught through all grades and German as a compulsory subject in the last two grades. This would not overload the pupils, as mathematics could be reduced.[292]

On July 12, 1877 the regulations were published, whereby mathematics was concluded after the fourth grade and Latin composition was to be continued in the fourth grade instead of being concluded after the third grade, as the board had suggested. Danish and religious studies became compulsory in all grades and German in the last two years.[293] Minister Nellemann sent the Governor a copy of his letter, dated July 12, on the day of publication of the regulations.

Repercussions

Governor Hilmar Finsen (1824–1886) was the grandson of the last bishop of Skálholt, Hannes Finnsson. He grew up in Denmark, and later became the mayor of Copenhagen and a minister in the Danish government. It seems odd for the Governor to write such a long letter about details of Icelandic school affairs, 35 pages. Certainly, the school had great weight in the finances of the country, but finances were not at issue here. It seems reasonable to think that some of the members of the School Affairs Board were discontented with the board's proposal, and had found an alternative way, through the Governor, to express their ideas. Discussions in the *Alþingi*'s sessions in 1877 and 1879 and two letters from 1882 could point to that conclusion.

The German teacher of the Learned School, Halldór Kr. Friðriksson (1819–1902),[294] was a member of *Alþingi*. In *Alþingi*'s session in summer 1877 he posed two questions to the Governor; firstly why the teachers and the administration of the school had not been given an opportunity to present their opinion about the new school regulations before they were adopted, and secondly how they were to be implemented this autumn. In his introduction Halldór Kr. Friðriksson criticized the arrangements that German had been transferred to the uppermost grade, that English and French started at the same time in the first class and moreover, that much of what had previously been taught in mathematics was now to be abandoned. One could say that not everyone was expected to become a mathematician, but by this act general education was reduced. Mathematics had a great role, as it was a kind of instruction in thinking for mankind. Halldór Kr. Friðriksson stated that no institutions in France, England and Germany on the same level as the present school, did not teach at least as much mathematics as had been done in the Reykjavík Learned School up to this time.

Fig. 4.9. Halldór Kr. Friðriksson.

The Governor answered Halldór Kr. Friðriksson's questions on August 13, 1877. The Governor said that he had not thought that it was necessary to present the resolution of the School Affairs Board to the teachers, as the headmaster had been a

[292] National Archives of Iceland: Skjalasafn landshöfðingja, LhJ 1877, N nr. 621
[293] *Stjórnartíðindi* 1877 A, no. 8, July 12
[294] *Skýrsla um hinn lærða skóla í Reykjavík skólaárið 1861–62*: 6

member of the board. Dr. Phil. Grímur Thomsen, a member of the board and of parliament, said that the board had expected that the headmaster would consult with his teachers about its work, and the Rev. Þórarinn Böðvarsson, another member of the board and parliament, agreed. Dr. Grímur Thomsen said that, as there was one more foreign language in Iceland to cope with than in Denmark, i.e. Danish, one language had therefore had to be dropped, for which German had been chosen.[295]

This remark suggests that it was Dr. Grímur Thomsen's idea to put French in the first place of languages, an idea that turned out to be difficult to realize. Discrepancies between the board's proposal and the regulations did not enter the discussion, so the board's proposals, which were published in a booklet, probably first appeared later in the year.

Switching suddenly between German, which had a long tradition in the Icelandic Learned School, and French, a more remote language, with practically no school tradition, is an example of the difficulties involved in changing school traditions in an educational system, governed by its own logistic rules and built-in inertia. In spite of considerable contacts with French fishermen on the Icelandic coasts, the French language had few proponents except Dr. Grímur Thomsen.

Further Discussions

In the following years the newspapers reflected some discontent about the regulations.[296] In 1879 the *Alþingi* resolved that the Governor should set up a board of all the teachers and two others to revise the 1877 regulations and propose alterations to it. Halldór Kr. Friðriksson brought up the matter.[297]

In 1882, the teachers wrote a letter, dated November 20, to the authorities, requesting as their main emphasis that German replace French as the first of the three new modern languages, and secondly that mathematics be restored to its previous status as a six-year subject. In their argumentation, they claimed that mathematics education was insufficient in itself without trigonometry and stereometry. They drew attention to the fact that trigonometry supported physics and astronomy, and that these topics "finalized and perfected" mathematics education. This would achieve the necessary preparation for those wanting to continue the study of mathematics at a higher institution. Secondly, the topics in question were, in their opinion, important for the country's "technical life", and

> … hyggjum vjer þeim mun meiri ástæðu til bera, að kenna þær í hinum lærða skóla, sem þær ekki eru kenndar í neinum öðrum skóla hjer á landi nú sem stendur, og landsmenn þannig ekki eiga neinn kost á að afla sjer þekkingar í þeim nema með sjálfskennslu.
>
> ... Eptir hinni eldri reglugjörð ... var danska að eins kennd 4 fyrstu skólaárin, og bar þó ekki á öðru, en að stúdentarnir væri fullfærir bæði að skilja dönsku og gjöra sig skiljanlega.

[295] *Alþingistíðindi* 1877: 636–643
[296] Heimir Þorleifsson (1975): 50
[297] Alþingistíðindi 1879: 408, 499–500

> … we think that there is the more reason to teach them in the Learned School, as they are not taught in any other school in this country currently, so our countrymen thus do not have any choice to acquire knowledge in them except by self-instruction.
>
> … By the older regulations … Danish was only taught through the 4 first school-years, and yet there was no indication other than that the students were fully capable of both understanding Danish and make themselves understood.[298]

Björn M. Olsen, a philologist and later headmaster, expressed himself in an appendix to the letter, saying that he considered it doubtful that the advantages of the alterations suggested would counterbalance the chaos and confusion they must cause while being implemented. However, he considered it right to immediately exchange German for French.

Headmaster Jón Þorkelsson, who referred to attached remarks, Björn M. Ólsen, Halldór Kr. Friðriksson, mathematics teacher Halldór Guðmundsson, Steingrímur Thorsteinsson, Benedikt Gröndal, who taught natural sciences, and Sigurður Sigurðarson signed the letter. In the letter, the teachers most strongly emphasized the exchange of German and French, even if their defence for mathematics teaching was well founded. Furthermore, one notices that Björn M. Olsen was, after all, not interested in introducing mathematics again.

Headmaster Jón Þorkelsson wrote a separate letter, where he said that in spite of the fact that he had signed the teachers' letter, he only agreed with the teachers in a few items, such as the exchange of German for French, and that he agreed with Björn M. Olsen's remarks. He reiterated that four new compulsory subjects had been introduced, and therefore instruction in mathematics had to be reduced. The present quantity of mathematics, which was nearly the same as required in the language-history stream in Danish learned schools, would suffice for all but those who were heading for the Polytechnic College [in Copenhagen]. Headmaster Jón Þorláksson claimed that hardly more than one Icelander per decade attended the Polytechnic College, and those few would have to seek private instruction in mathematics. The hours for more mathematics would inevitably have to be gained at the cost of the languages, and he, for his part, put the greatest emphasis on them. The headmaster also opposed the idea of reducing instruction in Danish, considering the relation Iceland had to Denmark, and in particular to the University of Copenhagen.[299]

The headmaster's letter was dated on November 27, 1882. It should be borne in mind that Headmaster Jón Þorkelsson was a member of the School Affairs Board that made the proposal for the regulations in 1876. The headmaster was a philologist and taught Latin. One is tempted to guess, after reading his letter, that he was in a minority on the School Affairs Board, and that he eventually shared his concerns with the Governor.

As the opinions of the teachers were unanimous only on the languages, the consequences were that the regulations were altered so that German became the primary modern foreign language alongside Danish, while mathematics was still terminated after four years. Its status and respect dropped, as illustrated by the fact that examination problems ceased to be printed in school reports after 1882, and printing first resumed in 1910.

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[298] National Archives of Iceland: Íslenska stjórnardeildin. S. VI, 5. Isl. Journal 15, no. 680 *Skólamál*
[299] National Archives of Iceland: Íslenska stjórnardeildin. S. VI, 5. Isl. Journal 15, no. 680 *Skólamál*

One also suspects some of the teachers of promoting their own interests in maintaining German as the second foreign language.

The Reykjavík Learned School thus became a language-history stream school in the Danish school system, with diminished mathematics teaching compared to previous times. No discussions in the following years could alter this situation. The pupils were mainly occupied with practical arithmetic.

The absence of higher mathematics education coincided with a period when Icelandic society was preparing to build up its infrastructure. However, at that time the need for mathematics was not yet perceived. No technical knowledge whatsoever existed in the country at this time, and there were few individuals who had the imagination to conceive that such knowledge could ensure a good position and a handsome income. There was e.g. no military with its immediate need for knowledge of engineering. Nor were students encouraged to seek the narrow road of mathematical education. That route did not open at the Learned School for 42 years, until 1919.

Arguments

It is noteworthy that all the main reasons mentioned by M. Niss concerning mathematics education were drawn into the debate. Halldór Kr. Friðriksson's arguments include mathematics' great role as instruction in thinking for mankind. This argument can be classified as contributing to society's cultural maintenance, and may also be thought of as providing individuals with prerequisites to cope with life in an educated way.

The arguments of the teachers also concern the fundamental reasons, i.e. that mathematics education

- contributed to society's cultural maintenance, as they considered the mathematics education, presently offered by the school, to be insufficient in itself without trigonometry and stereometry and that they would "finalize and perfect" mathematics education in the school;
- provided individuals with prerequisites for further mathematical studies at a higher institution;
- contributed to the technological development of society, in that it was important for the country's "technical life".

By mentioning the importance for the "technical life", the teachers reemphasised Björn Gunnlaugsson's arguments, 60 years earlier, that "every nation should have its *mathematicos* to send them out into nature to research its mysteries and who then point out to the nation where it should search to find the resources which are hidden in it." The process of utilizing nature's resources for "technical life" had not yet begun in Iceland. Neither the Governor nor the Minister of Icelandic Affairs in Copenhagen seems to have thought of that reason for mathematics education while they were exerting their influence on Iceland's school affairs. Icelandic society at that time lacked roads, bridges and harbours, and only a few buildings were made of durable materials. While authorities were beginning to realize that there was indeed a need for technical knowledge, there was no universal agreement that the origin of such knowledge should come from the Learned School.

The Governor's arguments concerned the present society. His arguments were that the pupils of Reykjavík Learned School were seeking qualifications for official examinations in theology, medicine, law or philology, and that anything else would be an extremely rare exception. In 1877, learned persons of other kinds, such as engineers, could not expect any official position in Iceland. However, educational administration requires a little foresight, as mentioned in Dr. Bahr's presentation to Icelandic educators in 1965. In 1886 attention of the authorities was brought to the fact that it might be less expensive to pay a salary to an Icelandic engineer than to a foreigner. In addition he might be better acquainted to Icelandic conditions than any foreign person and stay longer. That same year the first Icelandic engineer, Sigurður Thoroddsen, began his studies in Copenhagen.[300] In 1893 the office of National Engineer for Iceland was established.

Last Decades of the Reykjavík Learned School

After Björn Gunnlaugsson retired in 1862, Halldór Guðmundsson took over mathematics teaching at the Learned School. Halldór continued to use the same textbooks as Björn Gunnlaugsson had done. After the publication of Björn Gunnlaugsson's *Simple Land-Surveying* / *Einföld landmæling* in 1869, it was taught every other year in the uppermost class. No progress in mathematics teaching is attributed to Halldór Guðmundsson, and he had to submit, together with his colleagues, to the 1877 regulations. In 1877, the *Lærebog i den elementære Plangeometri* / *Textbook in Elementary Plane Geometry* by Julius Petersen was introduced. It survived until 1971 at the then Reykjavík High School, for the last 28 years in a translated form, as the original could not be obtained from Denmark during World War II.

For years, people discussed the regulations. Finnur Jónsson, later professor in Copenhagen, wrote in 1883 about the mathematics teaching:

> Stærðafræði er kennd að eins í 4 neðri bekkjunum; þessi fræði hefir, svo langt sem jeg man, ekki átt neinum vinsældum að fagna hjá hávaðanum af piltum, og optlega hafa þeir spurt að, hvað það ætti að þýða að kenna svona mikið í stærðafræði, og eru slíkar spurningar vottur um sorglega kennslu og sorglegan misskilning. Ef kennarinn getur ekki einu sinni komið lærisveinum sínum í skilning um gildi þeirrar fræðigreinar er hann kennir, þá er eitthvað veilt við kennsluna alla í heild sinni, enda veit jeg og að það hefir verið; *það sem vestu hefir gegnt, er skortur á skriflegum æfingum*; ... alla dýpri eigna skilning hefir vantað, öll verkleg notkun hefir verið lokuð úti, og þess vegna hafa menn verið að spyrja um, hvers vegna allt þetta skuli lært; það er eðlileg afleiðing fáfræðinnar. Síðan jeg fór úr skóla hefir þetta lítið breytzt hvað kennsluna snertir, – kennarinn er hinn sami enn –, en það sem kennt er, er ekki hið sama; nýja reglugjörðin hefir 1) kippt burtu – þríhyrningafræði, 2) lagt það fyrir, að stærðafræði sje að eins kennd 4 fyrstu árin (áður öll) og þar með sleppt til burtfararprófs, og 3) að rúmmálsfræði skuli byrja þegar í neðsta bekk; þetta þrennt er nú að hyggju minni jafnmörg axarsköpt; ...
>
> Mathematics is only taught in the 4 lower grades; these studies have, as far as I remember, not at all been popular among the majority of the pupils and they have often asked what sense there was in teaching this much mathematics, and such questions witness lamentable teaching and a lamentable misunderstanding. If the teacher cannot even make his students understand the value of the subject he teaches, then there is something wrong with the teaching on the whole, and I know also that this was the case; *the worst has been the lack of written exercises*; ... all deeper understanding has been missing, all practical use has been excluded and

[300] Sveinn Þórðarson (2002): 16–17

therefore people have been asking why all this has to be studied; that is the natural consequence of ignorance. Since I left school this has not changed markedly concerning the teaching, – the teacher is still the same – , but what is taught is not the same; the new regulations have 1) taken away – trigonometry, 2) prescribed that mathematics is only to be taught the 4 first years (previously all) and thereby dropped for the graduation examination, and 3) geometry shall commence already in the lowest class; these three items are as I conceive it equally many blunders; ...[301]

In continuation, the author explained the damaging alterations in the regulations. Judging from Finnur Jónsson's article, many pupils may not have felt the loss of mathematics at all. Prof. Finnur Jónsson published *Hauksbók* including *Algorismus* in a scientific edition in 1892–1896.

That same year, in 1883, Cand. Phil. Björn Jensson was appointed to teach at the school, and in 1885, when Halldór Guðmundsson retired, Björn Jensson became the main mathematics teacher. Instruction is said to have greatly improved on his arrival at the school.[302] Teaching mathematics was the only official position for a mathematician to be expected in Iceland. There was only one full-time position, at the Reykjavík Learned School, although there were several part-time assistant teachers during this period. Björn Jensson had good reputation as teacher, but even he was reported to be considered impatient by some of those who knew little, possibly due to his bad health.[303]

Theological Seminary teacher the Reverend Eiríkur Briem taught mathematics in the lowest classes for several years. A comment about the Rev. Eiríkur Briem is found in Engineer Jón Þorláksson's biography, saying that the Rev. Eiríkur Briem was a conscientious, venerable and strict teacher, who kept a distance from his pupils and emphasized rote learning.[304]

The Rev. Eiríkur Briem wrote a textbook, *Reikningsbók / Arithmetic* (1869), which was used in the period 1875–1883, but together with V. Bertelsen's *Regnebog for Seminarier og Realskole*r.[305] Eiríkur Briem's textbook was probably the first mathematics textbook in Icelandic to be used at the Learned School for more than a short period.

In 1888–1891 only books by Julius Petersen were used in all grades, in addition to V. Bertelsen's *Regnebog* in the first grade. Petersen's textbooks are of the latest editions when editions are mentioned in the school reports. In 1892 Halldór Briem's *Þykkvamálsfræði / Stereometry,* published in Reykjavík 1892, was taken up in the fourth grade. That book was taught until 1907, while in 1898 Bertelsen's book was superseded by Meyer's *Praktisk Regnebog II*. In 1905 the 5th ed. of 1902 was used.[306]

The Royal Directorate of the University and the Learned Schools continued to send books to the school's library, as did the Royal Danish Science Society and other institutes and individuals in Denmark, Iceland and other countries.

[301] Finnur Jónsson (1883): 115–116
[302] Sveinn Þórðarson (2002): 13
[303] Hannes Hólmsteinn Gissurarson (1992): 43
[304] Hannes Hólmsteinn Gissurarson (1992): 43
[305] *Skýrsla um hinn lærða skóla í Reykjavík skóla-árið 1875-1876*: 11; *1882–1883*: 13
[306] *Skýrsla um hinn lærða skóla / hinn almenna menntaskóla í Reykjavík./School reports* 1888–1905

Sigurður Thoroddsen was appointed National Engineer for Iceland in 1893. In 1904 he chose to apply for the post of mathematics teacher at the Reykjavík High School, rather than continue the thankless task of Iceland's National Engineer.[307] Sigurður Thoroddsen remained as mathematics teacher at the Reykjavík School until 1935.

The next Icelandic mathematician, Ólafur Daníelsson, studied mathematics under Björn Jensson. According to memoirs of Ólafur Daníelsson' friend, Björn Jensson opened his eyes to the wonders of mathematics. Ólafur Daníelsson taught Guðmundur Arnlaugsson and had as such great influence on him. Guðmundur Arnlaugsson was the main pioneer of "modern" mathematics teaching in Iceland in the 1960s. There are therefore hierarchical links within the very small Icelandic mathematical community. Björn Gunnlaugsson presumably studied the work of Stefán Björnsson. Later he influenced his own grandson, Björn Jensson, who influenced Ólafur Daníelsson, who in turn influenced Guðmundur Arnlaugsson.

4.6. Progress in Public Education

Public Education Legislation 1880

Throughout the 19th century, the regulations published after Harboe's visit in the 1740s were the basis for public education in Iceland. The families, supervised by the parish priests, were responsible for children's knowledge of reading and Christianity. A King's letter in 1790 forbade parish priests to confirm a child who was not able to read from a book. After *Alþingi* was granted legislative powers in 1874 and the country had acquired its own Treasury, there were frequent debates for education. In 1879, the *Alþingi* passed legislation concerning children's knowledge of writing and arithmetic. The King confirmed the legislation in 1880. However, implementation was still the responsibility of the families under the supervision of the parish priests.

The 1880 legislation is short, only four paragraphs, of which one concerns fines. The paragraph concerning arithmetic is the following:

> § 2. Reikningskennsla skal að minnsta kosti ná yfir samlagning, frádragning, margföldun og deiling í heilum tölum og tugabrotum.
>
> § 2. The teaching of arithmetic shall at least include addition, subtraction, multiplication and division in whole numbers and decimal fractions.[308]

Home-instruction was the official and accepted way of children's education until 1907.[309] The initiative for public education was mainly private enterprise, while the *Alþingi* allocated funds to public education as a special item in the Budget from 1878 onwards.[310] Around 1880 primary schools had been established in many towns, and itinerant schools (*farskólar*) in some rural areas.[311]

A lower secondary school was established at Möðruvellir in 1880, following the 1877 regulations. During the 1890s there were many debates in *Alþingi* on the Reykjavík Learned School. They concerned mainly its proposed lower secondary

[307] Sveinbjörn Björnsson (1981): ix
[308] *Alþingistíðindi* 1879: 521
[309] Ólafur H. Jóhannsson (1994): 13–14
[310] Sverrir Jakobsson (1994): 78
[311] Guðni Jónsson (1932): 12

school department for the general public, and its connection to the Möðruvellir School.[312]

Arithmetic Textbooks by Eiríkur Briem

In the second half of the 19th century, several textbooks were published for general education. One of them was *Reikningsbók / Arithmetic* by the Rev. Eiríkur Briem (1846–1929). The book was first published in 1869, and later reprinted several times, the last time in 1911, so it must have been widely used. Its first edition contained the four operations in whole numbers, common fractions and decimal fractions, *regula de tri* and area and volume computation for the first time in Icelandic printed textbooks. It was used at the Learned School in 1875–1883 and at the Möðruvellir School 1880–1905. Later it was divided into two volumes and extended. The second volume contained percentages, interests, equations, exponents, logarithms, area and volume calculations, and currency calculations. That volume was never used in the Learned School, but at Möðruvellir School, where the author's brother was mathematics teacher.[313]

In his foreword to the second edition of the second volume, the Rev. Eiríkur Briem said:

> Reikningsbók þessa hef jeg kostað kapps um að hafa svo ljósa, að menn, er löngun hefðu til að læra reikning og allgóðar gáfur til þess, gætu haft not af bókinni, þótt þeir hefðu litla eða enga tilsögn; á hinn bóginn vildi jeg eigi hafa bókina orðfyllri en svo, að hana mætti jafnframt nota til að kenna eptir; við kaflann um bókstafareikning, líkingar og reikning með logarithmum hef jeg þó gjört ráð fyrir að menn nytu nokkurrar tilsagnar; í kafla þessum hef jeg eins og annarstaðar leitt hjá mjer að færa rök fyrir reglum þeim, er settar eru; þar sem á stöku stað að vikið er frá þessu, þá er það af því, að röksemdin gat sjálf verið æfingardæmi eða hún lá svo ljóst fyrir, að hún gat verið til að festa regluna betur í minni.
>
> I have made efforts to make this arithmetic book so clear, that people, who had the desire to learn arithmetic, and had pretty good gifts for it, could use the book, even if they had little or no instruction; on the other hand I did not want to have the book more verbose than so that it could be used for teaching; at the chapter about algebra, equations and logarithmic calculations I have, however, expected that people had some instruction; in that chapter I have, as elsewhere, avoided supporting by reasoning the rules prescribed; where in several places I have made exceptions, then it is because the reasoning could as well be an exercise or it was so clear that it could be used to support the memorizing of the rule.[314]

The main emphasis is clearly on pupils memorizing the rules, and the author only brings up reasoning if it can support the pupils in their memorizing. He does not emphasize understanding, except that he warns that in most cases it is necessary to work through the preceding chapters to understand the following ones. Later in the foreword, the author emphasized the importance of exercise, in order to react effectively to problems. Knowing some arithmetic was unavoidable for every person. Parallel to progress in society, increased commerce and more diverse industry, the more need people had for knowing arithmetic well, and one would not have full use of the knowledge unless one computed easily without errors, which demanded

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312 Heimir Þorleifsson (1975): 54–68

313 *Skýrsla um Gagnfræða Skólann á Möðruvöllum/Möðruvallaskólann fyrir skólaárið 1881–1882, ..., 1904–1905*

314 Eiríkur Briem (1880): iii

considerable practice, so revising more than once or twice was of the utmost necessity,

> ... einkum hættir þeim, er tilsagnar njóta, mjög við, að gleyma aptur, ef þeir eigi hvað eptir annað rifja það upp, er þeir hafa numið.
>
> ... especially those who have had instruction are apt to forget again, if they do not regularly revise what they have learnt.[315]

This sentence is somewhat remarkable, as it expresses the author's suspicion that instruction from outside may be vulnerable, and not as effective as self-instruction.

The author said that, as he avoided giving reasons for the rules he put down, he was generally not able to use foreign books as models; some examples were taken from other books, mainly Ursin's and Bertelsen's arithmetic.[316] The author's avoidance of reasoning seems to be a reaction to what he saw as excessive formalism in the foreign textbooks.

Whatever the author says about his reasoning, some of his problems are illustrative and in line with society, as may be illustrated by the following one: a problem on equations in two variables, trying to estimate what to pay for feeding sheep, which was one of the main financial problems of a common farmer. The problem could well be a real problem in the farming of the time.

> Maður nokkur tók veturgamla sauði til fóðurs af kunningja sínum; borgunin var óákveðin, en átti að fara eptir því, hvernig veturinn yrði; um vorið bar þeim á milli um, hve miklu heyi sauðirnir mundu hafa eytt, og gat maðurinn ekki gert glöggari grein fyrir því en svo, að hann hafði gefið 40 sauðum og 36 ám af heyi, sem 116 hestar voru í, og var það þrotið; enn fremur hafði hann gefið 35 sauðum og 30 ám af heyi, sem 148 hestar voru í, en þriðjungur þess var óeyddur. Hve miklu heyi mun hver sauður og hver ær hafa eytt?
>
> A man took one-winter-old wethers to feed from his acquaintance; the payment was not decided upon, but was to depend on how the winter would proceed; in the spring they disagreed upon how much hay the wethers had consumed, and the man could not explain it better than that he had foddered 40 wethers and 36 ewes from hay measuring 116 horse loads and that was run out; furthermore he had foddered 35 wethers and 30 ewes from hay measuring 148 horse loads, of which one-third remained. How much hay would each wether and each ewe have consumed?[317]

The answer to this problem was realistic, also in that the ewes, carrying lambs, consumed more than the castrated wethers, and as such might have contributed to the understanding of mathematics as an economic tool.

Reikningsbók by Þórður Thoroddsen

In the second year of Möðruvellir School in 1881–1882, physician Þórður J. Thoroddsen taught there. Þórður Thoroddsen was the brother of National Engineer Sigurður Thoroddsen. He used his own *Reikningsbók / Arithmetic,* published in 1880 and republished in 1884 with *regula de tri*, in all essentials similar to Eiríkur Briem's book. After 1882, Þórður Thoroddsen gave up teaching for his medical practice.

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[315] Eiríkur Briem (1880): iv
[316] Eiríkur Briem (1880): iii–iv
[317] Eiríkur Briem (1880): 76–77

Geometry Textbooks by Halldór Briem

The Reverend Halldór Briem (1852–1929), teacher at Möðruvellir School from 1882, was the Rev. Eiríkur Briem's younger brother. Halldór Briem published his *Kennslubók í flatamálsfræði* / *Textbook on Plane Geometry* in 1889. In the foreword he said that he had aimed the book at what was important in "general working life" and left out what was less important in that respect. Therefore, while in other textbooks each statement was followed by a scientific proof, this was not his objective. Pages 1–22 contain definitions, section II concerns congruence and similarity in eight pages, and the remaining 38 pages concern area and volume computations. This book was never used at the Learned School.

In his foreword the author promised to publish a stereometry, if the plane geometry was considered handy for the common people. Three years later, in 1892, Halldór Briem kept his promise and published his *Þykkvamálsfræði* / *Stereometry*. Even though, strictly speaking, stereometry was not part of the Learned School's curriculum, this book was taught at the Learned School for 15 years until new regulations were introduced. The main content is volume computations, up to sectors of spheres. The five Platonic bodies are also introduced. The author thanked Björn Jensson cordially for reading both books in manuscript and offering good advice.

The two brothers Briem had lived abroad before they wrote their books, Eiríkur Briem in Copenhagen in 1879–1880 where he studied philosophy, and Halldór Briem in Canada in 1876–1882, but it is not known if they studied mathematics. As far as can be seen, the brothers published their textbooks at their own expense, in the Rev. Eiríkur's case together with the printer, Einar Þórðarson.

Teacher Training at Flensborg School

For the southwest area, the deanery of Kjalarnes, the Thorkillii fund, established by Jón Þorkelsson in the 18th century, was still in existence, intended for the upbringing of poor children in the southwest. The fund had supported the school at Hausastaðir on Álftanes and later the Reykjavík Primary School. At other times, the money was divided among the local authorities in the southwest area for the support of orphaned children in the area. The Rev. Þórarinn Böðvarsson (1825–1895), a member of the School Affairs Board in 1875–1876 and pastor of the Garðar parish, which included Álftanes and Hafnarfjörður, was grandson of the leader of Hausastaðir School, the Rev. Þorvaldur Böðvarsson. The Rev. Þórarinn Böðvarsson argued that the fund's goal was to run schools.

In 1875, a bill was proposed in *Alþingi* on a lower secondary school in Hafnarfjörður, but not passed.[318] The next year, 1876, in order to establish a primary school for his parish, the Rev. Þórarinn Böðvarsson bought a house in Hafnarfjörður, called *Flensborg*, from the Danish merchant Knudtzon, built in the late 18th century. The funding of the school was to be a donation from him and his wife in memory of their son, Böðvar Þórarinsson, plus funding from the Treasury, the Thorkillii-fund and the local authorities. The donation, dated in 1877, included the school building and a farm to support the running of the school. Teaching began in the autumn that same year. The school was formally established by a charter dated in February 1878 and published by the authorities, the Governor and the bishop.[319]

[318] Sverrir Jakobsson (1994): 78
[319] Kristján Bersi Ólafsson (1982): 6–10

Somewhat unexpectedly, Dr. Grímur Thomsen, another member of the School Affairs Board, supported in 1880 the establishment of another primary school on Álftanes, in the same parish. The reason was partly that the distance was considerable, but also some disharmony between Dr. Grímur Thomsen and the Rev. Þórarinn Böðvarsson,[320] more evidence of a personal disagreement within the School Affairs Board of 1875–1876. At least this new school would have the same rights to a share of the Thorkillii-fund. This resulted in the Rev. Þórarinn and his wife changing their donation to a lower secondary school, formally established in 1882 by regulations adopted by the Governor.[321] The first headmaster of Flensborg Lower Secondary School was Jón Þórarinsson (1854–1926), their son. The Governor confirmed his appointment and that of two more teachers.[322]

After graduation from the Learned School in 1877, Jón Þórarinsson went to Copenhagen, originally to study theology. Soon he transferred to pedagogy, for which he travelled to Germany and England. Upon arriving home in 1882, he was appointed headmaster of Flensborg School.[323] The Rev. Þórarinn Böðvarsson was a member of the *Alþingi* in 1869–1894, as was his son Jón Þórarinsson in 1886–1900. They supported each other in putting forward proposals on educational matters.

After the 1880 legislation, by which writing and arithmetic were required for confirmation, fewer people were qualified to teach at home to prepare children for confirmation, and no teacher training programme was available. Many graduates from the two lower secondary schools, at Möðruvellir and Flensborg, went on to be appointed as teachers. The need for teacher training was becoming more obvious.

The Rev. Þórarinn and his son Jón proposed in 1887 an ambitious bill to the *Alþingi*, which in many ways was a model for the legislation on public education passed in 1907.[324] The responsibility for supervision was to be in the hands of a school board, including the parish priests, and the costs paid by the local communities. This bill was rejected by a narrow majority, and one of its opponents was Dr. Grímur Thomsen.

The bill contained clauses on the establishment of two teacher training colleges, one at Flensborg and the other at Möðruvellir. Prospective primary school teachers were to pursue a three-year programme and secondary school teachers a four-year programme. Among the arguments against that kind of a college put forward at *Alþingi* was that the two lower secondary schools and the Learned School would soon produce enough teachers. Indeed, education in the country was progressing fast, and it was in fact a sign of impatience to expect any faster progress. Interest in education was general and no opportunity offering itself for self-education was left unused.[325] Another argument brought up was that the Flensborg School had no legislation and the *Alþingi* granted it yearly funds, which could be terminated at any time. It had its first legislation in 1930.[326]

[320] Guðni Jónsson (1932): 31–32
[321] Guðni Jónsson (1932): 33
[322] Guðni Jónsson (1932): 42
[323] Guðni Jónsson (1932): 42–44
[324] *Alþingistíðindi* 1887 C: 107–116
[325] *Alþingistíðindi* 1887 B: 678
[326] *Alþingstíðindi* 1887 B: 679

Primary mathematics education before the age of 10 was to include the little multiplication table, addition, subtraction and multiplication in whole numbers. From the age of 10 to 14, children were to learn decimal fractions, manifold numbers (multi-digit numbers) and simple *regula de tri*. In the lower secondary schools for youngsters above the age of 14, which were to be established for the counties, one or more, they were to learn common fractions, percentages, interests, partition and the simplest rules for area computations.[327] In the teacher training colleges there was to be arithmetic. There was no mention of geometry at any of these four levels, apart from the area computations.

The concept of special training for secondary-school teachers was not to be realized for more than six decades. However, Jón Þórarinsson kept on working on his ideas. He became one of the founders of the first society of Icelandic teachers, *Hið íslenzka kennarafélag*, established in 1889, and he published in company with his colleagues a journal, *Tímarit um uppeldi og menntamál / A Journal on Upbringing and Education* in 1888–1892. At the society's annual general meeting in 1891 the main subject of discussion was teacher training. That same year Jón Þórarinsson wrote an article on the matter in the journal. In 1892, a decision was made to run a six-week teacher-training course for the upper department of the Flensborg School and the Governor published regulations on the matter, dated February 1, 1892.[328] Such courses were run for four years until 1895. Jón Þórarinsson, in company with another teacher, wrote in 1895 to the authorities, the Bishop and the Governor (*stiftsyfirvöld*), on the issue of teacher training. In 1896, the course was not held and that same summer the two teachers, Jón Þórarinsson and his colleague, went to Copenhagen to learn about teaching and learning in Danish schools, which had much longer yearly terms than the Flensborg School. In the six-week period while the Danish schools were closed they attended a course for teachers.

Fig. 4.10. Headmaster Jón Þórarinsson.

The result of the request to the authorities was that the allocation to teacher training in Flensborg was increased and the course extended into a whole year's college course in continuation of lower secondary school. The majority of the college students had completed the Flensborg School or had enjoyed elsewhere more education than was customary. The Flensborg Teacher Training College was run in 1897–1908. The number of graduates from this programme was 121.[329] The mathematics content was contained in two Nordic textbooks, N. Meiers' *Praktisk Regnebog* (unknown), and P. Deinboll's *Praktisk Geometri* (Norwegian).[330]

[327] *Alþingistíðindi* 1887 C: 109–113
[328] Guðni Jónsson (1932): 65–67
[329] Helgi Elíasson (1945): 15
[330] Deinboll, P. V. Br. (1904)

The Flensborg Teacher Training College was never chartered, although among its funds there were official allocations. A bill to establish a formal teacher training college at Flensborg was proposed at *Alþingi* in 1895 by Jón Þórarinsson and others, but it was not passed. A similar bill was presented in 1897, and again not passed.[331] These bills were, however, useful in the sense that they kept the discussions going and paved the way for new legislation on public education and a teacher training college in 1907. Jón Þórarinsson became the first State Director of Educational Affairs (*fræðslumálastjóri*) in 1908 and thus continued his influence on Icelandic school affairs to the day of his death.

Textbooks for Children

After the 1880 legislation, textbooks were published, intended for children and their teachers. Jóhannes Sigfússon published *Reikningsbók handa byrjöndum* / *Arithmetic for Beginners* in 1885. It is mainly a collection of problems. According to the author, they are arranged by increasing complexity. At the back of the book, there is a little guide on counting and computation. The book is 32 pages, and probably printed and published at the expense of the author.

Eiríkur Briem published a list of answers to his arithmetic textbook in 1884, six pages, and attached to that a six-page guide to teaching children arithmetic and eight pages of suggested problems suitable for mental arithmetic.

In 1890 Morten Hansen published his *Reikningsbók handa alþýðuskólum* / *Arithmetic for People's Schools* textbook, in which he introduced the abacus, which he had been distributing in the country, according to his foreword. The guide to the abacus was mainly taken from a guide made by a Mr. J.A. Bonnevie in Trondheim in 1886. The textbook itself was modelled after the textbooks of arithmetic teacher Chr. Hansen in Odense, mainly *Tavleregning-Opgaver II* and *III*. Morten Hansen's textbook was republished many times (first ed. 1890, sixth ed. 1911). The guide to the abacus was omitted after the first edition. Instructions in the arithmetic textbooks are minimal, and are mainly directed to the teacher. The computation methods are conventional, as far as can be seen.

Conditions for Learning

Throughout the 19th century, Iceland was exclusively a rural society, and farming, including fisheries, was the main occupation. Acquiring education could be a hard struggle for many young people, not least in mathematics. In his memoirs lawyer Ari Arnalds (1872–1957) described his quest for education in the last decades of the 19th century.[332] His parents, a farmer and a midwife, could only afford a private teacher, a learned school graduate, for one winter for their nine children. The children read and wrote in all their spare time from their farm tasks, which could be weaving in wintertime or watching the sheep during summer. While they watched the sheep, they read or recited verses. Ari Arnalds went through Eiríkur Briem's *Arithmetic* by himself, and learned without external instruction about whole number operations, common fractions, decimal fractions, *regula de tri* in various forms, area and volume calculations, percentages and interests. He also learnt Danish by independent study.

[331] Guðni Jónsson (1932): 83
[332] Ari Arnalds (1949): 13–51

As the parish priest knew that Ari Arnalds had been working hard on arithmetic, his problem for his confirmation was to compute the sum to be expected if a penny had been carrying 4% interests from the year when Christ was born. Ari Arnalds only knew the method to find the length of the period needed to double the sum, which in this case were 17 years and several months. From this he calculated up to the year 1886, coming up with a figure with 30 digits, which took him more than an hour.

After his confirmation Ari Arnalds began to work to save up money for private instruction by a clergyman to prepare for the entrance examination into the Learned School. He began the private instruction at the age of 19 and entered the Learned School at the age of 21, graduating at the age of 26. He then sailed for Copenhagen to study mathematics and astronomy. He reached a professor in astronomy, Thiele, who helpfully informed him that astronomic studies were long and expensive, and most graduates became teachers. Ari Arnalds therefore decided to study law. He became county magistrate and a well-known person in the independence campaign, while he had to leave his mathematics dreams.

Icelandic Ethno-Mathematics

Up to the late 19th century, ordinary people had few opportunities to study mathematics and few reasons to apply it. In the wintertime people had the leisure time to spend on artistic activities, such as embroidery and woodcarving. The National Museum preserves a number of artistic objects with mathematical patterns. These, however, do not differ greatly from objects made in other Nordic countries.

Epic poetry is another field of artistic activity, more typical and special for the Icelanders. Poetry has deep roots in Icelandic history. Many Icelanders composed poetry for kings and chieftains in medieval times. All the Old Norse verse preserved, with the exception of short poems preserved in runic inscriptions, was later written down in Iceland, where poetry seems to have become a profession. Between the ninth and thirteenth centuries, over one hundred poets were known to have been in the service of foreign medieval kings.[333]

Icelanders continued to compose verses, and they are the only nation that has preserved the ancient Germanic rules of prosody. This form has been preserved by the popular pastime of composing and reciting Icelandic ballads, which are called *ríma*. *Ríma* tells a rhymed story, often about battles, love, nature, horses, etc. Composing *ríma*, as well as single verses, is still a popular pursuit in the 21st century.

The rhyming is governed by complex rules. For instance, alliteration in the four-lined verse form known as *ferskeytla*, the quatrain form, a four-line stanza, requires that two words in the first line must begin with a vowel or the same consonant, placed in the line according to fixed rules. The first stressed syllable in the second line must also begin with a vowel or the same consonant. This alliterative pattern is then repeated in the third and fourth lines.[334]

[333] Gylfi Þ. Gíslason (1990): 29
[334] Gylfi Þ. Gíslason (1990): 103

In addition, there are end-rhymes. Here is an example of this art in English:

She is fine as morn' in May
mild, divine and clever,
like a shining summer day
she is mine forever.

The alliteration in the two first lines is composed of the words *morn'*, *May* and *mild* and in the second one *shining, summer* and *she.* This verse has been further decorated with an internal circular rhyme, *fine*, di*vine*, *shin*ing and *mine* in addition to the regular end-rhyme, *May-day* and *clever-(for)ever*.

This artistic puzzle, to compose a meaningful text within the complex constraints of length of words, length of lines, rhymes and alliteration, is a mathematical activity, which may be considered an Icelandic ethno-mathematics. The skill of composing a verse has been a popular artistic activity and sport of the common Icelander through the centuries, exercised by young and old, men and women. No material was needed and no colour, only a skilful mind and the memory to rehearse the product in the dark winter evenings and long summer nights at work.

Society

A large proportion of farmers travelled to seasonal fishing stations or sent their workers there through the centuries.[335] The fishing stations consisted of what can be described as seasonal villages. Hardly anywhere until the 19th century did fishing develop into a year-round occupation with its own fishing towns.[336]

Since the settlement, Icelanders had sailed and fished from open boats. This changed in the 19th century. In 1853 there were 25 schooners or decked fishing boats in the country, in 1887 the overall number was 86 and during the 1890s the number of schooners went up to 140. The schooners contributed to modernising Icelandic society. They were the first large-scale enterprises in the country, run by a single employer with a hundred or more employees. The schooners could follow the shoals of fish out into the fishing grounds and they were often fishing for around half the year. Some of the remaining period was spent on maintenance of the ship and fishing gear, so that the ship-owners could provide their fishermen with a year-round occupation. In this way the schooners created a professional class of fishermen for the first time in the history of Iceland.[337]

Figure 4.11 shows the distribution of population in Iceland 1801–1901 by economic sectors. The majority of the population is still engaged in agriculture. However, the fisheries as a separate economic sector account for an increasing proportion. Included in "Others" are paupers, about one fourth of the group at first, while the total number increased only slightly. The greatest increase in "Others" is in manufacture, trade and transport.[338] The drop in population in 1890 is mainly due to emigration to America but also to extremely cold climate in the 1880s.

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[335] Gunnar Karlsson (2000): 163
[336] Gunnar Karlsson (2000): 109
[337] Gunnar Karlsson (2000): 239–241
[338] *Hagskinna* (1997): Table 3.4. Population by industry 1801–1901

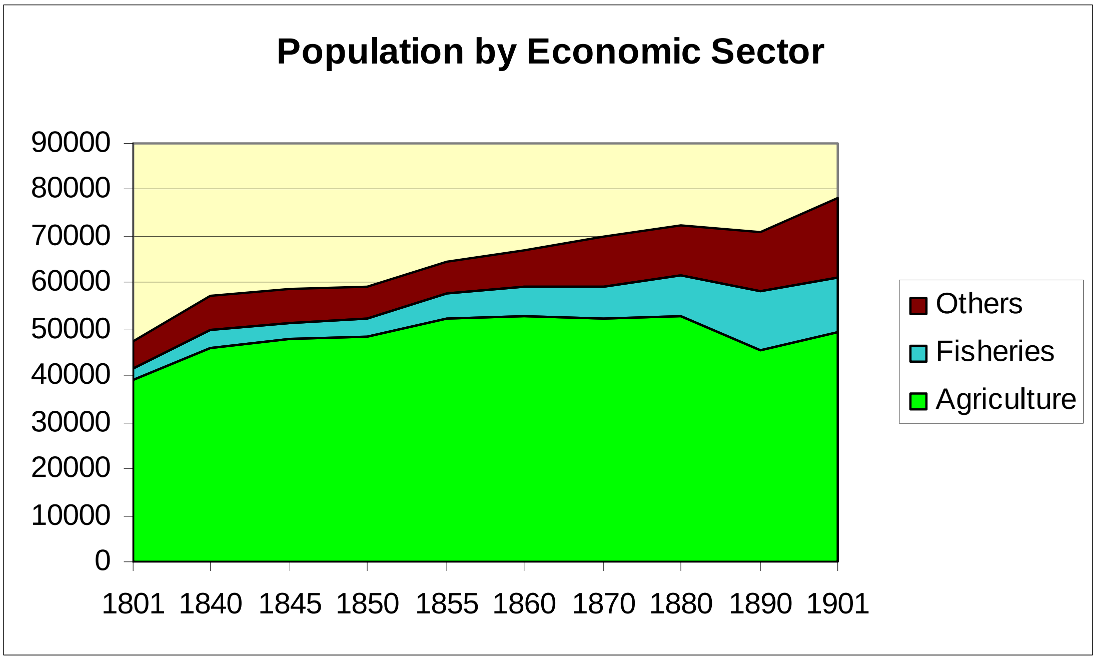

Fig. 4.11. Population by Economic Sector in 1801–1901.[339]

No statistical information about the division of the population between urban and rural areas is available until 1889, when 9,000 inhabitants, 13% of the population, lived in urban centres, towns and villages. The population of Reykjavík was a little more than 1,000 by the middle of the 19th century, while it was almost 4,000 in 1890.[340] The last decade of the 19th century turned out to be a period of rapid transformation in Icelandic society, which continued in the 20th century, not least in the first two decades.

[339] *Hagskinna* (1997): Table 3.4. Population by industry 1801–1901
[340] Gunnar Karlsson (2000): 239

5. The Early 20th Century

5.1. Introduction

The campaign for Iceland's independence continued through the last decades of the 19th century. On February 1, 1904, Iceland was granted Home Rule, i.e. an Icelandic Minister of Icelandic Affairs was appointed, situated in Iceland. The Danish monarchy introduced government by parliamentary majority in 1901, and in Iceland this was introduced simultaneously with Home Rule. Meanwhile, discussions continued in Denmark and Iceland about the learned schools, and in Iceland preparations were initiated for legislation on public education. The population was still small. Due to a number of cold climate years with ensuing pack ice in the 1880s and subsequent emigration to America, the number of inhabitants rose only slightly in the period 1880–1901, from 72,445 to 78,470.[341]

From the turn of the century onwards, the situation in the country improved greatly. The mechanization of the fishing fleet changed society radically. By the establishment of a bank with Danish and Norwegian sources, investment capital became available in the country for the first time, a middle class began to grow, and the basis for the present political parties was laid down in the 1910s and 1920s.

The 20th century was to see an ever-increasing population in urban nuclei vis-à-vis rural areas (figure 5.1.). All the increase in total population, and more, was absorbed by the towns.[342] In the year 1900, Reykjavík had 6,667 inhabitants, 8.5% of the total population, and in 1910 they numbered 11,600 or 13.6%.[343]

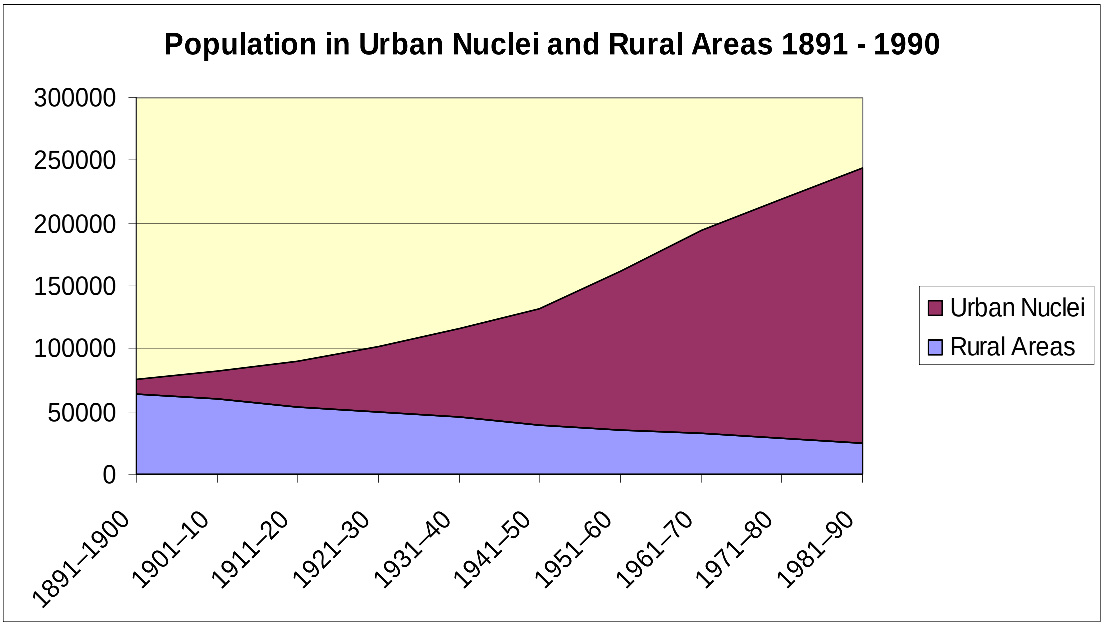

Fig. 5.1. Migration to Urban Nuclei.

[341] *Hagskinna* (1997): Table 2.1. Population in censuses 1703–1960 and according to the National Register of Persons 1953–1990
[342] *Hagskinna* (1997): Table 2.7. Population in urban nuclei and rural areas each year 1889–1990
[343] *Hagskinna* (1997): Table 2.4. Population by municipalities and administrative divisions 1901–1990

During the 1890s there were frequent debates in *Alþingi* about schools.[344] The Flensborg Teacher Training College was discussed, as was the lower secondary school at Möðruvellir. Many wished the pupils from Möðruvellir to be able to enter the upper grades of the Learned School. In order to make this possible, the Möðruvellir School would have to be extended from a two-year to a three-year programme, and its syllabus adapted to the Learned School. Meanwhile, the Möðruvellir School burnt down in 1902 and was replaced by a new school, established in 1904 in the nearby growing town of Akureyri. From 1908 the pupils from that school could enter the learned department of the Reykjavík School without entrance examination.[345] It was raised to the status of an upper secondary school in 1927, after having initated upper secondary teaching in 1924, and became formally *Menntaskólinn á Akureyri*, Akureyri High School, in 1930.[346]

The Reykjavík Learned School had a fair share of the discussions. *Alþingi* resolved in 1897 and 1902 that Greek should be abolished and Latin cut back. Similar discussions were going on in Denmark. The issue was put on hold, as the government in Denmark was reluctant to adopt the changes and said that it was not acceptable that the rules should be different for the entrance of Icelandic and Danish students to the University.[347]

In 1901 a young man, Guðmundur Finnbogason (1873–1944), completed a master's degree in philosophy and psychology at the University of Copenhagen. That same year *Alþingi* agreed to grant him a two-year subsidy to study pedagogical and educational matters abroad. It was stipulated that he would give a report of his research and make proposals about the arrangements of pedagogical and educational matters that he thought most suitable for Iceland. Guðmundur Finnbogason devoted one-and-a-half years to travelling in Denmark, Norway and Sweden in order to learn by experience about educational institutions in these countries.

On his arrival back in Iceland, Guðmundur Finnbogason did not begin by writing the requested report to *Alþingi*, but by writing a book called *Lýðmenntun / Education for the People*. Probably his intent was to seek support for his views from a wider audience than from *Alþingi*.

In 1903 Guðmundur Finnbogason was granted another subsidy for two years, to work on a report on the current situation of public education in Iceland. The report, *Skýrsla um fræðslu barna og unglinga veturinn 1903–1904 / A Report on Education of Children and Youth in the Winter of 1903–1904*, was published in 1905. The report provided important facts on which to base the forthcoming legislation.

In his work on preparing legislation on public education and regulations for the Reykjavík School, Guðmundur Finnbogason used many of the ideas brought up in the foregoing discussion period, e.g. from the education bill proposed by Þórarinn Böðvarsson and Jón Þórarinsson in 1887.[348] The ground had been prepared. Time had arrived to pass legislation on education, based on Guðmundur Finnbogason's research, both on schools in the neighbouring countries and on the current situation in Iceland.

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[344] See e.g. *Alþingistíðindi* 1895, 1897, 1899, 1901, 1902, 1903

[345] Helgi Elíasson (1945): 19

[346] Gísli Jónsson (Ed.) (1981): 181–205

[347] Heimir Þorleifsson (1975): 66. *Alþingistíðindi* 1901 C: 221–224

[348] Kristján Bersi Ólafsson, October 2002

5.2. The 1904 Regulations on the Reykjavík School

Proposal

On April 24, 1903, the Danish parliament passed new legislation on the learned schools. This opened the way for an amendment of the Icelandic Learned School's regulations. Guðmundur Finnbogason was assigned to write a proposal for new regulations. His proposal was dated in June 1904.[349] Its main features were:

- The school was to be divided into two departments, the middle school and the upper school, each lasting three years. Neither Latin nor Greek was to be taught in the middle school.
- The aim of the upper school was to prepare pupils to pursue higher scholarly studies at specialised educational institutions.
- In the upper school, pupils were to be offered a choice of two streams, with the main emphasis either on Latin or on mathematics and natural sciences.
- The school was to become coeducational, i.e. both for boys and girls.

The proposal contained a detailed syllabus in all subjects for the middle school, while a syllabus for the upper school was supposed to be announced later.

In Guðmundur Finnbogason's exposition of the proposal he explained that Cand. Polyt. Jón Þorláksson had informed him that Icelandic students had had to seek extra tuition in Copenhagen, 15–18 hours a week for a whole year, before they could enter the Polytechnic College (College of Advanced Technology). This had resulted in comparatively few students pursue engineering studies. In his letter, dated May 9, Jón Þorláksson said:

> Ef slíkri kenslu yrði komið á við skólann, mundu miklu fleiri stunda fjölvirkni en nú gerist, og hygg jeg að það væri gott, bæði fyrir stúdenta sjálfa – þeir komast betur áfram seinna meir – og eins frá almennu sjónarmiði, því að væntanlega líður þá ekki á löngu áður en hægt er að fá æfða menn, þ.e. ingeniöra sem hafa unnið erlendis nokkur ár að afloknu prófi, til starfa hjer á landi; það er ekki heppilegt frá almennu sjónarmiði að þurfa að taka mennina þegar er þeir hafa lokið prófi og fela þeim hvað vandasöm störf sem er. Námstíminn við fjölvirknisskólann er ekki nema 4 ½ ár og væri því fýsilegt að sækja hann ef ekki væri undirbúningsárið.

> If such tuition were to be established, many more would study engineering than is the case now, and I think that it would be good, both for the students themselves – they will be better off later – and also from the general point of view, as presumably then it would not be long until trained men will be available, i.e. engineers who have been working abroad for some years after their graduation, to work in this country; it is not practical from a general point of view to have to accept the men right after their graduation and entrust them with complicated tasks. The study time in the Polytechnic College is only 4½ years and therefore it would be desirable to pursue, if it were not for the preparatory year.[350]

[349] National Archives of Iceland: Skjalasafn stiftsyfirvalda. Stv. D.I. nr. 383-385 I. Dagb. I nr. 385
[350] National and University Library of Iceland: Bréfasafn Guðmundar Finnbogasonar

Jón Þorláksson (1877–1935) was one of the first Icelandic engineers, more precisely the fourth. He graduated from the Polytechnic College in Copenhagen in 1903, and therefore had not gained much experience. He was appointed as the first headmaster of the new Reykjavík Technical School (*Iðnskólinn í Reykjavík*) in 1904 and National Engineer in 1905, succeeding Sigurður Thoroddsen, who became mathematics teacher at the Reykjavík School in 1904. Jón Þorláksson went on to be Minister of Finance and Prime Minister of Iceland, and finally mayor of Reykjavík.[351] In 1904, however, Jón Þorláksson's words did not have as much weight as later.

Within a few days of Guðmundur Finnbogason submitting his proposal, the teachers of the Reykjavík Learned School must have had it in their hands, as on June 23 both Guðmundur Finnbogason's proposal and the teachers' revision of it was sent on from the newly- established Ministry in Iceland to the domestic directorate of school affairs, which still included the bishop (*stiftsyfirvöld*). The teachers had made several suggestions for alteration of the proposal, such as:[352]

- The school should be coeducational "when possible".
- The paragraph about two streams in the upper school was to be deleted.

Other suggestions for alterations were minor. Unfortunately, no record survives of the teachers' arguments for these changes, apart from their suggestions, neatly handwritten, without date and signatures. Only a draft to a letter from the directorate for the schools has been found. One wonders what the teachers' intentions were in demonstrating a slight resistance to the admission of girls. The directorate for the school agreed upon the teachers' suggestion, as accepting girls might entail greater costs. The teachers' resistance to a mathematics and science stream is more understandable, as they may have foreseen fewer teaching hours for themselves. The directorate also agreed upon this suggestion, as it would avoid increased costs.

There was no mathematics teacher at the Reykjavík Learned School in June 1904, as Björn Jensson died in February 1904. The Rev. Lárus Halldórsson stood in for him during his illness and after his death. The headmaster was Björn M. Olsen, who had not supported the revival of advanced mathematics teaching in 1882.

That was all, for the time being. The Reykjavík School was renamed *Hinn almenni menntaskóli í Reykjavík* and will hereafter be called Reykjavík High School. Greek was abolished and Latin teaching reduced, but the school remained a language-stream school. Somewhere in the process, the mission statement was specified as "to prepare pupils to pursue higher scientific studies at "the country's special educational institutions and the University of Copenhagen".[353] The end of the clause changed to "… at universities" in 1908,[354] even before University of Iceland was established in 1911. The horizons of the Icelandic ruling class were widening, following Home Rule.

[351] Hannes Hólmsteinn Gissurarson (1992)
[352] National Archives of Iceland: Skjalasafn stiftsyfirvalda. Stv. D.I. nr. 383-385 I. Dagb. I nr. 385
[353] *Stjórnartíðindi* 1904 A no. 13, September 9
[354] *Skýrsla um Hinn almenna menntaskóla í Reykjavík skólaárið 1907–1908*: 36

In spite of the proviso “when possible,” girls began to attend the school, few at first, but in growing numbers throughout the century. The proviso was omitted from the regulations in 1908.[355] The final names of the two departments of the school did not become the middle school and upper school, as in the original proposal, but a lower secondary department / *gagnfræðadeild* (literal translation: department of useful studies) and *lærdómsdeild*, learned department. This vocabulary may reflect that the intention was not to abandon the learned school altogether. And indeed for years influential intellectuals wished to return to the old days of a learned school with Latin and Greek.[356]

Mathematics in the Regulations

Guðmundur Finnbogason explained in his notes to the regulations that he had primarily made use of the new regulations for the Danish middle school when he prepared the Icelandic regulations. As the Danish middle school was to last four years and the Icelandic one only three years, it was unavoidable that the Icelandic school must have stricter entrance requirements. In addition, there were fewer schooldays a year in Iceland, and in many subjects there were inconvenient textbooks, written in a foreign language. The mathematics syllabus was intended to be the same as in the Danish schools, which had a total of 21–22 hours a week in the three years of the middle school, while the Icelandic one was to have 16 weekly hours. Mathematics hours in the Learned School had been 19. Mathematics therefore had to be taught in the first year of the upper school.[357]

The mathematics syllabus for the middle school in the regulations was divided into three parts: elementary arithmetic, advanced arithmetic and geometry. In elementary arithmetic, direct and compound proportions, partition, percentages, simple interests and the area and volume of simple bodies were to be studied, in addition to simple bookkeeping. The main topics in advanced arithmetic were the four operations in whole numbers and fractions, including decimal fractions, positive whole number exponents, prime numbers, divisibility and equations. Only square roots and their arithmetic were to be learnt, not cubic roots. The word “irrational” is mentioned in connection with square roots, but there was no mention of real numbers. Where a clause that no formula for solving quadratic equations should be learnt by heart had been taken from the Danish document,[358] the teachers suggested its omission, and it was omitted in the regulations.

In geometry, Guðmundur Finnbogason proposed that the possibility of incommensurability of two line segments should be proved. This item was not derived from the Danish document. The final version of the regulations did not contain that clause, at the suggestion of the teachers. There were angle measurements, congruent and similar triangles and parallel lines. The area of polygons was to be discussed, while the perimeter and area of circles were to be familiar to the pupils but not required to be mathematically derived, using exactly the same wording as in the Danish text.[359]

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[355] *Stjórnartíðindi* 1908 no. 4, March 13
[356] See e.g. Páll Sveinsson (1921): 20–34
[357] National Archives of Iceland: Skjalasafn stiftsyfirvalda. Stv. D.I. nr. 383-385 I. Dagb. I nr. 385. Uppkast að reglugjörð fyrir hinn lærða skóla í Reykjavík (June 1904): 13, 19
[358] Hansen, H.C. (2002): 50
[359] *Stjórnartíðindi* 1904 no. 13, September 9

On the whole, the teachers' version of the geometry section was nearly an exact translation of the corresponding section of the Danish curriculum for the middle school, published in 1904.[360] What Guðmundur Finnbogason had omitted from or added to the Danish version, the teachers had suggested should be revised, while they wished to keep the learning by heart of the solution of the quadratic equation.

The detailed regulations for the learned department of the school were published in 1908. From that time the total number of hours in mathematics increased slightly, distributed in such a way that there were two hours a week in all grades of the learned department. Trigonometry, which had been dropped in 1877, was now included in the syllabus. The hours were as shown below from first to sixth grade:[361]

1883–1884	6 – 5 – 4 – 4
1894–1895	5 – 5 – 4 – 5
1903–1904	5 – 5 – 4 – 5
1905–1906	5 – 6 – 4 – 5
1907–1908	5 – 6 – 5 – 2
1908–1909	5 – 6 – 5 – 2 – 2
1909–1910	5 – 6 – 5 – 2 – 2 – 2
1913–1914	5 – 6 – 5 – 2 – 2 – 2
1918–1919	5 – 6 – 5 – 2 – 2 – 2

Table 5.1. Number of weekly hours in mathematics at the Reykjavík School 1883–1919.

In spite of not achieving a division into two streams, there was a total increase of three weekly hours, and mathematics in all six grades, a step forward compared to the 1877 regulations.

Arguments and Reasoning

What can be deduced from the additions to and omissions from the original proposal? Philosophical speculations about incommensurability were superfluous, as they were not contained in the Danish regulations, while learning by heart was acceptable, even if it was not expected in the Danish regulations. The pedagogical view of mathematics education at that time is reflected in these two decisions. Björn Jensson had passed away, so there was no proponent of mathematics teaching at the Reykjavík School at that time.

Who influenced Guðmundur Finnbogason not to imitate the Danish curriculum completely? Philosopher Guðmundur Finnbogason, mathematician Ólafur Daníelsson and philosopher Ágúst H. Bjarnason were friends. Guðmundur Finnbogason and Ólafur Daníelsson graduated from the Reykjavík Learned School in 1896 and 1897. They were together at the University of Copenhagen, and lived at Regensen, the student residence where a number of Icelanders had their stipend. Jón Þorláksson, Ólafur Daníelsson's Learned School classmate, was there also. A photograph exists of all these men at the door of Regensen in 1898. Guðmundur Finnbogason and Ágúst H. Bjarnason graduated from the University of Copenhagen in 1901 and Jón Þorláksson from the Polytechnic College in 1903. Ólafur Daníelsson and Jón Þorláksson returned to Iceland in 1904. Ólafur Daníelsson's graduation was in April 1904, so Guðmundur Finnbogason and Ólafur Daníelsson may have been in contact while Guðmundur Finnbogason was preparing the regulations.

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[360] Hansen, H.C. (2002): 58

[361] *Skýrsla um hinn lærða skóla* / *Skýrsla um hinn almenna menntaskóla* / School reports 1883–1910

Someone must have been interested in the classical problem of incommensurability, which the teachers dropped. Possibly it was Guðmundur Finnbogason himself, who must have been knowledgeable about Greek mathematics. He was interested in mathematics, as witness his translating A. N. Whitehead's *Introduction to Mathematics* into Icelandic in 1930.

Figure 5.2. A photograph taken in 1898 at the door of Regensen, Copenhagen. In the front row, Ágúst H. Bjarnason is the 4th and Professor Finnur Jónsson 6th. In second row Ólafur Daníelsson is 3rd and Guðmundur Finnbogason 5th. In the middle row Jón Þorláksson is the 3rd.

There are no arguments available for the changes of the proposals. However, when considering the change of the aim of the school from preparing pupils for pursuing higher scholarly studies at specialized educational institutions to preparing them for some specific institutions, i.e. the available ones in Iceland and the University of Copenhagen, the changes of the mathematics paragraphs are of similar origin; to ensure the students' entrance to the University of Copenhagen, just as was the aim in 1822. Whatever ideas individuals, such as Guðmundur Finnbogason, might have about the aim of (mathematics) education for character training, or study of subjects such as mathematics for its own sake, for its philosophical value or as part of the world's cultural heritage, the main aim was to ensure that the students conformed with the current authorities' narrow horizon of further and higher education.

There is no trace whatsoever of any discussion of the aim of mathematical studies, such as can be found in Denmark at that time, and H.C. Hansen has explained in his book *Fra forstandens slibesten til borgerens værktøj*:[362]

> Principielt stod man frit i valget af pensum i matematik i mellemskolen, men traditionen fra latinskolen havde stor indflydelse. Der var enighed på tværs af skoleformerne om, at faget først og fremmest var formaldannende og specielt velegnet til udvikling af forstanden. Derimod var der stor debat om det euklidiske ideal, altså den aksiomatisk-deduktive fremstilling, der var kendt fra Euklid, var den bedste måde at fremme formaldannelsen på. Denne debat hang sammen med forskelligt syn på fagets natur. Var det grundlæggende et formelt fag, eller kunne det nærmere betragtes som et naturfag der studerede blandt andet rummet omkring os.

[362] Hansen, H.C. (2002): 41–59

> In principle, one was free in the choice of syllabus in mathematics in the middle school, while the tradition from the Latin school had great influence. There was animosity across the borders of the school forms about the subject being first and foremost formalizing and specially suited to train the intellect. On the other hand there was a great debate about whether the Euclidian ideal, i.e. the axiomatic-deductive presentation known from Euclid, was the best way to promote formalizing education. This debate was in connection with different view on the nature of the subject. Was it basically a formalizing subject or could it more be viewed as a nature subject, which e.g. studied the space around us.[363]

Such a debate did not exist in Iceland. The High School was an isolated institution where children entered at the age of 14, usually after private tuition, as there was an entrance examination. Lower secondary schools established during this period had no connection with the Reykjavík High School, except the Akureyri Lower Secondary School. These new schools accepted young people who had never been to school and needed basic arithmetic and bookkeeping. Geometry tuition there never went beyond area and volume computations. The teachers were usually theologians or learned school graduates, untrained in mathematical reasoning.

As a matter of fact, there was no one to carry on a debate. The main person capable of contributing to such a debate, Ólafur Dan Daníelsson, was excluded for some time, as we shall see. The Icelanders were busily occupied with building up their new society on their own premises, and the European debate in this field did not concern them for a long time.

Consequences of the Regulations

Reykjavík High School remained a language-stream high school for yet another one-and-a-half decades, and engineering students had to seek extra tuition in Copenhagen. In the period up to 1920, eleven engineers completed their studies in Copenhagen, two in Trondheim and one in Glasgow. During the same period, two Icelanders graduated from the University of Copenhagen in related subjects: Þorkell Þorkelsson, later director of the Meteorological Institute (*Veðurstofan*), in physics in 1903 and Ólafur Dan Daníelsson in 1904 in mathematics.[364] Both of them, Ólafur Daníelsson in particular, were to have a great impact on Icelandic mathematics teaching.

Meanwhile the Reykjavík High School operated under the new regulations. Although introductory trigonometry had been added to the mathematics syllabus for the learned department, in the second part of the regulations, published in 1908, the syllabus was weak. The problems were, however, hard enough, as may be concluded from the following examination problem from 1914:

$$\left(\frac{1\frac{92}{689} \cdot 4\frac{15}{142}}{\frac{32}{39} \cdot 1\frac{1}{14}} - \frac{\frac{707}{871} \cdot \frac{91}{101}}{\frac{21}{134}} \right) \cdot \frac{8190 \cdot 0{,}05^2 + 5{,}04}{11{,}34 : \frac{3}{4}} \cdot \frac{29 \cdot 0{,}12 \cdot 4{,}6 + 58 \cdot 7{,}164}{10{,}788}$$

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[363] Hansen, H. C. (2002): 41

[364] Sveinbjörn Björnsson (1981): ix–x

This was the first time that mathematics examination problems from the lower department were printed in the school reports and mathematics problems had not been printed in there since 1881. Before, each pupil had, according to the reports, a specially chosen problem for examination. A mathematics stream was first established in 1919.

The goal of the transformation of Reykjavík Learned School into the Reykjavík High School was to establish a more vital connection to other schools in the country and make the teaching more realistic and practical. This goal was not achieved, as the major steps required with respect to lower secondary schools had not been taken. The lower secondary department of Reykjavík High School also served as a general lower secondary school. That department and the Akureyri lower secondary school were the only ones qualified to prepare pupils for the upper, learned department of the Reykjavík High School. Considering the rigorous entrance requirements, it was no wonder that public discontent gradually rose. The general public saw the lower department of Reykjavík High School as being only a preparatory school for the learned department.[365]

Nearly all who passed the lower examination continued in the learned department. Of the 248 pupils who passed the lower examination in the period 1907–1916, only 16 dropped out, and the remaining 232 continued their studies in the learned department.[366] As Reykjavík grew, and no measures were taken to meet the needs for lower secondary schools, attendance at the High School became so large that restrictions were set in 1928 to limit the number of pupils admitted to the school.[367] The numbers were:[368]

Years	Pupils	Graduates
1919-1920	135	24
1929-1930	206	51
1939-1940	258	53
1945-1946	360	83

Table 5.2. Number of graduates from the Reykjavík High School 1919–1946.

The number of teachers grew more slowly, so the cost per pupil must have dropped. New education legislation in 1946 finally altered the school radically.

5.3. Technical Education

Times of Progress

The first two decades of the 20th century were the most progressive period up to that time in Icelandic history. This may partly be attributed to the optimism following Home Rule and a growing number of technically educated people, such as engineers. The main impetus was, however, the introduction of motorized vessels, and the establishment of the bank with foreign capital. Norwegians had great impact from 1880. They were active in catching and processing whale and herring in the West and East Fjords. They brought with them new technical knowledge, such as of steam engines and their maintenance.

[365] Heimir Þorleifsson (1975): 72–74
[366] Heimir Þorleifsson (1975): 72
[367] Heimir Þorleifsson (1975): 82
[368] *Skýrsla um hinn almenna menntaskóla* / *School reports* 1920–1946

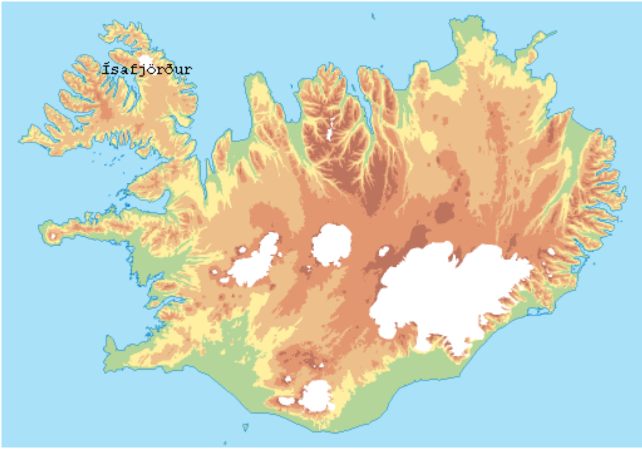

Fig. 5.3. Ísafjörður.[369]

The first steamboat in Icelandic possession was purchased and brought to Ísafjörður in 1890. The first fishing boat with an engine, fitted in 1902, belonged also to the people of Ísafjörður. In the period 1905–1910 the number of motorized boats in the whole country rose from approximately 60 in 1905 to about 380 in 1910. For maintenance of these machines, Danish and Norwegian blacksmiths and machinists settled in Iceland, and the Icelanders soon learnt from them, or were sent abroad for training.[370]

Figure 5.4. shows population in Iceland by economic sector during 1910–1960.[371] Manufacturing and trade had already become autonomous economic sectors at the beginning of the century. They were to grow throughout the period, especially during and after WWII. Construction, transport and financial, real estate and business services are counted together with trade. Ever fewer people worked in the fisheries, while for most of the period income in foreign currency was derived almost exclusively from the fisheries.

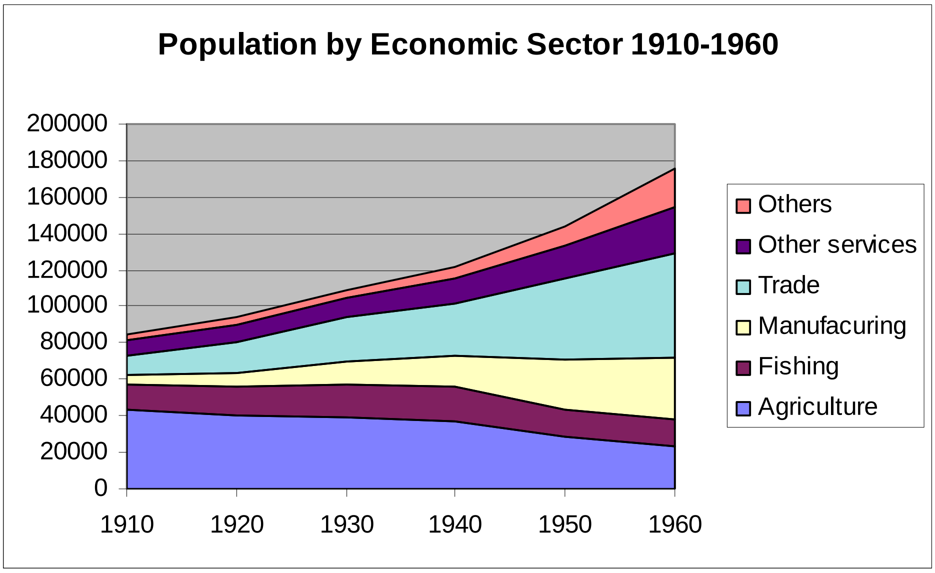

Fig. 5.4. Population in Iceland 1910–1960 by economic sector.

[369] National Land Survey of Iceland: website
[370] Sumarliði R. Ísleifsson (1987): 45–64
[371] *Hagskinna* (1997): Table 3.5. Population by industry 1910–1960

Jón Þorláksson's Contributions

The few engineers had many pioneering tasks to cope with. Jón Þorláksson established a metalworking firm together with a blacksmith and others, and while he was National Engineer he supported the National Metal Workshop (*Landssmiðjan*) in acquiring equipment, and he initiated the building of a bathhouse in Reykjavík.[372] He made a large contribution to the knowledge of the use of concrete. His opinion was that concrete, reinforced with iron, was the most suitable material for buildings in Iceland. He used this knowledge in many buildings and a number of bridges that he designed throughout the country as National Engineer.[373] He also contributed to the knowledge and implementation of the use of geothermal water.[374]

Fig. 5.5. Jón Þorláksson.

Certainly there were more people than Jón Þorláksson who contributed to Iceland's technical development. However, the above accounts show how one man's mathematical education can play a large role in the development of a small society. Another person worth mentioning is physicist Þorkell Þorkelsson, who did research on geothermal water. Þorkell Þorkelsson also made several contributions to the development of mathematics education at the Akureyri School, the Reykjavík High School and in the University, together with Ólafur Daníelsson, through the Association of Engineers.

Reykjavík Technical School

During the 19th century the number of craftsmen in various fields rose. They took on apprentices, and in 1893 legislation on crafts training was passed. After several experiments with evening schools and Sunday schools, formal tuition in technical drawing was initiated in 1893, since when it has been uninterrupted. In 1904 *Iðnskólinn í Reykjavík* / Reykjavík Technical School was established. Engineer Jón Þorláksson was its first headmaster until 1911, along with being National Engineer from 1905.[375]

The syllabus of the Reykjavík Technical School included arithmetic, Icelandic and Danish, but its main subject was technical drawing. The technical drawing was divided into drawing for carpenters, cabinetmakers and blacksmiths. As early as 1905 the school had acquired the form that it had for the following decades.

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372 Sumarliði R. Ísleifsson (1987): 71, 79, 99
373 Lýður Björnsson (1990): 76, 181
374 Sveinn Þórðarson (1998): 243–247
375 Ásgeir Ásgeirsson (1994): 7–14

Figure 5.6. Ólafur Daníelsson (wearing a black hat) in the middle of the group of teachers and pupils at Reykjavík Technical School in 1912.

The theoretical courses were short and elementary. However, the growing number of intellectuals had no other choice than to accept the teaching there was. Many renowned persons taught at the school. Among them were well-known poets and painters, philosopher Guðmundur Finnbogason, later professor at the University of Iceland and National Librarian, and mathematician Ólafur Daníelsson, who was the only applicant for the post of headmaster in 1916, but was not appointed.

5.4. Public Education

The 1907 legislation on public education is considered to have led to one of the greatest transformations of Icelandic society.[376] By that act the direction of school affairs was transferred from the church to local government. From Harboe's time in the 1740s the parish priests had had the duty of supervising home tuition. Home education did, however, not disappear completely until the last quarter of the 20th century.

The Icelandic authorities, having discussed public education through the 1890s, were probably influenced by new primary school legislation (*folkeskoleloven*) in Denmark in 1899. The legislation, however, was adapted to Icelandic circumstances, difficult transport, long distances and a near-total lack of school buildings, not to mention the fact that nearly all teachers had to be trained from scratch. As a consequence, 7- to 9-year-old children were to be taught at home, and the school requirements applied only to 10- to 14-year-old pupils.

Guðmundur Finnbogason and *Lýðmenntun*

Guðmundur Finnbogason prepared the legislation. His report on the situation of educational matters in the school year 1903–1904 served as background information. In the small towns and villages, schools had arisen with support from the National Treasury. In rural areas a system of itinerant schools, in addition to home tuition, was predominant. The teachers travelled from farm to farm, where the children gathered, according to the circumstances in each area. A total of 829 teaching locations were counted, while classrooms as such numbered 218. There were 415 teachers, 321 males and 94 females. Most of them were young people. Only 24 had been to a teacher training college, and one-third of the group, 134, were people who had never been to

[376] Ólafur Ásgeirsson, National Archivist, November 8, 2001

school themselves. Out of the other 257 were 99 graduates from a lower secondary schools, 11 high school graduates and 18 university graduates, mainly theologians. Most of the remaining 129 had some vocational education.[377]

In his book *Lýðmenntun* (1903), Guðmundur Finnbogason explained his vision of a school system in Iceland. In the first chapter the author expressed his ideas on education. The following 10 chapters were devoted to the various school subjects. A chapter about ways of establishing schools and their financial support followed, then a chapter on libraries, and the last one concerning a teacher training college. All this became useful when Guðmundur Finnbogason was assigned to write proposals for legislation and regulations for the Icelandic school system.[378]

In *Lýðmenntun*, Guðmundur Finnbogason expressed his thoughts about education. These are his thoughts about mathematics teaching:

> Lífið neyðir hvern mann til að reikna, hvort sem hann hefur lært nokkrar reiknisaðferðir eða þekkir nokkurn tölustaf eða ekki.
>
> Life compels every person to compute, whether he/she has learnt any computation methods or knows any digits or not.[379]

The author continued to explain how necessity compels a farmer, a housewife or a shepherd to compute in order to keep track of what they have acquired or are to share out.

> En ég skal undireins taka það fram, að eigi reikningskennslan að æfa og efla skilninginn og vekja sjálfstæða hugsun, má ekki haga henni eins og á sér stað víðast á Íslandi, þar sem ég þekki til. Börnin læra reikningsaðferðirnar og nota þær, án þess að skilja minnstu vitund, hvernig á þessum aðferðum stendur, *hvers vegna* farið er svona að því og ekki öðru vísi. En slíkt er niðurdrep fyrir allan skilning og sjálfstæða hugsun. Börnin verða líkust reiknivélum ... Atvikin rétta að oss reikningsdæmin „óuppsett", og þá þurfum vér fyrst að skilja hvað um er að vera, skilja þau stærðahlutföll sem um er að ræða, og þar næst að hafa næga leikni til að inna útreikninginn fljótt og rétt af hendi. Þetta tvennt, skilningur og leikni, útheimtist til að reikna vel, og það verður að haldast í hendur og styðja hvað annað. ... Og alstaðar þar sem reiknikennslan er komin í rétt horf er hún byggð á hlutskoðun. Börnin fá hugmyndirnar um tölurnar og sambönd þeirra við að athuga og fást við sýnilega og áþreifanlega hluti, telja þá og reikna með þeim.
>
> However, I must right away emphasize, that if arithmetic teaching shall train and increase the understanding and awake an independent thinking, then it must not be arranged as is done in most places in Iceland, where I know. The children learn the computation methods and use them without understanding the least how these have been created, *why* one does so and not otherwise. Such things destroy all understanding and independent thinking. The children begin to resemble computation machines ... The incidences offer us the problems in their context, and then we first have to understand what it is about, understand the quantities in question and then have enough skills to perform the computations quickly and correctly. These two things, understanding and skills, are required to compute well, and they must be hand in hand and support each other. … And everywhere where arithmetic teaching is as it should be, it is built on working with objects. The children form their concepts of the numbers and their connection to each other by

[377] Guðmundur Finnbogason (1905): 16
[378] Ólafur H. Jóhannsson (1994): 9–10
[379] Guðmundur Finnbogason (1994): 91

> investigation and working with visible and tangible objects, counting them and computing by them.[380]

Following this, the author describes how one can use simple things as aids in tuition. Furthermore he writes:

> Eflaust mætti farga miklu af reikningsreglum þeim sem enn standa í sumum kennslubókum í reikningi, með hátíðlegu yfirbragði, eins og þær væru stignar af himnum ofan. Sá sem kann að hugsa og nota með „skynsamlegu viti" hinar fjórar höfuðgreinir reikningsins, getur t.d. ofurvel leyst úr hverju þríliðudæmi, þó hann hafi aldrei heyrt þríliðu nefnda á nafn, eða heyrt getið um forlið, miðlið og afturlið, né reglurnar um meðferð þeirra.
>
> Without doubt one could throw out many of the computation rules which still are found in some arithmetic textbooks, with a pompous look, as if they had stepped down from heaven. He who can think and use common-sensibly the four main computing operations, can for example solve any *regula de tri* problem, even if he/she has neither heard of the *regula de tri*, a front term, middle term or rear term nor the rules about their use.[381]

This quotation indicates that the teachers and/or authors of arithmetic textbooks took the syllabus of the Learned School and transferred it to the primary level. It may also be understood as a critique on the syllabus of the Learned School. Probably Guðmundur Finnbogason is referring the Rev. Eiríkur Briem's arithmetic textbook.

Then this:

> Öll reikningsdæmi sem notuð eru við kennsluna ættu helst að vera tekin beint út úr daglegu lífi nemendanna, fela í sér spurningar, sem þá langar til að fá úrlausn á.
>
> All the problems posed in the tuition should preferably be drawn directly out from the children's daily life, and contain questions to which the children would like to know the answers.[382]

Clearly, here talks a pedagogue and an educator. *Lýðmenntun* is required reading for student teachers in the present day, so modern are his ideas.

1907 Legislation on Public Education

The legislation was passed in 1907. The tuition of children up to the age of 10 was to be undertaken and paid for by the children's families. Every 10-year-old child was to know how to read and write. Every child, who reached the age of 14 years, should have learnt:

> §4. fjórar höfuðgreinar reiknings með heilum tölum og brotum og geta notað þær til þess að leysa úr auðveldum dæmum, sem koma fyrir í daglegu lífi, meðal annars til þess að reikna flatarmál og rúmmál einföldustu hluta; það skal og vera leikið í því að reikna með lágum tölum í huganum.
>
> §4. in arithmetic the four operations in whole numbers and fractions and be able to use these in order to solve simple problems coming up in daily life, e.g. to calculate the area and volume of simple bodies; he/she should also be skilled in mental arithmetic with small numbers.[383]

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[380] Guðmundur Finnbogason (1994): 92
[381] Guðmundur Finnbogason (1994): 93
[382] Guðmundur Finnbogason (1994): 94
[383] *Stjórnartíðindi* 1907 no. 59, November 22

The country was divided into school districts and teaching districts. Towns and villages were to be school districts, where every child was to have tuition for at least six months every year. The period could be reduced by a month if the school building was to be used to teach children aged 8–10 for at least two months in the spring or autumn. Rural communities, which were not defined as school districts, were teaching districts. They had the duty either to maintain itinerant schools, which were arranged so that every child in each district would have at least two months' tuition a year, or, if the teaching commission preferred, to engage one or more teachers as needed, to supervise home tuition.[384]

Schools

In the early 20th century, 80–90% of the age cohorts 10 to 12 years attended schools (see figure 5.6).[385] While the arrangement that children began attending schools at the age of 10, probably did not encourage early numeracy, it did contribute to general literacy, e.g. of women, who overtook more of the parental duties with increasing migration from the agricultural areas to towns, where the fathers worked away from home.

The schools in general were still far behind the other Nordic countries. In the periodical on education, *Menntamál*, 1929 there is a short clause stating that the state paid 40 *krónur* (crowns) a child for its education, while in Denmark the amount was 211 kr., in Norway 245 kr. and in Sweden it was 214 kr.[386]

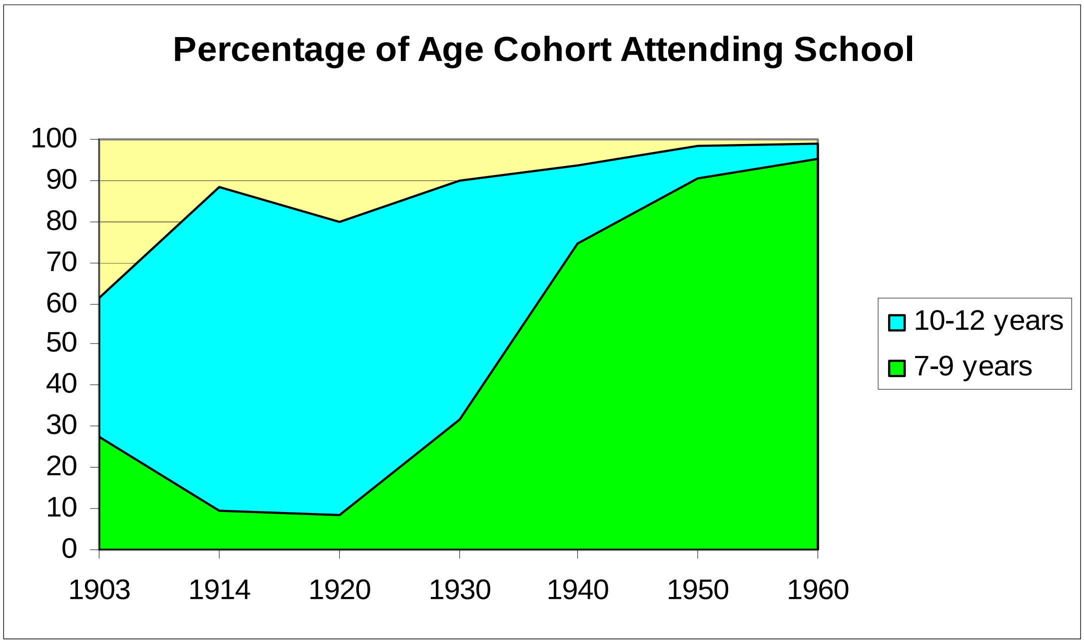

Fig. 5.7. Percentage of age cohort attending school. The figures 1903 are attained differently from the later ones. From 1920 the figures are attained at regular intervals.

[384] *Stjórnartíðindi* 1907 no. 59, November 22
[385] *Hagskinna* (1997): Table 18.2. Students by age cohort 1903–1990
[386] *Menntamál* (1929): 4 (3) 48

At the beginning of the century nearly 60% of children attended itinerant schools.

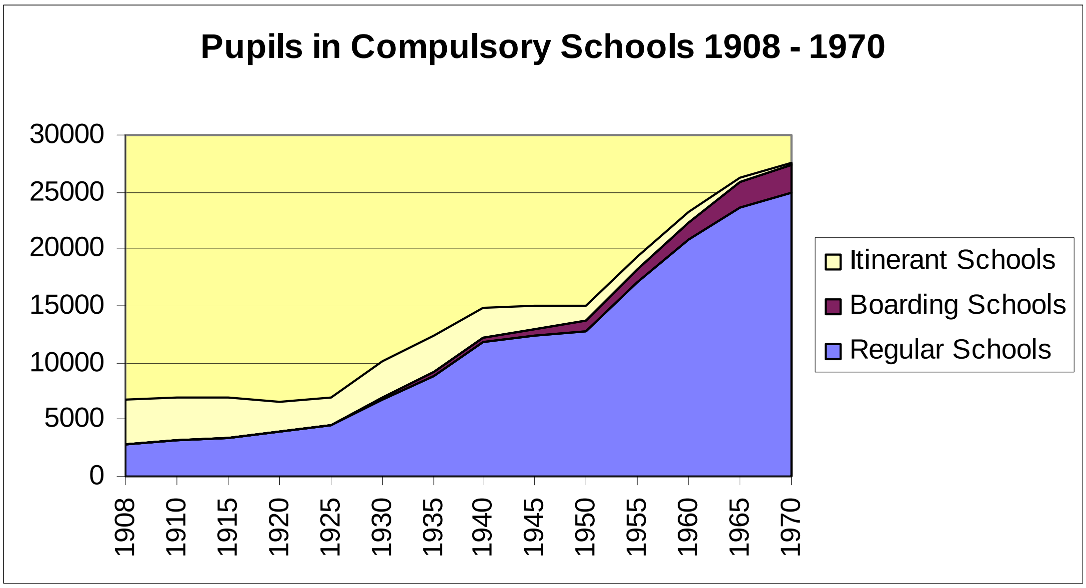

Fig. 5.8. Pupils in compulsory schools 1908–1970.

The itinerant schools were based on self-instruction for long periods, which was not always positive for mathematics learning. Presumably the majority of the pupils never progressed beyond the four operations in whole numbers (positive integers and zero), common fractions and decimal fractions, and probably had problems with that.

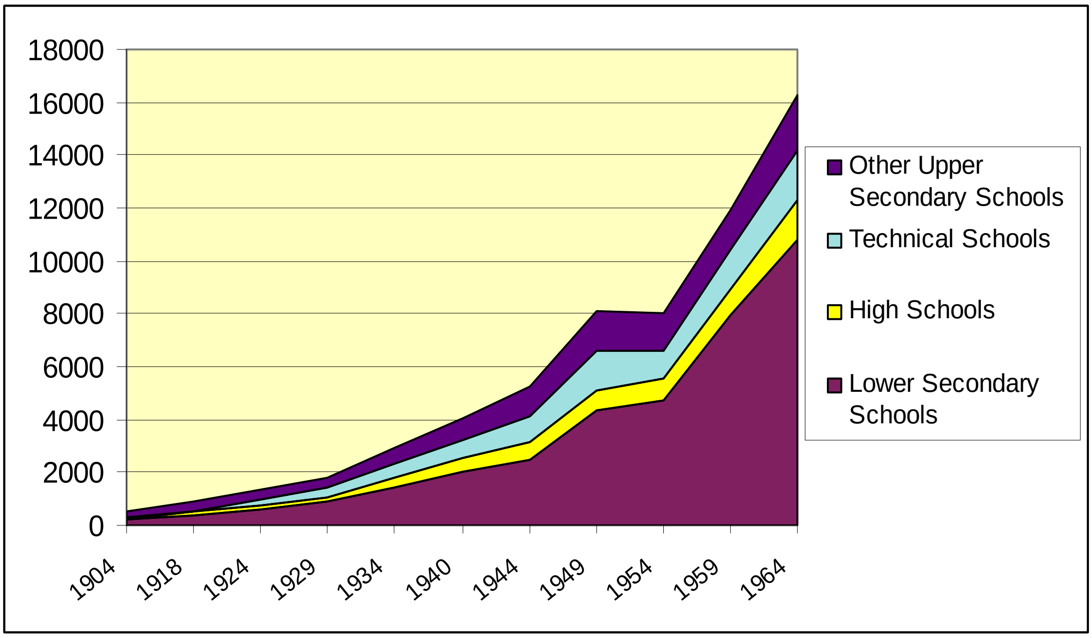

Fig. 5.9. Pupils in Secondary Schools in Iceland 1904–1960. The low number in 1954 is due to decreased birth rate in the late 1930s.[387]

A number of lower secondary schools for the general public were established in the early 20th century, especially around 1930. Figure 5.9. shows that during the first two thirds of the 20th century the number of pupils attending lower secondary schools increased much faster than the number attending other secondary schools.

[387] Hagskinna (1997): Table 18.8a. Values in 1924, 1934 and 1940 are partly attained by interpolation.

Included in “Other Upper Secondary Schools” are *Verzlunarskólinn*, a commercial school established in 1905, *Samvinnuskólinn*, the Co-operative Commercial College, established in 1919, *Vélskólinn*, a school for machinists, the Teacher Training College, navigation colleges, agricultural colleges etc. *Kvennaskólinn*, a school for girls established in 1874, is probably counted together with the lower secondary schools.

Distribution of Primary Level Textbooks in 1900s–1920s

Morten Hansen was the headmaster of Reykjavík Primary School 1890–1923. During that period his 1890 arithmetic textbook was dominant in his primary school. However, more textbooks came into being. Ögmundur Sigurðsson, teacher at Flensborg, published one in 1900, and Sigurður Jónsson, later headmaster at Reykjavík Primary School, another in 1906. Theologian Sigurbjörn Á. Gíslason published his *Reikningsbók / Arithmetic* in six volumes in 1911. Sigurbjörn Á. Gíslason, who later became renowned for his financial talents in his humanitarian work, certainly must have ensured some market for his book. He himself taught at various schools, such as *Kvennaskólinn*, *Verzlunarskólinn* and *Vélskólinn*, where he may have used the latter volumes, while the first volumes may have been used at some primary schools. The series was only published in two editions. Sigurbjörn Á. Gíslason’s books have far richer content than Morten Hansen’s books, and must thus have been more convenient for learning.

In 1914, Jörundur Brynjólfsson and Steingrímur Arason published an arithmetic textbook, *Reikningsbók handa alþýðuskólum*. This was deliberately of the concise type, as much arithmetic as possible up to proportions of the *regula de tri* type, on not too many pages. In 1928 the book was republished in a new and different edition, attributed to Steingrímur alone. For example, addition and subtraction were taught in parallel, as were multiplication and division. Steingrímur Arason had been studying in the USA, where he had become acquainted with the Winnetka movement and begun to think about learning in a new way.[388] However, teachers had no training in the new ways of work. The textbook by Steingrímur Arason and Sigurbjörn Á. Gíslason’s *Arithmetic* were two of the three arithmetic textbooks which were authorized for the primary schools by the ministry in charge of education in 1929. In spite of that Steingrímur Arason’s textbook did not gain distribution.

Jón Þórarinsson, headmaster of Flensborg School, became the first State Director of Educational Affairs in 1908. Guðmundur Finnbogason also applied for the post and was by many thought to deserve it after his work on the legislation and high school regulations. However, Jón Þórarinsson had been a strong proponent of education and deserved it no less, and he enjoyed seniority. In a letter from Director Jón Þórarinsson to a rural school board in 1909 it is clear that there was a choice of a variety of arithmetic textbooks for primary schools.[389] Jón Þórarinsson mentioned that Eiríkur Briem’s *Arithmetic* was considered difficult, which indicates that there was no sharp border between primary and lower secondary arithmetic textbooks.

[388] Kristín Indriðadóttir (1995): 9–33

[389] National Archives of Iceland: Skjalasafn fræðslumálaskrifstofunnar 1976-C/1 Bréfabók 1908-1909, 380-381

Another letter exists from Director Jón Þórarinsson, dated February 13, 1913,[390] to the Ministry, where he rejected Sigurbjörn Á. Gíslason's request for a grant for his publication. There was, Director Jón Þórarinsson said, already a choice of a good many arithmetic textbooks. Competition was healthy and necessary, while he doubted that the Treasury should support one textbook or another in that competition. The best would gain distribution and pay for itself.

The textbooks were in many respects similar in their introduction to computation algorithms. Their main differences lay in that authors such as Sigurbjörn Á. Gíslason (1911) and Steingrímur Arason in the second (1928) edition of his book, demonstrated that a choice of strategies might be used, especially in mental arithmetic, while e.g. Elías Bjarnason (1927) explicitly stated that he did not consider it advantageous to advise children to use various solution strategies.[391]

The Pioneer Elías Bjarnason

In 1919, Elías Bjarnason (1879–1970) began to teach at Reykjavík Primary School. He had been Dr. Ólafur Daníelsson's student at the Teacher Training College. *Reikningsbók / Arithmetic* by Elías Bjarnason was first published in 1927 and it was authorized in 1929. This first edition was two volumes, a little more than 100 pages each. Later, it was expanded into four volumes, 64–96 small pages each. In the forewords the author claims that his book was not modelled by any particular textbook, domestic or foreign. He had, however, tried to avoid inconsistency with Dr. Ólafur Daníelsson's *Arithmetic*. Furthermore he thanked Dr. Ólafur Daníelsson for hints that had improved the book, which indicates that the primary level textbook had Dr. Ólafur Daníelsson's sympathy.[392]

The textbook series by Elías Bjarnason was used at the upper primary level with minor alterations for over half a century, i.e. 1927~1980, while its distribution was disrupted for some years by "modern" mathematics. The author of the perennial textbook series, Elías Bjarnason, was mostly self-educated. He was a farmer's son who came from the countryside in southeast Iceland to the fishing village and commercial centre of Eyrarbakki in the southwest to fish in the wintertime in 1901.[393] In Eyrarbakki there was a good primary school, established in 1852, the oldest one in the country. The teacher allowed Elías Bjarnason to enjoy as much of his teaching as he could in his spare time, and he learnt to play the harmonium as well. Next winter, in 1902, Elías Bjarnason did not go to sea but established his own school with the support of his parish priest. After six years, in 1908, when the Teacher Training College was established, he attended it for one year, to graduate in 1909. By that time Elías Bjarnason was a married man, a father to three children and running a farm. After graduation, he went back to his school and farm, but in 1919 he became a teacher, and later head-teacher, at the Reykjavík Primary School. In his spare time he repaired and sold harmoniums. Elías Bjarnason retired in 1945. His eldest son was Helgi Elíasson, later State Director of Educational Affairs.

[390] National Archives of Iceland: Skjalasafn fræðslumálaskrifstofunnar 1976-C/2 Bréfabók 1909-1913, 978-979

[391] Elías Bjarnason (1927): 3

[392] Elías Bjarnason (1927): 4

[393] Marteinn M. Skaftfells (1945): 177–183. Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. I, 122.

Why did Elías Bjarnason's *Arithmetic* enjoy such a long life? One reason is the tight budget of the State Textbook Imprint which took over its publication at its establishment in 1937. As it had a usable set of textbooks, it may have been thought superfluous to publish new ones. Another reason might be that no teacher had the necessary educational background to write new textbooks, even though one could say that Elías Bjarnason, the pioneer, did not have much himself. A third reason to mention is that Elías Bjarnason's books had Dr. Ólafur Daníelsson' endorsement and they were consistent with his own *Arithmetic*. Dr. Ólafur Daníelsson had by then acquired dominant position in the Reykjavík High School, where the admission became highly restricted, as we shall see.

Mathematics Teaching at the Lower Secondary Schools

The border between the primary and lower secondary level was unclear for the first decades of the 20th century, as so many people had only little primary education. Before 1920 about 85–90% of the population did not receive any education after primary school. The majority of those who did, enjoyed education that was separated from the Learned/High School, so that only few pupils would have "truncated" mathematics education, as expressed by H.C. Hansen, i.e. the introductory parts to higher mathematics, offered by middle schools leading up to the high schools. As there was only one such school, the Akureyri School, besides the Reykjavík School itself, this would only concern a small minority. This was gradually changing when the Reykjavík High School's sphere of authority expanded.

Those who attended the two-year middle schools, first called a*lþýðuskólar* / folk high schools, later *héraðsskólar* / district schools and urban lower secondary schools, would usually have one of the more advanced arithmetic textbooks on the market. The last edition of the Rev. Eiríkur Briem's *Arithmetic* (1867) was published in 1911. Ólafur Daníelsson's *Reikningsbók* / *Arithmetic* appeared in its first edition in 1906 and so did the *Reikningsbók* / *Arithmetic* by the Rev. Jónas Jónasson of Hrafnagil. Gradually the Rev. Eiríkur Briem's book retreated for the more modern books.

The *Reikningsbók / Arithmetic* by the Rev. Jónas Jónasson had the same syllabus as Dr. Ólafur Daníelsson's book. A comparison of these two textbooks reflects that standardized Icelandic mathematics vocabulary had not yet been established. Jónas Jónasson offers a choice of methods. After explaining the standard digit-oriented addition algorithm, beginning by adding the units, as had been the custom since *Algorismus,* he then introduced the number-oriented approach, suitable for mental arithmetic, first adding the digits with highest place value, e.g. the thousands, then the hundreds, the tens and the units, in this order when adding up multi-digital sums. At a glance, Jónas Jónasson's book seems more flexible, while Dr. Ólafur Daníelsson's word problems may have been more inspiring and certainly more challenging. Jónas Jónasson's book was republished once, in 1911. Jónas Jónasson taught at Akureyri Lower Secondary School 1905–1917 and his *Arithmetic* was taught there until 1916, when the second edition of *Arithmetic* by Dr. Ólafur Daníelsson was introduced.

From 1908, physicist Þorkell Þorkelsson was appointed first teacher and main mathematics teacher at the Akureyri School. For some years he taught algebra from his own notes and in 1916 he published his *Stærðfræði handa gagnfræðaskólum / Mathematics for Lower Secondary Schools*.

Þorkell Þorkelsson's textbook is the first algebra textbook in Icelandic since Ólafur Stefánsson's *Stutt Undirvísun / Short Teaching* (1785), if *Tölvísi* by Björn Gunnlaugsson is not counted, as it was never used as a textbook. Þorkell Þorkelsson's book is straightforward and simple, without axiomatic approach. The book was used at Akureyri School until 1921, when the Danish textbooks used at Reykjavík High School were introduced. Þorkell Þorkelsson left Akureyri School in 1918.

From 1904 the mathematics requirements in Reykjavík High School were a nearly word-for-word copy of the requirements for the Danish middle school. Julius Petersen's *Plane Geometry* was used in the lower grades and his *Arithmetik og Algebra* was used in the upper grades. These books were rather formal, and reasoning and proofs may have been emphasized in oral examinations, but to judge from the written examinations, set by engineer Sigurður Thoroddsen, it seems that he mainly emphasized arithmetic. The Akureyri Lower Secondary School conformed with the requirements of the Reykjavík School, as it did after it became a high school, in the sense that the Akureyri School used mainly the books of the Reykjavík School, and from the 1920s exclusively those.

Except for these two schools in Reykjavík and Akureyri, there were no regulations or legislation on lower secondary schools until the late 1920s. A few privately operated schools taught mainly elementary arithmetic, as their pupils might previously only have had several weeks at an itinerant school. In the legislation on lower secondary schools, passed in 1930, the requirements were general arithmetic and basic area and volume computations.[394] No more detailed syllabus was published for the non-compulsory lower secondary level until 1968, except for the national examination, established in 1946.

The number of pupils attending secondary schools increased greatly from 1930. The formal establishment of lower secondary education was the result of the efforts of Minister Jónas Jónsson of Hrifla.

5.5. Dr. Ólafur Dan Daníelsson[395]

The mathematician Dr. Ólafur Dan Daníelsson (1877–1957) completed the Reykjavík Learned School in 1897, when there was no mathematics stream at the school. The main mathematics teacher was Björn Jensson, the grandson of Björn Gunnlaugsson. The Rev. Sigurbjörn Á. Gíslason, Ólafur Daníelsson's friend, who wrote his obituary, recalled that "he never forgot his joy and thankfulness when Björn Jensson opened up to him the labyrinth of mathematics". It seems that Björn Jensson was a devoted mathematics teacher, even if his syllabus was limited and he suffered from poor health.

Ólafur Daníelsson went to study mathematics in Copenhagen in 1897. A relative promised him financial support if he would study engineering, but he wanted to study mathematics.[396] At the beginning of his stay in Copenhagen he probably spent a year to complete the mathematics-natural science stream of the learned school. However, in 1900 he published his first article in the journal *Matematisk Tidsskrift* and the year after, in 1901, he earned a gold medal for a mathematical treatise, as Björn Gunnlaugsson had done 71 years before. The next Icelander to complete a degree in

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[394] *Stjórnartíðindi* 1930 no. 48, May 19
[395] The information in this section is drawn from Guðmundur Arnlaugsson and Sigurður Helgason (1996): 10–30, unless otherwise indicated
[396] Kristín Kaaber, September 28, 2004

pure mathematics was Sigurkarl Stefánsson in 1928, and the fourth Icelander to earn a gold medal at the University of Copenhagen was Sigurður Helgason, half a century after Ólafur Daníelsson. Ólafur Daníelsson's teachers at the University of Copenhagen were H. G. Zeuthen and Julius Petersen. Both of them were geometers. Ólafur Daníelsson's treatises were written under strong influences from Zeuthen and Petersen and his interest in geometry endured throughout his life.

In 1904 Ólafur Daníelsson completed his Mag. Scient. degree and returned to Iceland. As a promising young man he had been offered a position with an insurance company in Denmark, but he seems to have been determined to going home to create opportunities on his own, as positions there were scarce. The same year as Ólafur Daníelsson arrived in Iceland, Björn Jensson died, and Ólafur Daníelsson applied for his post as mathematics teacher at the Reykjavík High School. It was, however, granted to National Engineer Sigurður Thoroddsen who, after 12 years, had had enough of the difficulties as a pioneer in that position.[397] Over the next few years Ólafur Daníelsson published his first *Reikningsbók / Arithmetic* textbook in 1906 and prepared his doctoral thesis, defended in 1909. At the establishment of the Teacher Training College in 1908, Ólafur Daníelsson was appointed as the third teacher. In the first year he taught mathematics, natural science and geography, and after that mathematics, physics and Danish. In addition he taught at in-service courses for teachers, held for six weeks each spring.

Dr. Ólafur Daníelsson's *Arithmetic*

It was the good fortune of the Teacher Training College to have Ólafur Dan Daníelsson as its first mathematics teacher. His preface to his first edition of *Arithmetic* explains his thoughts about teaching. There Dr. Ólafur Daníelsson says:

> Þetta litla kver á, frá minni hálfu, að vera tilraun til þess að bæta úr tveim göllum, sem mjer þykja vera á flestum eða öllum reikningsbókum vorum; er annar sá að þær gefa alls engar skýringar, jafnvel ekki á einföldustu reikningsaðferðunum, og læra því margir aðferðirnar utanað, án þess að skilja hvernig á þeim stendur; og það því fremur, sem ýmsa af þeim er við kennslu fást mun vanta nægilega leikni í því að skýra eðli reikningsins frá rótum, án þess að hafa til þess neinn stuðning af kennslubókunum. En hinn gallinn er sá, að dæmin í þeim eru yfir höfuð helzt til ljett, og er hvert þeirra optast nær miðað aðeins við eina reikningsaðferð. Nemandinn getur því getið sjer til aðferðarinnar án þess að skilja dæmið. Þetta hef jeg viljað koma í veg fyrir. Jeg hef á eptir hverri grein haft nokkrar æfingar í því að beita þeim aðferðum jafnhliða, sem á undan eru komnar, og yfir höfuð að tala hef jeg leitast við að velja dæmin þannig, að þau ekki verði reiknuð, nema þau sjeu skilin til fulls.
>
> This little booklet is intended, on my part, to be an attempt to compensate for two drawbacks which I think characterize most or all of our arithmetic textbooks; one is that they give no explanations at all, not even of the simplest computation methods, and therefore many learn the methods by heart without understanding their reasons; and more so as many of those who work at teaching may lack sufficient skills to explain the arithmetic down to its roots, without having for that any support from the textbooks. But the other drawback is that their exercises are generally too easy, and each of them is most often aimed at only one computation method. The pupil can therefore guess the method without understanding the problem. This I have wanted to prevent. I have after each paragraph made several exercises in adopting simultaneously the methods that have previously appeared, and generally speaking I

[397] Valtýr Stefánsson (1958): 55–60

have tried to choose the exercises so that they may not be solved unless they are fully understood.[398]

The text in this first edition of *Arithmetic* thoroughly explains the basic elements of arithmetic, i.e. counting, the decimal place value system and the four operations: addition, subtraction, multiplication and division, in whole numbers and common and decimal fractions. Furthermore it contains a guide to the corresponding mental arithmetic. The written algorithms of addition, subtraction and multiplication are digit-oriented, i.e. begin from the right, while in mental arithmetic they are number-oriented, beginning from the right with the highest place value. Mental division is not introduced.

As emerges above, the author's intention was that the problems would require comprehension without becoming a routine. They concerned people mowing, selling hay, travelling on horses and boats and raising sheep, and thus were related to the daily life of the common people in the early 20th century.

In 1914 Dr. Ólafur Daníelsson published a new, more advanced, *Arithmetic*. It was altogether a different book. It now contained the metric system, proportions calculated by the *regula de tri*, in several variations, inverse proportions and the chain rule, percentages, interests, equations, area and volume. Basic algorithms and mental arithmetic are no longer included. This version of the book was, according to his foreword, adapted to his student teachers. Some of them may not have had any school education. Many people living in the rural areas throughout the first half of the 20th century acquired nearly all their basic education at home. Dr. Ólafur Daníelsson concluded the foreword by:

Það eru vandfengin einfaldari og snotrari viðfangsefni, heldur en lagleg reikningsdæmi.

It is hard to find simpler and neater tasks than artful arithmetic problems.[399]

When Dr. Ólafur Daníelsson began to teach at the Reykjavík High School in 1919, where the pupils were younger than the student teachers at the Teacher Training College, he needed both a basic and an advanced book. Pupils entered the High School at the age of 14 or so. An extended edition of his *Arithmetic* was published in 1920 to include some of the basic arithmetical rules from the first edition, especially on decimal and common fractions. Basic algorithms on whole numbers and mental arithmetic though were not included in this edition either. It seems that Dr. Ólafur Daníelsson regarded it as necessary to build up basic knowledge from scratch. In all editions of his books he explains thoroughly why things are done as they are, by simple examples from daily life, but there is no choice of methods; it is best to do things a certain way to avoid errors. Compared to e.g. the 1780 *Greinilig vegleiðsla / Clear Guidance* by Ólafur Olavius, there is much less flexibility in Dr. Ólafur Daníelsson's methods.

The 1920 edition of Ólafur Daníelsson's *Arithmetic* became a basis for arithmetic teaching in secondary education for half a century. It was on the syllabus of the Teacher Training College until 1952. It was last printed in 1956, and it was among the three textbooks from which a choice could be made in the syllabus for the national examination in 1974, even though at that time other arithmetic textbooks were more commonly used. They were of similar structure, while their applied problems reflected better the second half of the 20th century.

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398 Ólafur Daníelsson (1906): iii–iv
399 Ólafur Daníelsson (1914): Foreword

Geometry and *Trigonometry*

Um flatarmyndir / Geometry, a textbook by Dr. Ólafur Daníelsson, was also published in 1920, after he was appointed to the Reykjavík High School. In its foreword the author explained how difficult it was to compose a textbook in geometry which satisfied even the mildest requirements of logical reasoning and accuracy. Then he stated:

> Jeg ræðst ... í að gefa út bókina, einkum vegna þess að mjer finst mjer bera skylda til þess, þar sem jeg veit ekki til að aðrir núlifandi Íslendingar hafi lagt stund á þessa fræðigrein, svo teljandi sje.
>
> I take on this task to publish this book, mainly because I feel obliged to do this, as I do not know that other presently living Icelanders have studied this discipline to any degree.[400]

Dr. Ólafur Daníelsson had no support from a colleague, when it came to theoretical mathematics and geometry in particular. No wonder, that there were no documented discussions about geometry teaching in Iceland, as in Denmark.[401] He continued:

> Jeg þykist hafa orðið þess var, að ýmsum mentamönnum dylst gersamlega tilgangur stærðfræðinámsins í skólunum, halda, að takmark rúmfræðikenslunnar sje eitthvað í áttina til þess að kenna mönnum að mæla kálgarða eða túnskika. En þá væri illa varið löngum tíma og miklu erfiði, og held jeg fyrir mitt leyti, að betra væri þá að fá búfræðing til þess að mæla blettinn, en sleppa stærðfræðináminu í skólunum og losa þannig marga upprennandi mentamenn við mikið andstreymi.
>
> Að þetta sje ekki misskilningur minn, benda þau ákvæði skólareglugerðanna á, er mæla svo fyrir, að kenna skuli reglur um flatarmál og bogalínulengdir sannanalausar (sbr. orðin: „en ekki er heimtað að það sje stærðfræðilega rakið"). Það er í raun og veru sálarlaus stærðfræðikensla, sem á sjer engan rjett í hinum æðri skólum, nema ef vera skyldi þann, að æfa nemendur í reikningi. ... Nei, tilgangurinn ... er sá, að venja nemandann á þá nákvæmni í hugsun sinni og hugkvæmni um leið, sem engin önnur kenslugrein æfir hann í að sama skapi.
>
> I have been aware, that the purpose of mathematics education in schools is completely hidden from some intellectuals; they think that the goal of the geometry teaching is something in the direction of teaching people to measure cabbage gardens or grass fields. But then a long time and a lot of work would be badly spent, and I think, for my part, that then it would be better to have an agronomist to measure the piece of land and thus rid many of the future intellectuals of great adversity.
>
> That this is not my misunderstanding is indicated by the clauses in the school regulations which prescribe that the rules about area and arch lengths should be taught without proofs (cf. the words: "but it is not required to be mathematically derived"). It is really mathematics education without a soul, which has no right to exist in the higher schools, apart from perhaps the one to offer pupils exercise in arithmetic. ... No, the purpose ... is to train the pupil in precision of his thinking and at the same time his inventiveness, which no other subject trains him in to the same degree.[402]

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[400] Ólafur Daníelsson (1920b): iii
[401] Hansen, H. C. (2002): 41–59
[402] Ólafur Daníelsson (1920b): iii–iv

Then Dr. Ólafur Daníelsson continued by saying that mathematics education takes the pupil directly into the workshop of the sciences, where the tasks and the methods he/she uses are of the same kind as those of a scientist. This contains the main preparation for life on the part of mathematics. The “knowledge” it supplies was in Dr. Ólafur Daníelsson’s opinion less valuable.

Geometry tuition is here, for the first time in Icelandic discussion, directly referred to as a tool for training in scientific methods, not a tool for practical purposes. Björn Gunnlaugsson, the only preceding mathematician, probably thought more of mathematics as a practical tool on a high level, e.g. for his land-surveying, cf. his inauguration speech, where in fact he also mentioned training in logical thinking.

In *Lýðmenntun*, Guðmundur Finnbogason discussed the relation between the educational value and practical value of a subject. He took as an example the educational value of Latin studies, which he says are above discussion. However, if another language was found which offered the same educational value, and its literature was even closer to our world and it had more practical value in application in business, then Latin should give way to a language with more values. Concerning mathematical education, he emphasized that its aim should be to teach the pupils to think, well and clearly, to teach them to handle proportions of quantities which are found in daily life and draw correct conclusions from them.[403] Guðmundur Finnbogason and Ólafur Daníelsson’ views are here along similar lines.

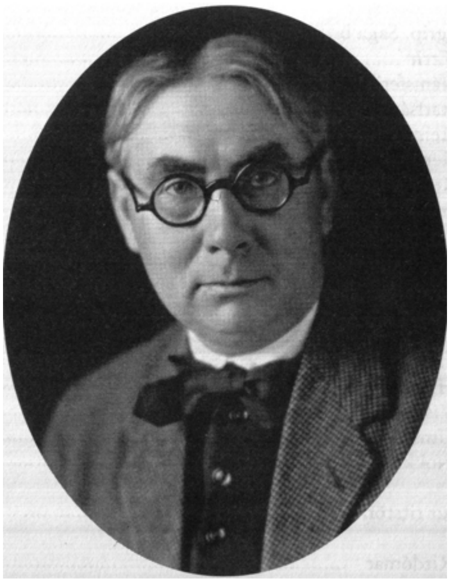

Fig. 5.10. Dr. Ólafur Daníelsson.

Dr. Ólafur Daníelsson had no mathematician to consult with. In the foreword to his *Um flatarmyndir* / *Geometry* he mentioned that he consulted on mathematical vocabulary with Medical Director of Health Dr. Med. Guðmundur Björnsson, with psychologist and philosopher Prof. Guðmundur Finnbogason, philosopher Prof. Ágúst H. Bjarnason, and especially theologian the Rev. Guðmundur Helgason, who had read most of the manuscript.[404] Those persons were the intellectuals available, and they had to suffice.

Dr. Ólafur Daníelsson’s *Um flatarmyndir* / *Geometry* was considered rather difficult, and it was not used after Jul. Petersen’s *Geometry* was translated into Icelandic in 1943.

In 1923 Dr. Ólafur Daníelsson published his *Kenslubók í hornafræði* / *Textbook in Trigonometry*. In his foreword he complained of the difficulty of expressing mathematical concepts in Icelandic, and inducing other mathematically educated people to adjust to the translations. Later in life Dr. Ólafur Daníelsson turned away from translating international mathematical concepts.[405]

[403] Guðmundur Finnbogason (1994): 61–63
[404] Ólafur Daníelsson (1920b): vi
[405] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 20

Algebra

One wonders why *Mathematics* by physicist Þorkell Þorkelsson, written for Akureyri School, was not introduced in Reykjavík School, which exclusively used Danish textbooks. That situation changed after Dr. Ólafur Daníelsson arrived at the school in 1919. Gradually, Dr. Ólafur Daníelsson exchanged the Danish textbooks for his own books in the lower grades. His *Kenslubók í algebru / A Textbook in Algebra* (1927) was considerably more formal than physicist Þorkell Þorkelsson's textbook, containing axioms and proofs.

In his foreword to *Algebra* Dr. Ólafur Daníelsson emphasized that mathematics was the only training in formal logic offered by the schools, and that it was not to be praised that Icelanders had to begin their training in logic in Danish as the case had been, instead of their own native tongue. Dr. Ólafur Daníelsson then once more expressed his opinions about teaching:

> ... nemendur, sem lært hafa utan skólanna, ... hafa komið til gagnfræðaprófs [í Menntaskólanum í Reykjavík] þannig undirbúnir í algebru, að þeir hafa kanski að eins leyst úr æfingunum, en þekkja alls ekki grundvöll merkjamálsins, hafa stundum enga tilsögn fengið í slíku. Þetta sýnir, að ýmsum þeim, sem við kenslu fást, er alls eigi ljós tilgangur þessarar greinar, halda að þýðing hennar sje fólgin í því, að nemendur verði færir í að leysa úr talnagátum ... En stærðfræðin er fyrst og fremst sjálfstæð vísindagrein, sú fullkomnasta sem til er, – og auk þess eru ýmsar aðrar höfuðgreinar vísindanna; einmitt þær sem mesta þýðingu hafa haft fyrir menningu nútímans, svo sem eðlisfræði, statistik, stjörnufræði o. s. frv., svo að segja ritaðar á merkjamáli algebrunnar, svo að þeim, sem eigi kann hana, eru öll þessi fræði að mestu leyti lokuð bók.
>
> ... pupils, who have studied outside the schools, ... have come up to lower department examination [in Reykjavík High School], so prepared in algebra that they have perhaps only solved the exercises, but do not at all know the basis of the symbolic language, have sometimes not had any tuition in it. This shows that some of those who work on teaching do not at all have a clear idea about the purpose of this subject, think that its importance is entailed in the pupils becoming able to solve number puzzles ... But mathematics is first and foremost an independent science, the most perfect existing, – and in addition several other main scientific subjects; exactly those which have been the most important for modern culture, such as physics, statistics, astronomy, etc., are so to speak written in the symbolic language of algebra, so that for those who do not know it, these subjects are in most respects a closed book.[406]

Dr. Ólafur Daníelsson's quotations indicate that he was a pioneer in writing mathematics textbooks in Icelandic, and that many teachers knew hardly enough mathematics to teach it. Algebra textbooks, and all higher mathematics texts, used up to that time, had been in Danish, intended for the educational elite.

When Ólafur Daníelsson wrote his *Algebra*, it was already two decades since he had been in Denmark. He was a man who went his own way, and it is unlikely that he was much influenced by anything else than his training at the University of Copenhagen and his own conviction. Danish educational currents after his stay in Copenhagen may not have influenced him much. Dr. Ólafur Daníelsson soon had a large family to support. Then the Great War came in 1914–1918 which prevented people from travelling for several years. On account of this he did not travel back to Denmark often or stay there for any period of time, except for mathematics

[406] Ólafur Daníelsson (1927): 3–4

congresses.[407] Julius Petersen was his teacher and probably his main source of influence. When Dr. Ólafur Daníelsson arrived at Reykjavík High School in 1919, Jul. Petersen's *Arithmetic and Algebra* of 1911 was in use. Comparing Ólafur Daníelsson's *Algebra* and Jul. Petersen's 1925 edition,[408] it is clear that Ólafur Daníelsson's algebra textbook is by no means a copy of the Danish one, and on superficial examination not a single exercise can be seen to have been copied. Ólafur Daníelsson's *Algebra* is rather a response to the other one, e.g. by adding a number of exercises on the associative law in multiplication and delaying exercises with algebraic fractions.

Dr. Ólafur Daníelsson's *Algebra* was to become one of the most influential textbooks of the 20th century. It even outlived his *Arithmetic*, as it was almost the only algebra textbook available up to the 1970s. Its last reprint was in 1971. Problems on the salary of a maid, paid in frocks and boots, or on the prices of horses, no longer appealed to pupils by the 1970s.

The mathematician Guðmundur Arnlaugsson wrote about Ólafur Daníelsson's work:

> Þessar bækur eru afrek, þær bera áhuga, bjartsýni og hagleik Ólafs við ritstörf órækt vitni. Ekki er heldur rétt að gleyma útgefendum bókanna. Íslenskir útgefendur voru ekkert stórveldi á þessum árum. Þurft hefur stórhug og dirfsku til að gefa út rit um nýstárleg efni, rit sem ekki var að vænta að seldust nema í nokkrum tugum eintaka á ári hverju.
>
> These books are a unique achievement; they testify to Ólafur Daníelsson's motivation, optimism and writing skill. The publishers must not be forgotten either. Icelandic publishers were no superpowers in these years. They would need ambition and daring to publish innovative works, works that could not be expected to sell in more than a few tens of copies each year.[409]

Dr. Ólafur Daníelsson's Mathematics Teaching Policy

Dr. Ólafur Daníelsson was to have a dominating position in the 20th century mathematics teaching, so it is interesting to try to analyse his textbooks to learn about his ideas on the connections of school mathematics to mathematics as a science, the most perfect science existing, as he said in one of his forewords.

Dr. Ólafur Daníelsson textbooks took over from the Rev. Eiríkur Briem's textbooks, from 1880. The Rev. Eiríkur Briem declared that memorizing was important. Dr. Ólafur Daníelsson on the other hand declared that in most current textbooks more explanations were needed; many people had had to learn by heart without understanding the reasons, and those who worked in teaching lacked sufficient skills to explain arithmetic from its roots. He emphasized the concept of *ratio* in *regula de tri*, while the Rev. Eiríkur Briem's emphasis was on the three terms: front term, middle term and rear term, even if he mentioned ratio. Dr. Ólafur Daníelsson added a more plausible explanation of the procedure, and a warning against using *regula de tri* on cases to which it did not apply, such as a free fall. The procedure continued to be presented in textbooks, as it had been for centuries, up to the 1970s, when it was finally dropped.

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[407] Kristín Kaaber, by e-mail from Svanhildur Kaaber, September 22, 2005
[408] Petersen, J. (1925)
[409] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 20

Dr. Ólafur Daníelsson did not attempt to formalize basic arithmetic, while he emphasized understanding of current procedures. He was obviously concerned with explanations and preventing rote learning. His strength lay in clear explanations and good and varied exercises, where the pupil had to choose his/her own solution methods, even if he did not consider problem-solving to be at centre of mathematics.

Yet one wonders what he meant by explaining and "understanding". In his *Arithmetic* (1906, 1914, 1920) he explained the basic operations fairly thoroughly by taking examples from pupils' daily life and e.g. how common fractions such as 1/2 and 4/8 have the same value. He also explained quite sensibly how fractions with different denominators must have a common denominator, divisible by all the denominators, to be added up. When, however, it came to finding the least common denominator, too high to be found mentally, he turned to an unexplained procedure, printed in a footnote. This might indicate that the author expected the pupils to find the common denominator mentally, rather than to use a procedure. However, the procedure, also found in older textbooks, became widely used.

The author explained that it was often sufficient to multiply all the denominators together, but for the least common denominator in the case of e.g. $\frac{1}{5}+\frac{5}{6}+\frac{2}{3}+\frac{4}{9}+\frac{5}{16}+\frac{3}{4}+\frac{7}{12}+\frac{3}{10}$ the denominators should be written in a line:

5 – 6 – 3 – 9 – 16 – 4 – 12 – 10

Then all the denominators that divided another one were to be ruled out, in this case 5, dividing 10, and 3, 4 and 6 dividing 12. Thereafter the remaining denominators were to be divided by the lowest number dividing two or more of them, and the others were to be copied unchanged:

2) 9 – 16 – 12 – 10
2) 9 – 8 – 6 – 5
3) 9 – 4 – 3 – 5
3 – 4 – 1 – 5

By now all the remaining numbers and the divisors were to be multiplied together: $2{\cdot}2{\cdot}3{\cdot}3{\cdot}4{\cdot}1{\cdot}5 = 720$. Done.

This unexplained procedure in Dr. Ólafur Daníelsson's *Arithmetic* was taken up in the primary arithmetic textbook by Elías Bjarnason, his student at the Teacher Training College. It prevailed as a standard procedure in Icelandic schools for half a century.[410]

The same applies to the greatest common divisor. In the first edition of Dr. Ólafur Daníelsson's *Arithmetic*, Euclid's algorithm was introduced in a smaller font size, with a note explaining that when a number *a* has a common factor with a number *b*, the common factor also divides the remainder when *a* is divided by *b*. In later editions (1920 and later), Euclid's algorithm is placed in a footnote without any further explanations.

This is the more regrettable as the fourth volume of Sigurbjörn Á Gíslason's *Arithmetic*, published in 1911, introduced prime factoring in order to compose the least common denominator and the greatest common divisor. Afterwards he showed

[410] Elías Bjarnason (1937–1975): Vol. 3, 32

the above-mentioned procedures, which he says were commonly used, presumably referring to Eiríkur Briem's 1869 book.[411]

Dr. Ólafur Daníelsson delayed all explanations involving prime factoring until after a thorough treatment of algebra. There at last, in *Algebra*'s last chapter, he mentioned prime numbers. He does not seem to have been ready to introduce prime factoring until after publishing a proof that the number of prime factors of a number could never exceed the number itself, which he does in a footnote, a fact that one might expect pupils to find unnecessary to prove. Thereafter he explained how the greatest common divisor and the least common denominator can be found by prime factoring. Then he proved Euclid's algorithm, and lastly proved the fact that if two numbers, which are relative primes, divide an integer, their product does so too.[412]

This indicates that Dr. Ólafur Daníelsson judged both topics incomprehensible for younger pupils. Euclid's algorithm was also repeated without explanations in Elías Bjarnason's textbook, volume 3.[413] One would like to express a regret that Dr. Ólafur Daníelsson did not consider it timely to introduce prime factoring earlier.

As these procedures were explained and proved in the last chapter of Dr. Ólafur Daníelsson's 1927 *Algebra*, which was taught neither for the national examination, nor in the Reykjavík High School's first grade, at least not in the early 1960s, many generations missed plausible reasoning for methods used to find the least common denominator and the greatest common divisor, and became stuck in incomprehensible procedural algorithms, introduced in Eiríkur Briem's textbooks for practical purposes.

In spite of Dr. Ólafur Daníelsson's wish to avoid rote learning, his rigid mathematical training hindered him in introducing more comprehensible procedures. Dr. Ólafur Daníelsson's syllabus prevailed for more than 50 years. Many of those who worked on teaching never became acquainted with anything else, and in addition may never have had any teacher training. Thus the tradition of unexplained procedures was carried on for decades.

This is the more regrettable as the majority of each generation received no education after primary school, only few entered lower secondary schools, and still fewer entered the mathematics streams where these secrets were at last revealed. It is not surprising that it was a common opinion among these generations that mathematics was not based on logical reasoning, but rather was a series of procedures without explanations.

The Pythagorean Theorem is supported with a nice geometric proof in the 1914 edition of the *Arithmetic*,[414] as is the fact that the sum of angles in a triangle is 180 degrees.[415] Also, the area formulae for geometric figures are supported by pictures.

In extracting square roots, Dr. Ólafur Daníelsson declared that he omitted all explanations. Sigurbjörn Á. Gíslason introduced in his book the square of a binomial, $(a + b)^2 = a^2 + 2ab + b^2$, in order to explain the square root algorithm. In passing, he used the opportunity to explain how the identity $a^2 - b^2 = (a + b)(a - b)$ may be used for mental arithmetic. Later in the same volume he treated the cubic root similarly to the square root.

411 Eiríkur Briem (1883, 4th ed.): 36–37, 40
412 Ólafur Daníelsson (1927): 138–143
413 Elías Bjarnason (1937–1975): Vol. 3, 28–29
414 Ólafur Daníelsson (1914): 82–84
415 Ólafur Daníelsson (1914): 77–78

Now it is likely that some of Sigurbjörn Á. Gíslason's explanations went over the heads of many teachers, and the pupils as well. Most teachers had little mathematical training, and his short outline of algebra probably did not suffice to act as an explanation of these procedures. Still, it was an attempt, which may have reached a few pupils.

Sigurbjörn Á. Gíslason and Dr. Ólafur Daníelsson were classmates at the Learned School and lifelong friends. However, their perception of the correct way of presenting mathematical topics appears to have been different. Dr. Ólafur Daníelsson's training undoubtedly contributed to a conviction that an inexact presentation was worse that none.

Dr. Ólafur Daníelsson as a Teacher

In the 50th-anniversary publication of the Teacher Training College, alumni wrote memories about their time in college. These quotations were found about Dr. Ólafur Daníelsson:

> Ólafur Daníelsson var yngstur kennaranna. Hann kenndi þrjár námsgreinar. Landafræði var ein þeirra. Í henni var hann ekki vel að sér. En í stærðfræði fannst mér hann afburða góður kennari, og ég held, að ég hafi aldrei lært eins mikið hjá neinum kennara og hjá Ólafi í einkatímum, sem hann bauð mér að hafa með mér í stærðfræði, sem kennd var í efri bekkjum menntaskólans. Og ekki tók hann neina greiðslu fyrir þessa aukatíma. En ekki voru öll bekkjarsystkini mín jafnánægð með kennslu hans og ég. Þeim þótti mörgum hann fara of fljótt yfir.

> Ólafur Daníelsson was the youngest of the teachers. He taught three subjects. Geography was one of them. He was not very knowledgeable about it. But in mathematics I found him a superb teacher, and I think that I have never learnt as much with any teacher as I did with Ólafur in private lessons he offered me in mathematics of the upper grades of the High School. And he did not take any fee for the lessons. But not all my classmates were as happy with his teaching as I was. Many of them thought that he went too fast through the syllabus.[416]

> Annar fastur kennari ... var dr. Ólafur Daníelsson, þá nýbakaður doktor í stærðfræði. Trúa mín er það, að vart fari aðrir framar dr. Ólafi í hnitmiðuðum og glöggum skýringum á flóknu efni. Var furðulegt, hvað honum tókst að láta sum okkar, sem litlum stærðfræðigáfum voru gædd, skilja flókin dæmi og reikning, sem við höfðum áður numið eins og páfagaukar, án skilnings. Er mér t.d. minnisstætt, að ég stóð eitt sinn uppi við töflu og átti að reikna dæmi, sem var brot deilt með broti. Dæmið reiknaði ég leikandi, hafði lært utanbókar, hvernig ég átti að reikna slík dæmi. En svo spurði doktorinn: Hvernig stendur á því að rétt er að reikna þetta svona?" Þar stóð hnífurinn í kúnni, og man ég ekki nú, hvort aðrir í bekknum kunnu svar við þessu. En víst er um það, að þetta og fleira í kennslu dr. Ólafs varð til að opna augu mín fyrir því, hve fánýtt er að kenna börnum og unglingum reikningsaðferðir, án þess að þeim skiljist, hvað þau eru að gera.

> Another tenured teacher ... was Dr. Ólafur Daníelsson, then having recently completed his doctoral degree in mathematics. It is my belief that hardly anyone will surpass Dr. Ólafur in precise and perceptive explanations of a complicated subject. It was astonishing how he succeeded in getting some of us, who were not supplied with much mathematical giftedness, to understand complicated problems and calculations, which we had previously learned parrot-fashion, without understanding. I, for example, recall when I once stood at the board and was to compute a problem where a fraction was divided by another fraction. I solved the problem easily, as I had learnt by heart how to solve such problems. But then the doctor asked: Why is it

[416] Þorsteinn M. Jónsson (1958): 183

> correct to solve the problem this way? This I did not know, and I do not remember if the other pupils in the class knew the answer either. But it is certain, this and other things in Dr. Ólafur's teaching opened my eyes for how futile it is to teach to children and young people computation methods without their understanding what they are doing.[417]
>
> Þá var og annað kennaraliö ágætt. Ég minnist Ólafs Dan, hins fræga stærðfræðings og kennara, en ekki hentaði honum að kenna þeim, sem lítið gátu skilið stærðfræðina.
>
> Then also other teaching staff were fine. I recall Ólafur Dan, the famous mathematician and teacher, but it did not suit him to teach those who could understand little in mathematics.[418]

Ólafur Daníelsson began to teach in the new mathematics stream at the Reykjavík High School in 1919. He continued to teach at the Teacher Training College until 1921, with some assistance. His contribution to high school education will be discussed later.[419]

5.6. Training of Primary Teachers

The Teacher Training College

In 1908 the Teacher Training College of Iceland was established in Reykjavík, under legislation passed in 1907 as a part of the education legislation, and in agreement with Guðmundur Finnbogason's suggestions in *Lýðmenntun*.

Kennaraskóli Íslands, the Teacher Training College of Iceland, operated for 63 years, until 1971 when it was raised to university level. About 15–30 primary teachers graduated from the college each year, and soon teachers became an established profession. By legislation in 1919, graduates of the Teacher Training College were given priority for tenure in teaching positions, with provision for exemptions for those who had taught for three years and had a certificate from the pastor and school board on good teaching talent and watchfulness in their work.[420] The traditional respect for home education had its advocates in the *Alþingi*, so the bill went through lengthy debate, culminating in the victory of the supporters of priority for college-educated teachers. The exemption was abolished by law on February 1, 1936.[421]

Entrance requirements for the Teacher Training College were in 1908 e.g.: The applicant should not be younger than 18 years old. The applicant had to know the four main operations of the arithmetic in whole numbers and fractions (also decimal fractions) and have skills to use them in simple problems that occur in a daily life. The entrance requirements concerning mathematical knowledge remained unaltered in 1924 when they were increased in other subjects, while in 1932, published in regulations 1934, new requirements were approved, in mathematics by adding proportions. However, they were still lower than for a lower secondary school examination, and the college still had its own entrance examination. The lower secondary school examination became a requirement in 1943.[422]

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417 Svava Þórleifsdóttir (1958): 188
418 Ingimar Jóhannesson (1958): 200
419 Guðmundur Arnlaugsson and Sigurður Helgason (1996): 12–17
420 *Stjórnartíðindi* 1919 no. 75, November 28
421 *Stjórnartíðindi* 1936 February 1
422 Freysteinn Gunnarsson (1958): 42

Mathematics Teachers, Syllabi and Weekly Teaching Hours[423]

The Teacher Training College training was a three-year programme. Until 1922, while Dr. Ólafur Daníelsson was teaching at the Teacher Training College, the students had 10 weekly hours in mathematics, four in the first year, four in the second year and two in the third year. After Dr. Ólafur Daníelsson left, a doctor in botany, Helgi Jónsson, took over the mathematics teaching. Helgi Jónsson was a tenured teacher in 1920–1923. After he stopped teaching mathematics in 1924, the number of weekly hours diminished, first to 3–4–2, and in 1925 to 3–3–2. In 1923, Helgi H. Eiríksson, a mine engineer educated in Glasgow, arrived at the college as a part-time teacher and he was tenured in 1925–1929. At the same time he was headmaster at Reykjavík Technical School, which at that time was a private school, run by the organization of skilled craftsmen in Reykjavík.

Sigurkarl Stefánsson, a mathematics graduate from the University of Copenhagen in 1928, arrived at the Teacher Training College in 1929. In 1932, the weekly number of hours decreased to 3–2–2. In 1933, it increased again to 4–3–2–2 in four grades for that one year, but decreased to 4–3–0 in 1934 and stayed so until 1945. Sigurkarl Stefánsson stayed at the College until 1942. He had throughout that period been a full-time mathematics teacher at the Reykjavík High School, a part-time teacher at the lower secondary schools, and had in 1941 started to teach at the new engineering programme at the University of Iceland. In the 1930s Sigurkarl Stefánsson and Dr. Ólafur Daníelsson were the only mathematicians in Reykjavík working in teaching, so it is no wonder that they were heavily occupied. Sigurkarl Stefánsson wrote a textbook in mathematics for the language stream of the high school, *Stærðfræði handa máladeildum menntaskóla / Mathematics for the Language Streams of High Schools*[424] and he translated Jul. Petersen's *Kennslubók í rúmfræði handa gagnfræðaskólum / A Textbook in Geometry for lower Secondary Schools*, published in 1943.[425]

In 1945, when new legislation was under way, Dr. Ólafur Daníelsson's *Algebra* was added to the syllabus. The college training programme was now four years. The number of weekly hours in mathematics had become 4–4–2–0, and stayed so until 1952.

In the period 1943–1947, mathematics teaching was provided by a young psychologist, Broddi Jóhannesson, who later became the headmaster of the Teacher Training College. In 1947 the school gained a new mathematics teacher, Hermann Jónsson, who had studied to become an actuary for one year, and thereafter for one year in a commercial college in Copenhagen (*Det handelsvidenskabelige Læreanstalt*). He stayed at the Teacher Training College until 1956. On the side he had a full-time position in an official institution.

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[423] The figures are taken from *Skýrsla um kennaraskólann í Reykjavík, 1908–1962*
[424] Sigurkarl Stefánsson (1946)
[425] Petersen, J. (1943)

After 1952, the syllabus was changed under the 1947 legislation. The number of weekly hours became 4–4–0–0. The number remained constant until 1962, when the school at last acquired a tenured mathematics teacher, Eiríkur Jónsson, who had been a part-time teacher since 1959.[426] Eiríkur Jónsson had taken the mathematics and physics programme for engineering students at the University of Iceland.[427] In 1967 there were 5–0–2–0 hours in the basic programme, while students could choose three or six hours in mathematics as electives. Below, this is collected in a table and compared to Icelandic, the other basic subject in primary schools:[428]

Period	Mathematics	%	Icelandic	%
1910–1924	4 – 4 – 2	10%	6 – 6 – 6	18%
1924–1925	3 – 4 – 2	8%	5 – 6 – 6	15%
1925–1932	4 – 2 – 2	7%	5 – 5 – 6	14%
1932–1933	3 – 2 – 2	6%	5 – 5 – 6	14%
1934–1945	4 – 3 – 0	6%	5 – 5 – 5	13%
1945–1953	4 – 4 – 2 – 0	7%	5 – 5 – 5 – 5	14%
1953–1962	4 – 4 – 0 – 0	5%	5 – 5 – 7 – 5	14%
1967–1971	5 – 0 – 2 + Electives			

Table 5.3. Number of weekly hours in mathematics at the Teacher Training College 1910–1971.

Icelandic phonetics was included in Icelandic from 1952, two hours a week, which kept Icelandic at the 14% level at least up to 1962. At the same time mathematics' share in the timetable of the Teacher Training College decreased from 8% to 5%.

Drawbacks of the Teacher Training College

The Teacher Training College did not have a full time tenured mathematics teacher, devoting his professional life to teaching, after Dr. Ólafur Daníelsson left in 1921, until 1962. Helgi Jónsson, the botanist who taught mathematics, was tenured 1920–23 but he was not a mathematician. Mine engineer Helgi H. Eiríksson was tenured in 1925–29, but he was headmaster of Reykjavík Technical School. After 1929 the school did not have a tenured teacher in mathematics until 1962. The school was too small to hire a full-time mathematics teacher, and in order to obtain tenure the teacher also had to teach something else. Later, suitable extra hours may not have been available, as it was easier to obtain teachers in other subjects. Mathematics teaching obviously suffered from these circumstances. This is indicated by the number of weekly hours, which dropped over the years and were fewer in 1962 than in 1921, even if the college was offering a four-year training programme in 1962 instead of three-year originally. Regulations and other authorities were too weak to hinder this unfortunate development.

As was the case in the only Learned / High School in the country in 1805–1919, there was only one mathematics teacher at work in the Teacher Training College in 1908–1967. The responsibility is heavy, and the respect for the subject greatly depends on whether there is a respected scholar, such as Björn Gunnlaugsson or Dr. Ólafur Daníelsson, conducting the work, or persons deeply engaged in other obligations.

[426] *Skýrsla um Kennaraskólann í Reykjavík 1961–1962*
[427] Ólafur Þ. Kristjánsson og Sigrún Harðardóttir (1958–1988): Vol I, 119, Vol. III, 253
[428] *Skýrsla um Kennaraskólann í Reykjavík 1908–1962*. Archives of the University of Education: Stjórnsýslugögn Kennaraskóla Íslands #150: Námsefni í Kennaraskóla Íslands, almenn deild 1968–1972

In-Service Courses

While Dr. Ólafur Daníelsson was at the Teacher Training College, six-week in-service courses were held for graduates from Flensborg and the new college, those who had completed high school and others who had taught for at least five years. The courses were held every year from 1909 until 1922.[429] Dr. Ólafur Daníelsson taught mathematics: arithmetic, volume computations, mathematics teaching and algebra, different topics in different years. In 1923–1927, the authorities did not offer support to these courses. Later there was not enough attendance to keep them going, until 1931 when no mathematics was taught and Dr. Ólafur Daníelsson was no longer present.

5.7. A Mathematics-Natural Science Stream at Reykjavík High School

Antecedents

The Icelandic school was not only different from schools in the other Nordic countries in that it was essentially still a six-year learned school. It also had only one stream and was not divided into ancient language, modern language and mathematics–natural sciences streams, like the Danish learned schools after 1871 and Danish high schools after 1903. Many thought that the pupils had to study too many subjects, and that the preparation offered by the school in mathematics, physics, chemistry and natural sciences was insufficient for those who wanted to study engineering, natural sciences, etc. [430]

Although the teachers had not recommended a mathematics–natural science stream in 1904, they did resolve on the establishment of such a stream at their meetings in the autumns of 1907 and 1908.[431]

Those who were most concerned about the mathematical subjects were the growing class of engineers. In 1912 thirteen engineers gathered to establish an association of engineers, *Verkfræðingafélag Íslands* / The Association of Chartered Engineers in Iceland. Its first chairman was National Engineer Jón Þorláksson. Members of the association were engineers and others on same level of education in mathematics and natural sciences.

During the first few years the members of the association discussed various issues concerning supply of water, electricity, roads, harbours and other modern conveniences which were rapidly spreading at this time, at least in urban areas. In 1917, Jón Þorláksson discussed the lack of engineers. This time he did not mention the High School, and he thought that it would take a possible technical college too long time to produce the necessary work force to assist the few engineers at work. He suggested a two-month course with basic tuition in land-surveying. The government granted the necessary funds.[432]

[429] *Skýrsla um Kennaraskólann í Reykjavík 1908–1931*
[430] Heimir Þorleifsson (1975): 70-78
[431] Heimir Þorleifsson, (1981): 62
[432] Jón Þorláksson (1918): 15. *Fundabók Verkfræðingafélags Íslands* 1918

In 1918 the Reykjavík School was discussed at *Alþingi*. A resolution was adopted containing the following items:

> Alþingi ályktar að skora á stjórnina:
>
> I. Að rannsaka, hvort eigi muni hollara að gera hinn almenna mentaskóla aftur að lærðum skóla, með líku sniði og áður var, en greina hann frá gagnfræðaskólunum.
>
> II. Að rannsaka, hvort eigi mundi rjettara að skifta þeim lærða skóla í deildir síðustu árin, málfræðideild og stærðfræðideild, eða jafnvel fleiri.
>
> III. Að gera sem fyrst ráðstafanir til þessarar breytingar, svá fremi rannsóknin leiðir til þeirrar niðurstöðu.
>
> Alþingi resolves to urge the government:
>
> I. To investigate whether it would not be more sound to alter the general High School back to a learned school, in a similar fashion as previously, and separate it from the lower secondary schools.
>
> II. To investigate whether it would not be more correct to split that learned school in the last years into a grammar stream and a mathematics stream, or even more.
>
> III. To arrange this alteration as soon as possible, if the investigation leads to that conclusion.[433]

Nothing was done that year. However, the matter had been brought up and been discussed, along with a nostalgic proposal that Latin and Greek should be restored to their former status in the education of the future ruling class.

In March 1919 physicist and then Meteorological Institute Director Þorkell Þorkelsson was admitted to the Engineer Association, and at that same meeting he explained that he and Dr. Ólafur Dan Daníelsson had applied to the government for a subsidy to establish and run a school of mathematics and natural sciences. Their plan was that the pupils could graduate from this school with a matriculation examination (*stúdentspróf*) which would provide them with the requirements to enter the Polytechnic College in Copenhagen, the University of Iceland and other universities. The matter had a lively debate, which resulted in a formal proposal to the government that it create a complete mathematics–natural science stream at the High School that next autumn, or, if it was thought more suitable otherwise, that the government provide for a complete preparatory education for entrance to the Polytechnic College in Copenhagen.[434]

During the summer session 1919 *Alþingi* made a resolution to urge the government to undertake investigation of all school affairs in the country.[435] Concerning the High School, the proposals were to split the three uppermost grades into two streams, a grammar–history stream on one side, while the other would be natural science–mathematics stream. Furthermore, the lower grades would be similar to those before 1904, and the same subjects taught, i.e. Latin and Greek, with the same entrance requirements. This meant that lower secondary education (the *gagnfræðadeild*) would not be a part of the High School.

[433] *Alþingistíðindi* 1918 A: 469
[434] *Fundabók Verkfræðingafélags Íslands* 1919: 72
[435] *Alþingistíðindi* 1919 A: 1853

Another event in this sequence was that philosopher Prof. Ágúst H. Bjarnason wrote an article in his journal, *Iðunn*, in which he discussed the recent sovereignty of Iceland in 1918, and the need for many kinds of specialists; no one knew anything in the subjects which were most needed: technical knowledge to produce electricity, to produce fertilizers and run machines. He would like to support Dr. Ólafur Daníelsson's and Þorkell Þorkelsson's suggestion of a mathematical school. However it would be more economical to divide the present learned department into a language-history stream and mathematics-physics stream, as he himself and others had suggested 10 years ago.[436] Possibly Prof. Ágúst H. Bjarnason was influential enough for his words to have great weight in the matter.[437] He had graduated from a learned school in Denmark, where streaming had existed for nearly half a century so he may have had a fresher view on the matter than others.

Snowball Effect

The result of these events was that a mathematics–natural science stream was established at the Reykjavík High School in the autumn 1919. Dr. Ólafur Daníelsson was appointed part-time mathematics teacher and Þorkell Þorkelsson taught physics.[438] Time had arrived for consensus on this matter. However, often a certain event starts a snowball rolling. In Prime Minister Emil Jónsson's memoirs there is a story concerning his studies at the Reykjavík High School in 1917–1919. Emil Jónsson took an entrance examination into the fourth grade of the Reykjavík High School in 1917 after having studied at Flensborg Lower Secondary School and had some private tuition from Dr. Ólafur Daníelsson. The teaching in Reykjavík High School was good, apart from mathematics:

> Í stærðfræði var aðeins kennt mjög lítið. Í Menntaskólanum var þá engin stærðfræðideild, en öll áherzla lögð á málin. En þó að „pensum"ið væri ekki stórt í sniðunum réði kennarinn ekki við það, og öll kennslan var utangarnar. Var þetta þeim mun sorglegra sem maðurinn var ágætur vísindamaður í sinni sérgrein, og hefur sennilega verið þvingaður til að kenna stærðfræðina. Mér er eitt tilfelli sérstaklega minnisstætt frá þessari kennslu. Við höfðum verið að lesa um óræðar tölur, en í kennslubókinni var kaflinn um þetta hvorki langur né margbrotinn. Síðan fengum við heimadæmi – úr bókinni – til úrlausnar. Ég reiknaði dæmið og skilaði því. Útkoman var $\sqrt{3} + \sqrt{6}$ og ég var handviss um að hún var rétt. Þegar ég fékk dæmið aftur hafði kennarinn skrifað aftan við útkomuna $\sqrt{3}+\sqrt{6}=\sqrt{3+6}=\sqrt{9}=3$. Ég varð alveg agndofa af undrun, því að þetta var slíkur „horror", að lengra varð ekki komizt, og sýndi algert skilningleysi. Ég bjó þennan vetur hjá dr. Ólafi Daníelssyni, og mátti alltaf leita til hans með hvað eina, sem ég vildi spyrja hann um í stærðfræðinni. Hann gaf mér líka oft dæmi, sem honum þótti gaman að láta mig spreyta mig við, og verulegan hluta af námsefninu til stúdentsprófs hafði ég farið yfir hjá honum, þegar á meðan ég var í 4. bekk.
>
> Strax að lokinni kennslu þennan dag hljóp ég því heim til að sýna dr. Ólafi það sem kennarinn hafði skrifað. Hann varð svo hneykslaður, að ég hélt að hann mundi hoppa uppí loftið. Ekki man ég nákvæmlega hvað hann sagði, en mig minnir, að það hafi verið eitthvað á þessa leið: „Já, svona vilja þeir hafa það." Dr. Ólafur var þá án alls samjafnaðar mestur stærðfræðingur á Íslandi, og raunar sá eini, sem það nafn gat borið með sóma. Hann hafði um mörg undanfarin ár verið kennari við Kennaraskólann, og orðið að sætta sig við að kenna einfaldasta barnaskólareikning, en aldrei kennt neitt við Menntaskólann, þó að hann hefði doktorsnafnbót og gullmedalíu frá

[436] Ágúst H. Bjarnason (1919): 80–88
[437] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 25
[438] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 26

Kaupmannahafnarháskóla. Mér skildist að hann hefði nú farið til Jóns Magnússonar, sem þá var forsætisráðherra, og skýrt málið fyrir honum, og það með, að þetta væri ófært. Hvort sem þeir hafa rætt þetta mál lengur eða skemur var dr. Ólafur orðinn kennari við Menntaskólann árið eftir og stærðfræðideild stofnuð við skólann upp úr því. Má segja, að ef athugasemdin við dæmið mitt hefir beint eða óbeint verið orsökin, var betur farið en heima setið.

In mathematics only very little was taught. At that time there was no mathematics stream in the High School and all emphasis placed on the languages. However, even though the syllabus was not large, the teacher could not cope with it and all the teaching was superficial. It was the sadder, as the person was a good scientist in his special subject, and he was probably forced to teach mathematics. One incidence from this teaching is especially memorable to me. We had been studying irrational numbers, and the chapter on them in the textbook is neither long nor complicated. Then we had a problem for homework – from the textbook – to solve. I solved the problem and handed it in. The answer was $\sqrt{3}+\sqrt{6}$ and I was absolutely sure that it was correct. When I had the problem back the teacher had written by my answer $\sqrt{3}+\sqrt{6}=\sqrt{3+6}=\sqrt{9}=3$. I was astonished as this was such a "horror" that it could not be exceeded, and it showed an absolute lack of understanding. That winter I was staying with Dr. Ólafur Daníelsson and I could always seek his assistance with whatever I wanted to ask him about in mathematics. He also often posed me some problems which he liked to let me try, and I had covered with him a considerable part of the syllabus for the final examination already, while I was in the 4th grade.

As soon as the school was over that day I therefore ran home to show Dr. Ólafur what the teacher had written. He became so shocked at this that I thought that he would jump into the air. I do not remember exactly what he said but I think it was something like: "Yes, this is how they like to have it." Dr. Ólafur was then, without any comparison, the greatest mathematician in Iceland, and indeed the only one who could carry that name with honour. He had then, for many previous years, been teacher at the Teacher Training College, and had had to settle for teaching the simplest primary school arithmetic and never taught anything at the High School, even if he had earned a doctoral degree and a gold medal at the University of Copenhagen. I understood that he went to Jón Magnússon, who then was Prime Minister, and explained the matter to him and added that this was not acceptable. Whatever they said, for a long or short time, Dr. Ólafur had become teacher at the High School the following year and a mathematics stream was established thereafter. One could say that, if my remark about my problem was directly or indirectly the cause, it was better done than not.[439]

These are memoirs about events, happening nearly half a century earlier, so they may be taken with a grain of salt. However, after graduating from the Polytechnic College in Copenhagen and practising engineering for a while, Emil Jónsson became a politician; he was a member of *Alþingi* for 37 years, 1934–1971, and a member of the government for 19 years, so he had a good knowledge of official decision processes. Dr. Ólafur Daníelsson and Þorkell Þorkelsson must have been the main proponents of this school matter. Their formal channel was to seek support from the Association of Engineers. Their schoolmate, National Engineer Jón Þorláksson, did not take a seat in *Alþingi* until 1921, but may have had contacts in the *Alþingi* to support the matter.

[439] Emil Jónsson (1973): 32–34

The Mathematics Stream

The main reason why this important alteration of the Reykjavík High School was made was, however, probably that the time had arrived in society for realizing the urgent need for technical knowledge. The provision of roads, harbours, bridges, water systems, electricity and building of durable material opened the eyes of people to all the tasks and problems waiting for engineers to solve. During the Great War the connection to Denmark was broken, and Iceland thus acquired in fact greater independence from Denmark than the Home Rule arrangement provided for. As a result, Iceland became a sovereign state in loose union with Denmark in 1918, with the King of Denmark the head of the Icelandic state.[440] Many things still had to be sought to Denmark, not least education, while the establishment of a mathematics stream in the High School was an important step towards independence in the field of education.

The alteration itself was not made by regulations; an advertisement from the Ministry sufficed. The new mathematics stream was to be taught according to Danish regulations on the mathematics–natural sciences stream. This was considered natural, as most students would go on to the Polytechnic College in Copenhagen.[441]

At the establishment of the mathematics stream, it had six hours in mathematics a week for three years. Danish textbooks were used, such as *Arithmetik og Algebra II* and *Metoder og Teorier til Lösning af geometriske Konstruktionsopgaver* by Jul. Petersen, *Trigonometri*, *Lærebog i Differential- og Integralregning*, *Lærebog i analytisk Plangeometri*, *Lærebog i Stereometri* and *Tillæg til Arithmetik og Algebra* by C. Hansen. In 1925 *Kenslubók í hornafræði / Trigonometry* by Ólafur Daníelsson was introduced. From 1937 *Lærebog i Arithmetik og Algebra* by Albert Kristensen and from approximately 1944 *Arithmetik og Algebra* by Jul. Petersen and *Lærebog i Stereometri* by Albert Kristensen were used.[442] C. The textbooks by C. Hansen and Albert Kristensen were elaborations of the late Julius Petersen's textbook series.

In 1929, mathematics was dropped from the language stream, and replaced with bookkeeping and French. By regulations of 1937[443] mathematics was reintroduced in the language stream, and Latin in the mathematics stream, where it had not been since its establishment. The 1937 regulations made the duration of studies in the learned department four years, and the lower department two years. This had probably to do with the preparation of pupils entering the Reykjavík High School learned department from other lower secondary schools, which were gradually growing in number. The following is stated about mathematics in the learned department, now that the mathematics stream finally had regulations:

> a. Í máladeild: Nemendur skulu læra notkun stærðfræði og æfast í exakt hugsun. Skal fyrst fara rækilega yfir undirstöðuatriði námsefnis gagnfræðadeildar. Síðan skulu nemendur læra algebru, nokkur undirstöðuatriði hornafræði og ágrip af hagnýtri stærðfræði.
>
> b. Í stærðfræðideild: Nemendur skulu læra rækilega um reellar tölur, föll og einfaldar rúmmyndir og fá leikni í talnareikningi og meðferð stærðfræðilegra formúlna. – Farið skal rækilega yfir undirstöðuatriði námsefnis gagnfræðadeildar. Síðan skulu nemendur

440 Helgi Skúli Kjartansson (1996): 90
441 Heimir Þorleifsson (1981): 63–64
442 *Skýrsla um hinn almenna menntaskóla í Reykjavík/ Reykjavík School Reports* 1920–1950
443 *Stjórnartíðindi* 1937 no. 3, February 8

læra algebru og geometríu og allýtarlegt ágrip af hornafræði, analytiskri geometríu og differential- og integralreikningi.

a. The language stream: The pupils shall learn the application of mathematics and be trained in exact thinking. Firstly the lower secondary department's syllabus shall be thoroughly revised. Thereafter the pupils are to study algebra, several basic concepts from trigonometry and an extract of practical mathematics.

b. The mathematics stream: The pupils are to study thoroughly real numbers, functions and simple three-dimensional objects and acquire skills in computations and handling of mathematical formulas. – The syllabus of the lower department shall be thoroughly revised. Thereafter the pupils are to learn algebra and geometry and a considerable extract of trigonometry, analytic geometry and differential and integral calculus.[444]

One notices that only pupils in the language stream are supposed to be trained in exact mathematical thinking. Probably it was taken for granted that this kind of training was naturally implied in the mathematics stream, or that its goal was primarily to prepare the pupils for studies at the Polytechnic College. These ideas seem to be derived from its model, the 1903 legislation for Danish schools.[445]

The number of weekly hours in mathematics at the Reykjavík High School was five hours in each of the lower department's first and second grades, while there were four hours in the undivided learned department, 3 + 3 hours for two years in the language stream and 4 + 6 + 7 hours a week for the four years in the mathematics stream.[446]

Guðmundur Arnlaugsson and Björn Bjarnason were appointed at Reykjavík High School in 1946 and 1948 respectively. As early as 1949 they began to test new textbooks for the mathematics stream of the school. They tried textbook series by the Dane V. A. C. Jensen for three years, to return to Jul. Peterson's system for two years, and finally settled on Juul and Rønnau's series from 1954. This series was to remain in use for a decade, until the advent of "modern" mathematics in 1964.[447]

No doubt the intention was to maintain the same standards of teaching in the mathematics stream as in Danish schools. Danish textbooks or their translations were used exclusively in the high schools' learned departments until 1964. It is difficult to say whether the results were similar to those in the Danish schools. Probably there was only little contact with Danish school authorities and e.g. Julius Petersen's series may have been considered outdated earlier in Denmark than in Iceland.[448] The complete loss of contact with Denmark during World War II delayed renewal of textbooks.

Furthermore, the Icelandic school year was shorter than in Denmark, from the beginning of October to the end of May, while the graduates were one year older, most of them at least 20 years old. The Icelandic mathematics stream pupils had to study more languages, Danish and Latin, in addition to English, German and French. The school was, in the period 1928–1946 and even up through the 1960s, highly selective, and may therefore have been able to maintain relatively high standards.

[444] Helgi Elíasson (1944): 73
[445] *Rapport fra landsmødet om matematikken i Danmark 1981*: 196
[446] *Skýrsla Menntaskólans í Reykjavík 1937–1938, ...*
[447] *Skýrsla Menntaskólans í Reykjavík 1949–1950, ...*
[448] Niss, M., October 2004

Dr. Ólafur Daníelsson in Retrospect

Dr. Ólafur Daníelsson had a strong influence on his pupils. Headmaster mathematician Guðmundur Arnlaugsson wrote an account of him:

> Hann minnti mig á listamann, frekast fiðlusnilling, eins og eg ímyndaði mér þá. Hann var léttur í spori og kvikur, ljúfur og kurteis. Hann gleymdi sér stundum gjörsamlega í glímunni við viðfangsefnið, en það bjó líka áreiðanlega í honum dálítill leikari, eins og í öllum góðum kennurum. En það sem einkum hafði áhrif á okkur var hinn eldlegi áhugi sem hann var gæddur og virðingin fyrir stærðfræðinni. Það var aldrei nein lognmolla kringum hann, heldur ferskur blær, hann hreif menn ósjálfrátt með sér. Hann gat skýrt flókin atriði á þann hátt að manni fannst að betur yrði naumast gert. Og frá honum seytlaði smám saman inn í mann virðing fyrir stærðfræðinni sem mikilli og göfugri vísindagrein – þetta var eins og að vera kominn inn fyrir þröskuld í vinnusal vísindanna. ... En hann var líka gæddur skapgerð listamanns, hann var hrifnæmur, fljótur til að gleðjast þegar honum fannst eitthvað vel gert, en vonbrigðin gátu líka verið snögg. Þolinmæðin entist honum ekki lengi ef hann varð var við skort á skilningi og áhuga. Til þess bar hann of djúpa virðingu fyrir stærðfræðinni. Hann kunni vel að meta hagnýtt gildi hennar, en fleyg orð þýska stærðfræðingsins Jacobi stóðu þó áreiðanlega hjarta hans nær: „Tilgangur vísindanna er fyrst og fremst sæmd mannsandans."
>
> He reminded me of an artist, rather like a violin genius, as I imagined them. He walked lightly and was lively, and gentle and polite. He sometimes completely forgot himself in his struggle with the tasks while he also contained something of the actor, like all good teachers. What especially influenced us was his enthusiasm and respect for mathematics. There never was a dull moment with him, rather a fresh breeze; he automatically carried one away with him. He could explain complicated topics in such a way that one thought that it could hardly be done better. And from him there seeped into one a respect for the mathematics as a great and noble science – this was like entering the workshop of sciences. ... his character was also that of an artist, he was impressionable, he was quickly delighted when he thought something was well done, while disappointment also could enter quickly. He did not have much patience if he sensed a lack of understanding and interest. For that he respected mathematics too deeply. He could well appreciate its practical value, while the well known words of Jacobi surely were closer to his heart: "The sole purpose of science is the honour of the human mind."[449]

In spite of his intensive activity at the Teacher Training College and the Reykjavík High School, Dr. Ólafur Daníelsson wrote several textbooks as cited earlier, a total of 12 scientific articles published in scientific journals and a number of book reviews and articles about the situation of mathematics education.[450] In 1931 he wrote a review praising Alfred North Whitehead's book: *Stærðfræðin / Introduction to Mathematics*, translated by Guðmundur Finnbogason and published by the Literary Society.

> Hananú, þarna kom frá Bókmenntafélaginu ný bók, sem á ekkert skylt við sýslumannaævir og prestatöl, né heldur rímnakveðskap eða fornfræði, bók sem ekki er í nema eitt kvæði og það í styttra lagi, læsileg bók um merkilegt efni, hver skyldi trúa?
>
> Já, hún er nú komin, og bókmenntaspekingarnir, þeir víðsýnu, fornfræðaþulirnir, allir eru þeir nú hneykslaðir alveg niður í tær, hvað á þetta að þýða, les nokkur þetta? segja þeir, innilega sannfærir með sjálfum sér, að árangurslaust muni vera að leita anda sínum svölunar annars staðar en í Hávamálum eða Íslendingasögum, þar er samankomið allt mannvit og málspeki, hvað þurfum við þá meira?

[449] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 22–23
[450] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 27–81

There now, there has come from the Literary Society a new book, which has nothing to do with biographies of county magistrates and priests, nor Icelandic ballads or antiquarianism, book that only contains one poem, and that a short one, a readable book about a remarkable subject, who would believe?

Yes it has arrived, and the wise literary men, the liberal ones, the scholars of ancient studies, they are all shocked all the way down to their toes, what does this mean, will anyone read this? they say, completely convinced that it is useless to seek satisfaction for their minds anywhere else than in *Hávamál* or the Icelandic Sagas, where all human knowledge and wisdom is gathered, what more do we need?[451]

One can sense Dr. Ólafur Daníelsson's battle as that of a voice shouting in the wilderness. The only branch of scholarship studied at the University of Iceland, apart from theology, medicine and law, was Icelandic philology and history, and it seems to have been considered as the only field of study worthwhile in the narrow learned community in Iceland in the first half of the 20th century. Gradually Dr. Ólafur Daníelsson became tired of teaching. He gave it up in 1941 at the age of 64 to continue as actuary, which he had been part-time for a while. He died in 1957 at the age of 80.

5.8. Early 20th Century Icelandic Society

University

The University of Iceland was established on June 17, 1911, on the centenary of the birth of independence campaigner Jón Sigurðsson, by uniting the Theological Seminary, established in 1847, the Medical School, established in 1876 and the Law School, established in 1908, and adding a Faculty of Humanities, where Icelandic studies, i.e. history and philology, were the main subjects.

Prof. Helgi Skúli Kjartansson says in his book *Ísland á 20. öld / Iceland in the 20th century* (2002):

Rannsóknir og fræði, voru … viðfangsefni sem Íslendingar hlutu að leggja rækt við ef þeir vildu taka sinn sess meðal menningarþjóða. … Rannsóknir við Háskólann beindust mjög að sögulegum efnum, þótt kennarastóll í sagnfræði væri lengi aðeins einn. Í embættismannadeildunum snerust rannsóknir að drjúgum hluta um fortíð þjóðarinnar, en málsaga og bókmenntasaga voru stundaðar við heimspekideild – bókmenntasagan með sérstakri reisn eftir að Sigurður Nordal, fremstur ungra fornsagnafræðinga og fjölmenntaður í bókmenntum og heimspeki samtímans, tók þar við kennarastóli 1918. …

Við hlið rannsóknanna á sögu, tungu og bókmenntum voru raunvísindin olnbogabarn. Þau voru ekki kennd við Háskólann, nema að því leyti sem heyrði undir læknisfræði, né stunduð við rannsóknarstofnanir, heldur af einstaklingum fyrir styrki og stuðning úr ýmsum áttum.

Research and studies were … tasks that Icelanders had to cultivate if they were to take their place amongst nations of culture. … Research at the University was very much aimed at historical topics, even if there was most of the time only one chair in historical studies. In the professional faculties research was to a large degree focussed on the nation's past, while language history and literary history were studied at the Faculty of Humanities, – the literary history at an especially high standard after Sigurður Nordal, the leading young Saga scholar with multiple education in modern literature and philosophy, took the chair there in 1918. …

[451] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 61–62

> Alongside research into history, language and literature, the natural sciences were the neglected children. They were not taught at the University, except within medical studies, nor cultivated at research institutes, but pursued by individuals, assisted by grants and support from various directions.[452]

History, language and literature were the tools to build up the self-esteem of a newly independent nation. This situation at the highest level of education and research, to direct the research to the nation's past, prevailed in the period between the world wars, and up to the 1960s. The natural sciences were not pursued within a separate faculty at the University until 1970.

As early as 1931 the issue of establishing preparatory studies for engineering students at the University was discussed, both in the *Alþingi* and within the Association of Engineers. One of the proponents of the issue was Meteorological Institute Director Þorkell Þorkelsson. He and his colleagues in the Association of Engineers had ideas of establishing preparatory studies in natural sciences, such as geology and marine biology.[453] Marine biology was first taught at the University in the 1990s.

In 1940 teaching of engineering students was initiated at the University, due to the broken connection to the Polytechnic College, later Technical University of Denmark, DTU, during World War II. A programme was tailored after that college, as most of the prospective teachers had studied there and they still had the textbooks from there. In the first year lack of textbooks was a great obstacle. In 1941 textbooks arrived from Denmark through Lisbon and America, a year after they were ordered.[454] Dr. Leifur Ásgeirsson and Sigurkarl Stefánsson were among the first teachers. The plan was to run only the first half of the programme. When the first group of six students had completed that first half in 1943, they were not able to go abroad to complete the studies. The second half of the programme was therefore run in 1943–1946. As this programme was expensive, it was not continued after the war had come to an end. The first half programme continued, but it was not until 1974 that engineers graduated again from the University of Iceland.[455]

For the same reasons the Faculty of Law started a programme in business administration, on the basis of a private school of economics. The programme in the Faculty of Law developed into a Faculty of Business Administration a few years later.

Jónas Jónsson of Hrifla

During most of the Home Rule period 1904–1918 there was only one Minister of Icelandic Affairs. In 1917 the number increased to four. From then onwards the cabinet numbered three to six persons until 1953. After the establishment of the mathematics stream at the High School, nothing special happened in educational affairs, which were part of the duties of the Minister of Justice most of the time. There was, however, the Office of Educational Affairs, busily building up the recently established compulsory school system. Jón Þórarinsson died in 1926, and theologian Ásgeir Ásgeirsson, later president of Iceland, was appointed director of the office.

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452 Helgi Skúli Kjartansson (2002): 158–159
453 Tímarit Verkfræðingafélags Íslands (1931): 46–53
454 Guðni Jónsson (1961): 152
455 Pálmi Jóhannesson, September 22, 2005

In 1927 Jónas Jónsson (1885–1969), named after his home farm Hrifla, became Minister of Justice and Education. Jónas Jónsson was a member of *Alþingi* in 1922–1949 and a member of the cabinet in 1927–1932 (with the exception of four months). In the 1920s he was very influential and his methods and actions in his office as a minister were, to say the least, controversial.

Fig. 5.11. Jónas Jónsson of Hrifla.

Jónas Jónsson was a farmer's son who graduated from Akureyri Lower Secondary School in 1905. He had by then become too old to be admitted to Reykjavík High School, so he taught for a year and then went abroad to study at Askov School in Denmark and *Danmarks Lærerhøjskole*, the Royal Danish School of Educational Studies in 1907–1908. He was granted a subsidy from the Treasury to study school affairs in Germany, France and England in 1908–1909. He taught at the Teacher Training College in 1909–1918 and afterwards at *Samvinnuskólinn*, the Co-operative Commercial College, as headmaster in 1919–1955, with a break while he was a member of the government.[456]

Jónas Jónsson and the Schools

Jónas Jónsson's opinion was that Iceland did not need a large number of officials, and that the stream of young people to the University was retrogression. In 1927 he granted the Akureyri School the right to offer upper secondary level education. At the same time he restricted the number of pupils admitted to Reykjavík High School in 1928 to 25 new pupils each year.[457] The Reykjavík High School was to restrict itself to the ancient languages, while at Akureyri High School, which was formally established in 1930, modern and general subjects were to be studied.[458]

The restriction of admissions to Reykjavík High School led to fierce competition for the few places available, and expensive private tuition blossomed. The restriction had converse effect to what Jónas Jónsson had anticipated; it contributed to excluding poor workers' children from the school. By restricting admission to Reykjavík High School, Jónas Jónsson wanted to direct young people to a new lower secondary school in Reykjavík where practical subjects could be studied, such as masonry, carpentry, net-making, simple sewing and cooking. This school was established that same year, in 1928,[459] and a left-wing priest, the Rev. Ingimar Jónsson, was appointed as headmaster of the school. This school, *Gagnfræðaskóli Reykjavíkur*, popularly called *Ingimarsskóli* or Ingimar's School, was the first real lower secondary school in Reykjavík.

[456] *Alþingismannatal* 1845–1995: 272
[457] Guðjón Friðriksson (1992): 24–27
[458] Guðjón Friðriksson (1992): 21–22
[459] *Stjórnartíðindi* 1928 no. 68, May 7

The middle class in Reykjavík was furious over the restriction. Out of the 42 who passed the entrance examination in 1928, only 25 were admitted. A private school was established by angry parents whose children had not gained admission to the High School. The name of the school was *Gagnfræðaskóli Reykvíkinga*, while it usually was called *Ágústarskóli* (Ágúst's School), from the name of its first headmaster, the philosopher Prof. Dr. Ágúst H. Bjarnason. Jónas Jónsson was out of office for four months in 1931, and meanwhile his allies decided that the final examination from *Ágústarskóli* sufficed for entrance into the learned department of the High School, which counterbalanced the restriction rules. Thus Reykjavík had two new lower secondary schools in 1928.[460]

Before Jónas Jónsson's period, four rural lower secondary schools existed, the so called *alþýðuskólar* / folk high schools. They were situated at Eiðar, Laugar, Núpur and Hvítárbakki, each in a separate quarter of the country, while there was no school in the southern lowlands. In 1928 Jónas Jónsson went to England to study schools, especially rural schools. He visited schools like Eton, Rugby and Harrow. That same year a lower secondary school was built at Laugarvatn in the southern lowlands on Jónas Jónsson's initiative. Legislation on district schools was passed in 1929.[461] In the next couple of years schools were established at Reykir and Reykholt,[462] while the nearby private school at Hvítárbakki was closed down.[463]

In 1930 Jónas Jónsson had passed legislation on lower secondary schools in the towns of Vestmannaeyjar, Hafnarfjörður, Reykjavík, Ísafjörður and Akureyri, and a provision for schools in Siglufjörður and Neskaupstaður. These schools changed considerably the future prospects of Icelandic youth, and society gradually grew similar to other Nordic countries. The school in Hafnarfjörður was Flensborg Lower Secondary School which finally had its legislation.

In parallel to the district schools, Jónas Jónsson established schools of domestic science for prospective housewives, which he saw as the natural future role of girls. Most of the rural schools were built in the neighbourhood of warm geothermal springs, so that the warm water could be utilized for heating of houses and swimming pools, as Jónas Jónsson had great belief in the educational value of swimming and in physical education in general. In this respect Jónas Jónsson was a pioneer in utilizing geothermal energy for economical heating of schools.[464]

The rural schools were of great value to people in the regions in the period between the Great Wars. When the new education legislation was implemented in 1946 their role changed, and after the 1974 education legislation they were gradually closed down.

[460] Guðjón Friðriksson (1992): 24–27
[461] *Stjórnartíðindi* 1929 no. 37, June 14
[462] Guðjón Friðriksson (1992): 32–42
[463] Helgi Elíasson (1945): 21
[464] Guðjón Friðriksson (1992): 36

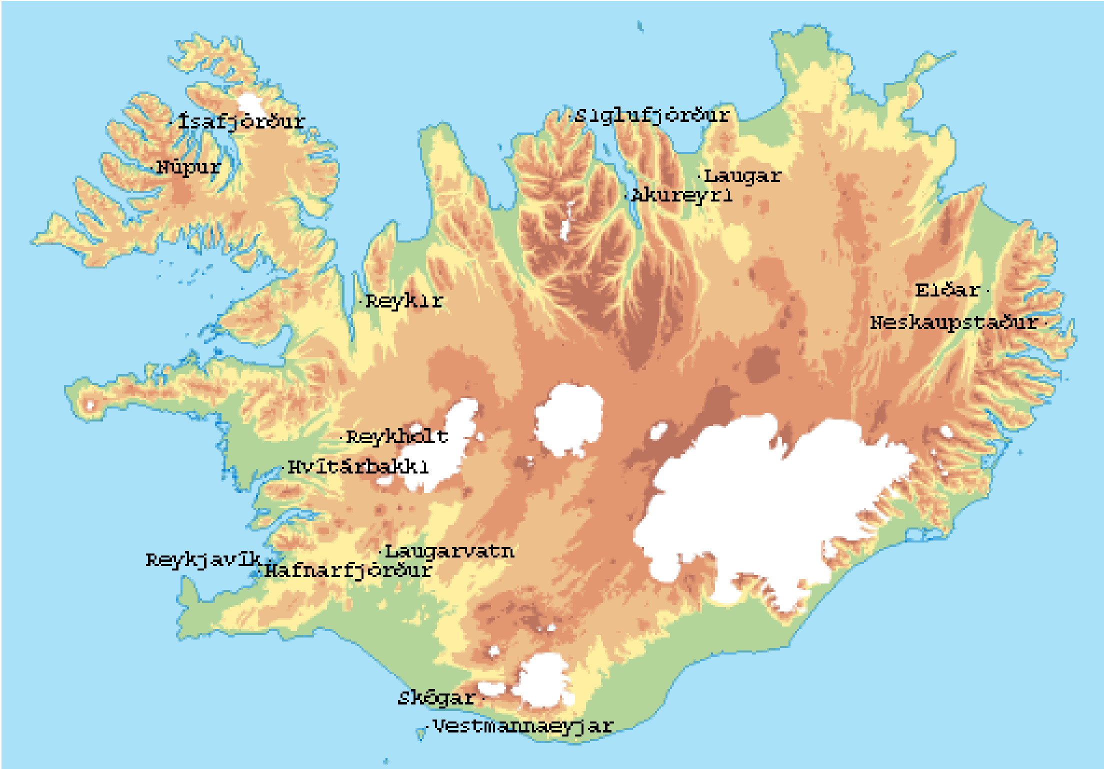

Figure 5.12. Secondary school sites in the early 20th century.[465]

Even if Jónas Jónsson's methods were highly disputable, great progress in the education available to the general public may be attributed to him. He shook up old habits of thought and created necessary discussions about the school system. He had schools built and textbooks published, if not written by others then by himself. Iceland at last had educational options for all adolescents. Children below 10 years of age were still to be educated at home, at least in rural areas. As road connections were difficult during winter in rural areas, that would take a long time to change, while in Reykjavík and all larger towns, schools from the age of seven became the norm in the 1930s. In 1936, legislation was adopted on compulsory schooling for all children in the age range 7–14 years, with allowance for exemptions up to the age of 10 in the rural areas.[466]

Jónas Jónsson's Legacy

Minister Jónas Jónsson did not stay long in office. However, his views persisted for a long time among the general public, such as that the Reykjavík High School and the University produced unnecessary officials, which were hindrances to his ideas of a utopian state. His dream was a romantic return of the Icelandic people to an imaginary Golden Age of the Saga period.[467] Jónas Jónsson and his friends had the idea that the countryside was a growth area for Icelandic culture where the general public worked at their home crafts, reading the Sagas, while in reality they themselves belonged to the educational elite and had no intention of returning to their homesteads.[468]

[465] National Land Survey of Iceland: website
[466] Helgi Elíasson (1944): 9
[467] Guðjón Friðriksson (1992): 43
[468] Sigríður Matthíasdóttir (2004a): 6; (2004 b): 123–131

Jónas Jónsson believed that growing up in the countryside would further the healthy race of the Icelandic nation, so he praised the rural life and home education. He himself had enjoyed home education and only had five weeks in school before his confirmation. His political party, *Framsóknarflokkurinn* (the Progressive Party, the party of farmers) resisted migration to towns and sought to maintain the *status quo* as long as possible, i.e. by ensuring that sparsely-populated rural areas were over-represented in *Alþingi* as against the increasingly more populous urban centres.

In fact Jónas Jónsson established two political parties at the same time, *Framsóknarflokkurinn* (the Progressive Party) for farmers and *Alþýðuflokkurinn* (the Labour Party, Social Democrats) for the workers. Both were established in 1916. Later he turned away from radicalism and devoted himself to the party of farmers.

Jónas Jónsson's nationalistic ideas had wide and long-lasting influence, not least through his textbook on Icelandic history for primary schools. The book's nationalistic message had immense influence on Icelanders' self-image. It was first published in 1915 and it was last printed in 1979.

Concerning mathematics education, Jónas Jónsson took a practical view. As one of the founders of the *Samvinnuskólinn,* the Co-operative Commercial College, and its headmaster, he favoured bookkeeping skills. His antipathy for people with in his view excessive higher and further education included engineers. He had more belief in common sense and simple reason than in highly educated engineers.[469] The ideas of establishing engineering education at the University probably did not conform in any way with Jónas Jónsson's ideas, and were only realized later. Naturally, Jón Þorláksson, being an engineer and a right-wing politician, was not among Jónas Jónsson's friends and was therefore, if not his enemy, his fierce opponent. However, as both were practical men, there were matters that they could agree upon.[470]

Didactic Discussions

One wonders what kind of didactic discussions took place in Iceland concerning mathematics teaching during this period. H.C. Hansen explains in his book *Fra forstandens slibesten til borgerens værktøj* discussions going on in Denmark by the turn of the century about the educational value of Euclidian approach to geometry and axiomatic approach to arithmetic.[471] It is not possible to find similar discussions in Iceland. Euclid, for example, has not been found mentioned at all in Icelandic discussions at that time.

The group of schoolmates, Dr. Ágúst H. Bjarnason, Dr. Guðmundur Finnbogason and Dr. Ólafur Daníelsson, expressed their views about true and superficial education in a similar fashion, so presumably they discussed these matters among themselves. Guðmundur Finnbogason explains in his *Lýðmenntun* in 1903 how true education is gained by the exertion of the gifts of the soul. Guðmundur Finnbogason said:

[469] Guðjón Friðriksson (1992): 21
[470] Hannes Hólmsteinn Gissurarson (1992)
[471] Hansen, H.C. (2002): 41–59

Almenn menntun stælir og hvessir sverð andans, sérmenntun kennir að beita því á sérstakan hátt.

General education toughens up and sharpens the sword of the spirit; specialised education teaches how to use it in a special way.[472]

In 1919, Dr. Ágúst H. Bjarnason wrote in *Iðunn*:

Ment er máttur. Alt er þó undir því komið, hver mentin er. Það getur verið sú yfirborðsmentun, er ekki sé nema hvarflið eitt og geri menn alls-ómáttuga bæði til hugsunar og starfs, af þvi að hún gerir þá ekki færa um að ráða við neitt eða fram úr neinu. En svo er til önnur ment, er gerir menn færa í flestan sjó, og það er sú mentun, sem hefur kent mönnum að glíma við örðug viðfangsefni, þangað til þeir gátu ráðið fram úr þeim, sú mentun, sem gefur mönnum þau fastatök á hlutunum og á hugsun sjálfra sín, að þeim skeiki sjaldan eða aldrei.

Education is power. Everything though depends on what kind of education there is. It can be the superficial education that is only fractional and makes people completely powerless, both for thinking and working, because it does not make them competent to deal with anything or to solve anything. But then there is another kind of education, which makes people capable of most accomplishments, the education which has taught people to exert themselves with hard problems until they can solve them, the kind of education that grants people a fast grip on things and their thinking so that they seldom or never fail."[473]

Dr. Ólafur Daníelsson and his younger colleague, Sigurkarl Stefánsson, wrote long articles to explain that mathematics belonged to general education, and even more so than the languages, in particular after mathematics was abandoned in the language stream in 1929 under the administration of Jónas Jónsson of Hrifla.[474]

Thus the discussions did not concern what kind of mathematics should be taught, but rather whether mathematics belonged to general education, and its effect on thinking. The discussions of Dr. Ólafur Daníelsson and Dr. Guðmundur Finnbogason were on a more philosophical level. Dr. Ágúst H. Bjarnason's article in *Iðunn* shows practical views on the nation's need for mathematical education and Jón Þorláksson as an engineer was convinced that mathematical education was needed for technical progress.

No evidence has been found that Dr. Ólafur Daníelsson and Jón Þorláksson discussed mathematics education between them.[475] They must have known each other well, however. They were classmates at the Learned School and both of them must have had some preparation in Copenhagen before they began their mathematical studies at the University and the Polytechnic College respectively. Both of them stayed at Regensen and they graduated at a similar time. Jón Þorláksson was the headmaster of the Reykjavík Technical School in 1904–1911. Dr. Ólafur Daníelsson taught mathematics there from 1912, and probably earlier too, so they must have been colleagues. However, their views of the nature of mathematics were probably different.

In a sense, Ólafur Daníelsson and Guðmundur Finnbogason may be thought of as spokesmen of mathematics as "forstandens slibesten", "the whetstone of the wits," in H.C. Hansen's terminology, and Jón Þorláksson, Ágúst H. Bjarnason and Jónas

[472] Guðmundur Finnbogason (1994): 58
[473] Ágúst H. Bjarnason (1919): 78–79
[474] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 67–81. *Stjórnartíðindi* 1929 no. 65, October 11
[475] Kristín Kaaber, September 28, 2004

Jónsson, each in their own way, as spokesmen of "borgerens værktøj", "the citizen's tool". However, all these men were very much preoccupied with the country's progress. Their views of the goal of education were coloured by what would best serve the country, and the great number of people with little or no school education. Ólafur Daníelsson, Guðmundur Finnbogason and Ágúst H. Bjarnason were among the few who knew that higher mathematics could belong to classical education. Mathematics was generally considered as a kind of an aid to common sense in everyday life.

Population

Icelandic society changed rapidly in the first half of the 20th century. At the beginning of the century more than 60% of the population earned their living by agriculture, while by the middle of the century the proportion was 25% and at the turn of the 21st century 5%. The changes were most rapid in the Home Rule Period and during the World War II, while the changes were slower in the late 1920s and 1930s,[476] at the time of the Depression. After 1925 the majority of the population lived in towns and villages with 200 inhabitants or above.[477]

Migration to towns meant a changed lifestyle in most respects. In the countryside the houses were mainly built of turf and stone, panelled with wood, more or less buried in the earth for insulation. Electricity and telephones with all their conveniences were not common in the countryside until after 1950, and not every farm had a road.

Migration to towns and villages also created a market for further education. The pattern in the rural areas was different. A poor country youth, thirsting for education, might save some money after confirmation at the age of 14, in order to be able to attend a district school for one to two years. In towns, they could attend school from home. Attendance at Reykjavík High School would have been much more had it not been for the restrictions. These things were going to change.

The society was swept from its dreams of a bucolic golden age into the harsh reality of the outside world, when it was occupied by British armed forces in May 1940, a month after Denmark and Norway were occupied by German troops. The long-lasting political connection with Denmark was broken. Denmark was unable to manage Icelandic foreign affairs as had been postulated in the 1918 treaty. This finally led to the declaration of full independence from Denmark and the establishment of the Icelandic Republic on June 17, 1944.[478] On July 1, 1944 the number of inhabitants in Iceland was 126,879. The number of inhabitants in Reykjavík was 46,578 in December 1945, 37% of the total population. In 1947 it had reached close to 40%, a proportion which has remained constant to the present day, while the neighbouring communities have grown enormously since then.[479]

476 Helgi Skúli Kjartansson (2002): 305
477 Hagskinna (1997): Table 2.7. Population in urban nuclei and rural areas each year 1889–1990
478 Helgi Skúli Kjartansson, (1996): 108
479 Statistics Iceland: website. Einar S. Arnalds (1989): 115–116

6. The 1946 Education Legislation

6.1. Preparing the New Legislation

The education legislation from 1907 was a great step forward in its time. In the fourth decade of the 20th century, however, it became evident that the educational system lacked coherence and that instruction at the upper school levels was perhaps not taking advantage of what the pupils had learnt at lower levels. In 1941 *Alþingi* resolved that the government should appoint a board of persons, knowledgeable in school affairs, working between the sessions of parliament, to examine the educational and pedagogical affairs of the nation and make suggestions about their organization.[480]

On June 30, 1943 the Minister of Education appointed a board to prepare the legislation, later called the School Affairs Board. The original board members were typical representatives of the educated class: Four of them were theologians, one philologist in classical languages, a woman teacher educated at Flensborg Teacher Training College, and one member had specialized in educational matters as psychologist.[481] Classical education was well represented and none of the members was familiar with natural sciences or mathematics through his/her education. There seems to have been a political consensus about the board. The government was extra-parliamentary, the only one in history. The ministers were appointed personally by the regent, appointed after the connection with Denmark was cut off in 1940. The members of the cabinet were drawn from *Sjálfstæðisflokkurinn*, the Independence (conservative) Party, and *Framsóknarflokkurinn,* the Progressive Party, the centre party of farmers, in addition to two officials. One of the board members was the social-democrat Headmaster Ingimar Jónsson, which indicates that his school, *Gagnfræðaskóli Reykjavíkur*, had earned respect, one-and-a-half decades after its controversial establishment.

The board wrote the parliamentary bills for the whole set of legislation on

1. The School System and the Duty of the State to Provide Education
2. Primary Education of Children
3. Lower Secondary Education
4. High Schools
5. Teacher Training
6. A Training and Experimental School (for student teachers)
7. Household Education

These bills were passed as law in 1946, except the bill on teacher training, which was passed in 1947, and the bill on a training and experimental school, which was not passed. That, and the fact that the legislation on teacher training was changed from what was first intended, might account for some of the discontent in the 1960s with the legislation and its implementation.

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[480] *Alþingistíðindi* 1941 A: 226
[481] Gunnar M. Magnúss (1946): 19. See Appendix C

The Aims of the New Educational Law

In October 1944 a new coalition government, known as the "Innovation Government", was formed by the right-wing Independence Party, the Labour Party (Social Democrats) and Socialists. The Socialists were in charge of education. In 1946 the Ministry of Education published a report called *Um menntamál á Íslandi 1944–1946. Greinargerð um löggjöf, framkvæmdir og næstu verkefni*. / *On Education in Iceland 1944–1946. A Report on Legislation, Realization and Future Tasks*.[482] The following information is based on that report.

In a speech to university students at a celebration on December 1, 1944 (the anniversary of Iceland's sovereignty in 1918), Minister of Education Brynjólfur Bjarnason, who was a natural scientist himself and a philosopher, declared the main future projects of the government in the matters of education to be the following:

> Meginverkefnin eru þessi: ... að breyta því [Íslandi] úr hálfgildings hráefnanýlendu, eins og það er nú, í iðnaðarland með nýtízku tækni, fyrst og fremst á sviði fiskveiða og fiskiðnaðar. ... Hvað er nauðsynlegt til þess að við getum framkvæmt þetta? ... Í fyrsta lagi þurfum við að fá efni og vélar frá útlöndum, og í öðru lagi þurfum við á kunnáttumönnum og vísindamönnum að halda, sem hafa fullkomnustu tækni nútímans á valdi sínu, sem eru færir um að rannsaka náttúru landsins og kunna skil á að hagnýta sér auðlindir þess. ... Af öllum verðmætum jarðarinnar er **maðurinn sjálfur**, með allri þeirri kunnáttu, sem hann hefur aflað sér um þúsundir ára, verðmætastar. ... Okkur vantar fjölda verkfræðinga í öllum greinum ... Við þurfum náttúrufræðinga til þess að rannsaka auðlindir okkar ... Okkur vantar fiskifræðinga, efnafræðinga og menn með sérþekkingu á hvers konar iðnaði ... Okkur vantar flugmenn og sérfræðinga um flugmál og flugvallagerð. ... Undirstaðan undir sérmenntuninni og hinni vísindalega menntun er hin almenna fræðsla. ... [Það] á að stofna verkfræðideild við háskólann ... Það þarf að byggja mjög mikið af skólahúsum ... margir unglingar, sem mesta ítroðslu hafa hlotið, [eftir aukakennslu, námsskeið, einkatíma og stúderingu í því hvernig eigi að taka próf], hafa að lokum mestar líkur til þess að komast inn fyrir [mennta]skólans dyr. ... margur gáfaður unglingur hefur útilokazt frá framhaldsnámi, sökum aðstöðumunar, sem einkum hefur skapazt af tvennu: fjárskorti og fjarlægð frá skólum. ... Upp af athugunum á þessu hafa sprottið þær skólamálatillögur, sem nú eru fram komnar.

> The main tasks are: ... to change it [Iceland] from a kind of a raw-material colony, as it is now, to an industrial country with modern technology, primarily in the field of fishing and fish industry. ... What is necessary in order to implement this? ... Firstly we have to have material and machines from abroad, and secondly we need knowledgeable people and scientists, who command the modern technology; who are capable of researching the country's nature and know how to utilize its resources. ... Of all the earth resources it is **man himself**, with all the knowledge he has gathered over thousands of years, who is the most valuable. … We need many engineers in all branches .... we need natural scientists to study our resources ... we need marine biologists, chemists and people with special knowledge of all kinds of industry ... We need pilots and specialists in aviation and in building airfields. ... The basis for specialised education and scientific education is general education. … Faculty of Engineering is to be established at the University ... a number of schools have to be built ... many young people who have been most stuffed [with extra teaching, special courses, private tuition and studying how to take exams] have in the end the most chance of entrance into the [high] school. ... many a gifted youngster has been excluded from secondary education by discrimination of circumstances, which particularly have been created by two items: lack of finance and distance from

[482] Gunnar M. Magnúss (1946)

> schools. ... The proposals on educational affairs, that now have been forwarded, are rooted in these observations.[483]

The intention of the government was to enhance education for the purpose of creating a new technological society. Equality amongst young people in access to education should be established, not least to find and promote the most able people to make their contribution to development of the country. The most urgent matter was to build schools. There were very few buildings for secondary schools, except the district schools which were to be merged into the new school system, and the Reykjavík High School had not had a new building for a century, since 1846.

A Technological Society

The report contains an estimate of the need for people educated in technological and natural sciences. In 1946 the total number of university-educated engineers was 74, and it was estimated that 92 more were needed in the next five years, 48 to work for the state, 31 for local communities and 13 for private enterprises. There were no land-surveyors and six were needed, all for the state and the local communities.[484] To fulfil this need the legislation for the University of Iceland was amended in 1944 to establish the Faculty of Engineering. For the next 30 years students could take the first half of the Cand. Polyt. degree in Reykjavík, while they had to complete the degree in the following 2–3 years in Copenhagen or in Trondheim, Norway.

There were 15 university-educated natural scientists: four botanists, one bacteriologist, five zoologists, four geologists and one geographer. Moreover there was one forest scientist, five meteorologists and one soil agriculture scientist. Twelve Icelanders were studying the above natural sciences abroad, or had graduated and had not yet returned.[485]

Demand for natural scientists was estimated to be 25 in the near future, including 10–12 meteorologists, due to growing air traffic. Only 3–4 natural scientists were needed to teach in the high schools. For the new lower secondary schools, teachers would not require full scientific education. The teachers were to be educated at a new teacher training department at the University. Others would be needed at the University – if a natural science department were to be established there – at the natural science museum, and in research institutes in connection with the country's main industries.[486]

No estimate was made of the need for mathematicians. In 1947, when *Íslenzka Stærðfræðafélagið*, the Icelandic Mathematical Society, was established on Dr. Ólafur Daníelsson's 70th birthday, there were fifteen founder-members. Five were mathematicians, four of them teaching at the high schools and the University, three were actuaries, five were physicists and two were engineers.[487]

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[483] Gunnar M. Magnúss (1946): 10–12
[484] Gunnar M. Magnúss (1946): 24–25
[485] Gunnar M. Magnúss (1946): 25–26
[486] Gunnar M. Magnúss (1946): 26–27
[487] Björn Birnir et al. (1995): 97–122. See Appendix H

In 1961 the Association of Engineers made an investigation of how many students were studying engineering or natural sciences abroad. Information had been acquired about 139 students who were studying engineering and natural sciences abroad, but they were known to be more numerous. Of those, 42% were studying construction engineering, 14% chemistry or chemical engineering, 13% electric engineering, 11% were studying physics, 8% mechanical engineering, 6.5% in non-specified engineering and 5.5% various other subjects. No-one was known to be studying mathematics. The majority were studying in Germany at that time, or 45%, 27% in Denmark, 10% in Sweden, 5% in Britain and 6.5% in the United States. The Association of Engineers had also investigated the need for teachers in the natural sciences, mathematics, physics and chemistry at secondary-level schools in the country, such as high schools, technical schools and lower secondary schools. Their conclusion was that 30 teachers were needed.[488]

This report was published in a celebration publication on the 50th anniversary of the University of Iceland in 1961. The author went on to state that the time had now arrived for education in these subjects at the University. This became a reality in 1970. A B.A. programme in the above-mentioned natural sciences had, however, existed at the University since 1951. But it had neither produced many teachers nor scientists. This will be discussed further in connection with teacher training.

Minister of Education Brynjólfur Bjarnason saw that technical and scientific education was needed to increase production, and that the basis of this was general education for all. But his ultimate goal was to strengthen the working class to take charge of society. In his speech at the inauguration of the Innovation Government in October 1944, Brynjólfur Bjarnason asked if the working class would gain from Iceland transforming from a kind of a colony to an industrialized society with modern technology. He answered himself that no-one would deny that the general public would gain from full employment, and that by doubling (or more) production along the coast, the strength of the labour movement would increase greatly and thereby its possibilities to gain the ultimate power to decide how society was organized.[489]

Fig. 6.1. Brynjólfur Bjarnason.

In Brynjólfur Bjarnason's three-volume, 780-page collection of essays and speeches, this seven-page speech of December 1, 1944 is almost the only mention of educational policy. A few items were echoed in a celebration speech on the occasion of the centenary of the Reykjavík High School in 1946. Otherwise there is nothing to be found about education in Iceland in his collection. In his speeches Brynjólfur Bjarnason upholds his Marxist views, and attacks capitalism as it appeared in Icelandic politics. Brynjólfur Bjarnason can thus hardly be seen as the heir of Jónas Jónsson and his vision for youth. One may rather say that it served Brynjólfur Bjarnason's other goals to promote education. However, a long time was to pass until

[488] Steingrímur Jónsson (1961): 351–352
[489] Brynjólfur Bjarnason (1973): 134

another Minister of Education could concentrate on educational affairs only. The viewpoint of using education as a tool to transform society was next brought from abroad, by Iceland's participation in the OEEC, later OECD, during Gylfi Þ. Gíslason's period as Minister of Education and Commerce in the 1950s to 1970s.

While Brynjólfur Bjarnason was in office, funds in foreign currency were available. This money had been earned during the war, when there was no opportunity to use it. The Innovation Government was in agreement upon using it to build up the new society through technology. All the ministers of education after Brynjólfur Bjarnason were also occupied with other ministries and the general problems of all post-war governments: economic instability, characterized by strikes and high inflation. The ministers do not seem to have been much concerned with the internal affairs of the school system, which later caused disturbance.

Primary and Lower Secondary Education

All children aged 7–15 years were now to attend school. As transport in rural areas was still difficult, rural communities could be exempted from running a school for children younger than 8, 9 or 10 years old, according to circumstances. In that case, education was the responsibility of the families. This schooling could also be completed at the age of 14.[490] Schools in communities of more than 100 inhabitants were to operate for nine months, while other schools were to operate for at least seven months. In boarding schools the school year could be split amongst the pupils, so that each child would have at least 3 ½ months' schooling a year.[491]

The legislation about the lower secondary level was detailed.[492] It could be

2 years' youth school (*unglingaskóli*) for pupils aged 13–15
3 years' middle school or (*miðskóli*) for pupils aged 13–16
4 years' lower secondary school (*gagnfræðaskóli*) for pupils aged 13–17

Theoretical and practical subjects were to be on an even footing. The school was to have two streams for the pupils to select between. When it came to implementation, pupils were usually either so few that they could not be divided into two groups, the practical subjects were too expensive to implement, or the parents and the pupils themselves preferred the theoretical stream. Only few pupils went through the practical stream. Admittedly, however, for some time in the 1950s and 1960s a relatively large practical lower secondary school was run in Reykjavík (*Gagnfræðaskóli verknáms*).

The main feature of the legislation was that the lower departments of the high schools were to disappear. The middle schools were to run a national entrance examination to the high schools, and the latters' entrance examinations were abolished. This examination was to be taken at the age of 16. The educational system had thus become coherent, where pupils could go through primary school, middle school and high school up to university, if they passed the national examination which was to be offered at a number of schools, all around the country.

The national examination was an important question of fairness to all the rural population, as only two high schools existed, in the largest towns, Reykjavík and Akureyri. In addition there was the Commercial School of Iceland (*Verzlunarskóli*

[490] *Stjórnartíðindi* 1946 no. 22, April 10
[491] *Stjórnartíðindi* 1946 no. 34, April 29
[492] *Stjórnartíðindi* 1946 no. 48, May 7

Íslands) in Reykjavík, which was granted a licence to graduate students with a matriculation examination in 1943. That school kept its six-year structure and its own entrance examination on the premiss that it was a private school and was not concerned with the national examination. It did not have a mathematics stream until 1984.

6.2. Teacher Training Legislation 1946–1947

Entrance Requirements to the Teacher Training College

In 1947 new legislation was passed on the training of teachers as a part of the integrated education legislation.[493] The law was in six sections:

I. About the Teacher Training College
II. About a Teacher Training Institute in Pedagogic Science at the University of Iceland
III. About a Training and Experimental School
IV. About the Physical Education Teacher Training College
V. About the Crafts Teacher Training College
VI. About the Domestic Science Education Teacher Training College
VII. About the rights of teachers etc.

Admission to the Teacher Training College had previously been decided by an entrance examination. Its requirements were lower than those of a final examination in a lower secondary school (see section 5.6.).[494] By legislation in 1943, lower secondary education was required for admittance to the Teacher Training College; otherwise the pupil was to take an entrance examination. The main innovation in Section I of the new law on the Teacher Training College was that the national middle-school examination was now an entrance requirement to the school, with a minimum grade 6 out of 10 in the average of the nine examination subjects. This was implemented in 1952. Yet an exemption from this condition was often granted by permission of the Director of Educational Affairs, as attendance to the school was often insufficient.[495] A good lower secondary school examination could thus suffice, as before, as the pupils would then be at least a year older and more mature. However, they certainly would have had less mathematics.

In time, when the national middle-school examination had established itself, the lower secondary school examination gradually lost its value, as many of the most competent pupils had already left the lower secondary school through the national examination. The lower secondary school examination was not standardized and it became as varied as the schools were many. The requirements in mathematics were much lower than for the national examination. For example, the pupils would not know any algebra apart from solving equations.

In 1958, on the 50th anniversary of the Teacher Training College, the principal claimed that in some subjects (Icelandic, Danish, Icelandic history) the requirements were similar to a matriculation examination from a high school (*stúdentspróf*), while slightly less in some other subjects (English, natural sciences and mathematics).[496] In

[493] *Stjórnartíðindi* 1947 no. 16, March 12
[494] *Stjórnartíðindi* 1934 (regulations) no. 82, July 19
[495] Freysteinn Gunnarsson (1958): 42–43
[496] Freysteinn Gunnarsson (1958): 45

fact, the mathematics was less than the mathematics course in the language stream of the high schools.

Further and Higher Education for Teachers

Section II of the Teacher Training Bill of 1945 proposed a teacher training department at the University of Iceland. Its role should be

1. að veita barna- og unglingakennurum framhaldsmenntun;
2. að búa þá, sem hafa lokið almennu kennaraprófi eða stúdentsprófi, undir kennslustörf við miðskóla og gagnfræðaskóla;
3. að annast rannsóknir og leiðbeiningar í þágu uppeldismála landsins, ...

1. to provide further education to primary and youth school teachers;
2. to prepare those who have completed teacher training or high school education for teaching in middle schools and lower secondary schools;
3. to carry out research and advice for the benefit of pedagogical affairs of the country, ...[497]

The department was to operate in close cooperation with the Teacher Training College. Graduates from the Teacher Training College or a high school were to have the right to enter the department. The students were to study pedagogy, didactics and educational theory and receive teacher training, according to their respective school level. In other respects, studies and examinations were to be as provided in regulations submitted by the University Council and ratified by the Directorate of Educational Affairs (*Fræðslumálastjórn*).

The intention of Section II of the bill was to guarantee further and higher education of teachers at the University. In the notes to the bill, it appears that the board feared that prolonged university education would not attract enough teachers to teach in the many schools that were being established. Conditions in the school system were difficult, even though salaries had improved greatly recently. It is explained that the reason for teacher training being within the University, and not a separate institution, was that university teachers could teach subjects like Icelandic in the Faculty of Humanities, and mathematics in the Faculty of Engineering, to the student teachers.[498]

According to the notes, the School Affairs Board had been studying an English report on school affairs: *Board of Education: Teachers and Youth Leaders* (London 1944). The board used that report to support its proposals; firstly, that the study of the principles and theory of education (pedagogy and didactics) was the most important factor in teacher training and, secondly, that it would lead to a great loss for the nation's pedagogical affairs if the profession of teachers were to be divided into two different groups, at the very time when the school system was being unified. This English report seems to have partly been a model for the bill.[499]

[497] *Alþingistíðindi* 1945 A: 366
[498] *Alþingistíðindi* 1945 A: 372–374
[499] *Alþingistíðindi* 1945 A: 364–378

The bill was neither debated nor approved that year, and was resubmitted the year after. According to the debates on the amended bill, the University had not agreed to accept students from the Teacher Training College for specialised subjects without a matriculation examination from a high school, but only for courses in pedagogy and theory of education, and only when suitable facilities were established for teacher training. Courses in specialist subjects for teachers must be provided in other schools, according to the opinion expressed by the University.[500]

However, no such schools existed in the country. The only professor in pedagogy, for whom provision was made under the law, was appointed in 1957, having been a part- time teacher since 1952. There is no indication that Teacher Training College graduates attended lectures in pedagogy and didactics at the University. Courses in some of the specialist subjects as a preparation for teaching were offered at that time for high school graduates, without the basis of regulations, however, until 1951.

In the Archives of *Alþingi* the following letter was found, from the University Rector to the education committee of the lower house of *Alþingi*:

> Heiðruð menntamálanefnd neðri deildar Alþingis hefir sent háskólaráði til umsagnar II. kafla frumvarps til laga um menntun kennara, er liggur fyrir Alþingi því, er nú situr, en fyrirsögn kafla þessa er: Um kennaradeild við Háskóla Íslands. Háskólaráð fól þremur háskólakennurum, prófessorunum Sigurði Nordal, Símoni Jóh. Ágústssyni og Leifi Ásgeirssyni að athuga umræddan kafla, og fer hér á eftir álitsgerð þeirra:
>
> > ... Til vandlegrar athugunar hefur ekki unnizt tími, en við viljum leyfa okkur að benda á þessi atriði.
> >
> > 1. Að vísu var gert ráð fyrir því, þegar hús háskólans var reist, að þar yrði rúm fyrir kennaradeild. En eftir að kennsla í hagfræði og verkfræði hefur verið tekin upp, er svo komið, að háskólinn er ekki aflögufær um viðunandi húsrúm fyrir alla þá starfsemi, sem frumvarpið gerir ráð fyrir.
> > 2. Kennsla sú í íslenskum fræðum, verkfræði og erlendum málum, er nú fer fram í háskólanum, er sniðin við annan undirbúning og aðrar þarfir. Yrði því að sjá nemendum kennaradeildar fyrir sérstakri kennslu að mestu ef ekki öllu leyti. Ósýnt er að unnt verði að bæta slíkri kennslu að teljandi leyti á þá kennara, er fyrir eru.
> >
> > ...
>
> Þótt háskólaráð sé því hlynnt, að framhaldskennslu fyrir kennara verði komið á fót við Háskólann, verður það, með skírskotun til framanritaðs nefndarálits að telja, að þetta mál þurfi frekari undirbúnings, áður en það er til lykta leitt, og leggur það því til, að umræddur kafli frumvarpsins verði eigi lögtekinn að svo stöddu.
>
> The respected education committee of the lower house of the Alþingi has sent the University Council for comment Section II of the bill on teacher training which now is proposed for the present Alþingi, the heading of this chapter being: On a Teacher Training Department at the University of Iceland. The University Council appointed three university teachers, professors Sigurður Nordal [Icelandic literature and history], Símon Jóhann Ágústsson [philosophy] and Leifur Ásgeirsson [mathematics] to investigate this section and the following is their report:

[500] *Alþingistíðindi* 1946 B: 1361

.... Time for thorough consideration has not been available, but we allow ourselves to point out the following items:

1. Certainly when the building for the University was erected it was assumed that it would house a teacher training department. But after teaching in economics and engineering was introduced, the University cannot now provide adequate accommodation for all the activities that are proposed in the bill.
2. The teaching in Icelandic studies, engineering and foreign languages which now takes place in the University is tailored for another kind of preparation and other needs. Therefore the students of a teacher training department would have to be provided with special teaching in most or all respects. It is hard to see how such tuition could be added to any considerable degree to the duties of the present teachers.

...

While the University Council is in favour of further education for teachers being established at the University, it must, with reference to the above board report, conclude that this matter needs further preparation before it can be concluded, and therefore it proposes that the section of the bill in question not be passed at present.[501]

To understand this comment it must be kept in mind that the bill only provided for one new professor, i.e. in pedagogy and didactics. The University seems to have been expected by the legislator to take over the task of further education for teachers at no extra cost, neither in housing nor tuition. Furthermore, the University was always very reluctant to give access to anyone who was not a high school graduate, as may be understood from referring to "another kind of preparation" mentioned in item 2. Concerning Icelandic and languages, this must be an allusion to lack of preparation in Latin. And certainly Teacher Training College graduates would not have fitted into the mathematics teaching for the engineering students, where only a select group of mathematics-stream graduates with good grades were admitted.

The Teacher Training College thus led only to primary teacher training, as the teaching qualification did not suffice for admission to the University. There was no straightforward road to a high school matriculation examination (*stúdentspróf*), which was the entrance condition for the University. There are cases of teachers who took this, but they were exceptions. The main route to further education was abroad. There are examples of teachers entering tertiary-level teacher training in the Nordic countries, England and the United States.

Survey on Two Groups of Teacher Graduates

A short glance at the teacher graduates in 1951, four years after the new legislation was passed, and the graduates in 1958, does not indicate that any of them took advantage of the limited offer of courses at the University.[502]

In 1951, 32 teachers graduated from the regular programme at the Teacher Training College. All but one became teachers in primary and/or youth schools. Ten of them sought university education later in life, many of them in the single sabbatical year that teachers are entitled to in their lifetime. Of those ten, five stayed abroad for

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[501] The Parliament Archives: *Alþingi*. Dagbók 45–46 no. 615

[502] *Skýrsla um Kennaraskólann í Reykjavík* 1950–51 and 1957–58; Ólafur Þ. Kristjánsson and Sigrún Harðardóttir (1958–1988)

one year, three of them in Nordic tertiary colleges of education (*Lærerhøjskoler)* and two in the United States. Of the other five, two took a degree abroad, after which they were accepted into the University of Iceland for a B.A. or Cand. Mag. degree. Another two completed a degree in education, one a Ph.D. degree in the USA and the other a Cand. Paed. Spec. degree in Oslo. Both of them became professors at the new *Kennaraháskóli Íslands* / Iceland University of Education, established in 1971.

In the same year, 14 teachers graduated from the Teacher Training College by taking a one-year programme intended for high school graduates. Of those, ten became teachers. One took a B.A. degree later at the University of Iceland. Another one of these teacher graduates, who had previously completed the mathematics stream in high school, became a successful mathematics teacher in one of the Reykjavík lower secondary schools.

In 1958 teacher training does not seem to have had the same attraction for young people. Only 13 students graduated from the regular programme. All but one of them became teachers. Seven of them sought university education later in life, of whom five spent their sabbatical year in Nordic tertiary colleges of education, one studied at a university in Spain to become qualified for entrance to the University of Iceland to complete a B.A. degree, and one completed a Cand. Mag. degree in pedagogy at the University of Oslo.

That same year, 1958, 19 teachers graduated from the one-year programme for high- school graduates. One of them never taught, and only one of them entered university later in life to earn a B.A. degree and to teach at an upper secondary school.[503]

This little survey of a sample of teacher graduates shows that the good intentions of the School Affairs Board, not to split the profession of teachers into two groups, at the same time as the school system was being unified and the different school levels connected, were not realized. Primary teachers became a united group with great solidarity but only limited training, not least in mathematics, and very limited opportunities for further education. By 1958, 1,383 teachers had graduated from the Teacher Training College, of whom 157 were high school graduates. Out of the 1,226 regular Teacher Training College graduates, only 20 had completed a high school matriculation examination afterwards to earn the right to university studies.[504]

Two of the 1958 graduates report in the teachers' biographical lexicon (*Kennaratal*) that they took a one-year course in mathematics at the Iceland University of Education in 1971–1972. Considering that the University of Education was only established in 1971, this was one of the first tasks of the new institution, so its authorities must have been aware that further education in mathematics for primary and lower secondary school teachers was much needed.

Secondly, this survey reflects the gap between the two subcultures of teachers, on one hand those educated at universities and on the other hand at teacher training colleges, which played a role in the introduction of "modern" mathematics in the 1960s and 1970s.

[503] Freysteinn Gunnarsson (1958): 158–163. Ólafur Þ. Kristjánsson and Sigrún Harðardóttir (1958-1988)
[504] Freysteinn Gunnarsson (1958): 143

Training of Lower Secondary School Teachers

The lower secondary schools established in many towns and rural areas during Jónas Jónsson's time in office (1927-32) were included in the legislation of 1946 as general lower secondary schools completed at the age of 16 or 17.[505] In 1946 it was clear to the authorities that there was a great shortage of teachers for that school level, as may be understood from the speech of Minister of Education Brynjólfur Bjarnason on December 1, 1944, cited earlier. The 1946 Lower Secondary Schools Act says of teachers:

> 37. gr.
>
> Til þess að verða skipaður kennari við skóla gagnfræðastigsins, þarf eftirtalin skilyrði:
>
> a) að hafa almenna kennaramenntun eða stúdentsmenntun að viðbættu námi í uppeldis- og kennslufræðum, er fræðslumálastjórnin metur gilt;
>
> b) að hafa stundað eins til tveggja ára nám hið minnsta við háskóla í þeirri fræðigrein, sem ætlazt er til, að verði aðalkennslugrein hlutaðeiganda, enda sýni hann skilríki fyrir árangri af háskólanáminu;
>
> c) að hafa kennt tvö til þrjú ár og hlotið meðmæli hlutaðeigandi skólastjóra;
>
> d) að vera hæfur til að kenna fleiri en eina námsgrein.
>
> Heimilt er og að gera að föstum kennurum þá, sem kennt hafa tvö ár hið minnsta með góðum árangri við skóla, sem svara til gagnfræðastigsins, þegar lög þessi taka gildi, enda komi meðmæli skólastjóra til.
>
> Nú sækir enginn, sem fullnægir þessum skilyrðum, um lausa kennarstöðu, skal þá skólanefnd og fræðslumálastjórn leitast við að fá til hæfan mann, og má að tveimur árum liðnum gera hann að föstum kennara, enda komi meðmæli hlutaðeigandi skólastjóra til.
>
> Article 37
>
> For a teacher to obtain tenure at a school at the lower secondary level the following conditions are needed:
>
> a) to have obtained a general teacher training or a final examination from a high school in addition to studies in pedagogy and didactics which the Directorate of Educational Affairs deems valid;
>
> b) to have studied the subject, which is supposed to be the main teaching subject of the person concerned, for at least one or two years at university, on condition that he/she submit documentation on the results of his university studies;
>
> c) to have taught for two or three years and obtained the recommendations of the headmaster concerned;
>
> d) to be qualified to teach more than one subject.
>
> It is also permitted to offer tenure to those teachers who have taught at least two years with good results at a school comparable to the lower secondary level, when this Act comes into effect, on condition that the headmaster concerned so recommends.
>
> Should no one fulfilling the above conditions apply for an available teaching post, then the school board and the Education Board and the Directorate of Educational Affairs shall try to obtain a qualified person, and he/she may after two years be

[505] *Stjórnartíðindi* 1946 no. 48, May 7

appointed with tenure, on the condition that the headmaster concerned so recommends.[506]

This paragraph stated the intention of the legislator to ensure that teachers at the lower secondary school level had at least some university education in their teaching subject. But there was no requirement for a final examination or a certificate, and the fact was that no institution in the country provided such education in most subjects. (Some language courses were offered for a B.A. degree at the University). The clause about the lack of qualified applicants was most certainly applied, not only in the early years but right up to 1974, when the 1946 legislation was superseded by the Compulsory Schools Act.

The only subject for which there was a sufficient supply of university-educated teachers was Icelandic. A number of people had graduated in Icelandic studies, i.e. Icelandic linguistics, literature and history. Many of them had not taken pedagogy and didactics. Therefore provisional legislation was hastily passed on September 19, 1947, before the *Alþingi* assembled, to offer those who had completed a Cand. Mag. degree or a master's degree from the University of Iceland an exemption from studies in pedagogy and didactics for tenure.[507] The offer of tenure must have been made to provide the lower secondary schools with as many teachers with university education as possible.

As mentioned before, the University authorities had other ideas about the training of lower secondary school teachers. In 1951 a change was made in the regulations for the University of Iceland. The following are important items of the amended regulations:

The number of subjects taught for a B.A. degree was increased, now including mathematics, physics and chemistry. A B.A. degree was to be granted for at least six or seven whole-year examination courses, where two courses were taken each year. The subjects could be three or four, including a single course in introductory philosophy. A maximum of three courses, of increasing difficulty, were to be taken in each subject. Combined with two courses in pedagogy and didactics, the outcome would be a B.A. degree with a teacher's qualification. The primary aim of B.A. training was to qualify for its graduates as teachers at lower secondary schools where the respective subjects were taught.[508]

A final examination in mathematics, physics and chemistry with a first class grade from a mathematics stream of a high school would be accepted as the equivalent of the first course in these subjects. The courses in introductory philosophy, pedagogy and didactics were common for students in all subjects, and the didactics of the specific subjects were not available.

These regulations were followed up by legislation on priority for those who had completed a B.A. degree with the required pedagogy and didactics, for teaching positions at lower secondary schools.[509]

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[506] *Stjórnartíðindi* 1946, no. 48, May 7
[507] *Alþingistíðindi* 1947 A: 2
[508] *Árbók Háskóla Íslands* 1950: 111–112
[509] *Alþingistíðindi*, 1951 A: 1005

The programme for mathematics and physics student teachers was almost completely included in the courses for engineering students, and required only little extra teaching cost. The two lower courses in mathematics were taught by high school teachers Björn Bjarnason, who taught linear algebra, and Guðmundur Arnlaugsson, teaching introduction to analysis. Björn Bjarnason was the supervisor of the B.A. students. One small extra course on the programme was run for student teachers only. The University authorities may have expected to extend the special teaching for student teachers when the attendance had reached some acceptable number. However, there were never more than 2–4 students at the time in this programme, from which the last students graduated in 1972.

Thus the will of the legislator was clear, but how did these regulations work? A total of 26 students graduated from the B.A. programme in mathematics and physics in the period 1952–1972, the majority with the required pedagogy and didactics.[510] The average was thus a little more than one graduate per year. Of these, all but two became teachers, and 19 became mathematics teachers. Only about half of the group ever taught at a lower secondary school for some period. Only two teachers remained in the lower secondary schools all their professional life. Every one of those graduating in 1962 or later moved to a higher school level when the upper secondary school level began to expand in the 1970s. About one third of the group completed a master's degree or a similar or higher degree at a foreign university later in life.[511] Nearly all the students graduating from the B.A. programme had been employed in teaching for some period during their study time.[512]

These teachers were the backbone of the mathematics teaching force in Iceland for three decades. The mathematical content of their studies was small, and even smaller than in the regular programme for the Part I engineering examination. However, these students were served by dedicated teachers who shared with them their mathematical thinking skills. Furthermore the students themselves had already begun teaching, and therefore received their tuition with that in mind. Yet the 19 teachers at the middle and high schools were far from meeting the demand for mathematics and physics teachers at that time.

Why did more teachers not graduate, and what was the situation in other subjects? One of the reasons was an underdeveloped student loan policy. The Student Loan Fund was aimed at students studying abroad. With the constant inflation, there were tight restrictions on foreign currency, which had an artificial exchange rate, and currency was furthermore exchanged at different rates for different purposes. Students abroad had a restricted amount, and might manage on that amount together with summer work salaries transferred at a low official rate. The same amount was of little use for students in Iceland, and they had to work alongside their studies. Most B.A. subjects were taught between 5 and 7 p.m., i.e. after work hours. There were more graduates in the humanities than in mathematics and physics, which had their courses in the morning, while that is not of course the only explanation for the smaller number of graduates in the science subjects.

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[510] See Appendix K
[511] Archives of the University of Iceland: The protocols of the Faculty of Humanities (*heimspekideild*)
[512] Ólafur Þ. Kristjánsson and Sigrún Harðardóttir (1958–1988)

6.3. The National Middle-School Examination

The tool to create equity in entrance to the high schools was chosen to be a centrally organized examination, *landspróf miðskóla* (the national middle-school examination). Normally the pupils would take it at the age of 16.

The national examination covered nine subjects: Icelandic, in two sections, Danish, English, history, geography, natural science, physics and mathematics. The requirements were to be similar to the previous examination between the lower and upper departments of Reykjavík High School. These requirements proved difficult to meet. During the first few years, only a small percentage of each year's cohort earned the right to admission to the high schools. The Reykjavík lower secondary schools had more experienced teachers to prepare their pupils, especially in mathematics and physics. However, schools all around the country did their best to gather competent people to prepare their pupils for the examination.

Provisional regulations on the middle-school examination dated January 22, 1946 state that it is an examination which is taken all over the country at the same time, and all middle schools and lower secondary schools have the right to offer it. Article 3 says:

> Þangað til ný lög um skólakerfi, gagnfræðanám og menntaskóla koma til framkvæmda, skulu námskröfur til miðskólaprófs samsvara sem næst námskröfum til gagnfræðaprófs við menntaskólann í Reykjavík, sbr. reglugerð, dags. 8. febr. 1937, samkvæmt nánari fyrirmælum fræðslumálastjórnar.
>
> Until new legislation on the school system, lower secondary education and high schools is fully implemented, the requirements for the middle-school examination shall correspond as closely as possible to the requirements for the lower secondary examination at the Reykjavík High School, cf. regulations dated February 8, 1937, according to further instructions from the Directorate of Educational Affairs.[513]

In view of the fact that many schools were already preparing their pupils for the first national examination held in May 1946, it is clear that the national examination was considered the most important factor of the 1946 legislation, thus abolishing the high schools' monopoly on the education of Icelandic youth.

The National Examination Board

On April 14, 1947 regulations were adopted on the national examination. Section I contains general provisions about the examination, which subjects should be examined and which schools could offer it. Section II concerns the National Examination Board, to be composed of specialists in the various subjects, familiar with teaching, and the curriculum required for the examination.[514] There do not seem to be any provisions for changes or revision of the curriculum.

The first National Examination Board was appointed in 1946, for one year initially. In the first few years there were some changes to the board, but from 1950 the board was fairly stable for one-and-a-half decades, until 1966. The board members during that period are listed in Appendix D.

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[513] Gunnar M. Magnúss (1946): 19

[514] See appendix D

All the board members had a university degree in their subject except two: the mathematics representative, and for a period a provisional member for geography. These two were theologians, graduates in 1917 and 1919, i.e. they had the 19th-century style higher teacher education, eligible for a position in the high schools. From the list one can conclude that the first priority was that the person had a university degree, preferably in their subject, and secondly had taught, preferably at the upper secondary level, or failing that at the lower secondary level. Four of the permanent members in 1950–1966 taught at Reykjavík High School, one at Laugarvatn High School, established in 1953, three came from the Teacher Training College, four taught at lower secondary schools, and two worked elsewhere, so the board can not be characterised as a group of high school teachers.[515]

The duties of the chairman were to be the overall supervisor of the examination and to be its executive director, chairing the board, which meant holding meetings to prepare and organize the examination, being in touch with the schools around the country and monitoring that the grades were recorded, computing averages and checking these with the schools and writing a final report.

The board had its mandate from the Minister of Education under the supervision of the Director of the Educational Affairs. There were no inspectors or counsellors for the specific subjects, until Guðmundur Arnlaugsson, the physics representative of the board, was appointed consultant in mathematics teaching in a 50% position, in the period 1964–1966, and Óskar Halldórsson Icelandic consultant 1964–1967.[516] In a telephone interview in January 2002 by the author of this study with former Minister of Education Gylfi Þ. Gíslason, he confirmed that the members of the board were chosen for their expertise, and that they were fully trusted to take all necessary decisions regarding their subjects.[517]

During the period 1950–1966 the examinations proved relatively similar from year to year. The board members had a mandate to develop their subjects, but they did not do so. From discussions that arose in the 1960s in the newspapers one may understand that their view was that the examination had to be realistic with respect to the requirements of the high schools, of which the Reykjavík High School was by far the largest. A great number of freshmen attending that school did not complete the first year, at least not at their first attempt. It would not be fair to the pupils to make any drastic changes in the examination. The three successive geography board members had differing views about the subject,[518] so the geography examination changed with the board members. It was only towards the end of that period that considerable changes were made in the mathematics examination, both in structure and content.

By the end of the period, the national examination, as an entrance examination to the high schools and many other ways to further education, came under considerable criticism. Finally the crisis was solved by opening up upper secondary education, with more high schools and introducing comprehensive schools, which in most cases were a combination of high schools and technical schools. This will be discussed in chapter 8.

[515] See Appendix D
[516] Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. III, 378; Vol. IV, 518
[517] Gylfi Þ. Gíslason, January 17, 2002
[518] Haraldur Steinþórsson, January 22, 2003

The National Examination in Practice

The first national examination was held in May 1946 in three schools in Reykjavík and nine schools in the regions. As might be expected, by far the best results were in Reykjavík High School, which had been preparing pupils for upper secondary level for one hundred years. Some of the schools were unprepared for the requirements, so a selected group of their students were allowed to repeat the examination in the autumn. The next year, two of the schools were not included and another two came in.[519]

By 1951 the national examination had established itself, and the numbers of schools and pupils become stable. In the board's reports a higher ratio of pupils from outside Reykjavík seemed to achieve the entrance requirements. The explanation lies in the fact that those schools did not send in the examination papers of pupils with low results, while the examination board examined all papers from Reykjavík and its neighbouring town Hafnarfjörður.

In the following years, around 25 schools outside Reykjavík held the national examination. This shows considerable ambition on behalf of the school authorities in many of the small places. In 1951 only 15 schools out of 24 sent 10 examination papers or more, and five schools sent papers for 20 pupils or more. It must have entailed considerable extra expense to teach so few students the extra curriculum that was needed for the national examination, in addition to the regular curriculum. The difference lay mainly in extra mathematics (algebra) and physics.

In the period 1952–1957 a stable 17–18% of the 16-year age cohort, born in 1936–1941, took the examination, and 12–13% of the same population met the requirements for high school.[520] From 1960 the number of pupils increased rapidly, both due to the rising birth-rate, and also due to an increasing rate of young people wishing to enter higher education. The demand for more education continued to increase during the next two decades. The growing number of pupils gradually exerted heavy pressure on the school system, and finally led to changes around 1970.

The Predictive Value of the National Examination

In 1961 Jónas Pálsson, later rector of Iceland University of Education, and Hjálmar Ólafsson, later deputy headmaster of the new Hamrahlíð High School, published their results from an investigation of the predictive value of the national examination.[521] Their conclusion was that it was sufficient to run examinations in Icelandic, English, Danish, mathematics and physics for the purpose of predicting the results in high school studies. Furthermore, it would probably be an advantage to the teaching of history, geography and natural science to be exempted from the national examination. These subjects might suffer in the national examination from one-sided emphasis on items to be memorized and less on understanding and independence in working methods.[522] This may have had influence on later choice of subjects in the compulsory school examination, which was established in 1976 and initially numbered six, but later four, and for some period of time only two subjects.

[519] See Appendix D

[520] Archives of the Ministry of Education: Landsprófsnefnd, reports (1952–1957). Jónas Pálsson and Hjálmar Ólafsson (1961): 206. See Appendix D

[521] Jónas Pálsson and Hjálmar Ólafsson (1961): 195–210

[522] Jónas Pálsson and Hjálmar Ólafsson (1961): 202

In the above-mentioned report the role of the national examination is considered to be the following:

- Í fyrsta lagi að tryggja ákveðna og samræmda lágmarksþekkingu í allmörgum námsgreinum, áður en nemendur hefja eiginlegt langskólanám.
- Í öðru lagi mun ætlunin að tryggja eftir mætti úrval hinna hæfustu með tilliti til ákveðinna eiginleika, sem nauðsynlegir eru taldir þeim, er með sæmilegum árangri vilja stunda nám í menntaskóla og háskóla eða öðrum framhaldsskólum. Er hér fyrst og fremst átt við það sem almennt er nefnt greind.
- Í þriðja lagi mun vaka fyrir skólayfirvöldum að veita öllum nemendum og aðstandendum þeirra nokkra tryggingu fyrir mati óvilhallra manna á prófverkefnum og að verkefni séu hin sömu fyrir alla. Af þessu leiðir líka bætt aðstaða unglinga, hvar sem er á landinu, til að búa sig þar undir framhaldsnám. Er þetta síðast nefnda vissulega mikilvægt, svo sem þeir vita gleggst, sem áður urðu að sæta inntökuprófum við menntaskólana, þótt þeir hefðu áður tekið gagnfræðapróf á landsbyggðinni.
- Firstly, to ensure a certain and standardized minimum knowledge in a number of subjects before the pupils commence true long-term education.
- Secondly, the intention appears to be to ensure as far as possible the selection of the fittest with respect to certain attributes, which are considered necessary for those who want to study in a high school and a university or other higher education. This is mainly a reference to what is generally called intelligence.
- Thirdly it appears to be the aim of the school authorities to offer all pupils and their relatives a certain assurance of an assessment by impartial persons of examination papers, and that the examinations are the same for all pupils. This leads also to improved conditions for young people, wherever in the country they may live, to prepare for further education. The item last mentioned is certainly important, as they best know, who in previous times had to take entrance examinations at the high schools, even if they had earlier taken a lower secondary school examination elsewhere in the country.[523]

The authors mention also certain disadvantages implied in the examination system. Some of them were unavoidable consequences of the organization, while others could be greatly reduced if the examinations were continually revised and improved in accord with changing circumstances and increased experience. They did not think it was desirable to abolish the national examination, even if it had no parallel abroad. However, it was necessary to allocate a considerable amount of money and work to research on the nature of the examination and its usefulness, as the examination determined the fate of young people today, more than other examinations.

This article was written in 1961. It contains many of the arguments which were brought up later in the 1960s when there was growing criticism of the school system and this examination in particular, such as the need for research. The only available information about the national averages in mathematics 1946–1955 is found in this article. These figures were used in tables D1–D3 in appendix D.

[523] Jónas Pálsson and Hjálmar Ólafsson (1961): 208

6.4. Mathematics in the National Examination

Responsibility

Cand. Theol. Steinþór Guðmundsson (1890–1973) was in charge of mathematics in the national examination in 1947–1962. He was a typical impoverished student at the beginning of the century. His father died when he was 11 years old. He seems to have studied at home to complete a lower secondary examination in 1909 and a matriculation examination in 1911 at the age of 21, both without attending school. Steinþór Guðmundsson studied introductory philosophy at the University of Copenhagen in 1911–1912 and mathematics and chemistry at the Polytechnic College in Copenhagen in 1912–1913. He completed a degree in theology at the University of Iceland in 1917, in order to be eligible for tenure at a high school as teacher of mathematics and natural sciences. Education to prepare for teaching these subjects was not available during World War I.[524] Steinþór Guðmundsson started to teach at the age of 16, in his home area. He taught and was headmaster at various schools in the period 1917–1942.

In 1942 Steinþór Guðmundsson was appointed mathematics teacher in *Gagnfræðaskóli Reykvíkinga*, *Ágústarskóli,* the only school after the Akureyri School allowed to offer lower secondary examination that conferred the right to entrance to the learned department of Reykjavík High School. This experience of a lower secondary school, comparable to the lower department of the Reykjavík High School, may have led to Steinþór Guðmundsson being entrusted to be in charge of the national examination in mathematics. Mathematician Björn Bjarnason took the task over in 1963. Björn Bjarnason was in charge of the mathematics examination for nine years, until 1972.

Content

Initially the national examination was to be equivalent in all respects to the lower department examination (*gagnfræðapróf*) of the Reykjavík High School in 1946. In all subjects the syllabus was a list of pages in textbooks to be studied. This must be due to the fact that the Reykjavík High School did not have a detailed curriculum, other than the existing law and regulations, and it had for a long time adjusted its syllabus to Danish high schools. A more general curriculum for the national examination was not published until 1968. For mathematics the list was as follows:

1. *Reikningsbók / Arithmetic* by Dr. Ólafur Daníelsson, pp. 1–87.
2. *Algebra* by Dr. Ólafur Daníelsson, pp. 1–115.
3. *Kennslubók í rúmfræði / A Textbook in Geometry* by Jul. Petersen, translated by Sigurkarl Stefánsson, the first 10 chapters and problems 1–130. Furthermore the chapters on area and volume in Dr. Ólafur Daníelsson's *Arithmetic*.[525]

Dr. Ólafur Daníelsson's influences are obvious. The syllabus is composed of his books and his master's, Jul. Petersen. After the first year the Euclidean geometry in item 3 was removed and transferred into the syllabus of the first year of high school. Considering that pupils who managed to enter the Reykjavík High School had had a year of preparation for a hard competitive examination for the 25 or 30 places, and

[524] Haraldur Steinþórsson, Steinþór Guðmundsson's son, January 22, 2003
[525] Bjarni Vilhjálmsson (1952): 25

then two years of teaching aimed at high-ability pupils, it is no wonder that this syllabus was hard to cope with for the inexperienced schools, teachers and pupils around the country. To take an example, 164 pupils attempted the entrance examination into the lower department in 1946, 38 failed, 126 passed and 30 pupils were accepted.[526]

The new schools offering the national examination must have had difficulties in finding teachers to teach this new subject, as no school in the country provided suitable education for them.

The (unpublished) mathematics curriculum for the examination in 1947–1966 was as follows:

- Proportions in fractions and percentage, taught with the aid of *regula de tri.*
- Proportional partition.
- Area and volume, including right cylinders, right cones and the sphere (its applications, not theorems).
- The rule of Pythagoras (the application, not the theorem).
- First degree equations with one, two and three unknowns.
- Algebra up to addition of fractions with sums in the denominator (only positive integer exponents, no graphs.)

The algebra was usually all taught in the same academic year as the national examination, except simple linear equations with one and two unknowns, which were taught the year before. The measurement topics – area, volume and the use of the Pythagorean Theorem – were usually taught the year before and revised in the examination year.

The mathematics examination in 1946–1965 was in two parts. Part I was a 3-hour examination with 6–8 composite problems, all of which the pupils had seen before, called *seen problems*. Part II also comprised 6–8 composite problems, not previously seen by the pupils. During the last years of this period, this part was taken in two separate sessions of 2 and 2½ hours on the same day. Often the pupils earned a considerably higher grade for the seen problems than for the second part (see table 6.3.).

While the problems were composite and complicated, 50–70% of them were in one way or another related to the "real world", i.e. were "word problems".[527] The ratio of "word problems" dropped to 10–15% after 1967, when the examination questions began to change. The problems became shorter and the number of unrelated tasks increased. It seems questionable whether this was an advantage for the pupils, or added to their interest in mathematics and improved their attitude. One reason may be that the number of pupils had increased to a degree that this measure was taken to lighten the burden of grading the papers. This will be discussed further in connection with the introduction of "modern" mathematics.

[526] Einar Magnússon (1975): 90
[527] See Appendix G

Mathematics Teachers

The B.A. programme in mathematics and physics at the University of Iceland was far from supplying enough teachers to meet demand. For the national examination classes the "best" people available were picked. They would often be persons who had one or two years' university studies, after which they had dropped out, in many cases because of lack of financial means for their studies. By adding pedagogy and didactics, they were qualified to become lower secondary school teachers. They might have studied law, medicine, economics, engineering or even theology, i.e. the subjects that were offered at the University of Iceland. Sometimes they were officials, bankers etc., coming in to teach part-time. In the first decade, no other teachers were available.

One might have thought that it was easiest to recruit teachers in Reykjavík and the vicinity and most difficult in the remote rural places. The urban schools could recruit officials with some university education to step in as part-time teachers, while the rural boarding schools would have to depend on finding resident full-time teachers. However, as the rural schools could offer inexpensive housing, which was in short supply in the capital area in the post-war period, they may generally have had more stable teaching staff than many of the urban schools.

In the following we will investigate the results in the national examination in relation to the background of the teachers.

Investigation of Sample Schools

The scene of schools changed during the period 1946–1976. Initially the high schools held the national examination. The last examination in the Reykjavík High School was held in 1949, while in the Akureyri High School the last one was held in 1964. After the first five trial years, the examination was held in 24 schools, and toward the end (in 1973) in 45 schools.[528]

<u>In 1951 there were</u>
4 schools in Reykjavík and the neighbouring town, Hafnarfjörður, with 250 pupils
12 schools in towns and villages with 137 pupils
8 rural boarding schools with 110 pupils
Total: 476 pupils

<u>In 1973 there were</u>
12 schools in Reykjavík and vicinity with 982 pupils
26 schools in towns and villages with 481 pupils
7 rural boarding schools with 164 pupils
Total: 1627 pupils

The number of pupils in urban schools thus was growing much faster than in the boarding schools, reflecting the changing population pattern in the country.

To investigate the status of the mathematics in the examination, a sample of eight schools was chosen. As the examination was not run throughout the period in all schools, and data are not available for the whole period, the investigation is divided into three periods with a different number of schools in each period. The first group includes five schools, the second one seven schools and the third also seven schools.

[528] Archives of the Ministry of Education: Landsprófsnefnd, reports 1946–1973

The criterion for choosing schools in this investigation was to pick:

2 schools from towns in different areas of the country

2 rural boarding schools, also from different areas

2 schools in Reykjavík and vicinity which offered the national examination for the whole period

and for comparison:

1 school in Reykjavík attended by most Reykjavík pupils for a short period

1 school in a small town which entered the group late but was known for exceptionally good results

Five schools are included in the first sample, as one of the other town schools only offered the examination later.

The third Reykjavík area school, School E, is included in the second sample and not in the first and the third. For several years nearly all students in Reykjavík preparing for the national examination were collected in that one school for one year. Results are only available for three years. All the schools except school B had some history of secondary education in their area.

List of the schools:

1.	School A	a selective school in Reykjavík
2.	School P	an urban school in the Reykjavík area
3.	School C	an urban school in Northern Iceland
4.	School R	an urban school in the East Fjords
5.	School D	a boarding school in Northern Iceland
6.	School S	a boarding school in Southern Iceland
7.	School E	a large Reykjavík school
8.	School B	an urban school in the West Fjords

The third sample will be looked at in connection with the introduction of "modern" mathematics.

Results

The results for all pupils taking the national examination were recorded each year in a single book of records, and the Examination Board wrote a report each year.[529] The grading scale was 0–10 with one decimal point. The report gave statistical information about how many pupils earned Distinction (average 9–10), Ist grade (7.25–8.99), IInd grade (6.00–7.24), IIIrd grade (5.00–5.99), and so forth, while there were no statistics about the individual subjects, such as mathematics. The results from the article on the predictive value of the national examination[530] are used for the percentage of age cohort in 1946–1961 and for the national average in mathematics in the period 1946–1955 in table 6.1. From 1968, averages for each subject were computed by the National Examination Board, and these were used in table 7.5.

Most of the schools served some defined geographical area, while the pupils could apply for other schools if they wished to do so. Soon a kind of competition formed between the schools about the results in the national examination. In most schools,

[529] Archives of the Ministry of Education. Landsprófsnefnd: Protocols 1946–1955, 1962–1973; Reports

[530] Jónas Pálsson and Hjálmar Ólafsson (1961): 195–210

pupils were advised not to attempt the examination unless they were properly prepared, and in some schools pupils not sufficiently prepared were not allowed to take it. Later, the pressure from pupils and parents increased, and in some schools, anyone who applied was let in. Schools from outside Reykjavík did not send in the papers for their pupils with the lowest grades, so the results from the various schools are not comparable in all respects.

In the tables the average grade in mathematics (Mth.) each year is compared to the average of averages of the nine grades in all subjects (Ave.) for each school each year. Where national averages are not available, weighted averages for the school groups were computed for mathematics and for the general averages.

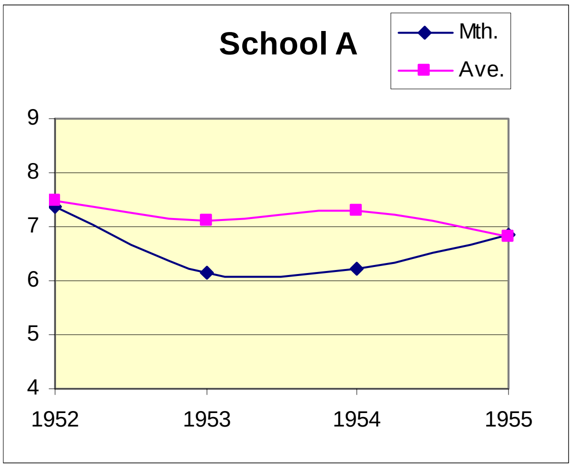
Table 6.1.

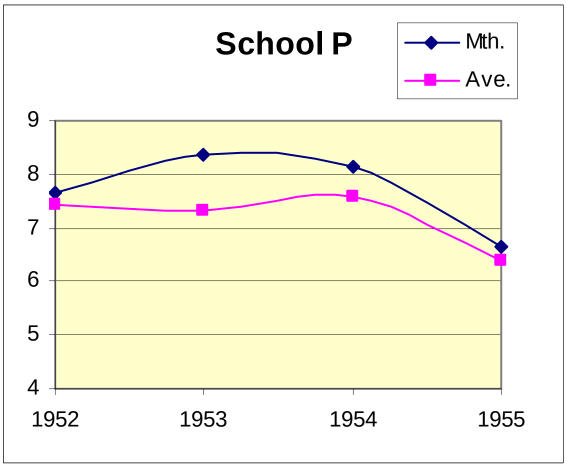
Year	School A			School P			School C		
	No.	Mth	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.
1952	4	7.1	7.47	16	7.4	7.43	18	5.4	6.30
1953	15	5	7.11	9	7.2	7.32	7	5.3	6.32
1954	7	5.8	7.29	5	7.7	7.59	13	5.5	6.39
1955	10	6.4	6.83	11	6.2	6.40	22	5.8	6.55

	School D			School S			Total			National		
Year	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	%	Mth.
1952	13	7.7	7.65	16	7.5	7.49	67	6.9	7.19	448	18.2	5.8
1953	12	5.0	6.52	12	6.0	6.51	55	5.6	6.78	396	17.2	5.6
1954	9	6.6	7.05	20	7.1	6.97	54	6.5	6.94	371	16.8	5.7
1955	11	6.8	7.11	13	6.9	7.08	70	6.3	6.76	418	18.7	5.4

Table 6.1. The number (No.) of pupils, the average grade in mathematics (Mth.) and the average of averages (Ave.) of grades in all the subjects in five schools in 1952–1955.

The results in mathematics were looked at in relation to the general averages. From the “Total” column it appears that the mathematics grade was usually lower than the general average, often considerably lower. In the graphs below the results in mathematics have been corrected for the difference between the general average (Ave.) in the five schools and the mathematics average (Mth.) as seen in the “Total” figures for the schools in question. The two curves should therefore be comparable.





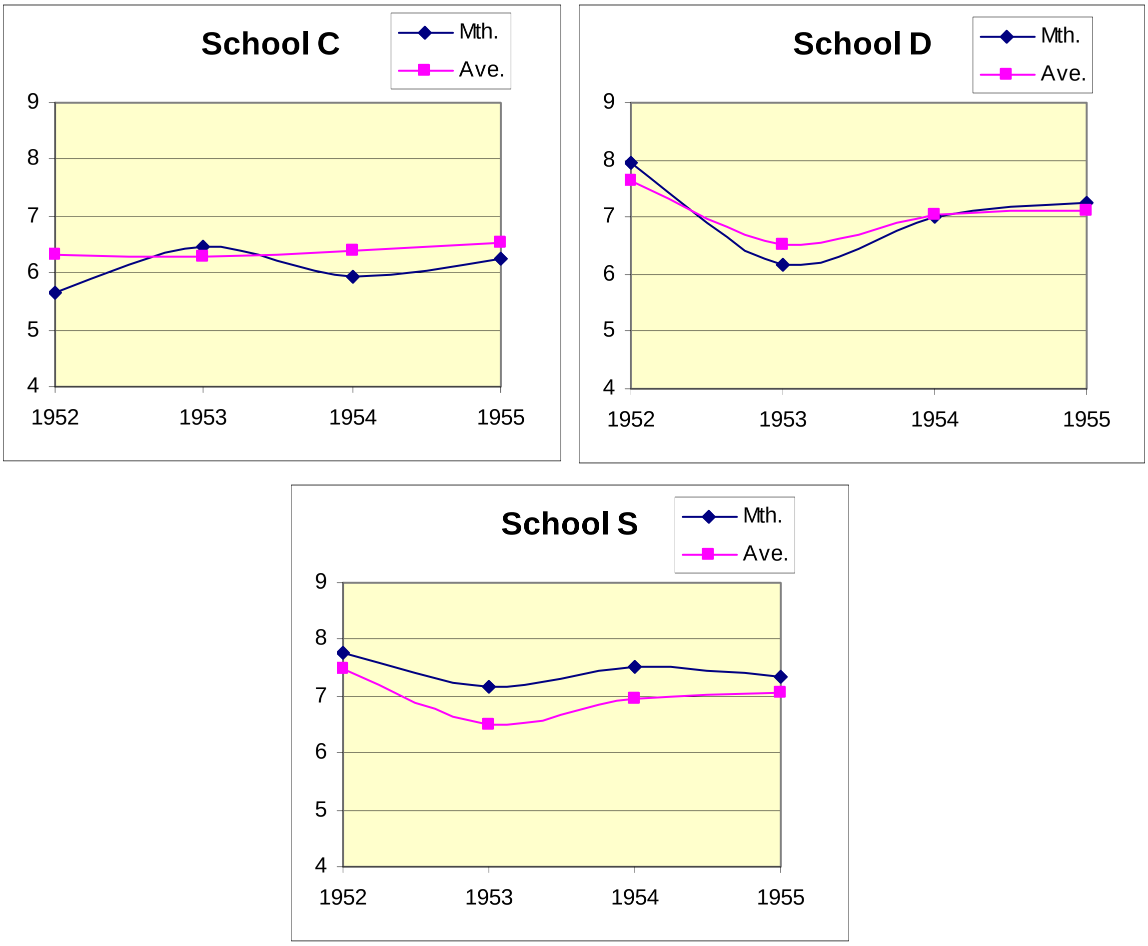


Fig. 6.2.–6.6. General results in national examination as compared to results in mathematics in five schools during 1952–1955.

Comparing the "Total" and the "National" columns, we see that the five schools chosen have most years a higher average in mathematics than the national average in mathematics. One possible explanation is that out of this small sample at least Schools A and P in the capital area selected their pupils from among the applicants. Another explanation is that the other three schools in the regions did not send all in their examination papers and they may weigh heavier here than in the national average. The third factor could be that the rural boarding schools (D and S) often had higher grades than the urban schools in the regions (C). In the rural boarding schools the pupils had already paid for their stay and could concentrate on their studies. In the urban schools in the regions the pupils might have to accept work periodically, and were less focussed on further and higher studies, and might only be trying their luck.

Comparing graphs of schools A and P, both of them capital area schools, which selected their pupils and have similar general averages most of the time, it emerges that the mathematics is well above the average in school P, while it is below average in school A. One of the explanations may lie with the teachers. School A was served by temporary substitute teachers for much of the period. Meanwhile a medical doctor and a mathematics textbook writer taught mathematics at school P for the entire period. Another explanation may be that school A was a school for girls.

Similarly, Schools D and S are both rural boarding schools and their results ought to be similar. In fact they are nearly exactly similar in their general averages, while they differ considerably in mathematics. Looking at the teachers' biographies, one finds that school S had a teacher who had studied mathematics teaching at the Royal Danish School of Educational Studies / *Danmarks Lærerhøjskole* in Copenhagen. The mathematics results in Schools C and D were more or less comparable with the general results.

The second sample in table 6.2. is from a decade later, the period 1962–1966, right before "modern" mathematics entered the syllabus.

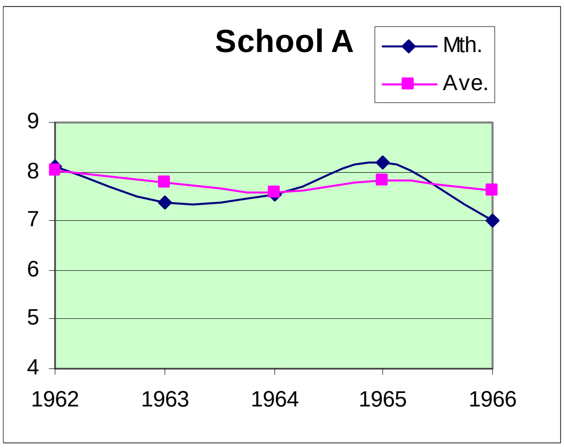
Table 6.2.

	School A			School P			School C			School R		
Year	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.
1962	27	7,9	8,03	11	5,5	6,06	35	5,5	6,19	6	5,3	6,14
1963	21	7,2	7,78	26	6,7	7,32	29	7,1	6,79	3	4,7	7,34
1964	23	6,5	7,57	28	6,5	7,12	30	5,1	6,34	6	6,1	7,49
1965	19	7,7	7,81	20	6,4	6,58	44	6,6	6,64	11	4,7	6,53
1966	26	6,5	7,63	28	6,3	6,99	52	6,7	6,66	10	6,0	6,95

	School D			School S			School E			Total			National
Year	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.
1962	11	7,3	7,60	18	7,8	6,67	187	6,5	6,33	108	6,7	6,86	740
1963	12	7,7	7,46	16	7,1	6,86	155	6,3	6,53	107	7,0	7,21	739
1964	13	5,3	7,29	15	6,7	7,11	112	5,1	5,86	115	6,0	7,04	785
1965	21	5,2	6,59	10	6,4	6,87				125	6,3	6,81	825
1966	21	5,7	6,84	16	7,0	7,04				153	6,4	6,97	918

Table 6.2. The number of pupils, the average grade in mathematics and the average of averages of grades in all the subjects in seven schools 1962–1956.

In the following graphs the mathematics grades are, as before, corrected for the difference between mathematics grades and the general average grades for all the schools, computed from the figures in the "Total" column. This difference is marginal in the first two years, while in 1964 and thereafter it is considerable. A new mathematics representative on the Examination Board took over responsibility in 1963. The consequent changes in the form of the examination will be discussed later.



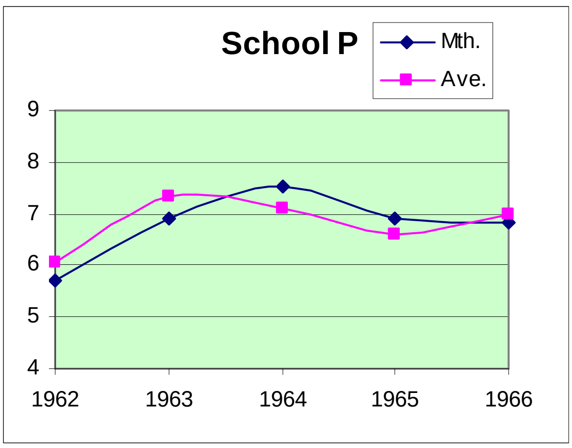


Fig. 6.7.–6.13. General results in national examination as compared to results in mathematics in seven schools during 1962–1966.

In the period 1962–1966 the mathematics teaching in School A was stable. A regular teacher taught in the national examination years, while junior classes were served by a series of part-time teachers. A new teacher with a B.A. qualification took over in School P in 1962 and the pupils were not selected any more. The results improved in time. The teacher suffered from periodic illness, during which the results of the pupils deteriorated.

The results in School C were as before, while a new school, School R, had come in. This was a typical urban school in the regions, where there were frequent changes of mathematics teachers. School S had good results in mathematics as before, until 1965, when the teacher suddenly died. The following year one of the teachers with B.A. education was appointed and the good results continued. The same teacher taught in all grades. According to the headmaster of the school at this time mathematics was always well handled, and it had more hours than officially prescribed.[531]

In school D the results deteriorated in and after 1964. This school was, as far as is known, served by the same teacher throughout the period. Possibly this teacher had become accustomed to examinations from the previous board representative, and he may have found it difficult to adjust his teaching to problems from a new representative.

For the period when School E was run, it had pupils of very varying abilities coming into the school for one year. In its last year, it had more pupils of lesser ability. It was divided into many groups according to the anticipated ability of the pupils. Due to lack of housing the school had two sessions. Some of the groups were morning classes, while others had to begin at 13.30 in the afternoon. The more able pupils had a choice of morning or afternoon classes, and thus had better conditions for their studies. From another collection of data it can be seen that pupils in higher-ability classes had a rather high average in mathematics compared with the general average, while the opposite was seen in low-ability classes, i.e., the range of mathematics was greater than the range of the general averages. The school had trained teachers, at least for the more able half of the pupils, and the results were stable.

The general conclusion is that the results in mathematics were sensitive to changes in the teaching staff. Where the results were exceptionally good, expertise was detected. On one hand there was the arithmetic textbook writer in School P, and later the B.A.-qualified teacher. On the other hand there were the two teachers in School S, both of whom had special training in mathematics teaching, one of them from the Royal Danish School of Educational Studies / *Danmarks Lærerhøjskole* in Copenhagen and the other with a B.A. degree from the University of Iceland. Illness and death of teachers affected the examination results, as may be seen in schools P and S. The investigation reveals that stability in the teaching force and teachers with training aimed at preparing for teaching mathematics are strong factors in successful mathematics teaching.

Clearly the national examination brought increased mathematics education to the regions. School authorities made a point of finding teachers who could cope with the mathematics teaching. Another point is that this new occupation, teaching introductory higher mathematics, made it more attractive for people specially

[531] Jón R. Hjálmarsson, November 2002

educated in mathematics to work in the regions, as they could find a suitable and worthy occupation.

The Seen Problems

In 1955 the national examination had become an established institution, and teachers become trained in coaching the pupils to take this kind of examination. Some data about the results in the seen and the unseen part of the examination are available from an experienced teacher, H, who taught pupils solely for the last year before the national examination.

What was the purpose of testing seen problems? Did they teach the pupils to study certain types of problems to help them cope with similar problems, according to Polya's theory: "Have I seen something similar before?"[532] or were they there so that those who were not good at mathematics would be rewarded for their effort in trying to remember the seen problems?

Posed otherwise: Were they helpful in teaching pupils problem-solving skills, or were they rather a life-belt they could cling to when they were not good at mathematics?

Data are available from the period 1956–1964, excluding 1963 for the classes the experienced teacher H taught in school E in Reykjavík. For each year the results in the seen problems are in the first column marked with S (if known), in the next column the unseen problems, and finally the average of those two as the final grade in mathematics in the national examination.

1956			Av.
3. A	6.2	6.0	6.1
3. B	7.2	7.4	7.3
3. C	6.1	5.9	6.0
Av.	6.5	6.4	6.5

1957			Av.
3. A	6.9	7.6	7.3
3. B	8.0	8.6	8.3
3. C	5.3	6.4	5.9
3. E	7.2	7.8	7.5
3. F	5.9	5.8	5.8
Av.	6.7	7.2	7.0

1958	S		Av.
3. A	8.6	6.9	7.8
3. B	7.9	6.2	7.1
3. D	7.1	5.4	6.3
3. E	8.5	6.9	7.7
3. G	5.6	4.7	5.2
Av.	7.5	6.0	6.8

1959	S		Av.
3. B	7.9	6.5	7.2
3. C	6.6	5.7	6.1
3. D	6.5	5.2	5.8
Av.	7.0	5.8	6.3

1960	S		Av.
3. A	9.5	6.7	8.1
3. B	9.1	5.9	7.5
3. C	7.9	5.1	6.5
3. D	8.1	6.0	7.0
3. F	6.9	4.7	5.8
Av.	8.3	5.7	7.0

1961	S		Av.
3. A	8.6	6.7	7.6
3. B	8.8	6.1	7.5
3. D	8.6	5.5	7.1
3. E	7.8	5.6	6.7
3. F	6.0	4.2	5.1
Av.	8.0	5.6	6.8

1962			Av.
3. B	7.8	7.5	7.6
3. C	8.0	7.5	7.8
3. D	7.1	6.2	6.7
3. I	5.6	5.2	5.4
Av.	7.1	6.6	6.9

1964	S		Av.
3. A	5.6	4.1	4.9
3. B	6.7	7.0	6.9
3. C	5.3	4.1	4.7
3. E	5.9	4.3	5.1
Av.	5.9	4.9	5.4

Table 6.3. School E – Seen Problems (S) as against Unseen Problems.

It cannot be seen from the data from 1956, 1957 and 1962 which group was the seen problems. However from the other data it is obvious that in most years in most classes the seen problems serve to lift the mathematics grade.

[532] Polya, G. (1973): xvi–xvii

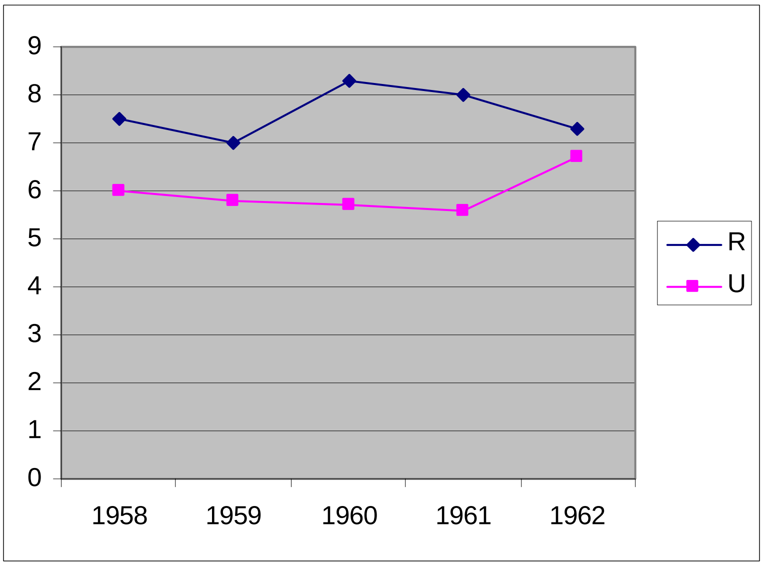


Fig. 6.14. Seen problems (R) against unseen problems (U).

In some years, such as 1960, it seems that the "high ability" classes earn more from the seen problems than the lower-ability classes. Possibly the "high ability" relies on working hard. In 1966, when the examination was composed of two parts, the first one in general skills and the second one in problem solving, H's data show nearly the same average in both parts (not shown on graph or table).

6.5. The State Textbook Imprint[533]

The Pioneering Years

The Union of Icelandic Primary Teachers / *Samband íslenska barnakennara, SÍB* was established in 1921, and from that time discussions about textbooks in Iceland intensified. Textbooks were expensive, and many children in primary schools did not have textbooks. In 1931 three social-democratic members of the *Alþingi* proposed a bill on a state textbook publishing house, on the initiative of the Union of Icelandic Primary Teachers. This bill was not passed, and for the next few years there were discussions of whether these matters were best handled by a state institution, or not.

In 1936, *Alþingi* adopted legislation on the State Textbook Imprint / *Ríkisútgáfa námsbóka*.[534] At that time *Alþingi* was led by a coalition government of the Social Democrats and the farmers' Progressive Party. An editorial board had the role of dealing with contracts with the authors, the design of the books and deciding the number of copies. The members of the first editorial board were:

Chairman: State Medical Director of Health Vilmundur Jónsson, appointed by the Minister of Education, a social democrat who had been the main proponent of the project, Ex-Minister of Education Jónas Jónsson for the teachers in district schools and lower secondary schools, and Primary School Headmaster Guðjón Guðjónsson from the Union of Primary Teachers. All these men were renowned for their commitment and idealistic work for public education.

[533] Ingvar Sigurgeirsson (Ed.) (1987). This section is drawn from this book unless otherwise indicated.
[534] *Stjórnartíðindi* 1936 no. 82, June 23

The publications were to be funded by a special textbook charge for each family with children, 5–8 *krónur* (crowns) in the first two years, thereafter 5 *krónur*. Headmasters were to provide the books free to all pupils at the compulsory level. It is not known whether children who did not attend school until the age of 10 were provided with schoolbooks from the age of seven.

The execution of the project was entrusted to the State Printing Press, Gutenberg, and its director became the executive of the project.[535] In 1940 another state institution, the Cultural Fund Imprint / *Bókaútgáfa Menningarsjóðs*, took over the distribution of textbooks. In 1937–1956 a total of 86 titles were published in 2,516,900 copies. Out of those, 63 titles were composed or worked out on the initiative of the State Textbook Imprint. Most of these new titles were, however, published in the period 1937 to 1941.[536] From that time until 1957 there were serious difficulties in funding the project. After 1941 increasing inflation became a great hindrance. *Alþingi* was reluctant to increase the textbook-charge, so the amount of available money declined in value.

Period of Decline

In 1953, Dr. Gylfi Þ. Gíslason, later Minister of Education for the Social Democrats, proposed in *Alþingi* a resolution on the State Textbook Imprint. Dr. Gylfi Þ. Gíslason was the son-in-law of physician Vilmundur Jónsson, the first chairman of the publishing board. Dr. Gylfi Þ. Gíslason had small children, the eldest in primary school, so he had many reasons to be concerned about the activities of the State Textbook Imprint. At this time the Social Democrats were, however, in opposition and could hardly expect to have adopted a resolution demanding increased funding.

Under the 1946 education legislation, compulsory education was extended by one year. The aim of the law on the State Textbook Imprint was to provide all children at the compulsory level with schoolbooks, so Dr. Gylfi Þ. Gíslason considered it natural and unavoidable to add one year to the Imprint's duties. He recalled that the textbook charge to each home with children was at that time only 15 *krónur*, which could not be regarded as high, while the average cost for children in the first year of the lower secondary level was 268.50 *krónur*, which was not a small sum. Dr. Gylfi Þ. Gíslason had heard criticisms that the books of the State Textbook Imprint were produced too cheaply, and that their finish was poor. Furthermore there were too many reprints, and money was not allocated to revise older editions or even have new material published. After all, saving money was not the only priority, and higher quality of the books could be advantageous. The matter was debated once more, but never put to the vote.[537]

In 1953 the consumer price index had multiplied by 7.5 since 1939, the first year with an index basis by Statistics Iceland, the national statistical institute. The currency had been devalued in 1938, so probably prices had increased by a higher factor since the family textbook charge of 5 *krónur* was adopted in 1936. The 15-*krónur* charge in 1953 had by no means kept up with the consumer price index, which explains the poor quality and the rarity of new publications.[538]

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[535] *Alþingistíðindi* 1936 A: 599–602
[536] Ingvar Sigurgeirsson (Ed.) (1987): 15, 76
[537] *Alþingistíðindi* 1953 D: 363–364
[538] Statistics Iceland: website, accessed May 2003.

In the same session of the *Alþingi* a proposal to amend the 1946 school legislation was debated. Statements were heard such as: every seventh *króna* from the State Budget went to the school system, there were too many compulsory school years, and it would be healthier for most 14- to 16-year-olds to join the work force, rather than sitting at a school desk. The academic year was too long, and it was absurd to keep children in schools into the summer and take them into schools early in the autumn, so that they were cut off from the natural contact with working life and deprived of a healthy outdoor life.[539] At this time the academic year was nine months only for 7- to 9-year-old children in towns; all other schools operated for eight months, or even less in villages, where exemptions were granted if pupils were helping with farming or fishing. This indicates that the general public was still in doubt about the education legislation, and the confidence in home education and early working life was still alive and kicking. This may explain the fact that Dr. Gylfi Þ. Gíslason was cautious about requesting increased funds for the State Textbook Imprint.

Partial Restoration

In 1956 new legislation on the State Textbook Imprint was passed,[540] under the leadership of Minister of Education Dr. Bjarni Benediktsson of the conservative Independence Party. The new law entailed that the state would provide one-third of the funding, while the textbook charge from the homes would fund two-thirds. Secondly, the State Textbook Imprint would have its own executive. Free textbooks were to be provided to all children in compulsory education, up to the age of 15. When Minister Bjarni Benediktsson presented the bill he said:

> … það hefur verið sá trafali í umbótum í þessum efnum, að ekki hefur verið hægt að greiða viðunandi ritlaun fyrir bækur, og þess vegna hafa menn verið sýnu tregari en ella til þess að taka það starf að sér. ... Þá verður einnig að játa, að bækurnar hafa hingað til verið ákaflega fábreyttar að ytra búnaði og sannast sagt svo, að úr hófi hefur keyrt. Að vísu er það rétt, að ekki hentar óhóf í þessum efnum, en vitanlega er það nauðsynlegt fyrir börn, ekki sízt þau yngstu, og verður þeim mun nauðsynlegra eftir því, sem fleira dregur áhugann frá námi, að bækur séu sæmilega úr garði gerðar, séu skreyttar myndum, helzt litmyndum, fyrir þau yngstu, eins og tíðkanlegt er um barnabækur. Slíkt gerir námið miklu fýsilegra fyrir börnin en ella.
>
> It has been a great obstacle to reform in these matters that it has not been possible to pay adequate royalties for books, so therefore people have been much more reluctant to undertake that task. … Then one must also admit, that the [text]-books have been very plain in their external finish and in fact, exceedingly so. Certainly it is right that luxury is not suitable in these matters. However, of course it is necessary for children, not least for the youngest ones, and becomes the more necessary as more things distract them from the studies, that the books be reasonably well made, and be illustrated, preferably in colour, for the youngest ones, as is the custom with children’s books. Such things make study much more desirable than otherwise for children.[541]

[539] *Alþingistíðindi* 1953 D: 366–368
[540] *Stjórnartíðindi* 1956 no. 51, April 7
[541] *Alþingistíðindi* 1955 B: 885–886

The minister had at this time three children in primary school, and thus knew what he was talking about.

Kristján J. Gunnarsson, who after the amendment of the legislation became in 1956–1979 the vice-chairman and later chairman of the governing board of the State Textbook Imprint, expressed the view in 1987 that the publishing house had been neglected or quite forgotten in the official system in the post-war period.[542] After 1956 the number of new publications increased considerably. They were five in 1957, 11 in 1960 and the average in 1960–1969 was 18 new publications a year.[543]

However, asked if he had had doubts about state publications of textbooks, Kristján J. Gunnarsson, who was a right wing politician and headmaster in a primary school, said in 1987 that there were advantages and disadvantages, but always when added up the advantages weighed more, considering the Icelandic context.[544] The small market and the common belief in the post-war period that monopoly would serve best in that context, be it milk, butter or textbooks, has contributed to a general consensus about state textbook publication up to present day.

Educational Material in Mathematics and Arithmetic

In 1927 the first of two volumes of a series of textbooks by Elías Bjarnason in arithmetic for the 10–14 age groups was published by a private enterprise. The State Textbook Imprint bought the publishing rights of a revised edition of the series, on its establishment in 1937. In 1938 the Imprint published a translated set of Danish arithmetic textbooks for the 7–9 age groups. The Danish series ran for 20 years, and the series by Elías Bjarnason was predominant in schools up to the 1970s, the time of the “modern” mathematics reform.

State Director of Educational Affairs, Helgi Elíasson was son of the textbook author Elías Bjarnason. As director, Helgi Elíasson worked closely with the State Textbook Imprint, and in 1956–1964 he was a member of its governing board.[545] According to Hörður Lárusson, Helgi Elíasson was open to mathematics reforms and supported the appointment of Guðmundur Arnlaugsson as mathematics education consultant in 1964.[546]

When the 1956 legislation was passed, the Imprint bought the rights to publish the lower secondary arithmetic textbook by Steinþór Guðmundsson and Jón Á. Gissurarson[547] which had already been in circulation for some time, and later that by Kristinn Gíslason.[548] The content of these textbooks were similar to Dr. Ólafur Daníelsson’s *Arithmetic*, but with easier and more up-to-date problems.

542 Ingvar Sigurgeirsson (Ed.) (1987): 49
543 Ingvar Sigurgeirsson (Ed.) (1987): 16
544 Ingvar Sigurgeirsson (Ed.) (1987): 53
545 Ingvar Sigurgeirsson (Ed.) (1987): 14–17
546 Hörður Lárusson January 10, 2003
547 Jón Á. Gissurarson and Steinþór Guðmundsson (1949–1950)
548 Kristinn Gíslason (1962)

Regrettably, the effect of the State Textbook Imprint seems for a long period to have been the opposite of what was intended, and social-democratic welfare thinking to have turned into its converse. The initiative to create new learning materials more or less disappeared, as, to quote Minister Dr. Bjarni Benediktsson, it was impossible to pay satisfactory royalties. This is very clear with regard to school mathematics. No new mathematics material for the 10- to 12-year-old age group was published from the time of the Imprint's establishment in 1937, until 1969 when the Danish "modern" mathematics material by Agnete Bundgaard et al. arrived, on the initiative of the Reykjavík Education Office. This situation contributes to explaining the mass excitement which emerged among teachers when this new material at last appeared.

6.6. Follow-up to the 1946 Legislation

The 1946 education legislation proved a difficult task for a nation taking its first steps in complete independence. As Minister Brynjólfur Bjarnason said, new schools had to be built all over the country. This became the main task during the following two decades, while not much seems to have been done to foster the internal activities of the school system.

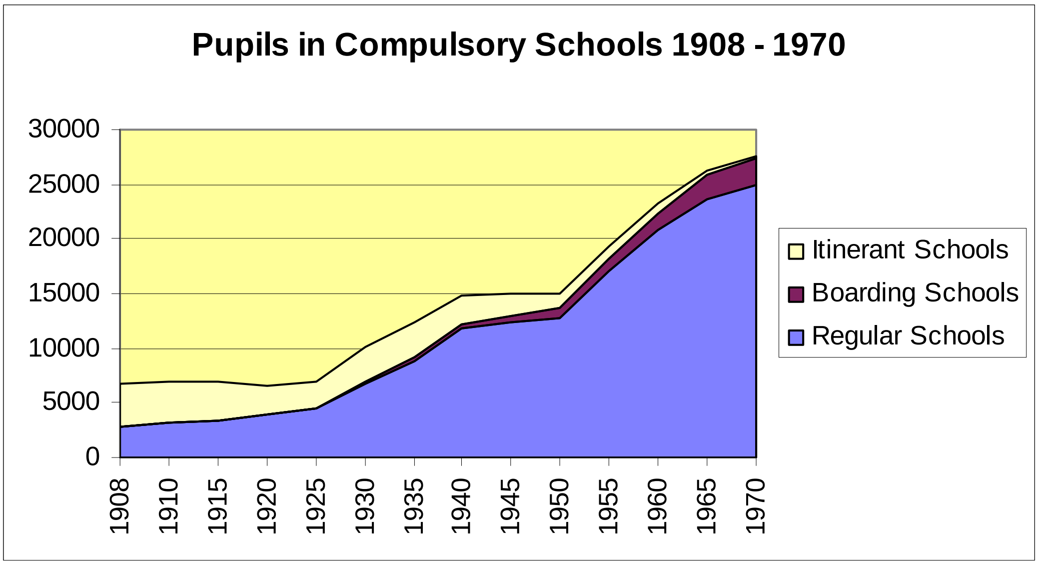

Figure 6.15. Pupils in compulsory schools 1908–1970.

Fig. 6.15. (Fig. 5.8. repeated) shows the enormous increase in the number of pupils in compulsory schools during the 20th century, especially after the 1946 legislation. Itinerant schools still existed, while the much more expensive boarding schools were taking over.[549]

Entrance into Reykjavík High School was restricted during 1928–1946 to a minimal proportion of the population. Less than 50 pupils a year graduated from the High School up to 1935, or 1–2.8 % of the age cohort. The rate was, however, nearly twice as high for boys, as the number of girls was minimal, 0.5% in 1930–1935. The rate increased after 1946. In 1955–1960 the total proportion had reached 8.7%, 5.6% for girls.[550]

[549] *Hagskinna* (1997): Table 18.5. Pupils and teachers of primary/compulsory schools 1908–1980
[550] Hagskinna (1997): Table 18.11. Students passing matriculation examination 1847–1990

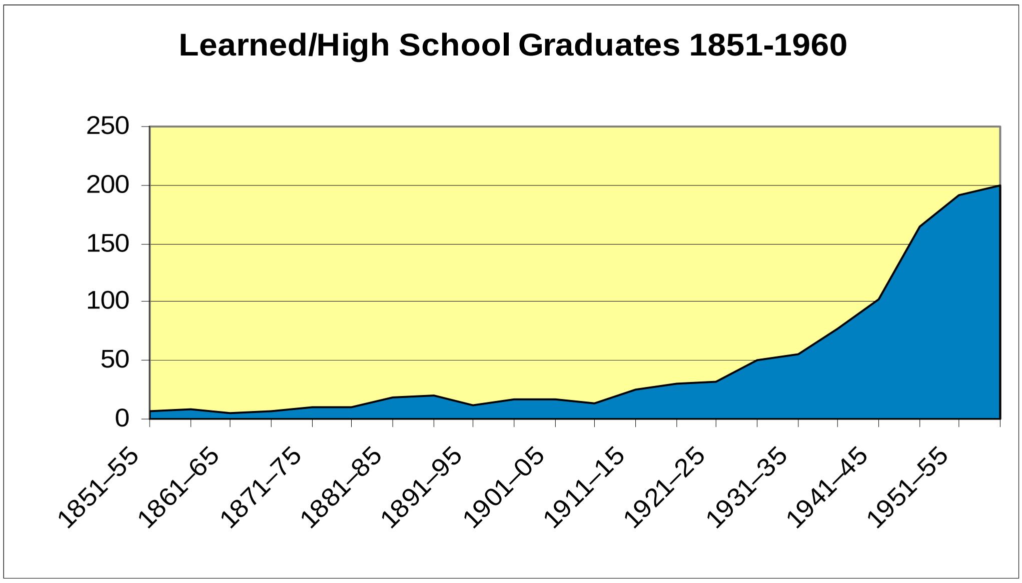

Fig. 6.16. Learned/high school graduates 1851–1960.[551]

Ministers of Education in the Post-War Period

Eysteinn Jónsson (1906–1993) became Minister of Education in 1947–1949 for the Progressive Party after the Innovation Government completed its term. Eysteinn Jónsson was a pupil of Jónas Jónsson from Hrifla, who handpicked him to serve in politics, on account of his great abilities. Eysteinn Jónsson was most often Minister of Finance, but not in 1947–1949. Financial problems began to emerge, when the Innovation Government's investments did not yield the expected revenue. In his 300-page collection of essays and speeches,[552] Eysteinn Jónsson never mentioned educational matters.

Dr. Bjarni Benediktsson (1908–1970), a law professor, served as minister in many governments for the Independence Party from 1947 to 1970. Most of the time he served as Minister of Justice or of Foreign Affairs, and in 1963 to 1970 he was Prime Minister. He was Minister of Education and Justice in 1953–1956, and for three months in 1949–1950. In Dr. Bjarni Benediktsson's time, important amendments to the 1946 education legislation were made by setting definite rules concerning the division of funding of educational affairs, school buildings etc., between the state and local government. The legislation on the State Textbook Imprint was amended in 1956 to give it a more firm financial support than before and separate it from the State Printing Press. Dr. Bjarni Benediktsson died tragically in an accident in 1970 and did not write any memoirs, but in a collection of essays about his life and work these two matters stand out in his work on educational issues.[553]

Dr. Gylfi Þ. Gíslason (1917–2004), a professor of economics, was a member of the government in the period 1956–1971 for the Social Democrats, in 1956–1958 in a left-wing government and in 1959–1971 in a coalition government with the Independence Party, the so-called Restoration Government. He was in charge of the Ministry of Education along with the Ministry of Commerce and/or the Ministry of Industry.

[551] Hagskinna (1997): Table 18.11. Students passing matriculation examination 1847–1990
[552] Eysteinn Jónsson (1977)
[553] Ásgeir Pétursson (1983): 87–101

During Dr. Gylfi Þ. Gíslason's long period of service, many changes were made to the school system and mathematics education. One of his first initiatives was to appoint a School Affairs Board of twelve members, known as the Apostles, to review the education legislation from 1946. The board concluded that it was unnecessary to change the legislation; changes could be made within the frame of the law.[554] This delayed the process of developing educational affairs, in the opinion of Andri Ísaksson, later director of the School Research Department.[555] In 1960 a national curriculum was published for the compulsory level.[556] During 1963–1968, increased criticism of school affairs emerged. The matters in concern were outdated legislation, which had still not been implemented in many parts of the country, and lack of long-term planning, research and adequate teaching material. Gylfi Þ. Gíslason reacted to this criticism in various ways.

Fig. 6.17. Minister of Education
Dr. Gylfi Þ. Gíslason.

Iceland had been a member of the OEEC, later OECD, since the post-war recovery period. When Dr. Gylfi Þ. Gíslason became minister in 1956 he was in charge of Iceland's relations with this organization. It was under the influence of the OECD that the establishment of the Technical College of Iceland was planned early in Dr. Gylfi Þ. Gíslason's period in office, and realized in 1963. In 1961 the Economics Institute was established, first as Ministry of Economics but in 1962 as a special institute, affiliated to the Ministry of Commerce. In 1966 the institute published a report on the University, which had a great influence on its future. The activities of the Economics Institute in the field of education and the influence of OECD will be treated separately.

Dr. Gylfi Þ. Gíslason's long period of service (1956–1971) as Minister of Education was a period of revolutionary change in educational affairs. From 1940/1942, when unemployment ceased to be a problem, the birth-rate rose, so the number of children passing through the schools grew rapidly. Also, as the general economy recovered during and after the war, people wanted more education for their children and, in the last couple of years of Gylfi Þ. Gíslason's period in office, foreign influences from the student uprisings in Europe may have had some effect.

Initially the national examination kept the door into the high schools, and thereby to further education, more or less closed to all but about 13% of the year cohort, so only the population growth had been met. This rate changed rapidly after 1960, and had grown to 36% in 1972, in addition to heavy population growth. This put increased pressure on the lower secondary schools, the whole of upper secondary level education and the University. No accommodation for a high school had been built in Reykjavík since 1846. Up to 1960 all high school pupils had to be housed in that more than century-old building. After 1960, buildings in the neighbourhood were rented to provide more space for teaching.

554 Gylfi Þ. Gíslason (1965): *Alþýðublaðið*, June 19
555 Andri Ísaksson, March 10, 2003
556 Menntamálaráðuneytið (1960)

Iceland's Education Policy in the Mid–20th Century

Many Icelandic politicians, such as Jón Sigurðsson in the 19th century, were aware that education of the people was a prerequisite for independence. It is probably not a coincidence that following the important landmarks of *Alþingi* becoming legislative body in 1874, the Home Rule in 1904 and the establishment of a republic in 1944 there were new education legislations in 1880, 1907 and 1946. But funds were scarce. The heritage of a small elite school for prospective officials, and confidence in home education supported by a healthy life in the countryside, characterized the attitude to education far into the third quarter of the 20th century.

Minister Jónas Jónsson of Hrifla represented this attitude. His largest contribution to Icelandic education was the district schools. In spite of his short period in office, his views remained influential for a long time. He sat on the first governing board of the State Textbook Imprint. His nationalistic Icelandic history textbook was first published in 1915, and last reprinted in 1979. Many textbooks were repeatedly reprinted by the State Textbook Imprint, but this one broke all records.[557] When at last it was replaced by a more modern textbook, people even asked what would be next, perhaps the Sagas would be rewritten, as if Jónas Jónsson's history book had attained the status of a literary classic.[558]

The 1946 education legislation was relatively modern, in that it prescribed compulsory education for children aged 7–15, going further than education legislation in the other Nordic countries. It had never come into effect in many rural areas and the stream of people lay anyway to the more densely inhabited areas. There the legislation was in full effect, while there was a great shortage of school buildings in the post-war period. Up to three sets of children came to each school room every day, at 9 am, 11.30 am and 2 pm. The state and the municipalities were busy building schools for compulsory education.

Meanwhile the purpose and the content of school activities were not much discussed. National curriculum documents were mainly in draft form, and although a national curriculum document for compulsory schools was published in 1960, 14 years after the legislation, no official curriculum existed for the non-compulsory lower secondary education, except for the national examination, which had priority in the implementation of the legislation. However, the Reykjavík High School with its accommodation problems, in cooperation with the much smaller Akureyri and Laugarvatn High Schools, practically controlled entrance to the upper secondary level.

The first ministers of the republic do not seem to have given the internal affairs of the school system much thought. The initiative for new thinking about internal school affairs came rather from educators themselves than from any of the ministers of education after Jónas Jónsson of Hrifla. It emerged through the annual meetings of the headmasters of the lower secondary schools, where Kristján J. Gunnarsson was a prominent figure, and the Reykjavík Education Office, where Jónas B. Jónsson was in charge. Dr. Gylfi Þ. Gíslason though dealt with many problems which may have been hidden in his first years of service. He channelled OECD's new thoughts on education into society, and took many measures to reform school affairs.

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557 Ingvar Sigurgeirsson (Ed.) (1987): 32
558 Gunnar Karlsson (1984): 409

In the following sections we will examine further discussions of educational matters in the 1960s, the history of national curricula, the effect of the OECD on educational policy, the activities of the School Research Department, established in the mid-1960s, and the expansion of the upper secondary level, all of which are intended to throw light on the tension that existed in the latter part of the 1960s and was relieved in the 1970s.

6.7. National Curricula

Amongst the complaints heard in the early and middle 1960s were that there were no curriculum documents and that the textbooks were outdated. Both claims were well-founded. We will now look at the situation.

In November 1948 provisional national curricula for the primary and lower secondary school levels were published. In the introduction it was explained that the Directorate of Educational Affairs wished to collaborate with as many people as possible and build on the experience of the teachers. These were only first drafts, and there was not full consistency between the various subjects. The headmasters of the lower secondary schools had discussed this matter thoroughly at their autumn meeting, which resulted in the drafting of a provisional curriculum for the lower secondary schools.[559] In fact, it was a list of available textbooks for use in the first three year-courses.

The drafts did not contain a syllabus for the national examination of the middle school, as it was run under special regulations.[560] Nor did it contain any information for the fourth grade of lower secondary school, as this had not yet been run anywhere under the new law. Nor did it say anything about what should be taught in the practical departments of the third year, as little experience was available of running their syllabus.[561] Twenty years were to pass before anything was laid down about the third and fourth grades intended for those who were not aiming at academic studies at a higher level.

Primary Level in 1948

The School Council of the Primary Schools had suggestions made for provisional curriculum for the primary level. The list of mathematics content was conventional. For the first grade there was the number concept, number notation, and addition and subtraction of numbers in the interval 1–20, together with mental arithmetic. In the following grades there was a gradual increase of complexity in the four operations in whole numbers, decimal fractions and common fractions up to the sixth grade. In the last three grades, the metric system and area computations were introduced. Area was expected to be taught in connection with the metric system.[562]

[559] *Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla* (1948): 3
[560] *Stjórnartíðindi* 1947 no. 51, April 14
[561] *Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla* (1948): 24
[562] *Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla* (1948): 3–15

There is a four-page epilogue to the primary level document, nearly all devoted to arithmetic teaching, written by Jónas B. Jónsson, director of Reykjavík Education Office, and Guðmundur I. Guðjónsson, trainer of student teachers at the Teacher Training College. They explained that the curriculum was not adapted to any available textbooks. Therefore, if the curriculum were validated, it would be necessary to revise the textbooks currently used and prepare new ones for the youngest age groups. The epilogue was otherwise written as a guide to good teaching practice by experienced teachers. The authors placed great emphasis on mental arithmetic, and suggested that it should be included in the curriculum. They emphasized the need for a handbook for teachers on this topic, which had been neglected for a long time. The use of models or objects was not mentioned, while the authors recommended that time should be devoted to independent work. The children should look at and count pictures from their own world, cut, paste, draw, etc.

Another item they mentioned was so-called "named numbers," which probably referred to changing quantities from one unit to another. The authors emphasized that it should not be expected that children learnt anything else than what everyone should know. They gave a list of monetary units, time units and commonly used units of the metric system suited for that purpose. This indicates that some effort had previously been made to have children learn outdated measuring units.[563]

The available arithmetic textbooks were translated Danish textbooks for the youngest age groups, and a series of textbooks for the fourth grade and above by Elías Bjarnason. The Danish series from 1938 was replaced only in 1955, when Jónas B. Jónsson himself wrote an alternative series for the three youngest age groups.

The series by Elías Bjarnason, first published in 1927 and in a revised version from 1937,[564] was reprinted unaltered until 1963, when Kristján Sigtryggsson began to revise it, and thereafter reprinted until 1978. In the period 1969–1979 it was used as an alternative to Agnete Bundgaard's "modern" mathematics textbooks.

Lower Secondary Level in 1948

This list of textbooks, which the provisional curriculum for the first three years of the lower secondary level consisted of, is as follows:[565]

Reikningsbók / Arithmetic by Elías Bjarnason, volume 3, for revision, and volume 4
Reikningsbók / Arithmetic by Dr. Ólafur Daníelsson
Kennslubók í algebru / Algebra by Dr. Ólafur Daníelsson
Dæmasafn / Collection of Exercises by Guðmundur Arnlaugsson and Þorsteinn Egilson
Dæmasafn / Collection of Exercises by Lárus Bjarnason

Recalling that Elías Bjarnason's *Arithmetic* was first published in 1927, Dr. Ólafur Daníelsson's *Arithmetic* in 1920, and his *Algebra* in 1927, these textbooks cannot be considered as recent.

[563] *Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla* (1948): 20–23
[564] Marteinn M. Skaftfells (1945): 177–183. Ólafur Þ. Kristjánsson et al. (1958): Vol. I, 122
[565] *Drög að námsskrám fyrir barnaskóla og gagnfræðaskóla* (1948): 28–29

Dr. Ólafur Daníelsson's books were to remain in use for the next two decades, the *Arithmetic* until the 1960s and the *Algebra* well into the 1970s, though mainly for the national examination, as the problems were rather complex. Elías Bjarnason's *Arithmetic* was soon confined to the primary level, and a new series was written for the first two years of lower secondary level in the early 1950s.

National Curriculum for the Compulsory School Level in 1960

The 1948 provisional national curricula never reached the status of regulations. The School Affairs Board appointed by Minister Dr. Gylfi Þ. Gíslason in 1958 held 47 meetings and made several proposals, e.g. about streaming pupils in the lower secondary schools.[566] Its conclusion was that no basic alteration of the legislation was necessary.[567]

Following this, a new national curriculum was written, to take effect in September 1960. The requirements in mathematics for the first six grades were the same as in the 1948 document, but were written in slightly more detail, e.g. the knowledge of the area and perimeter of a square and rectangle and the area of a right triangle were expected, instead of the general expression "area" in the 1948 paper. Mental arithmetic was now explicitly included in the curriculum, while no special methods were indicated.[568]

The list of requirements was followed up with three pages of instructions to the teachers about teaching. Use of "objects" and pictures was mentioned, both in connection to the introduction of whole numbers and fractions. The children should have them on their desk to be able to use them quickly. However, these were not detailed, i.e. what kind of "objects" were to be used for the specific connections. Problems concerning teaching the place-value system were not mentioned.

The two first years of the lower secondary level were compulsory. In the first year general arithmetic with whole numbers (without mentioning negative numbers) was practised, as were common fractions and decimal fractions. Percentages were introduced. In the second year, equations, *regula de tri*, interests, area and volume were the main topics and mental arithmetic was mentioned briefly. Nothing was said about the general third and the fourth year, which were not compulsory. The national examination was subject to its own regulations, as before.

Textbooks

No new mathematics textbooks were written for the primary level following these documents, neither the 1948 documents nor the 1960 national curriculum, until publication of a "modern" mathematics textbook series by Agnete Bundgaard and Eeva Kyttä began in 1966. So in that sense these documents had little influence on arithmetic and mathematics teaching.

The 1948 provisional curriculum for the lower secondary level simply presented a list of textbooks, which might not be thought to encourage the writing of new ones. However several textbooks were published for that level. They were sold on the free market until 1956. Lárus Bjarnason and physician Benedikt Tómasson published an arithmetic textbook in 1949, revised in 1953 by Benedikt Tómasson and Jón Á.

[566] Skýrsla til Menntamálaráðuneytis um störf skólamálanefndar (1959): 255–269
[567] Gylfi Þ. Gíslason (1965): *Alþýðublaðið*, June 19
[568] Menntamálaráðuneytið (1960): 20–25

Gissurarson. In their forewords the authors mention several Nordic textbooks as models.

Jón Á. Gissurarson and Steinþór Guðmundsson published an arithmetic textbook for the second and third year in 1949–1950. That textbook, called *Reikningsbók II A*, after the previous mentioned first year book, covered the same topics as Dr. Ólafur Daníelsson's textbooks. It was widely used, even if a teacher from the rural west coast, Gestur O. Gestsson, criticized it in educational periodical *Menntamál.* [569]

Guðmundur Arnlaugsson and Benedikt Tómasson published in 1950 an arithmetic textbook, *Reikningsbók II B*, to be used in the practical departments of the lower secondary level. It does not seem to have had much distribution, presumably as the practical departments were scarce.

After the publication of the 1960 national curriculum, Kristinn Gíslason wrote an arithmetic textbook with similar content to *Reikningsbók II A* but with a wider choice of easy exercises. It was published in 1962 by the State Textbook Imprint. Kristinn Gíslason wrote another arithmetic textbook together with Gunngeir Pétursson for the third year, published in 1963. These two sets of textbooks, by Jón Á. Gissurarson and Steinþór Guðmundsson and Kristinn Gíslason and Gunngeir Pétursson, were in use until the second half of the 1970s.

Gestur O. Gestsson

Little is found in public or professional discussion up to the mid-sixties about mathematics teaching in the newspapers. Most articles about such special subjects were published in the periodical *Menntamál.* They were infrequently in the form of a debate, but mainly for information. Only rarely did ordinary teachers express themselves in *Menntamál*. Gestur O. Gestsson (1895–1982) is an exception. From 1938 to 1962 Gestur O. Gestsson wrote constructive articles off and on about arithmetic and mathematics teaching. Gestur O. Gestsson enjoyed two years' education at a private folk high school in Núpur, after which he completed the three-year teacher training programme in 1915. Thus he must have studied mathematics with Dr. Ólafur Daníelsson. Throughout his life Gestur O. Gestsson taught at small schools in the West Fjords region, and worked as a carpenter in his summer vacations. He wrote as an outstanding mathematics educator. His two last articles were written in 1961 and 1962. In the former, Gestur O. Gestsson criticized the 1960 national curriculum. In the 1962 article he continued to criticize the curriculum, and added criticism on the textbook *Reikningsbók II A*.

In the 1961 article, Gestur O. Gestsson discussed the inadequacy of the national curriculum. In continuation he informed the reader that recommendations had been adopted on arithmetic teaching at international pedagogical congresses in Geneva in 1950 and 1956. Those should have been sent to the educational authorities of all nations. He wrote about the activities of the International Bureau of Education in Geneva and its director, Jean Piaget. He cited the recommendations of the congress in 1956 on the goals of mathematics teaching, requirements about timetables, curriculum, teaching methods and teachers.

Finally, Gestur O. Gestsson commented that it looked as if the [Icelandic] governors of educational affairs preferred to aim arithmetic teaching in compulsory schools at the lowest requirements.

[569] Gestur O. Gestsson (1962): 114–137

„Telpa þarf ekki að læra mikinn reikning til þess að geta afgreitt í brauðbúð." Þetta sagði þjóðkunnur maður á merkum stað, þar sem rætt var um reikningsbækur, og svo lítur út sem stjórnendur fræðslumálanna vilji miða reikningskennsluna í alþýðuskólum (skyldunámið) við lægstu kröfur, sem gerðar eru til afgreiðslufólks í sölubúðum. Þeir líti á alþýðu sem ódýr verkfæri til ákveðinna starfa. Þeir skilji það ekki, að stúlka, sem afgreiðir í brauðbúð sé maður, að hún eigi rétt á að lifa eigin menningarlífi, taka þátt í þjóðmálum og vera, eða verða, góð móðir, og að engu betur skilji þeir rétt annars vinnandi fólks. Þessa vanmats geldur öll þjóðin.

"A girl does not need to learn much arithmetic to be able to work in a baker's shop." This a nationally-known man said at an important place, where arithmetic textbooks were discussed, and it looks as if the governors of educational affairs want to aim arithmetic teaching in schools for the public (the compulsory level) at the lowest requirements which may apply to shop assistants. They look at the general public as cheap tools for certain tasks. They do not understand that a girl who works in a baker's shop is a person who has a right to live her own cultural life; to join the debate in society, and be or become a good mother, and no better do they understand the rights of other working people. The whole nation suffers from this underestimation.[570]

Comments about the right to learn arithmetic were rarely seen in Icelandic discussion. Probably most people at that time thought it more humane to lift the burden of arithmetic from the youngsters' shoulders. Gestur O. Gestsson was unique in his presentation of arithmetic studies as a human right.

The textbook Gestur O. Gestsson discussed in 1962 was *Arithmetic II A*, by Steinþór Guðmundsson and Jón Á Gissurarson. He judged the book to contain "recipe" methods, without giving adequate reasoning. Gestur O. Gestsson took the example of computing square roots. He showed several methods to find square roots; (i) by factorization, if possible; (ii) by *regula falsi*, that is dividing by a guess, then dividing by the average of the guess and the answer, such coming closer and closer to the solution; (iii) by interpolation of a graph and (iv) by Euclid's method, in addition to mentioning four other methods, including the one taught in *Hauksbók's Algorismus*. The computation of a cubic root he treated similarly, again referring to *Hauksbók*.[571]

Gestur O. Gestsson's writings reveal wide reading and extensive knowledge, which must have been more or less self-acquired. By his references he evidently read Danish, German and English books. Gestur O. Gestsson is one of the best examples of the "Turn of the Century Generation", fighting for a better society and education by using every opportunity to educate themselves and others.

It is noteworthy that Gestur O. Gestsson referred to *Hauksbók*. *Hauksbók* was published in Copenhagen in 1892–1896, immediately before Ólafur Daníelsson arrived in Copenhagen. We have seen a picture, taken in 1898, of Ólafur Daníelsson and his schoolmates together with Professor Finnur Jónsson, the editor of *Hauksbók*. Ólafur Daníelsson thus knew Prof. Finnur Jónsson. He surely studied *Algorismus* in *Hauksbók* and shared his knowledge with his pupils at the Teacher Training College.

[570] Gestur O. Gestsson (1961): 113–125
[571] Gestur O. Gestsson (1962): 114–137

The "Golden Age" of the Icelandic sagas motivated the "Turn of the Century" generation. It was encouraging for mathematicians to know that Icelanders had also been on even footing with other European people in mathematics at that time. *Algorismus* and the *Rím*-treatises are thus the *leitmotiv* that ties together the past and present history of Icelandic mathematics education.

6.8. Mathematics Teaching in the 1950s–1960s – A Case Study

Primary Years

As an example of mathematics teaching in the 1950s, I, the author of this thesis, will take my own school experience. In my primary school, *Austurbæjarskólinn*, there were up to eleven groups of 25–30 children in each of the six year-groups, a total of over 60 groups. In the early 1950s the number of pupils reached the historical maximum of 1839 children. There were 30 regular classrooms in the school. All classrooms were booked for two groups a day, some for three. In addition there were a number of rooms for special classes, such as drawing, singing, sewing, crafts, home economics, swimming and gymnastics and a cinema hall. The school, built in 1930, was a model school in that respect. Later the main emphasis would be on adequate number of rooms, and the special rooms might be established much later or never. This school was the first building in Reykjavík where geothermal water was used for heating.[572]

Children were grouped by "ability", measured by how well they could read when they came to school at the age of seven. Those who later met again in the Reykjavík High School mainly came from three groups, the G, H and I. This was a sign of the egalitarian policy of the school authorities, not to use A and B for the most able groups, but to pick the letters at random or in reverse order.

Mathematics teaching proceeded by the teacher explaining new topics and methods on the board and by talking to us, and we children continued, computing in our seats. The explanations still stay in my memory: the borrowing in subtraction built on the decimal place-value system, by taking a one from the seat to the left, splitting into a ten, and similarly keeping, by adding the tens to the seat to the left in addition. It was not all that easy in the beginning. I remember wanting to explain borrowing to my father when I came home from the first lesson, but then having forgotten it. This was soon remedied, and there was no doubt that I always understood what we were doing.

The multiplication tables were practised in the form of game, first by reciting them in order, then randomly. To ease the memorizing the teacher pointed out patterns, the 5s and 0s in the five times table and the transversal sums in the nine times table. I do not remember any reasoning for it, but certainly my friends and I made use of it and I, for my part, found it sensible and useful.

My teacher throughout the six primary school years was a woman in her fifties with 30 years' experience in teaching. According to the Teachers' Biographical Lexicon / *Kennaratal* she completed her teacher training in 1919, thus being a student of Dr. Ólafur Daníelsson. Since then she had been on one study trip to the Nordic countries in 1936. She was conscientious about her work, she knew her trade well, and the children were generally content. A friend, who attended the same school a year earlier, tells a similar story about his woman teacher.

[572] Austurbæjarskóli, website

During the first couple of the primary years the arithmetic teaching was based on the translated Danish 1939 arithmetic textbooks. From the middle of the third year Elías Bjarnason's textbooks took over. No extra sheets were needed, as far as I remember. I had usually done all homework at school. I did not give arithmetic much thought at home, and it certainly did not offer any painful experience.

Lower Secondary Years

I enrolled in a lower secondary school for girls in 1956 where I attended a selective class as well. The first year was largely a repetition of the primary syllabus, common and decimal fractions. The last quarter of the arithmetic textbook, written by Benedikt Tómasson and Jón Á Gissurarson, dealt with percentages. The teacher had a B.A. degree in geography and history. He disliked the procedures for adding fractions that my friend and I had learnt in primary schools. Later I discovered that our procedures were introduced in the *Arithmetic* by Dr. Ólafur Daníelsson, who had trained my primary teacher. Otherwise the studies were unproblematic.

Next year a part-time teacher, a grandson of Elías Bjarnason, educated in business administration, came in to teach mathematics. As before, the syllabus was easy: mainly *regula de tri*, area and volume computations and equations. The teacher explained some procedures at the board and the pupils computed in their seats.

In the national examination class, we had at last the head mathematics teacher of that school, who had ten years' experience. By that time, he had studied business administration and theology for a while and begun law studies. As was usual for university students at that time, he had to support himself by teaching. He had begun teaching so early that he was tenured without having completed the pedagogy and didactics, required later. The syllabus was mainly algebra, but also repetition of the most complex syllabus of the previous year.

The national examination syllabus was straightforward for me, even though more compact than previous years. I did not spend much time on mathematics homework; the two languages, Danish and English, were more demanding, or at least their teachers were. The school aimed at a regular lower secondary school examination, and those who wanted to take the national examination of the middle school had to attend extra classes in mathematics and physics on Saturday afternoons. Physics, taught by the mathematics teacher, was my favourite subject, and off and on I made fair copies of the physics notes in my notebook during weekends.

A friend of mine found the algebra more demanding than the earlier syllabus. She found the teacher impatient and she did not do well. It made a permanent impact on her, she felt. Another friend agreed that the syllabus was demanding, and she had spent more time on mathematics than earlier. She understood that the teacher had an extensive syllabus to cover, and considering that, she did not find him too impatient.

Reykjavík High School

My first year in Reykjavík High School, 1959–1960, the school was more crowded than ever before. There were groups from A to J, a total of ten groups of 25 pupils. The groups were reduced down to seven the following year. It was the policy of the Reykjavík School to regroup pupils from the same school. After long negotiations I could be with my friend by transferring into the J group. The next year, when we were split into streams, we had to part again, as she chose the language stream while I chose the mathematics stream.

All the new and part-time teachers were assigned group J. In no subject did we have an experienced, tenured teacher. A couple of them became university professors at a later time, however. All groups had afternoon-only classes, starting at 1:40 p.m. The mathematics teacher was a geologist who came in after 5 p.m. and had never taught before. The textbook was Jul. Petersen's *Geometry*, translated 16 years earlier. I made fair copies of my exercises in the weekends, thus keeping track of the syllabus. I do not remember that there was any emphasis on reasoning, but mainly on constructions. I recall a mid-term exercise problem, which the teacher deemed wrong where I knew I was absolutely right, and I spent some time on explaining to him. The discussion went on into the break, so my classmates became irritated. After that I stopped discussing with my teachers.

For the following three years, in the mathematics stream, the syllabus became more demanding. The Danish mathematics textbooks by Juul and Rønnau were more alien than previous textbooks. The teachers were the first I had who had prepared themselves to become mathematics teachers. The first one had studied in Copenhagen 1936–1939 and not returned, presumably due to the war. The second one was Björn Bjarnason, who later introduced "modern" mathematics. Björn Bjarnason was a jovial fellow, good-natured and caring. However, only five of the group of 23 achieved the first grade level, 7.25 out of 10, in written mathematics. All those who did became mathematics teachers, four of them with a master's or doctoral degree in mathematics, which indeed was unusual in the 1960s, when most university education in mathematics and natural sciences had to be acquired abroad. The fifth was educated as engineer, and three of the other four also did their first couple of years at the Faculty of Engineering at the University of Iceland. Probably we all were motivated by the "need" of Icelandic society for a mathematically educated workforce and mathematics teachers in particular.

The grade norm in mathematics lay between 4 and 6. A classmate, who reached this level, and later became a pharmacist, said when she brought her daughter to my school: "You remember how bad I was at math …" It seems a questionable process to be in the top 100 pupils in the year cohort who reach farthest in mathematics in the whole country, and afterwards consider oneself as an underachiever. Something must have gone wrong. Somewhere between the second lower secondary school year and the third high school year, many lost their confidence. One of my friends mentioned the foreign textbooks as an obstacle. He also mentioned the frequent regrouping in that period. Support from friends and pals diminished.

Guðmundur Arnlaugsson taught physics, my favourite subject, not as alien as the mathematics. Of course, there were many items that he mentioned which I understood better later. I remember him start a lesson by discussing that expressing force, F, in the form $F = m \cdot dv/dt$ was more general than $F = m \cdot a$ where m is the mass, a is acceleration and v velocity, something I did not grasp right away, but understood later. I also remember Björn Bjarnason having fun when introducing the Pascal triangle. It surprised me that a theoretical mathematician enjoyed adding up numbers in a triangle. These are examples of the many seeds they sowed which were later to bear fruit.

Guðmundur Arnlaugsson and Björn Bjarnason rarely called pupils up to question them at the board, and certainly not to humiliate them. They discussed the content by drawing figures and writing text in Icelandic on the board, and then took exercises as a demonstration. Once a month or so we were to hand in mathematics homework exercises and physics reports. Many tried to copy other people's work. I was reluctant to lend my answers, after I heard rumours that my solutions were too elaborate and far-fetched.

Guðmundur Arnlaugsson and Björn Bjarnason's friendliness and care for their pupils were unquestioned but the standards were strict. Only two years after my graduation Guðmundur Arnlaugsson was appointed as headmaster to Hamrahlíð High School, an altogether different school in its attitude to pupils. Instead of driving pupils away, all but the most "able", he attracted pupils, e.g. by establishing evening courses for older pupils. Adults could take evening courses to become high school graduates and earn the right of admission to university, a possibility not previously available in Iceland, of which women in particular took advantage.

Dr. Ólafur Daníelsson's Shadow

My generation studied Elías Bjarnason's *Arithmetic* in primary school and *Algebra* by Dr. Ólafur Daníelsson in the national examination class, while his *Arithmetic* was no longer used in Reykjavík, to my knowledge.

Two of my best teachers were pupils of Dr. Ólafur Daníelsson, i.e. my primary teacher, Sigríður Hjartardóttir, and Guðmundur Arnlaugsson. Their thorough understanding of the syllabus they presented is unquestionable in my mind. Björn Bjarnason was a pupil of Guðmundur Arnlaugsson, and the economist who taught me in lower secondary school was a grandson of Elías Bjarnason, the influential student of Dr. Ólafur Daníelsson. Dr. Ólafur Daníelsson's spirit was thus still very much alive in the early 1960s and still was in 1965 when I started teaching.

University

I enrolled in the University of Iceland in autumn 1963 and chose physics and mathematics as my field of studies. In short, for physics as a major subject, we took an introductory course in physics, split over two years. The third year we took special courses for engineers from recent American textbooks about fluid mechanics and basic engineering hydrodynamics, and a Danish textbook about electric engineering, definitely designed to cover the first section of the engineering studies at the Technical University of Denmark. Finally, the three teacher-students had a special course in modern physics from a recent American textbook.

The syllabus in mathematics, the minor, was a year-long course in mathematical analysis and another year-long course in linear algebra, both from Danish textbooks, presumably taught at the Technical University of Denmark. The teachers were high school teachers Guðmundur Arnlaugsson and Björn Bjarnason. In addition Björn Bjarnason gave an extra course for the student teachers in *Matematik 65* by in Bent Christiansen et al. "Modern" mathematics was underway.

The choice of textbooks reflected the background of the professors. Those who chose Danish books were educated in Denmark before or during the World War II, while the American textbooks were chosen by the younger professors who had studied in USA after the war. At that time, there was still academic freedom in the sense that the students had a choice of attending classes, and nothing was demanded from them until the final examination in spring. Several weeks were given for revision. This suited me well and I enjoyed university studies more than the high school studies. Personal relationships with the teachers were minimal. Therefore little remarks and praises stay in memory.

Conclusions

In spite of attending selective classes of the most able mathematics pupils in the age cohort, I did not have a mathematics teacher who had the required education during the secondary school years until the last two years of high school. At that time the syllabus had become much more alien than earlier and was presented in a foreign language.

The lower and mid-secondary level teachers had studied geography, history, geology, business administration, law and theology. Those who had teaching experience could explain the syllabus adequately. All of them were friendly and good-natured and wanted to be helpful. There were no conflicts, but there were no illuminations, nothing new or exciting. It seems that the lower secondary level syllabus was such that anyone could teach it. No special training was needed to introduce it to even the most able pupils. Possibly this was one of the reasons why the transfer into the high school mathematics stream was difficult.

In a group of 71 Reykjavík High School mathematics-stream graduates in 1963, only 11 reached the 7.25 level of a first grade in written mathematics, and most of them went on to complete doctoral degrees in mathematics or related subjects. The remaining sixty students were far below their averages in other subjects. The grade level was kept rigid possibly by the perceived demands of the Faculty of Engineering, which in turn must have felt itself governed by the requirements of the Technical University of Denmark which was to receive the engineering students for their second part of studies. Too many students went on with a perception of failure in mathematics after 13 years of the most demanding mathematics studies the country had to offer. Mathematics education seemed to be similar to a merry-go-round, which gradually flung off everyone except the most able ones.

The school setting I have described is composed of three different subcultures of people teaching mathematics. The first subculture is that of primary teachers, trained in a teacher training college, that of didactically aware teachers knowing well elementary arithmetic and how to teach it successfully. The second was that of university educated people who had not been trained in mathematics and almost none of whom had received teacher training. Precisely the pupils belonging to the selective group received their teaching from this subgroup of teachers. Others, not belonging to the selective group, may have had teachers trained in teacher training college. This was the case with one of my friends who did not go to high school. The very few mathematics teachers trained at the University of Copenhagen belonged to the third subculture.

The difficulties for even the most able pupils in the process of transferring from lower to upper secondary level were, and are, not unique to Iceland. They are for example mentioned in a recent Danish report about competences in mathematics learning.[573] The special circumstances in Iceland (see section 7.1.) were the group of people working on teaching mathematics that had received various kinds of education, different from mathematics, in some cases supplied by general pedagogy and didactics.

6.9. Public Discussion and Society in the 1960s

Introduction

In the 1960s there were five daily newspapers, each supporting its political party. *Alþýðublaðið* supported the Labour/Social Democratic Party, *Morgunblaðið* and *Vísir* the conservative Independence Party, *Tíminn* the Progressive Party, the farmers' centre party, and *Þjóðviljinn* the People's Alliance, previously the Socialists, the most radical party. Furthermore there was a weekly journal, *Frjáls þjóð*, originally supported by a left-wing social democratic party consisting of intellectuals, advocating that Iceland leave NATO and that the U.S. military base at Keflavík be closed down. By this time, the party had disappeared and its members had largely joined the Social Democrats or the People's Alliance. This journal was "orphaned" in 1964, as economist Jón Baldvin Hannibalsson expressed it. He was a history teacher in a lower secondary school in the 1960s, later a high school headmaster, member of *Alþingi*, Minister of Foreign Affairs, and ambassador to the U.S. and later to Finland. He became editor of *Frjáls þjóð*, succeeded by his brother, Ólafur Hannibalsson, a lower secondary school teacher. The third brother, Arnór Hannibalsson, a philosopher and a lower secondary school teacher at that time, later university professor, was a columnist on the journal.[574] So were a number of other intellectuals, university students who later became professors, politicians or both.

Discussions about educational issues can be found in these papers. However, those matters and other issues were often coloured by the political attitudes of the newspaper in concern, and the writers. Dr. Gylfi Þ. Gíslason was Minister of Education 1956–1971 for the Social Democrats, as well as Minister of Commerce from 1958. All progressive acts in the field of education on the part of the authorities were duly reported in *Alþýðublaðið* and to some degree in *Morgunblaðið*, whose party was in a majority in the government in 1959–1971. The parties of *Tíminn* and *Þjóðviljinn* were in opposition throughout this period, and took a critical view on the acts of the government. The same applied to *Frjáls þjóð*, even if it was between parties, so to speak.

In this period a number of people concerned about education wrote articles which shed light on the main problems discussed at that time. The main issue was the overcrowding of Reykjavík High School, the large gap between compulsory education and the high schools, bridged by the national examination, and outdated textbooks and curriculum.

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[573] Niss, M. et al. (2002): 23

[574] Kolbrún Bergþórsdóttir (2002): 185–187

Examination Strain and Overcrowded High Schools

Physician Benedikt Tómasson, mathematics textbook writer, mathematics teacher and headmaster at Flensborg Lower Secondary School, later School Health Inspector, wrote in 1961 about the mental strain on the pupils who failed in school examinations. The national examination had attracted more attention than any other examination taken in the country. A great number of pupils and their parents were "nerve-wracked" over it and Benedikt Tómasson suspected that the teachers were "nerve-wracked" too. They were also at a great risk, as the examination seemed to have become the main yardstick for lower secondary schools in the country.

However, the problems were not over when the examination was passed. Teachers at Reykjavík High School were originally not in favour of the national examination, which led to the first two years being cut off the school. The school had found it necessary to filter pupils out, especially in the first year. As against 184 pupils in the country who did not reach the 6.00 grade average at the national examination in 1960, 109 had not passed a year-course at the Reykjavík High School in spring 1961, one fifth of the group, the majority in the first year. Benedikt Tómasson said that the school was overcrowded. The first-year pupils had to attend school in the afternoon when the older pupils had gone home. This was not the teachers' fault, and he asked how long the authorities were going to leave this old and distinguished institution in this condition. It was no wonder that the pupils did not do well, tired and hungry after a long day.

On the contrary, at Akureyri High School only few pupils failed, the accommodation was adequate, and the pupils could study during normal working hours. Benedikt Tómasson suggested that by organized counselling, pupils lacking preparation or abilities should be guided away from the national examination. Then the high schools could, without any additional cost, give their drop-outs a lower secondary school examination certificate, even if they could not progress to the next year-course. By the present arrangement, where young people were given the harshest possible treatment when choosing schools, the nation was playing games with the mental health of the young, which Benedikt Tómasson doubted it could afford. [575]

These themes, the strain and the anxiety caused by the national examination and the overcrowded Reykjavík High School, were repeated throughout the decade.

Headmasters' Concerns in 1963

In June 1963, thirty headmasters of lower secondary schools had their annual meeting in Reykjavík together with State Director of Educational Affairs Helgi Elíasson, chairman of the National Examination Board, Bjarni Vilhjálmsson, Teacher Training College Headmaster Broddi Jóhannesson, and staff of the Office of Educational Affairs.

Minister of Education Dr. Gylfi Þ. Gíslason addressed the meeting and discussed planned changes in the training of lower secondary school teachers. Other speakers spoke of the necessity for more varied education in the lower secondary level, the necessity to standardize the lower secondary school examination, and of the general use of examinations, their advantages and disadvantages. The meeting then broke up into discussion groups about curricula and examinations.

[575] Benedikt Tómasson (1961): *Morgunblaðið* July 13

At the end of the meeting, Kristján J. Gunnarsson, on behalf of the headmasters, introduced proposals to the Directorate of Educational Affairs to implement the following:

1. Komið verði á fót fastri deild eða stofnun ... sem ... gegni því hlutverki:
  a) að fylgjast með og kynna sér á hverjum tíma nýjungar og tækni í námstilhögun og skólastarfi með öðrum þjóðum og meta, hvað af því kynni að henta við íslenzka staðhætti,
  b) að gera tilraunir með námsefni, námstilhögun og kennsluaðferðir í einstökum skólum eða bekkjardeildum, eða í sérstökum tilraunaskóla, sem stofnsettur yrði,
  c) að vinna stöðugt að endurskoðun námskrár barna- og gagnfræðastigsskóla og koma á þeim breytingum, sem tilraunir og athuganir hefðu leitt í ljós að nauðsynlegar væru,
  d) að fylgjast með því, að kennslutilhögun og prófkröfur skólanna séu ávallt í sem fyllstu samræmi við það meginhlutverk þeirra að stuðla að alhliða þroska nemendanna,
  e) að marka stefnu um, hvernig og að hve miklu leyti próf skulu notuð í skólum, ...

1. To establish a ... department or institution ... which is supposed ...
  a) to monitor what is happening at each time and study innovations and techniques in educational arrangements and school work among other nations and evaluate what of it might suit the Icelandic context,
  b) to experiment with educational material, educational arrangements and teaching methods in chosen schools or classes or in a special experimental school which would be established,
  c) to work continuously on revising curricula for the primary and lower secondary schools and implement the changes which experiments and studies have proved to be necessary,
  d) to monitor that the teaching arrangement and the schools' examination requirements are always fully consistent with their main role: to work for the pupils' all-round maturity and development,
  e) to form a policy on how, and to what degree, examinations should be used in schools, ...[576]

As a temporary measure the meeting suggested that the minister appoint a board to prepare a proposal to the Directorate of Educational Affairs, including that a curriculum document for the third and the fourth year would be available the following autumn, ensuring the same general requirements for the three kinds of lower secondary school examinations for the academic, commercial and practical departments at all lower secondary schools. The requirements for examinations in Icelandic, arithmetic and foreign languages [in lower secondary schools] should be standardized, eventually by a standardized examination. The yearly number of school hours should be increased. The current examinations should take less time than at present, slower learners in the youth school should have a special curriculum, and pupils who aim at higher education should take a special entrance examination into the national examination stream in their first year of lower secondary school.

[576] *Menntamál* 36 (2–4) (1963): 223–229

Finally, the meeting expressed its satisfaction with the new [1963] legislation for the Teacher Training College, and a wish to revise the legislation on teacher training for the lower secondary school level.

This report indicates several other themes of the decade. In the above proposals of the meeting, uneasiness of the headmasters about stagnation of the educational system is reflected in items 1.a) and 1.b) on experimental work, the lack of a standardized curriculum for lower secondary school, lack of monitoring, lack of policy regarding examinations, the short school year, and a wish to spread the burden of work over two school years for pupils preparing for the national examination. However, the wishes seem contradictory in proposing that time for examinations should be shortened, while a new entrance examination into the national examination stream should be established and a standardized examination established at the close of the general lower secondary school.

The minister spoke about teacher training and some plans that he had in mind. Subsequent changes were introduced in the mid-1960s in the training of mathematics teachers at the University of Iceland, such as more stringent requirements in the teaching subjects, but they did not have much effect, as not much was done to attract students to the programme.

Length of the School Year and the High School Crisis

In April 1964 five distinguished school leaders were asked in *Alþýðublaðið* if it was desirable to extend the school year. At that time the school year was eight months, from the beginning of October to the end of May, except for the 7- to 9-year-old pupils, who attended for nine months, at least in towns. The majority of the school leaders thought that an extension of the school year was necessary, provided that the syllabus was changed. The opinion was also expressed that a large group of pupils was bored at school, and that they should not suffer or dabble more than at present.[577]

In April 1964 *Alþýðublaðið* announced on behalf of the Minister of Education that a new high school in Hamrahlíð in Reykjavík was being designed, and a new annex to the old Reykjavík High School was being built. The minister had been asked in the *Alþingi* if there was any intention to raise the grade requirement for entrance to the high schools in the national examination. The minister promised that this would never happen in his time as a minister, and the grade requirement must never depend on the high schools' capacity in terms of their buildings. He reminded the parliament members that the original aim of the national examination had been to enable all young people, wherever in the country they came from, to have equal opportunities to enter high school. However, possibly the time had come to revise the national examination curriculum.[578]

In July 1964 "An Aged Student" wrote in *Frjáls þjóð* about the high schools. They should no longer give their pupils little bits of as many subjects as possible. For example what was the point of studying Latin, for a student aiming at physics studies in a German or American university? The high schools should stop shoving dates of wars down the throats of their pupils, and instead implement teaching in history of

[577] *Alþýðublaðið*, April 12, 1964
[578] *Alþýðublaðið*, April 18, 1964

culture, have pupils read literature, teach them to listen to music, show them nature and inculcate respect for mankind and life.[579]

In April 1965 *Alþýðublaðið* announced a bill, submitted to *Alþingi*, proposing that there should be a total of six high schools. Three high schools were present: in Reykjavík, Akureyri and at Laugarvatn. Three new schools would be added; one at Hamrahlíð, Reykjavík, one boarding school in the West Fjords, and another in East Iceland. These would be established when funds became available. The bill contained a provision that more high schools could be established in Reykjavík, and the lower secondary schools could run a high school department, where the circumstances were suitable.[580] This provision was to be much used in the 1970s. *Tíminn* devoted a whole page to the speech of the chairman of the Progressive Party, in the opposition, expressing his satisfaction with the bill, as it was identical with the Progressive Party's earlier bills.[581]

The Hamrahlíð High School opened in 1966 under the leadership of mathematician Guðmundur Arnlaugsson, the Ísafjörður High School in the West Fjords in 1970 under headmaster Jón Baldvin Hannibalsson, Egilsstaðir High School in eastern Iceland in 1976 under headmaster Vilhjálmur Einarsson, and the third school in Reykjavík, later called Sund High School, was established in 1970, led by mathematician Björn Bjarnason.

Critique of the National Examination

In spring 1965, a number of articles on education were published in *Alþýðublaðið*. Some debate had occurred on the national radio station. Kristján J. Gunnarsson, a primary school headmaster in Reykjavík, was said to have stated in a radio talk that exactly as many students passed the national examination as the high schools had space for; that this policy of the National Examination Board, of limiting the number of pupils, was the main reason for the small number of students graduating from the high schools.

The retiring chairman of the National Examination Board, Bjarni Vilhjálmsson, answered these allegations in an article in *Alþýðublaðið* and *Morgunblaðið* on April 14, 1965, and in *Tíminn* the next day. He was surprised that the honourable educator Headmaster Kristján J. Gunnarsson suggested such ideas. Bjarni Vilhjálmsson showed by statistics that the proportion of the population reaching the necessary average grade to enter high schools had increased from 11.9% to 15.1% in the period 1956–1964. In the same period of time only 8–10% of the yearly cohort graduated from the high schools. Bjarni Vilhjálmsson stated that no headmaster of the high schools had ever tried to have any influence on the examination, none of the four ministers of education during the period had ever intervened in the work of the board, and there had been no disagreements between those members of the board teaching at the high schools and the others. Admittedly the members of the board tried to compose the examinations in such a way that the pupils reaching the minimum entrance grade for the high schools would have some possibility of coping with their syllabus.[582]

[579] *Frjáls þjóð*, July 3, 1964
[580] *Alþýðublaðið* April 11, 1965
[581] *Tíminn*, April 21, 1965
[582] Bjarni Vilhjálmsson (1965): *Alþýðublaðið, Morgunblaðið*, April 14; *Tíminn* April 15

A week later, Kristján J. Gunnarsson wrote an article in *Morgunblaðið* in which he explained his arguments about the connection between the national examination, being an entrance examination to the high schools, and the accommodation problems of these schools. He expressed his hope that the anticipated revision of the high school legislation would overcome the faults, not only of the traditional high school but of the whole educational system.[583]

In 1970, when the legislation about high school was about being passed, Kristján J. Gunnarsson proposed to the Reykjavík Town Council a new experimental school for all, bypassing the national examination. The proposal became the basis for *Fjölbrautaskólinn í Breiðholti* / Breiðholt Comprehensive Multi-Stream School, established in 1975.

On April 22, 1965 Oddur Sigurjónsson, one of the lower secondary school headmasters at the above-mentioned meeting, stated in an article in *Alþýðublaðið* that the national examination was the greatest tool of justice and equality that had been given to young people in rural areas, remembering the times when even begging or plotting behind the scenes were the ways to admittance, especially into the Reykjavík High School. Oddur suggested that the preparation for youth examination (after the second year in the lower secondary school) be split into two streams. Those who were heading for higher education (after the national examination) would take the examination after two years, while the slower learners would take it after three years. Secondly he proposed that the lower secondary school curriculum be standardized.[584]

On May 11, 1965, in an interview in *Alþýðublaðið,* the new chairman of the National Examination Board, Njörður P. Njarðvík, expressed his view that the first and second years of lower secondary school were too easy for those pupils aiming at further and higher education. The national examination should be difficult, as it was supposed to select the best suited for higher education. It was indeed a fact that the first years of the high schools were another filter. However, the members of the board, of whom five out of ten were high school teachers, did not suggest a more difficult examination.[585]

The School Situation

The Union of University-educated Teachers, *Félag háskólamenntaðra kennara*, FHK, was established in November 1964. It immediately began to send out reports on its activities and concerns, the main issue being the revision of the educational system, teacher training, and salaries graded by qualifications.[586] In April 1965, a person writing as “A Teacher” claimed in *Alþýðublaðið* that teacher training did not matter at all, the problem of the lower secondary education was lack of discipline and that the tasks of the teachers were more like those of policemen. After all, the salaries were not so bad either, if one took account of overtime, teachers’ second jobs and summer vacation work.[587]

Ingólfur A. Þorkelsson, later chairman of FHK, took up the discussion on a broader basis and explained that in 1963 only 97 out of 264 lower secondary school teachers in theoretical subjects had a university degree, or 37%, and only 69 or 26% fulfilled

[583] Kristján J. Gunnarsson (1965): *Morgunblaðið,* April 22
[584] Oddur Sigurjónsson (1965): *Alþýðublaðið*, April 22
[585] *Alþýðublaðið,* May 11, 1965
[586] *Frjáls þjóð*, November 20, 1964
[587] *Alþýðublaðið*, April 2, 1965

the requirements of a university degree and courses in pedagogy and didactics. He rejected the idea of including in the income the slow–killing drudgery of extra work and summer vacation work, and said that soon the school year would be lengthened and the summer vacation shortened. On the other hand the lack of discipline in schools reflected society as a whole. With the sincere will of the teachers, led by the headmaster, good discipline could be established in every school. One could not blame new, untrained teachers for discipline problems. The school, as an institution, carried the main responsibility and a new teacher was working within a system he had not created himself.[588] This debate went on for six weeks. It is cited here, as throughout the decade very few ordinary teachers spoke up for themselves, and then mainly to criticize innovations. However, those who did so might reveal the real situation and not the optimal state, as it was often described in introductory or explanatory articles of specialists or political leaders.

In May 1965 *Tíminn*, whose party was in opposition, asked in an editorial whether the school system, school work and the curriculum were indeed suitable for those who were to inherit the country, and concluded that most people would answer in the negative. Schools had stagnated and that was the most dangerous obstacle to the nation's progress. The Progressive Party had proposed no less than five bills and parliamentary resolutions on education during the last session of the *Alþingi* to liberate school affairs from this dangerous stagnation. It proposed a revision of the 1946 legislation, and reiterated that continuous review of these matters had to take place. The matter of the high schools could not be isolated from the complete review of school affairs.[589]

Minister Dr. Gylfi Þ. Gíslason answered in June 1965 in *Alþýðublaðið* that in this time of rapid changes it was reasonable to investigate the school system scientifically and this was currently being worked at. But it seemed to him that it had escaped the attention of those writing about school matters in *Tíminn* that a new curriculum for the compulsory level had been published in 1960, so it was not true that the basis for all school work was 20 years old, even if the legislation was adopted two decades ago. Within the present curriculum the schools were free to plan their activities according to their special needs, but certainly the curriculum would be revised regularly.[590]

In June and July 1965, psychologist Ólafur Gunnarsson wrote a series of informative articles about research in the field of education. Ólafur studied and worked in Copenhagen in the period 1940–45, again in 1947–50 and in 1965–66 (in Herning). He was therefore knowledgeable on Danish school affairs. In one article he said:

> Um þessar mundir vinna skólasálfræðingar á Friðriksberg í Kaupmannahöfn að því að semja reikningsbækur, sem byggja á margra ára tilraunum, sem gerðar hafa verið með reikningskennslu af höfundum bókanna. Rannsóknir, sem gerðar hafa verið á reikningsaðferðum barna og orsökunum til þess að þau reikna rangt, eru svo umfangsmiklar að engin tök eru á að gera frekar grein fyrir þeim í þessu erindi.
>
> At present school psychologists in Frederiksberg in Copenhagen are working on writing arithmetic textbooks which are based on experiments over many years with arithmetic teaching, carried out by the authors of the books. Research, which has been done on children's computation methods and the reasons why they compute

[588] *Alþýðublaðið*, April 21, 1965
[589] *Tíminn*, May 16, 1965
[590] *Alþýðublaðið*, June 19, 1965

> incorrectly, is so extensive that there is no possibility of explaining them in a short article.[591]

This seems be the first time that mathematics reforms were mentioned in the daily newspapers.

In July 1965 *Tíminn's* editorial heartily agreed with Minister Gylfi Þ. Gíslason's address to the Nordic School Congress, where he expressed his concern that self-education and self-development continued to be very important. The task of the schools was above all to motivate pupils for self-education.[592] This is certainly true, while from the pen of *Tíminn* it may be taken as a praise of the home- and self-education tradition of former times.

Váli's Complaints

In the period July 1965 to February 1967 columnist "Váli," *alias* philosopher and lower secondary school teacher Arnór Hannibalsson, wrote weekly columns about politics of the day in *Frjáls þjóð*. His main themes were:

- The curriculum of the lower secondary school must be revised, as must the teaching methods and the examination arrangement. The teaching methods are so out of date that they hinder all school work. Many problems concerning curriculum and teaching methods cannot be solved without the aid of specialists in pedagogy and didactics. A research institute is needed (July 22, 1965).
- New people are needed to form a clear standpoint on pedagogical problems and modern school activities and lead the complex and difficult work in that area. … The Teacher Training College is still strolling along the road that the theologians destined it for in the early days (September 9, 1965).
- No institute for research of school and pedagogical matters exists. … The government's performance in building schools is not praiseworthy. The Reykjavík High School has used the method of cutting down the number of pupils in its first year course, only due to lack of adequate accommodation (October 7, 1965).
- No employee in the Ministry of Education has the function of computing the financial needs of the school system and finding out how the funds would be best utilized (October 14, 1965).
- The duty to supply education to children and young people has not been fulfilled in many areas in the countryside (November 11, 1965).
- Schools were first built in Iceland in the 1930s, and hardly any school was built until after World War II. Therefore comparing the funds allocated to school buildings in Denmark to that in Iceland has no meaning (December 23, 1965, not by Váli).
- No one has an overview of the real cost of the educational system now or in the future; or which school facilities are in use today and which will be needed in the future in proportion to the increase in number of children (February 10, 1966).
- No handbook for teachers is available in any subject taught in the schools of the state. There exists no literature about didactics, teaching methods, the use of teaching equipment or about pedagogy. At the same time the number of pupils in the country is 51,805 and the number of teachers at the primary and lower secondary level is 3,115 (March 3, 1966).

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[591] Ólafur Gunnarsson (1965): *Tíminn*, July 15

[592] *Tíminn*, July 25, 1965

- The State Textbook Imprint is in such a state of neglect and misery that it is a national shame (March 3, 1966).
- It is a great and grave fault in the Icelanders' school system that the pupils in the schools are not at all taught to respect the printed word as an aid for learning and work (April 28, 1966).
- The salaries of teachers are below all minimum levels for supporting a family, and teachers are forced to take on all kinds of extra work simply to survive (June 9, 1966).
- Courses, summer schools and a university of education are needed for lower secondary school teachers (July 7, 1966).
- The new Science Institute will become a great support for Icelandic sciences and industries. Specially, we want to mention mathematics, as nowadays there is no science that does not need the support of mathematics, and one can say that no execution or improvement of production methods is possible without the aid of mathematicians or people knowledgeable in mathematics (October 20, 1966).
- A new bill on the Student Loan Fund is praiseworthy, but is it clear that the purpose is to get the students back to Iceland after their studies? And will they then find any adequate facilities to work in the fields of their sciences? (February 23, 1967).

The OECD's Education Policy in Iceland

In the 1960s new ideas began to be heard in Icelandic society. In 1962 the State Economics Institute was established, affiliated to the Ministry of Commerce. The Economics Institute proved to reflect the OECD paradigm of that time, on manpower planning, especially in industry and education. The Economics Institute played a large role in 1960s policymaking in science, technology and education.[593]

The Economics Institute held a seminar in June 1965 on educational planning. At that time the Ministry of Education had assigned the Economics Institute to do long term planning of the nation's educational needs, and of the organizational and economical problems to be solved in this context. Two foreign speakers from the OECD were invited, Dr. Klaus Bahr, director of the Educational Investment and Planning Programme (EIP) of the OECD, and Kjell Eide, director of the planning department of the Norwegian Ministry of Education. Dr. Bahr was well familiar with the Icelandic school system, as he had visited Iceland repeatedly on behalf of the OECD.[594]

The results of this meeting are documented in its report, named *Nokkur efnisatriði erinda og umræðna frá fundum um menntaáætlanagerð 2. og 3. júní 1965 / Several Items from Presentations and Discussions in a Meeting on Educational Planning, June 2 and 3, 1965*.[595] The seminar was attended by many of the most influential persons in Icelandic education at that time. Among the participants were: Director of Educational Affairs Helgi Elíasson, Headmaster Dr. Broddi Jóhannesson of the Teacher Training College, Director of the Reykjavík Education Office Jónas B. Jónsson, Headmaster Jóhann Hannesson of Laugarvatn High School, later research adviser in the School Research Department of the Ministry of Education, and

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[593] Sigurjón Mýrdal (1989): 48
[594] Ingvar Sigurgeirsson (2004): 169. National Archives of Iceland: 1989 B/991 1966-1968 OECD ýmsar skýrslur: A letter from Torfi Ásgeirsson to Wolfgang Edelstein
[595] Efnahagsstofnunin (July 1965)

Guðmundur Arnlaugsson, mathematics consultant and headmaster-to-be of Hamrahlíð High School. Some of these participants were, following this meeting, appointed members of an advisory committee to the Minister of Education for the Educational Investment and Planning Programme (EIP) of Iceland. The OECD experts on EIP were Dr. Klaus Bahr and Dr. Wolfgang Edelstein.[596] The committee held its first meeting on April 28, 1966.[597]

Dr. Klaus Bahr said that while traditionally education had primarily been regarded as serving only cultural purposes, new concepts of the role of education had recently been developed i.e. that education contributed substantially to economic and social progress and stability, that it was as much a sector of society and national economy as traditional sectors and that education competed with other sectors for scarce financial, natural and human resources (see introduction to the research question, p. 27–28).[598]

As cited earlier, Dr. Bahr discussed education as a system governed by its own logistic rules, e.g. time-lags (teacher supply and demand, demographic development; student input and output), freedom of choice of the individual, efficiency of the system, co-ordination of educational decision-making, long-term financial planning and need for a continuous flow of information. Furthermore, Dr. Bahr stated that education is an integrated socio-economic sector of society, where the main problem areas are monopolistic position of the state, ad-hoc adjustment processes leading to cycles and bottlenecks, technical progress and economic policy targets demanding continuous adjustments and social objectives such as equal educational opportunities. The educational system supplied, while the labour market transmitted demand for human capital.

In the discussions following Dr. Bahr talk, Dr. Broddi Jóhannesson asked about methods of estimating needs for supply of teachers, to which Dr. Bahr answered that for short term planning the conventional rate of teachers / pupils could be used, given an estimate of the number of pupils. For long-term planning, it depended on the future role of the teacher in the school system. That kind of estimate had to be done in close cooperation and consultation with the teaching profession, and in fact all those who were concerned with education. Otherwise the planning would indeed have no value.[599]

Kjell Eide talked about experiences in Norway in the field of educational planning. He mentioned that nine-year compulsory schooling had already been implemented in one third of Norwegian schools. The plan was to complete the implementation around 1970, but it was not expected to be fully completed in all grades until 1980. In the follow-up discussions, the participants were mainly concerned about technical education and its planning in Norway, and the lessons Iceland could learn from it. Guðmundur Arnlaugsson asked how the Norwegians coped with the lack of teachers in mathematical sciences. Mr. Eide was familiar with the problem from Norway, but expected it to be resolved by 1970. The problem was, in his opinion, to shed light on the influence that unavoidable decisions of school authorities concerning the school system will have on the need for teachers. Teacher training had been coordinated [in

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[596] National Archives of Iceland: Skjalasafn Fræðslumálaskrifstofunnar 1989/E-10 Efnahagsstofnunin. Country Report. Submitted by the EIP-team of Iceland

[597] National Archives of Iceland: Skjalasafn Fræðslumálaskrifstofunnar 1989/E-10 Efnahagsstofnunin. Minutes of the meeting

[598] Efnahagsstofnunin (July 1965): 9

[599] Efnahagsstofnunin (July 1965): 16–17

Norway], so that the basic training of all teachers was the same, and simultaneously an attempt was made to make the way to further education easier. Wage scales for teachers at the various levels of the educational systems had been coordinated, so that teachers with the same qualifications would have the same salary, regardless of their level in the educational system.[600]

This seminar was probably one of the first times when many of these participants were exposed to this economic view of education. Dr. Bahr's introduction was therefore an important prelude to the coming reform activities. Mr. Eide introduced the Norwegian school system which faced similar problems to the Icelandic one: long distances and difficult transport. Norwegian teacher training was more developed than in Iceland, and possibly this visit may have influenced the future structure of Icelandic teacher training implemented in 1971. And certainly the Norwegian nine-year compulsory school may have contributed to a model for the 1974 compulsory education legislation.

The directly visible follow-up of the Economics Institute's work on education planning proved only to be a small report containing statistical analysis of the school population in 1966/67 and a comparison of the school population in 1960/61 and 1966/67.[601] The comparison showed a great increase, from 39% of the 15–19 year age cohorts attending schools to 49.5%. In addition the age cohorts were growing rapidly, 27% in the six-year interval, from 14,692 to 18,691. However, this report was only a draft, with no forecast or planning, and was never completed.[602] A complete report might have been a useful aid to forecast the rapid expansion of the upper secondary school level in the early 1970s.

In August 1965 Tómas Karlsson, later editor, wrote in *Tíminn,* when criticizing the government for little work on schools and decreased funds for school buildings:

> Í heiminum er nú að verða bylting á sviði skólamála. Tæknin, sem er undirstaða allra efnahagslegra framfara í nútíma þjóðfélagi, og aðrar breyttar þjóðfélagsaðstæður krefjast aukinnar skólamenntunar og nýrra kennsluhátta. Á sviði skólamála erum við að dragast stórlega aftur úr. – Ráðherrarnir opna þó varla svo munninn, að þeir lýsi því ekki yfir að erlendar þjóðir telji menntun æskunnar og aukningu skólakerfisins beztu fjárfestingu þjóðfélagsins og því séu þeir svo sannarlega sammála. Samt skera þeir nú niður það sem naumt hafði verið skammtað til þessarar fjárfestingar ...
>
> In the world, a revolution is developing in the field of schooling. Technology, which is the basis of all economic progress in a modern society, and other changed conditions in society, demand increased school education and new teaching methods. In the field of schooling we are lagging far behind. – The ministers, however, hardly open their mouths without announcing that foreign nations consider the education of young people and the expansion of the school system to be society's best investment and that they truly agree. Yet they cut down what had been stingily apportioned for this investment ...[603]

The OECD paradigm had started to echo in the public discussion.

[600] Efnahagsstofnunin (July 1965): 35
[601] Efnahagsstofnunin (December 1968)
[602] Sigurjón Mýrdal (1989): 55
[603] Tómas Karlsson (1965): *Tíminn*, August 6

7. “Modern” Mathematics

7.1. International Mathematics Reform Movements

Introduction

In the 1950s questions arose in many countries about mathematics teaching in the upper secondary school level. An international reform movement in mathematics education had at least three origins. In the United States of America there was discontent with mathematics teaching after the World War II. The induction testing for the armed forces in the United States presented evidence that many young people were incompetent in mathematics.[604] Furthermore, the war focused national attention on the growing need for trained personnel to serve an emerging technological society.[605] Wartime conditions involved problem solving, such as making and cracking of codes, which led to growth in the field of discrete mathematics, probability and statistics and operational research. These circumstances led the attention to school mathematics.[606]

During the 1950s several important reform projects were launched. At the time of the Sputnik Shock in 1957, nearly fully developed reform programmes already existed to respond to the national call for improvement in mathematics and physics education.[607] The largest and best-known school mathematics programmes were the School Mathematics Study Group (SMSG) project in Palo Alto and the University of Illinois Committee on School Mathematics (UICSM) project, the first American project.[608] There was also a broad reform movement in French-speaking Europe in the middle of the 1950s[609] and another from 1957 in England, where the School Mathematics Project (SMP) was developed.[610]

An important conference was held in Woods Hole, Massachusetts, in the United States in September 1959, where university professors in mathematics and natural sciences met professors of psychology and pedagogy for the first time to discuss the basis for further development of the reform projects in mathematics and natural sciences.[611] This was also the first time that the reform movement in the United States came into contact with European reform movements. The European contact was Bärbel Inhelder, a close collaborator of Jean Piaget in Geneva.[612] Piaget’s theories were to have great impact on the “modern” mathematics movement and its implementation in primary education.

The leaders of the SMSG and the UICSM projects were among the mathematicians present. The leader of the conference was psychologist Jerome Bruner, professor at Harward University. Its report is Bruner’s well-known and influential book, *The Process of Education*.[613]

[604] Osborne, A. R., and F. J.Crosswhite (1970): 231
[605] Osborne, A. R., and F. J. Crosswhite (1970): 238
[606] Gjone, G. (1983): Vol. I, 1
[607] Gjone, G. (1983): Vol. I, 60–61
[608] Gjone, G. (1983): Vol. I, 8–62
[609] Gjone, G. (1983): Vol. II, 55
[610] Cooper, B. (1985)
[611] Bruner, J. S. (1966): 2–3
[612] Gjone, G. (1983): 54
[613] Gjone, G. (1983): Vol. II, 16–21

Jerome Bruner gave the reforms a psychological and learning-theoretical basis in his book *The Process of Education*. The initial statement in its third chapter, Readiness for Learning, is a well-known hypothesis:

> … any subject can be taught efficiently in some intellectually honest form to any child at any stage of development.[614]

This hypothesis, its related theories and the spiral principle presented in the book were to have great impact on Icelandic curriculum development, promoted by pedagogy Professor Matthías Jónasson among others.

From 1959 reform started to expand – as psychologists and pedagogues became more interested in mathematics and natural science teaching – to new pupil-groups and new grades. In 1959, OEEC experts found that reform was necessary within the member countries to meet demands from industry and its new techniques. They knew about the movement which had started in the USA, and it was thought necessary to implement a reform of similar kind in Europe. While in the USA the reform began as mainly subject-related, in Europe its development was more balanced between subject-related and methodological aspects of the reform.[615]

The Organisation for European Economic Co-operation, OEEC, came into being on 16 April 1948. It emerged from the Marshall Plan and the Conference of Sixteen (Conference for European Economic Co-operation), which sought to establish a permanent organisation to continue work on a joint recovery programme after the World War II and in particular to supervise the distribution of aid. In September 1961 the OEEC was superceded by the Organisation for Economic Co-operation and Development (OECD).[616]

Different Directions in the Reform Movement

The reform was not a uniform movement. Howson, Keitel and Kilpatrick have identified five directions in their book, *Curriculum Development in Mathematics*. The best known is the set-theoretical "New Math" approach, in this book called "modern" mathematics. Furthermore, there were behaviourist, structuralist and formative approaches, all of which had their specific characteristics.[617]

1. <u>The Behaviourist Approach</u> was based on Skinner's theories that what is to be learnt is reinforced by immediate reward. This led to programmed learning material, where the material is presented through a string of information and questions. Formulating and grouping objectives bulks large in the behaviour-oriented development of curricula. The approach is content-neutral and could as well be suited to New Math oriented curriculum as traditional mathematics.[618]
2. <u>The New Math Approach</u>. The background for this movement was basic research in mathematics and the effort to represent mathematics with unifying elements, such as the work of the Bourbaki group. The Bourbaki group was a group of French mathematicians, who worked at a kind of mathematical encyclopaedia, where the borders between the different mathematical topics were abolished. The Bourbaki group's central concept was "structure". When describing the structures

[614] Bruner, J. S. (1966): 33
[615] Gjone, G. (1983): Vol. II, ii–iii
[616] OECD, website, accessed October 11, 2005
[617] Howson, A.G. et al. (1982): 93–125
[618] Howson, A.G. et al. (1982): 94– 95. Gjone, G. (1983): Vol. II, 13–14

the importance lay in the elements' relationships, determined by axioms. The axiomatic method became essential for the mathematics and its language was that of the set theory.[619] In the New Math approach the subject was given a new form, but there was less emphasis on the teaching practice.[620]

3. The Structuralist Approach was based on cognitive psychology and on organizing the subject on the basis on its structure, synthesized in Jerome Bruner's theories of the "structure of disciplines" and his concept of the "spiral curriculum", providing a united philosophy. In the spiral model the mathematical objects themselves only function as examples and rank lower than processes, methods and working procedures. An emphasis was laid on a (guided) discovery approach.[621]
4. The Formative Approach developed from the structuralist approach. It laid emphasis on learning theories, especially with a background in Piaget's theories on children's development. There was a greater emphasis on the teacher's role in teaching, and the pupils being stimulated by different types of material. It may be considered as a reaction to other directions of the reform. Open-endedness became a basic characteristic of curricular units, which then served to initiate learning processes but not to determine them. Since one cannot ensure what will emerge from the autonomous activity of the pupil, progress of the teaching process is somewhat uncertain and materials cannot be produced in the form of ready-to-use teaching units.[622]
5. The Integrated-Teaching Approach came up around 1970 at the same time as the Formative Approach and is less relevant in this context.[623]

The original projects may have been initiated according to one specific approach, but gradually, as the projects developed, they absorbed ideas from other approaches and borders became unclear. For example, the Nordic projects were on one hand a mixture of New Math approach and the structuralist approach, and on the other hand based on the behaviourist approach.[624] In Iceland, the "New Math" or "modern" mathematics was to have a great influence. It was brought into all school levels during a short period, 1964–1966, essentially by one person, Guðmundur Arnlaugsson, in cooperation with his collaborator Björn Bjarnason. The structuralistic and the formative approaches were later introduced as reactions to the "modern" mathematics. The behaviourist approach never reached Icelandic curricula to any marked degree, even if local experiments of that kind were made.

The Royaumont Seminar in 1959

An important seminar on mathematics teaching was held in Royaumont in France, arranged by the OEEC in November 1959. Each member country and the United States and Canada were invited to send three delegates: an outstanding mathematician, a mathematics educator or person in charge of mathematics in the Ministry of Education, and an outstanding secondary school teacher of mathematics.[625] The seminar was attended by all the invited countries, among them

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619 Skovsmose, O. (1980): 28–29
620 Howson, A.G. et al. (1982): 100–101. Gjone, G. (1983): Vol. II, 13
621 Howson, A.G. et al. (1982): 107–109. Gjone, G. (1983): Vol. II, 14
622 Howson, A.G. et al. (1982): 116–117. Gjone, G. (1983): Vol. II, 14
623 Gjone, G. (1983): Vol. II, 14
624 Gjone, G. (1983): Vol. II, 15
625 OEEC (1961): 7

Yugoslavia, except Portugal, Spain and Iceland. The Royaumont meeting can be seen as the beginning of a common reform movement to modernize school mathematics in the world.[626]

In the first paragraph in the preface of the report of the seminar, *New Thinking in School Mathematics*, which also includes the result of a survey on mathematics teaching in OEEC member countries, it says:

> Early in 1959, two activities – a survey and a seminar – were initiated by the O.E.E.C. for the purpose of improving mathematical education. While it was anticipated that the survey would serve as a basis for the seminar, the difficulty of framing a questionnaire suitable for reporting the different educational systems and the time required for securing responses made it necessary to hold the seminar before the data from the survey were available.
>
> ...
>
> The seminar was held from November 23 to December 4, 1959, at the Cercle Culturel de Royaumont, Asnières-sur-Oise, France.[627]

The seminar was divided into three sections devoted to the following topics:

(I) New Thinking in Mathematics;
(II) New Thinking in Mathematical Education;
(III) Implementation of Reform.

Many scholars have written about the seminar and its consequences.[628]

In 1959 there was one mathematics professor at the University of Iceland, Leifur Ásgeirsson, and three assistant lecturers, the Reykjavík High School mathematics teachers Guðmundur Arnlaugsson, Björn Bjarnason and Sigurkarl Stefánsson. No-one was appointed as mathematics educator or person in charge of mathematics at the Ministry of Education. These three high school teachers and a couple of younger teachers, Jón Hafsteinn Jónsson and Skarphéðinn Pálmason of Akureyri High School, were the most outstanding upper secondary school mathematics teachers, but none of them was chosen to attend the seminar. At that time, such a journey from Iceland to France was time-consuming and, as the school year was short, schools could hardly spare their teachers for two weeks or more.

A Survey Questionnaire

Attached to the report from the Royaumont Seminar were results from a survey questionnaire:

> Work on a survey questionnaire was initiated in March, 1959, and completed in September. In December, 1959, the final form, bearing the title "Survey of the Present Status of Mathematical Education in the Member Countries of the O.E.E.C.", was sent to each country participating in the programme. Returns were received from almost all of the 21 countries that were canvassed.
>
> The information derived from these questionnaires was intended to answer four main questions:
>
> 1. To what extent is mathematics study required?
> 2. What are the training and certification requirements of teachers?
> 3. How is the curriculum determined, who produces the textbooks and what reforms – if any – are under way?

[626] Gjone, G. (1983): Vol. II, 57
[627] OEEC (1961): 7
[628] Gunnar Gjone (1983). Ole Skovsmose (1980). Barry Cooper (1985)

4. What mathematics is or is not taught in the present curriculum, and what are the purposes and kinds of mathematics given?[629]

The Secretary General of the Ministry of Education, Birgir Thorlacius, answered the questionnaire on Iceland's behalf. The result is interesting reading for comparison of Iceland to other OEEC countries at that time.

An explanation of the survey says:

> The purpose of the survey was to give for each country, in concise form, a picture of the present mathematics programme, information as to how that programme is administered, and what recognisable trends or changes are taking place in subject matter and organisation. In addition, it was also meant to show how many qualified pupils get to the university, and to describe the training – both in mathematics and pedagogy – of mathematics teachers in secondary schools.[630]

The results of the survey revealed that Iceland had the shortest school year after Greece and Italy, and its pupils received, together with Canada, the fewest hours of mathematics teaching in the age range 12–17 years, after Greece and Portugal. Spain did not participate in the survey.[631]

The report contains a survey of the education and training of teachers. From the report it is clear that there were different requirements for teachers teaching at the lower and upper secondary level.[632] The table containing information about qualifications for permanent certification for teaching in secondary schools in the individual countries, however, does not indicate any difference. The qualifications listed for Icelandic teachers are the following:

> Graduation from the university with a master's degree in mathematics. One year of general pedagogy and practice teaching, followed by a state examination.[633]

These requirements are similar to those in Denmark for what here is called the scientific stream of the upper secondary level. Traditionally, prospective Icelandic teachers for that level would acquire their education abroad, usually in Denmark, as the University of Iceland did not offer such a degree. This arrangement had produced two new mathematics teachers for Iceland in the period 1947–1959, Jón Hafsteinn Jónsson and Skarphéðinn Pálmason.

The lower secondary level in Iceland, as in other Nordic countries, did not have a scientific stream, except perhaps the national examination class. This is not accounted for in the survey, due to the different systems in the OEEC countries.

In an attached table showing the percentage of teachers with full certification requirement at the scientific upper secondary level, Iceland is the lowest, with 30%.[634] Only four out of the 20 countries participating in the survey stated a percentage below 80%, i.e. Iceland 30%, Turkey 38%, Portugal 60% and the Netherlands 70%, while Denmark had 95%, Germany 100%, Norway 100% and United Kingdom 80%. In spite of that, no shortage is reported in Iceland to be foreseen for the years 1960–1965, while Denmark expected a shortage of 200 and Norway of 1000 teachers in 1965.[635] Either there is a printing error, or the reporter misunderstood the question.

[629] OEEC (1961): 7–8
[630] OEEC (1961): 129
[631] OEEC (1961): 140
[632] OEEC (1961): 146–148
[633] OEEC (1961): 155
[634] OEEC (1961): 158
[635] OEEC (1961): 158

The Ministry of Education can hardly have intended to maintain the current situation of having only 30% qualified teachers.

The fact that the report, *New Thinking in School Mathematics*, does not exist in Icelandic university libraries, invites the suspicion that its contents did not become widely known in Iceland, neither by the authorities nor in the mathematical community. It must, however, have been received by the Ministry of Education, but now, more than 40 years later, it is not available. The shocking comparison of the numbers of qualified teachers must have stirred those who saw it. The question remains, however, whether it was ever studied. In 1961, no one in the Ministry was appointed to be responsible for teaching in upper secondary schools. The only persons active for the teaching subjects at the lower secondary level were the members of the National Examination Board, who were appointed for that temporary task. They were not the employees of the Ministry, and had their offices in their own living rooms.

The survey is composed of 21 questions designed to elicit answers to the four main questions. To the query about how the curriculum was determined, who produced the textbooks, if reforms were under way and standards for selecting textbooks, the answers from Iceland were presented in the following way, presumably after editing:

> Question 14–16. Trends, Changes and Major Movements …
> Iceland … No changes or trends in the programme. Recently, emphasis on geometry was diminished and emphasis on calculus increased.[636] …
>
> Question 18. Standards for Selection of Secondary School Textbooks …
> Iceland … Danish books are used and are selected by the headmaster.[637]

These answers apply to the high schools. In question 21 there is a detailed survey of the age at which pupils were introduced to the various arithmetical topics. Generally the answers from Iceland reported a higher age than the other countries, e.g. an exercise such as $684 \cdot 342$ was reported to be introduced at the age of 13. This is not credible, and may reflect the unfamiliarity of the person answering the questions. Introduction of percentages, e.g. finding the figure of which 15% is 6, is correctly reported at the age of 13, later than in all other countries.[638] To the question: “Is the teaching of arithmetical operations such as the above, supplemented by theoretical discussions intended to help understanding?” this answer is reported on behalf of Iceland:

> Depends on the teacher; meaning is supposed to be given.[639]

As the little text contained in the textbooks was not suitable for the children to read, and in some cases aimed at the teacher, this answer was natural. Concerning the scientific stream, i.e. the curriculum of the national examination and the high schools, most topics were treated at similar ages as in other OEEC countries. A few topics were taught after the age of 18 in Iceland, which was an exception, as high school in Iceland was a four-year school, to be completed at the age of 20.[640]

This was the situation in Iceland before the mathematical education reform in the 1960s. It supports a hypothesis of the author of this thesis that the main focus of the

[636] OEEC (1961): 180–181
[637] OEEC (1961): 183
[638] OEEC (1961): 199, 233
[639] OEEC (1961): 199
[640] OEEC (1961): 233–236, 199–202

authorities was on pupils aiming at further education through the national examination, and the others were not given any detailed attention.

Recommendations of the Royaumont Seminar

The final recommendations of the Royaumont Seminar included a combined syllabus, introducing mathematics as a unity, and that modern algebra should be the basic and unifying item in the subject of mathematics. In the teaching of all secondary school mathematics, modern symbolism should be introduced as early as possible, as it represented concepts that bring clarity and conciseness to thinking and are unifying. In addition to arguing for the inclusion of modern algebra in school courses, the conclusions also included arguments for[641]

- the unification of traditional branches of elementary mathematics, where the difference between algebra and geometry is made gradually to disappear, possibly through a vector approach or "motion geometry", where deductive geometry is treated by the use of vectors and later merged with algebra
- increased use of "modern symbolism" (i.e. from set theory)
- removing much of the traditional school geometry and algebra
- ending isolated study of trigonometry
- introducing probability and statistics
- more attention to be paid to preparing pupils for axiomatic approaches.

The reforms were primarily conceived for a select group of pupils, but on the other hand there are indications that a broader group was also borne in mind. This may be read from this introductory note to the summary and conclusions of the seminar:

> Although the seminar focused most of its attention on the education of university-capable students, its efforts included reform in mathematics education as related to current needs of society and its enterprises.
>
> Thus mathematical education, especially at the secondary-school level of instruction, is not to be aimed directly at producing future mathematicians. The uses and applications of mathematics, the needs of scientists in all fields of knowledge, the needs of laymen, the co-ordination of mathematics instruction with that in the other sciences, and the great need for articulating secondary-school and university study are the principal factors considered as a new mathematics education programme is unfolded.[642]

The anticipated needs of society were thus the main aims of the planned reform. The needs of the individual pupils were not mentioned. In a section about teaching of arithmetic, we find:

> Psychological implications of learning procedures used in primary-schools and the shift of school aims to developing concepts and modes of thinking (as well as skills) must bring a corresponding change in arithmetic instruction. The learning must be the result of understandings arising from guided experimentation and discovery, probably with the use of physical objects of one sort or another.
>
> In this way, the child must be led to the abstraction of the *quality of a set* called its *number*.[643]

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[641] OEEC (1961): 105–125. Cooper, B. (1985): 161
[642] OEEC (1961): 105
[643] OEEC (1961): 108–109

While this is presumably a reference to Piagetian theories about how children think and learn mathematics, this seems not least to be intended as preparation for a more advanced study:

> Such study must certainly be under the supervision of a teacher who understands all the implied mathematical relations and the relation of the material to the subsequent study of mathematics. Finally, generalisations of arithmetical relations through the use of literal symbols can serve as an informal introduction to algebra.[644]

In the final section of a summary of the report from the seminar it says that

> ... we see a *change in purpose* that a reform must emphasize – namely, a building of mathematical concepts, structure and understanding as paramount to manipulative skills, although the latter must be adequately developed. We see another *change in the use of ideas* that have hitherto not been in our secondary-school programme.
>
> … We recognize also a *different organisation and treatment of the several branches* (based on psychological knowledge of mental growth), to the end that each is linked to the others so as to bolster understanding and provide a more common basis for the continued study and application of mathematics.
>
> ...
>
> The aim of all these programmes is two-fold: Firstly to provide a better preparation for university study; secondly, to give to all pupils an instrument for use in daily life.[645]

Also here it seems that the second aim was very much a secondary aim, while the first and primary aim was to provide a better preparation for university study. The need for mathematics for the general public is mentioned earlier, whence the justification for the second aim seems to be sought. In the section of the summary, The Case for Reform Summarized, it says:

> The changes in cultural, industrial, and economic patterns of many nations call for a basic change in educational patterns. More people must be better trained in scientific knowledge. Even laymen must come to understand science; today, knowing mathematics is basic to understanding science.[646]

This sounds more like a justification of those who have in mind the needs of the society for scientifically-educated intellectuals, rather than a desire for providing individuals with tools to cope with their life on their own premises.

In Barry Cooper's opinion, clearly, at this seminar the dominant participants took it for granted that "mathematics" meant "university mathematics". Furthermore it was this "mathematics" that school pupils should necessarily study.[647]

Each country could reform its mathematics teaching according to its own needs, but it was recommended to establish as much cooperation as possible:

> Each country will have its own unique way of making the reform – of introducing new material, of organising the sequential study and of experimenting with possible programmes. There should be provided channels for communicating the results of these programmes and experiments between all our countries, so as to enable us to use the best thinking of all countries in stimulating new ideas.[648]

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[644] OEEC (1961): 109–110
[645] OEEC (1961): 124–125
[646] OEEC (1961): 107
[647] Cooper, B. (1985): 160–161
[648] OEEC (1961): 125

For the Nordic countries the Royaumont Seminar was a central event. The Nordic participants agreed upon trying to organize Nordic cooperation on reform of mathematics teaching.[649]

Nordic Regional Cooperation

The ideas about Nordic cooperation were presented to governmental bodies, and the issue was taken up in the Nordic Council, which decided to set up a committee under its Culture Commission. Each of four countries – Denmark, Finland, Norway and Sweden – appointed four persons to the committee, *Nordiska kommittén for modernisering af matematikundervisningen* (The Nordic Committee for Modernizing Mathematics Teaching), abbreviated as NKMM. Finland was not a member of the OEEC, and therefore it did not have a representative at Royaumont. This committee dominated mathematics instruction in the Nordic countries for most of the 1960s.[650] Iceland did not participate in the NKMM cooperation. However, all the Danish representatives made an impact in Iceland through their writings, so their names will be mentioned: Agnete Bundgaard (primary level textbooks), Bent Christiansen (textbooks for teacher training), Erik Kristensen and Ole Rindung (textbooks for the high school level).[651] The secretary of the committee, Matts Håstad from Sweden, also had an impact through a high school textbook by Bergendal, Håstad and Råde.

The committee was active from 1960 until 1967, when its report was ready in the autumn. The members of the committee were mathematicians and mathematics teachers, or they came from school administration. The programme for Nordic reform was to analyse the situation found within mathematics education in each country, to work out preliminary and revised curriculum plans, and furthermore writing of experimental texts.[652]

The committee appointed several teams of writers. Its focus was on the mathematical content, and the teaching of seventh to twelfth grades was its main object. Consequently its main contribution was in this field. However it was decided to handle mathematics courses throughout the school and the committee contacted for that purpose extra experts for the first to sixth grades.[653]

Writing sessions were arranged in summer 1961. Some texts were ready that autumn, and the others were to be so successively until the beginning of 1966. Joint Nordic manuscripts were planned. Several persons from each country would translate and adapt the joint publications to each language. This was only implemented for the seventh to twelfth grades. Norway, for instance, introduced translated series from the School Mathematics Study Group, SMSG, in USA for the fourth to sixth grade at primary level.[654] Iceland, on the other hand, adopted the Nordic primary-level material, as we shall see.

The report, published in 1967, contained a description of the school situation and mathematics teaching in the Nordic countries, a chapter on the aims of mathematics teaching, an account of experiments and developmental work, and a proposal on content of mathematics syllabuses. The chapter on the aims of mathematics teaching

[649] Gjone, G. (1983): Vol. II, 62
[650] Gjone, G. (1983): Vol. II, 78
[651] Skovsmose, O. (1980): 35–36
[652] Gjone, G. (1983): Vol. II, 80
[653] Gjone, G. (1983): Vol. II, 78
[654] Gjone, G. (1893): Vol. II, 79–80

is close to an extract from Bent Christiansen's book, *Mål og midler i den elementære matematikundervisning* (1967). Bent Christiansen's discussions thus became NKMM's official aims.[655]

The committee specified that the goal of teaching for all school subjects was to be:

a) that the subject shall make its contribution to the preparation of the individual student for life in general, regardless of his possible function in society and of his possible view of life.

b) that the subject shall make its contribution to the preparation of the individual student for subsequent education – in the subject itself and in other subjects – both at school and afterwards.[656]

The twin goal led to three choices:

1. Choice of curriculum.
2. Choice of mathematical procedures.
3. Choice of pedagogical methods.[657]

The committee proposed the general aims for school mathematics teaching to be as follows:

1. To give the students insight into the curriculum adopted, an understanding of its basic concepts and recognition of their interaction.
2. To give the students insight into the aesthetic values of mathematics and a wealth of opportunity to experience pleasure through (productive) work on the subject.
3. To give the students the opportunity to experience mathematics as a living and ever-open subject, the conceptual structures of which are often established on the basis of inspiration from experiments with objects from the physical world, or from previously established mathematical theories.
4. To acquaint the students with the part played by language in definitions and reasoning, both in mathematics and in other subjects.
5. To help the students to master new mathematical topics through independent mathematical study.
6. To give the students the opportunity to recognize the deductive nature of mathematics and, in particular, the special role played by the axiomatic method.
7. To enable the students to perceive mathematics as a means of describing actual problems theoretically, whereby they may come to recognize the principles for the practical application of mathematics.[658]

The needs of society for scientifically-educated intellectuals are not mentioned, and one notices more concern for the subject mathematics itself and the pupils' acquaintance with it than e.g. providing individuals with mathematics as prerequisites for further studies and daily life, as stated in the main goal. However, the aims presented in the proposal on content of mathematics syllabuses are:

- Pupils should experience the pleasure of working with mathematics.
- Instruction must aim above all at making the pupils understand basic concepts.
- Pupils should be able to apply their mathematical knowledge in practice.[659]

If the desirable insight into and understanding of mathematics were to be achieved it must constantly be made apparent, during the construction of increasingly

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655 Gjone, G. (1983): Vol. II, 82–83
656 *New School Mathematics in the Nordic Countries* (1967): 14
657 *New School Mathematics in the Nordic Countries* (1967): 16
658 *New School Mathematics in the Nordic Countries* (1967): 28
659 *New School Mathematics in the Nordic Countries* (1967): 48–49

complicated concepts, that the mathematical considerations bear the very closest relationship to reality.[660]

In the view of Norwegian scholar Gunnar Gjone, the NKMM can be seen as the carrier to the Nordic countries of the spiral principle, an important part of Jerome Bruner's theories:[661]

> Most mathematical topics have to be dealt with at several different junctures during school life. This type of teaching is known as the spiral system. Each individual treatment of a certain subject or field of material calls for its particular degree of insight and absorption.[662]

The very use of the concept of sets was expected to permit a particularly flexible variation in connection with sorting out basic concepts for later building of mathematics, such as *variable*, *open statement*, *solution set of an open statement*, the basic concepts of logic, *ordered pair* and *sets of ordered pairs* (*relations* and *functions*). But certainly it had to be carefully ensured that each individual concept was suitably accepted and incorporated before being used as a basis for further study.[663]

The author(s) of the report repeatedly mentioned the importance of concrete material which played a major part in the first grades and the "principle of discovery" in connection with the inductive type of teaching.[664] The inductive way of work, by which the student is given a wealth of opportunity to discover for himself connections between different parts of the mathematical structures, was believed to be the most likely method to permit a transfer from mathematics to other fields of study or to the field of practice.[665]

This goal-setting chapter is central to the NKMM's views and is the background for the work of the committee.[666] It is based on the general goals put forward at the Royaumont Seminar. However, even if the Nordic goals are much concerned with the subject of mathematics *per se*, and the relation of the pupil to the subject, the emphasis in the Nordic report is more on the personal development of the pupil than in the report of the Royaumont Seminar. The needs of society are discussed less in the Nordic report; it is rather the pupil's need to cope with life in society, than the need of society itself to produce mathematically and technically educated persons. The extensive discussion of the contribution of mathematics teaching to general education, and the pleasures of working with mathematics that the pupil should have the opportunity to experience, bear witness to a pedagogical philosophy, concerned with the individual pupil,[667] not displayed to the same degree in the report from the Royaumont Seminar.

Concerning the curriculum itself, the committee is explicit in saying that this is based on foreign experience, especially the American SMSG and UICSM projects and the English SMP project, as mathematics is an unusually international subject.[668]

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660 *New School Mathematics in the Nordic Countries* (1967): 32
661 Gjone, G. (1983): Vol. II, 88
662 *New School Mathematics in the Nordic Countries* (1967): 32
663 *New School Mathematics in the Nordic Countries* (1967): 34
664 *New School Mathematics in the Nordic Countries* (1967): 16
665 *New School Mathematics in the Nordic Countries* (1967): 22
666 Gjone, G. (1983): Vol. II, 87
667 *New School Mathematics in the Nordic Countries* (1967): 16–23, 34–37, 48–50
668 *New School Mathematics in the Nordic Countries* (1967): 52

The curriculum for the basic course in the first to ninth grade for all pupils was to be in the spirit of "modern" mathematics and make use of basic concepts such as sets etc. Not only was the way of presentation new, but there were also new topics, such as probability. The number concept was developed in the first grade by pairing elements in sets, i.e. producing a one-to-one correspondence, so that e.g. the number 3 is defined as the common property of all sets with three elements. In the eight and ninth grade, the consideration of preparation for further education is dominant, e.g. by introducing advanced topics such as vectors and trigonometry. In the ninth to twelfth grade the idea was to narrow the gap between the mathematics teaching of the university and the high schools by transferring some of the university syllabus down to high school.[669]

After NKMM had completed its work in 1967, the MUNK–committee / *Matematikk-undervisningens Nordiske Komité*, successor to the NKMM, was established. There Iceland had its representative, Guðmundur Arnlaugsson.

7.2. Influences in Iceland

Events Arranged by the OEEC and the OECD

At some point of time in 1960 or 1961, Icelanders started to become aware of reform activities in mathematics teaching in the neighbouring countries.

The report of the OEEC seminar on "The Mathematical Knowledge required by the Physicist and Engineer" (Project STP 17) held in Paris in February 1961, reveals that one of those who attended was Magnús Magnússon, a young professor of physics at the University of Iceland.[670] He was Iceland's representative at the European Nuclear Energy Agency (ENEA), an OEEC institute in Paris.[671] Professor Magnús Magnússon taught the B.A. students physics within the group of the engineering students. In an interview with the author of this study, he said that the University had been invited to send a delegate to the conference and that he had been chosen in connection with his work with ENEA in Paris.[672] The purpose of the seminar was to define the mathematical knowledge and practices to be emphasized and developed, in order to produce engineers and physicists of a quality level compatible with the needs of modern science and technique. Among other topics discussed was the shortage of teachers.[673]

In June 1961, Matthías Jónasson, professor in pedagogy, and Sveinbjörn Sigurjónsson, headmaster in a lower secondary school, attended the OEEC seminar "Ability and Educational Opportunity in a Modern Economy", held in Kungälv, Sweden. The topics of the seminar were resources of talents in the population, particularly in the fields of science and technology, where the need for talented individuals was expanding more rapidly than in most sectors, and how talents could be discovered early enough.[674] The OEEC/OECD's influence was beginning to filter into Icelandic education.

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[669] Gjone, G. (1983): Vol. II, 88–91
[670] OECD (1961): 217
[671] Magnús Magnússon (1987): 95
[672] Magnús Magnússon, November 21, 2003
[673] OECD (1961): 6–14
[674] Matthías Jónasson and Sveinbjörn Sigurjónsson (1962): 74–79

Mathematics To-Day, proceedings of an International Working Session on New Teaching Methods for School Mathematics, held in Athens in November 1963 by the OECD, reveal that Reykjavík High School mathematics teacher Björn Bjarnason, later one of the main proponents of mathematics reform in Iceland, was a participant in the session.[675] All the Nordic participants were those most involved in the NKMM activities, and Sweden and Denmark presented their "modern" curricula. If Björn Bjarnason had not already known of these, he became acquainted with them in Athens.

The direct involvement of the OECD in mathematics reform came to an end in 1963/1964. Evaluating the contribution by the OEEC and the OECD to the reform, Gunnar Gjone's interpretation was that the concern about economic development in its member countries was the main driving force in its engagement. The organization grasped the current reform projects, especially in the USA, and saw them as an expression of modernizing in order to satisfy modern society's needs. In the same way as federal authorities with regard to the Sputnik Shock in the United States, the OEEC and the OECD hardly had any influence on professional policy. On the other hand, it had an enormous influence on the reform's volume and diffusion in Europe.[676] The Danish scholar Jens Høyrup has also expressed similar opinions. The university mathematicians were those who determined policy.[677]

Nordisk Matematisk Tidskrift

One of the channels of information about the "modern" mathematics movement was the Nordic mathematical journal, *Nordisk Matematisk Tidskrift* (The Nordic Mathematical Journal, NMT). In its first issue in 1960 there is an article giving a short account of the Royaumont Seminar, the speech of the president of the seminar Dr. Marshall H. Stone in Norwegian translation and the resolutions of the seminar, signed by K.P., the Norwegian Kay Piene.[678] Dr. Stone was professor at the University of Chicago and chairman of the International Commission on Mathematical Instruction (ICMI) of the International Union of Mathematicians.[679]

In the second issue that same year, Kay Piene wrote an article where he discussed the resolutions of the seminar about each of the topics: arithmetic, algebra, geometry, analysis, probability and statistics.[680]

In the second issue in 1963, Piene wrote about teacher training in those times when university mathematics was to be brought down into the high schools.[681] That same year, in 1963, there is a review of new textbooks for high school, already published in Denmark, *Matematik I*, by Erik Kristensen and Ole Rindung (1962), and *Matematik for gymnasiet I*, by Poul O. Andersen, Stig Bülow and Hans Jørgen Helms (1963). Both were the beginning of series of textbooks, introduced in Iceland in the following years.[682]

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[675] OECD (1964): 393
[676] Gjone, G. (1983): Vol. II, 70
[677] Høyrup, J. (1979): 50–51
[678] Piene, K. (1960a): 53–62.
[679] OEEC (1961): 12
[680] Piene, K. (1960b): 65–71
[681] Piene, K. (1963): 49–55
[682] *Nordisk Matematisk Tidskrift* (1963): Vol. 11(2), 70; Vol. 11(4), 184

Early in 1964, Matts Håstad wrote in NMT an account of the activities of NKMM.[683] At that time it had published a textbook for the first part of the first grade of primary school, which was being tested in the school year 1963-64. The second part for the first grade was underway and texts for the second and third year were being planned. These were probably the texts written by Agnete Bundgaard and Eeva Kyttä for the first two grades, and thereafter by Agnete Bundgaard alone.

For the fourth to sixth grades the American SMSG texts were on the plan. The first part for the fourth grade had already been published and was being tested in a few classes.

For the seventh to ninth grade, algebra and geometry texts were being written and tested. *Algebra I* had been tested since 1962–63. The testing of *Algebra II* started in spring 1963, and it was too early to comment on it. *Algebra III* was to begin being tested in 1963–64. *Geometry I* had been tested in Finland, Norway and Sweden in 1961–62. Tests on *Geometry II* had begun last spring. From the Icelandic point of view this series looks similar in content to the one which was translated from Swedish in Iceland in 1970–1972, and it may be its prelude.

For high school, experimental texts were being written or planned for *Algebra I–II*, *Geometry I–III*, *Function Theory I–II*, *Differential Equations* and *Probability and Statistics*.

In issue 1–2, 1965, of *Nordisk Matematisk Tidskrift* there is a review of the teacher training textbook *Almene Begreber fra Logik, Mængdelære og Algebra* by Bent Christiansen, Jonas Lichtenberg and Johs. Pedersen, published in 1964.[684] A list of contents had been published in NMT in 1964. The following summer, 1965, this book was used as a basis for a teacher in-service course, chaired by Guðmundur Arnlaugsson, to introduce "modern" mathematics to Icelandic teachers. In the same year, 1965, a list of contents of a textbook with a similar purpose, *Matematik 65*, was published in NMT, and it was reviewed the year after, in 1966.[685] Björn Bjarnason introduced it to his student teachers at the University in the autumn of 1966.

In 1966, the series by Erik Kristensen and Ole Rindung and the series by Poul O. Andersen, Stig Bülow and Hans Jørgen Helms had both been published for all the three-year courses in Danish high schools, and in 1967 thorough reviews on both series appeared in NMT.[686] This same year, in 1967, the Kristensen & Rindung series was introduced by Guðmundur Arnlaugsson at his new Hamrahlíð High School, while Jón Hafsteinn Jónsson at Akureyri High School introduced the Andersen, Bülow & Helms series in 1966. Three years later, in 1969, the Akureyri High School introduced the Kristensen & Rindung series.[687] Jón Hafsteinn Jónsson said in an interview with the author of this study[688] that he studied a review in the NMT which influenced him in the decision to reject the Andersen, Bülow & Helms series. Possibly this were these paragraphs:

> Det er bøger, som mange lærere og elever er glade ved at arbejde med. ...

[683] Håstad, M. (1964): 8–16
[684] Hansen, E. (1965): 51–52
[685] Solvang, R. (1966): 118–119
[686] Jørgensen, K. W. (1967): 104. Møller, S. (1967): 108–114
[687] Gísli Jónsson (Ed.) (1981): Vol. II, 157–158
[688] Jón Hafsteinn Jónsson, August 21, 2003

Man kan konkludere, at mens forfatternes matematiske samvittighed måske er lidt grumset, så er deres pædagogiske samvittighed næsten ren.

These are books that many teachers and pupils are happy to work with. …

One can conclude that while the authors' mathematical conscience is perhaps a little filthy, their pedagogical conscience is nearly clean.[689]

In that same issue, 3–4 in 1967, a more positive review of the Kristensen & Rindung series was published, and this may have influenced Jón Hafsteinn Jónsson's decision as well. This clause is found in the review:

Det er overmåde meget at glæde sig over ved læsning af forfatternes behandling af denne omfattende stofmængde. Den skønsomme brug af symboler, specielt fra mængdelæren og logikken, er med til at skabe den klarhed i fremstillingen, som virkelig er beundring værd.

There are immensely many things to enjoy when reading the authors' treatment of this voluminous amount of material. The perceptive use of symbols, especially from the set theory and logic, contribute to create a clarity in the presentation, which really is praiseworthy.[690]

The reviewer measures the textbook series on the yardstick of the "modern" mathematics; the use of symbols creates a praiseworthy clarity. This approach probably appealed to Jón Hafsteinn Jónsson, who was a former student of Svend Bundgaard in Copenhagen and followed closely what was going on.[691] That same year, in 1969, the Hamrahlíð High School introduced the SMP-series for the natural science line, but retained Kristensen & Rindung for the physics line.[692]

In the fourth and last issue of NMT in 1967 a review was published of six new series for the first year-course in mathematics according to a new plan for the high schools in Sweden.[693] One of the two series which received the most positive response was *Matematik för Gymnasiet*, *na/te* by Bergendal, Håstad and Råde, a series that was introduced at Hamrahlíð High School in 1969, first for the uppermost natural science line, and in 1971 at Reykjavík High School. The Reykjavík High School took up Kristensen & Rindung series for the physics line in 1980.

The review about the series by Bergendal et al. discussed the set-theoretical content and use of symbols in the textbooks, and also praised accessibility for the pupils. This series turned out to be more accessible for Icelandic pupils than the Kristensen & Rindung series, even if Swedish was less familiar to them than Danish. At any rate it endured for most of the 1970s in the upper secondary level schools. In the review it says:

Detta är en mycket väl genomförd bok som synes lagom utförlig (utom möjligen kapitlen om tal som är något för omfattande och därför svåroverskådliga). En detalj som förhöjer bokens värde är de regelbundet återkommande sammanfattningarna av genomgångna avsnitt. Dessa ger god oversikt och torde vara av särskilt värde vid repetition.

This is a very well processed book, which seems to be suitably concise (except possibly the chapter on numbers which is somewhat too extensive and therefore

689 Jørgensen, K. W. (1967): 99–104
690 Møller, S. (1967): 108–114
691 Jón Hafsteinn Jónsson, August 21, 2003
692 Hildigunnur Halldórsdóttir, e-mail September 2, 2003. Skýrsla Menntaskólans við Hamrahlíð, 1969–1970: 38–39
693 Hanner, O. (1967): 157–166

difficult to overview). A detail that elevates the value of the books is the regularly repeating summaries of the previous sections. These offer good overview and must be of special value at revision.[694]

Thus the *Nordisk Matematisk Tidskrift* channelled information about the Royaumont Seminar, the activities of the Nordic Committee for Modernizing Mathematics Teaching (NKMM) and the latest textbooks written in the new style. Sigurkarl Stefánsson, then a senior mathematics teacher at the Reykjavík High School, was a member of the editorial board, so certainly the staff room at Reykjavík High School and the small mathematical community were well informed.

Kristján Sigtryggsson's Study Trip

In 1963 the wave of mathematics reform was known in the circles of primary education. The Reykjavík Education Office, under the leadership of Jónas B. Jónsson, sent primary teacher and inspector Kristján Sigtryggsson to the United States to study mathematics education reforms. Kristján Sigtryggsson was appointed as an inspector for mathematics teaching in Reykjavík in 1960. He went on a short study trip in 1961 to Norway, where he may have learnt about the reforms. In the academic year 1963–64 he spent six months in the United States on a Fulbright scholarship to study experimental programmes in mathematics. In 1964 he wrote an article in *Menntamál* about the Madison Project in Syracuse, New York, experiments at Stanford University with Patrick Suppes and at the University of Illinois (UICSM), and the SMSG (School Mathematics Study Group) experiment in Stanford, Palo Alto.[695]

A quotation from Kristján Sigtryggsson's article about his trip may shed light on the self-esteem of the Icelandic people at that time, but with an anticipation that something might change:

> Við eigum því láni að fagna að búa nú þegar við góða alþýðumenntun og teljast þannig til mestu menningarþjóða heimsins. En vandi fylgir vegsemd hverri. Nú þurfum við að gæta heiðurs okkar og hagsmuna í menningarsamkeppni nútímans, gæta þess, að kröfurnar eru aðrar en þær voru. Einangrun landsins er úr sögunni. Samskipti við aðrar þjóðir aukast árlega, og fræðslustofnanir okkar eru vegnar og metnar til samanburðar við þær beztu erlendis.
>
> We are fortunate enough to have good public education already and thus be counted among the world's culturally most advanced nations. But every glory is problematic. Now we have to protect our honour and interests in the cultural competition of today; be aware that the requirements are different from what they were. The isolation of the country no longer exists. Interaction with other nations increases every year and our educational institutions are weighed and evaluated in comparison with the best ones abroad.[696]

Possibly the Icelandic concepts, *menntun* (education) and *menning* (culture) have interchangeable meanings in the mind of the author. At any rate, Kristján Sigtryggsson brought back from the USA a number of books, which he studied through the winter 1964–1965 together with Director Jónas B. Jónsson.

The initiative for mathematics reform thus did not come from the Ministry of Education but from the Reykjavík Education Office and from interested individuals. Even if Iceland was not involved in the NKMM cooperation, Director Jónas B. Jónsson probably knew about it and its curricula. The NKMM committee was explicit

[694] Hanner, O. (1967): 162
[695] Kristján Sigtryggsson (1964): 148–151
[696] Kristján Sigtryggsson (1964): 146

in saying that they are based on foreign experience, the SMSG, UICSM and the SMP projects. Jónas B. Jónsson may have thought it wisest to learn about the original sources in the United States. The American projects also were ahead of the Nordic ones. He had already seen that changes were needed before any Nordic material was ready for use.

In retrospect, one wonders how much Kristján Sigtryggsson was influenced by "modern" mathematics. From 1966 onwards he rewrote Elías Bjarnason's textbooks for the upper primary level. This new series did not contain any new philosophy or methodology. Some effort was made to explain more than in Elías Bjarnason's older version, while in many cases still only a method is shown, such as in finding the least common denominator as discussed earlier (see section 5.5.). In the last five pages of the last volume the concepts set, element, union, intersection, set difference and subset were listed, without any further elaboration. The acquaintance was there, but no noticeable pedagogical influence.[697]

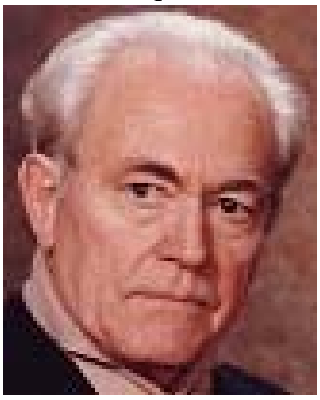

Fig. 7.1. Director Jónas B. Jónsson.

Sveinbjörn Björnsson's Comparison of Nordic Science Syllabi

One of the first signs of the OECD's influence on education in Iceland was the establishment of the Technical College of Iceland in 1963. Sveinbjörn Björnsson, later physics professor at the University of Iceland, prepared the establishment of the new college by writing a report on similar schools in Denmark, Norway and West Germany, their structure and teaching methods, after having studied them on a grant from the OECD.[698]

The Technical College was supposed to offer further education to those who had completed technical education in the conventional apprenticeship system and a corresponding technical school. The college was eventually to prepare them for university studies. Sveinbjörn Björnsson became a part-time teacher at the Technical College in 1964. In November 1965, the principal of the college asked him to make a comparison of studies in mathematics, physics and chemistry in Danish, Norwegian and Icelandic lower secondary schools. The pupils in the preliminary courses of the college had had great difficulty with the syllabus, and the reason seemed to be that Icelandic lower secondary school graduates had much less grounding in these subjects than lower secondary school graduates in Denmark and Norway.

Sveinbjörn Björnsson compared the school systems, number of hours per year, number of hours per week and the syllabus in each of the three subjects. His conclusions were that the mathematics syllabus of 16-year-old Icelandic pupils was comparable to the syllabus of 14-year-old pupils in Norway and Denmark. In physics the 16-year-old pupils were comparable to 15-year-old Danes and 14-year-old Norwegians, and the Icelandic pupils hardly had any demonstrations and experiments, which were greatly emphasized in the other countries. The Icelandic pupils had learnt

[697] Elías Bjarnason (1965): 108–112
[698] Sveinbjörn Björnsson (1966): 100

next to nothing in chemistry. The school year in Iceland was much shorter than in the other countries.[699]

This report caused disturbance among educators and politicians. Its result was the initiation by the Ministry of Education of a new curriculum and learning material in physics and chemistry for the lower secondary level. This curriculum was written under the influence of the Woods Hole Conference and the British Nuffield physics textbook system. Reform in the curriculum and learning material for physics became the first task of the School Research Department of the Ministry of Education, established in 1967.

When this report was published, Guðmundur Arnlaugsson had already for two years been mathematics teaching consultant appointed by the Ministry. He had made a survey among lower secondary pupils, which regrettably is no longer available, but which according to Director of School Research Department Hörður Lárusson showed deficiencies in mathematics education.[700] Following that, Guðmundur Arnlaugsson published a new textbook for lower secondary school in 1966, *Tölur og mengi / Numbers and Sets*. Together with his colleague, Björn Bjarnason, he had also already begun a reform in 1964 in the high school, both based on ideas on "modern" mathematics. People were becoming aware that Icelanders were lagging behind in the educational respect, at least in mathematical subjects. New currents in education were underway.

An Earlier Voice of a Natural Scientist

In 1958, a bacteriologist, Sigurður H. Pétursson and a part-time teacher at the learned department of *Verzlunarskólinn*,[701] wrote in *Menntamál*:

> Hér á landi hefur náttúrufræðum þegar verið gefið talsvert rúm í skólunum, en lítið fer fyrir þeim í blaða- og bókaútgáfu og í útvarpi. En það er eins og fólk hér hafi ekki áhuga fyrir þessum fræðum og telji sig geta komizt af án þeirra. …
>
> Mér virðist það vera áberandi hjá Íslendingum, hversu sljóir þeir eru fyrir einföldustu undirstöðuatriðum raunvísindanna, en hneigjast mikið að húmanískum fræðum. Held ég, að þetta sé ein aðalástæðan til þess, hversu óraunsæir þeir eru. En óraunsæi og rómantíska í hugsun og háttum hefur verið mjög áberandi með Íslendingum í seinni tíð. Helztu áhugamál þeirra virðast vera skáldskapur, þjóðleg fræði og stjórnmál. ...
>
> Það er oft illmögulegt að kenna Íslendingum, sem komnir eru af unglingsaldri, vegna þess að þeir halda sig vita svo mikið. ...
>
> Þetta vanmat íslenzkrar alþýðu á raunvísindum stafar fyrst og fremst af því, hversu mjög hinum áðurnefndu húmanísku fræðum hefur verið haldið að þjóðinni. Þegar íslenzk alþýða vill fræðast, þá eru henni alltaf fengnar í hendur nokkrar bækur, sem hér voru skrifaðar í fornöld og prentaðar eru nú upp ár eftir ár.
>
> In this country, the nature subjects have already had considerable space in the schools, but they are not extensive in newspaper- and book-publication and on the radio. It is as if people here are not interested in these studies and think that they can manage without them. ...
>
> I think that it is striking in Icelanders, how obtuse they are concerning the simplest basic topics of the natural sciences and much inclined to the humanistic studies. I think that this is the main reason for how unrealistic they are. Unrealism and romanticism in thinking and conduct have been very prominent among Icelanders in

[699] Sveinbjörn Björnsson (1966): 118
[700] Hörður Lárusson, March 26, 2002
[701] Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. II, 197

> recent times. Their main interests seem to be poetry, folkloristic studies and politics. …
>
> It is often wellnigh impossible to teach Icelanders, who are out of their teens, because they think they know so much. …
>
> This undervaluation by ordinary people of natural sciences is mainly caused by how much the previously mentioned humanistic studies have been thrust upon the nation. When ordinary Icelandic people want to learn they are always given several books, which were written in ancient times and are now reprinted year after year.[702]

This article continued with a comparative study, much on the same lines as Sveinbjörn's Björnsson's study, eight years later in 1966. For two reasons, this article probably had no impact. Firstly, the OECD paradigm, about the contribution of education to economic progress, had not arrived. Secondly, the tone was rather haughty. Simply to speak of Icelanders as "they" was a sign of arrogance. This did not contribute to sympathy on the part of colleagues, authorities or the ordinary reader of *Menntamál*, the rank-and-file teacher. However, Sigurður H. Pétursson mentions the main trends in society, the ignorance and indifference about natural sciences and the dominant emphasis on traditional national studies. It is as if the nation had to pass through that period to see itself as a people among peoples, a nation among nations.

Halldór Elíasson's Views

In the second issue of *Menntamál* 1966, the same issue as published Sveinbjörn Björnsson's report, there is an article by mathematician Halldór Elíasson, where he explained the changes in mathematical thinking and interpretation which had been going on in recent years. Halldór Elíasson had just recently arrived from his doctoral studies in Göttingen, Mainz and Princeton.[703] He explained that changes that began at the universities had started to filter down to the lower secondary schools and primary schools too, in many places abroad. In this country changes had taken place in the high schools, and the textbook, *Numbers and Sets,* by Guðmundur Arnlaugsson was a step in the right direction. Halldór Elíasson emphasized how important it was to introduce as soon as possible the three main concepts of the mathematics: an element, a set and a mapping. He said that the main core of mathematics teaching was to teach how mathematics should be studied, how one should think about mathematics.

Halldór Elíasson then explained the main faults which he saw in the present syllabus and teaching.[704] The faults included: Skills were too highly appreciated as they were bound to deteriorate when people used machines. Proportions in the form of *regula de tri* were an insult to common sense. Mathematical thinking was not nurtured. It was worth absolutely nothing to learn the seen problems in the national examination by heart, and they had negative influence on the teaching methods. What was first and foremost needed was a university department which educated teachers for high schools, teacher training colleges and lower secondary schools.

In this article, Halldór Elíasson brought together many items that were coming into the discussion. Skills vs. understanding, the textbooks, the teaching methods, the *regula de tri*, and the seen problems were brought into debate, followed up by proposals on teacher training and participation of teachers in research. Backed up by his high level of education, his words were taken seriously.

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[702] Sigurður H. Pétursson (1958): 58–59
[703] Halldór Elíasson (1966): 91–99
[704] See Appendix E

At this time Halldór Elíasson had taught for one year at Reykjavík High School, where he was among the first to teach a textbook with "modern" mathematics content, *Principles of Mathematics* (1963) by C. B. Allendoerfer and C. O. Oakley, under the supervision of the head teachers Guðmundur Arnlaugsson and Björn Bjarnason. After that he had pursued advanced research at Princeton. Thus he did not have much actual experience with teaching, but he saw things with a fresh view. He was probably influenced by discussions in the staffroom at Reykjavík High School, where Guðmundur Arnlaugsson and Björn Bjarnason were working on reforms. Halldór Elíasson worked with Hörður Lárusson on an experiment with programmed mathematics learning material in a lower secondary school for a while,[705] but he soon left the scene of lower level teaching and became a university professor.

In the spring of 1966 there were no seen problems in the national examination of the middle school. This was probably announced to the schools no later than in September/October 1965. This may have been the result of the general discussions among the group of mathematicians at the Reykjavík High School, rather than due to the sole influence of the young mathematician Halldór Elíasson. All the same, the article had great impact, for example on Professor Matthías Jónasson's didactics teaching at the University of Iceland, where the author of this book was student.

7.3. Reform Measures in Iceland

Proponents of a Reform

In 1946 there were two high schools in Iceland with mathematics streams, whence about 400 students had graduated since 1919. These were the only people who were qualified to teach algebra for the newly established national examination of the middle schools. Only few went abroad for advanced studies in mathematics, in order to qualify to teach in high school or at a university. In 1947, fifteen persons established the Icelandic Mathematical Society / *Íslenzka stærðfræðafélagið*.[706] By 1959, one of the five mathematicians, Dr. Ólafur Daníelsson, had passed away, and two had been added to the group. The six mathematicians were:

Dr. Leifur Ásgeirsson	Professor, University of Iceland
Sigurkarl Stefánsson	Reykjavík High School and University of Iceland
Guðmundur Arnlaugsson	Reykjavík High School and University of Iceland
Björn Bjarnason	Reykjavík High School and University of Iceland
Jón Hafsteinn Jónsson	Akureyri High School
Skarphéðinn Pálmason	Akureyri High School

All these men except Dr. Leifur Ásgeirsson were educated at the University of Copenhagen. Dr. Leifur Ásgeirsson returned home from Göttingen in Germany immediately after completing his thesis in 1933, to become headmaster at Laugar District School. He became the first university professor in mathematics in 1943. He was 60 years old in 1963, and he was not interested in "modern" mathematics.[707] Sigurkarl Stefánsson was a member of the editorial board of *Nordisk Matematisk Tidskrift* in the 1960s. He knew about the international "modern" mathematics reform wave through that task. He was 61 years old in 1963, he had been teaching for 35 years and did not become involved in the reform activities.

[705] Hörður Lárusson, March 26, 2002
[706] See Appendix H for founding members
[707] Jón Ragnar Stefánsson, July 15, 2003

Jón Hafsteinn Jónsson and Skarphéðinn Pálmason were comparatively young, but situated in Akureyri, away from the cultural mainstream. Guðmundur Arnlaugsson and Björn Bjarnason were half a generation younger than Dr. Leifur Ásgeirsson and Sigurkarl Stefánsson and were in their forties when the "modern" mathematics wave broke out. They had experience of approximately fifteen years of teaching each of them. They had both stayed in Copenhagen during World War II and had many personal contacts within the Danish mathematical community. In 1962–1964 these two men were the most experienced ones capable of knowing what "modern" mathematics was about, and introducing to their countrymen its high promises on both sides of the Atlantic in the early 1960s.

In other countries, mathematicians and educators had come together to discuss the new ideas. In Iceland, the mathematicians were only two, but they were to mobilize others, especially Hörður Lárusson, and also Halldór Elíasson for a short while. Several other young people were influenced by Guðmundur Arnlaugsson and Björn Bjarnason in the early 1960s, but they did not return home until the early 1970s, and did not participate in this project to any marked degree, except Anna Kristjánsdóttir.

The physicists were also influenced by the Woods Hole Conference. They were not many more in number than the mathematicians. Five physicists were founding members of the Icelandic Mathematical Society. Two had passed away, one had moved abroad and two had become professors at the University. None of them were involved in the lower level reforms. In the early 1960s there were five young physicists: Magnús Magnússon, Páll Theodórsson, Örn Helgason, Þórir Ólafsson and Sveinbjörn Björnsson. All of them became professors at the University of Iceland or Iceland University of Education at some point, Magnús Magnússon as early as 1960. The other four were involved in a school physics reform.

Pedagogy professor Dr. Matthías Jónasson was also interested in the reform activities which he discussed with his students. When psychologist Andri Ísaksson arrived home from his studies in France he was encouraged to join the movement.

So, all in all, there were three mathematicians and four physicists, supported by a pedagogue and a psychologist, who supplied the ideology for the reform of the Icelandic school system with special reference to the science subjects, mathematics, physics and chemistry. The powerful Director of Education for Reykjavík, Jónas B. Jónsson, had been on the outlook for reform, and the Minister of Education, Culture and Commerce, Gylfi Þ. Gíslason, was influenced by the OECD theories about education contributing substantially to economic and social progress.

In short: In the mid-1960s, half a score of influential people and a small number of students and teachers in their radius of influences were ready to react to the bad news that Icelandic pupils were behind their contemporaries in other Nordic countries, knowing the good news that a reform movement had risen in the United States, accumulating in Woods Hole, joined to a similar European movement in Royaumont, and that people in the Nordic countries were now busily rearranging their teaching.

A group of eleven persons is not large, and only three of them were mathematicians. Still, comparing to the United States, where the number of inhabitants is 1000 to 1 Icelander, ten thousand people would be a large group. In Woods Hole there were 35 scientists altogether. Even if each of them had 20 influential collaborators, that would only be 700. In Denmark the number of inhabitants is 20 to 1 Icelander. Were there 200 influential Danes involved in the reform movement, or sixty mathematicians? Hardly.

Guðmundur Arnlaugsson

Fig. 7.2. Guðmundur Arnlaugsson.

Guðmundur Arnlaugsson (1913–1996) grew up in Reykjavík, where he attended the Reykjavík High School in 1927–1933. He has told the story of mathematics studies in the school. In the first year there was arithmetic, algebra in the second year and geometry in the third year. The textbooks for all grades were written by Dr. Ólafur Daníelsson. After the third year the school was divided into language and mathematics streams. Dr. Ólafur Daníelsson taught in the mathematics stream, which was Guðmundur Arnlaugsson's choice after a period of hesitation. He has described Dr. Ólafur Daníelsson's teaching, which strongly influenced his choice of career, as follows:

> ... það sem einkum hafði áhrif á okkur var hinn eldlegi áhugi sem hann var gæddur og virðingin fyrir stærðfræðinni. ... frá honum seytlaði smám saman inn í mann virðing fyrir stærðfræðinni sem mikilli og göfugri vísindagrein – ...
>
> ... what mainly influenced us was his fiery interest and his respect for mathematics. ... from him there gradually filtered into one a respect for mathematics as a great and noble science – ...[708]

Guðmundur Arnlaugsson was a talented student, one of the few graduates who received a four-year national scholarship for university studies abroad. He studied mathematics as his major and physics, chemistry and astronomy as minors, at the University of Copenhagen in 1933–1936. Then only one year was left of the scholarship, while three years were left of study. So he went home and taught at Akureyri High School 1936–1939, after which he returned to Copenhagen (via Buenos Aires) in 1939 and completed his studies in 1942.

During World War II when Iceland was completely isolated from Denmark, Guðmundur Arnlaugsson taught mathematics in Copenhagen high schools until the end of the war in 1945. Upon his arrival home, he taught mathematics and physics, first at Akureyri High School and then at Reykjavík High School in 1946–1965, after which he was appointed headmaster of the Hamrahlíð High School in Reykjavík. Guðmundur Arnlaugsson taught mathematical analysis at the University of Iceland in 1947–1967 and worked part-time as mathematics consultant in 1964–1966.[709]

As a young man, Guðmundur Arnlaugsson was a well known chess-player. He competed on behalf of Iceland in the International Chess Olympiad in Buenos Aires in 1939 and became Iceland Chess Champion in 1949. In 1972 he became an international chess arbiter, the first in Iceland.

In 1950 Guðmundur Arnlaugsson wrote together with physicican Benedikt Tómasson an arithmetic textbook for the practical stream of lower secondary schools, which was, however, not much used. He was a member of the National Examination Board from 1948 to 1966, for physics. He wrote articles about school activities and

[708] Guðmundur Arnlaugsson and Sigurður Helgason (1996): 22
[709] Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. I, 178–179; Vol. III, 378

mathematical subjects in newspapers and journals, and a book for the general public about physics: *Hvers vegna? – Vegna þess* / *Why? – Because*. He had a regular radio talk show and wrote regular columns in periodicals about chess.

Guðmundur Arnlaugsson, having lived in Copenhagen for nine years and taught in Danish high schools, knew and was in contact with many Danish mathematicians and mathematics teachers. Early in 1961, an article written by Guðmundur Arnlaugsson was published in *Menntamál*. In this article he explained changes which had recently been made in the Danish high school system and were to be initiated in autumn 1963, recalling that Icelandic high schools originated in the Danish system and that no school system was more similar to the Icelandic one than the Danish.[710]

In the autumn of 1963, Guðmundur Arnlaugsson was granted a sabbatical. He travelled widely in the United States by invitation of some official body, possibly Fulbright.[711] Being free from the daily duties of teaching, he used his time to gather information about school planning and the latest in mathematics teaching. Both his study of the Danish schools and the trip to the U.S. were of great use to him when he was appointed headmaster of the Hamrahlíð High School and could supervise its design right from the beginning. In discussions in the periodical *Samvinnan* in July-August 1967 there is a separate chapter about school buildings with criticism about general lack of planning in school affairs. Then this clause, presumably written by the editor, Sigurður A. Magnússon:

> Ein gleðileg undantekning frá þessari leiðu reglu er Guðmundur Arnlaugsson rektor Menntaskólans við Hamrahlíð, sem veit hvað hann vill, hvernig skólahúsnæði hann vill fá og hvaða starfsemi á að fara þar fram, enda vekur byggingin athygli fyrir einfaldleik og hagkvæmt fyrirkomulag.
>
> One pleasant exception from this regrettable rule is Guðmundur Arnlaugsson, headmaster of the Hamrahlíð High School, who knows what he wants, what kind of a school building he wants to have and what kind of activities will go on there, and therefore the building draws attention for its simplicity and functionality.[712]

In 1964, Guðmundur Arnlaugsson, according to Minister Dr. Gylfi Þ. Gíslason, turned to the minister to offer himself as a consultant in mathematics teaching for the Ministry in a half-time position.[713] At the same time, in 1964, another distinguished teacher, Cand. Mag. Óskar Halldórsson, was engaged to be a consultant in Icelandic for the compulsory schools. The purpose was to "freshen up" these main school subjects, mathematics and Icelandic.[714] Dr. Gylfi Þ. Gíslason told the author of this study in an interview that he had the fullest confidence in Guðmundur Arnlaugsson and gladly appointed him. The confidence, he said, was best witnessed by the fact that the following year he appointed Guðmundur Arnlaugsson as headmaster at Hamrahlíð High School. However, he now had some doubts about the "modern" mathematics implemented by Guðmundur Arnlaugsson.

This is the first evidence of Guðmundur Arnlaugsson's intention to work on mathematics reforms at the state level. Previously, the pupils of Guðmundur Arnlaugsson and his colleague Björn Bjarnason in high school, the author of this book among them, had noticed their interest in "modern" mathematics when they used a

[710] Guðmundur Arnlaugsson (1961): 71–75
[711] Arnlaugur Guðmundsson, March 16, 2003
[712] Sigurður A. Magnússon (1967): 25
[713] Dr. Gylfi Þ. Gíslason, January 17, 2002
[714] Andri Ísaksson, March 10, 2003

new set-theoretical notation in the conventional Danish textbook syllabus. In 1964 they introduced the aforementioned “modern” mathematics textbook, *Principles of Mathematics* in the mathematics stream of the Reykjavík High School.

In 1965, Guðmundur Arnlaugsson published two editions of an algebra textbook for high schools, the latter one extended with an introduction to trigonometry. The 40-year-old algebra textbook from 1927 by Ólafur Daníelsson contained a short section about quadratic equations. Guðmundur Arnlaugsson’s treatment was fuller and more pedagogic. However, later accounts indicate that he was not pleased with it.[715] At this time, no textbooks were available in Icelandic about any mathematical topic above the level of Ólafur Daníelsson’s *Algebra*. Traditionally, Danish textbooks were used, but from this time on, American, English and Swedish textbooks were coming into use in the high schools and gradually more booklets were written in Icelandic or translated.

Björn Bjarnason

Björn Bjarnason (1919–1999) attended Akureyri High School and graduated from there in 1939. He studied mathematics as his major, with physics, chemistry and astronomy as minors, at the University of Copenhagen whence he graduated with a Cand. Mag. degree in 1945. He taught mathematics and physics at Akureyri High School in 1946–1948, Reykjavík High School 1948–1969 and linear algebra at the University of Iceland 1951–1971. Björn Bjarnason was a member of the National Examination Board as the representative for mathematics in 1963–1970. Having known each other from their university years in Copenhagen, and also from the Akureyri High School where Guðmundur Arnlaugsson was Björn Bjarnason’s teacher, they were close collaborators, at the Reykjavík High School, at the University, on the National Examination Board and, as will be seen, in introducing the “modern” mathematics in Iceland.

Fig. 7.3. Björn Bjarnason

Björn Bjarnason did not write any textbooks, but he translated the book *Mathematics* in the series Life Science Library, published by Time Inc., New York, in 1963 and in Icelandic in 1966.[716] In 1970 Björn Bjarnason was appointed headmaster of the third high school in Reykjavík, *Menntaskólinn við Tjörnina,* later *við Sund* (Sund High School), where he remained until his retirement.[717] Björn Bjarnason attended the OECD conference Mathematics To-Day in Athens in 1963, at the same time as Guðmundur Arnlaugsson was travelling in the United States. In early 1964 they were well prepared to lead the reform in Icelandic mathematics teaching.

The Mathematics Consultancy

At the time when Guðmundur Arnlaugsson began his work as mathematics consultant, education was handled by the Office of Educational Affairs under the supervision of the Ministry of Education and Culture, which at that time was very small. Guðmundur Arnlaugsson was the only person working in this field and he must

[715] Ingvar Ásmundsson, March 2003
[716] Bergamini, D. (1966)
[717] Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. I, 76–77, Vol. III, 180

have been expected to organize his work himself. His first task was to investigate the mathematical knowledge of pupils in compulsory education. Secondly, he began to organize in-service courses for teachers. Thirdly he wrote a textbook, *Tölur og mengi* / *Numbers and Sets* for the lower secondary schools, which was published in 1966 and used for the national examination for the next decade. Fourthly he cooperated with the Reykjavík Education Office to select the new Bundgaard set of textbooks for primary school. A detailed report of his work does not exist, but through various evidences one can trace his work to some degree. Unfortunately, a report on his investigation of children's and adolescents' mathematics knowledge has not been found.

In-Service Courses for Lower Secondary School Teachers

The first large course for mathematics and physics teachers at the lower secondary level, led by Guðmundur Arnlaugsson, was held September 2 to 16, 1965 from 9.00 to 16.00 each day.[718] At this time the school year started at the beginning of October. Two hours were allocated for mathematics, two for physics and two for films, slides, experiments and demonstrations. Guðmundur Arnlaugsson and Björn Bjarnason shared the mathematics teaching. The subjects were number notation and number systems, real numbers and their computing, approximation and practical calculations, the slide rule, logic, sets, open sentences, several simple functions and graphs, relations and functions. The basis for the course was partly the textbook *Almene begreber fra logik, mængdelære og algebra* / *General Concepts from Logic, Set Theory and Algebra* by *Danmarks Lærerhøjskole* Professor Bent Christiansen et al. A total of 52 teachers attended the course, the author of this book among them. She recalls that they were generally interested, even though one teacher said that the textbook was the most boring he had ever read. The physics part of the course may have been still more interesting. A new American textbook, *Physics,* from the Physical Science Study Committee was introduced and the Woods Hole conference was mentioned more than once.

Another merit of the course was that teachers in these subjects were working together for two weeks. The fora for teachers to meet were the teachers' unions, which discussed various issues aside from the working conditions of teachers, in one- to two-day study meetings, about general school affairs; professional associations had not yet been established. Many teachers taught more than one subject, so that interest groups for separate subjects had not yet found their place, at least not for mathematics teachers. At this time, in 1965, no new textbooks for the lower secondary level had yet arrived, so the purpose of the course was not to prepare the teachers for any special new courses, but their overall preparation and support.

A similar course was held the following year, 1966. A total of 33 teachers attended it regularly, and 10–15 irregularly. Björn Bjarnason and Guðmundur Arnlaugsson taught the mathematics as before, now with the assistance of Hörður Lárusson. In November that same year, Guðmundur Arnlaugsson is reported to have given a lecture at a two-day study meeting of teachers in southern Iceland.[719] Guðmundur Arnlaugsson´s textbook, *Tölur og mengi / Numbers and Sets*, had now been published, and was already to be taught in several national examination classes.

In 1967, Guðmundur Arnlaugsson had become headmaster in Hamrahlíð High School, and was no longer mathematics consultant. That autumn he gave lectures at

718 *Menntamál* (1965): 38 (3) 320–321
719 *Menntamál* (1966): 39 (3) 300–301

in-service courses, which were now held mainly for teachers at the primary level, where a new textbook series had been taken up. The teachers' union in Mid-West Iceland held a study meeting where Björn Bjarnason gave a lecture about numbers and sets. Probably his talk was based on Guðmundur Arnlaugsson's new textbook. At the same meeting the teachers concluded that there was a need for a special consultancy in mathematics and physics. Furthermore there was a need to speed up reorganizing of teaching and textbook writing for the above-mentioned subjects, and allocate more time for them in the standardized timetable.[720]

The Textbook *Tölur og Mengi / Numbers and Sets*

Guðmundur Arnlaugsson's experiences from his travels in 1963, as mathematics consultant in 1964–66, and from his investigation of children's and adolescents' mathematics knowledge, in addition to the general debate, probably led him to write his book, *Tölur og mengi / Numbers and Sets*, published in 1966. The content of this textbook was new to Icelandic teachers, even though the number part of it was contained in Björn Gunnlaugsson's book, *Tölvísi* (1864), published a century earlier, and Ólafur Daníelsson's textbooks also touched on the matter. That part of *Numbers and Sets* was partly a translation of a book by W.W. Sawyer, *Vision in Elementary Mathematics* (1964).[721] It covers divisibility and prime numbers, patterns, square numbers, divisibility by nine, the decimal place-value system, the binary, octal and hexadecimal systems. The second part of *Numbers and Sets* contains an introduction to set theory, the union, intersection and set difference, the use of propositions, one-to-one correspondence, scientific notation, and some counting problems of the inclusion-exclusion type.

The book's style was lucid, and it was written in good language, printed in a handy small format: a mathematics textbook that teachers could urge their pupils to read for themselves.

Guðmundur Arnlaugsson wrote in the foreword to his *Number and Sets*:

> Áhersla á leikni og vélrænum vinnubrögðum hefur þokað fyrir kröfum um aukinn skilning. Þessi þróun hefur ýtt nokkrum grundvallarhugtökum úr rökfræði, mengjafræði og algebru niður á barnaskólastig. Reynsla víðs vegar að bendir til þess, að börn – og það jafnvel á unga aldri – eigi tiltölulega auðvelt með að tileinka sér þessi hugtök, sem áður voru eigi kynnt fyrr en á háskólastigi, og hafi gaman af þeim. Enn fremur virðast þau stuðla að auknum skýrleik og nákvæmni í hugsun og reikningi.
>
> The emphasis on skills and mechanical ways of work has moved aside for demands for increased understanding. This development has pushed several basic concepts from logic, set theory and algebra down to primary level. The experience from many places indicates that children, even very young children – can easily adopt these concepts, which previously were only introduced at university level, and enjoy them. Furthermore, they seem to be conducive to increased clarity and exactness in thinking and arithmetic.[722]

Here Guðmundur Arnlaugsson suggests that the basic concepts of logic and set theory would facilitate understanding, even for small children. These words indicate that he was aware of theories by the Swiss psychologist Jean Piaget, on which the idea of implementing abstract algebra into school mathematics was based. Piaget wrote in

[720] *Menntamál* (1967): 40 (3) 253–254
[721] Sawyer, W.W. (1964)
[722] Guðmundur Arnlaugsson (1966): 4–5

his “Comments on mathematical education” (in Howson, A.G. (ed.) (1973): *Developments in Mathematical Education.* Cambridge, Cambridge Univ. Press), that:

> … having established the continuity between the spontaneous actions of the child and his reflexive thought, it can be seen from this that the essential notions which characterize modern mathematics are much closer to the structures of "natural" thought than are the concepts used in traditional mathematics.[723]

This quotation explains the expectations people had of “modern” mathematics in the 1960s. The structures of “natural” thought were closer to the essential notions of “modern” mathematics than to the concepts used in traditional mathematics. The news had spread that children could easily adopt the “modern” concepts, and even enjoy them. Guðmundur Arnlaugsson’s foreword is dated in 1966 when he was mathematics teaching consultant. In April-May 1966 he advised that the set-theoretical syllabus be adopted for the primary level, which agrees with the ideas put forward in his foreword.

Guðmundur Arnlaugsson’s Views of Mathematics Teaching

The visit to the American schools was a revelation to Guðmundur Arnlaugsson,[724] and it is clear that after his sabbatical in 1963 he launched plans to influence mathematical teaching in Iceland on all levels, together with his colleague and friend, Björn Bjarnason. Guðmundur Arnlaugsson certainly wrote *Numbers and Sets* with the purpose in mind of altering the syllabus for the national examination.

Guðmundur Arnlaugsson’s and Björn Bjarnason’s discussions are not documented. However, they are probably reflected in Björn Bjarnason’s curriculum paper in 1968 (see section 7.5.) and in Guðmundur Arnlaugsson’s article in 1967, “New Views in Arithmetic Teaching”,[725] in *Menntamál*, the main forum for educational discussions. Guðmundur Arnlaugsson also wrote a chapter in Dr. Matthías Jónasson’s book, *Nám og kennsla* / *Learning and Teaching*, published in 1971, where he expressed his views of learning and teaching mathematics. Several excerpts will be picked out from these two articles to learn about Guðmundur Arnlaugsson’s views of teaching. In the article in *Menntamál*, Guðmundur Arnlaugsson says:

> Mörgum hefur verið ljóst að reiknings- og stærðfræðikennslu er verulega ábótavant hjá okkur, og skýr samanburður Sveinbjörns Björnssonar ... á námsefni í þessum greinum hérlendis og í grannlöndum okkar sýnir, að ástandið er okkur enn óhagstæðara en flesta hafði grunað. ... Gleðiefni er það hve margir kennarar hafa sýnt þessu máli áhuga. ...
>
> Aðalgalli reiknings- og stærðfræðikennslu hérlendis hygg ég hafi verið sá að hún hefur verið of vélræn, beinzt um of að vissri tegund leikni, en ekki lögð nóg áherzla á yfirsýn og skilning. Aðferðir sem ekki hafa nægan skilning að bakhjarli gleymast fljótt aftur og koma að litlu haldi.
>
> It has been clear to many that our arithmetic and mathematics teaching has been considerably defective and the clear comparison by Sveinbjörn Björnsson … of the syllabi in these subjects in this country and in our neighbouring countries ... shows that the situation is still more unfavourable to us than most people had suspected. ... It is pleasing to notice how many teachers have shown an interest in this matter. …
>
> The main drawback in arithmetic and mathematics teaching in this country I think has been that it has been too mechanical, too much aimed at a certain kind of skills

723 Gjone, G. (1983): Vol. II, 54
724 Þorsteinn Þorsteinsson, March 27, 2003
725 Guðmundur Arnlaugsson (1967): 40–51

> and not much enough emphasis on overview and understanding. Procedures which are not backed up by enough understanding are quickly forgotten and of little use.[726]

Guðmundur Arnlaugsson listed several disadvantages of the arithmetic teaching, the disjunction between the various topics, the synthetic applications, the lack of respect for the aesthetic value of mathematics and the dominance of the examinations:

> Mér virðist … að námsefnið hafi verið um of hólfað sundur í rækilega aðgreinda flokka. Nemendum hefur verið kennd sérstök aðferð á hvern flokk, en samhengið milli flokka hefur orðið ósýnilegt með öllu, þótt oft sé um að ræða náskyld efni, skógurinn hefur ekki sést fyrir einstökum trjám. ... sjónarmið sem talin hafa verið hagnýt hafa ráðið of miklu. Reikningur virðist einkum kenndur til þess að menn láti ekki hlunnfara sig í viðskiptum, geti mælt sér út kálgarðsskika ef á þarf að halda ... en ekki vegna þess að hann hafi gildi í sjálfum sér.

> It seems to me … that the curriculum has been too much partitioned into definitely separated groups. The pupils have been taught a separate method for each group, while the connection between different groups has been totally invisible, even if often the subjects are closely related; the wood has not been seen for the trees. … views, which have been considered practical have been too dominating. Arithmetic seems to have mainly been taught for the purpose that people will not let themselves be cheated in business, can measure their own vegetable garden if needed … and not because it has its own value.[727]

These remarks remind the reader of those of his mentor, Dr. Ólafur Daníelsson, that future intellectuals could be rid of great adversity by having agronomists measure their vegetable gardens.

> ... mönnum hefur að mjög verulegu leyti sést yfir innra gildi stærðfræðinnar. Tækifæri til að kynna börnum og unglingum þá fegurð sem býr í stærðfræðilegri hugsun, jafnvel þegar hún fjallar um einföldustu frumatriði, hafa verið vanrækt. Börnin hafa stundum verið þreytt á síendurteknum fangbrögðum af sama tagi við lífvana tölur í stað þess að eignast nýja og góða kunningja í lifandi tölum, kynnast sérkennum einstakra talna, sjá hvað liggur að baki þeim reikningsaðferðum, er þeir læra. Prófin eiga þarna sjálfsagt nokkra sök. Í stað þess að kenna „fyrir lífið" hefur áherzlan verið lögð á hluti sem líta vel út á prófi.

> … people have largely overlooked the inner value of mathematics. Opportunities to introduce to pupils the beauty hidden in mathematical thinking, even when it concerns the simplest basic things, have been neglected. Children have sometimes been tired out with endless repeated struggles with lifeless numbers, instead of making new and good friends of living numbers, learning to know the characteristics of individual numbers, seeing what lies behind the computation methods that they are learning. The examinations are probably partly to blame. Instead of learning "for life", the emphasis has been placed on things that look good in an examination.[728]

Guðmundur Arnlaugsson was concerned about this attitude:

> Þetta viðhorf [að kenna fyrir próf] hefur ráðið full miklu og þess vegna er ég hræddur um, að sumir átti sig ef til vill ekki nógu rækilega á því nýja, sem nú er á döfinni, sjái þar aðeins eina aðferðina enn til viðbótar við allar þær gömlu.

> This attitude [teaching for the tests] has been too predominant, and therefore I am afraid that some people perhaps do not understand well enough the new things coming up, and only see one more method, in addition to all the old ones.[729]

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[726] Guðmundur Arnlaugsson (1967): 40–42
[727] Guðmundur Arnlaugsson (1967): 42
[728] Guðmundur Arnlaugsson (1967): 42–43
[729] Guðmundur Arnlaugsson (1967): 43

It is likely that many teachers enjoyed teaching for the national examination the inclusion-exclusion counting examples that Guðmundur Arnlaugsson introduced in connection with the set algebra in his book, *Numbers and Sets.* Otherwise they may not have seen the point of the introduction to the new concepts. Guðmundur Arnlaugsson worried about the teachers and their preparation:

> Margir kennarar á barna- og unglingastigi hafa aldrei á námsferli sínum kynnzt stærðfræðilegri hugsun ... og á íslenzku er tilfinnanlegur skortur á bókum er geta bætt úr þessu, svo að hér er um alvarlegt vandamál að ræða.
>
> Many teachers in the primary and lower secondary schools have never in their studies met mathematical thinking ... and there is a severe shortage of books in Icelandic that can improve this situation, so this is a serious problem.[730]

In Guðmundur Arnlaugsson's view, the purpose of teaching mathematics concerned the basis of logical thinking, and hence its value extended far beyond ordinary computing. The teachers had to be trained for that purpose:

> Hlutverk reikningskennslunnar ætti að vera að verulegu leyti frá upphafi að kenna barninu að hugsa. … reikningurinn og stærðfræðin mega með engu móti verða viðskila við aðra rökrétta hugsun, þau eiga einmitt að vera tæki til þess að þjálfa barnið í rökréttri hugsun. Ef kennaranum er þetta ljóst og hann hefur yfirsýn yfir samhengið í þeim greinum reiknings sem hann er að kenna, getur hann áreiðanlega náð betri árangri en margir gera nú, og það jafnvel þótt um litlar breytingar á sjálfu námsefninu sé að ræða.
>
> The role of arithmetic teaching should be to a considerable degree from the start to teach the child to think. … the arithmetic and the mathematics must not part from another kind of logical thinking, they should precisely be the tool to train the child in logical thinking. If this is clear to the teacher, and he/she has the overview of the coherence of the topics of arithmetic that he/she is teaching, he/she could doubtless achieve a better result than many do now, even if there were few actual changes in the syllabus.[731]

And he stressed the shift of aims into studying mathematical structure:

> Hinu er svo ekki að leyna að þær breytingar sem nú eru á döfinni hagga að ýmsu leyti sjálfum grundvelli reikningskennslunnar. Reikningur fjallaði áður fyrr nokkurn veginn einvörðungu um tölur, en nú beinist reikningskennslan í æ ríkara mæli að *strúktúr* stærðfræðinnar … tölurnar eru að vísu mikilvægar enn, því að talnakerfi eru forvitnileg og skemmtileg á margan hátt, en tölurnar eru ekki lengur hið eina sem um er fjallað.
>
> One still cannot hide the fact that the changes that are now going on, shift in some ways the very basis of arithmetic teaching. Previously, arithmetic was mainly concerned with numbers, but now arithmetic teaching is increasingly focussed on the *structure* of mathematics … the numbers are surely still important, as number systems are intriguing and interesting in many ways, but numbers are not the only things that are now discussed.[732]

In this article one can notice a certain warning against the plain "dressing up" of the syllabus:

> Ef þessar breytingar sem nú eru á döfinni eiga að verða nokkuð að ráði meira en nafnið tómt, þarf dýpri viðhorfsbreytingu en þá sem felst í uppdubbun námsefnis, þótt góð sé í sjálfu sér. Námsefni okkar í reikningi hefur staðið nokkurn veginn í stað um langt skeið og er á margan hátt orðið úrelt, svo að breytingar á því eru

[730] Guðmundur Arnlaugsson (1967): 43
[731] Guðmundur Arnlaugsson (1967): 43–44
[732] Guðmundur Arnlaugsson (1967): 44

væntanlega til bóta. En ef menn hugsa sér þetta sem eina breytingu nú og síðan ekki söguna meir, er stöðnunin komin aftur og þar með er hnignunin vís á ný.

If these changes, which are now being introduced, are supposed to be in anything more than name, a deeper change in attitude is needed than implied by an update of a syllabus, even if that is good in itself. Our arithmetic syllabus has been at a standstill for a long period and is in many respects out of date, so a change is presumably an improvement. However, if people think of this as a one-time change and then never again, the stagnation is back, and thereby the decline is sure to resume.[733]

Guðmundur Arnlaugsson put a great emphasis on a change of attitude to mathematics itself. However, *understanding* was most important:

Menn mega ekki einblína um of á ný orð og hugtök. Gildi stærðfræðikennslunnar fer ekki einvörðungu eftir því hvort það er nýtt efni eða gamalt, sem er á boðstólum. Það fer fyrst og fremst eftir því, að hve miklu leyti *skilningurinn* er með í förinni. Gildi hinna nýju hugtaka er framar öðru fólgið í því að auðvelda skilning á atriðum sem áður hafa oft verið óljós og legið í láginni.

One should not focus too much on new words and concepts. The value of mathematics teaching is not solely a matter of whether the syllabus offered is new or old. It depends first and foremost on to what degree *understanding* follows. The value of the new concepts lies more than anything in facilitating the understanding of matters that previously have been unclear and hidden. [734]

Here Guðmundur Arnlaugsson echoes the theories that the unifying concepts would facilitate understanding.

Guðmundur Arnlaugsson's remarks, that the examinations are to blame, deserve attention. He had been a member of the National Examination Board since its establishment. The board was autonomous in its choice of syllabus and form of examination. However, it did not see it as its task to take any initiative in suggesting changes to the examination system. Also, the examination existed anyway, and for example the only opportunity to teach Guðmundur Arnlaugsson's examination subject, physics, was to teach it under the pressure of a national examination. Later, when there was an option to teach either biology or physics for the later compulsory-school national examination, physics more or less faded away in compulsory school education.

In Guðmundur Arnlaugsson's article in 1971 he mentioned Jerome Bruner's theory of the spiral approach, which was widely discussed and used in experimental textbook writing, which had been initiated within the School Research Department. By the end of his article, Guðmundur Arnlaugsson summarised his main views in six items:[735]

1. Frá fornu fari hefur áhugi Íslendinga einkum beinzt að bókmenntum, skáldskap og þjóðlegum fróðleik af ýmsu tagi. Af þessum sökum njóta stærðfræði og raunvísindi lítils stuðnings í umhverfi nemandans.
2. Stærðfræðin er vandasöm í kennslu og gerir því miklar kröfur til kennarans. Allt of mörg dæmi eru til um nemendur, sem hafa gefizt upp við námið og misst alla trú á að þeir gætu lært stærðfræði. Því er mjög mikilvægt að kennslan efli sjálfstraust nemandans og glæði áhuga hans á viðfangsefninu.
3. Einn regin-misskilningur í kennslu er sá að auðveldara sé að læra utanað heldur en komast til skilnings. Þessu er öfugt farið, en vitaskuld þarf að nálgast viðfangsefnið á þann hátt er hæfir þroska nemandans.

[733] Guðmundur Arnlaugsson (1967): 41
[734] Guðmundur Arnlaugsson (1967): 41–42
[735] Guðmundur Arnlaugsson (1971): 317–318

4. *Form* kennslunnar er mikilvægara en *innihald* hennar. Það skiptir meira máli, *hvernig* kennt er, heldur en *hvað* kennt er.
5. Að því er *innihald* varðar, ætti frekar að stefna að *dýpt og samfellu* í kennslunni, heldur en því að komast yfir einhvern tiltekinn fjölda þekkingaratriða.
6. Að því er til *formsins* tekur skiptir meginmáli að *virkja* nemandann, vekja áhuga hans og fá honum jafnframt viðfangsefni til að glíma við, leyfa honum, eftir því sem unnt er, að fara sínar eigin götur, reyna að örva hann til þess að uppgötva stærðfræðileg sannindi. Sannindi sem maður uppgötvar sjálfur (eða fær leiðsögn til að uppgötva) koma að innan og verða manni ósjálfrátt hugstæðari en þau, sem koma að utan.

Í heild má segja: Kennsla þarf að vera opin, þannig að utan þess svæðis, þar sem allir vinna saman og full birta fellur á, sé dálítill jaðar í hálfskugga, handa duglegustu nemendunum til að hætta sér út á.

Kennslan þarf einnig að vera opin á þann hátt, að komið sé oftar en einu sinni að sama viðfangsefninu, og þá á hærra stigi og fyllri hátt í hvert sinn.

1. Traditionally, Icelanders have mainly studied literature, poetry and folk knowledge of various kinds. Due to this, mathematics and science have little support in the pupil's environment.
2. The mathematics is a difficult teaching subject and it imposes great demand on the teacher. There are too many cases of pupils, who have given up their studies and lost all belief that they could learn mathematics. Therefore, it is very important that the teaching supports the pupil's self-confidence and awakens his/her interest.
3. It is a widespread misunderstanding that it is easier to learn by rote than to reach understanding. The opposite is true, while of course the topic needs to be approached appropriately to the pupil's maturity.
4. The *structure* of the teaching is more important than its *content*. It is more important *how* one teaches than *what* is taught.
5. Concerning the *content*, one should rather aim at *depth and continuity* in the teaching than to cover a certain number of items of knowledge.
6. Concerning the *structure* of the teaching the most important thing is to *activate* the pupil, awake his/her interest, let him/her find his/her own ways, try to stimulate him/her to discover mathematical truth. The truth that one discovers on one's own (or is guided to discover) comes from within and is automatically more memorable than that which comes from outside.

In general one can say: Teaching needs to be open, in such a way that outside the area where all are working together in full light, there should be a little margin in half-shade, for the most able pupils to venture into.

The teaching should also be open in the sense that one should return more than once to the same topic, and then at a higher level and fuller mode each time.[736]

In the first item one notices views similar to Sigurður H. Pétursson's article from 1958 and in the last words one notices reference to Bruner's theories about the spiral curriculum. Items 2 and 6, to support the pupil's confidence and to activate him/her, are basic principles in all education, but it may have been a widespread opinion that they did not apply to mathematics. The reason might be the short tradition for mathematics, indicated in item 1, and the fact that many people, who themselves did not have self-confidence regarding mathematics, had to teach it. Therefore there may have been too much emphasis on rote memorizing, indicated in item 3. Items 4 and 5 concern the basic curriculum itself, especially item 5, which individual teachers could not change, in the context of standardized examinations in the lower secondary schools and the requirements of the university at the upper secondary level.

[736] Guðmundur Arnlaugsson (1971): 317–318

Mathematics education was not a subject taught at this time, neither at the Teacher Training College nor at the University of Iceland. There were no books available in Icelandic on the subject either. Therefore Guðmundur Arnlaugsson's articles, which most certainly reflect his talks at courses and study meetings, were helpful to promote discussions and reflections among mathematics teachers.

Analysing Guðmundur Arnlaugsson's views on the basis of these articles, and of some personal acquaintance, he was to some degree influenced by the "modern" mathematics wave's ideas of a unifying structure, having noticed through the years that pupils did not see connections between topics in which he saw a strong relation. However, what seems to have been his strongest concern was to bring the pupil to appreciate and respect mathematics and its aesthetic values, to stimulate children and youngsters to active thinking, and appeal to their curiosity and interest, e.g. for the beauty implied in the simplicity of structures and patterns. Guðmundur Arnlaugsson knew a wealth of ancient and recent problems and theories, which he enjoyed presenting to pupils, long after his retirement. The same applied to his chess activities. People who knew only the basic moves in chess enjoyed his explanations on television, where he pointed out the elegance, brilliance and simplicity of specific moves in great international chess tournaments.

During winter 1966–1967 Guðmundur Arnlaugsson gave a weekly show on "modern" mathematics on television. The aim was to reach parents with the new concepts taught in the new programme. These shows were probably not as successful as his chess shows. Apparently many people did not understand him at all.

A Reading List for Mathematics Teachers

An important part of Guðmundur Arnlaugsson's mathematics teaching consultancy in 1964–66 was the introduction of foreign books for teachers to read. In his 1971 article Guðmundur Arnlaugsson listed 28 books, which he recommended as reasonably easy reading for mathematics teachers, admitting that the choice was coloured by his own knowledge and taste. There were only two books in Icelandic, Whitehead's *Introduction to Mathematics,* translated in 1932, and *Mathematics* by D. Bergamini, translated in 1966 by Björn Bjarnason.[737]

This meagre list describes the situation. Two books on mathematics for the general public had been translated at 30 years' interval, and no theoretical books or textbooks for teacher training or further education were available.

Icelandic teachers in the 1960s did not travel much abroad and did not have extensive training, and thus especially mathematical literature in English was hard for them. Six books written in Nordic languages were counted, among them the two books by Bent Christiansen mentioned earlier, used at in-service courses and at the University, and another three were translated into Danish or Norwegian, two of them recently in connection with the "modern" mathematics wave.

Amongst the books in English were several classics, such as Courant and Robbins' *What is mathematics?* (1941), Morris Kline's *Mathematics in the Western Culture* and George Polya's *How to solve it* (1945). In addition there were four popular books by W.W. Sawyer, first published in 1943, 1955 and 1964, the 1957, 1964 and 1969 yearbooks of NCTM in the USA and *Basic Concepts of Elementary Mathematics* (1960, 1965) by W.W. Schaaf, which Guðmundur Arnlaugsson and Björn Bjarnason

[737] See Appendix I

recommended for the Teacher Training College when it offered a mathematics course as an elective in 1967. In addition, the list gives Guðmundur Arnlaugsson's sources on general pedagogy, which include two books by Jerome Bruner, one of them *The Process of Education*, which had great influence, when the time came to create home-made textbooks.

So, all in all, there were two books in Icelandic, six books in Nordic languages, nineteen in English and one in German. In order to understand the value of this list, one has to keep in mind that someone had to have the initiative to ask the bookstores to order these books. No bookstore would do that on its own or without a certain clientele. Many teachers bought these books to broaden their horizons. The author of this book bought at least ten of them then, and more later.

The Decision Process for Primary Level Reform

When Kristján Sigtryggsson was appointed headmaster of a primary school in Reykjavík in 1965, Kristinn Gíslason (1917–) took over his work as an inspector for mathematics teaching in Reykjavík. He has written an account of the events that lead to the implementation of "modern" mathematics in the primary schools.[738]

Initially Kristinn Gíslason was expected to assist mathematics teachers in Reykjavík primary schools, especially regarding slow learners. In addition, Director of Education Jónas B. Jónsson asked him to look through the material that had been ordered after Kristján Sigtryggsson's study visit in 1963–1964. Kristinn Gíslason recognized the same principles in this material as he had become acquainted with on the teachers' in-service course with Guðmundur Arnlaugsson in the autumn of 1965. The problems with the textbook samples were that most of them were not complete sets, but parts from here and there, and it was difficult to create a coherent picture.

Jónas B. Jónsson, Kristján Sigtryggsson and Kristinn Gíslason discussed this at many meetings. In the spring semester 1966 there were several meetings with Andri Ísaksson, the newly-appointed consultant on school research at the Ministry of Education, Guðmundur Arnlaugsson, still part-time mathematics teaching consultant at the Ministry, and Björn Bjarnason. The topic of these meetings was whether it was desirable and timely to work for changes in mathematics teaching in Icelandic primary schools by turning it towards "modern" mathematics. There were no doubts or opposition to this, and an early agreement emerged to experiment with few pupils and a chosen group of teachers. The problems to solve were: firstly, to find suitable learning material; secondly, teachers were generally not acquainted with the ideas of "modern" mathematics; and thirdly, it seemed important to ensure as far as possible a positive attitude of the parents of pupils participating in the experiment.

For the learning material there were two choices: either to use foreign material or to compose it in Icelandic. The latter choice was rejected, as neither knowledge nor experience was present in Iceland at this time. The American material was only in scattered parts and for a while the possibility was discussed of ordering more of the SMSG material. In April or early May 1966, Guðmundur Arnlaugsson reported that he had heard of experimental teaching with new material in Denmark, where the author was a teacher named Agnete Bundgaard in Frederiksberg, Copenhagen.

Where did Guðmundur Arnlaugsson learn about the Danish material? From his years in Copenhagen he knew Prof. Svend Bundgaard at Aarhus University, one of

[738] Kristinn Gíslason (November 1978): An unprinted report in the author's possession

the guest speakers at the Royaumont Seminar and one of the prime proponents of "modern" mathematics in Denmark. Prof. Svend Bundgaard was Agnete Bundgaard's brother. At some point Prof. Bundgaard informed Guðmundur Arnlaugsson about his sister's experiment, and Guðmundur Arnlaugsson passed this information to the Reykjavík Education Office.

It was decided that Kristinn Gíslason would visit the school in Frederiksberg, where he stayed for ten days. He attended Agnete Bundgaard's classes and had discussions with her and other teachers. Jónas B. Jónsson must have been in contact with the NKMM cooperation. After Kristinn Gíslason's visit to Agnete Bundgaard he went to Hässelby-palace in Sweden where Matts Håstad, NKMM's secretary, gave a lecture on proposed innovations in mathematics teaching and an experiment with new material in Sweden. Kristján Sigtryggsson was also present there. This was a year before the NKMM completed its work. Kristinn Gíslason says, however, that he learned only later that Agnete Bundgaard was one of the Danish representatives in the NKMM.[739]

Kristinn Gíslason brought home Bundgaard's textbooks. All of them were in experimental form. The first one was being prepared for a final edition after three years of experiments, the second one had been tested for two years and the third one had had one year. Kristinn Gíslason discussed the material with Kristján Sigtryggsson. Their conclusion was to propose to Jónas B. Jónsson that this set of textbooks be tried in the next school year with a limited number of pupils. The reasons for this choice were that the basis for the material was "modern" mathematics, and that the material was easily available, as Agnete Bundgaard most graciously gave her permission and allowed the use of her stencils. No translations were needed, except for the teachers' handbook, which Kristinn Gíslason translated quickly. Jónas B. Jónsson acquired the necessary permission from the Ministry of Education.

Reform Experiment

The experiment took place in two schools, a total of seven first grade groups. Kristján Sigtryggsson taught one of them in his school, *Hvassaleitisskóli*, and Kristinn Gíslason another in his school, *Laugarnesskóli*. There were weekly meetings of the teachers, where progress was discussed and Kristinn Gíslason and Kristján Sigtryggsson explained the basic ideas. After the first school year, the teachers were pleased and did not wish to return to the old material.

Kristinn Gíslason and Kristján Sigtryggsson then suggested that the selected groups would continue with the experiment, and new groups come in. The appropriate authorities approved of this suggestion. When it came to deciding how many would participate in the continuing experiment, the choice was between selecting some teachers and offering the opportunity of participation to those who volunteered. The latter option was chosen. In May 1967 this was presented to the headmasters in Reykjavík primary schools at their meeting, with the intention that they would present the project to their teachers. When teachers had registered for the autumn's teacher in-service courses, they turned out to number 86.[740] Not all of them were too keen, but had registered at the urging of their headmasters. By this time it became clear that the majority of Reykjavík 7-year-old pupils would study the new material. This numerous attendance made the execution of the project much more difficult than in the first

[739] Kristinn Gíslason, March 3, 2003
[740] *Menntamál* (1968): 41(1) 95

year, but according to Kristinn Gíslason, it was pleasing how many teachers achieved a good grasp on the task. As some teachers had not participated on their own initiative, but been ordered to by their headmasters, they were not all equally happy with the increased work involved in the project.[741]

From 1967 onwards, large in-service courses were held for primary teachers every year until 1971. More and more teachers attended the courses. In 1968 they were in two groups for the two year-groups, the next year in three and so on. In 1968 only 59 came back, out of the 86 attending the course in the previous year. Out of the 62 women, 44 came back, and out of 24 men, 15 came back.[742] That same year 75 teachers, of whom 56 were women, attended a course for those who had not taught this syllabus before. In the first course, Kristinn Gíslason was the organizer and Guðmundur Arnlaugsson and Björn Bjarnason taught the mathematics part. Agnete Bundgaard herself came up to Iceland to teach how to work with the new material, and an effort was made to intertwine these two factors, content and methods. In the following years new instructors came in: Karen Plum came with Agnete Bundgaard from Frederiksberg, and Anna Kristjánsdóttir, Ingvar Ásmundsson, Kristján Sigtryggsson, Hörður Lárusson, Ragnhildur Bjarnadóttir and more were added to the group. Meetings with parents were held in 1966 and 1967, but were not continued.

7.4. School Research and Public Discussion

A School Research Specialist

Andri Ísaksson, a young psychologist, was appointed a specialist in school research at the Ministry of Education in July 1966. At the same time two advisers were appointed.[743] One was Dr. Wolfgang Edelstein, a German specialist in school research at the Max Planck-Institut for Bildungsforschung in Berlin, who grew up in Iceland during World War II. The other was Headmaster Jóhann Hannesson at Laugarvatn High School, who had studied linguistics at Berkeley and been librarian and university teacher at Cornell, Ithaca, NY in the United States in 1950–1960.[744]

In the Budget Bill presented to *Alþingi* in the autumn of 1966, 800,000 *krónur* were allocated for the item "Scientific Research of the School System". In the notes to the Bill this clause is found:

> Upp er tekinn nýr liður, kostnaður vegna *fræðilegrar rannsóknar á skólakerfinu*. Hér er um að ræða rannsókn þá, sem hafin var að tilhlutan menntamálaráðherra á þessu ári, að nokkru í samvinnu við Efnahags- og framfarastofnunina í París.
>
> A new item has been introduced, expenses of *scholarly research on the school system*. This refers to the research that was initiated on the initiative of the Minister of Education in this year, partly in cooperation with the Organisation for Economic Cooperation and Development in Paris.[745]

The influence of OECD on this educational research project is obvious. Initially the activities of the school research specialist were mainly interviews with educators, to learn about their view of the drawbacks of the school system in Iceland and the most urgent tasks. Soon the activities turned to revising curricula, teaching material

[741] Kristinn Gíslason (November 1978): An unprinted report
[742] *Menntamál* 41(3): 272–273
[743] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 7
[744] Ólafur Þ. Kristjánsson et al. (1958–1988): Vol. I, 336
[745] *Alþingistíðindi* A 1966: 99

and methods of instruction. The main reasons were, according to the interviews, that many teachers were discontented with old and outdated teaching material and the State Textbook Imprint offered little new material. This project was considered promising for effecting quick changes and desirable development in school activities.[746]

Meeting Arranged by Teachers in Reykjavík

In April 1967 a meeting was arranged by the Union of Lower Secondary School Teachers in Reykjavík.[747] Director of Education in Reykjavík Jónas B. Jónsson, State Director of Educational Affairs Helgi Elíasson, and Andri Ísaksson, the school research specialist, were invited. The matter for debate was whether the school building policy of the educational authorities in Reykjavík was desirable and in accordance with education legislation.

Mathematics teacher Haraldur Steinþórsson initiated the discussion and recalled that the 1946 education legislation said that lower secondary schools should be separate institutions and operate in their own building. At present, only four lower secondary schools in Reykjavík were operating according to this law. Others had second to fourth year, third and fourth year or only the third year for the national examination. Only about half the number of second-year pupils were attending a full four-year school. Keeping pupils for only one or two years in each school was more similar to running courses than school activities.

Director Jónas B. Jónsson responded that there was no special policy on this matter, but that he was not sure that it was that bad to run a one-year national examination school, and that in Reykjavík this was an inevitable necessity. In the coming years many changes must be expected and those could be implemented within the existing buildings. (Possibly the idea of a nine-year school had come into consideration at this time. The preparation for new compulsory school legislation began in 1969)[748]. On the other hand Jónas B. Jónsson stated that at least five high schools were needed in Reykjavík. At that time there were two high schools and the Commercial School of Iceland, *Verzlunarskóli Íslands*. The number rose to four in 1970, and by 1975 there were seven upper secondary level schools in the capital area.

Andri Ísaksson emphasized that a condition for progress in educational affairs was that a plan be made on teacher training in accordance with the goals of the school activities, and furthermore that educational affairs had to be given priority by the authorities instead of being marginalized. As we shall see, Andri Ísaksson's words were soon to have weight. The coming decade was probably the period when educational affairs were least marginalized in Iceland.

Discussions in 1967–1968

Through 1967 to the end of 1968, when the periodical *Frjáls þjóð* ceased to be published, more intellectuals wrote in it about education, on such subjects as teaching modern literature, the situation of the University and its lack of funds, and the huge dropout rate of university students. The following are excerpts from their notes:

[746] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 7
[747] Hörður Bergmann (1967): *Frjáls þjóð*, April 6
[748] Kristín Indriðadóttir (2004): 20

- The schools were urged to allow for some democracy and personal freedom. In other fields of society people have mangaged to keep discipline within such a frame and the schools will have to adjust to that too (Nov. 67).
- In May 1968 Gunnar Karlsson wrote:

> Fyrir skömmu drap ég hér í blaðinu á sérfræðingaskort dreifbýlisins og rakti hann til námsaffalla á háskólastigi. Ég gizkaði á, að núverandi árlegur fjöldi stúdenta gæti nægt okkur sæmilega, ef hann nýttist að miklu leyti sem efni í háskólamenntaða sérfræðinga. … En ef við viðurkennum, að stúdentar eigi aðeins að litlu leyti að vera sérfræðingaefni, eins og nú er í raun, þá þurfum við á miklu fleiri stúdentum að halda. Þá þarf að vinna stórátak á menntaskólastiginu. … Framsýnir menn og stórhuga eru þegar teknir að spá því, að hér á landi verði tekinn upp nokkurs konar menntaskóli fyrir alla. … sá félagslegi aðskilnaður „alþýðu" og „menntamanna", sem hér hefur verið að myndast síðustu áratugi, gæti horfið aftur. … Hér hefur raunar alltaf verið lögð meiri áherzla á að takmarka fjölda menntaskólanema en að laða fólk þangað. Sú skoðun hefur lengi verið talsvert útbreidd, að alltof margir legðu leið sína inn í menntaskólana.
>
> Some time ago I mentioned here in these pages the shortage of specialists in the rural areas and blamed it on the drop-out rate at the university level. I guessed that the present yearly number of high school graduates would suffice if the majority became university educated specialists. ... if we acknowledge that only a small part of the high school graduates are to become specialists, as is now the case, then we shall need many more high school graduates. Then a great effort has to be made at the high school level. ... Prescient and visionary people have already forecasted that in this country a high school for all will soon be taken up. ... the social difference between the "public" and the "intellectuals" that has come into being during the last decades might disappear again. ... Here, more emphasis has indeed always rather been on limiting the number of pupils in the high schools, than on attracting people to them. The opinion has been considerably widespread for a long time, that far too many attended the high schools.[749]

- The salaries of the teachers were unacceptable and decent textbooks were needed instead of decades-old ones, and in some cases the obsolete rubbish, that teachers were forced to use as a basis for their discussions with their pupils (June 68).[750]
- The 1946 education legislation had not yet been implemented in the whole country. In some areas only 55% of 13- to 14-year-old pupils attended school while attendance was 95 % in Reykjavík (Oct. 68).[751]
- Two large devaluations of the *króna* doubled the cost of studying abroad (Dec. 1968).

Gunnar Karlsson mentioned many of the trends of the time. "We shall need many more high school graduates" refers to the OECD paradigm of society's human resources, while his last sentences about rejecting pupils from high schools refers to Jónas Jónsson of Hrifla's policy of limiting the number of intellectuals and to the national examination's limiting effects, in addition to the attitudes within Reykjavík High School on the basis on its accommodation. The "prescient" people were so prescient, that within less than a decade anyone who so wished could enter an upper secondary level school.

[749] Gunnar Karlsson (1968): *Frjáls þjóð*, May 16
[750] Hjörleifur Guttormsson (1968): *Frjáls þjóð*, June 13
[751] Svavar Sigmundsson (1968): *Frjáls þjóð*, October 10

The year 1967 was a general election year. *Morgunblaðið* published an article about progress in education in the period of the Restoration Government, which had been in office since 1959, and listed its accomplishments: School research – an important innovation, strengthening of the University, new buildings for the high schools and new high schools, research and science institutes, a new building for the Teacher Training College, a new Technical College, technical schools in each constituency, a new agricultural college, greatly increased student loans and grants, repairs to domestic-science school buildings and a number of new, more economical school buildings.[752]

Certainly this was good progress, but far from enough. The increase in student loans was probably the one item that had the greatest influence on the conditions of students for completing university studies. The Teacher Training College had experienced a crisis because of the increasing number of pupils when it had by legislation in 1963 been changed in order to qualify its graduates for university entrance. The new annex for Reykjavík High School did not solve the problems of upper secondary education, and nor did the establishment of Hamrahlíð High School. In the period 1959–1969 the number of pupils reaching the minimum entrance grade into high schools more than doubled, i.e. increased from 393 to 861, or 119%, and was to grow still further before the national examination was abolished in 1976.[753] The establishment of the school research project, however, proved to be the channel for important innovations and reforms until it was suddenly closed down in 1984.

In June 1967, two *Morgunblaðið* editorials discussed the national examination. They must have been written by Editor Matthías Johannessen, who participated in the debate about the examination in this period. The editor expressed his concern that the proportion of the yearly cohort completing high school and university degrees was lower than in other countries, and that the national examination was too great an obstacle for young people on their way to high school and university.[754] Furthermore the editor said that it was widely known that in exactly those years when youngsters were taking the examination, unusually many interests were expanding their minds, and they then missed their opportunity for higher education, which they might regret all their lives. One even suspected that the shortage of buildings for the high schools had the influence of making the national examination and the first year of high school more difficult. It was therefore a great necessity to reconsider the national examination.

Discussions in *Samvinnan* 1967

When an old periodical of the cooperative movement, *Samvinnan*, was refreshed by hiring a new editor, Sigurður A. Magnússon, and giving it a new look, the first matter to be put under debate in the July-August issue 1967 was education.[755] Those participating in the debate, in addition to the new editor, were Andri Ísaksson, Arnór Hannibalsson, Hörður Bergmann, Matthías Johannessen, Guðmundur Hansen and Jón R. Hjálmarsson. Some of the views of Arnór Hannibalsson, Hörður Bergmann and Guðmundur Hansen had appeared in *Frjáls þjóð*. Arnór Hannibalsson, alias *Váli* in *Frjáls þjóð*, defined the Icelandic school as being still in the medieval phase inherited from Skálholt and Bessastaðir, and called for an integral educational policy. Hörður

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[752] *Morgunblaðið* (1967): June 8
[753] Archives of the Ministry of Education: Landsprófsnefnd (1959–1969), reports
[754] *Morgunblaðið* (1967): Editorials June 16 and June 29
[755] Sigurður A. Magnússon (Ed.) (1967): 10–31

Bergmann mentioned the necessity for school to emphasize understanding, training of logical thinking, developing judgement, creativity, imagination and emotional maturity. Guðmundur Hansen discussed the salaries of the teachers.

Andri Ísaksson reported on the measures that he and his assistants had taken on educational research since July 1966.[756] He divided the tasks into two parts, firstly investigations or questionnaires among school directors about their opinion of the present status and needs, and secondly experiments on innovations in schools. Preparations for the experiments began in August 1966, when the Minister of Education and the Government approved of allocation of funds for the experiments.

Concerning the future the following issues would be on the agenda:

1. To choose specialist teachers to write reviews on chosen groups of textbooks.
2. To look into the role and purpose of national curricula, how they should be made and worked on and what they should prescribe.
3. The research staff would like to investigate the relation between the social background of pupils and their achievement in school. It had also begun to look into pre-school/kindergarten education and into teacher training.

Finally Andri Ísaksson discussed at some length the present status of the examination system in Icelandic schools. In the first place it had been an extensive and expensive factor in the educational system. Another reason for investigating the examinations further was that the form and the nature of the examinations had an unequivocal influence on the teaching methods. Often the teaching was aimed more at preparing for the examination than for life or the maturing of the pupil, and this applied possibly nowhere more than in the lower secondary schools and the high schools. The nature of the national examination was for example such that it was well suited to that kind of working method, and the many complaints from teachers, pupils and parents this spring were evidence of this. The syllabi were extensive, the examination requirements were rather strict, and the examination itself a condition – and in many cases an invariable one – for admission to higher education.

Furthermore there was little correspondence between the content of the various examination certificates bearing the names "general middle" and "lower secondary" school examinations. Andri Ísaksson posed several questions on this matter for further thought, such as whether lower secondary school examinations in Icelandic, mathematics and Danish, and possibly English, should be standardized. Even though standardized curricula for the two last years of the lower secondary level were very desirable and possibly necessary, they would take a long time to create. He asked if these examinations could be implemented by defining minimum requirements and regulations about assessment in cooperation with experienced teachers and counsellors.

The above quotations reveal the first plans for the activities of the school research project, which expanded greatly in the following one-and-a-half decade. They also reveal the paradoxes in educational discussion. There is some contradiction in explaining all the disadvantages of the national examination of the middle school, which was only intended for those who were aiming at higher and further education, and at the same time initiating discussion about the same kind of examinations for all the other pupils. Certainly, both views were relevant. The Technical College, the

[756] Sigurður A. Magnússon (Ed.) (1967): 10–13

Teacher Training College and other vocational colleges, such as the Nursing College, accepted pupils with a general lower secondary school examination and they needed to know what was behind the grades. On the other hand, the national examination could not stay unaltered any longer.

We will see that in the near future, two measures were taken to meet the needs of the vocational colleges. One of them was to establish a standardized examination for general lower secondary education, and the other was to extend the lower secondary school level with two-year continuation departments.

Dr. Matthías Jónasson's Research

Dr. Matthías Jónasson published in 1967 an extensive study on a group of children to investigate the relationship between their results in primary school, in the national examination of the middle school, at Reykjavík High School and his measurements of their intelligence quotient. The research showed that there was a correlation between the results on the IQ tests and the national and lower examinations. However, amongst those who did not pass the first year of the Reykjavík High School there were relatively many, 17%, with IQ 135 or higher. From his results he deduced that the national examination was a better tool to choose the "fittest" than the methods used by Reykjavík High School.[757]

Dr. Matthías Jónasson participated in the public discussion in an article he wrote in *Morgunblaðið*'s weekend edition in May 1968 where he said:

> Inngönguprófi að æðra námi hefur löngum verið ætlað eins konar síu- eða úrvinzunarhlutverk. Sú framkvæmd er ekki sársaukalaus og mun seint reynast óskeikul. Samt á hún sér nokkra réttlætingu. Hjá þjóðum, sem halda æðri menntastofnunum í sífelldri fjárhagskreppu, virðist sú viðleitni ekki óskynsamleg að beina helzt þeim unglingum inn í æðri skóla, sem virðast hæfastir til að notfæra sér menntunarskilyrðin, sem þar bjóðast. ... Með þessa staðreynd fyrir augum virðist skynsamlegt að afnema ekki prófin algerlega, nema að undangenginni vísindalegri tilraun með hæfilega stóran og fjölbreyttan nemendahóp, enda leiddi sú rannsókn þá skýrt í ljós yfirburði hins próflausa skipulags.
>
> An entrance examination to higher education has for a long time had the role of filtering or selecting. It has never been painless and a long time will elapse before it will become infallible. Still it can be justified. In nations which keep their educational institutions in a constant funding crisis it does not seem unreasonable to channel into higher education schools preferably those youngsters who seem fittest to utilize the educational provisions offered there. ... With this fact in mind it seems sensible not to abolish the examinations completely without a prior scientific experiment with a suitably large and varied group of pupils, if that revealed the superiority of an examination-free arrangement.[758]

Dr. Matthías Jónasson suggested that the preparation time for the national examination become at least two years, for three reasons. Firstly, the teachers would gain more time to learn to know the capacity and the diligence of their pupils; secondly the teachers could have more opportunity to offer guidance to their pupils; and thirdly the studies could be more carefully planned. One year was too short, both for pupils and teachers. It led to too tight a time schedule, pressure and hurried work, which a youngster in a formative period could not easily sustain. Dr. Matthías Jónasson's opinion was also that the national examination would have to be changed

[757] Matthías Jónasson (1967): 218–241
[758] Matthías Jónasson (1968): *Lesbók Morgunblaðsins*, May 5

from the root. The host of incoherent details that the pupils were expected to remember was horrifying. Would the answers to such questions be the correct measure of the capacity of youngsters for higher education? What about inventiveness, judgement, reasoning and creativity?

In a later article, Dr. Matthías Jónasson explained his research where he investigated the relation between grades in the national examination and the final matriculation examination (*stúdentspróf*) in the Reykjavík High School around 1960. In the language stream the pupils generally achieved the same grade in mathematics at the national examination and the matriculation examination, while in the mathematics stream their grades were considerably lower than at the national examination; for a chosen group a decrease in average from 7.40 to 5.98 out of 10.[759] This agrees with the conception of the author of this study about the results of her classmates on graduation from the mathematics stream in the Reykjavík High School in 1963. The question arises if it was necessary to leave a large proportion of the elite group attending the mathematics stream with a lowered self-esteem in mathematics.

Dr. Matthías Jónasson furthermore quoted Dr. Þuríður Kristjánsdóttir's research, where she had found out that in the period 1957–1959 only 80% of those, who entered the Reykjavík High School, graduated from there.[760] Taking into account that only 20% of the yearly cohort ever tried the national examination and that two-thirds of the pupils taking it reached the minimum entrance level to a high school, the facts that four out of five graduated from high school and only 36% of those who matriculated graduated from the University of Iceland in 1950–1958,[761] suggested firmly that the road to higher education in Iceland in the 1950s and 1960s was a narrow one.

FHK's Declaration on Basic Principles in School Affairs

The Union of University-educated Teachers, FHK, sent out in January 1969 a report, *Yfirlýsing um grundvallarsjónarmið í skólamálum / Declaration on Principal Points of View on School Affairs*, signed by economist Jón Baldvin Hannibalsson as chairman. The report is based on discussions within the FHK Union in the previous years and worked out by the presidium of the Union in the school year 1967–1968. Those discussions, which are reflected in the writing of *Váli*, alias Arnór Hannibalsson, in *Frjáls Þjóð* 1965–1967, are also coloured by the OECD aspects. In its chapter about schools and society it says:

> 1.4. Menntun: Félagsleg nauðsyn, en ekki forréttindi. ... komin [eru] til sögunnar ný grundvallarviðhorf, sem ... hljóta að gerbreyta allri hugsun um skólamál í þróuðum iðnaðarþjóðfélögum: Í fyrsta lagi er ljóst, að menntun getur ekki framar skoðazt sem lúxus, forréttindi fárra útvalinna, heldur þjóðfélagsleg nauðsyn hverjum einstaklingi. Í öðru lagi vex óðum skilningur á því, að fjárfesting í vísindum og menntun er grundvallarforsenda efnahags- og félagslegra framfara. Í þriðja lagi blasir það við, að þær þjóðir, sem ekki gera sér grein fyrir þessum grundvallarstaðreyndum og haga skólastarfi sínu í samræmi við það, hljóta að dragast aftur úr og verða stöðnuninni að bráð.
>
> 1.4. Education: Social necessity and not a privilege. ... new principal views have arrived, which ... must completely change all thinking about school affairs in developed industrial societies: In the first place, it is clear that education can no longer be considered as a luxury, a privilege for the chosen few, but a social

[759] Matthías Jónasson (1971): 190–191
[760] Matthías Jónasson (1971): 193
[761] Morgunblaðið (1967): Editorial, June 16

> necessity for every individual. Secondly, an increased understanding is emerging that investing in science and education is a basic prerequisite for economic and social progress. Thirdly, it is clear that nations that have not realized these basic facts and arranged their school work according to them must lag behind and fall prey to stagnation.[762]

The spirit of OECD's views of education appears clearly in this declaration. The report went on to emphasize the necessity of new textbooks, renewal of equipment and continuing education of teachers and recalling the neglected subjects, mathematics, physics and chemistry, according to Sveinbjörn Björnsson's report.[763]

Follow-Up to Sveinbjörn Björnsson's Investigation and the OECD

The article by Sveinbjörn Björnsson was followed up by some discussion. We have previously quoted the following paragraph that Guðmundur Arnlaugsson wrote in *Menntamál* in 1967 to explain the reasons for a reform of mathematics teaching:

> It has been clear to many that our arithmetic and mathematics teaching has been considerably defective and the clear comparison by Sveinbjörn Björnsson … of the syllabi in these subjects in this country and in our neighbouring countries ... shows that the situation is still more unfavourable to us than most people had suspected.[764]

Physicist Páll Theodórsson wrote in the same issue of *Menntamál,* Vol. 40(1) about physics teaching in the lower secondary schools, and how it had been forgotten in the 1946 education legislation, when the uppermost primary grade was transferred to the lower secondary level. When it was introduced again by the national curriculum in 1960, much of its traditions and experience had been forgotten after 15 years of oblivion. He referred to a recent OECD report [not dated] on the teaching of physics, about a recommended minimum of 400 hours of physics, while Icelandic pupils were assumed to have 150–180 hours at the lower secondary level. Then Páll Theodórsson discussed the lack of textbooks and equipment for experiments and teacher shortage, and lastly suggested some ways for reforms.[765]

One notices that the OECD is frequently mentioned in connection with proposed educational reforms. The OECD supported Sveinbjörn Björnsson when he studied the structure and teaching methods in technical colleges in Denmark, Norway and West Germany, and Páll Theodórsson studied a report on physics teaching by the OECD. The year before, in 1966, in the Budget Bill for 1967, there was a note previously referred to, that the school research was partly initiated by the OECD. An advisory committee for educational planning had also been initiated including Dr. Wolfgang Edelstein and Dr. Klaus Bahr as OECD-experts to EIP, Educational Investment and Planning Programme of Iceland (see section 6.9.).[766]

The School Research Department

At first, Andri Ísaksson was appointed as a school research specialist, eventually with some assistance, but soon the school research activities increased, and in 1968 a special department within the Ministry of Education was established. The department

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[762] Félag Háskólamenntaðra kennara (January 1969): 2
[763] Félag Háskólamenntaðra kennara (January 1969): 6–8
[764] Guðmundur Arnlaugsson (1967): 40–41
[765] Páll Theodórsson (1967): 62–75
[766] National Archives of Iceland: Skjalasafn Fræðslumálaskrifstofunnar 1989/E-10 Efnahagsstofnunin. Country Report. Submitted by the EIP-team of Iceland

was called the School Research Department, SRD, even if its activities were mainly of a developmental and experimental nature rather than research.

Sveinbjörn Björnsson's revelation, that traditional education did not keep mathematics and physical sciences up with the neighbouring countries, had the effect that SRD initiated preparation for revision of the curriculum and teaching of physics and chemistry in primary and lower secondary schools as its first project. A committee of five members was appointed on August 14, 1967 to make a study of the current teaching and proposals for reform. The committee, chaired by Sveinbjörn Björnsson, returned a report with a plan of action and a detailed budget in May 1968.[767] The educational and financial authorities approved of its execution. The well-prepared proposal and the report on the poor standing of Icelandic pupils were sufficient to convince authorities to allocate funds for it. This later became the model for a parallel reform of other subjects: biology, Danish, sociology and music, and on different lines, mathematics and Icelandic.[768] The budget for the "scientific research of the school system" was modelled after the physics reform budget.[769]

The School Research Department and its activities enjoyed support from *Alþingi*, remarkably enough. Earlier, the *Alþingi* had doubts about the value of the new education legislation since its adoption in 1946, probably because the majority of members of parliament came from rural areas where the legislation had not yet been implemented due to lack of school buildings and difficult transport during wintertime. The OECD theories that education contributed substantially to economic and social progress seem to have won general support.

The department grew very fast. Its size can be estimated by the funds allocated to scientific research on the school system in the Budgets of the following years when Dr. Gylfi Þ. Gíslason was the Minister of Education.[770] The figures in the following tables are recalculated according to the consumer price index at 1968 values.[771]

Scientific Research of the School System	**1967**	**1968**	**1969**	**1970**	**1971**
Consumer price index	90.2	100	121.5	139.6	159.8
Funds in thousands of *krónur*	800	1,368	1,790	2,503	5,770
Funds recalculated to 1975 prices	3,526	5,438	5,856	7,127	14,353

Table 7.1. Allocations to Scientific Research of the School System 1967–1971.

In the Budget for 1968, 1969 and 1970, the allocation for this item was raised by the *Alþingi*'s Budget Committee from the proposal in the Bill, in spite of a financial crisis during that period. In 1971 a new government took over. The Minister of Education was a left-wing social democrat. The SRD continued to grow under the aegis of this government. Under the heading of "reconstructing learning materials", in the Budget, omitting the salary of the school research specialist, this is found:

Reconstructing Learning Materials	**1972**	**1973**	**1974**	**1975**
Consumer price index	168.5	194.8	264.6	397.5
Funding in thousands of *krónur*	6,227	11,375	16,316	24,807
Funds recalculated to 1975 prices	14,690	23,211	24,511	24,807

Table 7.2. Allocations to reconstructing learning material 1972–1975.

767 Menntamálaráðuneytið. Skólarannsóknir (1968): 25
768 Andri Ísaksson (1972): 5
769 Sveinbjörn Björnsson, November 5, 2004
770 *Alþingistíðindi* 1967–1971: Budget Bill. *Stjórnartíðindi* 1967–1971: Budget
771 Statistics Iceland: website

There is no sign that the allocation of these funds met with any objection in the *Alþingi*. Further, in 1973 and 1974 the Budget Committee of *Alþingi* raised the figures from the original Budget Bill to the final Act. Only in 1975 it was lowered slightly.

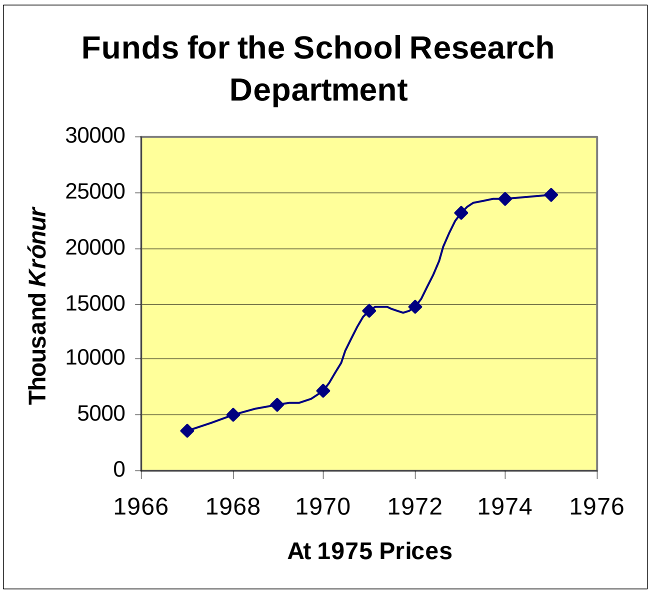
It was not sufficient to compose and reconstruct the learning material; it had to be published by the State Textbook Imprint / *Ríkisútgáfa námsbóka*. The amount allocated for this in the Budget increased every year, as can be seen from this table:

Publishing	**1967**	**1968**	**1969**	**1970**	**1971**	**1972**	**1973**	**1974**	**1975**
C. price index	90.2	100	121.5	139.6	159.8	168.5	194.8	264.6	397.5
Thousand *kr.*	3,475	10,417	11,687	13,647	28,125	32,172	39,963	52,615	106,029
1975 prices	15,314	41,408	38,235	38,859	69,960	75,895	81,547	79,042	106,029

Table 7.3. Allocations to the State Textbook Imprint 1967–1975.

In the period of the Restoration Government, the state only met one-third of the cost. Since 1956 two-thirds were met by the textbook charge on homes with children. This charge was abolished by the left-wing government from 1972[772] as a part of its measures to simplify the taxation system. The *Alþingi* never lowered the Ministry's proposals, and in 1973 and 1974 they were raised slightly from the original Bill.

Below are graphs of the above mentioned figures for parliamentary allocations to the SRD and the State Textbook Imprint during the years 1967–1975, recalculated to 1975 prices.



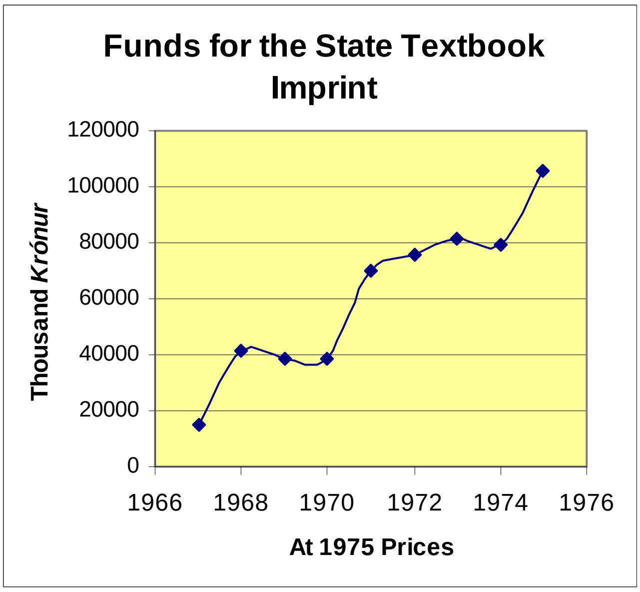


Fig. 7.4. Funds for the State Textbook Imprint. Fig. 7.5. Funds for the School Research Department.

The Imprint now only had the role of publishing the products of the department, and no longer the responsibility of initiating publishing projects. The country suffered an economic crisis in 1968 and the 1973 oil crisis. There was a catastrophic volcanic eruption on Heimaey, the only inhabited island of the Westman Islands, in 1973. All these events contributed to increased inflation and devaluation of currency. None of them affected the ever-increasing support to the School Research Department, and neither did changes of government from conservative to left-wing in 1971, to conservative again in 1974.

772 *Alþingistíðindi* 1971 A: (Budget Bill) 146

Throughout that period, Birgir Thorlacius, an influential man, was permanent secretary of the Ministry of Education. According to Hörður Lárusson, he was a strong supporter of the SRD.[773] By the end of 1978 the School Research Department had 28 employees, in about 20 full-time positions. In addition there were 54 teachers, working part-time writing learning material.[774]

Assessment

In March 1967 the director of the School Research Department, Andri Ísaksson, became the chairman of the National Examination Board. After conducting the first examination in spring 1967, he organized the writing of a more detailed description of the requirements for the examination, together with fairly detailed guidance for teaching. Upon the director's proposal, the detailed grading scale of 100 steps was changed to a whole-number scale from 1 to 10, and examination could be repeated the following autumn. A standardized examination for the four-year lower secondary school was implemented in 1968, together with corresponding standardizing preliminary curricula documents, written by the School Research Department. In 1974–1976, an examination for the four-year lower secondary school became the same as the national examination of the middle school, so that practically the whole cohort took the examination, at the age of either 16 or 17. At the same period, 1974–1976, discussion within the Examination Board and the staff of the department led to less emphasis on standardized examinations, and a new scale was introduced, with five grades from A to E, distributed on the Gaussian curve. The last national examination of the middle school was held in 1976 and in the four-year lower secondary school in 1977. In February 1977 the first compulsory-school examination was held. The schools were to give their own grades, which were to weigh evenly against the examination grade for entrance to the continuing schools at the upper secondary level.[775] By this time, these schools were not only classical high schools.

Alongside these changes, a more detailed analysis of the results of the examination was implemented at the same time, as the number of pupils taking the standardized examination multiplied. The examination, previously organized and administered by a committee of ten persons, with minimal office assistance, run from their homes, had now become a voluminous body as can be seen from the following figures taken from the Budgets 1968–1972:

Standardized Examinations	**1968**	**1969**	**1970**	**1971**	**1972**
Funds in thousands of *krónur*	250	405	500	750	2,126
Funds recalculated to 1975 prices	954	1,325	1,424	1,866	5,015

Table 7.4. Allocations to Standardized Examinations 1968–1972.

In 1973 the funds were no longer attached to the main office of the Ministry, and cannot be traced. They may be part of the total sum allocated to the SRD.

[773] Hörður Lárusson, March 26, 2002
[774] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 79–81
[775] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 30–31

7.5. “Modern” Mathematics in the National Examination

Changes in Mathematics Examination

Björn Bjarnason was the mathematics representative on the National Examination Board from 1963. After the first couple of years he began to alter the examination. Njörður P. Njarðvík, the departing chairman who left the board after the examination in 1966, has confirmed that he did not intervene in any way with the individual subjects.[776] Bjarni Vilhjálmsson, the previous chairman who had left after the examination 1964, steered the board for about half a year from autumn 1966 until early 1967. Björn Bjarnason discussed with Bjarni Vilhjálmsson the plans of offering two alternative syllabi and examinations in the academic year 1966–1967.[777] That academic year and the following one in 1967–1968, a choice of two mathematics syllabi was offered, and two types of national mathematics examinations, with a conventional and “modern” mathematics syllabus. SRD Director Andri Ísaksson, the chairman of the National Examination Board from 1967, told the author of this study that when Björn Bjarnason proposed alterations of the mathematics syllabus he had had the full support on the board from Guðmundur Arnlaugsson, the physics representative, and Andri Ísaksson himself, being a promoter of school reforms.[778]

The mathematics examination had been in two parts, seen and unseen problems, all composite, taken in three sessions. In 1966 the examination was different in such a way that there were no seen problems. Part I now comprised 12–13, and later up to 25, comparatively simple (non-composite) problems, taken in one three-hour session, while part II had six composite problems taken in one three-hour session. Data are available from two groups in one school in 1966 which indicates that the average of the result of the two parts is about the same in high-ability classes.[779]

In the first group the average was 8.2 in part I and 8.1 in part II.
In a second group the average was 7.9 in part I and 7.3 in part II.

The lower the average is, the more difference exists between the average in the “small” problems and the “large” problems, so this arrangement may have helped the less able pupils.

In 1967 and 1968 there was a choice between the two syllabi, the “old” conventional syllabus with area and volume, proportions (*regula de tri*) and percentages, and the “new” one with topics from Guðmundur Arnlaugsson’s new textbook, *Numbers and Sets*. In 1968 the decision was taken to have only one kind of national examination in mathematics, mainly based on Dr. Ólafur Daníelsson’s classical algebra textbook and on *Numbers and Sets* by Guðmundur Arnlaugsson. At the same time the examination was reduced to one session of three hours.

This was a change from the classical syllabus that had been used for over two decades. However, up to this ninth school year the pupils had had a conventional syllabus, based on a textbook tradition from the first quarter of the 20th century. There was no change until the ninth year, and then only in about 40% of the year-course. There is no evidence that teachers resisted this, and it can be seen from the writings of Björn Bjarnason that the changes were prepared cautiously.

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[776] Njörður P. Njarðvík, an email, January 17, 2003
[777] Vilhjálmur Bjarnason, January 2003
[778] Andri Ísaksson, March 10, 2003
[779] Unpublished, confidential sources. See section 6.4

Preliminary Curriculum

The School Research Department's main activities soon became to write curriculum documents and, following this, experimental textbooks, and one of the first tasks was to have curriculum documents prepared for the subjects in the national examination.

In September 1968 the paper *Drög að námskrá í landsprófsdeildum miðskóla / A Draft Curriculum for the National Examination Departments* was published. In its introduction, signed by Andri Ísaksson, it says that the main form of the curriculum draft was decided upon in one meeting, and thereafter the subject specialists made the preliminary copies, each for their subject. The subject specialists were indeed the National Examination Board members at that time, including Björn Bjarnason in mathematics. In this study the document will be called preliminary curriculum as it acted as regulations for some period.

In this preliminary curriculum, explicit goals for the mathematics teaching for the national examination were stated for the first time.

The introduction to the mathematics curriculum is the following:[780]

> A good many years ago the curriculum of the secondary schools was subjected to revision under the leadership of the O.E.C.D., the Organization for Economic Cooperation and Development in Paris. This revision has led in most places to radical changes taking place in mathematics curriculum and instruction.
>
> The aim is
>
> 1. to base school mathematics on the basic concepts of the set theory, which simultaneously are simple and general
> 2. to put more emphasis on the meaning and the nature of numbers and of number computations than has been customary.
>
> Four years ago changes in the [mathematics] curriculum in Icelandic high schools were initiated in accordance with these new aims. At once it became clear that these changes could not be successfully implemented unless a different approach was applied in the national examination classes, where the basis is laid for algebra, one of the most important basic topics of high school mathematics. But as no textbooks of an appropriate form existed in Icelandic and only very few teachers had had an opportunity to study the new views, it proved completely impossible to implement such changes. By the publication of Guðmundur Arnlaugsson's textbook, *Numbers and Sets*, and with in-service courses for mathematics teachers, the attitude changed so much for the better, that it proved, two years ago, possible to modify the syllabus partly into the new directions, and the individual school authorities were offered the option of introducing a new syllabus, but were not instructed to do so. This was evidently not a long-term solution and it was only done in order to give the mathematics teachers time to familiarize themselves with the new ideas. – Last winter the majority (more than 2/3) of the national examination pupils benefited from instruction in the new material, and as it now can be expected that the mathematics teachers have had time to adjust to changed customs, the implementation of two kinds of syllabuses will now come to an end and the same syllabus will apply to all.
>
> However, it was not considered possible to avoid adapting the syllabus almost wholly to those textbooks already available in Icelandic, even if some of them are rather old and are not suited to the desired requirements.
>
> The syllabus is based on the use of the following textbooks:

[780] Landsprófsnefnd (September 1968): 56–60. See the Icelandic original version in Appendix F

1. *Tölur og mengi | Numbers and Sets* by Guðmundur Arnlaugsson, the whole book.
2. *Kennslubók í algebru | A Textbook in Algebra* by Ólafur Daníelsson, up to a text in front of exercises XI, and omitting more complicated problems in exercises VI (such as exercises 29-44).
3. Some material on direct proportions and partition in *Reikningsbók | Arithmetic* by Ólafur Daníelsson or *Reikningsbók fyrir framhaldsskóla | Arithmetic for secondary schools*, II. A, by Jón Á. Gissurarson and Steinþór Guðmundsson or *Reikningsbók handa framhaldsskólum | Arithmetic for secondary schools*, vols. II and III, by Kristinn Gíslason and Gunngeir Pétursson.

In his introduction, Björn Bjarnason is of course referring to the Royaumont Seminar. Obviously, it was his conviction that changes were not before time; the problem had been a lack of textbooks and the unfamiliarity of the teachers. When these two items were partly solved, the decision was taken to implement the new syllabus for the whole population who aimed at further and higher education.

After this rationale Björn Bjarnason listed the goals of mathematics teaching for the national examination and its syllabus. The aim was to base the teaching on set-theoretical concepts.

It was especially stressed that the goals were aimed at that target group and thus did not concern the general population in middle school. (In 1968, 30% of the age cohort took the national examination, and in 1969 the proportion was 33.8%, so the target group was certainly growing fast).

Goals

Mathematics teaching in a national examination class shall be on the assumption that the pupils studying there have in mind further education in high schools and other schools to which the national examination will entitle them.

The aim should be that pupils will obtain

1. increased confidence in general number computations,
2. understanding of our number notation and of the nature of computational operations,
3. reliable ideas about the basis of algebra, understanding of the symbolic language and its rules,
4. skills in rewriting statements, written in the symbolic language of algebra,
5. skills in solving first degree equations with one, two or three unknowns,
6. skills in rewriting sentences from ordinary language to the symbolic language of algebra.

The syllabus

1. Direct proportions and partition.
2. The basic concepts of the set theory: a set, an element, Venn diagrams, one-to-one correspondance of sets, cardinality, a subset, a union and an intersection, an empty set, a set difference.
3. Propositions, a universal set and a complementary set, open sentences and their solution sets.
4. N_0, the set of the natural numbers and zero. Patterns in N_0, partition of N_0 into residue classes, prime numbers, number notation in the decimal and other place-value systems. The operations of addition, subtraction, multiplication and division in other place-value systems than the decimal, especially the binary system.
5. The four operations: Addition, subtraction, multiplication and division, in the set of rational numbers, the basic laws of the operations and the computation rules deducted from them.
6. Factoring (rewriting polynomials to products), division in the set of polynomials.
7. First degree equations with one, two or three unknowns and their solutions.
8. Word problems leading to equations of the same kind as mentioned in article 7.

9. The order of magnitude of numbers and scientific notation.

Remarks to the teachers:

1. ... In compulsory education, pupils have become acquainted with a great number of examples of relations between two variables such that one variable is proportional to the other. Even if right proportion is not mentioned in any of those, they have found e.g. price proportional to the weight or number of items, weight proportional to volume, interest over a certain interest period proportional to the principal sum, a distance travelled at an even velocity proportional to time. Probably only few have noticed the common item in such relations, the real mathematical relation, which is described by the phrase "proportional to" and later in their education will be expressed by an open sentence of the form y = ax. (a : a number) ...

   By implementing "right proportion" into the syllabus, it is expected that the pupils will be encouraged to sense what these words imply about number relations. …

It seems that the ghost of *regula de tri* was to be quashed once and for all, and indeed, it was. The *regula de tri* was a topic in all the textbooks mentioned in article 3 of the textbook list, but it did not appear in any textbook published after this time.

2. It is very important that the pupils obtain clear ideas about the basic concepts of the set theory and their relations and acquire a mastery of its symbolic language.

   These basic concepts emerge in the basis of every branch of mathematics, and therefore they must often be referred to and used. The symbolic language allows ideas and their relations to be expressed in an exact and clear way. It is desirable not to begin working with the algebra of numbers (i.e. the conventional algebra) until the pupils have acquired mastery of the relations of sets and the introduction to set theory, to be found ... in the textbook.

   Concerning set algebra the following should be clearly expressed:

   1. To form a union and an intersection can be considered as <u>binary operations</u> in a set of sets, that is a union and an intersection are each time formed from <u>two</u> sets (which still could be the one and same set) ....
   2. For these operations the following rules apply [a list of the commutative, associative and the distributive laws for sets].

3. If pupils have already had the experience of solving equations, their attention should be drawn to the fact that each such equation is an open sentence on a universal set, which in most cases is the set of rational numbers.

The idea of set theory as a unifying structure emerges from the above paragraphs.

4. Number notation and number operations in other place-value systems than the decimal system are intended to awaken the pupils to consider the merits of our number notation and teach them to distinguish between the number concept and the symbols with which numbers can be expressed. ... every natural number expresses a characteristic which is the common property of sets with one-to-one correspondence. Thus the number five, in whatever system it is written, expresses the common property of all sets which have a one-to-one correspondence to the fingers on a correctly-formed human hand. …

The topics on numbers and number notations in the various place-value system turned out to be rather successful and awaken the pupils' interests.

5. There are various algebras. But a common feature of all is that they deal with the nature of some kind of "computing". The basis is a certain set, which is not an empty set. From every two elements of this set, a new element of this same set is made by

> an operation... [Continued with parallels between the algebra of sets and the usual algebra of rational numbers] ...[781]

This long citation has the purpose firstly to show how the mathematics education authorities (i.e. the National Examination Board member) tried to make contact with teachers through this first official curriculum document for this stage and introduce to them the new way of thinking about conventional algebra with the aid of set theory, and secondly to show the optimism of mathematics educators at that time about the expected remedies to be achieved by this way of thinking.

Later in this curriculum handbook the author saw reason to remind teachers of the use of letters like a, b, c and x in mathematical expressions, by taking an example such as that 2d + 3d = 5d did not represent that 2 donkeys + 3 donkeys equalled 5 donkeys. The letter d represented a number, so it really was 2·d + 3·d etc. One wonders if this rectified misunderstanding on behalf of some teachers at that time. At least, possibly no institute they had attended had taken this "self-evident" fact up to discussion.

In retrospect, this short introduction did not suffice to convince many teachers of the merits of the set-theoretical approach. On the other hand, they may have enjoyed new types of problems, like the inclusion-exclusion counting problems, solved with the aid of Venn diagrams. A fuller text would have been needed to help the teachers integrate set theory and conventional algebra. In at least two rather large schools in the Reykjavík area, these two sections of the syllabus, the set theory and the introduction to algebra, were taught as two separate subjects, algebra four hours a week and the textbook *Numbers and Sets* three hours a week. It was regarded as necessary to start the classical algebra training as early as possible in the autumn. The number chapter of *Numbers and Sets* came first, so numbers were taught in the first semester and the set theory in the second semester. Then the teacher might point out the similarities of the set operations to the algebraic operations, or he/she might not. At any rate, the set algebra did not precede the conventional algebra.

In some cases two teachers might teach the two sections, the older teacher the classical algebra and the younger one the *Numbers and Sets,* and never discuss the relation between the two sections. This was the case of the author of this book and her older colleague.

Curriculum Trends

The effect of this new curriculum was that the syllabus changed from emphasis on solution of "applied" and "practical" problems, to understanding the numbers and number systems. It is hard to say if the pupils became more motivated. The national examination changed from a few large, composite problems to solve, to a number of small items without coherence, while there was much to remember. The ratio of word problems to the total number of problems changed from about 60% in 1957–1965 to about 35% in 1969–1972 and to less than 30% in 1973–1975.[782] One explanation could be that while the population was not over-large, the teachers and the examiner could use a grading system whereby they graded well all first attempts at each problem, and then demanded a full solution for a full grade.[783] When the examination papers became too numerous for one examiner to grade all papers, fill-in sheets,

[781] Landsprófsnefnd (September 1968): 56–60
[782] See Appendix G
[783] Haraldur Steinþórsson, January 22, 2003

multiple choice and *right*/*wrong* answers were introduced and became a part of the examination.

Recalling Dr. Matthías Jónasson's remarks about the host of incoherent details that the pupils were expected to remember being "horrifying," one would like to ask with him: Would the answers to such questions be a correct yardstick of the capacity of youngsters for higher education? What about inventiveness, judgement, reasoning and creativity? Admittedly, though, usually one or two large and neatly composed problems at the end of the examination might test these factors to some degree.

In conclusion: New general goals for mathematics teaching were formulated in preliminary curriculum documents, dated 1968. The aim was to introduce set theory as a frame for unifying concepts of mathematics, as had been the aim of the "modern" mathematics movement abroad.

Even though the aims and objectives were now formulated explicitly and a list of topics was given, there were still references to particular textbooks. This may be a remnant from a tradition of non-specialists, when people may not have been too certain on how to express or interpret a syllabus in general terms, but there was also the fact that the choice of textbooks was a very limited.

When it came to the implementation of the curriculum, the textbooks were for a while largely the same as before. The teaching methods did not change radically, nor did the habits of the teachers. After all, one may suspect that the main models for their teaching were their own teachers and habits would take longer time to change.

This national examination curriculum and syllabus remained the same for all schools for only a four-year period, 1969–1972.

Björn Bjarnason and Guðmundur Arnlaugsson in Retrospect

While Guðmundur Arnlaugsson looked like a serious person, an ancient thinker, as he was often kindly caricatured by his pupils, Björn Bjarnason's manners were more jovial, and his pupils couldn't miss it when he enjoyed a mathematical idea that he was teaching. This description of those two very distinguished and friendly teachers is what stays in memory, four decades after their great and long lasting impact.

Björn Bjarnason seems somehow to have been more fascinated by the set-theoretical approach than Guðmundur Arnlaugsson. Björn Bjarnason is the only person who more-or-less explicitly mentioned the set algebra as a unifying structure. Guðmundur Arnlaugsson mentioned in his foreword to *Numbers and Sets* that concepts from logic and set theory were expected to support increased clarity and exactness in thinking and arithmetic. He spoke about "structure" in his 1967 article, and that the value of the new concepts was primarily to facilitate the understanding of concepts that previously had been unclear and remained tacit, while he never mentioned the unifying aspect of sets. The texts compared were aimed at slightly different target groups. Guðmundur Arnlaugsson's texts were aimed at all teachers, primary and secondary, and the general reader. The aim of Björn Bjarnason's text was to inform the national examination teachers about the requirements for the examination.

Survey of Six Schools

We continue the survey of several lower secondary schools offering the national examination. Previously, we have seen how the national examination results in mathematics were dependent on stability in the teacher force and the qualifications of the teachers. The large Reykjavík school, school E, no longer features, while school B, in the West Fjords, has entered the scene.

Schools A and P were situated in the capital area; School A was a selective school, attended by especially able pupils.

Schools B, R and C were typical urban schools in towns in the West Fjords, the North and the East Fjords.

Schools D and S were boarding schools in rural areas.

Year	School A			School P			School B			School R		
	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.
1967	27	7.6	7.64	30	6.0	6.58	7	7.8	6.15	12	7.1	7.23
1968	27	6.9	7.92	29	6.0	6.19	11	6.9	6.27	11	6.6	6.71
1969	37	6.7	7.81	32	5.4	6.82	6	8.4	6.35	5	5.4	6.66
1970	32	7.0	7.40	41	6.3	6.3	7	8.4	6.3	14	7.6	6.64
1971	48	6.9	7.45	41	5.6	6.14	12	7.4	5.98	13	5.0	5.77
1972	46	7.4	7.34	69	6.0	5.46	10	6.8	5.47	16	5.6	6.20
1973	47	7.6	7.56	93	5.6	5.58	9	6.9	6.19	15	5.3	5.78

Year	School D			School S			School C			Total		
	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.	No.	Mth.	Ave.
1967	13	5.8	6.50	18	6,6	6,62	45	7.0	6.51	152	6,8	6,78
1968	25	5.9	6.70	29	7.5	6.92	78	6.3	6.03	210	6.5	6.55
1969	16	4.8	5.75	35	7.2	7.01	85	5.6	5.82	216	6.0	6.53
1970	23	5.9	6.50	26	7.4	6.99	83	6.4	6.30	226	6.7	6.58
1971	27	4.9	5.87	31	5.1	6.20	76	5.3	6.12	248	5.7	6.34
1972	23	6.5	6.84	28	5.9	6.20	69	5.9	5.92	261	6.3	6.16
1973	13	4.9	6.05	30	5.6	5.99	66	6.2	6.12	273	6.1	6.15

Year	National[784]				
	No.	%	Mth.	Ave.	Diff.
1967	963	24.5			
1968	1175	29.9	6.21	6.4	0,19
1969	1408	33.8	5.51	6.1	0,59
1970	1416	33.7	6.1	6.2	0,10
1971	1465	34.2	5.5	6.3	0,80
1972	1584	36.0	6.3	6.1	-0,20
1973	1627	35.8	5.9	6.1	0,20

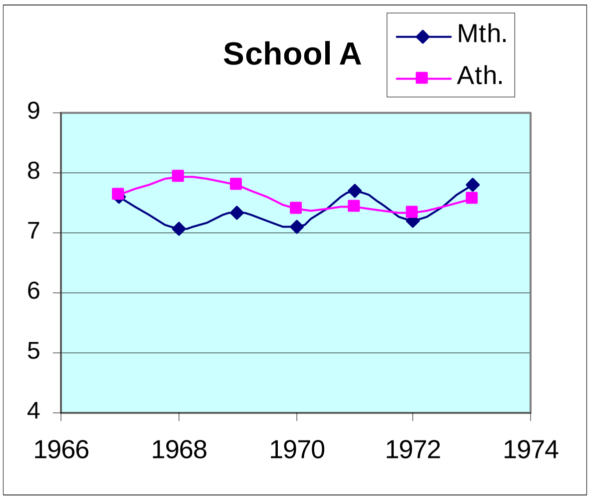
Table 7.5. The number of pupils, the average grade in mathematics and the average of averages of grades in all the subjects in seven schools 1967–1973.

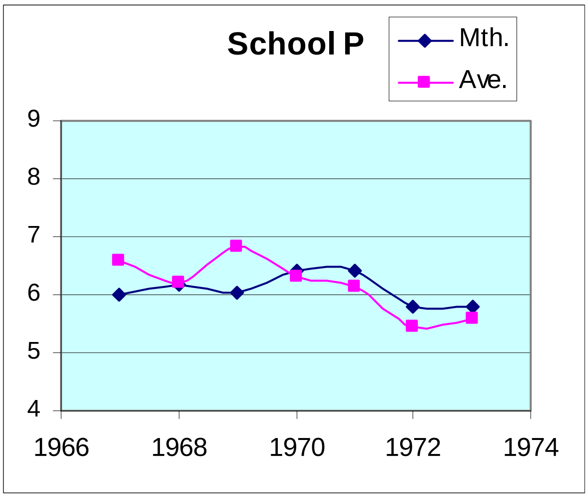
As seen in the above table, national averages were available from 1968. From that time the mathematics averages in the graphs have been corrected on the graph below by the difference of the Ave.- and Mth.-columns under the heading "National". The 1967 mathematics grades have not been corrected, while e.g. in 1971 the mathematics grades have been corrected by 6.3 – 5.5 = 0.8 on a scale of 0–10.

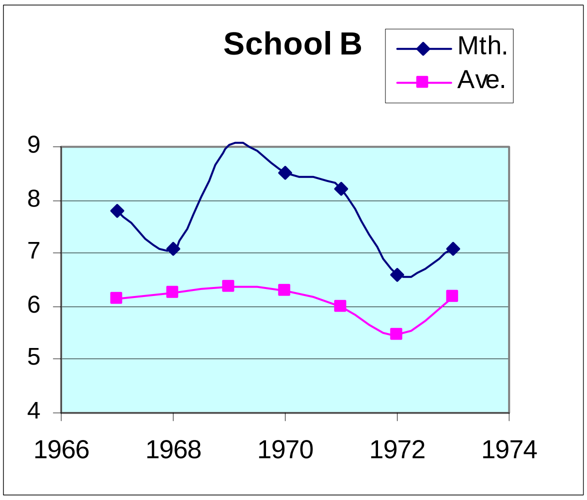
784 Percentage of the year cohort

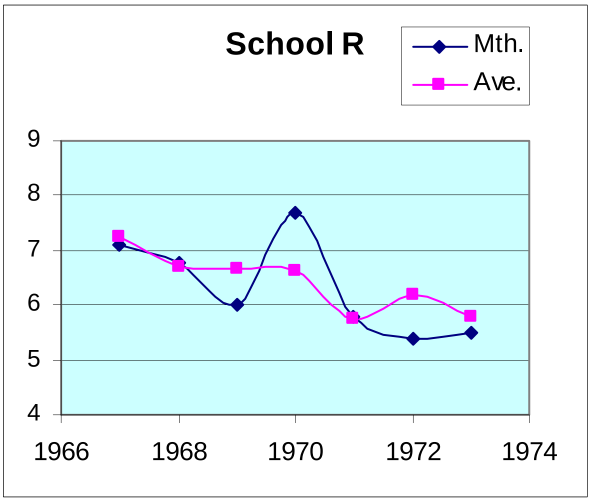
In 1967 several schools began teaching "modern" mathematics as a part of the syllabus. In 1969 all schools were obliged to take up "modern" mathematics, which may explain the sudden drop in the mathematics average.

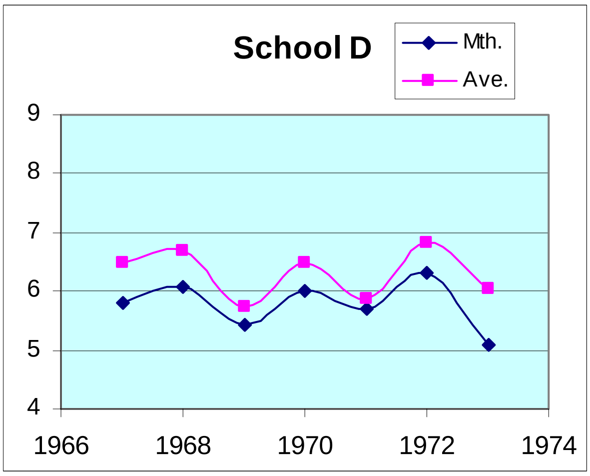
The results are shown in the graphs below.[785]

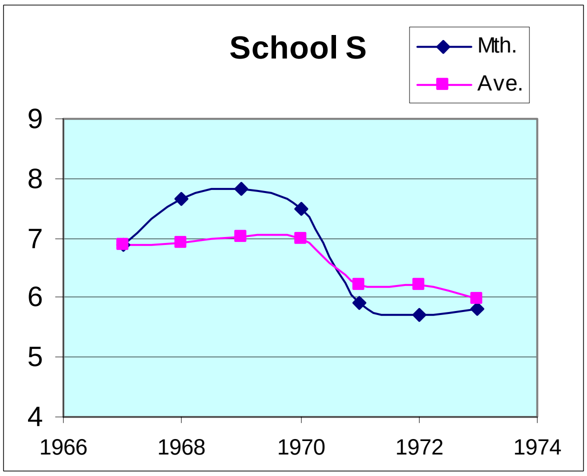












785 The Archives of the Ministry of Education: Landsprófsnefnd. Prófabækur (Protocols for the national middle school examination) 1967–1973

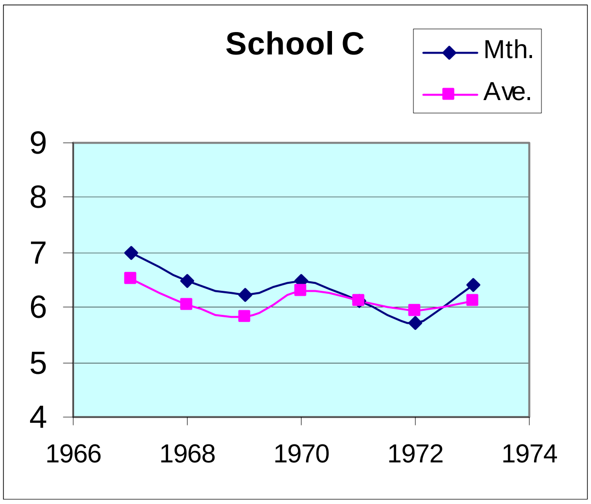


Fig. 7.6.–7.12. General results in national examination as compared to results in mathematics in seven schools 1967–1973.

In school A the mathematics teacher, law graduate, was on leave in 1967 to 1969, while non-mathematics university students and other teachers taught mathematics. After the lawyer came back and shared the mathematics teaching with a B.A.-qualified teacher, the gap between mathematics and the other subjects narrowed.

In school P, a B.A.-qualified mathematics teacher was on sick leave in 1967 and 1969. Otherwise the results were above average. The school no longer selected its pupils into the national examination stream. The headmaster said (in 2002) that the results had been better earlier when the entrance to the national examination had been restricted. The results with the present mathematics teacher had been affected by his illness. The headmaster had not realized that they were above the average when the teacher's health was good.

In school B, a B.A.-qualified teacher taught the pupils for all three years of lower secondary education and the results were by far the best in the country. In 1969, when the mathematics average in the country was 0.6 below the general average, it did not affect his pupils. The headmaster said (in 2003) that he knew that the results in mathematics were better than in the neighbouring schools. However he believed that this had been the case in most subjects in his school, as he had had excellent staff. The mathematics teacher, however, had managed to motivate his pupils to work industriously. In the author's interview with the teacher it emerged that he had the capacity to introduce ideas without being limited by the textbook. His main concern, however, was to keep the pupils working.

School R suffered from frequent changes of mathematics teachers. In 1968–1970 an engineering student taught mathematics. The first year the results dropped, while in the second year the results were exceptionally good.

School D was a typical rural boarding school with some changes of mathematics teachers with no special training, even though they avidly attended in-service courses in "modern" mathematics. The results in mathematics were generally about 0.4–0.6 below the school's average of all subjects.

School S was also a rural boarding school, where a B.A.-qualified teacher died suddenly in 1970, and a university student took over. The headmaster said in an interview (in 2002) that mathematics had been taken good care of in his school and it always had had extra hours compared to the standard number, recommended by the Ministry of Education (see section 6.4.). The teacher had been an excellent person, while the headmaster did not seem to connect the results in mathematics with his specialised education in mathematics.

School C was a school in a comparatively large town. It had a stable teaching force, and two of the mathematics teachers were in the small group nation-wide qualified with a B.A. degree. For periods the results were above the average. It has not been possible to find out who taught the national examination classes, but there was probably more than one teacher.

Before this survey, only the exceptional results in school B were known to the author of this study. The good results in school S were a surprise. They indicate that a teacher with good education and teaching skills, given sufficient time, can obtain good results. This sample of teachers is not typical for schools at that time, and by coincidence, more schools with B.A.-qualified teachers were chosen than average. The Reykjavík schools, with the majority of pupils taking the national examination, were not included in the survey, as the examination moved between institutions within the national examination period 1946–1976 and it might have been difficult to identify the teachers.

The survey shows that in this period of "modern" mathematics reform with new and unfamiliar mathematical language, the few B.A.-qualified teachers seem to have had the advantage of easier adaptation to new syllabus.

7.6. Problems in Icelandic Education in the 1960s – A Summary

Narrow Educational Road and Lack of Pedagogical Aims

From the quotations from newspapers and the professional periodical *Menntamál* in the 1960s it is clear that the narrowness of the educational road was causing growing unrest. Ordinary people could now increasingly afford to support their children for education, but education was hard to attain. The lack of material, such as curricula and textbooks, was also becoming more obvious, and the teachers were increasingly discontented with their conditions. We will now recall several of the quotes from that decade.

In the quotations from 1965 and earlier, it emerges that it was a common belief that high school studies were difficult and that it was necessary that the national examination be rather tough, to select the qualified pupils (Bjarni Vilhjálmsson, Njörður P. Njarðvík and Matthías Jónasson). Others discussed how to protect those who were not qualified from the mental strain of failing (Benedikt Tómasson). Many spoke of the need for two years' preparation time. This was counteracted by the accommodation problems of the lower secondary schools in Reykjavík which had resulted in all the national examination pupils being brought together for a period in one building for one year.

However, there were growing doubts over whether the national examination was the best tool for filtering out the right pupils (Hörður Bergmann, Matthías Jónasson) and more people wished for further education for young people without the mental strain (Matthías Johannessen). Ideas about a high school for all entered the discussion in 1968 (Gunnar Karlsson). Extreme drop-out rates of high school pupils and university students were pointed out in public and professional discussion.

One more factor discussed was the length of the school year. Eight months a year was a short time, and a large proportion of it was spent on examinations. This was counted as one of the reasons why Icelandic pupils had learnt less than pupils in other Nordic countries (Sveinbjörn Björnsson). However the matter was controversial, and still is. Pupils could usually find employment in the summer vacation, and in that way become active members of the "real world", which in itself has always been considered as education.

The lack of pedagogical staff, ongoing policymaking and support was becoming increasingly evident in the 1960s. The lower secondary school headmasters proposed at their meeting in June 1963 to establish a department or institution which was supposed to study innovations and techniques among other nations and evaluate what of it might suit Icelandic conditions. This body was to experiment with educational material, educational arrangements and teaching methods, revise curricula for the primary and lower secondary schools and implement the consequent changes. Furthermore it was to monitor that the examination requirements were always fully consistent with the schools' main role: to work for the pupils' all-round development, and it was to make proposals about the education legislation.[786]

All this was repeatedly mentioned in the general debate until Andri Ísaksson was appointed specialist in school research. In his earliest days in office he emphasized that it was a condition for progress in educational affairs that they were given priority by the authorities instead of being marginalized. *Frjáls þjóð* commented: What will one person be able to do? This turned out to be unnecessary pessimism.

The next decade saw changes in all these factors. Every pupil was then expected to take a national examination. That examination would not, however, close the doors to further education, but it rather streamed pupils, after the upper secondary level was opened up for everyone. Through a better student loan funding policy, the drop-out rate at the University reduced. The new University of Education was established. We will discuss the altered situation in next chapter.

The Back-Wash Effect of the National Examination

Evidently some method or another was needed to find the right pupils to be allowed entry to the narrow road to further and higher education. In the first decade or so after 1946, people remembered the difficulty of entering high schools in previous times (Oddur Sigurjónsson), but after 1960 people began to doubt the pedagogical value of the examination. The headmasters' meeting in June 1963 urged the Directorate of Educational Affairs to form a policy on how, and to what degree, examinations should be used in schools.

Several of the comments from the most responsible persons are cited again below.

Andri Ísaksson, in *Samvinnan* 1967, discussed at some length the current state of the examination system in Icelandic schools. It was an extensive and expensive factor in the educational system. Another reason for investigating the examinations further was that the form and nature of examinations have an unequivocal influence on the teaching method itself; often the teaching was aimed more at preparing for the examination than for life or the maturing of the pupil and this applied possibly nowhere more than in the lower secondary schools and the high schools. The nature of the national examination was such that it was well suited to that kind of approach, and

[786] *Menntamál* (1963): 36 (2–4) 223–229

the many complaints from teachers, pupils and parents that spring were evidence of this.[787]

Halldór Elíasson in *Menntamál* 1966:

> Finally I would like to express myself about the much-discussed examinations. I do not think that it is right to examine pupils in a syllabus which there is no reason for them to know. If this view were accepted, then for example one would not run examinations in seen problems. It is worth absolutely nothing for the pupils to learn these problems by heart. The less able students are tempted to do exactly so, while they ought to spend their time in a more sensible way. Another reason why such examinations are undesirable is that they have very negative influences on teaching methods. The teaching for the national examination is a sad example of this.[788]

Guðmundur Arnlaugsson in *Menntamál* in 1967:

> Opportunities to introduce to pupils the beauty hidden in mathematical thinking, even when it concerns the simplest basic things, have been neglected. … The examinations are probably partly to blame. Instead of learning "for life", the emphasis has been placed on things that look good in an examination.[789]

Dr. Matthías Jónasson's opinion, expressed in 1968, was that the national examination would have to be changed from the root. The host of incoherent details that the pupils were expected to remember was horrifying. Would the answers to such questions be the correct measure of the capacity of youngsters to study at a higher level? What about inventiveness, judgement, reasoning and creativity?[790]

The above quotations suggest that the national examination had a serious backwash effect. This fact was universally admitted, and many teachers even thought it was natural or at least necessary. An experienced mathematics teacher in the 1960s said in an interview with the author: "One was even more preparing the pupils for the examination than teaching mathematics".[791] Another mathematics teacher said: "In those years I was a specialist in hammering routines into children!"[792]

Weaknesses in Teacher Training

We have seen that the results in the national mathematics examination depended heavily on the combination of experience and good training of the teacher. But only few teachers had received special training in mathematics. No doubt the many people who taught mathematics in the post-war years did so with the best will and intentions to improve mathematics education in the country. But their basis was weak.

In 1927 Ólafur Daníelsson wrote:

> … pupils … have come up to lower secondary department examination [in Reykjavík High School], so prepared in algebra that they have perhaps only solved the exercises, but do not at all know the basis of the symbolic language, have sometimes not had any tuition in it. This shows that some of those who work on teaching do not at all have a clear idea about the purpose of this subject, think that its importance is entailed in the pupils becoming able to solve number puzzles ....[793]

787 Sigurður A. Magnússon (Ed.) (1967): 10–13
788 Halldór Elíasson (1966): 97–98. See Appendix E
789 Guðmundur Arnlaugsson (1967): 43
790 Matthías Jónasson (1968): *Lesbók Morgunblaðsins*, May 5
791 Haraldur Steinþórsson, January 22, 2003
792 Stefán Árnason, January 28, 2003
793 Ólafur Daníelsson (1927): 3–4

In 1967 Guðmundur Arnlaugsson wrote in *Menntamál*:

> Many teachers in the primary and lower secondary schools have never in their studies met mathematical thinking ... and there is a severe shortage of books in Icelandic that can improve this situation, so this is a serious problem. ...
>
> The role of arithmetic teaching should be to a considerable degree from the start to teach the child to think. ... arithmetic and mathematics must not part from another kind of logical thinking, they should precisely be the tool to train the child in logical thinking. If this is clear to the teacher, and he/she has the overview of the coherence of the topics of arithmetic that he/she is teaching, he/she could doubtless achieve a better result than many do now, even if there were few actual changes in the syllabus.[794]

Not much seems to have changed in 40 years, except the number of pupils enjoying questionable mathematics teaching. Again, the attention is turned to teacher training. How could it be improved? All teacher training underwent changes around 1970, soon after these discussions, with the establishment of the Iceland University of Education and the new B.Sc. programme in mathematics and sciences at the University of Iceland.

The Late 1960s

In the 1960s, a number of problems were defined in Icelandic education. Arithmetic and mathematics teaching was one of the problems which were attacked early. Already by the end of the decade, disappointments regarding "modern" mathematics teaching had begun to show up. The School Research Department had then been established. It had laid down a certain procedure for adopting school reforms; i.e. to set goals, write national curricula, and from there compose learning material on an experimental basis. In the crisis that came up when it transpired that the mathematics teaching experiments in the primary schools had become far too voluminous, too difficult to run in respect to guidance to teachers, and even in a few cases close to being disastrous,[795] the department decided to skip the step of setting goals and writing a curriculum, and go directly ahead to create a new set of mathematics textbooks.[796] No national curriculum document in mathematics for the compulsory level appeared until 1989.

There was also a greatly increased demand for education. New high schools were established and new options in upper secondary education were introduced, the comprehensive multi-stream schools, so the problem of shortage of well-qualified mathematics teachers was not yet solved.

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[794] Guðmundur Arnlaugsson (1967): 43–44
[795] Ragnhildur Bjarnadóttir, September 16, 2003
[796] Andri Ísaksson, March 10, 2003

8. Redefinition of the School System

8.1. Primary Level Mathematics

The Bundgaard Textbooks

Soon after its establishment, the School Research Department took up cooperation with the Reykjavík Education Office on the edition of Agnete Bundgaard and Eva Kyttä's textbooks for primary school. The publication of the first books in the series was simple, as the Danish stencils were copied and the text to be translated was minimal. The amount of text increased considerably up to the sixth grade. Kristinn Gíslason translated what was needed. The books were initially mimeographed at the Reykjavík Education Office, but the publication later moved to the State Textbook Imprint.

The content of the series was highly theoretical.[797] The commutative and associative laws, Roman numerals and place-value notation to the base five, prime numbers, permutation of three digits, the transverse sum and its relation to the nine times table were all introduced before the close of the third grade. In the fourth year the whole set theory was introduced, with pairing, subsets, intersection and union, in addition to various place-value systems and geometry with points, lines and planes introduced in a set-theoretical framework.

In short, the basis was laid for serious theoretical concepts.

The National Curriculum and the Bundgaard Material Ideology

Compared to the national curriculum, published in 1960, the Bundgaard series included all the topics required, except area computations, which were replaced by non-metric geometry in a set-theoretical framework as a considerable part of the syllabus. Some topics were introduced a little later than prescribed in the national curriculum. This is explained in the teachers' handbooks.

The teachers' handbook for the fourth grade contains a notice saying that what cannot be found in the textbooks for the first four grades required by the national curriculum in force, would be provided in the textbook for the fifth grade, except the common fractions, which, according to a curriculum for the schools in Frederiksberg in Copenhagen, was considered a difficult topic and would not be touched before the sixth grade.[798]

However, such topics as time unit computations, mentioned in the 1960 curriculum, are not touched upon in the Bundgaard series, and money is mentioned only a few times, rather marginally, in connection with the metric system. The translator suggested in the handbook for the second grade that teachers make up their own problems about money.[799] These topics were probably considered as applications and not pure mathematics, which was the intention to teach the children according to the "modern" mathematics' philosophy. Possibly it was thought that they were also treated within other subjects, could be supplied by the teacher or even would emerge naturally as a consequence of the pupils' training in mathematical thinking.

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[797] See Appendix J
[798] Bundgaard, A. (1970): 6–7
[799] Bundgaard, A. (1968): 4–5

These are examples of the textbook series having another aim than the curriculum in effect. The current national curriculum aimed at teaching children to cope with arithmetic in everyday life, while the aim of the pure "modern" mathematics textbooks was to train mathematical thinking, and the children's understanding of their environment must have been expected to arrive as a consequence.

The greatest difference from the conventional syllabus is probably the language, the vocabulary and the demands on the pupils, to read carefully the highly theoretical text. In the teachers' handbook for the first year the translator, Kristinn Gíslason, added a section about the use of language. There he says:

> Þáttur stærðfræðinnar í þroskun málkenndar hjá nemendum hefur vaxið drjúgum með tilkomu þeirra nýju viðhorfa til stærðfræðikennslunnar, sem þessi kennslubók er byggð á. Samtöl kennara við nemendur um viðfangsefnin verða nú stórum meiri þáttur í kennslunni en víðast mun hafa átt sér stað við reikningskennslu fram til þessa. Í annan stað gerir stærðfræðin strangar kröfur um skýra og rökrétta notkun máls, enda er rökvísi frumskilyrði stærðfræðilegrar hugsunar.
>
> ... gildi stærðfræðikennslu í almennum skólum er ekki einvörðungu fólgið í því, að nemendur læri að leysa tilteknar tegundir viðfangsefna, sem líkjast ýmsum vandamálum daglegs lífs. Hitt er mun mikilvægara, að glíman við stærðfræðileg viðfangsefni getur þroskað með nemendum þá nákvæmni og rökvísi í hugsun, sem gerir þeim kleift að bregðast á réttan hátt við ýmsum viðfangsefnum og taka þau réttum tökum.
>
> The role of mathematics in developing language sense has increased greatly by the introduction of the new attitudes to mathematics teaching that this textbook is based upon. Teachers' conversations with pupils about the topics will now become a much greater factor in the teaching than it has been in most places in arithmetic teaching up to this time. Secondly, mathematics makes strong demands for clear and logical use of the language, as logical thinking is the prime condition for mathematical thinking.
>
> ... the value of mathematics teaching in compulsory schools is not only concerned with the pupils learning to solve specific types of problems that simulate various problems of daily life. The other purpose is much more important: that the effort put into solving mathematical problems may develop the pupils' accurate and logical thinking, to enable them to react correctly to various problems and handle them correctly.[800]

This quotation reflects the ideology on which this new syllabus and "modern" mathematics was based. The main goal was to change the pupils' thinking, so that they could cope with any kind of problem. Simulating practical problems was of less importance than the way of thinking.

Motivation

The pupils' motivation was expected to be mainly dependent on the motivation of the teachers, which put great responsibility on them. In a letter attached to the handbook for the second grade, Agnete Bundgaard expressed her discontent that the Icelandic edition of the textbook was illustrated with drawings that were irrelevant to the text. At the end of the letter Ms Bundgaard said:

> Kære islandske kolleger. Det er Dem, der skal pröve at vise börnene, at faget i <u>sig selv</u> er morsomt og til dette formål kan jo kun benyttes ting, det er <u>relevante for faget.</u>

[800] Bundgaard, A. (1967): 7

> Dear Icelandic colleagues. It is you who shall try to show the children that the subject in itself is fun and for that aim one can surely only use items that are relevant for the subject.[801]

The handbook for the fourth grade says:

> Ef kennarinn hefur sjálfur mikinn áhuga á því efni, sem hann er að kenna, hrífast börnin ósjálfrátt með. Þess vegna getur það verið mjög svo ákjósanlegt, að kennarinn auki sjálfur við efni bókarinnar, ef honum dettur eitthvað skemmtilegt í hug.
>
> If the teacher him/herself is highly interested in the topic he/she is teaching, this will naturally be transmitted to the children. Therefore it can be very desirable for the teacher him/herself to add to the content of the textbook, if he/she thinks of something interesting.[802]

Few primary school teachers had had opportunities for further education, so the majority of them were not acquainted with these mathematical ideas. For at least some of them this was an intellectual challenge which they enjoyed.[803] For others, this may have brought great difficulty or even been a disaster, especially for those who entered the programme at a later stage than the first grade. One of the great obstacles was that the algorithms for operations with multi-digit numbers were different from those which most adults knew. The parents were advised not to assist their children in mathematics (or reading), which led to the loss of an important link in the upbringing of children, and the help needed, when the teachers could not cope with assisting all the children, was not available.[804]

Implementation

In a circular from the SRD written in 1977, signed by mathematics teaching consultant Anna Kristjánsdóttir, there are some statistics about how many pupils used the Bundgaard material, when taking a national examination at the end of the sixth grade, and the proportion of pupils still in primary school using the material. In 1976 the Bundgaard era had come to an end in the first grade of the primary schools, and it was not used for pupils born in 1969. The first group to use the material was born in 1959, so 10 year-groups had this material. The estimated proportion of the total year-groups was as follows as illustrated in Figure 8.1:[805]

1959	4%
1960	28%
1961	30%
1962	42%
1963	40%
1964	38%
1965	38%
1966	26%
1967	14%
1968	8%

Table 8.1. Estimated proportion of the age cohort taking the Bundgaard material.

801 Bundgaard, A. (1968): A letter attached to the handbook
802 Bundgaard, A. (1970): 7
803 Interviews with primary teachers
804 Anna Kristjánsdóttir (1996): 27–28
805 Menntamálaráðuneytið, skólarannsóknadeild (May 5, 1977): Circular, 1

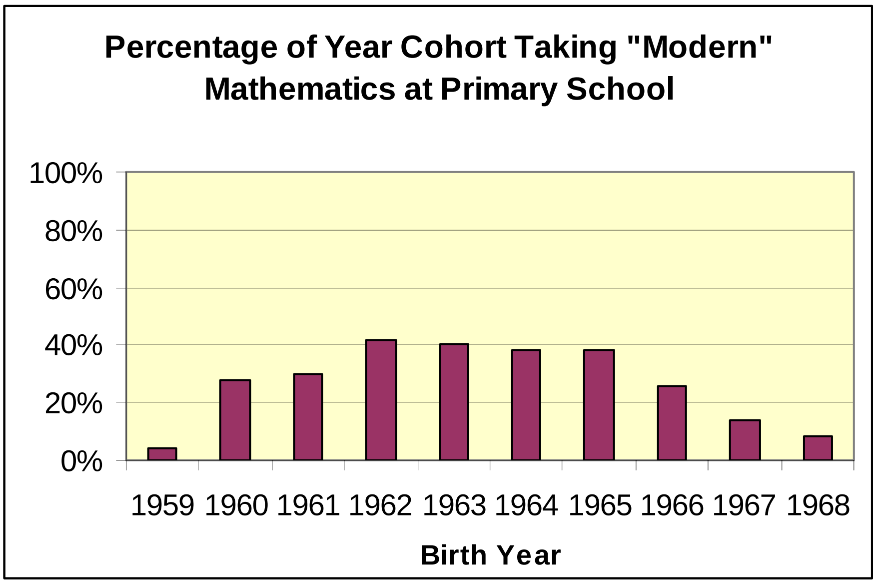

Figure 8.1: Percentage of year cohort taking Bundgaard material.

In some cases schools may have changed from the Bundgaard material to the conventional material after the 3rd grade, which indicates that about half the population of the four year-groups born in 1962–1965 in Iceland may have had the Bundgaard material.

The Content and Emerging Difficulties[806]

When Kristinn Gíslason and Kristján Sigtryggsson decided to adopt the new material, they had seen only the material for the first three year-courses, and only the first one in a nearly-final version. The first Icelandic year-group had an experimental version, while the following year-groups had the final version. It seems that the first three year-courses were easier for teachers than the latter three. The place-value system is treated very carefully in the first two grades, preparing the pupils for addition and subtraction with multi-digit numbers. Methods based on the associative law help pupils to add and subtract mentally. Multiplication is prepared with Venn diagrams together with emphasizing the special role of zero in multiplication. Attention is drawn to the commutative law.

In the second half of the third grade, more difficult items arrived, such as prime numbers and the commutative and distributive laws to help with multiplication of two two-digit numbers. When it came to algorithms to multiply multi-digit numbers, the method was unfamiliar to teachers and parents. The third year-course concluded with arithmetic to the base five.

As nearly all the teachers had studied only at the Teacher Training College, where mathematics education was minimal, they had to work hard to cope with this new material. The in-service courses were a great support for them. A teacher, who only taught the youngest pupils, the first to third year, reported that she enjoyed it and she thought it was much better than the material she had taught before. In her school there was a special teacher to guide the teachers through the material.[807]

[806] Bundgaard, A. and E. Kyttä (1967–1968). Bundgaard, A. (1969–1972)
[807] Vilborg Dagbjartsdóttir, February 12, 2002

More difficulties emerged in the latter three years. Firstly, the classes often changed to new teachers after three years. The new teachers who taught in the middle years, fourth to sixth grade, did not know the background of this new project, and had a hard time to learn it all at once. Secondly, the material became increasingly complex. The division algorithm was scrupulously prepared, after which there was a long section with the whole set theory and its notation. Prime numbers vs. composite numbers were connected with the union and intersection of e.g. the set of divisors of 21 and 45, in the first semester of the fourth grade, for 10-year-old children.

In the second semester of the fourth grade, geometry was introduced in a set-theoretical framework. This finally led to measurements and decimal fractions. The four operations in decimal fractions were introduced quickly, and the problem of multiplying two decimal fractions was solved by multiplying and dividing by an appropriate power of 10.

The metric system was introduced, to be continued for much of the first semester of the fifth grade, which concluded with computations modulo nine and division by decimal fractions. The second semester opened with functions, relating volume to mass etc. to introduce rate and proportions. Thereafter geometry was revisited to study the unions and intersections of points, lines, planes and half-planes. Finally, there was estimation and rounding of decimal fractions. In the final textbook, for the sixth year, there were common fractions followed by the concept of a multiplicative inverse etc. The numbers were minimally represented with pictures, only the number line.

In conclusion, there were many theoretical concepts with which the teachers may not have been familiar. Furthermore, there was more and more for the pupil to read and understand in order to be able to continue, and too little that average pupils or below could work on by themselves. In the handbooks the pupils were divided into the "efficient" or "quick" pupils, as opposed to the "slow" or even "slow-witted", who were not expected to grasp every idea.[808]

Ragnhildur Bjarnadóttir, now professor in psychology at the Iceland University of Education, studied mathematics teaching at *Danmarks Lærerhøjskole* / the Royal Danish School of Educational Studies after high school graduation and her teacher's certificate. Ragnhildur Bjarnadóttir taught mathematics to one group through the fourth to sixth grade. This was a group of able pupils and she said that they had studied the material together. Many of them had become experts in mathematical subjects later. But had the pupils been an average group, she did not think that the result would have been good. When the time came to replace this material, Ragnhildur Bjarnadóttir became one of the authors of the new series of textbooks.[809]

Discussions

When the Bundgaard project had been tested for a couple of years, many switched to the old syllabus after the third grade, when a basis of the number concept had been built up. However, most of the schools continued with the material, eventually with some extra exercises to train skills.

The "modern" mathematics approach was much discussed and in a recent history of Iceland in the 20th century, this clause is found:

[808] Bundgaard, Agnete (1970): 7
[809] Ragnhildur Bjarnadóttir, September 16, 2003

> Risu … deilur um nýtt námsefni í stærðfræði, þar sem áhersla á rökræn viðfangsefni vék til hliðar þjálfun í reikningsaðferðum, og um Íslandssöguna …
>
> Disputes rose about a new syllabus in mathematics where emphasis on reasoning tasks pushed training in computation algorithms aside, and about Icelandic history …[810]

On asking the author about sources for this comment, he said that he based it on his own memory from discussions, e.g. in school staff rooms. A clause from an article in the first issue of *Menntamál* in 1972 by Hörður Lárusson, who was mathematics teaching consultant at that time, situated in the new School Research Department, points to such disputes:

> Meðal foreldra, kennara og fleiri, sem fylgzt hafa með störfum skólanna síðustu árin, hafa farið fram miklar umræður um nýtt námsefni í stærðfræði, sem fyrst var tekið upp í barnaskólum Reykjavíkur haustið 1966. Menn hafa haft mjög skiptar skoðanir á þessu nýja námsefni, og er ekki nema gott eitt um það að segja. Námsefnið í heild hefur ekki verið kynnt almennt, né heldur þau markmið, sem stefnt er að, og má vera að það valdi nokkru um hluta þeirrar gagnrýni, sem fram hefur komið. Útbreiðsla þessarar nýjungar varð miklu meiri og örari en ráð var fyrir gert, og nú í vetur, þ.e. skólaárið 1971-72, munu næstum öll börn í 1. bekk barnaskólanna í Reykjavík, auk fjölmargra annarra úti á landi, læra þetta nýja námsefni.
>
> Among parents, teachers and others who have observed the activities of the schools during the last few years, there have been many discussions about a new syllabus in mathematics, which was first introduced in Reykjavík primary schools in the autumn of 1966. People have had very different views on this new syllabus, and that is all well and good. There has been no general publicisation of the syllabus as a whole, nor of its goals, and this may partly cause the criticism which has emerged. This innovation became far more widespread far quicker than was planned, and now in this school-year, 1971–72, nearly all children in the first grade in Reykjavík primary schools, in addition to a great many in the regions, are studying this new material.[811]

The year cohort in question was born in 1964. That year, on December 1, 1971, exactly 40% of the population lived in Reykjavík,[812] so according to Hörður Lárusson's account considerably above 40% of the population began their mathematics within the new syllabus, compared to the 38% of that age cohort reported in 1977. This points to a fact, generally recognized, that many switched over to the conventional syllabus after the first three years.

Another article about the new mathematics in the second issue of *Menntamál* 1972 was written by an elderly mathematics and physics teacher, Magnús Sveinsson. He praised Guðmundur Arnlaugsson and his book *Numbers and Sets*. Then he said:

> Þótt þessar nýju reikningsaðferðir séu ágætar í sjálfu sér og rökfræði þeirra sé til skilningsauka og létti að einhverju leyti annað nám, þá verður að telja það mjög hæpið að láta fræði þessi flæða hömlulaust á örskömmum tíma yfir allt fræðslukerfið. Ég hef heyrt, að t.d. Danir taki þessu með varúð, reyni kerfið fyrst í nokkrum aldursflokkum, en séu ekkert á leið með að steypa því yfir allt skyldunámsstigið. Einnig hef ég heyrt, að til séu skólar bæði í Danmörku og víðar á Norðurlöndum, sem ekki hafi innleitt þessar reikningsaðferðir hjá neinum aldursflokki.
>
> Although these new computation methods are good in themselves and their logic increases the understanding and eases to some degree other studies, it must, however, be deemed very questionable to let these studies flood without restrictions

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810 Helgi Skúli Kjartansson (2002): 421
811 Hörður Lárusson (1972a): 9
812 Statistics Iceland: website

in a very short time over the whole education system. I have heard that e.g. the Danes take this cautiously, test the system in several age groups, but are not going to throw this over the whole compulsory school level. I have also heard that there are schools, both in Denmark and elsewhere in the Nordic countries, where these computation methods have not been introduced for any age group.[813]

Magnús Sveinsson went on to claim that a young person completing school at the age of 15 that spring would know less general arithmetic than one who graduated some years before, and that he would prefer a pupil from the old system to serve in a shop. And his view was that new textbooks were needed, where new and older methods were connected in a simple way, so that slow learners had no less knowledge in general arithmetic than before the changes.

In *Alþýðublaðið* on July 24, 1973, Hörður Lárusson is cited that the set-theoretical concepts had had abnormal emphasis instead of being used as help concepts to traditional methods. There was no doubt that this had reduced pupils' number skills, which was detrimental to their later studies in secondary schools and at work. Therefore, alterations were being made. These alterations would without doubt contribute to pupils' broader perspectives and deeper understanding of mathematics at all levels and spheres of mathematics. Abroad, people were making voluminous experimentations on the content and presentation of textbooks with special consideration to those who have difficulties in studying mathematics.[814]

This piece of news is not written by a knowledgeable person, as Hörður Lárusson is titled historian. However, it reflects that at that point of time the set-theoretical emphasis was over. Hörður Lárusson and Andri Ísaksson from the School Research Department were now presenting a new project which was expected to remedy the mistakes made at the primary level. The reorganization was to be along guidelines made after a Nordic committee's two years work, mainly for the lower secondary level. Possibly this is the MUNK-cooperation.

The news release confirms that the "modern" mathematics at primary level was now conceived as mistake. However, now when a new plan was being made, the new promoters, Andri Ísaksson, the outgoing director of the SRD, and Hörður Lárusson, the incoming director, succeeded in persuading those who supplied the sources to continue the financial support (see section 7.4.).

Otherwise it is difficult to find any public or professional discussion about mathematics teaching. But the fact remains that as early as 1971, five years after the first implementation of the Bundgaard textbooks and before they were introduced in the sixth and last grade, a workgroup was established to prepare a new series for the primary level, a series that was intended to replace the Danish textbooks by Agnete Bundgaard et al.

[813] Magnús Sveinsson (1972): 89
[814] Skjalasafn Fræðslumálaskrifstofunnar 1989/S-56 Skólarannsóknir

Reasons for Difficulties

There are several reasons for why the Bundgaard material caused disappointment. Anna Kristjánsdóttir has listed some of them,[815] and her points will be included in the following list.

1. Although "modern" mathematics had been discussed for a couple of years after Kristján Sigtryggsson travelled to the United States, only four months passed from the time when news arrived about the Danish material until it had been implemented in seven first-grade classes. The material was still operated on an experimental basis in Denmark, and the fourth- to sixth-grade material had not been composed at the time of the decision. As the material for the first and second years was comparatively easy to handle, the later problems were not foreseen.
2. It was an unfortunate decision to allow for 86 teachers to take the new material in their classes already in the second year. It was too much for one person to offer the guidance and support needed for all of them. Some teachers were not even especially interested in participating in this project, which cost them a lot of extra work.
3. The teachers' mathematical background from the Teacher Training College was weak. This may not have been fully realized at the time of the decision, as the second half of the project had not been published and was not known. There was little reading material for the teachers to support them concerning the content, and they would have to rely on the two-week in-service courses each autumn and the monthly meetings. Teachers entering the middle of the project needed special support which probably could not be offered.
4. The algorithms presented in the Bundgaard material, especially in multiplication, and also in subtraction and division, were different from the ones known by Icelandic adults. Therefore both new teachers, stepping in, and the parents, with good intentions, might at a sensitive moment impose different methods on the pupils without trying to find out what they had been learning.
5. Previously, teaching arithmetic was teaching algorithms, while the main emphasis in the Bundgaard material was to teach the pupils to think. The teachers had no training in teaching the pupils to think, and may not have understood that purpose, which made their task the more difficult.
6. The changes were implemented by the authorities. The teachers themselves were in many cases not aware of the stagnant situation, and that changes were needed. They concentrated on the new content but many of them may not have changed their teaching habits.
7. It was Agnete Bundgaard's policy not to let children take their mathematics home, in order to avoid the risk of their parents confusing them with their older methods. By this, the opportunity was lost for individual pupils who lagged behind to receive help at home, and an important connection to the home was cut off.
8. The word *mengi*, meaning a set, was a problem in itself. Unlike the English word *set* and the Danish word *mængde*, the Icelandic word was a neologism,

[815] Anna Kristjánsdóttir (1996): 26–28

coined for the purpose. It did not have any previous meaning and therefore had no concrete reference. In the minds of the public, this word came to signify the new algorithms which the children were supposed to use and their parents did not understand.

9. Parts of the syllabus were too difficult for average pupils and below. It demanded good reading ability and maturity to think over hard questions. It therefore required much of the teachers and for many of them it probably was too difficult a task.

Anna Kristjánsdóttir mentioned that teachers were unaware of the need for changing the ways of teaching.[816] The Bundgaard material was accompanied by a booklet with instructions to the teachers for each year course, something that the teachers had not had before. According to the handbooks, there was to be increased emphasis on communication and discussing the topics with the children, and the teachers were guided in that task. A handbook and a two-week in-service course every year were a novelty, but in many cases that did not suffice to change the old habits of teaching, having pupils compute a long series of similar problems, without constructive discussions. As pointed out in item 5, the teachers had no training in teaching children to think. The changes were in content rather than in pedagogy.

The Bundgaard material was not the right choice for inductive way of work, investigations or discovery learning. The handbooks did not contain any instructions about investigative work, so that kind of instruction was not part of this new programme. Other projects, like the UICSM projects prepared at the University of Illinois, had different tactics. The UICSM has been analysed as being characterized by high pupil activity and an inductive way of work.[817] Kristján Sigtryggsson mentioned this project in his article, and primary school mathematics might have developed differently if the material he brought back from his U.S. trip had been studied further, and chosen instead of the Bundgaard material.

Concerning teaching methods, “modern” mathematics brought changes in the high schools. Jón Hafsteinn Jónsson at Akureyri High School says that he stopped calling pupils “up to the board,” as questioning pupils was called, and started lecturing.[818]

Anna Kristjánsdóttir also mentioned that discussions had hardly begun about the use of computers and pocket calculators and their potential. The teachers did not foresee the development of new teaching methods that they would entail.[819] This was the case in all countries. Not until nearly a decade later did pocket calculators become common, and another half decade passed before it was practical to talk about computers in mathematics education.

Preliminary Mathematics Curriculum for Primary School in 1970

In December 1969 a committee of four persons was appointed by the Ministry of Education to write a preliminary mathematics curriculum for the primary school level. The chairman was Kristinn Gíslason, primary-school teacher, the translator of the Bundgaard material. Other members were Ásdís Steinþórsdóttir, primary-school teacher (daughter of Steinþór Guðmundsson, the former mathematics representative

[816] Anna Kristjánsdóttir (1996): 27
[817] Gjone, G. (1983): Vol. I, 15
[818] Jón Hafsteinn Jónsson, August 21, 2003
[819] Anna Kristjánsdóttir (1996): 28

of the National Examination Board), Eiríkur Jónsson, mathematics teacher at the Teacher Training College, and Ragnhildur Bjarnadóttir, primary-school teacher, who had become involved in teacher training in the "modern" mathematics in-service courses. The committee collaborated with Hörður Lárusson, SRD's mathematics teaching consultant. The report containing proposals for a curriculum was handed in on June 11, 1970.

In an introduction to the proposal, the committee declared that it had unanimously agreed to adhere to the "modern mathematics wave".[820] The committee studied textbooks and national curriculum documents from other countries.[821] In its list of textbooks there are four sets of American textbooks for the first six grades, among them textbooks by Patrick Suppes [in Stanford] et al., books from the School Mathematics Study Group, SMSG, a set of English textbooks by L. G. Marsh, the Danish series by Agnete Bundgaard et al. for the first four grades and *Matematik M1–M3*, published by the NKMM.

The committee had of course studied the current national curriculum from 1960, in addition to the following documents:

- *Undervisningsvejledning for folkeskolen*. Undervisningsministeriet, Copenhagen.
- *Læreplan for forsøk med 9-årig skole.* Forsøksrådet for skoleværket, Oslo [1st ed. 1960, 2nd ed. 1964].[822]
- *Läroplan for grundskolan*. Skolöverstyrelsen, Stockholm.
- *Nordisk skolmatematik*. Nordiska kommittén för modernisering av matematik-undervisningen [NKMM]. Stockholm 1967.
- Bent Christiansen: *Mål og midler i den elementære matematikundervisning*. Munksgård, Copenhagen 1967.

Not all these documents were written under the influence of the "modern" mathematics wave, e.g. not the Norwegian one, which was first published in 1960. A lot of work had been done in Norway in the period since 1960. However, a provisional Norwegian national curriculum in two versions or "alternatives", a conventional one and a "modern" one, M 71, was not published until 1971, and a final version, M 74, in 1974.[823] Knowledge of the work of the Nordic Committee for Modernizing Mathematics Teaching, NKMM, is obvious, as the committee studied both the Bundgaard textbooks, based on the experimental texts of the NKMM[824], and its final report.

Naturally, the committee looked to the other Nordic countries to prepare their proposal, recalling that Iceland had inherited its educational system from its relationship to Denmark. With reference to these documents, the committee said that it was difficult to cover the same syllabus as the other Nordic countries, since the school year in Iceland was so much shorter than there, so either the school-year would have to be longer or more weekly hours allocated to mathematics. However, the committee did not look upon it as its task to make recommendations about the number of teaching hours in mathematics. Secondly, the committee noted that if this proposal for a curriculum were to be accepted, no textbooks would be available in Icelandic

[820] *Drög að námsskrá í stærðfræði fyrir barnafræðslustig. Nefndarálit* (June 1970): 2.
[821] *Drög að námsskrá í stærðfræði fyrir barnafræðslustig. Nefndarálit* (June 1970): 19
[822] Gjone, G. (1983): Vol. VIII, 33
[823] Gjone, G. (1983): Vol. VIII, 34
[824] Gjone, G. (1983): Vol. II, 92

and new ones would have to be written, which indicates that the Bundgaard textbooks were not a model for this curriculum proposal.

At a closer look, the emphasis on set-theoretical concepts is greatly reduced as compared with the Bundgaard material. Admittedly, the list of items is written in set-theoretical notation. However, set-theoretical concepts are only mentioned in two grades, the first grade and the fourth grade. The concept *set* was to be used for explanations as a help concept in the first grade, where the following concepts were to be introduced: Equal sets, union, subset and empty set, and furthermore the concepts and corresponding symbols "greater than" and "less than". In the fourth grade, these concepts were refreshed, and the intersection and set difference together with the corresponding symbols added to them. These concepts were, according to notes attached to the list of topics, expected to be used regularly in the teaching, even together with the concepts "relation" and "mapping". However, knowledge of these concepts was not to be an aim in itself. The commutative, associative and distributive laws were all supposed to be introduced.

Applications, such as money and time computations, were to have a place in the curriculum, and common fractions were to be introduced as early as the third grade. The size of fractions and comparison of two fractions were to be examined in the fourth and fifth grade, whereas addition of fractions with the same denominator was to be taught in the fifth grade, and with different denominators, prepared in the fifth grade and continued in the sixth grade, as was provided for in the 1960 national curriculum.[825]

Geometry appeared in the curriculum proposal without any further definition. The committee said in its introduction that geometry should be taught in the fourth to sixth grade, and it should be well prepared by giving the children an opportunity to view the tasks on their own, but it did not feel confident enough to make any suggestions about topics in geometry, as no experience was available in Iceland. When this was written, early in 1970, the first groups of pupils using the Bundgaard material had not yet reached volumes 4b, 5a and 5b of the textbooks, containing geometry, and 5b had not yet been published. Geometry, with the exception of measurements and computations of length, area and volume, had never been included in the compulsory syllabus in Iceland.

In general, the committee emphasized understanding, as the previous curriculum papers had done. It also emphasized plausible explanations, often within the frame of sets of touchable objects, followed up by Venn diagrams of sets of visible objects. There seems to have been a choice between the conventional multiplication and division algorithms, where the method is more digit-oriented, and the algorithms introduced in the Bundgaard material, where the methods were more number-oriented, beginning with the first digit in multiplication. Either way, the algorithms were to be prepared and built on previous investigations and plausible explanations. Measuring units were also to be introduced carefully by measuring. A clear understanding of the place-value system was discussed without further reference. Multiplication of two common fractions was expected to be taught in the sixth grade, preferably by dividing squares or rectangles into smaller parts, thus expecting pupils to know area.[826]

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[825] *Drög að námsskrá í stærðfræði fyrir barnafræðslustig. Nefndarálit* (June 1970): 4–12

[826] *Drög að námsskrá í stærðfræði fyrir barnafræðslustig. Nefndarálit* (June 1970): 13–18

Primary Level Textbooks after 1970

This preliminary curriculum never reached the status of regulations. In the next couple of years, the School Research Department started to work on a new series of textbooks.[827] Book A for the first grade was published in 1972. At that time Ragnhildur Bjarnadóttir, one of the members of the committee, had joined the workgroup to publish the textbooks B and C for the first grade, book A for the second grade in 1973 and more books in the following years. In the final editions of these textbooks, sets are hardly mentioned, except as help concepts in connection with solutions to an equation. Geometry is treated in various ways, and area and perimeter have their proper place. Admittedly, the series developed through subsequent editions, away from the set-theoretical approach towards a more conventional one.

As the curriculum paper was only preliminary and not very detailed, one could say that the content of the new series of textbooks was not at odds with it. However, the spirit is different. The remark that the committee had unanimously agreed to adhere to the "modern mathematics wave" is only slightly reflected in the textbooks, although Ragnhildur Bjarnadóttir was a member of the textbook writing group. She has said that she was facing a dilemma, assisting Ms Bundgaard at her in-service courses for teachers, while Ragnhildur Bjarnadóttir herself had begun to write new textbooks for SRD.[828] Enthusiasm for the "modern mathematics wave" seems to have passed its peak at the primary level in Iceland before 1972, even though the first experimental group of the Bundgaard material had not yet reached the sixth primary grade.

The SRD series brought a new approach to primary school mathematics. With their great variety of tasks, they are very different from the traditional syllabus of Jónas B. Jónsson's and Elías Bjarnason's books. The new series had other limitations which will not be discussed here. One of its merits was few and varied problems, which would offer opportunities to discuss and promote thinking and wondering about the various properties of the concepts in question. As time passed, more and more support material was published, both for stimulating mathematical thinking and problem solving, and others with a number of skill training exercises, which many teachers favoured. This raises questions about policy in mathematics education. Why is it that many teachers prefer a number of problems to discussions and cogitation? Is it their lack of training or background knowledge, or is it a more successful procedure to let the pupils compute and hope that tacitly they will discover relations and properties and develop mathematical thinking?

The SRD series was in use in the primary grades for nearly three decades. By 2004 it was fully replaced by a new set of textbooks published by the National Centre for Educational Materials, NCEM, the former State Textbook Imprint.

Hörður Lárusson, who had been mathematics teaching consultant, became the director of the SRD in 1973. He stayed in that position until 1984 when the SRD was dissolved. Anna Kristjánsdóttir, who had been mathematics consultant for teachers in Reykjavík, employed by Reykjavík Education Office in 1972–1975, became a leader of mathematics activities within the SRD in 1975, a post which she held until 1981. One of Anna's last efforts at Reykjavík Education Office was to set up an exhibition

[827] See Appendix L
[828] Ragnhildur Bjarnadóttir, September 16, 2003

on mathematics teaching, where she presented new teaching methods, e.g. use of tools, models and games and practice of open-ended problems.[829]

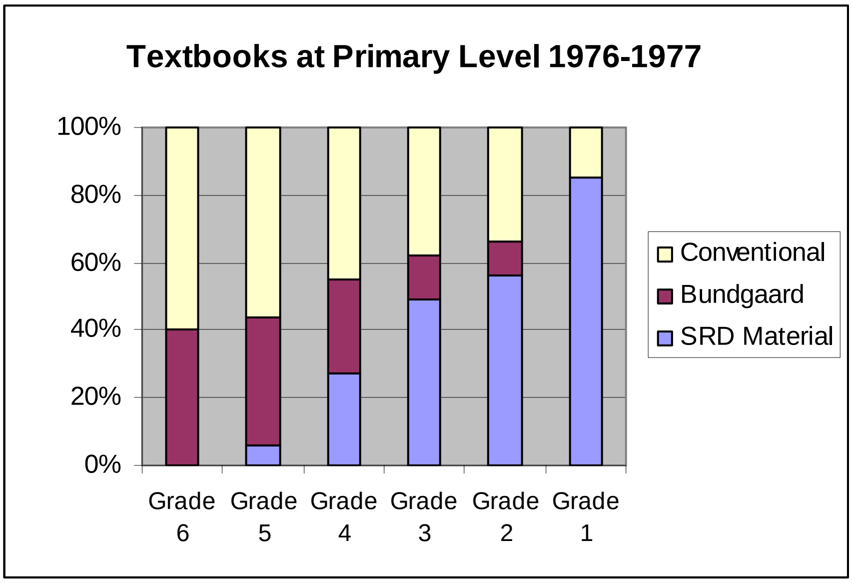

Figure 8.2. The use of mathematics textbooks in primary grades in the school-year 1976–1977. The Bundgaard material has completely disappeared in the first grade.

In a circular from the SRD, dated May 5, 1977 and signed by Anna Kristjánsdóttir,[830] it appears that in the preceding school year, 1976–1977, the textbooks by Agnete Bundgaard had disappeared in the first grade; 85% of the first grade used the SRD material, while about 15% were still expected to run the conventional syllabus, as opposed to about 60% taking the conventional syllabus in the sixth grade and 40% using the Bundgaard material. In the fifth grade about 6% used the SRD material, and an increasing proportion of each year-group down to the first grade. The Bundgaard material might have disappeared even more quickly if the authorities had not been very careful to keep the number of teachers and classes in manageable size while the new material was being developed and implemented.[831]

8.2. Lower Secondary Level Mathematics

Hörður Lárusson

Hörður Lárusson was a promising mathematics teacher at Reykjavík High School from 1958. He completed a B.A. degree in mathematics at the University of Iceland in 1960. Hörður Lárusson was supported by the educational authorities to study mathematics in the United States, to complete a master's degree in 1966, so that he could join the reform group in Iceland.[832] After returning home, he taught at Reykjavík High School until 1972 and worked as mathematics teaching consultant at the Ministry of Education, 1969–1973, first in a part-time position. When Andri Ísaksson left the Ministry in 1973, Hörður Lárusson was appointed director of the School Research Department. During the 1970s, Hörður wrote several series of textbooks for the upper and lower secondary level.

[829] *Skólaskýrsla : barna- og gagnfræðaskólar Reykjavíkur : skólaárið 1975–1976*: 27–30
[830] Menntamálaráðuneytið, skólarannsóknadeild (May 5, 1977)
[831] Guðný Helga Gunnarsdóttir, May 18, 2006
[832] Hörður Lárusson, March 26, 2002

Preliminary Curriculum for Lower Secondary School

The National Examination Board issued a syllabus every year with lists of pages in textbooks which were to be studied. It was not until 1968, at the initiative of the SRD, that a formal preliminary curriculum was published, containing general aims and a list of topics in addition to comments of advice to teachers. This document for mathematics, written by Björn Bjarnason, has previously been discussed in detail (see section 7.5.). It is obviously written under strong influence from the "modern" mathematics wave, suggesting that set algebra should be studied before the regular number algebra, the main topic in the national examination school year. The idea was that the use of set algebra and its concepts would act as unifying structure, and thus help the pupils to unify the various branches of mathematics, which they met.

That same year a preliminary curriculum for the new proposed standardized lower secondary school examination was published. The authors of the mathematics part were Þórður Jörundsson, a lower secondary school mathematics teacher, and Hörður Lárusson, then a high school teacher. This draft also presented general aims for mathematics teaching and a list of topics where each of them is treated with comments to the teacher. The general aims were:

Stærðfræðikennslunni skal haga þannig, að hún veiti nemendum:

1. Hagnýta þekkingu á tölum og meðferð þeirra.
2. Leikni í almennum reikningi.
3. Nokkra þekkingu og skilning á stærðfræðinni.
4. Þjálfun í rökrænni hugsun.
5. Þá þekkingu í stærðfræði, sem nauðsynleg er til inngöngu í aðra skóla.

The mathematics teaching shall be organized in such a way that it provides pupils with:

1. Practical knowledge of numbers and their treatment.
2. Skills in general arithmetic.
3. Some knowledge and understanding of mathematics.
4. Training in logical thinking.
5. The knowledge in mathematics, which is necessary for entrance into other schools.[833]

The list of aims is different from the list presented for the national examination. The list for the general lower secondary school examination is more concerned with skills in general arithmetic and practical knowledge of numbers and their treatment. The general aims for the national examination of the middle school contain understanding of our number notation and the nature of arithmetic operations, solid ideas about the basis of algebra (set theory?), understanding of the symbolic language and its rules and skills in rewriting algebraic sentences, as well as translating sentences from everyday language to the symbolic language of algebra. Set theory was not mentioned for the lower secondary school examination, but it entered the topic list for the next year (1969).[834] In contrast to the preliminary curriculum for the national examination classes, here the main requirement is that pupils become familiar with the set-theoretical notation, which is said to be both simple and clear. There is no comment that the set algebra is of importance or should precede number algebra.

There are some explanations for the difference of attitudes to the understanding of the nature of mathematical operations, the basis of the number system and the basis of

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833 Samræmingarnefnd gagnfræðaprófs (September 1968): 34

834 Samræmingarnefnd gagnfræðaprófs (August 1969): 44

algebra in the preliminary curricula for the two standardized examinations, which within five years had merged into one.

One reason could be that the national examination of the middle school was intended for pupils aiming at high school and higher education, i.e. university. The purpose of the lower secondary school examination was to offer a standardized education for those pupils who were e.g. going to attend "other schools", understood to mean vocational colleges, e.g. the Technical College, the Nursing College or the Teacher Training College. These two examinations were thus originally intended to be organized according to a certain differentiation which had established itself in the lower secondary level during the two decades when the national examination of the middle school had been run as a continuation of the previous six years' high school heritage.

Another explanation is simply differing views of the writers. Hörður Lárusson, for example, wrote a series of textbooks for the three grades of lower secondary education up to the national examination, later compulsory school examination, in strict set-theoretical notation but essentially without set algebra.

At a conference held in August 1968, preparing the standardization of lower secondary school mathematics examinations, where the main speaker was Hörður Lárusson, the view was put forward by one of the teachers that teaching in the fourth year in lower secondary school should aim at those who were going into working life after completing the lower secondary school, and not at the small group that was preparing for the Teacher Training College or the Technical College.[835]

From the above it is clear that there were three groups leaving the lower secondary level: Those who were aiming at university studies through the high schools, those who were aiming at the vocational colleges, and those who had no plans for further study and were aiming at "working life". Each group was supposed to have its special needs. During the next few years other social aspects came into effect, concerning equality and that no one should be deprived of education. Influences from the 1968 student uprisings in Europe were reflected in the train of events in the following years. Within only a few years, such classification into groups in the lower secondary schools was no longer mentioned.

The two national examinations merged in 1974, to remain so until 1976. However, those taking the national middle school examination after three years generally did better than those who completed a lower secondary school examination after four years of study. In 1974 the national average for the 1,515 pupils in the examination for the middle school in mathematics was 6.22 on a 0–10 scale, while for the 2,242 pupils taking the examination for the lower secondary school the national average was 3.69.[836] Noting that a total of 3,757 pupils took the examination and the 17-year-old population in 1974 numbered 4,469,[837] only few did not take that examination. A large group of the 700 who were missing was probably the rural population who left school after confirmation, a hidden resource of talented people.

[835] *Fundargerð ráðstefnu samræmingarnefndar gagnfræðaprófs* (August 30, 1968)
[836] Archives of the Ministry of Education: Prófanefnd [1974a]: *Dreifingar einkunna á lands- og gagnfræðaprófi vorið 1974. Stærðfræði*
[837] Statistics Iceland, website

The SRD and Lower Secondary Level Textbooks

The Bundgaard primary school project had already been prepared when the SRD was established, so mathematics was not much discussed there at the beginning. The main cost of the project was presumably the expense of in-service courses which were held every year with increasing number of teachers. For the teachers this was probably a welcome opportunity for further education and for enjoying the company of fellow teachers.

It was important to be prepared to meet the primary school pupils who had studied the Bundgaard material, when they entered the lower secondary level. Hörður Lárusson taught part-time in 1970–1972 at Hagaskóli, a lower secondary school, where he ran an experiment with individualized programmed material in cooperation with Halldór I. Elíasson. However, this project did not turn out to be promising and soon the activities took another direction.[838]

The Reykjavík Education Office initiated the reform in 1966. When the School Research Department was established these bodies took up cooperation on the publication of the primary textbooks by Bundgaard et al. and the organization of the teacher's in-service courses. The Reykjavík Education Office continued, however, to work on the reform. In February 1969 Prof. Bent Christiansen at the Royal Danish School of Educational Studies came to Iceland to discuss mathematics teaching on the lower secondary level and a possible publication of his books for that level[839] with Director Jónas B. Jónsson. A plan was made to arrange a month-long in-service course each year for three years for about 45 teachers before a new syllabus would become compulsory. Subsequently Director Jónas B. Jónsson discussed possible financial support from OECD with Dr. Wolfgang Edelstein, who wrote to the OECD to request funding of $5000,- a year for three years. In that letter Dr. Edelstein said that even if only the first year could be ascertained, they should go ahead, the project being an innovation project of the sort they should be interested in. This information appears in a letter, and its attachment, from Director Jónas B. Jónsson to Hörður Lárusson in the School Research Department. In the letter, Jónas B. Jónsson stated that a decision was urgent upon further cooperation between the Directorate of Educational Affairs and Reykjavík Education Office about the organization and expenses of such courses.[840] It is not known what became of these plans. A Swedish educator came up to Iceland to give a course on the so-called Swedish textbooks, mentioned below. However, the School Research Department soon began to publish its own home-made textbooks as we shall see.

As an intermediary step, the State Textbook Imprint published *Algebra* by Már Ársælsson in two volumes in 1971.[841] It had previously been published in manuscript in 1969. According to the forewords, its vocabulary was adjusted to Guðmundur Arnlaugsson's *Numbers and Sets*. Presumably, the intention was to replace Dr. Ólafur Daníelsson's *Algebra*. This edition, however, did not achieve great distribution as the School Research Department was taking action for the whole secondary level.

The first step was to translate a set of Swedish textbooks for the seventh to ninth grade.[842] The first book was *Rúmfræði / Geometry* by G. Bergendal, O. Hemer and N.

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[838] Hörður Lárusson, March 26, 2002
[839] Christiansen, B. et al. (1967–1968). Christiansen, B. et al. (1969)
[840] National Archives of Iceland: Skjalasafn Fræðslumálaskrifstofunnar 1989/S-56, Skólarannsóknir
[841] Már Ársælsson (1971)
[842] See Appendix M

Sander, published in 1970 as a provisional publication. It was translated by Anna Kristjánsdóttir, who at that time was Bent Christiansen's student at *Danmarks Lærerhøjskole* / the Royal Danish School of Educational Studies. In a foreword, Guðmundur Arnlaugsson expressed regrets that until that time the only geometry available for Icelandic lower secondary schools had been the practical part of it; i.e. a collection of rules about area and volume. The part of geometry concerned with observation, understanding and logical reasoning had been missing, and it was expected that this book would fill that gap. This geometry textbook was one of the innovations of the NKMM. It was supported by the basic concepts of set theory and it aimed at opening the eyes of the pupils to the structure of geometry, training him/her to think and reason, and in that way creating a basis for further studies.[843]

This geometry textbook was clearly conceived as a continuation of the Bundgaard textbook series, both composed by the initiative of the Nordic cooperation committee, NKMM.

In 1970 or 1971, Anna Kristjánsdóttir and Hörður Lárusson translated *Algebra unglingaskóla / Algebra for Youth Schools* in two volumes.[844] In 1972, *Stærðfræði handa gagnfræðaskólum / Mathematics for Lower Secondary Schools* was published in translation by Helga Björnsdóttir.[845] The authors of these three volumes are not mentioned in the textbooks but are probably Ove Hemer and Nils Sander.[846] *Algebra I* is missing in libraries, and it may be the same book as the *Geometry*. Anyhow, it seems plausible that these two or three books belong to the same series as the *Geometry*, and that these are the books mentioned in Hörður Lárusson's article in *Menntamál* (see pp. 310–311). Compared to the conventional syllabus, much less emphasis is placed here on proportions, conventionally practised with *regula de tri*, while there is increased emphasis on factoring and prime numbers, exponents, the place-value system and approximation.

This translated set of textbooks was only used for a couple of years or so, as Hörður Lárusson explained, and probably not for a large proportion of the year cohort. He began as early as 1971 to write his own set of books on behalf of the SRD. The first experimental series by Hörður Lárusson for the lower secondary school was published by the State Textbook Imprint in 1971–1973. In the next few years, final editions were published, now intended for the future nine-year school, under compulsory school legislation of 1974. The books were on similar topics to the Swedish ones but arranged differently. In Hörður Lárusson's books the coordinate system was introduced earlier than in the Swedish books, and in Hörður Lárusson's ninth grade book probability and statistics appeared for the first time in Icelandic lower secondary textbooks. The author has explained that he mainly picked out ideas from American textbooks, while some influences can also be traced from the Swedish textbooks, e.g. some pictures from the *Geometry* by Bergendal et al. were reproduced in the series. [847] Hörður Lárusson's textbooks were used for more than a decade at the lower secondary level and some of them longer.

Hörður Lárusson's series comprised textbooks for reading and books of exercises. Some teachers dropped the textbook, which had much more reading text than the

[843] Bergendal, G., Hemer, O and Sander N. (1970): 3
[844] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 63
[845] See Appendix M
[846] Emanuelsson, G., e-mail May 18, 2006
[847] Bergendal, G., Hemer, O and Sander N. (1970): 24–25

previous conventional ones, and had their pupils use only the exercise book. On teachers' in-service courses the author stressed that this was not the intention: the pupils were primarily to study the text, and the exercises were of secondary importance. If either book was to be dropped, it should be the book of exercises.

In these exchanges of opinion the different attitudes crystallized. The teachers may have wanted to interpret the topics on their own as they used to do. By pupils studying the text, the teachers' role was mainly to explain what the pupils could not find out on their own and motivate them, which sometimes proved harder than to explain, as not all the teachers were enthusiastic about this new syllabus themselves.

Hörður Lárusson's Reflections in 1972

In Hörður Lárusson's article in *Menntamál* 1972, "Hugleiðingar um stærðfræði og stærðfræðikennslu" / "Reflections on Mathematics and Mathematics Teaching," he discussed at length the origin of "modern" mathematics, the reasons for implementing it in Iceland, the demands on teachers, their role in teaching and the planned innovations at the lower secondary level.[848]

In his introduction to the article, Hörður Lárusson mentioned cooperation between mathematicians, psychologists, teachers and school specialists, thus referring to meetings such as those at Woods Hole and Royaumont, and the NKMM cooperation.

> En hverjar voru meginorsakir þessara breytinga [á stærðfræðikennslu í heiminum um og eftir miðjan þessa öld]? … ég læt nægja að benda á hinn gífurlega mismun, sem er á þekkingu nemendanna, þegar þeir ljúka námi, annars vegar og þeim kröfum, sem þjóðfélagið gerir til stærðfræðilegrar þekkingar þegnanna hins vegar. ...
>
> What are the main reasons for these changes [in mathematics teaching in the world in and after the middle of this century]? … I will content myself with pointing out on one hand the colossal difference between the knowledge of the pupils when they complete their studies, and, on the other hand, the demands that society makes on the mathematical knowledge of its subjects. …[849]

Hörður Lárusson recalled the establishment of the Technical College, where the requirements were established on a Danish model, taking account of the tasks that technologist would meet. Then he mentioned the problems with the college syllabus in mathematics, encountered by ordinary pupils who had completed lower secondary school. The question was whether to aim the teaching at the level of the pupils, or at the requirements of their future role as technologists and of the labour market. The latter had been the choice, and the result had been that only 30% had passed the test and the remainder had failed, especially in the preparation department.

The establishment and requirements of the Technical College are still at issue, and the fact that the students there were not able to cope with the same syllabus as the Danish students. The question remains, whether ordinary Danish lower secondary school pupils also had difficulties in fulfilling these requirements.

Hörður Lárusson said that some might say that the requirements were too strict, but the fact was that there had been colossal progress in mathematics, which could not be said of school mathematics, which lagged far behind. This had caused problems in many countries and some countries were expanding their syllabus, while others were trying to omit all unnecessary topics or transfer them between school levels.

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[848] Hörður Lárusson (1972a): 9–13
[849] Hörður Lárusson (1972a): 9

What was needed was to reconsider the whole system of mathematics teaching and not least teacher training. One could in fact achieve much better results by changing strategy. Then Hörður Lárusson described an example from a mathematics lesson, where the teacher built up motivation by modelling a problem about the oil-consumption of a big ship through discussion with his pupils. He then continued to explain that an investigation, which had been done in Icelandic lower secondary schools, had revealed that the time was badly utilized, there were many repetitions, teaching was monotonous and purposeful preparation for further studies was fragmentary. The first step planned towards improvement was to establish a standardized examination for the lower secondary schools in four subjects, one of them in mathematics.

However, the main progress would emerge from efforts of the teachers themselves. For the purpose of supporting them, a new set of textbooks had been published, a translation of the new joint Nordic project, made in the previous decade [the material promoted by the NKMM]. There were three aims: firstly to offer opportunities to the teachers to study by their own experience the innovations which were being introduced; secondly for use on in-service courses for teachers; and thirdly to gather domestic experience to use for creating a specifically Icelandic set of textbooks. Experimental work had already been established towards that task. Seven teachers were now testing the material for grade seven, which would then be rewritten, and the experiment would continue over the following years for grades eight and nine.

What Hörður Lárusson described here is his own creation of textbooks. Even if he gave a good description of how it depended on the teacher to awaken the pupils' interest, such as by modelling mathematical problems from ordinary life, his own textbook did not exactly encourage that kind of approach. The textbooks for the first lower secondary grade, grade seven, especially, were loaded with definitions of set-theoretical and algebraic concepts. When those had been collected and implemented by the second half of grade eight, the material began to come closer to real life, e.g. with topics such as proportions, presented without *regula de tri*, and probability and statistics in the ninth grade, but it hardly lent itself to modelling. That kind of work had to be on the initiative of teachers, most of whom did not have the knowledge or training to depart very far from the textbook. In addition, by then all the lower secondary level had been tied down to standardized examinations, at which the teaching had to be aimed.

The method Hörður Lárusson described may lead to initiative and autonomy on behalf of the pupils. It is a real challenge to create suitable learning material for open-ended tasks, i.e. curricular units that serve to initiate learning processes but not to determine them. Since it is not possible to ensure what will emerge from autonomous activity of the pupils, progress of the teaching process is to some extent uncertain. For that reason, projects of that kind cannot produce ready made units.[850] One cannot say that this kind of approach has been practised in Iceland to any marked degree, however worthwhile it might be.

Transition Period

No formal national curriculum in mathematics was published following the Compulsory Schools Act no. 63/1974, while national curriculum documents were published for most other subjects. A preliminary mathematics curriculum for

[850] Howson, A.G. et al. (1982): 116–117

compulsory school was published in August 1974. The document starts by presenting a list of pages in textbooks for the seventh to tenth grades. The tenth grade was a remnant of the previous lower secondary school. The final three pages concern general aims and instructions to teachers to set detailed objectives for each topic. The general aims were:

Með kennslu stærðfræði í grunnskóla er m.a. stefnt að því að nemandinn hafi öðlast

1. öryggi í almennum talnareikningi og nokkra leikni í umskriftum á fullyrðingum sem skráðar eru á táknmáli algebrunnar,
2. þekkingu á táknmáli stærðfræðinnar bæði til lestrar og tjáningar,
3. leikni í að umrita setningar úr venjulegu máli á táknmál stærðfræðinnar,
4. nokkra þjálfun í rökleiðslu og meðferð hlutfir[r]tra hugtaka,
5. nokkra hugmynd um möguleika til beitingar stærðfræði við dagleg viðfangsefni og þýðingu hennar fyrir aðrar greinar,
6. góðan undirbúning til framhaldsnáms.

Teaching mathematics in compulsory school aims among other things for the pupil to have gained

1. confidence in general arithmetic and some skills in rewriting sentences written in the symbolic language of algebra,
2. knowledge of the symbolic language of mathematics, both for reading and expression,
3. skills in rewriting sentences from ordinary language into the symbolic language of mathematics,
4. some training in logical reasoning and the use of abstract concepts,
5. some idea of possibilities in applying mathematics in daily tasks and its significance for other subjects,
6. good preparation for further studies.[851]

The general aims for mathematics teaching seem in fact to be simply the aims for the combined national examinations of the middle school and the lower secondary school, as they basically only concern the syllabus of the final year and not the general activities in the three last years of the nine-year school. They are similar to the 1968 aims for the national examination of the middle school, excluding ideas about set theory as the basis of algebra, understanding of our number notation and the nature of arithmetic operations, while including training in logical reasoning, as the 1968 lower secondary school curriculum did, in addition to providing some idea of how to use mathematics in daily tasks and its significance for other subjects.

To illustrate how to set detailed objectives for each topic, examples of objectives for simplifying rational fractions of increasing complexity are presented. A national curriculum of that form for the compulsory school was first presented in 1999.[852]

Concerning the content, the pupils are supposed to read either the "old" conventional syllabus using Dr. Ólafur Daníelsson's *Algebra* and Guðmundur Arnlaugsson's *Numbers and Sets* in the final year, or take one of the two "modern" syllabi: the translated Swedish textbooks or the textbooks by Hörður Lárusson. [853] As said above, sets as such are not mentioned in the general aims and set theory is not mentioned as a separate topic. Those who studied the "modern" syllabus must, however, have used set-theoretical notation and those who studied *Numbers and Sets*

[851] Menntamálaráðuneytið, skólarannsóknadeild (August 1974): 4–5
[852] Menntamálaráðuneytið (1999a)
[853] Prófanefnd (1974b): *Stærðfræði. Samræmd próf vorið 1975*

must have studied set algebra, as at least 6% of their national examination was concerned with set algebra.[854]

The last national examination for middle schools was held in 1976, and in 1977 for the lower secondary schools. No new general aims for lower secondary level mathematics were published on the occasion of a new standardized examination starting in 1977. Only the "new" syllabus was to be studied from 1975 in the seventh to ninth grade of the new compulsory school. Both the Swedish series and the "old" syllabus had been removed.[855] The compulsory school examination no longer contained any set theory or set-theoretical notation. In spite of the fact that the available textbooks were written in set-theoretical notation, "modern" mathematics in the lower secondary school level had come to an end, about a decade after it started.

Lower Secondary Level Textbooks in the Nine-Year School

In 1977, the whole cohort of the lower secondary school's ninth grade was thus supposed to take the same final examination, with the same goals as in the previous years, but with only one option of textbook series. This examination would influence the possibilities for entrance into the upper secondary level. However, there were now many upper secondary schools with different entrance requirements, as opposed to the three high schools and the Teacher Training College in 1965, so the effect of the examination was not as serious in 1977 as a decade earlier.

After Anna Kristjánsdóttir was appointed consultant in charge of mathematics in the SRD in 1975, she brought new emphasis to mathematics teaching. In 1976 the SRD published a book written by Anna Kristjánsdóttir, containing a collection of single topics in geometry with an investigative approach[856], different from the set-theoretical approach in the textbook by Bergendal et al. that she translated in 1970. In the same year, the SRD published a book by Anna Kristjánsdóttir and Rúnar Þorvaldsson, containing topics from commercial arithmetic[857] with investigative approach of the same kind as the geometry collection.

In 1976 it was considered necessary, with such a wide spectrum of population, that the pupils had more options than the algebra introduced in Hörður Lárusson's series, which had finally superseded Dr. Ólafur Daníelsson 1927 *Algebra*. A choice was offered between algebra, geometry and commercial arithmetic. According to a circular in May 1977 the proportion of pupils choosing the three options were:[858]

Algebra	51%
Geometry	19%
Commercial arithmetic	30%

The choice concerned a part-time syllabus for the first four months of the ninth grade. The choice topics first weighed 40%, then 34%, in 1979–1981 30%, and in 1982–1991 20%, of the examination. The remaining 60–80% was based on the syllabus of the first two years and basic algebra and probability and statistics in the ninth grade. The syllabi for the geometry and commercial arithmetic were the new

[854] Unprinted sources: Landspróf miðskóla og samræmt gagnfræðapróf 1975
[855] Menntamálaráðuneytið, skólarannsóknadeild (May and October 1975)
[856] Anna Kristjánsdóttir (1976)
[857] Anna Kristjánsdóttir and Rúnar Þorvaldsson (1976)
[858] Menntamálaráðuneytið, skólarannsóknadeild (May 5, 1977): 3

textbooks made by Anna Kristjánsdóttir and Rúnar Þorvaldsson, while the algebra was taken from Hörður Lárusson's book.

In 1979 a workgroup chaired by Anna Kristjánsdóttir began to prepare a new series of textbooks, turning away from the set-theoretical approach. Some starting points were used to introduce classical topics by an investigative approach, where applicable. The group of authors included Anna Kristjánsdóttir, Guðmundur Arnlaugsson and Kristín Bjarnadóttir.[859] Books that influenced the creation of the first year-course of this textbook series included *Starting Points* by C. S. Banwell, K. D. Saunders and D.G. Tahta and the English SMP-series for lower secondary level.

This new SRD set of textbooks may be considered as having belonged to the structuralist approach, characterized by investigation, as defined by Howson et al. (See section 7.1.). These books mostly abandoned the set theory and will therefore not be categorized in the "modern" mathematics wave but rather as a synthesis, taking the "best" out of the "modern" mathematics reform and fitting it into a more conventional content, thus creating more varied material than before the reform wave.

For financial reasons, this textbook series in its first experimental publication covered only half of the three seventh- to ninth-grade year-courses. When the time came for a final edition for whole year-courses, the SRD had been closed down. In 1988–1991 five out of six planned textbooks for the three last years of the compulsory school, now called eighth to tenth grades, were published by the National Centre for Educational Materials, NCEM. The last book in the series was never completed. By that time a Swedish series by Björk, Björksten, Brolin, Ernestam and Ljungström, with numerous exercises, had been translated and published. They were probably closer to the "back-to-basics" movement than the SRD series, "back-to-basics" meaning returning to emphasis on computational skills, leaving out open-ended problems and guided discovery learning. The first Swedish textbook for the eighth grade was published in 1987. This series soon took over the market. Only few teachers chose the Icelandic series, which may have been considered more demanding in teaching. It also suffered from uncertainty about its completion.

Again, this raises the question of teacher-friendly textbook series. As at the primary level, in the end many teachers favoured a skill-training approach. It is worth researching what kind of approach provides the most useful tools for the future adult life as a citizen and for further studies, lasting understanding and the most positive attitude to mathematics, and how teachers can adopt and assimilate that approach.

"Modern" Mathematics in Compulsory Schools – Summing Up

The set-theoretical approach at the lower secondary school level began in 1966 as half the year-course in the national examination programme of the third grade of lower secondary school (later ninth grade of compulsory school) by introducing Guðmundur Arnlaugsson's *Tölur og mengi / Numbers and Sets*. This was followed up by a radical change in the national examination. In 1970 the Swedish NKMM series of set-theoretical textbooks was introduced, and in 1971 Hörður Lárusson's set-theoretical textbooks were introduced. In 1975-1977 it had reached around 90% of the total eighth-grade syllabus. After that, a choice of an investigative approach was offered as a part-time programme in the ninth grade for the new compulsory school national examination. From 1979 the SRD work-group's material, also of

[859] See Appendix L

investigative nature, was used in many schools as a part-time programme. In the early 1990s, “back-to-basics” approach textbook series had taken over.

An important sign of change was the alteration in the national examination of the middle and lower secondary school. From 1967 a set-theoretical examination was an option, and from 1969 there were no other options. In 1975 the questions were generally posed with set-theoretical notation and in one of the three different types of examination with questions about set algebra. In the compulsory school examination 1978 hardly any trace of set-theoretical notation is found. It seems natural to conclude that the set-theoretical emphasis greatly diminished in the new nine-year compulsory school, and that the set-theoretical wave lasted approximately a decade. The set-theoretical textbooks gradually disappeared in the 1980s.

The last primary school group with pupils born in 1968 taking the Bundgaard material completed primary school in 1981. At that time the latter SRD material for the seventh to ninth grade had arrived on the market. It seems therefore that at the combined primary and lower secondary level the period of “modern” mathematics lasted from 1966 to 1981, or about 15 years.

The year-groups born in 1962 to 1965, who were most exposed to the Bundgaard material, had, when they completed their compulsory school education in 1978 to 1981, a lower secondary school syllabus where the emphasis on the set-theoretical approach had already diminished, especially if the national examination is taken into account. According to the theory of “modern” mathematics, one would expect that these pupils’ thinking had by that time been shaped by the set-theoretical approach, so that they might discover by themselves the similarities of the various branches of mathematics.

The reason why a choice of geometry and commercial arithmetic was set up may, however, have been a fear that the results in algebra for the whole year cohort would not be good. In spite of all efforts, the average for the whole country was 31 points out of 100 in the first compulsory school mathematics examination in February 1977.[860] In the 1978 examination the average number of points had reached 45.[861] One of the reasons, besides that the test may have been easier, could be that teachers realized that the pupils had to be trained for this examination, as had been the case for the national examination of the middle school, even if the Examination Board had stressed that the examination should be held just like any other event on an ordinary February day.

In one of his many articles about the activities of the SRD, Andri Ísaksson made the comment that scientific knowledge of learning and teaching is as yet rather limited.[862] These words stay in one’s mind when one looks over the process of mathematics teaching in compulsory schools in Iceland during the turbulent years in 1960s and 1970s.

860 Menntamálaráðuneytið, prófanefnd (August 1977)
861 Menntamálaráðuneytið, prófanefnd (June 1978)
862 Andri Ísaksson (1972): 2–3

8.3. The Teacher Training College

The Upper Secondary Level in the 1960s and 1970s

One of the reasons for the turbulence in the second part of the 1960s was the accommodation problems of the high schools, and lack of options for further education at the upper secondary school level. Four important steps were taken to meet these needs:

The first step taken was to open the Teacher Training College up in 1963 to be equivalent to the high schools. Thereby a route was opened to university studies for many who did not meet the standard requirements to enter the high schools, and thereby to further and higher studies.

The second step was the establishment of several new high schools of which that of Hamrahlíð High School in 1966 is the most important.

The third step was the creation of continuation departments at the lower secondary schools (*framhaldsdeildir gagnfræðaskóla*) by a provisional law in August 1969.[863] These departments were primarily intended for those passing the recently standardized national examination of the lower secondary school.

The fourth step was the creation of a new type of upper secondary level schools, the multi-stream comprehensive schools. The first schools of this type were established in 1975. They were *Fjölbrautaskólinn í Breiðholti* / the Breiðholt Comprehensive Multi-stream School in Reykjavík, established on the initiative of the Reykjavík Education Office, in accordance with a proposal accepted in the Reykjavík Town Council in January 1970, and *Flensborgarskólinn* / the Flensborg School in Hafnarfjörður, based on the 1882 lower secondary school.

The Last Years of the Teacher Training College

The legislation for the Teacher Training College was amended in 1963. It had become clear that, educationally, it led to a dead end. In the new legislation this was remedied by adding a one-year course to the teacher training for those who wanted to take a matriculation examination, *stúdentspróf*, equivalent to the high school examination, and thereby gain access to university. This department was first operated in 1967 and it continued until 1974.[864] A general high school department was opened within the college in 1968, but only for one year-group. Soon this resulted in a flood of pupils pouring through the school. On the 50th anniversary of the Teacher Training College in 1958, it had 116 students. In 1952–1953 the number had reached a high point of 141.[865] In 1964 the number had become 292 and in 1969 it was 954 pupils.[866]

Traditionally, the Teacher Training College offered exemptions to pupils with lower average grades in the national examination than applied in high school (5.50 instead of 6.00) on certain conditions, and also to pupils with the lower secondary school examination only. Those groups now made their way through the Teacher Training College, many without intending to become teachers. A new building, replacing in 1962 the old one that had housed the Teacher Training College for over

[863] *Stjórnartíðindi* 1969 no. 84, August 27
[864] Lýður Björnsson (1981): 44
[865] Freysteinn Gunnarsson (1958): 142
[866] Sigríður Þ. Valgeirsdóttir (1987): 24

50 years, was soon overcrowded and the school was overwhelmed. However, because of this increased attendance it became possible to divide the pupils into groups so that they could choose selective courses, including mathematics. The reasons for this heavy pressure on the Teacher Training College were, of course, the increased demand for education opportunities, and lack of other options.

In 1971 teacher training transferred to the tertiary school level by the establishment of Iceland University of Education, whose role was to educate teachers for the soon-to-be-established nine-year school, serving pupils in the age groups 7–16 years, later 6–16 years. Thereby the problem of shortage of licensed teachers for the lower secondary level was solved. Another matter was whether sufficiently many received sufficient mathematics education to cover the nine-year syllabus.

Mathematics Education in the Teacher Training College

Psychologist Dr. Phil. Broddi Jóhannesson, headmaster of the Teacher Training College 1962–1971 and rector of Iceland University of Education 1972–1975, was, as a young teacher in 1943–1947, appointed to teach mathematics, because there was no one else available at that time. He was interested in improving mathematics education in the college, and the elective courses, starting in 1967, were a part of a scheme for that purpose. But this was a difficult period for the college. The majority of the students were not preparing for teaching, but were taking advantage of this open route to university. At that time no one had specialized in teaching mathematics education, and the college had to largely make do with the available university students.

The mathematics educators, Guðmundur Arnlaugsson and Björn Bjarnason, chose the textbook *Basic Concepts of Elementary Mathematics* in 1967–68 for those who chose mathematics as a special elective course. The book focussed on the nature and structure of mathematics. The author, W. Schaaf, says in the preface to the book:

> In short, teachers of arithmetic and junior high school mathematics require more than a conventional course in methods of teaching arithmetic. They need a *content* course in mathematics. Such a course should not be a simple review or refresher course in seventh and eighth year arithmetic, or a traditional course in algebra, geometry, trigonometry, and analytics. Nor should it be an experience designed to achieve desirable computational proficiency. On the contrary, this course should strive to give some insight into the nature and structure of mathematics, including not only arithmetic, but algebra and geometry as well.[867]

This was probably the kind of material the students needed. However, studying this book proved difficult for many of them, as it contained a number of new concepts in a foreign language. The teacher was a university student, the author of this book, who had completed two first year-courses in mathematics for engineering students and a small course on “modern” mathematics. She had a hard time keeping ahead of them, but probably learned the most. She was offered a position the next year but chose to quit for of lack of training, by her own perception. More university students and part-time instructors were appointed the next year, when the number of pupils had doubled. This account reflects the conditions of mathematics education in the late 1960s when two-thirds of the 20th century had passed. In the 1970s, when the new University of Education had settled down with a manageable number of pupils, one of its first tasks was to establish courses in “modern” mathematics, both for its own students and in-service continuing education courses.

[867] Schaaf, W. L. (1966): x

8.4. High Schools

The Purpose of High Schools

Until the 1960s, the main purpose of the high schools was generally considered to be to produce competent candidates for university studies to become physicians, clergymen, engineers, county magistrates, judges, etc., to fill the need for the professional classes. Certainly a few became scholars, especially in Icelandic studies, and many became teachers, some with a degree required for that purpose, while others, who did not manage to complete a degree, often for financial reasons, also turned to teaching. Only late in the 1960s did the idea of high school for all appear in public discussion.[868]

In discussions in the 1960s, e.g. in the periodical of the Association of Engineers, this opinion of the high schools' main purpose, to prepare for university, is accepted. However, there was growing criticism of their outmoded curriculum and study material, for instance in chemistry and physics. Furthermore, there is an interesting discussion about the role of the education of the schools:

> Hvaða kröfur eru gerðar til skóla hlýtur að ákvarðast fyrst og fremst af því, hver þörf þjóðfélagsins er fyrir menntun þá, sem í skólanum er veitt. … Fyrir nokkrum áratugum höfðu vísindi fyrst og fremst menningarlegt gildi; þau voru þáttur í menningu hverrar þjóðar og að því leyti hliðstæð og sambærileg við bókmenntir, myndlist og aðrar listir. Hagnýtt gildi vísinda var næsta lítið. ...
>
> Nú hafa raunvísindi ... beinlínis hagnýtt gildi ... Þýðing menntunar í nútímaþjóðfélagi hlýtur því að vera öll önnur og meiri en áður. Í öllum greinum atvinnulífsins er þörf mun meiri þekkingar en áður. Menntun þjóðarinnar er orðin þjóðhagsleg nauðsyn samfara tæknivæðingu atvinnulífsins.
>
> The demands made on a school must be determined primarily by the need of society for the education that the school offers. … Some decades ago, the sciences had primarily a cultural value; they were factors in the culture of each nation and thus parallel and comparable to literature, the visual arts and other arts. The practical value of science was rather small. …
>
> Now, the natural sciences have … directly practical value … The importance of education in a modern society must therefore be vastly different from and greater than before. In all sectors of the economy there is more need for knowledge than before. The education of the nation has become a economic necessity parallel to the technological development of society.[869]

The curriculum reforms in the high schools in the 1960s were based on this idea of their purpose, the need of society, "the economic necessity", for people well educated in the physical sciences. The mathematicians and physicists, some of whom were the same people, were the first to revise their teaching material.

Recalling the fundamental reasons for mathematics education as identified by M. Niss, one notices the ideological shift of reasons for science and mathematics education, from the second reason, contributing to society's cultural maintenance, to the first, contributing to the technological and socio-economic development of society. These young physicists and engineers certainly speak under the influence of the discussions initiated by OECD in Iceland and other countries where they had studied.

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[868] Gunnar Karlsson (1968): *Frjáls Þjóð*, May 16
[869] Hinrik Guðmundsson, Jakob Björnsson and Páll Theodórsson (1964): 34–35

A New Kind of High School

In 1966 the Hamrahlíð High School was established with the mathematician Guðmundur Arnlaugsson as headmaster. It was a classical high school, but in the spring of 1967, when the pupils chose their streams for the next three years, it became apparent that the Latin "wall" of the Icelandic high school had been broken down. There was no Latin in the mathematics stream, and there was a choice between Latin and French in the language stream. The mathematics stream was to be undivided in its first year, but in the second year it was divided into a natural-sciences line and a physics line, with more mathematics in the physics line. This division was also new. Reykjavík High School took the structure up the following year.

In 1961 Guðmundur Arnlaugsson wrote an article in *Menntamál* where he explained the changes that had been made in the high schools in Denmark. He mentioned how the structure of the Danish high school had remained the same since 1903. In 1959 a committee had been appointed to look into its structure and curriculum. It had submitted a document, *The New High School* / *Det ny Gymnasium, Betænkning nr. 269*, (Statens trykningskontor), where a new division into streams was explained.[870] In his new high school Guðmundur Arnlaugsson seems to have been allowed to experiment with a new stream division modelled after the new Danish structure.

It is noticeable that the mathematics stream at Hamrahlíð High School was divided into two sub-streams as early as 1968, and at Reykjavík High School in 1969, although new high school legislation was first passed in 1970[871] and the concomitant regulations about the streams, termed "fields of electives" / *kjörsvið*, was issued in January 1971[872]. Also, from 1968 Latin was at last abolished in the mathematics stream of the Reykjavík High School. These changes may be considered as part of the high schools' effort to adjust to the different needs of the growing number of pupils attending high school, and also as a sign of fresh ideas following the establishment of a new school, breaking the history of 120 years of monopoly by Reykjavík High School in the capital area. The new legislation and regulations were adjusted to this new model.

At similar time, in 1970, the learned department of the Commercial School of Iceland, *Verzlunarskólinn*, was also divided into two streams, the economics stream and the language stream. A mathematics stream was first established there in 1984. For that reason, *Verzlunarskólinn* will not be treated further in this study.

Mathematics Reform in the High Schools

Guðmundur Arnlaugsson and his colleague Björn Bjarnason took the initiative to alter the curriculum and teaching material in mathematics in Reykjavík High School according to the "modern" mathematics. This was their first step towards the reform, occurring before their efforts at the lower secondary level.

In 1964 Guðmundur Arnlaugsson and Björn Bjarnason chose a new textbook for the mathematics stream of the Reykjavík High School, *Principles of Mathematics* by Carl B. Allendoerfer and Cletus O. Oakley, first published in 1955, 2nd edition in 1963. Previously, the Danish series by E. Juul and E. Rønnau had been in use, the

[870] Guðmundur Arnlaugsson (1961)
[871] *Stjórnartíðindi* 1970 no. 12, March 25
[872] *Stjórnartíðindi* 1971 no. 12, January 22

choice that they had made together with Sigurkarl Stefánsson after they took over the leadership of the mathematics at the school in the post-war period.

Allendoerfer was one of those behind the *Program for College Preparatory Mathematics*, which was one of the central documents within the "modern" mathematics movement in the United States.[873] He was also one of six mathematicians present at the conference in Woods Hole in 1959.[874]

The *Principles of Mathematics* had typical "modern" mathematics content. The first four chapters were on logic and sets, number fields, integers and groups, a total of 123 pages. That was unconventional content for a whole first-year course in the mathematics stream, and in general, the ratio of calculus to the total text was smaller than previously.

Traditionally Euclidean geometry was taught to all in the first semester of the first year in the high schools. In the late 1960s the Danish *Kennslubók í rúmfræði* / *A Textbook in Geometry* by Jul. Petersen, which had been taught for about 100 years, was abandoned. A new Danish *Geometri – 1. og 2. realklasse / Geometry* by C.C. Andersen et al.,[875] written with a more modern approach where reflection was a basic operation, was taken up in 1966 at the Reykjavík and Hamrahlíð High Schools as well as at the Teacher Training College.

Algebra and introduction to trigonometry were taught in the second semester of the first year. Ólafur Daníelsson's *Algebra* was dropped in 1964 in favour of *Algebra* by Guðmundur Arnlaugsson.[876]

The work with the *Principles* was not too successful. In a four-year period 1964–1968, Guðmundur Arnlaugsson and Björn Bjarnason tried three sets of "modern" mathematics textbooks from the three mainstreams: the American set-theoretical approach in the *Principles*, the English semi-applied oriented SMP-series, and the Nordic set-theoretical approach in Kristensen & Rindung's series.[877] At the University, the Kristensen & Rindung series was the most favoured (See section 7.2.).[878]

The search went on for several years for suitable series of textbooks to introduce "modern" mathematics in the high schools. As there were only two high schools with mathematics streams in Reykjavík, their teachers could make experiments and share experiences, also with the schools in Akureyri and Laugarvatn. The teachers wanted to modernize their syllabus, but at the same time they were concerned that their students should be prepared for university studies, in particular for the Faculty of Engineering at the University of Iceland. It was also a problem to use textbooks in foreign languages. There was a tradition for using Danish textbooks in the mathematics streams, while textbooks in English and Swedish brought difficulties. The first year's textbooks were preferably to be in Icelandic. The syllabus which emerged from this period of experiments and continued for most of the 1970s was the following:

[873] Gjone, G. (1983): Vol. II, 17
[874] Bruner, J.S. (1966): vi
[875] Andersen, C. C., S.A. Bo, G. Nielsen and J. Damgaard Sørensen (1963)
[876] See Appendix N
[877] See Appendix N
[878] Jón Ragnar Stefánsson, July 15, 2003

First year: *Rúmfræði handa 1. bekk menntaskóla / Geometry for the First Grade of High School* by Hörður Lárusson and *Stærðfræði handa 1. bekk menntaskóla / Mathematics for the First Grade of High School* by Hildigunnur Halldórsdóttir, both written in Icelandic.

The second to fourth year in the mathematics-natural science streams: The Swedish series *Mathematics for the High School, na/te*, by Bergendal, Håstad and Råde.

All these textbooks were written with a set-theoretical approach. According to Hörður Lárusson, his *Geometry* was an introduction to geometry, looking away from a strict axiomatic approach.[879] Hildigunnur Halldórsdóttir has said that she wrote her book as an introduction to the Swedish *na/te* textbooks.[880] Taken together, the textbooks for the four-year high school offered reasonable training in algebraic skills, acceptable for the University, which had complained about severe lack of skills during the experimental time.[881]

According to an interview with one of the university teachers involved in a complaint in March 1971, the subject of complaint was not the textbook but an inadequacy of some teaching, but so well wrapped up that the teachers took both factors for serious reconsideration, their teaching and textbooks.[882]

Influences on Reform

There are several remarks to be made on the frame of the mathematics reform in Icelandic high schools:

Firstly, there was a long tradition of a single Latin high school, understood to be a doorway towards professional positions, and on the basis of that status, highly selective. The official attitude was that only the "most qualified" were to pass through into the elite group. An overbooking into the class of officials was to be avoided. Secondly, the accommodation of the Reykjavík High School, heir to the elitist tradition, was completely inadequate for the growing number of applicants. As a consequence of both reasons, the first year was extremely difficult for many pupils and the drop-out rate was high.

Social trends, related to the 1968 student uprisings, changed the public attitude as to who were eligible for attending high schools and who were not. A high school for all was the emerging idea. During the reform period, an increasing proportion of the population passed the national examination and was attending high school. In effect, the teachers at Reykjavík High School may have been searching for a new textbook to suit a broader population than before.

The pioneers of "modern" mathematics, Guðmundur Arnlaugsson and Björn Bjarnason, became headmasters of new high schools in the course of the reform period, Guðmundur Arnlaugsson at Hamrahlíð High School in 1965 and Björn Bjarnason at Sund High School in 1970. They had been the senior mathematics teachers, and taken all decisions regarding the curriculum. When problems began to emerge, such as complaints from the University, Guðmundur Arnlaugsson and Björn Bjarnason were teaching neither at the Reykjavík High School nor at the University. Their collaboration, which had been so fruitful, became less extensive. Those who

[879] Hörður Lárusson, March 26, 2002
[880] Hildigunnur Halldórsdóttir, September 10, 2002
[881] Skarphéðinn Pálmason, March 23, 2003
[882] Jón Ragnar Stefánsson, July 15, 2003

took over in the Reykjavík High School soon reverted to a more conventional curriculum.

When the idea of upper secondary schools for all had won support, more and more schools were established, offering access to university studies. The schools were not standardized to any detailed degree, even if a short common curriculum existed in regulations, and the examinations were not standardized. While Hamrahlíð High School became renowned as a progressive school and was the most attractive school for a while, Reykjavík High School gradually regained its status of attracting the most able pupils, and hence it could offer the most demanding syllabus with a moderate set-theoretical approach.

8.5. The New Upper Secondary School

Continuation Departments

While the high schools were adjusting themselves to new views of mathematics teaching and a multiple number of pupils, probably less homogeneous in learning skills than in earlier times, still keeping in mind their main purpose of preparing for university studies, new upper secondary study options were coming up.

In the 1960s there were still teachers at the Reykjavík High School who remembered the “good old days” of twenty years earlier, when a limited number of pupils could be picked out of hundreds of applicants who attempted the entrance examination (See section 6.4.).[883] This school, and the new Hamrahlíð High School, where many of the teachers had previously taught at Reykjavík High School and therefore may have had a similar attitude, may have been reluctant to make extensive adjustments to their syllabi and teaching methods to cater for the numerous crowd of pupils passing the national examination of the middle school, and from 1968 the national examination of the lower secondary school too. Furthermore they did not have the necessary facilities to accept this number of pupils. The Teacher Training College had been expanded to its limit, and was by the end of the 1960s preparing to step up to tertiary school level. Something had to be done.

In August 1969 provisional law was enacted on continuation departments of the lower secondary schools, providing up to two years of additional studies.[884] According to an introduction to the legislation, the SRD had for some time worked on studying continuing education in lower secondary schools for those who had completed the national examination of the middle schools or lower secondary schools. It was modelled after a newly established *Højere forberedelse*, HF, higher preparation programme, in Denmark.[885] That same year a provisional curriculum and a syllabus list were published for the first and the second year, which indicates that some experiments had already been performed the year before.[886]

The curriculum in the continuation departments was aimed at those who completed the two-year programme to be able to enter various vocational training in the fields of health, education, commerce or the Technical College, which is specially mentioned in the curriculum pamphlet, or a high school.

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[883] Einar Magnússon (1975): 90
[884] *Stjórnartíðindi* 1969 no. 84, August 27
[885] Kristín Indriðadóttir (2004): 30
[886] Menntamálaráðuneytið. Skólarannsóknir (1969)

Mathematics in the Continuation Departments

Concerning mathematics in this first year, 1969, the syllabus contained *Book T* (1964), the first book in the SMP-series.[887] However there was an option of a new syllabus being produced in Icelandic. The following content was taken: Sets, inequalities, transformations and geometry of the plane, statistics, trigonometry, the coordinate system and graphs, and, as an elective, the slide rule, several items from algebra and three-dimensional geometry.

Recommendations to the teachers in the provisional curriculum pamphlet say that the textbooks' expositions were well suited to stimulate free discussion about the topics under the supervision of the teacher. It was recommended as desirable to nurture the basic concepts themselves so well that methods and operations built on them would appear natural and self-evident to the pupils.[888]

For the second year the basis was *Book T4* (1966) from the SMP-series with transformations, trigonometry, statistics and probability, geometry in the plane and coordinates and as selective topics, matrices and vectors, practical items from algebra, geometry in three dimensions and modern algebra.

In the revised provisional curriculum, dated 1974[889], a new textbook is introduced, *Stærðfræði fyrir framhaldsdeildir gagnfræðaskóla / Mathematics for the Continuation Departments* written by Hörður Lárusson.[890] The content for the first year was sets, inequalities, geometry in a set-theoretical framework, transformations, statistics, volume and surface area, enlargement, trigonometry and a choice of the slide rule and systems of linear equations. This seems to be an extract from *Book T* and *Book T4*, adapted and translated into Icelandic. In the second year Hörður Lárusson's book was continued, together with Hildigunnur Halldórsdóttir's *Stærðfræði handa 1. bekk mennaskóla / Mathematics for the 1st Grade of High School,* mainly treating functions, up to the exponential and logarithmic functions.

Obviously the textbooks from the SMP-series must have been difficult to cope with for the pupils in the continuation departments. Both the Reykjavík high schools had turned away from them, due to language problems and also to lack of rigor, and it was only natural that this would be the case here too. However, the syllabus in the continuation departments continued to be mainly an extract of the SMP books translated into Icelandic, with set-theoretical notation. Probably it was their practical orientation with statistics and probability that made this syllabus more advantageous to the vocational training that the pupils were preparing for, rather than a more theoretical syllabus. The SMP books were generally more practically oriented than the Nordic textbooks for upper secondary level.

Multi-Stream Comprehensive Schools

The continuation departments of the lower secondary schools can hardly be considered as a pedagogical innovation, but rather an effort to provide those pupils who had earned rights to further education with some reasonable, temporary solution.

In January 1970, Kristján J. Gunnarsson, who in 1965 had accused the authorities of the national examination of adjusting the examination to the accommodation

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[887] *School Mathematics Project* (1964)
[888] Menntamálaráðuneytið. Skólarannsóknir (1970): 24
[889] Menntamálaráðuneytið. Skólarannsóknir (1974): 12
[890] Hörður Lárusson (1970)

available at the high schools,[891] proposed in the Reykjavík Town Council that an experimental school be established at the lower secondary school and high school levels. The proposal was accepted and in July 1971 detailed proposals had been worked out by the Reykjavík Education Office, signed by Chairman of Education Board, Kristján J. Gunnarsson, and Director Jónas B. Jónsson.[892] The idea was that the school would accept pupils in the age range 13 to 19 years, bypassing the national examination. The aim of this school would be on one hand to prepare pupils in the best possible way for working in the various vocational fields, and on the other hand to provide those pupils who wished to head for further and higher education with sufficient preparation for entrance to university or other institutes at university level.

Jóhann S. Hannesson, who had been the headmaster at Laugarvatn High School and was one of the two advisers of the SRD, was appointed to prepare this new project on behalf of the Ministry of Education. A detailed account of the project is beyond the scope of this account, while in April 1973 legislation was passed in order to permit the Ministry of Education and the Municipality of Reykjavík to establish a comprehensive multi-stream school.[893] This was the beginning of Breiðholt Comprehensive Multi-Stream School, established in 1975, intended for the same age group as the high schools, 16–20 years, but with a much greater variety of educational options. As the nine-year compulsory school legislation was adopted in 1974, there was no longer need to bypass the national examination. Simultaneously, the Flensborg School with its long history was elevated to upper secondary level as a comprehensive multi-stream school in 1975.

Expansion in the 1970s

In the following decade a number of schools of this kind were established in Reykjavík, its vicinity and all around the country, in many cases either on the basis of the lower secondary schools continuation departments or the existing technical schools, or both. These schools were all intended for the 16- to 20-year age group at the upper secondary level. The following is a list of upper secondary schools in Iceland established in and after 1966:

1. Menntaskólinn við Hamrahlíð, Reykjavík, 1966
2. Menntaskólinn við Tjörnina / Sund, Reykjavík, 1970
3. Menntaskólinn á Ísafirði, 1970
4. Menntaskólinn í Kópavogi, 1973
5. Fjölbrautaskólinn í Breiðholti, Reykjavík, 1975
6. Flensborgarskólinn í Hafnarfirði, 1975
7. Fjölbrautaskóli Suðurnesja, Keflavík, 1976
8. Fjölbrautaskóli Vesturlands á Akranesi, 1977
9. Menntaskólinn á Egilsstöðum, 1978
10. Fjölbrautaskólinn Ármúla, Reykjavík, 1979
11. Fjölbrautaskóli Norðurlands vestra, Sauðárkrókur, 1979
12. Framhaldsskólinn í Vestmannaeyjum, 1979
13. Kvennaskólinn í Reykjavík, 1979
14. Fjölbrautaskóli Suðurlands, Selfoss, 1981
15. Verkmenntaskóli Austurlands, Neskaupstaður, 1981/1986

[891] Bjarni Vilhjálmsson (1965): *Alþýðublaðið*, April 14
[892] *Sameinaður framhaldsskóli. Tillögur og greinargerð fræðsluráðs Reykjavíkur um stofnun tilrauna-skóla á gagnfræða- og menntaskólastigi* (July 1971)
[893] *Stjórnartíðindi* 1973 no. 14, April 13

16. Fjölbrautaskólinn í Garðabæ, 1984
17. Verkmenntaskólinn á Akureyri, 1984
18. Framhaldsskólinn á Húsavík, 1987
19. Framhaldsskóli Austur-Skaftfellssýslu, Höfn, 1987
20. Framhaldsskólinn að Laugum, 1988
21. Borgarholtsskóli, Reykjavík, 1996
22. Fjölbrautaskóli Snæfellinga, Grundarfjörður, 2004

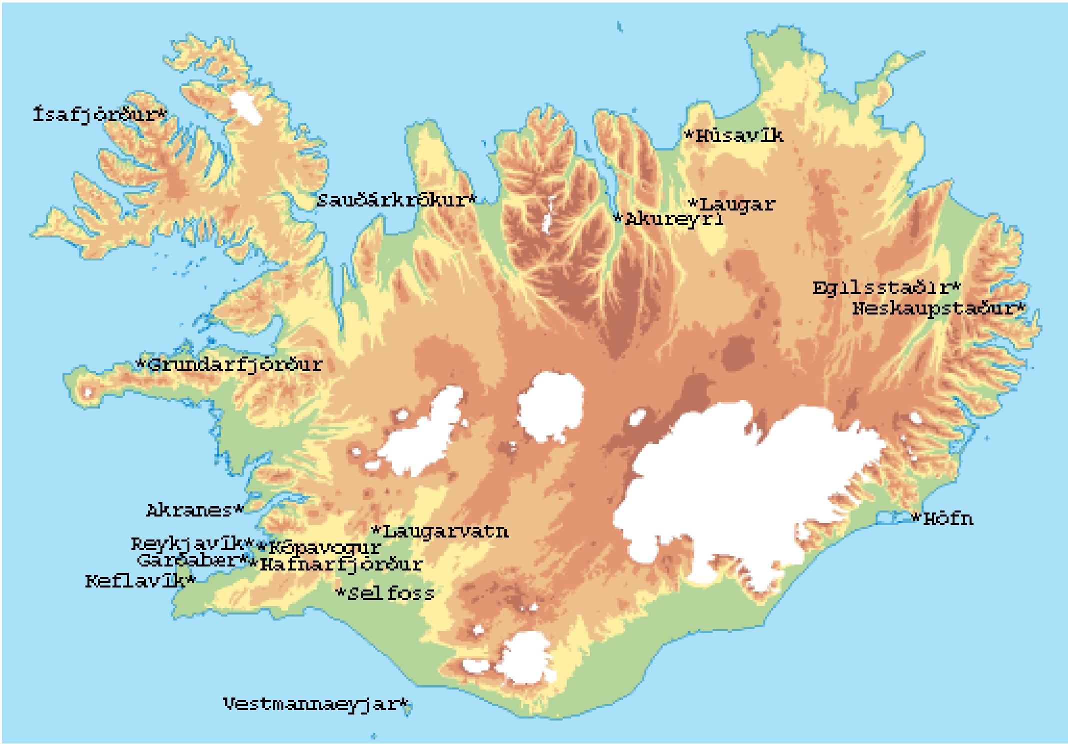

Fig. 8.3. Location of Upper Secondary Schools 2005.

The first four schools, named *menntaskóli*, high school, were, at least in their first years, intended as classical high schools, preparing for university studies. The schools named *fjölbrautaskóli* (comprehensive, literally multi-stream, school), *framhaldsskóli* (continuation school) or *verkmenntaskóli* (craft education school), as well as *Flensborgarskóli* / Flensborg School, growing up from continuation departments of lower secondary schools, had many streams. They were run on the basis of a modular course system, which the Hamrahlíð High School prepared and introduced in 1972 and the Breiðholt Comprehensive School too on its foundation in 1975.

Looking at this long list, one notices that out of the eight new schools established before 1978, five are in Reykjavík or its vicinity, Hafnarfjörður and Kópavogur. In 1967 the Director of Education in Reykjavík, Jónas B. Jónsson, predicted that at least five high schools were needed in Reykjavík.[894] Within a decade his prediction was more than fulfilled. Certainly he had made his share of the effort by his work at preparing proposals on the Breiðholt School. Flensborg School had a new role after a history of nearly a century. The high schools in Ísafjörður (no. 3) and Egilsstaðir (no. 9) were the result of political promises made in 1965 in the *Alþingi* to help the rural areas struggling with ever-decreasing population. Their establishment relieved the pressure on the small boarding high schools in Akureyri and Laugarvatn, rather than

[894] Hörður Bergmann (1967): *Frjáls þjóð*, April 6

in the capital area. On the other hand they, and the comprehensive multi-stream schools, proved to be a strong social support of their communities.

Legislative Framework

By law it should have been the *Alþingi* that took the initiative to establish and build high schools. It passed legislation in 1965 on a total of six high schools.[895] Three high schools were in existence, and Hamrahlíð High School in Reykjavík was already being prepared, while legislation on the other two was passed, contingent on available funding. Ísafjörður High School did not become a reality until 1970, and was then housed in an old primary school building, and the Egilsstaðir High School was established in 1978. Meanwhile a number of continuation departments were established on the basis of provisional legislation adopted in August 1969, when the *Alþingi* was not in session.

The continuation departments were often housed within the existing lower secondary school buildings, and their operating costs were funded by the municipalities, while the teaching costs were funded by the state, as had been the case with the lower secondary schools. Technically, the continuation departments were at the lower secondary level. Especially after the 1974 Compulsory School Act, when the fourth year of lower secondary school was abolished, many schools may have had extra accommodation and staff available to establish a continuation department. Those departments often had rather able and mature pupils compared to the regular upper division of compulsory school, and were therefore desired by the local school authorities. All operating expenses other than the salaries of teachers were borne by the municipalities, sometimes subject to some reimbursement from the state.

The Breiðholt Comprehensive Multi-Stream School was established on the initiative of the Reykjavík Municipality with the consent of the Ministry of Education. The state was to pay the expenses of teaching, half of other operating expenses and 60% of the building cost, while for the high schools the state had borne all expenses. The 1973 Act on Breiðholt School[896] allowed for the establishment of more comprehensive upper secondary schools in cooperation with municipalities. Many schools of that kind were established within that framework until legislation was passed in 1988, whereby the sharing of expenses was settled such that the state paid all operating expenses and 60% of the building costs.[897]

Sharing of costs between the state and the municipalities was considered to be a security valve against the danger of unrealistic pressure on the part of municipal authorities to establish a continuation department or a comprehensive school, even in a vanishing community, to save its life, regardless of whether there were any pupils there in need for education. If the municipalities carried some financial responsibility, such matters would be less likely to enter the discussion.[898]

Coordination of Syllabi

By this new system of many schools at the upper secondary level, there were naturally fewer opportunities to coordinate the mathematics curricula and the syllabi of these schools by personal contact, as had been done while there were only four

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[895] *Stjórnartíðindi* 1965 no. 56, May 6
[896] Stjórnartíðindi 1973 no. 14, April 13
[897] *Stjórnartíðindi* 1988 no. 57, May 19
[898] Þorsteinn Þorsteinsson, e-mail June 12, 2003

schools, in 1966-1970, or earlier when they were even fewer. The new multi-stream schools were of course preparing their pupils for a much wider range of vocations than the classical high schools. The schools continued to confer about textbooks, while each school held its own examinations. The modular-system schools gathered into groups to hold coordination meetings, where they made up their own syllabi with detailed descriptions of their modules/courses.

The aims of all the schools were that their pupils, who earned admission to university by their final examination, would not be given false promises. However, there was no authority to coordinate the syllabi or to provide guidance on their level of difficulty.

During the time from the mid-1970s to the mid-1980s, when the legislation on the upper secondary level schools was being prepared, the schools had time to develop their internal functions. As could be expected in such rapid expansion there were many young teachers and many young headmasters. Young and enthusiastic people worked long hours on the structure and on the content of their syllabi.

The set-theoretical approach in mathematics declined as the 1970s drew to a close. More textbooks came on the market. Hörður Lárusson's and Hildigunnur Halldórsdóttir's textbooks, which were coloured by the authors' American education in the 1960s, disappeared, and a translation of the American programmed teaching oriented *Algebra fyrir framhaldsskóla / Intermediate Algebra* by Carman and Carman[899] was used in many schools for a while in the 1980s. An Icelandic textbook series by Jón Hafsteinn Jónsson et al. was first published in the early 1980s and has since then been republished in several editions. Jón Hafsteinn Jónsson was Svend Bundgaard's student in Denmark and has retained "modern" mathematics' formal axiomatic approach. Teachers at Hamrahlíð High School took the initiative of translating or writing their own textbooks, and most modular-system schools took Hamrahlíð High School as their model. Reykjavík High School, however, went its own way. It was able to choose able pupils from many applicants, and it put emphasis on thorough reasoning and rigid proofs. Finally, in the 1990s, its teachers had rewritten their entire syllabus, as no Icelandic, Danish or other textbooks fulfilled their requirements.

New Preliminary National Mathematics Curriculum

In October 1979 the Minister of Education appointed a committee to write a national curriculum in mathematics. The committee, originally consisting of Hildigunnur Halldórsdóttir, Anna Kristjánsdóttir, Guðmundur Hjálmarsson, Halldór I. Elíasson, Hjálmur Flosason, Jón Hafsteinn Jónsson and Steinþór Kristjánsson, sent out a questionnaire to mathematics teachers in the various schools in order to gather information about their current syllabus and learn about their views on matters concerning mathematics teaching. Furthermore the committee studied national curricula from Denmark, Norway and Sweden.

The result of the work of the committee was a detailed analysis of the whole possible mathematics curriculum in the upper secondary level. The curriculum was divided into courses with one, two or three credits each, together with a diagram of prerequisites.[900]

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[899] Carman, R. A. and Carman, M. J. (1983)
[900] Menntamálaráðuneytið, skólarannsóknadeild (April 1981)

Only one course was allocated to the theory of sets and logic and this was only supposed to be the prerequisite for a course in probability, combinatorial analysis and descriptive statistics. The set-theoretical approach was no longer in the foreground of all mathematics learning. The main emphasis was on algebra. An attempt was made to define a kind of difference in action radius and technical level for the various streams, depending on the vocation that the pupils were aiming at.[901]

In the introduction about mathematics in the school system, the authors mention its twofold role, its practical applications and its role as recreation or art, which has the principal aim of training logical thinking.

The main aims for teaching mathematics in the first year of the upper secondary level were defined as follows:

1. Að auka skilning nemenda á hlutverki stærðfræðinnar við mótun hugtaka og rökstuðning ályktana, sem nátengd eru daglegu lífi og samskiptum fólks.
2. Að efla reikningslega færni nemenda, auka kunnáttu þeirra í táknrænni túlkun og veita þeim yfirsýn yfir almennar reiknireglur, m.a. þannig að þeir nái tökum á öðrum námsgreinum þar sem stærðfræði er beitt.
3. Að auka rökrænan þroska og alhæfingarhæfni nemenda og búa þá þannig undir frekara stærðfræðinám.

1. To increase pupils' understanding of the role of mathematics in the formation of concepts and in logical support for conclusions concerning people's daily life and affairs.
2. To strengthen the pupils' arithmetic skills, increase their knowledge of symbolic interpretation and provide them with overview of common rules of arithmetic, among other things in order that they can master other subjects applying mathematics.
3. To increase the pupils' reasoning maturity and ability to generalize and thus prepare them for further studies in mathematics.[902]

The aims, only related to the first year's study, concern the pupil him/herself, his/her daily affairs, applications in other subjects and prerequisites for further studies. Nothing in these aims or elsewhere in the document referred to the importance of mathematics for the socio-economic or technological development of society. Emphasis on the needs of society was no longer on the agenda, at least not among the respectable mathematics teachers who comprised the group.

A booklet containing the preliminary or draft curriculum (*drög að námskrá*) as it was called, a total of 62 pages in A4, was published in April 1981. It had no official status. No general legislation had as yet been passed for the upper secondary level, and the curriculum document was adapted neither to the year-long courses of the traditional high schools, nor to the modular system of the more recently-established schools. In 1986, when the 1988 legislation was about to be adopted and a concise national curriculum was published,[903] this 1981 draft was a useful background document. However, the 1986 curriculum, which was amended in 1987 and 1990, only had advisory status for all but the first year of study, so the individual schools' curricula remained fairly diverse until the publication of a new national curriculum in 1999,[904] following the adoption of new legislation in 1996.

[901] Menntamálaráðuneytið, skólarannsóknadeild (April 1981): 20–21
[902] Menntamálaráðuneytið, skólarannsóknadeild (April 1981): 24
[903] Menntamálaráðuneytið, framhaldsskóladeild (1986)
[904] Menntamálaráðuneytið (1999b)

Availability of Mathematics Teachers

There were never many mathematics teachers with the complete required qualifications in the high schools. The head teachers in each school, however, usually had a firm basis in mathematics. This was bound to change in the great expansion of the 1970s. During the very first years of the newly established Faculty of Engineering and Natural Sciences at the University of Iceland in 1970, it produced a fair number of teachers graduating with a B.Sc. degree in mathematics and physics, but soon most of these students went abroad for further studies. On their arrival back home they were usually overqualified for teaching at the upper secondary level.[905]

In 1987 the Ministry of Education had made a report on mathematics education in upper secondary schools. The report revealed that 38% of mathematics teachers in upper secondary schools had earned 60 university credits or more in mathematics: whereas a bachelor's degree required 90 credits.[906]

In 2004 the Ministry of Education had another survey made,[907] in which 46% of mathematics teachers on the upper secondary level are reported to have a B.Sc. degree or higher in mathematics, while many of the remaining 54% had 15–30 credits in mathematics included in a B.Ed. degree from the University of Education, in addition to a B.Sc. degree in some natural science or commercial studies. About 36% of the teachers of the basic first year upper secondary courses had a B.Sc. degree or higher, and many teachers at that level had a B.Ed. degree. The figures for the mathematics stream courses were 56–59% with a B.Sc. degree or higher.

While less than half with a B.Sc. degree or more is certainly too little, one may say that there is an improvement as compared to 17 years earlier. It seems that a B.Sc. degree is on the way to becoming realistic as a minimum requirement for teaching in the mathematics stream at the upper secondary level, while one should recall that an equivalent to a master's degree was supposed to be required before the expansion.

Creation of Mathematics Textbooks during the Reform Period

The first sign of a reform of mathematics teaching at the upper secondary level was the introduction of new foreign textbooks, belonging to the new movement of "modern" mathematics, beginning in 1964. This entailed no expense, but a lot of inconvenience and unpaid extra work for the teachers and the students, as well as some enthusiasm, at least among the teachers.

When writing in Icelandic began, much of it was more or less translated, shortened and adapted to the Icelandic context. While Guðmundur Arnlaugsson was mathematics consultant he wrote the textbook *Numbers and Sets* for the lower secondary schools and *Algebra and Trigonometry* for the first year of high school.

Hörður Lárusson wrote textbooks when he became consultant and joined the staff of the SRD. He wrote a series for the lower secondary schools, published by the State Textbook Imprint, for high school, published by himself, and for the continuation departments, published by the SRD. Later Anna Kristjánsdóttir joined the SRD primary textbook writing group and still later she established another group for writing lower secondary textbooks.

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905 See Appendix K

906 Benedikt Jóhannesson (1987): 19

907 Menntamálaráðuneytið (February 2005): 12

Foreign textbooks were translated by teachers in the schools, and new books were written, especially in Hamrahlíð High School. Hildigunnur Halldórsdóttir wrote her book for the first year, while she taught there. Ingvar Ásmundsson and Ragna Briem later wrote *Rauntölur og föll* / *Real Numbers and Functions*[908] for the first-year course as well. At that time, Hildigunnur Halldórsdóttir, Ingvar Ásmundsson and Ragna Briem and other teachers in Hamrahlíð High School working on translations were around their thirties, building up a new school with an encouraging headmaster, Guðmundur Arnlaugsson. He himself also wrote several books while he was headmaster, such as *Vektorar og hornaföll / Vectors and Trigonometric Functions*, *Föll, afleiður og heildi / Functions, Derivatives and Integrals*[909] for the non-scientific streams of the high school and *Rökfræði / Logic*[910] for the more ambitious mathematics pupils, a book that went onto a small market but has been republished and used up to the present day (2004), thereby proving its validity.

Thus the "modern" mathematics reform encouraged initiative and creativity in textbook writing, preceded only by Dr. Ólafur Daníelsson in the 1920s.

8.6. Beyond the Reform Period

The Awakening Effect of the Reform

All facts considered, the introduction of "modern" mathematics into the Icelandic school system was a reform in the sense that it awakened people – teachers, parents, educators and politicians – to think about mathematics education. It aroused feelings: fear, respect, anger and enthusiasm. The attitude was introduced that mathematics was about thinking, rather than training skills, even if the reformers may not have hit upon the best way to lead all pupils to the clearest thinking about mathematics.

The teachers also had opportunity to exert their creativity and initiative. Compulsory School Headmaster Hörður Zóphaníasson was one of the teachers that joined an SRD workgroup to write textbooks for the early primary years to follow Agnete Bundgaard's textbooks. In his opinion the Bundgaard textbook series contained too much theoretical mathematics and it did not support the number work well enough. However, he claimed:

> Það þarf alltaf að vera stöðug endurnýjun. Það gefur kennurum nýjan kraft að vera að fást við eitthvað nýtt. Kennarinn þarf að hafa frjálsar hendur, svo að hann hafi trú á að hann sé að gera eitthvað gott.
>
> There is always a need for a constant renewal. It empowers teachers to work on something new. Teachers need to have a free hand, so that they have faith that they are doing something worthwhile.[911]

The Legislative Framework

The 1974 Compulsory School Act created a new framework for education. There were no longer any exemptions for children below the age of 10, and a number of boarding schools were built in rural areas. This was not a good system for young children, and the operating expenses were high. The boarding schools were gradually superseded by a school-bus system, as the road system improved. National curricula

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[908] Ingvar Ásmundsson and Ragna Briem (1977)
[909] Guðmundur Arnlaugsson (1976b). Guðmundur Arnlaugsson (1976a)
[910] Guðmundur Arnlaugsson (1984)
[911] Hörður Zóphaníasson, June 2, 2003

in all school subjects in the compulsory school were published in one concise volume in 1989.[912]

Throughout the 1970s, proposals for legislation for an upper secondary school system were discussed. The main obstacle was disagreement on how to divide expenses between the state and the municipalities. The system of municipalities essentially dated back to medieval times. The local communities numbered over 200, with populations ranging from 50 to 100,000, and their ability to undertake major tasks varied immensely. Finally in 1988 upper secondary school legislation was passed.[913]

In the 1990s, efforts were made to merge small communities with larger ones, and the municipalities overtook the compulsory schools under new compulsory school legislation in 1995. New upper secondary school legislation was adopted in 1996. Both acts had concomitant national curricula in 1999.[914]

In the early 1990s, a minor economic recession resulted in a decrease in number of school hours for children at primary level, already suffering from a short school year and the tradition of double use of classrooms by one set of pupils in the mornings and another in the afternoons. This habit had prevailed since the post-war period, and possibly longer in some places. Following the 1995 legislation the number of hours was increased again. Later in the decade, the "all-day school" became an issue in the general election campaign, possibly in connection with perceived poor results in the 1995 TIMSS survey. Since 2000 double shifts in compulsory schools have disappeared.

One might say that at the turn of the 21st century the Icelandic school system had at last become "normal" in the European sense; all children attended school in the morning, there were official curriculum documents, and qualified teachers with adequate education were the rule rather than the exception.

Mathematics Teaching

The School Research Department mathematics textbooks series of the 1970s, revised and amended, continued to be the basic material for grades one to seven until the turn of the century. They had more reading text than exercises, as they were originally intended for investigative work. Gradually more exercise-booklets were published, both for training skills and problem solving.

From 1974 the three year lower secondary level was part of compulsory schooling. The requirements for teachers were no longer B.A. or B.Sc. degree in mathematics, but a B.Ed. degree from the University of Education, preferably but not obligatorily with mathematics as an elective. Gradually more teachers became qualified, but not enough to cover the lower secondary level. In a recent survey, 35% of mathematics teachers in the eighth to tenth grades had a B.Ed. degree with mathematics as an elective or a B.Sc. degree in mathematics, in the school year 2003–2004. Teachers with a general teaching certificate comprised half of the group of the mathematics teachers in grades eight to ten.[915]

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[912] Menntamálaráðuneytið (1989)
[913] *Stjórnartíðindi* 1988 no. 57, May 19
[914] Menntamálaráðuneytið (1999a); (1999b)
[915] Menntamálaráðuneytið (February 2005): 16

Mathematics educational material for the primary and lower secondary level initially developed in the field of investigative approach. Many teachers found that approach too demanding, and not conducive to developing skills. A more "back-to-basics" approach has become prevalent at the lower secondary level since the early 1990s, through the Swedish textbook series by Björk et al. (See section 8.2.).[916]

Mathematics was allocated a similar share of the compulsory school curriculum in the 1974 legislation to what it had had throughout the century, while compared to other Nordic countries the school year was much shorter and consequently the syllabus was more limited. In the new compulsory school legislation in 1995 and the concomitant regulations, mathematics was allocated an increased share of hours, partly having in mind the future plans to reduce the duration of upper secondary level studies to three years in the near future. In 2005 that matter was on the agenda of the Minister of Education, but has met resistance from various bodies.

Mathematics was allocated a considerable share of the 1986/1987/1990 national curriculum of the new upper secondary education for all, a curriculum that had been developing in the schools since the mid-1970s. In recent years, the level has developed according to the 1999 national curriculum and concomitant regulations, where compulsory mathematics was reduced and transferred to fields of electives. Following the new curriculum, some official support was offered to curriculum development. The series by Jón Hafsteinn Jónsson et al.[917] is used in several schools, and a translated Swedish series for upper secondary schools by Björk et al.[918] is used in many schools.

School Research Department

The School Research Department kept up its developmental work until 1984, when materials for the social sciences, i.e. history and geography, were being developed after a 15-year process. In 1983 this suddenly caught the attention of the *Alþingi*, resulting in a heated public debate in 1983–1984. The Icelandic history project became a catalyst for a serious dispute about national heritage education. The obstacle was a new presentation of the history of Iceland, to succeed the 1915 *Íslandssaga / Icelandic History* by Jónas Jónsson of Hrifla. While the discussions about Icelandic history were long overdue, they were not all well-founded. Furthermore, there were suspicions that the new geography concealed a political agenda.

After a long "dark winter" dispute, the School Research Department was abolished.[919] It is beyond the scope of this study to discuss the reasons for that fact. Its tasks were divided between the National Centre for Educational Materials (NCEM), formerly the State Textbook Imprint, which resumed the development of educational materials and the Ministry itself, which undertook the policymaking. Counselling was provided by state educational centres in the regions and after 1995 in the various municipalities. In the 1980s there were consultants working at the Ministry of Education, but after 1992 there were none. Many of the former staff of the School Research Department had already joined the staff of the Iceland University of Education, and some of them also at the NCEM. Thus, the spirit was nurtured at the

916 Björk, L., et al. (2000–2001)
917 Jón Hafsteinn Jónsson, Níels Karlsson and Stefán G. Jónsson (1997–2002)
918 Björk, L., et al. (2000–2004)
919 Gunnar Karlsson (1984): 405–415

University of Education and elsewhere, while there was less central assistance to the mathematics teachers around the country to be received.

International Context

From 1985 to the present day the teaching community has been disturbed by frequent strikes. An examiners' report: *Review of Educational Policy in Iceland*, made in 1986 on behalf of the OECD, described the situation as

> The rather relaxed atmosphere of the secondary schools that we observed went along with the pro-forma quality of their main offerings, putting students through academic courses in preparation for university with little reference either to actual individual student outcomes or to the needs of the Icelandic labour market. Teachers bring their income close to that of skilled labourers by working typically one-third more hours overtime and/or on another job. … Only 58 per cent are qualified - - even much less in the science and maths fields - - and teaching is predominantly lecture-question-answer style. … many schools are obliged to work on double shifts.[920]

and

> ... this denigration of the teaching profession may be due less to an egalitarian social setting than to the deeper cultural notion that teaching is merely a common-sensical activity, until recently performed within the family, that requires no special training. For example, when a staff writer for Iceland's leading newspaper wrote (Morgunblad, 22 February 1986) that the professional training of teachers is irrelevant to teachers' success in their jobs, he was representing a widely-held view that the development of people's intelligence and culture does not rest on the quality, or even the existence, of schools. This attitude harks back to a time when parents were at home for their children and before Iceland entered the modern world economy. Attitudes and priorities toward the teaching profession, that now must act both in loco parentis and as the locomotive of technical excellence required by future generations, cannot remain at this point. Otherwise the quality of teaching in schools could actually have a negative impact on the country's culture. [921]

These remarks did not persuade the authorities to improve the conditions of teachers to any marked degree, and strikes have continued regularly up to the year 2004.

In the Third International Mathematics and Science Study in 1995, TIMSS, Icelandic eighth-graders' achievement in mathematics did not differ significantly from that of pupils in Denmark, the United States, Scotland, Latvia, Spain, Greece, Romania, Lithuania and Cyprus. The international average was 515, while Icelandic pupils scored 487, its placement no. 32 of 41 countries.[922] As the score was below the average and significantly lower than that of such countries as Norway, Sweden and England, this achievement was considered unsatisfactory by educational authorities and the public and sparked considerable discussion.

Gradually, mathematics education in Iceland has begun to be measurable on the same scale as that of any other nation. One possible explanation is that the unfavourable results in TIMSS were a challenge to the teachers to focus more on mathematics. In spite of the mathematical background of many teachers being weak, the nature of their general education is tailored to respond to new challenges.[923] This

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[920] OECD Education Committee (1987): 22 (§ 70)
[921] OECD Education Committee (1987): 29 (§ 91)
[922] International Association for the Evaluation of Educational Achievement (November 1996): 22–23
[923] Ólafur Proppé, January 2003

may have been proved by satisfying results in the Programme for International Student Assessment (PISA) comparison study in 2003. In the mathematics part of the PISA 2003 study, Iceland's placement was no. 13–17 out of 41 countries and no. 10–14 out of 29 OECD countries,[924] on similar terms as Czech Republic, Denmark, France, Sweden and Britain, and significantly better than Norway, in addition to being well above the average, which was a relief to the educational authorities.

The PISA results indicate that the position of Icelandic mathematics education is parallel to that of neighbouring countries. It showed egalitarian tendencies, in that there was little difference between the results in the individual schools, in fact the smallest in the study. The number of pupils in the lowest and highest proficiency levels was also low, indicating even achievement. However, the Icelandic results have a special profile that has to be investigated. In no other country is the performance of girls in mathematics significantly better than that of boys at the age of 15. In most geographical areas in the country the girls' performance in mathematics is considerably better. Icelandic girls' placement compared to all girls in the study was no. 8, while the boys were no. 20, exactly in the middle of all boys in the study.[925]

It is necessary to explain this gender difference. An increased understanding of the social and educational factors which may influence these results is needed. Schools, and mathematics education in particular, do not seem to reach boys in many areas. One could conjecture that one factor is the culture in fishing villages, where the economy is not based on theoretical knowledge or education but on a specialised training in highly technical fishing. However, the boys' lowest performance is found in the southern agricultural areas, so this explanation is not completely satisfying.

Another factor to be explained is a comparative lack of truly excellent performance, e.g. compared to New Zealand and Switzerland with the same percentage of pupils in the lowest achievement groups.[926]

[924] Námsmatsstofnun/Educational Testing Institute, website, accessed June 14, 2005
[925] Námsmatsstofnun/Educational Testing Institute, website, accessed July 16, 2005
[926] Námsmatsstofnun/Educational Testing Institute, website, accessed July 16, 2005

9. Modern Mathematics in the Neighbouring Countries

The Icelandic mathematics reform movement was a part of an international wave. In section 7.1. we outlined briefly the origin of the reform movement in the United States and its diffusion with the European reform movement at the Royaumont Seminar in 1959 with the aid of the OEEC.

We shall now look into examples of the reform processes in England, Norway and Denmark, described in scholars' works, and extract similarities and differences to the Icelandic context. Accounts from all three countries indicate that the idea of implementing abstract algebra into school mathematics was based on theories of the Swiss psychologist Jean Piaget. These theories were the basis for the expectations that the professional people had of "modern" mathematics in schools.

9.1. England[927]

Introduction

Barry Cooper has written the book *Renegotiating Secondary School Mathematics. A Study of Curriculum Change and Stability*. It is a study of the initial processes of the redefinition of English secondary school mathematics that occurred during the late 1950s and early 1960s, with particular reference to the School Mathematics Project (SMP) and the Midland Mathematical Experience (MME).[928] In his study Cooper discusses the related issues of power and resources, the stratified nature of the educational system, the roles of the individuals and segments within the subject, and the relationship between the subject and other arenas and communities.[929]

Initially, discontinuity existed between school mathematics and some variety of university mathematics, and also between the subject perspectives of many mathematicians in educational institutions and some "users" of mathematics in an industrial context. In addition, there was increasing extra-subject concern regarding scientific manpower and the supply of mathematicians at the time.[930]

Exchanges between mathematicians, mathematics teachers and representatives from industry at several conferences created a basis for support from industry for reform of school mathematics. Initially, the emphasis was on applied mathematics.

On the other hand, the leadership of one of two influential teachers' associations, the Association for Teaching Aids in Mathematics, ATAM, campaigned for the introduction of post-1800 algebraic ideas into syllabi, while it also campaigned for pedagogical changes, legitimising both elements of its mission in terms of improving the child's "understanding" of mathematics.[931] ATAM was established in 1953 under the influence of the International Commission for the Study and Improvement of the Teaching of Mathematics (ICSITM), and ATAM's emphasis on abstract algebra reflected the involvement of ATAM's member in the Europe-wide meetings of ICSITM, where university-based modern algebraists had much influence.[932]

[927] Cooper, B. (1985).
[928] Cooper, B. (1985): 2
[929] Cooper, B. (1985): 26
[930] Cooper, B. (1985): 273–274
[931] Cooper, B. (1985): 151
[932] Cooper, B. (1985): 70–77

Many members of ATAM were supporters of Piaget's ideas on psychology. The editorial of ATAM's journal, *Mathematics Teaching*, in April 1958, p. 3, says:

> Indeed much modern research in pure mathematics has been directed towards putting the foundations of the subject on a simpler logical basis, and much of the psychological work of Piaget suggests that many of the essential notions of modern algebra (which are regarded as a university study) have to form in the pupil's mind before he is even ready to undertake the study of number … Such topics as the algebra of sets or relations might be taught with a profit not merely in the sixth form but lower down the school as well. In other countries they are learning how to do this, and unless we learn too we shall be left behind.
>
> Of course, such ideas have to be presented in a suitable way. The formal axiomatic way in which groups, rings and fields are presented to pure mathematicians at university would never do in school. The idea must be presented in terms of concrete applications with a similar structure.[933]

This citation describes the ideological background for the introduction of "pure" mathematics, i.e. post-1800 algebraic ideas, into school mathematics, in the "modern" mathematics reform wave, promoted by the ICSITM. It also reveals the concern for not being "left behind."

It is not known if there was any contact between the ICSITM and the International Bureau of Education (IBE) in Geneva, where Jean Piaget was director. It seems not unlikely, as the ICSITM was a European carrier of the idea of implementing "modern" mathematics in school mathematics on the basis of Piaget's ideas. ICSITM, which no longer exists, may have been one of IBE's standing commissions. Gestur O. Gestsson made a report on IBE's recommendations to Icelandic teachers in 1961 (see section 6.7.), and was thus a carrier of Piaget's theories about children's mathematics learning to Iceland, while Prof. Matthías Jónasson introduced Piaget's theories in his pedagogy courses at the University of Iceland. We have also seen that Piaget's theories were detected in the foreword to Guðmundur Arnlaugsson's textbook *Numbers and Sets* (see section 7.3.).

In spite of vastly different societies in heavily-populated England and extremely sparsely-populated Iceland, there are some parallels between the implementation processes of "modern" mathematics in these countries, which will be highlighted in the following. The Icelandic parallels occurred about five years later than in England. However, there is no reason to infer that there was a direct impact from the events in England on the Icelandic evolution. International currents from the Royaumont Seminar spread to both countries, but to Iceland through the Nordic cooperation, NKMM.

Stratification

About the stratified nature of the educational system, Cooper says:

> It is well-established that within the secondary sector, as one moved from secondary modern through grammar to independent schools, that is from low to high status schools in terms of social prestige, the occupationally-defined class origins of both teachers and pupils shifted upwards. ... Teachers in the selective schools tended to have been prepared by one professional route – the high status graduate route; those in the non-selective schools by the lower status training college route. ... Simplifying, teachers in secondary schools at this time can be seen as having been members of one of two distinctive educational subcultures of different social status.

[933] Cooper, B. (1985): 76

> ... Other constraints and opportunities arose from the typically different origins and destinations of their pupils.[934]

The Icelanders have often prided themselves on having a non-stratified society. There was some flow between social layers in the 19th and early 20th century, from poor tenants to the land-owners and from the land-owners to the professional class, in fact more so than in other countries, i.e. Icelandic society seems to have been socially mobile.[935] However, the learned class was certainly an upper class in society, the group exclusively entitled to the highest positions by law. That social stratification was clearly reflected in the educational system, and manifested on one hand in the Reykjavík School, the successor of the cathedral schools, the only school of its kind until 1930 and a dominant school until the 1960s. On the other hand there were the Teacher Training College, the technical schools, the district schools and the general lower secondary schools.

Similarly, on the learned/high school path the teachers were generally university-educated, at least in some university subject, while for the non-college-bound pupils there were college-trained teachers, more pedagogically oriented but lacking mathematical education. According to a study made in 1975, university students in the 20th century were more likely to originate in the uppermost layer of society, i.e. families of officials and businessmen, than those attending the Teacher Training College. However, more than 40% of both classes were descendants of farmers.[936]

Mathematics Content

The two major subcultures seen to exist within secondary-school mathematics in 1950s England involved different versions of "mathematics" or drew different selections of contents from the corpus of "mathematics". That of the selective schools, of higher prestige, looked forward to the further study of mathematics and science, while that located in the secondary modern sector looked to the workplace and the home for the legitimation of its selection.[937]

The curriculum of the selective schools was an amalgam of "academic" and "practical" mathematics, with more emphasis on the former, classical mathematics, rather than arithmetic, i.e. algebra, geometry, trigonometry and possibly calculus. The primary function of mathematics courses was perceived as being to prepare pupils for further study of mathematics and science. The non-selective schools were concerned almost entirely with the "practical", while this simple picture was complicated during the 1950s by the increased tendency for pupils in the secondary modern schools to be entered for GCE (General Certificate of Education) examinations.[938]

In the field of curriculum one can also see similarities to the Icelandic context. The high schools, led by the Reykjavík High School, had their rigid syllabus. Their mathematics programme aimed at preparing their pupils at least for entrance into engineering studies in Denmark or Germany, while the general lower secondary education had no defined curriculum, and a practically-oriented syllabus. The picture became more complicated with the introduction of a new national examination for general lower secondary schools in 1968, and the establishment of the continuation

[934] Cooper, B. (1985): 27
[935] Gunnar Karlsson (2000): 166
[936] Helga G. Halldórsdóttir and Kristín Waage (1975): 54–55
[937] Cooper, B. (1985): 275
[938] Cooper, B. (1985): 36–42

departments and comprehensive schools in 1969, a kind of parallels to the English GCE programme and the HF-programme in Denmark (see section 9.3.).

The OECD Paradigm and the SMP

The source of development of the School Mathematics Project, SMP, can be traced back to 1957, when a conference was convened in Oxford on a personal initiative for the purpose of bringing together, for the first time, those who taught mathematics in schools and those who used mathematics in real life.[939] The conference was attended by university people, mainly mathematicians from the applied segment of the subject, by representatives from private and nationalized industry, and by mathematics teachers from selective schools.

The government and opposition in England were at that time increasingly concerned with the adequacy of arrangements for teaching and research in scientific and technological areas, and in particular with potential shortages of manpower in these fields. Many politicians and commentators assumed that Britain's economic success would depend on the application of scientific research to industrial processes. Concern was expressed by many in educational organizations about possible and perceived shortages of specialist teachers of the subject, and about the mathematical education of non-specialists.[940] Another important concern was an increase in the number of pupils at the upper secondary level. These trends were discussed, in Oxford and elsewhere, in the context of media-supported concern with a threat, both economic and military, from other industrial societies.[941]

The above-cited debates echoed in early 1960s Iceland, influenced by the OEEC and the OECD. In spite of the absence of highly technical industry, with its need for scientifically-educated manpower, there were concerns about shortages of teachers, which must be considered well founded, and recently implemented ideas about education contributing substantially to economic and social progress and stability, much needed in Iceland. Furthermore, there was greatly increasing pressure on the upper secondary level, which had not yet been resolved in 1965 when the OECD theories were being implemented.

Major Shifts in Access to Resources

The active interest developed by major companies in 1950s England in mathematical and scientific education, and in particular the involvement of a number of them in the 1957 Oxford conference and another in Liverpool in 1959 on similar topics, represented a major shift in the resources potentially available to those promoting various missions within mathematics. According to Barry Cooper, not only were funds now possibly available, but as a result of the conferences "authoritative" statements supporting curriculum reform had become available from an influential source.[942]

The "influential source" is probably the influential newspaper *The Times,* widely read by politicians. *The Times* reported the address of the president of the Mathematical Association to the Liverpool conference in April 1959, expressing concerns about teacher shortage as a serious threat to government plans for technical

[939] Cooper, B. (1985): 91
[940] Gooper, B. (1985): 91
[941] Cooper, B. (1985) : 92–93
[942] Cooper, B. (1985): 153

education.[943] *The Times* continued in 1959–1961 to promote messages questioning current school knowledge, linking the perceived shortage of mathematics and science teachers with the standard of living, and suggesting a need for the rewriting of "outdated textbooks".[944]

The major shift in Iceland occurred in 1966 when the *Alþingi* decided, upon the initiative of Minister Dr. Gylfi Þ. Gíslason and in cooperation with the OECD, to allocate funds to the School Research Department, the body that became the prime channel for the redefinition of the school system and school mathematics in particular. This occurred following several visits by specialists from the EIP, Educational Investment and Planning Programme of the OECD, and their meeting with all the most influential persons in Icelandic education at that time.

In Iceland, where the educational system is much more centrally organized, and industry and the university were much weaker, than in England, the external influences were thus mainly exerted by the international body OECD, while in England the redefinition was due to cooperation between external bodies, such as certain university teachers and influential people from industry, and educators, especially from high-prestige selective schools. In addition there were influences from the OEEC / OECD and their theories about needs for scientific and technological manpower and the OEEC-promoted Royaumont Seminar in 1959.

Power Groups

In his conclusions on the movements for change realized at the two conferences in Oxford and Liverpool, Cooper said: "… once the resources seem fairly secure, various groupings within the subject, perceiving opportunities for the development of their missions and careers, struggle to win a share. Here, competing groups promote specific versions of possible change."[945] In such a situation it becomes possible that the initial instigators of the events, notwithstanding their control over important resources, will become relatively marginalized.

Following the Liverpool conference in England there was the Royaumont Seminar in November 1959, where stress was laid on the discontinuity that existed in most countries between school and university mathematics.[946] At this seminar, the dominant participants took it for granted that "mathematics" meant "university mathematics", i.e. modern algebra. The conclusion was that it was this "mathematics" that school pupils should necessarily study,[947] while those who supplied the resources in fact had applied mathematics in mind.

In 1961, under the influence of European and American algebraists felt through ICSITM and the seminar in Royaumont, some members of the ATAM leadership had further modified their mission, putting a much greater emphasis on introducing "modern" mathematics into schools, and they had succeeded in diffusing their version of "mathematics" to some English school teachers.[948]

When *Alþingi* agreed to allocate funding to "school research" in 1966, it certainly was not aware of the nature of the changes ahead concerning mathematics. Decisions

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[943] Cooper, B. (1985): 129
[944] Cooper, B. (1985): 167–171
[945] Cooper, B. (1985): 153
[946] Cooper, B. (1985): 159
[947] Cooper, B. (1985): 160–161
[948] Cooper, B. (1985): 165–167

on content were left to specialists who, due to their high prestige, could promote their version of change, which in turn was highly influenced by the theories promoted at the Royaumont Seminar, diffused through Danish and American contacts and textbooks. When "modern" mathematics had been implemented, a leading politician and a member of *Alþingi* – an engineer and a good mathematician, Emil Jónsson, who fifty years earlier had urged Dr. Ólafur Daníelsson to work on implementing mathematics stream at the Reykjavík School – expressed his concern that he would no longer be able to cope with even a lower secondary school mathematics examination.

"Modern" Curriculum and Its Diffusion into the School System

According to Cooper, by April 1961, as a result of the importation of the notion of curriculum development from the USA via the School Mathematics Study Group, SMSG, and similar projects, the possibility of developing similar structures, with their associated career opportunities, was beginning to be canvassed within the mathematical community in England.[949] The prospective project leaders were careful to stress the possibility of "modern" abstract ideas being taught in a child-centred, practical fashion.[950]

In 1963 Kristján Sigtryggsson brought home to Iceland samples of materials from the SMSG and similar projects. Concurrently with the consideration of the SMSG material, a basis was laid for "modern" abstract ideas being taught in a child-centred fashion, by presenting in *Menntamál* ideas by Prof. Patrick Suppes at Stanford University and his experiments with teaching small children about sets.[951] The Icelandic educational community felt itself unable to initiate developmental work from scratch. The SMSG project was considered for a while and later the Danish Bundgaard project, highly influenced by the Royaumont ideas, was chosen for translation.

In Cooper's opinion, "the very success of the School Mathematics Project, in ensuring its diffusion "downwards", brought a curricular selection originally developed within … selective subculture, and only modified by teacher-writers experienced with "less able" children, into contact with many teachers whose subject and pedagogical perspectives derived from within the non-selective subculture, in which criteria for selecting mathematical content differed considerably. This, together with the dissatisfaction of members of various university disciplines and sub-disciplines, ensured that SMP would be continuously subject to criticism."[952]

In Iceland the Danish primary school material became subject to much criticism when it came into contact with many teachers whose pedagogical perspectives derived from within the non-selective subculture, i.e. the Teacher Training College, where there was only little emphasis on mathematics. Those teachers' criteria for selecting mathematical content differed considerably from the criteria of the university persons who had selected and composed it (i.e. Guðmundur Arnlaugsson, Björn Bjarnason, Svend Bundgaard and Agnete Bundgaard).

Case studies suggested that the SMP's materials, when used in the comprehensive schools in England, might often be used within a well differentiated curricular

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[949] Cooper, B. (1985): 199
[950] Cooper, B. (1985): 203
[951] Kristján Sigtryggsson (1964): 159
[952] Cooper, B. (1985): 265-266

framework and that, pedagogically, there may have been little change in secondary mathematics education since the 1950s.[953]

The SMP material was used in Iceland with limited success. It was differentiated, in the sense that the "selective" school, i.e. the Reykjavík High School, used the most advanced version, *Advanced Mathematics*, while the "comprehensive" schools, i.e. the continuation departments, used *Book T* and *Book T4* in a translated, shortened and adapted version. And as Guðmundur Arnlaugsson expressed it concerning the compulsory school level, the teachers may not have changed their attitudes or teaching methods, but only seen some new computation methods in addition to the old ones. Thus the ultimate change was in content, not in pedagogy. This is also suggested by Cooper:

> [In curriculum change] … innovations operate on three fronts: at the level of the syllabus, at the level of the teaching methods employed, and at the level of theories of the whole purpose of education. …
>
> Industrial employers were concerned with content, pedagogy and the "attitude" of their future graduate employees, university mathematicians shared these concerns in respect of their students and potential successors, and teachers, at all levels, were likely to find their practice legitimated or otherwise by the outcomes of any process of redefinition once begun in earnest. It was probably this wide range of occupational interests that grounded the breadth of a debate in which content, pedagogy, psychology and ultimate purpose all featured. ...
>
> At classroom level … the redefinition finally achieved by the actions of those involved in the social movement of the late 1950s and early 1960s has probably been primarily one of content … mathematics teachers remain "transmission" orientated but new content is, in many cases, being transmitted.[954]

Summing Up

We have seen that there are clear similarities between the implementation of modern mathematics and the redefinition of school mathematics in England and Iceland, in spite of only slight direct mutual influences. The main influence was from the Royaumont Seminar, where a certain group of university mathematicians, supported by pedagogical ideas, won support. Their ideas were to be implemented by people with different pedagogical perspectives and different views on mathematics. In many cases the result was more of a change in content than in pedagogy.

Those who supplied the resources were unaware of the nature of redefinition which was to be implemented. In England their intention was increased emphasis on applications and practical mathematics. The SMP material was made under the strong "modern" mathematics influence. However, the text may be considered as a compromise, embracing the various paradigms of both pure and applied mathematics, as well as elements of traditional school mathematics.[955] Compared to the Nordic "modern" mathematics syllabus for high school the English SMP material was more related to applications, and consequently, the SMP material in Iceland was used primarily for upper secondary level streams preparing for vocational colleges.

In Iceland, the resources were provided by politicians who left detailed decisions to subject specialists. They were unrestrained by any content requirements and they

[953] Cooper, B. (1985): 266
[954] Cooper, B. (1985): 279–281
[955] Cooper, B. (1985): 275

chose to implement "modern" mathematics in its most orthodox fashion, at least at the primary level. In both cases, in England and Iceland, the actual implementation caused disappointments, negative reactions, criticism and various changes in conditions, such as beliefs concerning the economic efficacy of education, which promoted further redefinition.[956] This process led to permanent redefinition of school mathematics.

9.2. Norway

Introduction

Gunnar Gjone has written *"Moderne matematikk" i skolen. Internasjonale reformbestrebelser og nasjonalt læreplanarbeid*, I–VIII / *"Modern Mathematics" in School. International Reform Efforts and National Curriculum Work.* I–VIII (1983),[957] an account of the international process of implementing "modern" mathematics and the concurrent and consequent curriculum development in Norway. The following will be based upon Gjone's work.

An experiment with nine-year compulsory schooling was going on in Norway in the 1960s. When "modern" mathematics reform arrived there, it was closely related to Nordic cooperation in NKMM. After some experiments with NKMM and other material, a committee was appointed, with boards for each subject to prepare a "normal" curriculum plan for primary school. The proposals then went into a discussion process which lasted several years.

In short, there was a developed process from controlled experiments within a limited number of schools, to proposals from a subject committee, to a proposal from a curriculum plan board to the School Council, reconsideration and subsequent debate in newspapers and parliament. This went on while the worldwide excitement about "modern" mathematics reached its peak. Final decisions were not taken until after that, and "modern" mathematics was first formally introduced nationwide when the curriculum plans had undergone this process. The most abstract concepts had retreated into the background, even if the underlying idea was still that the best way to learn mathematics was to grasp the structure of mathematics, its main lines and the dominating concepts.[958]

Framework

In the 1960s, a curriculum plan for an experiment on nine-year compulsory schooling was gradually taking over from two plans; a so-called *Normal Plan* from 1939, with seven-year compulsory schooling, and the curriculum plan for the theoretical lower secondary school, *realskole*, for those who were aiming at higher and further education.[959]

Compared to Iceland, the lower secondary *realskole* in Norway was more theoretical than the Icelandic lower secondary *unglingaskóli/miðskóli/gagnfræðaskóli,* as eight-year compulsory schooling for all had already been established in Iceland in 1946. It was therefore only the ninth year of the national examination of middle

[956] Cooper, B. (1985): 279
[957] Gjone, G. (1983)
[958] Bjørnestad, Ø. (1998): website accessed February 27, 2005
[959] Gjone, G. (1983): Vol. III, iii–1

school/*miðskóli* which can be considered theoretical, and comparable to the Norwegian lower secondary *realskole*.

The new experimental plan in mathematics for the nine-year school was accepted in Norway in 1959. In comparison to a later plan introduced in 1971, this 1959 plan was rather conventional. However, the intentions of the authors were progressive, in the sense that there was some "mathematizing" of the syllabus in comparison to the *Normal Plan*: early use of letters representing numbers and quantities, and introduction of geometrical concepts. Mathematical topics were brought down to lower grades, while the more difficult practical computations were moved upwards.[960] This "mathematizing" is the clearest example that the plan's intentions were in harmony with ideas implied in the reform movement. But there was little follow-up of the intentions contained in the plan. It turned therefore out to be rather controversial and was perceived as traditional.[961]

The Implementation Process of "Modern" Mathematics

The "modern" mathematics reform movement arrived in Norway from abroad via the Royaumont Seminar, where American reform thinking and projects were presented under the direction of the OEEC.[962] The first developmental projects were worked out together with the Nordic countries through the NKMM cooperation. This period may be considered to have come to an end in 1967, when the NKMM completed its report. The experiments were not officially debated. They were important in that the persons involved also became involved in the compulsory school's national curriculum reform work which followed.[963]

The year 1967 saw the beginning of official developmental work and expansion of the experimental work. Projects from the USA (SMSG) and Sweden (Individualized mathematics instruction / *Individualisert matematikkundervisning*, IMU)[964] were adjusted to the Norwegian context. In 1971 the development was completed, and some of the reform projects also. Instead of a following period of implementation and realization, there was a period of reaction, discussion and new development of teaching material.[965]

In 1967 to 1973 several curriculum plans were made. The first proposal was made by a subject committee to a *Normal Plan* Board, which changed it and included elements from both directions: "modern" mathematics and the behaviouristic-inspired method, introduced in IMI. The result was therefore a kind of a compromise between many different views.[966] That proposal underwent treatment by the School Council, calling for comments, and another subject committee. The resulting plan, later called *Alternative 2*, was a "modern" mathematics plan.[967] *Alternative 1* was by and large a reproduction of the curriculum plan for the nine-year school produced in 1959.[968] A provisional pattern plan *M 71* in 1971 comprised the two alternatives of a plan.

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[960] Gjone, G. (1983): Vol. III, 2–3
[961] Gjone, G. (1983): Vol. III, 5–6
[962] Gjone, G (1983): Vol. VIII, 7
[963] Gjone, G. (1983): Vol. III, 17–20
[964] Gjone, G. (1983): Vol. III, 15–17
[965] Gjone, G. (1983): Vol. VIII, 8–9
[966] Gjone, G. (1983): Vol. III, 93–100
[967] Gjone, G. (1983): Vol. IV, 19–41
[968] Gjone, G. (1983): Vol. IV, 83

The question remains, why two alternatives were introduced. They were presented as a temporary need for a choice. It was clear that *Alternative 2* would demand a great deal of adaptation from most teachers, who had no experience of “modern” mathematics. Criticism regarding “modern” mathematics had also emerged in other countries. However, there would be obstacles for pupils moving between the two alternatives. It seems that *Alternative 1*, prepared after *Alternative 2*, was presented as a kind of security back-up. It later turned out that *Alternative 1* became very important and played a crucial, unexpected role.[969]

Public debates about *Alternative 2* were going on in 1971–1973.[970] In January 1972 the School Council found it necessary to revise *Alternative 2* to reduce the syllabus. The reason given was that the number of teaching hours was too small for it. However, the changes, put on hold until after parliament had treated the two alternatives, witness that the council had become aware that *Alternative 2* might have had a somewhat unfortunate form of representation.[971]

The education committee of parliament wished for a new, wide, “composite” board for further development and evaluation of the curriculum plan in mathematics. The new board’s solution was to take in the more practical aspects of Alternatives 1 and 2, and leave out the logic and set theory in the form presented in *Alternative 2*.[972] A new proposal shows a break with “modern” mathematics and NKMM, while it gave relatively great methodical freedom for teachers and textbook writers, and could therefore hardly exclude relatively modern learning material.[973] That proposal became basically the new curriculum pattern plan in mathematics, M 74, in 1974.[974]

This long decision process, going through a subject committee, a curriculum board and consultations, from the School Council to parliament, back to a new subject committee, to board and council, is the main difference from the process in Iceland, which was much less developed. In Norway, the teaching is in many ways decided centrally by curricula, textbooks, legislations and regulations,[975] and this was and is also the case in Iceland. In both countries there was thus a centralized government of educational affairs, while in Iceland it was underdeveloped: at each point of time there were at most a couple of subject specialists, to whom it was left to take necessary decisions. In both countries, one might say that “modern” mathematics did not go beyond the experimental stage at primary level, while the process at primary level in Iceland spurred a new nationwide school mathematics reform.

Summing Up

The development period in Norway came to an end in 1973. It was influenced by the same social currents as in the USA, with critique of a lack of basic skills in arithmetic.

At the upper secondary, college-bound level, the reform movement brought a necessary adjustment to university mathematics. No one would e.g. question the implementation of vector computations. The difficulties were primarily within

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[969] Gjone, G. (1983): Vol. IV, 88–90
[970] Gjone, G. (1983): Vol. V, 54–114
[971] Gjone, G. (1983): Vol. V, 41–43
[972] Gjone, G. (1983): Vol. VI, 1–26
[973] Gjone, G. (1983): Vol. VI, 55–56
[974] Gjone, G. (1983): Vol. VI, 76–85
[975] Gjone, G. (1983): Vol. VIII, 8

primary education. The central question throughout the reform period was: what kind of mathematical knowledge was necessary for the individual leaving school? The reactions were similar to that in other countries, but the centralized Norwegian school system gave the subject committees great power and influence on the resulting proposals for a national curriculum.[976]

Similarly, the reform movement brought many useful mathematical ideas to the Icelandic upper secondary level, while the main problems emerged at primary level.

Individuals had great influence on the process in Norway, as in Iceland. There were three Norwegians present at the Royaumont Seminar and four to five people worked out the curriculum proposal, *Alternative 2,* for the basic school. For the first four to five years, there were individuals, who had the gift to engage other persons in "modern" mathematics teaching. In the curriculum plan work also, individuals were the driving force.[977]

In Norway it was difficult to find persons who were "neutral" to evaluate and comment on the proposals, due to the small size of the community.[978] In Iceland it was practically impossible. No one in Iceland either had the authority or perspective to question the activities and decisions taken by the proponents of "modern" mathematics, due to their education and the superiority of their position in the dominating Reykjavík High School.

The implementation of "modern" mathematics reform in Norway was clearly an influence from other countries. In the 1960s optimism with respect to what could be gained by teaching technology prevailed. General optimism about technology, strongly emphasized by OECD – that technology could bring economic development of society – was probably an important factor in bringing the authorities' attention to what was happening in other countries. The cooperation and solidarity between the Nordic countries was another important factor.[979]

Developments in Iceland can also be ascribed to both of these factors. The authorities, the government and the parliament were influenced by the OECD theories, and the school authorities were aware of the Nordic NKMM cooperation and the meetings at Woods Hole and Royaumont. Iceland was not directly involved in Nordic cooperation, but Icelanders were aware of its work in the 1960s. Direct influences from the United States may, however, have been stronger in Iceland than in Norway concerning the high school reform.

Yet another important factor for Norway was the development of a nine-year compulsory school, which was to combine grade seven from the primary school and the theoretical lower secondary school. This was to include the latter's connection to the upper secondary level, which many felt that should be for all, and not only an elite high school. In that respect *Alternative 2* looked very theoretical. There was a kind of clash between two directions within the school system, each with a long tradition, between a movement originating from "below" in beginners' education and a movement from "above" from higher education, each supported by its own teacher organization.[980]

[976] Gjone, G. (1983): Vol. VIII, 11
[977] Gjone, G. (1983): Vol. VIII, 15–16
[978] Gjone, G. (1983): Vol. VIII, 16
[979] Gjone, G. (1983): Vol. VIII, 13
[980] Gjone, G. (1983): Vol. VIII, 14–15, 18–19

Although the pattern of the general school reform in Iceland was different, similarities may clearly be identified in the clash between the two directions within the school system – or subcultures as B. Cooper expressed it – the elite high school and the general education for the common people, terminating by the year after the confirmation, or continuing for one or two years with "practically" oriented education. In both countries there was a growing wish for an egalitarian "school for all", which exerted strong pressure on the school system. It was not yet formalized or manifested in Iceland, rather an underlying current which the authorities may not have completely realized, while they, as representatives of the elite class, were trying to maintain streaming and differentiation in various forms. In Iceland this was solved to a great extent through the activities of the School Research Department, while in Norway it may have been in a more conventional channel of a more developed decision process.

9.3. Denmark

Introduction

In Denmark, the external roots of school mathematics reform lay in the wish to acquire economic strength.[981] In the 1950s there was growing influence from the developmental work in the field of teaching mathematics and natural sciences, financed by the OEEC. These currents entered Denmark through rather narrow channels. One of the first signs was the establishment of *Danmarks Matematikundervisningskommission* / Mathematics Teaching Commission of Denmark, – where Svend Bundgaard was the driving force – as a national committee for the International Commission for Mathematics Instruction (ICMI).[982] Several mathematics teachers and textbook authors, both at the high school and primary level, began making experiments and renewals on their own, inspired by the international reform movement.[983]

Another sign of international reform trends was the demands from the technical and industrial sphere for a better-qualified working force. A need for increased expertise was emphasised, simultaneously with an economic up-swing. These factors among others led to the establishment of a commission of technicians, on the initiative of the Prime Minister. This commission demanded in 1959 a considerable strengthening of the natural-scientific content of education to achieve a more thorough and differentiated education of technicians.[984] The commission saw it as necessary (as did the OEEC) to improve mathematics teaching as early as in the primary school, which demanded intensive re-training of primary school teachers. The commission suggested the establishment of a chair in mathematics at the Royal Danish School of Educational Studies / *Danmarks Lærerhøjskole*, which became a reality in 1958.[985]

In Iceland, these currents were felt from two sides, but considerably later than in Denmark. On one hand they were brought in by individuals, influenced by pedagogical theories, based on Piaget's ideas, and partly channelled to Iceland through personal contact with Danish educators, like Svend Bundgaard. On the other hand there were direct influences from the OEEC / OECD on governmental bodies, realized in the School Research Department in the mid1960s.

[981] Høyrup, J. (1979): 50
[982] *Rapport fra landsmødet om matematikken i Danmark 1981*: 193
[983] Høyrup, J. (1979): 58
[984] *Rapport fra landsmødet om matematikken i Danmark 1981*: 193
[985] Høyrup, J. (1979): 56–57

Primary and Lower Secondary Level

New legislation was passed in Denmark in 1958, after which the *Blue Report / Blå Betænkning*, guidelines for teaching in the primary and lower secondary school, was published in 1960. The social background for its innovations and recommended changes to the form and content of the mathematics teaching had a certain relation to the international reform movement. The technological and scientific revolution was also hovering in the background, as well as a wish to adjust the school to needs defined by a changing society. Also here, there was a revolt against outdated material and rote learning, as opposed to "understanding".[986]

However, the *Blue Report* differed from the "modern" mathematics movement in that it was prepared by educators, not by university professors. In addition to a stable school tradition, the teaching handbook was a product of a social-liberal pro-industry policy, together with the national Grundtvigianism and moderate foreign pedagogical currents à la Dewey. The border between what was called *regning* / arithmetic and mathematics was to be dissolved, and so was the border between geometry and arithmetic, while there was not a word on sets, logic or abstract algebra. Relevant applications, pupil activity, and inductive ways of work were on the agenda.[987] The *Blue Report* was thus being prepared without direct influence from the international reform movement. No similar developmental work was going on in Iceland at that time and discussion about needs for changes had not yet been initiated.

Simultaneously and independently, developmental activities in the Mathematics Teaching Commission and in the commission of technicians were going on, and mathematics reform activities in the high school.[988] The combined effect of the work of the commissions, high school mathematics reforms, private initiatives and activities within the Royal Danish School of Educational Studies was a wide consensus that primary and lower secondary mathematics could not stay within the guidelines of the *Blue Report,* and that the international reform movement was the right way forward. Yet, it did not result in a direct adoption of foreign reforms into the actual compulsory school mathematics, but in a powerful inspiration.[989]

Professor Bent Christiansen was from 1960 a leader of the mathematics institute at the Royal Danish School of Educational Studies. The NKMM, established in 1960, arranged and coordinated experimental teaching on the basis of the reform movement in the whole of Scandinavia. The mathematical institute arranged an experiment in cooperation with the NKMM, whereby 100 sixth and seventh grades in Denmark were taught abstract algebra. Furthermore, Bent Christiansen wrote the goals chapter of the NKMM report, as cited earlier. At the same time Agnete Bundgaard, Svend Bundgaard's sister, and Eeva Kyttä worked on experimental teaching in the first and second grade on behalf of the Nordic Committee, NKMM. Both experiments involved teachers educated at the University's new mathematics programme.[990]

Agnete Bundgaard and Eeva Kyttä started to publish their work in 1967, a year after its first experiments in Iceland. Bundgaard's books were characterized as the most direct adjustment of the university professors' demands of the Danish school system. While another series for the primary level by Cort and Johannessen was full

[986] Høyrup, J. (1979): 54–55
[987] Høyrup, J. (1979): 55–56
[988] Høyrup, J. (1979): 56
[989] Høyrup, J. (1979): 58
[990] Høyrup, J. (1979): 58–59

of playfulness, knowledge of a lot of simple mathematics, practical experience in teaching and ideals close to the *Blue Report*, the pedagogical problem of the Bundgaard material, which had such a great impact in Iceland, was that it was too dry and formal.[991] One may wonder about the course of the reform in Iceland if the textbook series by Cort and Johannessen had been chosen for translation into Icelandic.

According to Jens Høyrup, the consequences of implementing "modern" mathematics in Denmark for primary and lower secondary level education were of two kinds. One concerned teacher training. In the 1969 regulations on teacher training, the mathematics content included "modern" mathematics in its most orthodox international form, admittedly also with probability and statistics. The other concerned regulations and curriculum for the compulsory school. The reform put through under the *Blue Report* could with good will contain much, but definitely not "modern" mathematics. It was, however, not until after new legislation in 1975 that a new national curriculum document, aiming at "modern" mathematics, was worked out.[992] According to Jens Høyrup, at no point of time did the process in Denmark entail one-sided "modern" mathematics; at each moment "modern" mathematics contained many different facets, with their own demands, possibilities and dangers.[993]

Gradually the reform amalgamated with more realistic pedagogic reflections, through

- textbooks written as compromises between the "modern" Bourbakian basic concepts and basic computation skills,
- an ongoing debate in the mathematics teachers' journal,
- activities on the part of the School of Educational Studies, working against what it joined in starting, moving away from Bourbakian ideas towards teaching that was less defined in terms of mathematics as a subject, and more on the basis of the children themselves and their cooperation,
- the actual implementation by the teacher trainers and primary school teachers, too varied to be characterized.

Similarly, new compromising textbooks were written in Iceland for the compulsory school level in the early 1970s. A professional debate did not proceed on a formal platform and therefore cannot be detected, except from the memoirs of those involved, but certainly teacher trainers and many teachers were deeply engaged in developmental work at that time.

Upper Secondary Level

At the upper secondary level, mathematics studies up to the late 1950s were merely based on the 1903 legislation, which was partly the model for the 1904 regulations for the Reykjavík School. The high school had a double goal, to offer young people general education, which was also to serve as preparation for further studies. The high school's structure, the three three-year streams and a yearly admission of below 5% a year, proved to be very stable and altered only marginally in this period.[994] The same

[991] Høyrup, J. (1979): 59
[992] Høyrup, J. (1979): 60
[993] Høyrup, J. (1979): 61
[994] *Rapport fra landsmødet om matematikken i Danmark 1981*: 191

may be said of the Icelandic high school. Danish pupils had however, more educational opportunities within the vocational education system than Icelanders.

In 1953, mathematics was removed from the modern language stream. This soon caused problems, e.g. for medical students, and was generally considered a mistake.[995] At the same time, there was discussion of the placement of lower secondary level education and the possibility of separating it from the high school, an action that had been taken in the Icelandic 1946 legislation.

In 1958, and in a more detailed manner in 1961, the Danish high school became subject to considerable alterations. It was now divided into two streams, a mathematics stream and a language stream. These streams were further divided into lines. These are the changes that Guðmundur Arnlaugsson wrote about in *Menntamál* in 1961 and seem to have been model for the 1970 high school legislation and 1971 regulations (see section 8.4.).[996]

As was the case in Iceland, a quantitative expansion of the high schools in the 1960s was neither foreseen nor planned by the central authorities. This caused a shortage of facilities and teachers, which led to reduction of hours, but also to the so-called Higher Preparation Examination/*Højere forberedelseseksamen, HF*, in 1966. The HF was initially planned as a preparation for teacher training, but became a channel for adults to return to the educational system.[997] The HF became a model for the Icelandic continuation departments.[998] Contrary to their Danish model though, the continuation departments were only temporary arrangements, and were superseded by the multi-stream comprehensive schools.

The high school mathematics content up to about 1960 was similar to what has previously been described in Iceland: arithmetic and algebra, plane geometry, trigonometry, stereometry and analytic plane geometry with increasing emphasis on the function concept and calculus (*infinitesimalregning*).[999] The similarities were natural, as Danish textbooks for the mathematics stream were used in Icelandic high schools up to 1964.

The idea of introducing "modern" mathematics reform was to use the expected up-swing of natural-science, mathematics and technical education in the educational system to modernize high school mathematics, and bring it closer to the content and form of modern university mathematics. It was not a critique of the current content, but a wish to modernize with the aid of concepts such as set, mapping, relation, composition and, in the context of analysis, the neighbourhood, as binding elements throughout the syllabus. This was expected to contribute to the unification of mathematics, by having only few, general concepts available, but also to make mathematics more transparent and understandable to the pupils.[1000]

This reform was made possible by the late 1950s revision of the school system. The highest leadership of the high school system, as well as the mathematics consultants, became convinced that it was necessary for Denmark to follow the international currents in the field of mathematics and that considerable resources

[995] *Rapport fra landsmødet om matematikken i Danmark 1981*: 192
[996] Guðmundur Arnlaugsson (1961). Stjórnartíðindi 1970, no. 12, March 25; 1971, no. 12, January 22
[997] *Rapport fra landsmødet om matematikken i Danmark 1981:* 194–195
[998] Kristín Indriðadóttir (2004): 30
[999] *Rapport fra landsmødet om matematikken i Danmark 1981*: 196–198
[1000] *Rapport fra landsmødet om matematikken i Danmark 1981*: 199

should be allocated for national implementation, such as refresher education for teachers.[1001]

Consequently, "modern" mathematics was to be taught in all high school streams and lines, channelled not so much by official documents as by a textbook series, written by Kristensen and Rindung and promoted by Svend Bundgaard. The series became the norm in the following decades for the mathematics stream, while no similar model emerged for the language stream.[1002] The series was also adopted in Iceland for a while (see sections 7.2. and 8.4.).

The above information about the high school level is based upon a report from 1981, made by a committee that had the task of giving an overview of the previous and present content of mathematics teaching in Danish high schools and its underlying ideas. The committee responded by the report containing its considerations, views and recommendations.[1003]

The committee investigated material from neighbouring countries, where it emerged that the "modern" mathematics, introduced in the 1960s, became the basis for mathematics curriculum documents in the 1970s, not only there but in most of the world. The experiences gained by implementing "modern" mathematics in addition to other factors – such as the explosion in the attendance of the upper secondary level, changes in the pattern of further education, ideas about keeping pupils on the same educational path as long as possible, new demands to provide individuals with prerequisites to participate in socio-political discussion, and the use of computers – had channelled into several main trends.[1004]

The trends referred to are for example *the "back-to-basics" trend*, originating in the U.S., emphasizing computing skills. It seemed to be retreating in the early 1980s, in face of a trend characterized by *educating the "whole person"*, so that the pupil, in addition to achieving knowledge and skills, was to acquire a general humanistic-oriented development of his/her personality and general prerequisites for a private and professional life and life as a citizen. The third trend is *the relevance trend*, where the emphasis is on pupils achieving knowledge, skills and methods in connection to topics that are relevant outside the subject, such as in life outside or after school. None of these trends were dominating in the early 1980s, but were expected to contribute to a complex pattern of influences on mathematics instruction.

In the period 1960-1980, alterations of regulations and official circulars adjusted the mathematics teaching to actual circumstances, in such a way that the emphasis on the unifying concepts was reduced. This led to their binding effect being weakened, and the underlying ideas of the regulations could not be realized in a satisfying manner. At the same time there was a growing wish for increased emphasis on the pupils' intuitive understanding and their sense of mathematics as a subject applied within many areas.[1005]

The recommendations of the committee for future high school mathematics education in Denmark consisted of four items. It was the committee's conception that all pupils should receive mathematics instruction with respect to their personal and

[1001] *Rapport fra landsmødet om matematikken i Danmark 1981*: 198–199
[1002] *Rapport fra landsmødet om matematikken i Danmark 1981*: 201
[1003] *Rapport fra landsmødet om matematikken i Danmark 1981*: 174–175
[1004] *Rapport fra landsmødet om matematikken i Danmark 1981*: 207–208
[1005] *Rapport fra landsmødet om matematikken i Danmark 1981*: 178–179

social life, and that each systematic mathematics instruction must offer the pupils insight into

1. *the special nature of mathematics*, e.g. expressed in a process of intuitive understanding of coherence, of a formulation of a theorem and of its proof,
2. several *mathematics topics, central for their appearance in many applications*, and examples of such,
3. some authentic applications of mathematics, treated for their *social importance*,
4. topics from the history of mathematics and *mathematics in a cultural, philosophical, historical and social context.*[1006]

To be able to realize mathematics education according to these four articles, it was considered necessary that mathematics be presented partly as an independent subject and partly in an interdisciplinary context.[1007]

The committee was led by Mogens Niss, then assistant professor at Roskilde University. The committee's report and its recommendations contain the seeds of a larger report, written in 2002, also under M. Niss's chairmanship: *Kompetencer og matematiklæring. Ideer og inspiration til udvikling af matematikundervisning i Danmark. / Mathematical Competencies and the Learning of Mathematics. Ideas and Inspirations for Development of Mathematics Teaching in Denmark.*[1008] The ideas presented in the 1981 report were elaborated on and deepened in the 2002 report, while the spirit of both reports is a balanced consideration of mathematics in the context of the pupil himself and his needs, the needs of the society and of preservation of cultural values, formulated in more detail in M. Niss's identification of fundamental reasons for mathematics education,[1009] cited earlier (see section 1.4).

Summing Up

It is difficult to identify similarities between the evolution of "modern" mathematics in Iceland and Denmark, while there were direct influences. In the 1960s and 1970s there were still strong impacts on Icelandic education from Denmark, e.g. reflected in the Icelandic 1970–1971 high school reform and in the establishment of the continuation departments in Iceland. There were also direct personal contacts between Danish and Icelandic leaders in mathematics education, personified e.g. in Guðmundur Arnlaugsson and Svend Bundgaard, who together channelled Agnete Bundgaard's primary school material and Kristensen & Rindung's high school textbooks to Iceland. Svend Bundgaard also influenced his former student Jón Hafsteinn Jónsson, who has been publishing high school mathematics textbooks with a formal axiomatic approach up to the present time. Anna Kristjánsdóttir studied mathematics education with Bent Christiansen in 1969–1972 at the Royal School of Educational Studies, which contrary to the Icelandic teacher training institutions, was took a leading part in the reform process.

The Faculty of Engineering at the University of Iceland was also tailored as the first part of the Technical University of Denmark until 1971. However, exactly at that point in time, influences on the Icelandic educational system became more global. Iceland followed only partly the same path as Denmark in the 1970s. Kristensen &

[1006] *Rapport fra landsmødet om matematikken i Danmark 1981*: 179
[1007] *Rapport fra landsmødet om matematikken i Danmark 1981*: 179
[1008] Niss, M. (2002)
[1009] Niss, M. (1996): 13

Rindung's textbooks, for instance, remained standard high school textbooks only for a short time, and were finally replaced by a Swedish series, a more moderate one with respect to "modern" mathematics.

In the late 1990s, Danish textbook series for the lower primary level, *Factor*, by Silla Balzer Petersen and Arne Mogensen have been translated and adapted to the Icelandic context. Cultural currents from Denmark have thus continued to exert an impact in Iceland up to the present time.

10. Mathematical Education in Iceland – A Summary

10.1. Introduction

The Research Question

A fairly thorough description of the Icelandic society and of its ways to promote education through its eleven centuries has been provided. An attempt has been made to estimate the share of mathematical learning in the education available to the inhabitants from the 10th century to the present day.

Sources are not available about mathematical matters for long periods. About the first few centuries we have only scattered information in manuscripts, usually written some centuries, or occasionally decades, after the events took place.

We have accounts of the mathematical learning of prominent people, such as the bishops during the first centuries after the Reformation. We know the mathematical books they possessed and presumably read. Yet other evidence is scarce that they did any work of a mathematical nature, and only one person is known to have been appointed specifically to teach mathematics before the 19th century.

In accord with the sparse sources and remoteness from present times, the period up to 1700 will only be discussed briefly, mainly in order to throw light on the special features of Icelandic culture in comparison to its neighbours.

From the early 18th century manuscripts have survived of substantial arithmetic textbooks whose origins and use remain unknown. Printed arithmetic textbooks first appeared in the mid-18th century. They were written to inform the public and meet their mathematical needs, particularly in trade. From that time on, an increasing number of printed sources on mathematical activities is available and allows drawing a fairly plausible map of the situation of mathematics education in Iceland up to present time.

Thus, even though an attempt is made to survey the whole period of Iceland's habitation, the availability of sources should be kept in mind when turning to the research question:

To what extent has mathematics education developed similarly or differently in Iceland from that in other northern European countries, and what explanations can be offered for this?

The Northern European Countries

Which countries would it be reasonable to compare with Iceland? There is geographical closeness and there are cultural and historical relationships. One might e.g. recall that the Danish Kingdom ran the Royal Greenlandic, Icelandic, Finnmarkish and Faroese Trade as a monopoly enterprise, and thus these four geographical areas constituted one commercial area. This relationship draws attention to the Inuit people in Greenland and the Sami in the northernmost part of Norway, Finnmark, both of whom have adjusted to even more difficult living conditions than the people of Iceland. The Icelandic people might have benefited from learning about their ways of living and assimilation with nature. However, their origin and their languages were different, and there is only little evidence of cultural or commercial exchanges with these people after the contact with the Norse people in Greenland was lost in the 15th century.

One might also think of the Norse people in Mid- or Northern Norway, the Faroe Islands or even the people in the Orkney Islands and the Hebrides. Certainly we know about contacts with all these people during the first few centuries. In that period the Icelanders were of comparable number to these groups of people: more numerous than the inhabitants of the small archipelagos, and a fair proportion of the number of the Norwegians: they are believed to have numbered about one-seventh of the Norwegian population.[1010] The two groups of Norse people in Norway and Iceland looked upon themselves as the same, and the Icelanders submitted to the Norwegian King in 1262. Originally, they spoke the same language and adhered to the same religion. The same can be said of the people in the Faroe Islands.

In the late 14th century, Iceland became more culturally isolated, as the language of the other Norse groups had developed differently from that spoken in Iceland, and the Icelanders had become dependent on others for ocean transport. At a similar time Norway and Iceland became tributaries of Denmark. From the 16th century onwards Iceland's cultural and commercial exchanges were almost exclusively with Denmark. This relationship of a ruling nation and its tributary, and later what might be called a protectorate, persisted until the beginning of World War II. Cultural influences from Denmark remained at least until the 1960s when various international currents began to exert influence in Iceland. From the 1960s there were rising international influences in cultural respect, like those channelled by the OEEC, later OECD.

The special language contributed to Iceland's isolation and at the same time to its cultural independence. Following the Reformation, the Bible was translated into Icelandic and thus created a basis for Iceland's own cultural life. The other Norse people amalgamated with their dominant neighbours; the Orkneys and Hebrides with Scotland, Finnmark was part of Norway, and the Faroe Islands became much more culturally dependent on Denmark than Iceland by accepting Danish as the official language in schools and churches. Iceland also differed from its small neighbour, the Faroe Islands, in that the administration was largely manned by Iceland's own landed gentry.

Subsequently, for the period up to 1550 the comparison will be confined to Norway and people in the Nordic countries, and to Denmark during the modern period up to 1960. For the period 1960–1975, which is characterized by massive changes, accounts from Denmark, Norway, England and the United States will be drawn upon. For the recent time some international comparative studies will be cited.

[1010] Gunnar Karlsson (2000): 45

In the following summary leading to the research question, the long period of history will be divided into sub-periods according to the historical situation, i.e.:

Middle Ages 900–1550

Early Modern Period 1550–1800

Society in Transformation 1800–1900

Urbanization, Industrialization and Independence 1900–1944

Challenges of a Newborn Republic 1944–1964

The Educational System in Flux 1964–1975

In each period the status of mathematics education will be related to what is known about parallels in other countries, and the reasons for the existence or lack of mathematics education will be evaluated in light of M. Niss's identification of fundamental reasons (see section 1.4.).

10.2. Middle Ages 900–1550

During the middle ages the main mathematical tasks concerned chronology, including ecclesiastical computations, trade with its measuring and currency units, taxes such as the tithe, and the introduction to the Hindu-Arabic number system. These tasks were carried out by all the Norse people and the Danes.

Mathematical activities

It is known that the Icelanders made some important independent astronomical observations in the 12th century, questioning the established chronology of the Christian Church, the Julian calendar. They also developed their own special week-based calendar, which was gradually adjusted to the Julian and later the Gregorian calendar.

With Christianity at the beginning of the 11th century the Icelanders followed the European custom of placing young boys in cathedrals and monasteries as novices for the priesthood. The Icelandic system differed from the European one at the beginning in that the churches were run privately by the ruling chieftains. Their sons received education as well, whether they were to be ordained as priests or not. Therefore reading and writing may have been more common amongst laymen in Iceland than at least in the other Nordic countries. Little is known of the *quadrivium* in their training. One would, however, expect chronological computations to have been studied, as they were necessary for keeping track of ecclesiastical computations of the church calendar for the liturgical year. The computations for Easter were particularly complicated, as they still are.

In the 12th and 13th century, Europeans were translating ancient knowledge, preserved in the Islamic World, into Latin. Some of this Latin knowledge was brought to the Nordic countries and studied there. *Algorismus*, a translation of *Carmen de Algorismo* by the Frenchman Alexander de Villa Dei, was written in Iceland, while one of its extant copies is written in the Norwegian version of the Norse language. This indicates how the Norse people, the Icelanders and the Norwegians, shared some cultural resources.

At a similar time the Dane Peder Nattegal or Petrus Philomeni de Dacia, wrote a commentary on a parallel treatise, *Algorismus Vulgaris* by Sacrobosco, a voluminous and important Nordic contribution to European mathematical development,[1011] but unlike the Icelanders Peder Nattegal wrote in Latin. The Icelanders were thus working on similar tasks to people in the other Nordic countries but in their own way, translating into the vernacular.

The tax implemented by the church, the tithe, was taken up in Iceland on a foreign model, but in a different form. While in the neighbouring countries the tithe was a 10% income tax, it was a 1% property tax in Iceland. Income was dependent on the value of the pieces of land which were the main properties, so the concept of tax seems to have been adjusted to domestic circumstances. As Iceland's economy was almost exclusively agricultural and the farmers had their own fishing grounds, there was very little internal trade.

Foreign trade in the first centuries was mainly with other Norse people, so they used the same kind of currency and measuring units or at least were familiar with each other's units. Furthermore, the trade was mainly in the form of barter. The currency depended on the export goods, in Iceland woollen cloth until around 1300 and after that stockfish (dried fish), which was also known in Norway. There was increased trade with Englishmen and the Hanseatic League in the 15th century, but the export goods were also stockfish.

Conclusions

Given the peculiarities of Icelandic society, e.g. concerning the tithe, the mathematics used at commercial tasks such as the tithe and trade should have been on similar level of difficulty in Iceland and Norway in the first few centuries. Therefore, during the Middle Ages up to the 15th century, the Icelanders' mathematical tasks were similar to those of the inhabitants in their neighbouring countries, while their currency was adjusted to their own special circumstances.

The Icelanders were at first on comparable level to the inhabitants of other Nordic countries in acquiring mathematical learning, both by their own observations and by reading and translating some of the latest acquirements in the learned Latin world. In the late 14th century the governmental centre of Norway and Iceland moved to Copenhagen, with the unification of Norway, Sweden and Denmark in the Kalmar Union. When universities were established in the late 15th century in Copenhagen and Uppsala, followed by faculties of mathematics and science in the 16th century, higher mathematical learning in those countries had exceeded that in Iceland. From that time on, the Icelanders concentrated on preserving their medieval mathematical heritage.

[1011] Pedersen, O. (1966): 498

10.3. Early Modern Period 1550–1800

Introduction

Many of Iceland's most important historical landmarks have their origin or similarities in other countries. These international currents amalgamated into the current culture and contributed to new trends, parallel to those in the neighbouring countries. The Icelanders shaped them in their characteristic way of translating foreign sources, such as the Bible, into the vernacular. Icelandic culture was based upon the European heritage, and the cultural influences that reached the country were of European origin, usually in a Danish fashion.

Important European cultural currents such as Humanism and the Enlightenment had invaluable effect; Humanism concerning preservation of medieval cultural values and the Enlightenment movement in educating the common people, not least in a mathematical respect.

The Modern Age in Iceland is marked by the introduction of the Protestant Evangelical Lutheran faith in 1550. During the following two hundred and fifty years there were two cathedral schools, whose primary goal was to educate young boys for the clerical profession. The cathedral schools were situated at Hólar in northern Iceland and at Skálholt in southern Iceland.

The period 1550–1800 is characterized by worsening living conditions. There was a gradual deterioration compared to the neighbouring countries, both in population and education. The main mathematical achievements in Iceland were astronomical: measuring the latitude and longitude of Iceland as a basis for locating Iceland on the world map and a publication of a calendar, but also important publications of arithmetic textbooks during the last part of the period.

The Icelandic economy remained stagnant into the 19th century, while in the neighbouring countries the trade flourished and grew increasingly complex. For centuries after towns were established in the other Nordic countries, Icelandic society remained persistently rural, marked by absences of towns and trade.

Bishop Guðbrandur Þorláksson

Geography and navigation were two related aspects of mathematics, extremely important to the world of the sixteenth century.[1012] Bishop Guðbrandur Þorláksson (1541/42–1627) at Hólar see was thus working on the same kind of tasks as mathematicians in the European world, contributing to the world's knowledge of its geography by his map, introduced to the European learned world through the mediation of a Danish researcher, Andreas Sørensen Vedel.

Bishop Guðbrandur Þorláksson was an adherent of Humanism and a proponent of education, who saw and utilized the new technology of the printing press as a prime channel for educating the people. The bishop's eagerly pursued theological activities, such as publishing the Bible, may be attributed to his desire to ensure the power and influence of the recently-established Protestant Church.

[1012] Katz, V. J. (1993): 360

His valuable promotion of the Icelandic language in his publications may also be seen as a move in his power game. As long as the Icelanders kept their own language they would retain some independence from foreign rule by Denmark. For both ends, the publication of theological works was useful and the printing press an excellent tool. Mathematical publications had no such purpose. And mathematical publications for the general public may not have been so widespread at the turn of the 16th century that there would be foreign models for that task, except for calendars, such as the bishop's *Calendarium*. Other mathematical books might have been aimed at merchants or university professors, professions not found in Iceland.

Mathematics was probably studied at the Hólar Cathedral School during Bishop Guðbrandur Þorláksson's long term in office, from 1568 when he became headmaster of Hólar Cathedral School until his death in 1627. However, Latin was the main subject of the school. It was the *lingua franca* of the European world; it was the thread that kept Iceland in contact with the civilized world, and that had to be a priority.

Bishop Brynjólfur Sveinsson

Bishop Brynjólfur Sveinsson (1605–1675) attempted to lift the Skálholt Cathedral School in southern Iceland to Danish standards by appointing a special mathematics teacher, Gísli Einarsson, the only one in the school's history. However, no new mathematical knowledge, except to determine the latitude of Skálholt, is attributed to Gísli Einarsson or any of his Icelandic contemporaries. Bishop Brynjólfur Sveinsson's library included Euclid's *Elements* with commentary, a work that is otherwise unknown to have been in the possession of Icelanders until the late 19th century. Efforts to elevate the mathematical education in Skálholt Cathedral School declined again after his day.

The reasons for the decline are presumably that there was no internal need for higher mathematical education. Mathematics was not a requirement for admission at the University of Copenhagen. What was needed was enough knowledge of the four arithmetic operations, currency and scales in order to run the assets of the church and cope with the limited trade. Some calendars were made and printed. Maps published at that time and up to the mid-18th century were based on Bishop Guðbrandur Þorláksson's map.[1013]

The two cathedral schools at Hólar and Skálholt were theological seminaries. It was not necessary to educate the whole clergy in the mathematical sciences: geodesy and computations of calendars. That kind of education could be confined to a few scholars and be the responsibility of the Danish authorities.

The contributions of Bishop Brynjólfur Sveinsson and manuscript collector Professor Árni Magnússon to mathematical education in Iceland were to save the ancient mathematical heritage preserved in manuscripts, which at that time was of mainly historical interest.

Not much is known about learning during the early 18th century. At least three manuscripts in Icelandic exist from that time whose content is arithmetic. These manuscripts show that some studies were made. They are presumably translations of, or written under the influence of, some foreign sources.

[1013] National and University Library of Iceland: website, accessed February 3, 2005

All the manuscripts had a similar form, prevailing in arithmetic textbooks into the 1920s or later: an introduction to the four operations in whole numbers (positive integers and zero) and fractions, usually with measuring units and currencies, extraction of square roots, in some cases arithmetic and geometric progressions and series in primitive form, and *regula de tri*.

One of the manuscripts bears witness of theoretical interests and has some resemblance to *Algorismus*, which suggests some contact with the medieval heritage. It was possibly worked out by an Icelandic mathematician working on land surveying in the Danish Navy or was a free translation of the Danish *Frommii Arithmetica*.

The Enlightenment and Mathematics Textbooks

The formal origin of public education in Iceland is Harboe's ordinance on literacy in the 1740s. The cathedral schools were situated at the two sees at Hólar and Skálholt, and the clergy was to supervise instruction in reading and Christian knowledge provided to children at home. Thus all education was governed from the episcopal sees. Furthermore, the only printing press was situated at the Hólar See until 1799, although temporarily at the end of the 17th century at Skálholt. From Bishop Guðbrandur Þorláksson's time, the church published vast amounts of books, which made it possible for people to read devotional writings, and preserve the written language.

With the Enlightenment the publication of secular books greatly increased. With the aim of enlightening the general public they taught working methods, and also how to cope with trade and avoid being cheated by foreign merchants.

Of the total of six mathematics textbooks published in Icelandic in a 100-year period, 1746 to 1841, three of them, published in 1746, 1782 and 1841, were specifically published to inform farmers and other "simple-minded" people, as indicated in one of the textbook titles, "about all kinds of computations in … purchases and sales".[1014] These books gave lists of the measures and currencies and their relationships valid in Iceland. The first and third of these three books also demonstrated the four arithmetic operations in whole numbers, and in frequently used measuring units and currency.

The other three textbooks, published in 1780, 1785 and 1841, contained the same kind of information, together with more substantial arithmetic, covering 250–375 pages of the four operations in positive integers and fractions, in addition to *regula de tri* and some more advanced arithmetic, such as progressions and algebra. The algebra contains a short survey of linear equations with one and two unknowns and an introduction to quadratic equations. These books were intended for the pupils at the learned schools and were also addressed at laymen, especially young people. One of them, Olavius' *Greinilig vegleidsla / Clear Guidance,* claims a relationship to a Danish Enlightenment textbook by Chr. Cramer and to the German *Demonstrative Rechenkunst* by Christlieb von Clausberg.

All the mathematics textbooks printed in this period are practical. They explain the operations which are necessary to solve ordinary problems associated with trade. They were intended for the ordinary layman's information and enlightenment. As farms were measured in terms of *landaurar* and not by area, there was no reason to teach area computations. A hint of area is found in one of the early 18th century

[1014] Jón Jónsson (Johnsonius) (1782)

manuscripts, *Limen Arithmeticum*, while area computations were not seen in printed Icelandic textbooks until in the Rev. Eiríkur Briem's textbook in 1869. One may conclude that the main purpose of mathematics education in this period was of an economic nature.

In the forewords of the above-mentioned 18th-century arithmetic textbooks there are claims that Icelanders had lagged behind their neighbours, in particular the Danes, in the art of computing. The textbooks were generally good, even by modern standards. However, there may have been few reasons for young people to study their content, beyond measures and currencies. Internal trade was limited, and mainly barter. The small minority of the population that entered the learned school had of course to be provided with prerequisites to cope with their future education or occupation, but there were no definite requirements in mathematics education at the University of Copenhagen until the early 19th century. That fact might indicate that in Denmark too, mathematics education before the 19th century was confined to the few that could expect to work on mathematical tasks, such as astronomical observations, geodesy and teaching, in addition to those concerned with trade.

Higher Mathematics Learning

Stefán Björnsson was the only Icelandic 18th-century mathematician. Living in Denmark, his important theoretical work in the field of geometry had no effect on Icelandic mathematics education. However, Stefán Björnsson's publication of the ancient mathematical treatises *Rímbegla* and *Oddi's Tale* may have contributed to society's political and ideological development, through the impression that the ancient heritage later had on Danish authorities in Iceland's campaign for independence. Thus, what once had been an Icelandic mathematical achievement up to the European standard was mainly of historical interest in the late 18th century, and possibly of political interest in the 19th century.

Stefán Björnsson did, however, make an important contribution to mathematical education in the 19th century through his treatises intended to inform the common farmers about technical matters, based on modern knowledge of physics. By his conceived influence on the mathematician of the following generation, Björn Gunnlaugsson, this contribution was long-lasting.

There are no signs that contemporary higher mathematics was known in Iceland after Bishop Brynjólfur Sveinsson's time, up to the 1820s. Euclid is mentioned here and there, but only in connection with arithmetic, and there is no evidence that Euclidian geometry was known or discussed in Iceland.

Conclusions

The main permanent progress in mathematical education in Iceland, up to 1800, was therefore the introduction of Hindu-Arabic number notation and its algorithms, and the efforts to measure Iceland geodetically and put it in its place on the world map, both of which belong to utilitarian aspects. During the 17th and 18th centuries Icelanders fell markedly behind their neighbours in mathematical education, both in higher learning and in public education.

10.4. Society in Transformation 1800–1900

The Learned School – Björn Gunnlaugsson

While ordinary people needed to be able to cope with their economic tasks, the small group of pupils in the learned school, 24 at the time at the beginning of the 19th century, were to be provided with prerequisites for their future studies. When these were expanded to include mathematics in the early 19th century, the Icelandic school needed a mathematics teacher, who by a stroke of luck was a scion of the Enlightenment movement, Björn Gunnlaugsson.

Björn Gunnlaugsson's views, declared in his inauguration speech, were mainly utilitarian. Björn viewed mathematics as a tool to explore nature, while he also argued how mathematics could train people in logical thinking, as nowhere else was truth as easy to research and easily distinguished from falsehood.

The Icelandic learned school remained a theological seminary up to 1847, far longer than the Danish schools, and it was so small that up to three year-courses were taught together in one group. However, it managed to follow the mathematics syllabus of Danish learned schools during Björn Gunnlaugsson's time, and to fulfil the requirements of the University of Copenhagen. Another great feat of Björn Gunnlaugsson was his geodetic measurement of Iceland and a consequent map.

> One may therefore state that Björn Gunnlaugsson's contribution to Icelandic mathematics education was to keep mathematics education in the Icelandic learned school up to Danish standards in his time, and to survey Iceland and thus create a basis for a new map of Iceland, which endured far into the 20th century. His remarkable mathematical book *Tölvísi* does not seem to have had much impact, and none of his pupils became a mathematician.

Regulations in 1877

When the Danish learned schools, by legislation in 1871, could be divided into a language-history stream and a mathematics-natural science stream, the Icelandic school authorities decided to follow the language-history stream in regulations adopted in 1877, in spite of initial proposals by a school affairs board, suggesting a combination of these two streams. This was presumably due to lobbying by the Governor of Iceland on the part of the headmaster and his allies among the teaching staff, while another group of teachers argued for the initial proposals. In the controversy, that lasted five years, various arguments were drawn into the debate.

All the fundamental reasons for mathematics education cited by Niss (see section 1.4.) were included in the arguments of the proponents of unaltered mathematics teaching at the Learned School. By stating the importance of mathematics education for the "technical life", the teachers re-emphasised Björn Gunnlaugsson's utilitarian arguments, made 60 years earlier. They also mentioned the training of the mind, providing individuals with prerequisites to cope with life in an educated way, prerequisites for further studies and cultural maintenance, as the missing topics would "finalize and perfect" the mathematics education in the Learned School.

The Governor represented the opponents, who argued that the Learned School pupils were seeking qualifications for professional examinations in theology, medicine, law or philology, and anything else would be an extremely rare exception. The need for engineers in a country devoid of roads and bridges was not yet considered relevant, and even so, it might not have been thought unnatural that an extra year in Copenhagen would be necessary for those who were inclined to become pioneers of that kind.

The basic reasons for excluding the mathematics-natural sciences stream were of an economic nature. It was not financially possible to divide the school, of about 80 pupils in six age groups, into two streams. The school was already a substantial item in the country's budget, which was run at a deficit, and paid for by the Danish government. As the number of hours could not be increased, some of the teachers must have been afraid that the hours for their subjects would be cut down and hence their own share of work. They were therefore also thinking of their own personal interests.

Also politically, more people would be immediately content with reducing the workload in mathematics instead of cutting back the amount of teaching in the ancient languages, Greek and Latin, even if Latin's role as *lingua franca* had by now greatly declined in importance. The classical languages were considered necessary prerequisites for the most common professional occupation, the priesthood, as well as Latin for medical studies, in addition to their often cited qualifications in training the mind. By comparison, mathematics had no immediate application. Furthermore, evidence exists that it was taught in such a manner in the 1870s in the Reykjavík Learned School that its purpose was invisible, and its popularity among pupils minimal.[1015]

By transforming the school into a language stream, the school was in one sense assimilated to the Danish learned schools by adopting one of their streams more-or-less wholesale, instead of running a combination specially adapted to Icelandic circumstances. However, seen from the aspect of mathematics education, it deviated from the Danish model for nearly half a century, in an important period of technical innovations in society and progress in education. The Reykjavík Learned/High School thus hardly participated at all in the country's transition from a structure, which was predominantly rural, towards a modern industrial society.

Another aspect of the train of events in 1877 is well known, all the way from nurseries up to the highest level of political decisions. When an agreement cannot be reached and the solution of a dispute is left to a superior authority, that same body is apt to take its share. When the Icelanders left their dispute to the Governor and the Danish Minister of Icelandic Affairs, they were given more Danish to learn, to replace the mathematics. And it was only after the authorities had realized that it would be less expensive and more advantageous to hire Icelandic engineers than foreigners, that Icelanders were encouraged to study engineering (see section 4.5.)

[1015] Finnur Jónsson (1883): 97–135

19th Century Lower Secondary Education

In the absence of any legislation on public education for most of the 19th century, lower secondary education was confined to the lower part of the Reykjavík Learned School, attended by 1–2.5% of the male population, and two lower secondary schools, at Möðruvellir in the north and Flensborg in the south, established in the 1880s. In the language-history stream of the Learned School, some mathematics was taught, using predominantly Danish mathematics textbooks together with the Rev. Eiríkur Briem's 1869 *Arithmetic*. The two lower secondary schools taught textbooks by the two brothers Briem.

Eiríkur Briem stated that he avoided complexity by what he found unnecessary reasoning and the Rev. Halldór Briem aimed at what he found necessary in "general production", and not following each statement with a scientific proof as was done in other [foreign] textbooks. These author brothers were theologians, and thus educated neither as mathematicians nor as teachers. Hence they were probably not involved in didactic discussions about views on mathematics as a discipline exclusively to train the mind. Their first aim was to meet the immediate needs of young people for practical knowledge in a country devoid of theoretical mathematical studies. One might even conjecture that the authors thought that bothering about proving self-evident facts was an intellectual luxury (or adversity) that educationally-deprived youth were not to be disturbed with.

One can therefore state that the two series of textbooks suitable for adolescents, in school or in self-study, published in the last decades of the 19th century, contained fairly practical computations and aimed to help people to adjust to increasingly complex society and industry without bothering too much about formal reasoning. Their foreign models were used more with regard to the problems than to the text itself or its pedagogy. Percentages and interests, the practical matters of all citizens, were introduced by the use of *regula de tri*. The Rev. Eiríkur Briem's definition of the front term, middle term and back term of the *regula de tri* proved to be tenacious, even if later educators pointed out that common sense was more useful than memorizing such definitions.

Again we see the utilitarian aspects. The purpose of arithmetic instruction, measured to M. Niss's classification, was to provide the individuals with prerequisites which might help them to cope with life in their occupation and life as a citizen in a developing society. Prerequisites for further education were no longer on the agenda. The focus was on society, what was useful for an individual in a society aiming at industrialization and independence.

It seems that the choice of textbooks depended on the position of the author. The textbooks of the Briem brothers, published in a number of editions, may have survived due to the position of their authors as teachers at the two recognised secondary schools, in Reykjavík and Möðruvellir, while the book by Þórður Thoroddsen, who left teaching, was only published in two editions.

19th Century Primary Education

After the 1880 legislation on requirements in writing and arithmetic, three publications were aimed at teaching children arithmetic. The most widely-distributed of them, Morten Hansen's book, was modelled after the Danish textbooks by Chr. Hansen, which dominated Danish arithmetic teaching from 1860.[1016] In that sense the Icelanders adapted themselves to Danish textbooks and pedagogy in their few primary schools towards the close of the 19th century.

These publications for primary education were more-or-less handbooks for teachers, and less aimed at the children themselves. Their purpose was to introduce practical arithmetic in positive integers and fractions. People were experimenting with ways to teach arithmetic in the new context of compulsory arithmetic education without compulsory schools. The age cohort numbered around 1600 in 1901.[1017] The total number of educated primary school teachers was only 24, and one third of the 415 persons employed in teaching had never been to a school themselves,[1018] so it was no wonder that an experimental situation persisted for some time.

Conclusions

Throughout the underdeveloped Icelandic education system in the late 19th century, people were collecting knowledge and raising the status of arithmetic. The syllabus was mainly arithmetic up to *regula de tri* and practical geometry with area and volume computations. Either Danish textbooks were used without translation, if possible, or dominant Danish textbooks were models for textbooks for children and adolescents. In many cases the theory was left out, sometimes for relief, avoiding formal argumentation for self-evident facts, while in other cases it may have left the teachers and the pupils with memorizing rules without any sensible reasoning.

The Danish influences were obvious, while adjustments to Icelandic circumstances were necessary, at least outside the Learned School, where the requirements of the University of Copenhagen were primary aims.

By the end of the 19th century, the main goal of mathematics education was providing individuals with prerequisites to cope with their tasks in a society, changing from being exclusively rural toward urbanization. The emphasis was on grass-roots education, built upon individual initiative, where the prominent players were the Briem brothers and Members of Parliament Þórarinn Böðvarsson and his son Jón Þórarinsson. The highest institute of learning, the Reykjavík Learned School, was totally absent from this process. Björn Gunnlaugsson said in 1822/3 that "every nation should … have its *mathematicos* to send them out into nature to research its mysteries and who then point out to the nation where it should search to find the resources which are hidden in it." No official preparation of pupils for that task was connected to mathematics education. Providing learned-school pupils with prerequisites for engineering and natural sciences studies was still a remote idea, and had to wait for another two decades.

[1016] Hansen, H.C. (2002): 19
[1017] *Hagskinna* (1997): Table 2.11. Population by sex and age group 1703–1990
[1018] Ólafur H. Jóhannsson (1996): 15

Mathematics did, however, show up in the role of contributing to society's political and cultural development when in 1892–1896 Finnur Jónsson edited the medieval manuscript *Hauksbók*, including *Algorismus,* on behalf of the Royal Nordic Ancient Writings Society / *Det Kongelige Nordiske Oldskrift-Selskab*. There was a genuine interest in the old Norse heritage to which *Algorismus* belonged, and this heritage had a political role in the struggle for independence. Kr. Kålund and N. Beckman's edition in 1914–1918 of *Alfræði Íslands*, containing the *Rím*-treatises, was of similar value.

During the 19th century the educational currents lay towards developing similarly to the neighbouring countries, especially Denmark. The Learned School was part of the Danish school system and as such it conformed to Danish requirements. Regrettably, this was possible simultaneously to reducing mathematics education in 1877. The highest mathematics education did therefore not develop similarly to Denmark, but was instead entrusted to Denmark.

Primary education, on the other hand, was developing towards norms in other countries. Requirements in basic arithmetic, establishment of schools and publication of textbooks were modelled on Danish education. However, there was a long way to go. The absence of technology, like in the form of roads and bridged rivers, was a great obstacle to proper public education.

10.5. Urbanization, Industrialization and Independence 1900–1944

The period 1880–1930 was a crucible for mathematical education and formation of textbooks at the primary and secondary school levels. For several decades the border between the primary and lower secondary level was blurred. Most people, children, adolescents and adults, needed instruction in basic arithmetic. Various textbook series were produced and published. The market was unsaturated but uncertain, as schools were few. The state's expenses for each Icelandic pupil on compulsory level were less than one fifth of corresponding expenses in the other Nordic countries.[1019]

Lower Secondary Mathematics Education

There was no definite syllabus in Minister Jónas Jónsson's new lower secondary schools, apart from practical arithmetic, and bookkeeping was the main topic. The syllabi were to be unrestricted by the high schools. The personal and national needs for mathematical subjects were expected to emerge from the needs of the pupils, and the schools were to adapt their syllabus to them.

In the society of the early 20th century there was no definite need for more than the arithmetic prescribed in the 1907 Education Act. Tradition was also lacking, and no educational institutions were capable of identifying a syllabus which might contribute to mathematics as a tool to enrich the pupils' cultural life, or unfold the cultural values of mathematics. The Ministry of Education hardly existed except as a niche in the Ministry of Justice or Industrial Affairs. The Directorate of Educational Affairs could offer only limited professional support for the various subjects, and few teachers were likely to know more than basic arithmetic facts. However, Teacher Training College teacher Dr. Ólafur Daníelsson seems to have exerted strong influences on a number of his pupils and students through his exceptional attitude to and knowledge in mathematics.

[1019] *Menntamál* (1929): 4 (3) 48

Distribution of Primary Level Arithmetic Textbooks

There was a choice of a variety of arithmetic textbooks for primary schools in the 1910s although they were printed in a small format.[1020] Director of Educational Affairs Jón Þórarinsson said in a letter in 1913[1021] that time would sort out the best book without official interference. In retrospect, it is doubtful that the only factor ensuring distribution was quality. The textbooks were in many respects similar in their introduction to computation algorithms. Their main differences lay in the demonstrations of some of the authors that there could be a choice of strategies, especially in mental arithmetic.

Primary textbook author Elías Bjarnason claimed to have tried to avoid major inconsistencies with Dr. Ólafur Daníelsson's *Arithmetic*, confirmed by warm thanks to the latter in the forewords and the fact that both books contained the same procedures without explanations. Elías Bjarnason's *Arithmetic* was selected in 1929 as one of three arithmetic textbooks authorized for compulsory school by the Ministry of Education (See section 5.4.).[1022] Time favoured Elías Bjarnason's series. At the lower secondary level Dr. Ólafur Daníelsson's *Arithmetic* dominated. While both textbooks were well written, one may wonder if Ólafur Daníelsson's influence as the leading mathematician in the leading high school was not a substantial factor in the decision process. Children were being prepared for admission to the Reykjavík High School, so it might be advantageous for them to study textbooks which had the consent of the authorities in that school.

Public Perception of Mathematics

Dr. Ólafur Daníelsson was the undisputed master of Icelandic mathematics education in the first half of the 20th century. In his arithmetic textbooks he presented simple examples from daily life to explain commonly used procedures, while there is little indication that he considered it useful to offer alternative methods. He mentioned different procedures to use in mental arithmetic in his 1906 edition of *Arithmetic*, published before he began to teach, but that had disappeared in the 1914 and later editions. Elías Bjarnason, his former student at the Teacher Training College, did not mention mental arithmetic in his upper primary level textbooks.

The topic of Dr. Ólafur Daníelsson's doctoral thesis was geometry. In his *Geometry*, geometry instruction is, for the first time in Icelandic discussion, directly referred to as a tool for training exact thinking, not a tool for practical purposes, although Björn Gunnlaugsson had hinted at that in his inauguration speech in the 1820s.

The views of Dr. Ólafur Daníelsson, that the purpose was to train the mind, was unique among his fellow countrymen. A satisfactory equivalence to a proof was to be presented if possible. If that was not possible and the procedure was necessary, he seems to have preferred to present it without an explanation than by reasoning that led to doubts. This is also followed in Elías Bjarnason's textbooks. As their textbooks dominated the market for decades, many generations may have missed out on plausible reasoning while trained in unexplained procedural algorithms. This may

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[1020] National Archives of Iceland: Skjalasafn fræðslumálaskrifst. 1976-C/1 Bréfabók 1908-1909: 380–381

[1021] National Archives of Iceland: Skjalasafn fræðslumálaskrifst. 1976-C/2 Bréfabók 1909-1913, 978-979

[1022] Helgi Elíasson (Ed.) (1944): 32

have contributed to the widespread perception that mathematics was not expected to be understood; only imitated.

Choice of Secondary Level Textbooks

Dr. Ólafur Daníelsson gradually phased out the mathematics textbooks of Jul. Petersen's series for his own at the lower level of the Reykjavík High School. In the mathematics stream he used solely Julius Petersen's system in the editions of Albert Kristensen and C. Hansen, where his own books did not cover the syllabus.

Ólafur Daníelsson may not have kept track of changes in syllabus in Danish high schools. His term of office in the Reykjavík High School coincided with Iceland's first years of sovereignty. The school was no longer part of the Danish school system and thus did not acquire information automatically. His education was rooted in Jul. Petersen's philosophy, and his system undoubtedly suited Ólafur Daníelsson's taste. When he retired from teaching during World War II there were difficulties in acquiring foreign textbooks and as a consequence Icelandic teachers probably kept faith with the late Julius Petersen's textbook series far longer than their Danish colleagues. The syllabus did not change until 1949, when Guðmundur Arnlaugsson and Björn Bjarnason, belonging to the next generation mathematics teachers, had both been appointed at the school.

The distribution of arithmetic textbooks and their consequent influence depended as before on the position of the author and his range of influence. Jónas Jónasson's 1906 *Arithmetic* was used in the Akureyri School while Jónas remained there. Once Dr. Ólafur Daníelsson and Þorkell Þorkelsson were installed at the secondary schools, they were able to implement their ideas and create a market for their textbooks. While Þorkell Þorkelsson remained at Akureyri School in 1908–1918, his *Mathematics* was in use there. He then left teaching, and thus had only minor influence on mathematics education. After Dr. Ólafur Daníelsson took charge of mathematics at Reykjavík School in 1919, the syllabus of that school was closely followed in Akureyri, presumably to ensure the admission of the northern pupils to the learned department of the Reykjavík School. Dr. Ólafur Daníelsson thus governed the choice of his books in his own school as well as those schools leading up to it or wishing to help their pupils take an entrance examination into its learned department. The State Textbook Imprint chose Elías Bjarnason's series to be supplied free to compulsory school pupils in 1937, and the National Examination Board took Ólafur Daníelsson's textbooks as the basis for the new national entrance examination to the high schools in 1946. After that, the position of their textbooks and the influence of Dr. Ólafur Daníelsson were assured, long after he left teaching.

Not much of importance has been found about trends in mathematics education in the pre-war period in public discussion or journals on education. The available arithmetic textbooks were rather similar, while Dr. Ólafur Daníelsson's book had the richest choice of exercises. None of them came close to Ólafur Olavius' *Clear Guidance* of 1780 in length and versatility in explanations and demonstrations, or *Short Teaching* (1785) by Governor Ólafur Stefánsson for that matter. Instead, economical frugality prevailed. Concerning algebra, only Þorkell Þorkelsson and Ólafur Daníelsson wrote books on that subject in Icelandic. Þorkell Þorkelsson's book might have suited more youngsters. The Reykjavík School, however, led the course, and there Dr. Ólafur Daníelsson and his interpretation of advanced arithmetic and algebra dominated the scene.

The Dominating Position of Reykjavík High School

Through the centuries the learned schools had their special place in Icelandic society as the only entrance to official positions. In that respect they hardly differed from Danish learned schools, except that there was only one Icelandic school from early 1800s until 1930. When public secondary education became more common in the first decades of the 20th century, the dominance of Reykjavík High School in the education system gradually emerged. Even though Akureyri School was the only lower secondary school which could offer a direct entrance examination to the upper department of the Reykjavík School, there was a long-standing tradition that pupils, for example from rural areas and from Flensborg School in Hafnarfjörður, tried to enter Reykjavík School at various stages. The syllabus and the requirements of Reykjavík School were therefore bound to have an effect on primary education and the upcoming lower-secondary schools.

While there was no mathematics stream at Reykjavík High School this effect on public mathematics education was not substantial. It intensified after a mathematics stream was established in 1919 and Dr. Ólafur Daníelsson, a man of strong character and superior knowledge, was appointed. Not only did he rearrange the entire mathematics syllabus in that school in the period up to 1927, but schools leading up to Reykjavík High School adjusted their syllabus to its requirements. Akureyri High School was established in 1930 to counterbalance Reykjavík High School, but it followed the mathematics syllabus of the older school. It did not have a mathematics stream during the first years, but could allow those who wished to take the mathematics stream to do so, by the recommendation of Dr. Ólafur Daníelsson,[1023] who was thus a protector of the mathematics stream education there. His advice was likely to be followed and not opposed. Restricted admission to Reykjavík School in 1928 further intensified the urge to adapt to its requirements. This dominance might have been challenged and more discussion taken place, if there had been another school or schools of similar size and reputation to Reykjavík School.

One of the first acts of Guðmundur Arnlaugsson and Björn Bjarnason after being appointed mathematics teachers at the Reykjavík High School in the post-war restoration period was to modernize the mathematics stream syllabus. One notes the growing independence and conservatism of Akureyri School, in that it only abolished Jul. Petersen's system in the 1960s, in favour of "modern" mathematics.

As the physics and mathematics representatives of the National Examination Board, Guðmundur Arnlaugsson and Björn Bjarnason also took the initiative of altering the syllabus there in the 1960s, when the latter had entered the National Examination Board. The main obstacle was lack of suitable textbooks in Icelandic. Guðmundur Arnlaugsson's sabbatical leave in 1963 had probably been in preparation for some time, and by then the news about international mathematics reforms was beginning to filter in. At that time, he in company with Björn Bjarnason had acquired a strong position to exert influence, not only on mathematics education on all levels but on all secondary level education. As successors of Dr. Ólafur Daníelsson, their authority was unquestioned.

[1023] *Skýrsla um Menntaskólann á Akureyri 1930–1931* (1931): 35

University and Research

The University of Iceland was established in 1911. When Iceland gained sovereignty in 1918, Icelanders lost their privileges at the University of Copenhagen and the Regensen student residence. From then on, the *Alþingi* granted scholarships to a limited number of students to study abroad. Through increased opportunities for further studies within Iceland and fewer privileges at the University of Copenhagen, Icelandic intellectuals may have become more isolated.[1024] The choice of tasks for research contributed also to isolation and self-centredness. To quote Prof. Helgi Skúli Kjartansson:

> Alongside research into history, language and literature, the natural sciences were the neglected children. They were not taught at the University, except within medical studies, nor cultivated at research institutes, but pursued by individuals, assisted by grants and support from various directions.[1025]

The emphasis was on what was particular to Iceland, and the global cultural heritage, which could be cultivated by other nations, was secondary for the time being. The natural sciences were not pursued within a separate faculty at the University until 1970. Mathematics did not have a place in the higher and further education that was being developed in early 20th century Iceland.

Conclusions

The first decades of the 20th century were a period of a rapid progress, new education legislation and regulations. Many people, mainly teachers and clergymen, suggested ways and methods to contribute to increased education by publishing arithmetic textbooks. There were also mathematician Dr. Ólafur Daníelsson, and physicist Þorkell Þorkelsson. There was an awakening in the country for more education, and there was a market for inexpensive books for the home, the simpler primary-level books and the more intermediate, lower-secondary-level books as well. There were only few pupils in each school and grade, which of course was not a sufficient basis for publishing a book, so the distribution of textbooks depended very much on the authors' range of influence.

In 1929 the Icelandic compulsory school system allocated less than one fifth of the expenses of its Nordic counterparts per pupil, but was rapidly catching up. There were gradual improvements of the external conditions of primary and lower secondary education. Restricted admission to higher education contributed to the strengthening of stratification based on educational status. The small higher status educational elite appreciated the values of mathematics for technological development to some extent, but rather the cultural value of mathematical subjects as a part of world's heritage.[1026] The lower status group studied arithmetic for their personal needs.

The fact that Reykjavík High School's second grade syllabus was chosen as a model for the national examination in 1946 confirms its dominating position. The national examination was introduced in order to abolish the power of the two high schools to select their pupils. Apparently, the act of copying their syllabus was a compromise to meet discontent in the high schools over their new position.

[1024] Aðalgeir Kristjánsson, December 6, 2004
[1025] Helgi Skúli Kjartansson (2002): 158–159
[1026] Hinrik Guðmundsson, Jakob Björnsson and Páll Theodórsson (1964)

10.6. Challenges of a Newborn Republic 1944–1964

Introduction

The history of Icelandic education from the late 1920s to 1960s was in many respects characterized by a stand-still situation, with the exception of the 1946 education legislation and the introduction of the national examination. The centre of young intellectuals moved from the University of Copenhagen to the University of Iceland, a much smaller community. Previous circulars from Danish school authorities were the more important considering the geographical distance from other countries. A state monopoly institution, State Textbook Imprint, established to ensure textbooks for all, contributed to a reduced choice of textbooks. The Depression of the 1930s, and restricted access of the public to foreign currency in the post-war period, contributed to minimal exchanges with other countries in the field of education, as well as other cultural exchanges, lasting up to the 1960s.

The 1946 school system was established with a calculated need to achieve a certain number of technicians and natural scientists in order to create a foundation for a new technological and egalitarian society. The proposed investments in trawlers and factories, expected to fund the new society, did not render the revenue expected. Mechanization of agriculture was an expensive investment which did not turn out to be as profitable as had been hoped. Expectations of the school system and other living standards soon turned out to be too high.[1027]

Throughout the post-war period, governments struggled with runaway inflation, which was the result of an underdeveloped and vulnerable economy not capable of generating enough income. Educational projects, not least the State Textbook Imprint, producing textbooks, were at the mercy of the economic situation up to the 1960s.

Higher Education

In 1960 the total proportion of each year-cohort graduating from high school in Iceland had reached 8.7%. Those who carried on to the University of Iceland had few choices except Icelandic studies, in addition to the options of professional training. Some went abroad, mainly to Germany and Denmark, for natural sciences and engineering studies (see section 6.1.) or studies in social sciences. Others, not belonging to the educational elite, went through an educational system, unrelated to the learned path that emphasized practical education, in some cases acquired in Denmark or other neighbouring countries.

This time was a period of progress in the main industries, agriculture and fishing. Roads, water and electricity gradually improved living conditions through the efforts of the few engineers, most of whom completed their education in Copenhagen or Germany. Roads were needed to transport milk, and to some degree other agricultural products, while fish products were transported by sea. Trains with their precise, planned schedules, were never introduced in Iceland, so that sector did not offer opportunity for any developed mathematical activity.

[1027] Helgi Skúli Kjartansson (2002): 266–269

Practical arithmetic was considered sufficient mathematics up to the 1960s, except for the very few who were to satisfy the country's need for engineering and science. The mathematics that was exercised was in preparation for entrance to the Polytechnic College in Copenhagen, and after 1940 to the Faculty of Engineering at the University of Iceland. Other mathematics was not studied there and not needed. However, the increased complexity of national economics, for instance, was gradually to make more demands on public mathematical knowledge. Statistical concepts in relation to the labour market in the volatile inflation 1940–1990 were for example extremely relevant for the general public.

In other countries, like in England and the United States, debates about school mathematics at the college-bound secondary level were initiated after World War II. The main concern was what kind of mathematics was of primarily needed in industry and for national defence.[1028] Iceland had no native military forces. Industry was primitive and on small scale. It had not defined any specific needs demanding mathematical skills.

High school mathematics teachers were required to have qualifications equivalent to a Cand. Mag. degree, unavailable in the country. During the entire period from Ólafur Daníelsson's graduation in 1904 until 1960, this system only produced six more mathematics teachers with these qualifications, a few more if the physicists are counted, much less than was needed to teach at the growing number of high schools, which in 1955–1960 produced 200 graduates a year. The number of graduates was to grow dramatically in the following decade.

The lack of formal mathematics curriculum except for a limited number of elite pupils, the lack of mathematically-educated teachers, and the 40- to 50-year-old textbooks, all contributed to a crisis situation in the 1960s.

The National Examination

The restricted admission to Reykjavík High School eventually became to be considered a social injustice. The national entrance examination into the high school level, intended to ensure equal opportunities, was run on the Reykjavík High School's terms, its syllabus in all subjects defining the syllabus for the examination. This condition, and the fact that there were a limited number of textbooks in Icelandic, may be the reasons why the syllabus was only prescribed by certain pages in given textbooks.

Euclidean geometry as part of the mathematics syllabus proved to be impossible to implement in the general lower secondary schools. One of the reasons must be the lack of a teacher training programme for mathematics teachers at the lower secondary level. Most of the teachers did not have any specialized education in mathematics, and so had to depend on their high school training. Also, the available teachers of the scattered groups in 20–30 schools could not manage to teach introduction to algebra and introduction to Euclidean geometry in one school year. Apart from the geometry and some other topics, such as square roots, the mathematics examination problems up to the 1960s were modelled on Danish lower secondary school examinations.[1029]

[1028] Osborne, A. R. and F. J. Crosswhite (1970): 237
[1029] Haraldur Steinþórsson, January 22, 2003

Through 25 of the 30 years of the examination's existence, the Reykjavík High School held the national examination trapped between reasonable demands on the pupils at a vulnerable age, and the threat to send them to a drop-out bin in their first school year. The Reykjavík High School was therefore the body that actually governed the examination, even if the National Examination Board claimed absolute autonomy. The school had the extenuating circumstances that it was situated in a building from 1846, originally built for 60–80 boarding pupils, but by 1965 accommodating 1060 pupils.[1030] *Alþingi* was reluctant to increase the number of high schools in the capital area, as its schools policy was in a trap between political demands for schools in the rural constituencies with falling population, and the need for schools in the growing urban areas.

Another factor was the attitude, inherited from the learned school era, and strengthened by Jónas Jónsson of Hrifla and his followers, that higher education should be limited to the few that were to become government officials, and that too many of them would result in a burden on society. People discussed whether there was a need for more high school graduates and whether they would find a suitable occupation. The number of high school graduates as a proportion of the age cohort may have been no higher in Denmark, while vocational and technical education in Denmark was more advanced than in Iceland. But was there a need for such detailed training programmes in Iceland, a country with limited variety in industry? There was no research and there seemed to be no policy to decide where to aim. Meanwhile the pressure on the upper secondary level increased.

Teacher Training

After Dr. Ólafur Daníelsson left the Teacher Training College in 1921, the share of hours devoted to mathematics began to decline. It did not have a tenured teacher of mathematics from 1929 until 1962, i.e. for most of the period up to 1971, when the college was elevated to tertiary level, and those who taught did not have any special education in mathematics. Mathematics' share in the timetable of the Teacher Training College decreased from 8% to 5% during the same period, compared to a steady 14% allocated for Icelandic. One can therefore claim that mathematics did not have priority in the college.

Another drawback of teacher training was that there were no guaranteed paths for the college graduates to acquire further education at the University of Iceland, as the legislator had planned. In this respect the otherwise ambitious 1946 education legislation was a great disappointment.

The Teacher Training College was modelled on Danish teacher training colleges, and it cultivated that heritage in most respects. However, the special circumstances in Iceland, – the fact that there was only one college, and the general lack of mathematically-educated people in Iceland – probably contributed to primary school teachers having less mathematical training than the average Danish teacher. Regulations applying to many colleges would hardly have cut down the hours for mathematics whenever there was no mathematics teacher available, or when more hours were needed for other teachers or subjects.

[1030] *Skýrsla Menntaskólans í Reykjavík, 1965–1966*: 9

When university entrance for the Teacher Training College' graduates was finally assured in the 1963 legislation, the college acted as a safety valve on the pressure for entrance to the upper secondary level, by being a high school option for those who did not gain access to the regular high schools. Hundreds of pupils went through the teacher training programme without intending to become teachers. This was a shrewd move on the part of the Ministry of Education, to solve the problems of the upper secondary level, while it also made a choice of electives possible, where mathematics had its place. This unfortunate situation, with its consequent lack of properly educated teacher trainers, created however an opportunity for increasing the mathematics syllabus at the Teacher Training College, not preceded since Dr. Ólafur Daníelsson's period.

Only few graduated from the training programme in mathematics for lower secondary education teachers at the University of Iceland in 1951–1973.[1031] However, a survey of results of the national examination at a few schools, made by the author of this study, indicates that the pupils of many of them did better in mathematics compared to other subjects, than the pupils of other teachers. Other factors, such as stability in the teaching force and teachers' illness, were shown to affect the results.

The total lack of a training programme for secondary mathematics teachers until 1951, and then only as a part of an engineering training programme, also bears witness to the "pioneer" situation in Iceland in the mid-20th century. What was available had to suffice, at least for a while. New legislation was passed on compulsory teacher training in 1971 and the University in 1970, adjusting the programmes more to European standards.

National Curriculum Documents

One may wonder about the lack of proper curriculum documents in mathematics for most of the 20th century. The Reykjavík High School kept its 19th century Danish learned school form up to the 1970s, with minor upgrading in regulations in 1904, in 1919 when the mathematics stream was tailored after the Danish mathematics stream, in 1937 when the mathematics stream finally had proper regulations, and in the 1946 legislation when the first two grades were chopped off the school. There were no definite curriculum documents except the regulations published in 1904, 1908 and 1937. Tradition was as powerful as any curriculum document.

The status of the non-compulsory lower secondary school reflects the perplexity of the education authorities. On one hand there was the rigid lower department of the high schools and later gradually stagnating national entrance examination into the highly selective upper secondary school level, and on the other hand there was the unstructured two-year programme where nothing was formally defined and the choice was up to each school and each teacher. This situation prevailed in the two uppermost grades of the lower secondary schools through the 1946 legislation into the late 1960s, reportedly due to an experimental state of that level. To illustrate this point, non-metric geometry was nowhere contained in the syllabus except in the high schools' lower department during the whole period up to the 1970s.

The compulsory school level for the 7- to 15-year age group at last received a national curriculum document in 1960, 14 years after the legislation. The arithmetic curriculum was more detailed than the curriculum for primary level published in

[1031] See Appendix K

1929, including some guide to teaching, while the content was similar: the four arithmetic operations in whole numbers and fractions in addition to area and volume computations. This was continued by equations, percentages and *regula de tri*.

Therefore, when it came to restoration through the School Research Department, its first task was to develop curriculum documents and teaching material in all subjects of the compulsory school and the two grades following it. The need in mathematics, however, was considered so urgent that work on teaching material was initiated before the curriculum documents, which only existed in preliminary or draft editions until 1989.

Primary School Level after 1946

The older Icelandic custom of teaching 7- to 9-year-old children at home declined rapidly in the period between the world wars, although it did not disappear completely until the 1970s. When, after the second war, the new republic wanted to create a new school for all children from the age of 7, no one saw any reason to reconsider primary arithmetic teaching enough to change it, until the Director of Education in Reykjavík, Jónas B. Jónsson, himself wrote arithmetic textbooks for first to third primary grades in the 1950s.

The textbooks by Elías Bjarnason for the fourth to sixth grade were considered adequate for another two decades. No one seems to have had any idea of what such books should be like, and the economic conditions of the State Textbook Imprint did not offer itself for a choice of two different syllabi, even though the legislation allowed for it. One might also blame this fact on the inadequate teacher training in mathematics and the limited opportunities teachers had to keep up with what was happening in other countries. Travel was still expensive and time-consuming. Primary arithmetic in which methods, algorithms and rules were prescribed in a certain way, and where it was considered confusing to the pupils to introduce options of procedures, was prevalent in public education in Iceland for another while.

Iceland before 1960 Compared to Denmark

Through the centuries, Iceland had been part of the Danish realm and hence Copenhagen been its centre for higher education. Iceland was a peripheral part, which demanded autonomy and its own system, ever more insistently from the 1830s to 1918. Even if it did not wish to be like other parts of the Danish kingdom, such as by joining the Danish islands in their consultative assembly in the 19th century, its situation was similar to theirs. The population, for example, of Northern Jutland was easily comparable in size, just over double, and the economies were similar, fishing and agriculture. One would expect the mathematical needs of Northern Jutlanders to be of an economic nature, in connection with weights and measures, selling and buying their products. An exception may have been the shipbuilding companies and cement production in Aalborg, needing some engineering knowledge. Similarly in Iceland, initial efforts at industrialization were to establish a shipbuilding industry and factories on a small scale to meet domestic demands in the production of cement and fertilizers. Higher mathematical needs were in the field of engineering. All development of higher mathematics was left to the educational centre in Copenhagen, both on the part of Northern Jutland and of Iceland.

From 1940 a decreasing proportion of the population in Iceland was engaged in agriculture and fishing, and the increase was received by manufacturing and trade.[1032] The trends were towards a more complex society. Figure 10.1. illustrates the trends in Iceland up to 1990 and figure 10.2. the trends in Denmark until 1970.

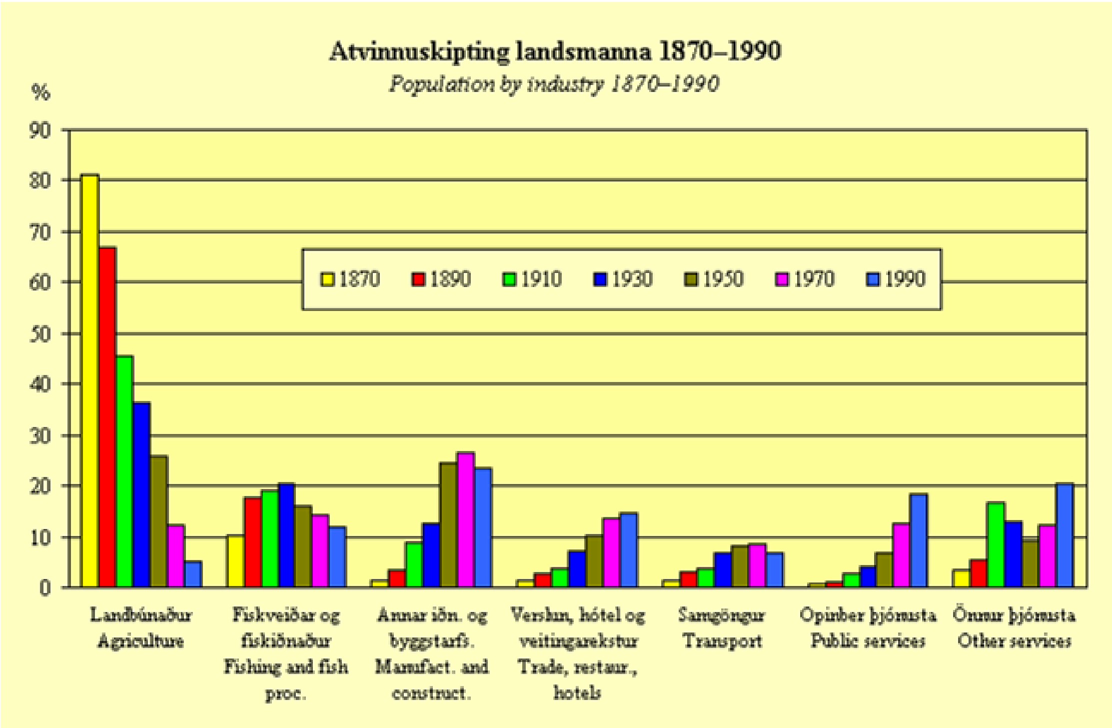

Fig. 10.1. Population in Iceland by economic sector 1870–1990.[1033]

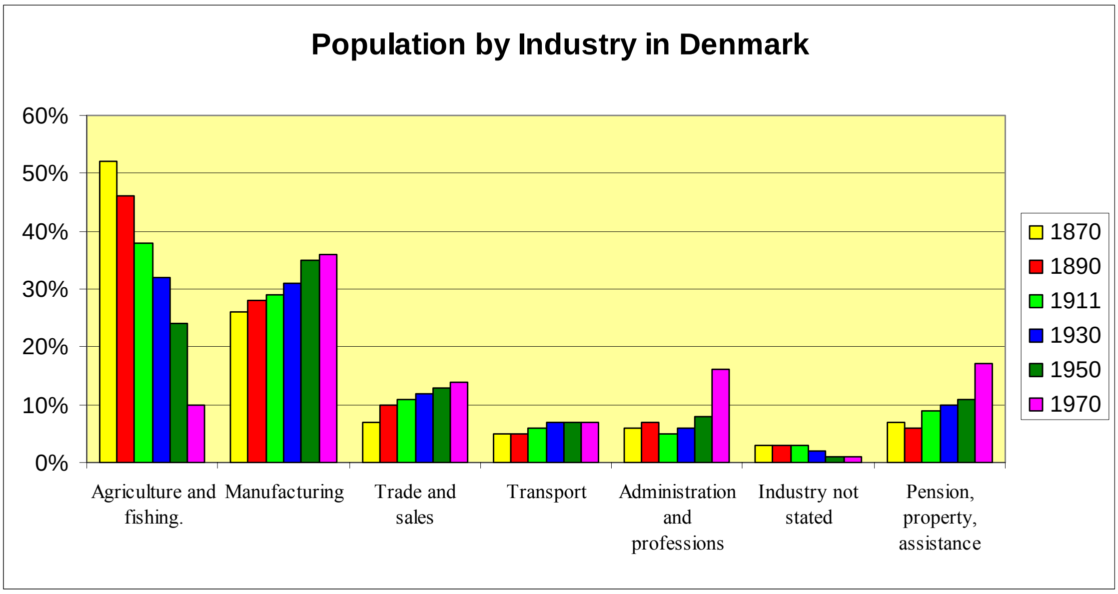

Fig. 10.2. Population in Denmark by economic sector 1870–1970.[1034]

[1032] *Hagskinna* (1997): Table 3.5. Population by industry 1910–1960
[1033] *Hagskinna* (1997): Figure 3.1.
[1034] Statistics Denmark: website, *Statistical yearbook 2004*. Tables 132, 133. Population by industry in the censuses.

In figure 10.2., fishing is included in agriculture. Even if the official classification of the statistics bureaux in Iceland and Denmark are not exactly the same, the trends in the economy in Iceland and Denmark are similar: towards increased service, and proportionally less employment in the basic sectors, agriculture and fishing. Obviously, the development occurred earlier in Denmark than in Iceland.

The increasing proportion of the population employed in services may be further illustrated by the following graph of the sectoral distribution of the working population in Iceland in 1870–1990 in figure 10.3.[1035]

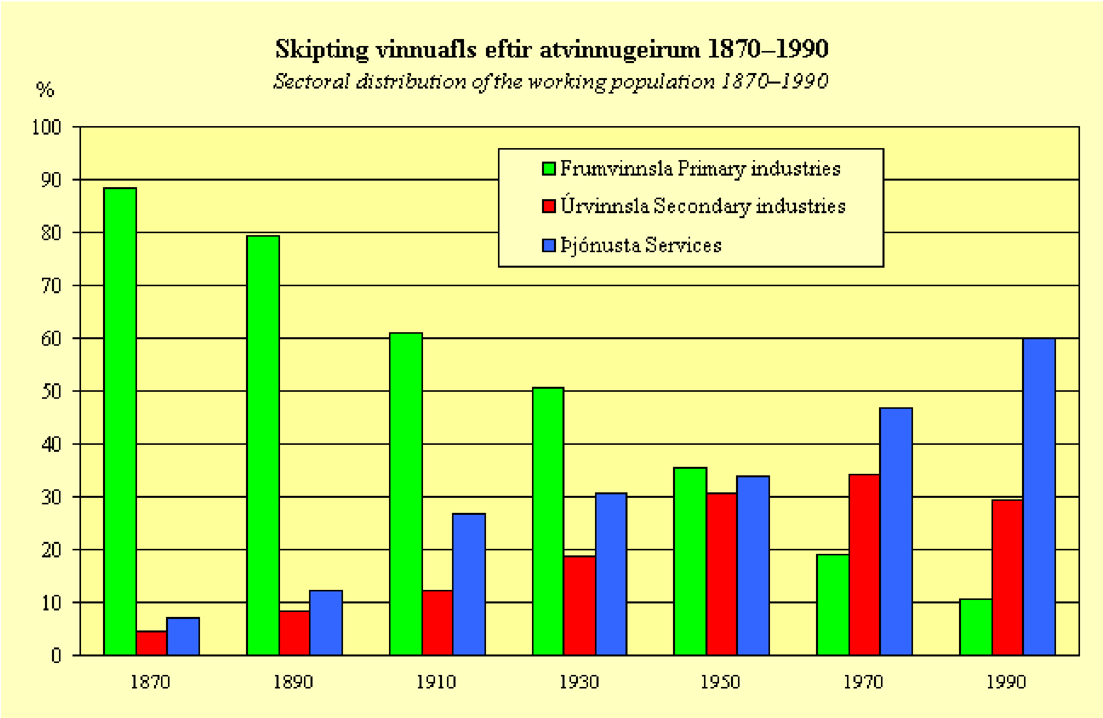

Fig. 10.3. Sectoral distribution of the working population in Iceland in 1870–1990.

The needs of society in the second half of the 20th century were no longer the same as they had been in the first half.

Conclusions

In the early 1960s the content of mathematics education was in most respects similar to what it had been since the 1920s, except that a greater number of people were receiving instruction. The focus of the authorities was on pupils aiming at further education, and others were not given any detailed attention. No development had taken place and there was little initiative, in the compulsory education and elsewhere. The upper secondary level still adhered to the requirements and standards of the Danish school system. Demands for adequately trained teachers were not met, as was confirmed by an OEEC survey in connection to the Royaumont Seminar in 1959 (See section 7.1.).

[1035] *Hagskinna* (1997): Figure 3.2.

During this post-war period one can therefore say that mathematics education did not develop similarly to that in the neighbouring countries, as considerable developmental activities had already been initiated in Denmark, Norway and England in the late 1950s.[1036]

Trends between 1950 and 1970 show clearly that Iceland was on its way from a society of primary industries where only primary mathematical skills were needed, towards a more industrialized one. One might therefore conjecture that the international mathematics education reform movement hit Iceland at a particularly appropriate moment, when it was developing away from a self-sustaining society of agriculture and fishing of the pre-1960 period towards more international aspects of life. Its sectoral distribution of the working population was becoming more similar to that of other Northern European countries, at the same time as the nation was introduced to Western theories about education as an important factor in economic development, through its membership in the OEEC, later the OECD.

10.7. The Educational System in Flux 1964–1975

The OECD's Influence

In the early 1960s Icelandic educators, mathematicians, physicists and psychologists had learnt about the "modern" mathematics reform movement promoted by the OEEC, and later by the OECD. The movement had by that time reached most countries in the western world, including the Nordic countries, England and the United States. A move was made towards implementing similar reforms in Iceland. Reform experiments, made by individuals, were initially exclusively aimed at university-bound pupils, as had also been the original plans in the U.S. and England and at the Royaumont Seminar in 1959.

At the same time, the Icelandic authorities were introduced to the OECD's theories about education contributing substantially to economic and social progress and stability, and education being as much a sector of society and of the national economy as the traditional sectors.[1037] The Technical College of Iceland was established on the encouragement of the OECD[1038] on these terms, and this was reiterated at a meeting in Reykjavík in June 1965 of OECD representatives with Icelandic educators. A report, dated in January 1966, revealing that the lower secondary school syllabus in mathematics, physics and chemistry in Iceland was behind that of other Nordic countries, may, like the 1957 Sputnik Shock in the USA, have been the catalyst that initiated the willingness of the authorities to allocate funds to reforms that had already, in the case of mathematics, been prepared and realized on a small scale.

At this particular time, reforms seemed feasible. The economic benefits had been clarified by the policy of the OECD, and experts were ready to propose solutions along similar lines as had proved promising in other OECD countries, both in the USA and the Nordic countries, with which Iceland identified itself. The physicists proposed a reform of the physics syllabus in the spirit of the Woods Hole Conference and the mathematicians a reform in the spirit of the Royaumont Seminar, in the style reforms had been developed in the Nordic countries.

[1036] Høyrup, J. (1979). Gjone, G. (1983). Cooper, B. (1985)
[1037] Efnahagsstofnunin (July 1965)
[1038] Sveinbjörn Björnsson (1966): 100

Experiments with "modern" mathematics at all levels of the school system were initiated and initially led by mathematician Guðmundur Arnlaugsson and his collaborators, and strongly supported by Minister of Education and of Commerce Gylfi Þ. Gíslason, influenced by the OECD paradigm of the economic value of education.

After the first experiments, the School Research Department, SRD, established on the encouragement of the OECD, became the framework for the reforms on compulsory level. The very first subjects to be reformed by the SRD were the science subjects: mathematics and physics. The physicists' budget for the reforms was used as a model for the School Research Department's budget for the reforms in other school subjects.[1039] Eventually, "modern" mathematics reform reached most schools at all school levels.

The School Research Department established reforms in all subjects, and it actually turned out to become a school development department. When the framework for the reforms had been created, the politicians left the scene, except for ensuring parliamentary support for a budget necessary to run the reform projects, and the specialists overtook their creation. The budget for the SRD was whole-heartedly supported for more than a decade by the rural-policy-oriented *Alþingi* and both right-wing and left-wing governments.

Through the establishment of continuation departments at the secondary level and the SRD's influence on the development of the experimental school in Breiðholt, the SRD also had a considerable influence on the mathematics curriculum and teaching-material development of the upper secondary level.

"Modern" Mathematics at the Upper Secondary Level

The initial purpose of the reforms in the mathematics stream was to provide the pupils with better preparation for university studies, even though probably not more than one-fourth of the group was ever to study mathematical subjects. The mathematics teachers at the Reykjavík High School and in the first year courses at the University were the same persons, the prime proponents of the reform, so they must have intended to ensure coherence between the school levels.

As was the case in other countries, the upper secondary level went through a process of implementing and developing a "modern" mathematics syllabus, and its subsequent retreat, without much conflict.

The process was initially a measure towards implementing university mathematics in the college-bound stream of the upper secondary level. In the course of the events it contributed to dissolving the rigidity of the upper secondary level, by testing a variety of textbooks which suited different needs, and by adapting the syllabus better to the Icelandic language, thus making mathematics more accessible for a wider variety of pupils. By the end of this period, this school level had a wide choice of streams instead of the traditional two streams, with some purposeful mathematics implemented in each stream, if not completely adapted to each stream's estimated needs, then at least an improved offer of mathematics education to that group.

[1039] Andri Ísaksson, March 10, 2003. Sveinbjörn Björnsson, November 5, 2004

The number of young teachers increased proportionally to the number of pupils. There was no institute yet to educate mathematics teachers for upper secondary level. However, several of the teachers were well educated, while others may not have had much training in mathematics. The reform period gave all those teachers good opportunities for initiative. What they might have lacked in formal training they could gain by self-education through developmental activities. Thus the system benefited from the reform, in that more enthusiastic people were recruited and had the opportunity to exert their initiative and creativity.

In one sense, Icelandic upper secondary level schools were freer to search for suitable reform projects than lower level schools and schools in other countries with small markets, as they did not consider themselves restricted by the language of the textbooks, at least not initially. Nor were they restricted by a detailed curriculum, although they obviously had to ensure that the university level was provided with adequately prepared pupils.

"Modern" Mathematics at the Lower Secondary Level

For the lower secondary level mathematician Guðmundur Arnlaugsson wrote a good, lucid and easily readable Icelandic textbook, not disturbing the average teacher in spite of exotic topics like set theory. The reform created a release from earlier ways of approaching mathematics. This first new textbook was conducive to discussion and thinking, rather than instructions on how to do things.

Set theory as an introduction to algebra of real numbers, however, was to many teachers a foreign idea and a waste of time. In spite of the intentions, expressed in a provisional national curriculum from that time, to introduce set algebra as a unifying structure before the general arithmetic algebra, this did not work any better in Iceland than in other countries that were experimenting with "modern" mathematics at the lower secondary level. Set algebra soon retreated into the background to become a collection of help concepts.

A negative side of the "modern" mathematics reform at the lower secondary level was that the national entrance examination into high school developed towards a collection of incoherent details. The weight of the word problems declined, and those which remained were in words without a familiar context. They were increasingly short, and the number of problems increased inversely to the brevity of the problems. While this may have made it easier to tell whether pupils knew the most basic algebraic rules, even though they could not cope with complex ones, there was less opportunity for the pupils to display their creativity and strategic skills, so their perspective of mathematics may have become narrower than it could otherwise have been.

The School Research Department developed more textbooks for that level. After a period of a set-theoretical approach, a series following the structuralist or even formative approach, including more investigative work than before, was published around 1980. Around 1990, a "back-to-basics" series arrived on the market, and was favoured by the majority of teachers, except a few who were inclined towards investigative work.

As at the upper secondary level, the set-theoretical "modern" mathematics approach gradually retreated without any major conflicts, towards a more traditional, yet broader, mathematical syllabus than the previous arithmetic and introduction to algebra.

Both the set-theoretical and the structuralist approaches were unfamiliar to most teachers, so a number of in-service courses were offered to teachers to introduce the new content, and in the latter case also new teaching methods. Many teachers on this level also benefited from developmental work by testing new material, attending in-service courses and even joining work-groups developing the new material.

"Modern" Mathematics at the Primary Level

The primary level reform turned out to be the most problematic, with the translated Danish Bundgaard textbooks from the Nordic NKMM cooperation. Translation and adaptation of foreign textbooks was the only option, as it was not considered possible to compose a completely Icelandic series at that time. A Danish series was chosen because it was the most easily accessible for translation. It was in its initial phase at that time. Only three year-courses had been tested and one was about to be completed. Later volumes in the series turned out to be extremely set-theoretical, mainly suited for the ablest pupils and teachers with a high inclination for mathematics. At the outset, teachers and headmasters were excited about the new material and were eager to join the experiment. A great number of mathematics in-service courses were held for primary school teachers as well.

The new set-theoretical mathematics syllabus aroused debates and reactions. Parents and the public realized that school mathematics teaching was changing radically. Different computation algorithms were one of the side effects. Consequently, parents had difficulties in assisting their children, and indeed were not expected to, as they might confuse them. More problems emerged when new teachers, who were not familiar with the material and the mathematical language, took over grades four to six.

In fact, similar problems occurred in other countries. Introduction of "modern" mathematics in primary schools in the USA proved to be the beginning of the end.[1040] The difference in Iceland was that the primary level experiment probably reached a greater proportion of the population than in many other countries, such as the United States.

The School Research Department quickly initiated writing of a new textbook series for the primary level. Only four years after the introduction of foreign translated textbooks, when there had been no way to create an Icelandic series from scratch, it was possible to recruit a number of experienced teachers to write the new series. The reform movement thus developed initiative and self-confidence in teachers, and new teachers, unknown to all, entered the scene of writing. The new series was a kind of synthesis, with less emphasis on set-theoretical language and notation, yet a more varied coverage than the traditional material, as was the case at the lower secondary level. This changed the mathematics syllabus in Icelandic compulsory schools permanently from simple arithmetic to varied material, with non-metric and metric geometry, introductory statistics, counting and advanced arithmetic.

[1040] Gjone, G. (1983): Vol. I, 53

“Modern” Mathematics in Teacher Training

Teacher training was the only area with which the School Research Department was not directly concerned. The Teacher Training College suffered until 1971 from being used as a bypass to the overcrowded high schools (See section 8.3.). University students and other part-time teachers were appointed to inform the prospective teachers about the “modern” mathematics reform, in some cases without any philosophical speculations about its goals.

One of Gylfi Þ. Gíslason’s last acts in office was to raise the college to university level. At that time the peak of “modern” mathematics, in the sense of set-theoretical approach, had passed. As a result, the ideology of “modern” mathematics did not originate there, nor was it discussed to any marked degree.

An Educational Crisis

Mathematics curriculum reform in Iceland in the period 1965-1975 was intertwined with a serious crisis in the Icelandic school system. The crisis had several reasons:

1. A long-standing tradition of a high school as a learned school for the professional elite, accompanied by a fear of the professional class becoming a social burden. Extremely strict standards were maintained in order to limit admission to the school, in addition to reluctance to respond to the need for improvement in accommodation for the leading Reykjavík High School.
2. An enormous increase in the number of pupils earning access to higher education in the period after World War II, due to the establishment of the national examination, a rising birth-rate, a better economy in the 1960s and an increasing demand for education for all.
3. A constant stream of people moving from rural to urban areas throughout the 20th century, illustrated in fig. 5.1.
4. Neglect of nurturing the inner function of the schools on all school levels, appearing in a lack of proper national curriculum documents, outdated teaching material and a chronic shortage of adequately educated teachers.
5. Conflict between foreign ideas, seeing education as one factor in economic resources, and traditional ideas about the benefits of education provided by the family and early entrance into working life. One reason for the conflict was that education in the field of mathematics and natural and physical sciences, rarely acquired outside schools, had a very short tradition.

The conflict appeared, for example, in the ways *Alþingi* acted to meet the demand for education at the upper secondary level. The composition of the *Alþingi* was not in correct proportion to the composition of the population, as the rural areas were over-represented. The need for resolving serious oversubscription to urban schools was pushed aside, in favour of ideas about reversing the flow of population from rural to urban areas, by means of passing provisions for high schools in the regions, in the 1965 legislation.

This tension was resolved through the cooperation of the OECD-promoted School Research Department, Reykjavík Education Office and Reykjavík Town Council, by the establishment of continuation departments and the Breiðholt Comprehensive Multi-stream School. In establishing these new kinds of schools the municipalities had to take the initiative and carry their share of the economic responsibilities, thus relieving *Alþingi*'s burden. The establishment of comprehensive multi-stream schools around the country in the 1970s in itself delayed and even in some cases stopped migration from the regions to the capital area, while migration from the agricultural rural areas continued.

The fourth item, concerning the inner functions of the schools and in particular the lack of curriculum documents and outdated teaching material, was also resolved on the initiative of individuals and the activities of the School Research Department, which soon turned its attention and efforts to revising the curriculum and teaching material, an initiative which by legislation had been assigned to, and should have been undertaken by, the State Textbook Imprint. That institution, after its first five years, had neither funding and staff, nor an active governing body needed for its proper operation. The lack of teachers was partly solved by transferring the Teacher Training College to the tertiary level, and by a new mathematics teacher training programme at the University.

There were several reasons why mathematics textbooks, teaching material and teaching methods were allowed to become so outmoded:

1. The State Textbook Imprint was dependent on a personal tax, the textbook charge, levied by the *Alþingi*, and thus linked to the price index. For that reason, *Alþingi*, which struggled with economic problems and strikes throughout the period, was reluctant to raise the tax. The lack of funding, and rigid official procedures, hindered individual initiative to compose new material. The State Textbook Imprint may even be taken as a textbook example of how an official body may, after some years of initiative, act as such a hindrance.
2. The meagre mathematics education of teachers. Too many had never studied any kind of mathematics topics or teaching other than the arithmetic presented in the outdated textbooks.
3. The very short tradition of mathematics education in the country, compared for example to a widespread public tradition of enjoying literature.

That this crisis coincided with an international movement for mathematics reform was in a sense a coincidence. The mathematics textbooks for the primary and lower secondary schools were basically rooted in the 1920s, and therefore long obsolete. Possibly, if this had not so obviously been the case, the reform experiment at the primary school level might have been more deliberate, and might not have spread so rapidly throughout the population without adequate preparation.

Dr. Gylfi Þ. Gíslason's Period

During Dr. Gylfi Þ. Gíslason's period at the Ministry of Education 1956-1971, many remarkable innovations were initiated, such as the Technical College in 1963, legislation on the Teacher Training College in 1963, the elevation of teacher training to tertiary level by establishing the University of Education in 1971, legislation on the University of Iceland in 1957 and 1970, when the Faculty of Engineering was expanded to the Faculty of Engineering and Natural Sciences, and legislation on student loan funding in 1967.

The establishment of the School Research Department was Dr. Gylfi Þ. Gíslason's greatest accomplishment in educational affairs, and it may have been a cleverly contrived way of bypassing the reluctance of *Alþingi* to face reality in school affairs. Dr. Gylfi Þ. Gíslason travelled widely during his days in office, and he had become aware that the Icelanders were lagging behind other countries in educational matters.[1041] His contact with OECD through his role as Minister of Commerce, channelled foreign ideas into discussions about education, elevated them from stagnated tracks, and created a favourable climate for increased funding. Dr. Gylfi Þ. Gíslason showed genuine interest in attempts at innovation, such as the mathematics experiment initiated by the Reykjavík Education Office, and his sympathy for art and culture was unquestioned.

Generally, however, the authorities do not appear to have foreseen the rapid development, in spite of initiatives towards long-term planning by the Economics Institute.[1042] This may have been a common problem at the international level. No one foresaw the 1968 student uprisings or their social consequences.

Legislation, such as on high schools, was enacted in accordance with political wishes, but the real needs had to be met by other means. The School Research Department did minimal research, but started to work on what it considered the most urgent tasks, to prepare continuation departments and the multi-stream comprehensive schools to relieve the pressure on the high schools, to create curricula and learning materials, and to coordinate the lower secondary school examination with the national examination of the middle school.

The treatment of the Teacher Training College is also very questionable. It was probably a shrewd move in the power game with *Alþingi*, which wanted the upper secondary schools elsewhere than in the capital area, to convert the Teacher Training College more-or-less into a high school. Or perhaps it was a desperate move, in a vulnerable situation when hundred of pupils had earned the right to enter high school and had nowhere to go.[1043] In this connection, one should note that by the time the bypassing of high school via the Teacher Training College had worked, Dr. Gylfi Þ. Gíslason had provided new legislation on the University, ready for young people thirsty for education.

Reasons for Support to School Mathematics Reform in the 1960s

The OECD theories that education, in particular mathematical and technical education, contributed substantially to economic and social progress, seem to have won support. Neither domestic nor international problems nor changes of governments occurring in the first half of the 1970s disturbed the growing support for the SRD and the State Textbook Imprint, the frameworks for the school mathematics reforms.

It is interesting to study the sudden increase in public education funding after the findings of the survey, revealing that the mathematical knowledge offered to Icelandic lower secondary level pupils was inferior to the corresponding syllabi in Norway and Denmark. What were the real reasons for official authorities to devote funds to education, and mathematics education in particular? Why was it that the

[1041] Hörður Lárusson, March 26, 2002
[1042] Efnahagsstofnunin (December 1968)
[1043] Þuríður Kristjánsdóttir, July 30, 2005

implementation of “modern” mathematics in the 1960s gained such massive financial support?

We consider again M. Niss’s identification of fundamental reasons for mathematics education in various societies (see section 1.4.).[1044] By tradition, literary studies were considered to contribute to the cultural maintenance of Icelandic society,[1045] while few thought of mathematics education in that light. Similarly, a limited number of individuals were expected to need mathematical training as a prerequisite for their occupation, and only limited needs had yet, in the late 1960s, been identified in society to provide the general public with education in mathematical subjects beyond basic arithmetic as prerequisites for their private or social life.

It must therefore have been the OECD’s paradigm – that mathematics education would contribute to the technological and socio-economic development of society at large – which changed the authorities’ attitude towards education, and caused the total reversal of policy on financial support to education in general, and to mathematical subjects in particular.

Individuals’ Initiative and Impact

The population of Iceland is small, and so was its intellectual community for most of Iceland’s 1100 years, up to the mid 1970s. The mathematical community was still smaller. In the whole of the 19th century there was only one Icelandic mathematician, Björn Gunnlaugsson, whose work as teacher and land-surveyor was an admirable and unique achievement.

Dr. Ólafur Daníelsson was Björn Gunnlaugsson’s successor in the 20th century, being a pupil of his grandson, Björn Jensson. Dr. Ólafur Daníelsson’s influence on Icelandic mathematics education through his textbooks persisted for more than six decades. He was alone for a quarter of a century. After that his pupil, Sigurkarl Stefánsson, was his collaborator. Another pupil, Guðmundur Arnlaugsson, together with his pupil and collaborator, Björn Bjarnason, became the most influential persons in mathematics education in the second half of the 20th century. They took the charge of the dominating Reykjavík High School and modernized it in the 1950s and again in the 1960s. They in turn influenced their university students Hörður Lárusson and Anna Kristjánsdóttir, who took over the lead in reform activities in the late 1960s and 1970s.

Thus there was a long-standing tradition of individual authority in the field of mathematics education. Guðmundur Arnlaugsson was the initiator of new trends with the support of Björn Bjarnason, his collaborator. In the 1960s they promoted mathematics reform at all school levels, including the primary level. They were the persons, who were most dedicated to the ideology of “modern” mathematics as it appeared in the set-theoretical approach. They had a clear vision for it in school mathematics, rooted in Piaget’s theories. They had the capacity to identify the needs, to introduce and present new ideas, and to persuade and mobilize people to participate in a reform project, by the authority of their position, by their genuine learning and gentle manners and the confidence that Guðmundur Arnlaugsson enjoyed as a media personality.

[1044] Niss, M. (1996): 13
[1045] Helgi Skúli Kjartansson (2002): 158

When Guðmundur Arnlaugsson and Björn Bjarnason were no longer in their influential positions as members of the National Examination Board, heads of the mathematics department in the influential Reykjavík High School and associate professors at the Faculty of Engineering at the University, the emphasis on structure and set-theoretical concepts declined, which was in accordance with the international trend of the time. Their successors and the ordinary primary and lower secondary school teachers, who were to implement their ideas, were not all so enchanted by "unifying concepts," nor were some of the other university mathematicians. Teaching, textbooks and curricula, if available, evolved into more conventional channels, but not at all identical to what had gone before. The reform movement had an irreversible impact on mathematics teaching to come.

Through Guðmundur Arnlaugsson's influence, the dominating and partly monopolistic position of one particular school was abolished with the establishment of Hamrahlíð School with different streams, the abolition of Latin, and the introduction of a modular system. It was a catalyst for establishing a wide choice of options at the upper secondary level in the following years, continuation departments, and multi-stream comprehensive schools. The modular system contributed to the breakdown of barriers between vocational schools and high schools, with its detailed curriculum description.

Guðmundur Arnlaugsson and Björn Bjarnason also contributed to the dissolution of the stagnated national examination of the middle school by a choice of several syllabi in mathematics. They thus introduced reconsideration of its mathematical content, lasting until 1976 when the national examination was abolished and a number of upper secondary level education opportunities had been created.

Within such a small nation, a small group can have great influence, as was the case with Björn Gunnlaugsson, Ólafur Daníelsson with support from Sigurkarl Stefánsson, and Guðmundur Arnlaugsson together with Björn Bjarnason. They were the right people in the right place in the transition processes of their time. If they had opponents, those did not have the authority or knowledge to hinder or even question the process.

The impact of the presence or absence of influential individuals versus official reasons for redefinition of school mathematics in Iceland may be summarized as follows:

- In 1822, when mathematician Björn Gunnlaugsson offered himself to become mathematics teacher at Bessastaðir Learned School, the official reason for his appointment and for enhancing mathematics was to ensure prerequisites for admission to the University of Copenhagen, while Björn Gunnlaugsson brought up utilitarian arguments and cultural aspects of mathematics education (see section 4.2.).
- In 1877, mathematics was no longer required for admission to the University of Copenhagen. No mathematician existed at the Learned School to present cultural or utilitarian arguments for the subject, and the mathematics syllabus was reduced (see section 4.5.).
- When a mathematics stream of the Reykjavík High School was established in 1919 on the urge of the Association of Engineers in Iceland and mathematician Dr. Ólafur Daníelsson, the official reason was to ensure prerequisites for engineering studies, i.e. utilitarian reasons for a rapidly industrializing society. Dr.

Ólafur Daníelsson's arguments for mathematics education were, however, mainly cultural, presenting mathematics as the most perfect science existing (see sections 5.5. and 5.7.).

- In the mid-1960s when "modern" mathematics was implemented as part of the revision of the Icelandic school system, the official arguments were that education would contribute substantially to economic and social progress. The leader of the activities, Guðmundur Arnlaugsson, had ideological arguments in mind, that the new concepts would be conducive to increased clarity and exactness (see section 7.5.).

One may therefore conjecture that a redefinition in mathematical education can take place when both the official body that is to decide upon it and the persons that are to provide the pedagogical leadership have their own vision. They may not be identical but in all cases they may be classified among the fundamental reasons identified by M. Niss (see section 1.4.). The pedagogical leadership of influential individuals is of crucial importance.

Reform or Confusion? – Conclusions

The introduction of "modern" mathematics gave Icelandic teachers a new view on the subject of mathematics. Instead of teaching from the books they themselves had learnt from, they discovered that there were various ways to express and explain mathematical topics, both through their own experience and by studying a variety of books. The reform released a resource of teachers' own creativity and initiative.

Most likely the difficulties and turmoil that followed the introduction of the international "modern" mathematics in Iceland would have been smaller if more teachers had had solid mathematical training and the general public more insight into mathematics. More people might then have had the self-confidence to join a meaningful debate on the reform experiments before they spread to a large part of the population.

Still, the material for the primary level and the upper secondary level had been tested and accepted abroad, and was transferred to Iceland in good faith. Both the increased demand for school education, due to a better economy and changed attitudes to rights to education, and the mathematics reforms were parts of international movements that happened to reach Icelandic schools at a time when the educational system had drifted into stagnation, and society was developing away from basic primary industries toward more complexity. All the attention and funding had gone into building schools, while inner activities, such as curriculum and textbook development, training of teachers etc., had been kept on their pre-war tracks and thus neglected or forgotten. When in 1966 the authorities felt obliged to act, the "modern" mathematics reform movement enjoyed general acceptance, e.g. in the Nordic countries and by the OECD.

Even if Iceland is geographically relatively large, the population is small and homogeneous, and news spread rapidly. New trends and currents either merely pass the country or are more-or-less accepted by all. The need for renovation of curriculum and textbooks had become so strong that many more teachers wanted to join the reform experiment than anyone expected. This created turmoil and confusion.

The positive aspects of the reform attempts were that teachers, who had had no opportunity of further education within the country, received a variety of educational options, and were presented with mathematics with which they had not previously been acquainted. A number of young teachers, both in mathematics and other subjects, had the opportunity to translate, create and test new teaching material. New textbooks were written, revised and rewritten, handbooks for teachers and preliminary national curricula were published, and counselling to teachers was greatly increased.

The adventure of "modern" mathematics changed the scene of Icelandic mathematics education permanently. On the whole, "modern" mathematics, with all the turmoil and confusion it brought, must be considered to be a reform.

10.8. International Similarities and Differences in the Reform

Did "modern" mathematics develop similarly or differently from other countries? Clearly, it was a foreign influence, but was it received in a special way in Iceland? The School Research Department had a unique role in Iceland, and probably "modern" mathematics was not imposed upon pupils in a foreign language in many other places. However, the motivation for its implementation had parallels in many countries.

England[1046]

In England the population is about 170 times that of Iceland. There are a great number of universities, and a number of secondary schools of higher and lower status, influenced by a deep-rooted class division of society. Yet, there are some similarities.

By the 1950s there were two broad traditions in England, of selective and non-selective school mathematics. Two versions of mathematics were taught to two different categories of pupils, largely in different types of schools, by teachers who, broadly speaking, had been educated in two different types of post-school institution: the university and the teacher training college.[1047]

Considerable class stratification existed also in schools at the secondary level in Iceland, between the high schools and the general lower secondary school, both before and after 1946. It was demonstrated by differently rigid syllabi and different requirements for qualifications of their teachers. At the high schools and their entrance examination grade, university education was required and fulfilled if possible, while teacher training college education was more likely to be accepted at the general lower secondary level.

Redefinition of Icelandic school mathematics happened three times, first by Björn Gunnlaugsson, then by Dr. Ólafur Daníelsson and later by Guðmundur Arnlaugsson et al., all situated in the dominating Reykjavík School. As quoted by B. Cooper about the success of the SMP group: "… success was conditioned by both the academic and social status of those involved and by their structurally conditioned access to resources".[1048] Björn Gunnlaugsson, Dr. Ólafur Daníelsson and Guðmundur Arnlaugsson had the structurally conditioned status and academic legitimacy to redefine school mathematics. Guðmundur Arnlaugsson was able to promote the "modern" mathematics in Iceland and ensure the resources, first from the Reykjavík Education Office and consequently from the Ministry of Education through its School Research Department.

[1046] Cooper, B. (1985)
[1047] Cooper, B. (1985): 63
[1048] Cooper, B. (1985): 278

Proposed changes in both countries were legitimized by reference to the nation's need for scientific and technological manpower. There was no pressure in Iceland from any industry, but the common hope was economic gain in line with the OECD's paradigm of (mathematical) education as an economic resource. In both countries, university people had most to say about the content, and after its implementation, university people were among the first to react negatively.

In both countries, problems emerged when the redefinition became established "lower down". The age level 11–13 was a common vulnerable area. In this section there was a clash between the perspectives of the two types of teachers belonging to the two subcultures, trained at universities vs. teacher training colleges, where the former were the initiators and the latter were expected to implement the university version of mathematics. In many cases Icelandic primary school teachers missed the point of the reform, and saw only yet another method in addition to the old ones.[1049]

Even if curriculum change innovations operate at many levels, and those involved are concerned with content, pedagogy and the "attitudes" established, the actual redefinition finally achieved by the actions of those involved in the movements in both countries has probably been primarily one of content. That is, mathematics teachers in both countries remained "transmission" oriented but new content was, in many cases, being transmitted.

However, in both countries, the redefinition was permanent, in the sense that the mathematics syllabi changed, even if the "market" steered the content and the pedagogy partly into more conventional tracks. For example, the weight of probability and statistics, vectors and graphical representation increased.

Norway[1050]

The population of Norway is about 20 times that of Iceland. As in Iceland the population is spread over a large area, and in many areas living conditions are similar. However, as a nation Norwegians were more self-sufficient than Icelanders in the post-war period, in the sense that they had had a university with mathematics research since 1811, several technical colleges and developed industry, such as some mining and technological industry based on good access to electricity.

The similarities between the Icelandic and Norwegian education contexts lay in their centrally organized structure. Textbooks, curricula, law and regulations were, and still are, centrally organized in Iceland as they were in Norway. Also, they had in common the influences from the OEEC and the OECD educational policy towards the implementation of "modern" mathematics education, to some degree via Nordic cooperation.

The differences in the reactions to the foreign educational currents lay in the decision process. In Norway, the implementation of "modern" mathematics underwent a process of experiments, proposals for curricula and discussions about them in boards and committees, in addition to official discussions in parliament and public newspapers.

Compared to Norway, important steps in the implementing process in Iceland were missing. The decision process was underdeveloped, only few persons were knowledgeable, and still fewer were involved in the decision process. The process

[1049] Guðmundur Arnlaugsson (1967): 43
[1050] Gjone, G. (1983)

went from one experimental stage to another, while one might conjecture that that the process in itself created more knowledgeable personnel, which would lead the developmental work of the following decade. The primary school experiment went out of control, and no national curriculum document included the "modern" mathematics until a preliminary one was produced at about the time that the experiment was coming to an end.

Denmark

Comparing the process of implementing "modern" mathematics in Iceland to that in Denmark, the influences are obvious, through the two sets of Danish textbooks introduced, the Bundgaard material and the Kristensen and Rindung series, and Bent Christiansen's teacher training textbooks. Guðmundur Arnlaugsson was in personal contact with the prime promoter of the reform in Denmark, Svend Bundgaard, who pointed out his sister's textbook series for the primary school to him. Both Kristinn Gíslason and Anna Kristjánsdóttir studied at the Royal Danish School of Educational Studies with Bent Christiansen, the author of the goals for the Nordic NKMM project. He turned later away from the orthodox policy, as did Anna Kristjánsdóttir. Moreover, Guðmundur Arnlaugsson closely monitored the 1961 high school reform in Denmark, having been a mathematics teacher there two decades earlier.

However, the context was different, and the similarities are more doubtful. There had been no national movement towards any reform in the field of mathematics education in Iceland in the post-war period, and indeed not since the 1920s. In Denmark a new handbook for teachers, the *Blue Report/Den Blå Betænkning* prepared by school people had recently been published. It was tailored to the Danish system of primary and lower secondary education, but its implementation was disrupted by the "modern" mathematics reform wave. In Iceland there was nothing to disrupt. No one was considered qualified to write teaching material from scratch, and thus people were on the lookout for suitable material to translate. Therefore it was a crucial coincidence that the Bundgaard material was offered, with minimal difficulties in translating and adaptation. It was hardly a choice built on debate or a broad unanimity among a number of people.

Concerning the secondary level, similar problems were dealt with in both countries, more due to changing social settings than by direct influence. In both cases, preparations were made to unify primary and lower secondary education, thus separating lower secondary education from the high school and its immediate influences. In both countries ways and means were established in the 1960s to cope with mounting attendance to the upper secondary level, and for young and adult people to bypass the traditional high school, to acquire higher education and enter vocational training, such as teacher training.

During the first few years, Icelandic classical high schools were "saturated" with "modern" mathematics, as were Danish high schools.[1051] While Guðmundur Arnlaugsson and Björn Bjarnason were the leading mathematics teachers, there were direct Danish influences, but also Anglo-Saxon and Swedish. The orthodox series by Kristensen and Rindung was quickly abandoned for a more moderate approach, except with the most university-bound pupils, and the Danish influences gradually dwindled.

[1051] Niss, M., October 2004

In Denmark there was a long-standing tradition of high schools to build upon. The explosive increase in attendance to the high school level was handled there in a formal way, with legislation, regulations and curriculum documents. In contrast, a series of provisional laws, regulations and curricula were passed in Iceland, and the upper secondary level was legally in an experimental state until 1988, while experience and various influences were shaping it. No Icelandic mathematics textbook series won the market, and translated Norwegian and Swedish textbooks prevailed. A formal policy was not stated, while an increased number of hours for mathematics in 1988, and a reduced number in 1999, exerted their effects.

While the Royal Danish School of Educational Studies / *Lærerhøjskolen* in Copenhagen was influential in implementing "modern" mathematics at the compulsory level in Denmark, the Teacher Training College in Iceland had other issues to deal with, its being used as a bypass to high school. The college played a minimal role in the implementation of "modern" mathematics.

One of the main differences in Iceland from Denmark is the small discussion forum in Iceland, which is so dependent on leading individuals. There is a saying "A man replaces a man," but Icelandic society is very vulnerable in that respect.

The persons who really adhered to the philosophy of "modern" mathematics, Björn Bjarnason and Guðmundur Arnlaugsson, worked hard at teacher in-service courses for only a few years. Compared to Denmark, one could say that in a sense "modern" mathematics in its orthodox form never reached the heart of the Icelandic mathematics education community. Two, three or four persons were preaching, several tried to imitate their ideas, but the majority of ordinary teachers may have considered it as much ado about nothing, cumbersome methods, wordy explanations, and the result a decline in computation skills, disregarding the educational opportunities it brought.

The United States

The reform movement in the United States and its influences on the Nordic reforms was discussed in sections 7.1. and 7.2. The Icelandic mathematical community is not comparable to that in the United States, which has 1,000 inhabitants to one Icelander. University mathematics hardly existed in Iceland, while the majority of the "modern" mathematics experimental projects in the U.S. in the 1950s originated in universities. There were influences from American projects in Iceland, through a high school textbook early in the reform process, and by materials from School Mathematics Study Group for the middle school, and fragments from other projects.

At a national call for improvement in mathematics and science education after the Sputnik Shock in 1957, the mathematical community in the U.S. had an answer already nearly fully developed in the form of "modern" mathematics projects.[1052] In Iceland a similar national call for improvement occurred after a report was published in 1966 on the deficiencies of the lower secondary mathematics and science syllabus. The reaction was similar; funding for improvement of the educational system, and in particular science and mathematics education, became available. The reasons were socio-economic, to ensure economic progress, and nationalistic, so that the nation

[1052] Gjone, G. (1983): Vol. I, 60–61

would in no way be inferior to, and preferably gain some superiority over other nations.

Reactions of parents in the two countries and probably in many other countries were also similar. Parents did not understand the procedures and algorithms their children applied, and worse, they did not observe any improvement in understanding or skills; in fact, many saw only confusion.

The International Reform Movement – Discussion

In comparing the implementation of "modern" mathematics in the above-mentioned five countries, England, Norway, Denmark, the U.S. and Iceland, the conjecture emerges that its original intentions were of egalitarian nature. People from different social classes had fought side-by-side in the World War II, which had contributed to increased communication across social borders. Social-democratic ideology and egalitarianism became widespread in Northern Europe, and the Nordic countries had social-democratic governments for long periods in the post-war era. With an improving national economy there was a growing grass-roots wish for "education for all" which naturally implied "mathematics for all". Furthermore, experiences from the war had created a demand for different content of mathematics to serve an emerging technological society.

These currents, to dissolve social stratification, to improve public education and to improve and alter mathematics education, were developing and amalgamating during the 1940s and the 1950s. They were e.g. realized in the GCE examination for secondary modern schools in England, the HF programme in Denmark and in new nine-year compulsory school legislation in the Nordic countries in the 1960s and 1970s. Nor should the 1968 student uprisings and their social consequences be forgotten in this respect.

That the mathematics reform coincided with the school reforms was therefore only natural. The disturbing elements were the radical ideas of implementing university conceptions of a unification of the various branches of mathematics, through logic and set theory, into school mathematics. These ideas influenced or even disturbed the internal development. In Denmark they disturbed the implementation of a modernized mathematics curriculum, prepared within the school culture. In Norway they went through a long decision process, which finally swayed the national curriculum away from its most orthodox form. In Iceland they caused disturbance, while they also gave rise to reconsideration and new creation during the reactive action. "Modern" mathematics caused, in all the countries in question, conflicts between different cultures within education, on one hand that of the universities, and on the other hand of the teacher training colleges, possibly harsher than in other school subjects in this egalitarian process.

While the borders between these cultures may not yet be fully dissolved, the events in 1960s and 1970s contributed to a dialogue and communication flow. And in Iceland, where school mathematics had not had any attention since the 1920s, the implementation of "modern" mathematics in a context of a meeting of different educational currents, however unfortunate in many respects, contributed to the creation of a long-needed channel for initiative and creativity on the part of the teachers belonging to both cultures.

Comparisons to Other Countries after the Reform Period

There were many similarities in the implementation of "modern" mathematics in Iceland and in other countries. Encouragement by the OECD, economic expectations, enthusiasm on the part of some university mathematicians, and later disappointments, and reactions from parents and the public when implemented at lower levels, were all common features.

As in other countries, Icelandic mathematics teaching evolved into more conventional channels after the reform period. The question of whether Iceland had "caught up" with other countries remained unanswered for years. The fact that only 30% of upper secondary level mathematics teachers had the required training in 1959, and that in 1987 38% had had two years' university studies or more in mathematics, does not indicate adjustment to international standards. Recalling that a B.Sc. programme in mathematics and science at the University of Iceland was established only in 1969, and that the great expansion period of upper secondary schools lasted into the 1980s, one should consider the natural time-lag in education, mentioned in an earlier citation from Dr. Klaus Bahr. In 2004, 46% of upper secondary level teachers have a B.Sc. degree qualification or more in mathematics: a major improvement, which indeed confirms the time-lag.

When the 1995 TIMSS results were revealed in 1996, it was generally acknowledged in Iceland that Icelandic pupils were one year behind their Nordic contemporaries. The survey was considered by many to be a sign of the incompetence of the teachers, regardless of their working and salary conditions. During the following decade various improvements to the school system were made: new national curriculum documents, increased number of school hours, and abandoning of the custom of double-booking of schoolrooms, in addition to initiation of new mathematics textbooks, whose effects will not be felt until later.

Satisfactory results in the mathematics part of the 2003 PISA study may have been a surprise to the public and the authorities, as people had become accustomed to, even if unsatisfied with, being inferior in school mathematics and natural sciences. The results indicate that the Icelandic educational system has achieved an even footing with other European countries, the Nordic countries in particular, and will henceforth deal with educational questions parallel to those in other countries.

Compared to the primary and lower secondary level curriculum up to the 1960s – the four operations with whole numbers and fractions, area and volume computations, percentages, *regula de tri* for ordinary pupils and algebra for the elite – the mathematics content is now varied; balanced between arithmetic, algebra, geometry, probability and statistics and proportions, with an even emphasis on processes: use of language, problem solving, reasoning and applications, a structure that has some similarities to the TIMSS definition of curriculum.[1053] Theoretically, similar emphasis should be laid on the role of processes on the upper secondary level. In practice the emphasis has been on content, under time pressure, revision of compulsory school topics as well as presentation of new ones, such as functions and calculus, in the context of decreasing number of hours and pressure from the authorities "to move topics down" to compulsory school.

[1053] Robitaille, D.F. et al. (1993)

10.9. Discussion on the Research Question

In the foregoing sections the main features of mathematical education in Iceland through the centuries have been considered. Its similarities and differences compared to that of other countries have been highlighted, as well as the main characteristics of Icelandic society at each period of time and its perceived need for mathematical education. Particular attention has been paid to the period during and after the middle of the 20th century with respect to the international mathematics reform movement and its implementation in Iceland.

It is now appropriate to return to the research question:

To what extent has mathematics education developed similarly or differently in Iceland from that in other northern European countries, and what explanations can be offered for this?

Icelandic society has always belonged to the European cultural sphere and attempted to adopt cultural novelties. Humanism and the Enlightenment brought the Icelanders in touch with European mathematics, and their proponents initiated mathematical activities. However, Icelandic society did not keep up with its neighbouring countries in mathematical learning. Without urban centres and due to late industrialization, the most important site of higher learning was Copenhagen, hundreds of miles away. The meagre mathematics introduced was legitimized by use in trade, which, however, was minimal. Land surveying was the only field for advanced mathematics practiced by the Icelanders.

It was not until the late 19th century, when roads, bridges and durable buildings began to be built, that the need arose for more advanced mathematical computations. This task was initially provided by Danish and Norwegian engineers, and only gradually did it become domestically available. Paradoxically it was decided a few years earlier that the only learned school were exclusively to offer a language stream, while a mathematics-natural sciences stream had been established in Denmark. During decades of rapid technological progress from the 1890s until the 1910s, higher mathematics education was unavailable in Iceland. Other subjects than mathematics had a higher priority. The most advanced educational institutions, the Reykjavík School and the University, seemingly did not participate in this progressive development.

A need for furthering higher mathematical education was not perceived in Iceland. Mathematicians Björn Gunnlaugsson in the 19th century and Ólafur Daníelsson in the early 20th tried to persuade their fellow countrymen that mathematics would strengthen logical thinking. Their belief was not widely shared, and composing verses and practising Latin declensions were considered more suitable for that purpose. Future professionals were to be provided with the necessary prerequisites for university studies. Higher mathematics was seldom practised for other purposes.

Arithmetic textbooks for primary and lower secondary education produced from the 1890s until the 1910s were not connected to the programme of the Learned School. They are rather a testimony to a grass-root need for elementary mathematical knowledge, which was to a large degree met by non-mathematicians. In 1919 a mathematics stream was established at the Reykjavík High School, in order to enable students to acquire higher mathematical-technical knowledge abroad.

The establishment of the mathematics stream entailed that the Reykjavík High School dominated mathematical education nationwide, at all school levels. This structure and the establishment of the monopolistic State Textbook Imprint led to declining vitality of mathematics textbooks and absence of other material, and may have contributed to a perception of mathematics as a collection of incomprehensible rules.

Several factors made the Icelandic school system unique and distinguished it from other countries, from the 1920s until the 1960s. There were vestiges of the two-century-old tradition of home-education; there was the dominating high school; there was the sole Teacher Training College, which did not support mathematics education and was not supplemented by continuing education; and finally there was the fact that university mathematics did not exist until 1940, and only on a very small scale until the 1970s. The tradition of home- and self-education achieved general literacy, while elementary mathematical literacy was in many respects beyond the reach of the Icelandic educational system.

Up to the 1970s, training and qualifications of mathematics teachers were inferior to that of the countries with which Iceland is compared. The gradually shrinking mathematics education in the Teacher Training College, the absence of further education for teacher training graduates, and the late and inadequate mathematics teacher training programme on university level had negative consequences. The high schools became training centres for secondary mathematics teachers, a situation unknown in the comparison countries. The 1959 OECD survey reveals that nowhere in Northern Europe was there a comparable shortage of mathematics teachers.

Icelanders' traditional centre for specialized education remained in Denmark for a long time after political independence in 1944. Considered as a "stand-alone" society, Iceland lagged behind other countries in the first two thirds of the 20th century in the context of mathematics education and mathematics teacher training. Regarded as a part of the Danish educational system, the situation might be considered differently, such as that mathematics education eventually developed in line with rural and remote provinces of Denmark or Norway. As the Icelanders were a separate cultural unit with own language, they had to supply officials and professionals themselves. While there was a flow of trained professionals between the various provinces in Denmark and Norway, the flow was mainly one-sided from Iceland to Denmark, with exception of some technicians and engineers. The Icelandic language resulted in a barrier, as well as it contributed to Iceland's cultural independence. Qualified teachers could not be foreigners. What was available had to be accepted and was in many respects below the standards of the countries with which Iceland is compared.

The reasons for the late development of higher mathematics education in Iceland as compared to developed countries were primarily economic, as industry did not require specific mathematical skills from more than a limited number of professional engineers and technicians. This is linked to the late industrial development of Iceland. In the first two-thirds of the 20th century it was a rapidly developing country. However, at the mid-century Icelanders considered themselves better off than most other nations in the world, and not having anything in common with colonial subjects. Mathematics was hardly seen as a missing component of its culture.

The “modern” mathematics reform movement hit Iceland at a time of domestic crisis in education. The crisis resulted from a rapid transformation of society from an agricultural tradition to industrialization and an internal disquiet associated with urbanization. In addition there were economic adolescence problems of a newly independent nation. Economists trained in mathematics might have improved the government of the new republic’s economic affairs, but other political and cultural aspects prevailed.

Due to the influence of the OECD in the 1960s, it was widely believed that a progressive general mathematical education would transform Iceland into a prosperous society. The OECD paradigm had similar influence in neighbouring countries, but hardly in any of them was society so devoid of mathematical education in the Western traditional sense, where e.g. any precision industry was lacking.

Several characteristics were common to the countries with which Iceland is compared in the implementation process of “modern” mathematics. There was an enormous increase in resources available, believed to provide economic advantages for industry and society. Those who supplied the resources may have had applied mathematics in mind, while the prospective project leaders of the reform projects stressed the advantages of “modern” abstract ideas being taught in a child-centred, practical manner. When skills deteriorated and improvement in understanding was not detected, resources declined, e.g. in England and the United States. In Iceland resources were channelled through the OECD-promoted School Research Department, whose activities eventually comprised all school subjects. The fact that the department enjoyed considerable support after the “modern” mathematics reform wave had peaked, indicates that in spite of the disappointments of the first primary school projects, the resources were not withdrawn, but were available to new school mathematics projects.

Another characteristic in common was a political vision of an egalitarian nature. The trend of the post-WWII period was removal of stratification, stressing general education, including mathematics education for all. In the countries with which Iceland is compared, certain stratification persisted in the educational system, between the elite and ordinary people. The two educational paths of teachers aimed at university and the teacher training colleges. The introduction of “modern” mathematics confronted mathematics teachers from the two subcultures with each other, each interpreting and implementing school mathematics according to their own education and professional experience. The implementation of “modern” mathematics into compulsory education, on behalf of the elite subculture, meant that the ultimate change was in content, rather than in pedagogy or understanding mathematics as an integrated structure, unified by set theory. However, in Iceland the meeting of the subcultures contributed to a dialogue and increased communication across their border, even though the subcultures have continued to exist.

The reform in Iceland signalled a break from a long-lasting situation characterized by stagnation. The challenge to cope with “modern” mathematics stimulated teachers’ creativity and initiative, and became a long-needed opportunity for further education. Contrary to the countries with which Iceland is compared, e.g. Denmark, the introduction of “modern” mathematics in Iceland became a positive reform, in spite of the unrest it caused. No alternative plans had been developed, so the reform movement stimulated discussions about meeting a long-overdue need to renew curricula and syllabi.

The “modern” mathematics reform alerted Icelanders to the existence and importance of mathematics for technological and economic progress and its cultural value. The reform catapulted Icelandic mathematical education into the global community.

A combination of improved general education, increase in mathematical education and economic diversification has characterized Icelandic society since the 1960s. Recent international comparative studies indicate that Icelandic adolescent pupils are on even level with pupils in the neighbouring countries in mathematics and science. The “modern” mathematics reform was a crucial event in this process.

10.10 Reliability of Conclusions

A number of facts has been collected from various sources in order to compose this study. The sources have been cited as honestly as possible. Distinguished scholars have been consulted about the historical framework within which the history of mathematics education is told. Hundreds of reliable sources have been consulted. To take an example of sources, all printed mathematics textbooks in Icelandic have been duly registered in libraries and most of them were explored. They witness what was studied and what was not accessible, at least for the majority who only read Icelandic. Legislations and regulations contain indisputable facts about what was decided and what was excluded. Many persons, who were involved in the events discussed or were knowledgeable about them, have been interviewed. Consequently the facts cited in the thesis are fairly reliable and the conclusions drawn seem plausible.

However, work on sources has its reservations. The testimony of my interviewees must be understood in the light of their position in relation to the events when they occurred and the time passed since then. Furthermore, certain sources have been chosen, while others have been considered irrelevant. In some cases, those have later been found to be valuable. That fact leads to the suspicion that something else, which has been disregarded, might include good contributions as well. Yet a full stop must be put somewhere. Contrary to theoretical mathematics, an infinite number of sources are beyond human capacity.

This thesis has developed into a social study. It has been necessary to make an attempt to analyze the development of mathematics education within the framework of education policy and the characteristics of Icelandic society at each particular time. My education lies, however, in the field of mathematics and my working experience within school organization, both away from the experience of sociological and economics studies and knowledge of their theories and vocabulary. I have therefore had to keep within the framework of what recognised scholars have published, relevant to the content of my study. The main references have been to historians who have analyzed the history of Icelandic society in a modern way, different from the romantic and nationalistic views which dominated the mid-20th century when I attended school. It certainly has been a worthy effort to adopt that more realistic attitude. If the reader senses a nationalistic bias in my work, it may be ascribed to my background.

Having mentioned these reservations, it must also be stated that this study could not be undertaken without a mathematical background. Mathematics has it own vocabulary and concepts, which can only to a limited degree be investigated by a non-mathematician. It has been a pleasure to combine my mathematical knowledge and historical interests to compose this book. It is my sincere hope that this survey of mathematical components in Icelandic culture may become of use for the international as well as the national research community.

10.11. Future Research Projects

The following topics, related to this thesis, deserve scholarly research and elaboration:

The *Algorismus* treatise. A comparison to *Carmen de Algorismo* and Sacrobosco's works.

The three *Arithmetica* manuscripts from the early 18th century and their connection to European literature.

The Enlightenment arithmetic textbooks by Ólafur Olavius and Ólafur Stefánsson.

The complete works of Björn Gunnlaugsson.

Grímur Thomsen's influence on school affairs.

A study of tradition in arithmetic textbooks from *Algorismus* to Ólafur Daníelsson.

Comparison of Icelandic 18th and 19th-century arithmetic textbooks to foreign textbooks, such as *Greinileg vegleiðsla* by Ólafur Olavius to his Danish and German sources, and Eiríkur Briem's textbook to V. Bertelsen's book.

A study of arithmetic and mathematics textbooks through history, comparing them to contemporary international educational trends.

Why do many teachers prefer skill-training approach with a number of exercises to discussions and cogitation? What kind of approach provides the most useful tools for future adult life as a citizen and for further studies, lasting understanding and the most positive attitude to mathematics, and how can teachers adopt and assimilate that approach?

The PISA results, such as their correlation to the education and background of teachers, size of schools and other social context, the relative absence of excellent performance and the lower average performance of boys than girls.

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Andri Ísaksson, former director of School Research Department, March 10, 2003. † August 7, 2005.

Arnlaugur Guðmundsson, son of Guðmundur Arnlaugsson, March 16, 2003.

Árni Stefánsson, former chairman of National Examination Board, May 18, 2005. † April 16, 2006

Baldur Sigurðsson, Associate Professor at Iceland University of Education, e-mail September 16, 2004.

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Guðmundur P. Sigmundsson, former mathematics teacher in Bolungarvík Lower Secondary School, January 26, 2005. † August 25, 2006

Guðný Helga Gunnarsdóttir, Assistant Professor at Iceland University of Education, May 18, 2006.

Gunnar Ragnarsson, former headmaster at Bolungarvík Lower Secondary School, December 2003.

Gylfi Þ. Gíslason, former Minister of Education, telephone conversation, January 17, 2002. † August 18, 2004.

Haraldur Steinþórsson, high schools mathematics teacher, January 22, 2003. † August 16, 2005.

Helgi Skúli Kjartansson, history professor, Iceland University of Education, e-mails, November and December 2004.

Hildigunnur Halldórsdóttir, mathematics teacher in Hamrahlíð High School, September 10, 2002, e-mail September 2, 2003.

Hörður Lárusson, head of department, Ministry of Education, former director of School Research Department. March 26, 2002, January 10, 2003.

Hörður Zóphaníasson, ex-headmaster in a primary school, primary textbook author, June 2, 2003.

Ingvar Ásmundsson, former mathematics teacher and ex-headmaster, March 2003.

Jónas B. Jónsson, former Director of Reykjavík Education Office, March 24, 2002. † April 1, 2005.

Jón R. Hjálmarsson, former headmaster in Skógar District School, November 2002.

Jón Hafsteinn Jónsson, former mathematics teacher at Akureyri High School, May 25 and August 21, 2003.

Jón Ragnar Stefánsson, associate mathematics professor at University of Iceland, July 15, 2003.

Kristinn Gíslason, former primary school teacher, March 3, 2003.

Kristín Kaaber, daughter of Dr. Ólafur Daníelsson, September 28, 2004.

Kristján Bersi Ólafsson, ex-headmaster in Flensborgarskólinn, October 2002.

Magnús Magnússon, physics professor emeritus, November 21, 2003.

Niss, Mogens, professor at Roskilde University Center, Denmark, September 2001– October 2005.

Njörður P. Njarðvík, former chairman of National Examination Board, e-mail, January 17, 2003.

Ólafur Ásgeirsson, National Archivist, former headmaster at Fjölbrautaskóli Vesturlands, Akranesi, November 8, 2001.

Ólafur Proppé, rector of Iceland University of Education, January 10, 2003.

Pálmi Jóhannesson, adm. manager, Department of Engineering, University of Iceland, September 22, 2005.

Ragnhildur Bjarnadóttir, primary textbook author, psychology professor, Iceland University of Education, September 16, 2003.

Skarphéðinn Pálmason, mathematics teacher in the Reykjavík High School 1963–2003, March 23, 2003.

Stefán Árnason, mathematics teacher, Fjölbrautaskólinn í Garðabæ, January 28, 2003.

Svanhildur Kaaber, granddaughter of Dr. Ólafur Daníelsson, e-mail, September 22, 2005.

Sveinbjörn Björnsson, physicist, former rector of the University of Iceland, November 5, 2004.

Vilborg Dagbjartsdóttir, primary teacher, poet, February 12, 2002.

Vilhjálmur Bjarnason, son of Bjarni Vilhjálmsson, chairman of the National Examination Board, January 2003.

Þorsteinn Þorsteinsson, headmaster in Fjölbrautaskólinn í Garðabæ and former chairman of the Icelandic Chess Federation, March 27, 2003, e-mail, June 12, 2003.

Þuríður Kristjánsdóttir, professor emeritus, Iceland University of Education, July 30, 2005.

Primary teachers: Arnfinnur U. Jónsson (July 13, 2005), Dagný Albertsson (June 2003), Ragnhildur Bjarnadóttir, Kristinn Gíslason, Sigrún Jóhannesdóttir (July 2003), Vilborg Dagbjartsdóttir.

Appendices

The appendices contain information which the author has collected as bases for facts, assumptions and conclusions. More detailed information is to be found in their primary sources referred to here or in the main text.

Appendix A – Mathematics Textbooks in 1846–1856

In 1846–1847 Björn Gunnlaugsson taught mathematics in three grades. In the first grade there was practical arithmetic as before, using Ursin's *Arithmetik* / *Arithmetic* and Ursin's *Mathematik / Mathematics*. In second grade he taught geometry using Ursin's *Mathematics*. In third grade he taught Svenningsen's *Geometri* / *Geometry*, of which he complained that it was much more complex than Ursin's book.[1054]

In 1847–1848, there were already four grades, 1, 2, 3a and 3b. Björn Gunnlaugsson taught Ursin's books in 1, 2 and 3a, while he repeated Svenningsen's book in 3b, and began teaching trigonometry as a support for natural science, i.e. physics, which he taught using Örsted's textbook. In 1848–1849, Ursin's *Arithmetic* and *Mathematics* were taught as before. Svenningsen's book was not mentioned anymore in reports, while Ursin's *Plantrigonometri / Plane Trigonometry* appeared.[1055]

In 1851–1852 Ursin's *Arithmetic* was exchanged for Fallesen's *Arithmetik* / *Arithmetic* in the first grade, and geometry from Ursin's *Mathematic*s in second grade exchanged for Ramus' *Geometri* / *Geometry*. Ursin's *Geometry, Arithmetic, Plane Geometry* and number theory from Ursin's *Mathematics* were taught in the third and fourth grades. The next year, 1852–1853, Fallesen's book was replaced by Ursin's *Arithmetic* in the first grade once more, while other grades went on as previous year. In 1853–1854, algebra is reported taught in second grade by Assen's number theory. From 1855 the two third grades have mathematics only partly in common, two out of four weekly hours. Ramus' *Geometry* appeared again in grade 3a in 1855–1856.[1056]

[1054] *Skólaskýrsla fyrir Reykjavíkur lærða Skóla árið 1846– 47*: 11
[1055] *Skólaskýrsla fyrir Reykjavíkur lærða Skóla árið 1847–48*: 12 – 13; *1848–1849*: 9–10
[1056] *Skýrsla um hinn lærða skóla í Reykjavík skóla-árið 1851–56* (1853)

Appendix B – Details from Letters 1876–1882

Details from letters concerning the proposal for regulations for the Reykjavík Learned School, 1876–1882.

Minister Nellemann forwarded Governor Hilmar Finsen's proposals to King Christian IX with a letter, dated July 10 1877, where he said:

> Hvad ... angaar det af Kommissionen udarbejdede Regulativ for Undervisningen i den lærde Skole har Ministeriet i det Væsenlige kunnet tiltræde samme, idet man dog, tildels i Overensstemmelse med hvad Landshövdingen har ytret, har anset adskillige Ændringer for nødvendige, af hvilke jeg fornemmelig skal tillade mig at fremhæve fölgende, der ere af en mere indgribende Betydning:
>
> - at der undervises i Dansk igjennem alle Klasser idet dette Sprog maa anses for det nærmeste af de fremmede Sprog, og det er af den største Vigtighed, at det læres grundigt, da det er Forretningssprog for mange af de islandske Embedsmænd;
> - at Religion læres igjennem alle Klasser, ... ;
> - at Naturlære ogsaa læres i øverste Klasse i Forbindelse med Astronomi.
>
> Optagelsen af disse 3 Fag som Gjenstand for Undervisningen i överste Klasse, medens Kommissionen har foreslaaet dem afsluttede med Udgangen af 4 Klasse, vil efter Ministeriets Mening ikke kunne medføre nogen Overlastelse for Disciplene, da det paa den anden Side maa anses for rettest
>
> - at Undervisningen i Mathematik i Stedet for at meddeles igjennem alle Klasser, afsluttes med 4 Klasse.
>
> Fremdeles foreslaas fölgende Ændringer i Kommissionens Forslag:
>
> - at Tydsk gjöres til Gjenstand for tvungen Undervisning i överste Klasse, medens Kommissionen overlader det til Disciplene, om de ville deltage i Undervisningen i dette Sprog;
> - at Øvelserne i latinsk Stil fortsættes i 4 Klasse i Stedet for at slutte med 3 Klasse ...;
> - at Skrivning optages som Fag i de laveste Klasser;
> - ...
> - at der som Fag ved Afgangspröven i Henhold til det Foranstaaende optages Dansk, Tydsk, Religion og Naturlære med Astronomi, medens Geometri og Arithmetik udstedes;[1057]
>
> What … concerns the regulations for the instruction in the Learned School, worked out by the commission, the Ministry has, in the main items been able to agree with the same, as one though, partly in agreement with what the Governor has expressed, has considered various alterations necessary, from which I first and foremost will allow myself to emphasize the following which are of high importance:
>
> - that there will be instruction in Danish through all the grades as that language must be considered the closest foreign language, and it is of greatest importance that it is studied thoroughly, as it is a working language for many of the Icelandic officials;
> - that religion (exegetic) is studied through all grades;
> - that nature sciences will be studied in the uppermost grade in connection to astronomy.

[1057] National Archives of Iceland: Skjalasafn landshöfðingja, 621

Taking up these three subjects as objects for instruction in the uppermost grade, while the commission has proposed them completed by the end of the fourth grade, will by the understanding of the ministry not bring any overload to the pupils, as on the other side it must be considered most correct

- that the instruction in mathematics instead of being informed through all grades will be completed by the end of the fourth grade.

Furthermore the following alterations to the commission's proposal are suggested

- that German will be object to compulsory instruction in the uppermost grade while the commission leaves it to the pupils whether they choose to participate in the instruction of that language;
- that the exercises in Latin writing will continue in the fourth grade instead of terminating in the third grade ...;
- That writing will be taken up as a subject in the lowest grades;
- ...
- That there as subjects to the final examination, in agreement with the above, Danish, German, exegetics and nature sciences together with astronomy will be taken up, while geometry and arithmetic will be left out;

A letter from the teachers at the Learned School, dated November 20, 1882, contained proposals to alterations of the 1877 regulation:

Hinar verulegustu breytingar á skiptingu námsgreinanna, sem vjer höfum stungið upp á, eru þær, að stærðfræði verði látin halda áfram í gegn um allan skólann, og þarnæst að þýzka verði kennd 4 fyrstu árin í staðinn fyrir 2 hin síðustu, en að franska aptur á móti verði að eins kennd 2 síðustu árin. Það eru þær breytingar, sem kennararnir fyrir sitt leyti leggja mesta áherzlu á. Að því er snertir stærðfræðina, þá eru kennararnir á því, að það sje mjög svo óheppilegt, að skólapiltar, eins og nú stendur, eigi fá neina kennslu í hinni almennu flatþríhyrningafræði nje heldur neina vísindalega kennslu í þykkvamálsfræði og bera til þess ýmsar ástæður. Fyrst er það, að vjer álítum menntun þá í stærðfræði, sem skólinn veitir, vera ónóga í sjálfu sjer, ef þessar greinir hennar eru eigi kenndar og skulum vjer sjerstaklega taka fram, að flatþríhyrningafræðin styður mjög kennsluna í eðlisfræði og stjörnufræði og að hún bindur enda á og fullkomnar stærðfræðiskennsluna í skólanum, svo að þar við fæst nauðsynlegur undirbúningur fyrir hvern þann, sem vill halda áfram stærðfræðisnáminu við æðri menntastofnun. Þar næst eru hinar umræddu greinir stærðfræðinnar að áliti kennaranna mjög svo þýðingarmiklar fyrir hið verklega líf, enda hyggjum vjer þeim mun meiri ástæðu til bera, að kenna þær í hinum lærða skóla, sem þær ekki eru kenndar í neinum öðrum skóla hjer á landi nú sem stendur, og landsmenn þannig ekki eiga neinn kost á að afla sjer þekkingar í þeim nema með sjálfskennslu. En til þess, að skólinn geti veitt kennslu í almennri flatþríhyrningafræði og þykkvamálsfræði, er nauðsynlegt, að láta stærðfræðiskennsluna halda áfram í gegnum allan skólann.

... bæði danska og trúarbrögð gæti að skaðlausu sjeð af þeim tíma, sem til þeirra gengur í 2 hinum efstu bekkjum. Eptir hinni eldri reglugjörð (§4,2) var danska að eins kennd 4 fyrstu skólaárin, og bar þó ekki á öðru, en að stúdentarnir væri fullfærir bæði að skilja dönsku og gjöra sig skiljanlega. Reynslan hefur því sýnt, að óhætt muni, að láta dönskukennsluna hætta við lok hins 4. skólaárs eins og vjer höfum lagt til.

The most important alterations of the division between the subjects, which we have suggested, are those that mathematics will continue throughout the school, and secondly that German will be taught for the first four years instead of the last two, while French on the other hand would only be taught the last two years. These are the alterations, which the teachers most strongly emphasize. Concerning mathematics, the teachers consider it very unfortunate, that the pupils, as the situation is now, do not receive any instruction in the general plane trigonometry or any scientific instruction in stereometry, and that is for several reasons. Firstly, our opinion is, that the mathematics education, which the school offers, is insufficient in itself, if these topics are not taught, and we shall especially make it clear that plane trigonometry very much supports the instruction of physics and astronomy and that it concludes and perfects the mathematics instruction in the school, so that thereby is acquired the necessary preparation for every person who wants to continue mathematics studies at a higher educational institution. Secondly these topics are, by the opinion of the teachers, very important for technical life, and we think that there is the more reason to teach them in the learned school, as they currently are not taught in any other school in this country, so our countrymen thus do not have any choice to acquire knowledge in them except by self-instruction. However, in order that the school can offer instruction in general plane trigonometry and stereometry, it is necessary that the mathematics instruction will continue throughout the school. …

Both Danish and religion could, without any harm, do without the time allocated for them in the two uppermost classes. By the older regulation (§4,2) Danish was only taught through the 4 first school-years, and yet nothing else was noticed than that the students were fully capable of both understanding Danish and making themselves understood. Experience has thus shown that that there is no harm done to let the Danish instruction conclude by the end of the fourth school year, as we have suggested.[1058]

Extracts from Jón Þorkelsson's letter, attached to the teachers' letter:

1. Kennararnir álíta, að það kvantum í stærðfræði, sem reglugjörðin ákveðr að læra skuli, sé ónógt, og vilja því bæta því við, er á þykir vanta, og til þess að nógr tími fáist til að nema það, fara þeir fram á, að stærðfræðiskenslan gangi gegnum allan skólann, einnig í gegnum 5. og 6. bekk. Eg skal með tilliti til þessa leyfa mér að taka fram, að með reglugjörðinni 1877 vóru innleiddar fjórar nýjar námsgreinir sem skyldugreinir (obligatoriske Fag), nefnilega, skrift, teiknun, enska, frakkneska, (enska og frakkneska vóru áðr kjörfrjálsar námsgreinir). Af þessu leiddi, að nauðsynlegt varð að takmarka kensluna í stærðfræði. Það kvantum, sem nú er heimtað eftir reglugjörðinni, mun vera nálega hið sama, sem kent er í dönskum skólum <u>í hinni mállegu og sögulegu deild</u> (den sproglig-historiske Afdeling), sem ávalt er miklu fjölmennari enn hin <u>stærð- og náttúrufræðiliga</u> (den mathematisk-naturvidenskabelige Afdeling). Þetta kvantum er að minni ætlan þeim nægilegt, sem eigi ætla að ganga á polytechniska skólann í Kaupmannahöfn. Þeir Íslendingar, sem hingað til hafa á hann gengið, eru mjög fáir, og ef ályktað er af tölu þeirra, má gera ráð fyrir, að varla muni meira enn einn Íslendingr ganga á polytechniska skólann á hverjum tíu árum. Þeir hinir fáu, sem á hann gengi, yrði að útvega sér aukakenslu í stærðfræði.

   Ef sú stundatafla, sem nú gengr til kenslu í stærðfræði, yrði aukin um 6 stundir (frá 19 stundum til 25) þá hlyti það að mestu leyti að verða á kostnað málanna; enn eg fyrir mitt leyti legg mesta áherzlu á kensluna í þeim; enn því færri stundir sem þeim eru ætlaðar, því ófullkomnari verðr kenslan í þeim. Eg get því eigi lagt til, að kenslan í stærðfræðinni verði aukin frá því sem nú er.

2. Kennararnir fara fram á, að danska gangi eigi í gegnum allan skólann heldr hætti kenslan í henni við árspróf í 4. bekk. Mín skoðun er, að eitt af hinum lifandi málum, fyrir utan móðurmálið, eigi að kenna svo vel, að stúdentar geti talað það og ritað nokkurn veginn vel. Ef nú litið er á það samband, sem Ísland stendr í við Danmörku, og sér í lagi á það samband, sem hinn lærði skóli stendr í við háskólann í Kaupmannahöfn, þar sem kenslan fer fram á dönsku, þá getr varla verið nokkur efi á því, að danska sé það af hinum útlendu nýju málum, sem eigi að kennast bezt við hinn lærða skóla, og svo vel, að Íslendingar geti haft

[1058] National Archives of Iceland: Íslenska stjórnardeildin. Skólamál. Isl. Journal nr. 680

fullkomin not af kenslunni við háskólann, jafnskjótt sem þeir koma til hans. Af þessum ástæðum þykir mér óráðlegt að takmarka kensluna í dönsku með því að ætla henni færri kenslustundir enn hún hefir nú. Mér virðist því eigi ráðlegt, að taka dönskukensluna burt úr 5. og 6. bekk.

1. The teachers consider that the quantity in mathematics, which the regulation decides to be studied, is insufficient and want to add what they believe is lacking, and in order to that sufficient time will be gained to study that, they request, that the mathematics teaching continue through all the school, also fifth and sixth grade. I shall, with respect to this, allow myself to draw attention to, that by the 1877 regulation four new compulsory subjects were introduced, namely writing, drawing, English, French (English and French were previously electives). This meant that it became necessary to limit the teaching of mathematics. The quantity that now is required by the regulations is supposed to be nearly the same as is taught in Danish schools in the language and history stream, which is always more numerous than the mathematics and nature science stream. This quantity is, by my opinion, sufficient for those who do not intend to enter the Polytechnic College in Copenhagen. Those Icelanders, who have hitherto attended it, are very few, and if one concludes from their number one can expect that there hardly will be more than one Icelander every tenth year attending the Polytechnic College. The few that would attend it would have to find themselves private instruction in mathematics.

   If the timetable, which now is allotted to mathematics teaching, would be increased by 6 hours (from 19 hours to 25 hours), then it would have to be mainly at the expense of the languages; while I, for my part, put the greatest emphasis on them; as the fewer hours are allotted to them, the more imperfect their instruction will be. I therefore cannot suggest that the instruction in mathematics be increased from the present situation.

2. The teachers suggest that Danish were not to be taught throughout the school but cease after the fourth year's examination. In my opinion, one of the living languages, besides the mother tongue, should be taught so well that the students can speak it and write reasonably well. If one now looks at the relation which Iceland has to Denmark, and especially the relation, which the learned school has to the University in Copenhagen, where the instruction is in Danish, then there can hardly be any doubt that Danish is the one of the foreign languages which should be best taught at the learned school, and so well that the Icelanders can have complete benefit from the instruction at the University, as soon as they arrive there. For these reasons, I consider it unwise to reduce the instruction in Danish by allotting to it fewer hours than it presently has. I therefore do not regard it as advisable to remove the Danish teaching from the fifth and sixth grade.[1059]

[1059] National Archives of Iceland: Íslenska stjórnardeildin. Skólamál. Isl. Journal nr. 680

Appendix C – The 1943 School Affairs Board

On June 30th 1943 the Minister of Education appointed the following persons to take seats on the board, which later was called the School Affairs Board:

Jakob Kristinsson Cand. Theol., Director of Educational Affairs.

Mrs. Aðalbjörg Sigurðardóttir, former teacher, educated at Flensborg Teacher Training College, authority in pedagogical matters.

Ármann Halldórsson, headmaster at Reykjavík Primary School, Mag. Art. at University of Oslo in psychology, biochemistry and history of philosophy. Considered to have been the main ideologue of the group.[1060]

Ásmundur Guðmundsson, theology professor, later bishop.

Ingimar Jónsson, Cand. Theol., headmaster at *Gagnfræðaskóli Reykjavíkur* lower secondary school.

Kristinn Ármannsson, Cand. Mag., Latin teacher at Reykjavík High School, later headmaster.

Sigfús Sigurhjartarson Cand. Theol., member of *Alþingi*, former lower secondary school teacher and an editor of *Þjóðviljinn*, the socialist newspaper.

Later in 1943 Helgi Elíasson, a primary teacher with further education at Danish and German universities, the new Director of Educational Affairs, replaced Jakob Kristinsson, his predecessor.

[1060] Árni Stefánsson, May 18, 2005

Appendix D – National Middle School Examination

Regulations no. 51, April 14 1947

I. Almenn ákvæði.

1. gr. Miðskólapróf í bóknámsdeild skal vera landspróf í þessum greinum: íslenzku, dönsku eða öðru Norðurlandamáli, ensku, sögu, landafræði, náttúrufræði, stærðfræði og eðlisfræði. Þó getur menntamálaráðuneytið fjölgað eða fækkað landsprófsgreinum, ef fræðslumálastjóri og landsprófsnefnd mæla með því. Landspróf skal vera skriflegt að mestu leyti og haldið samtímis um allt land.

...

3. gr. Um prófkröfur fer eftir gildandi lögum og fyrirmælum á hverju tíma.

II. Prófnefnd og prófdómarar.

4. gr. Menntamálaráðuneytið skipar 9 manna prófnefnd til allt að fjögurra ára í senn. Felur ráðuneytið einum þeirra formannsstörf eða skipar sérstakan formann, sem þá skal eigi hafa með höndum prófdómarastörf í nefndinni. Formaður annast framkvæmdastjórn fyrir nefndina. Í prófnefndinni skulu vera sérfróðir menn í þeim greinum, sem landsprófið nær til, og vel kunnugir kennslu og námsefni því, sem krafizt er til prófsins. ...

5. gr. Landsprófsnefnd gegnir prófdómarastörfum, að svo miklu leyti sem því verður við komið. Að öðru leyti skipar ráðuneytið prófdómara að fengnum tillögum fræðslumálastjóra og landsprófsnefndar. ...

I. General Provisions

Art. 1. The Middle school examination in the academic department shall be a national examination in the following subjects: Icelandic, Danish or another Nordic language, English, history, geography, natural science, mathematics and physics. However, the Ministry of Education can increase or decrease the number of subjects if the Director of Educational Affairs and the National Examination Board so recommend. The national examination shall be written for the most part and held at the same tame all over the country.

...

Art. 3. The requirements for the examination are decided upon according to the laws and instructions valid at any time.

II. The National Examination Board and External Examiners

Art. 4. The Ministry of Education appoints an examination board of nine persons for up to four years at the time. The ministry allocates to one of them the role of a chairman or appoints a special chairman, who then shall not act as an external examiner in the board. The chairman acts as an executive director for the board. On the examination board there shall be specialists on the subjects that are covered by the national examination, and are well familiar with teaching and the curriculum which is required for the examination. ...

Art. 5. The national examination board shall act as external examiners, as far as is possible. Otherwise the ministry appoints external examiners after receiving recommendations from the Director of Educational Affairs and the national examination board.

The National Examination Board Members for most of the Period 1950-1966

The members and their schools or work places at the time concerned:[1061]

Bjarni Vilhjálmsson, chairman 1947-64, and 1966-67. The Teacher Training College and *Gagnfræðaskólinn við Vonarstræti* lower secondary school.
Ólafur Briem, Icelandic 1946-71, Laugarvatn Lower Secondary School, Laugarvatn High School.
Steingrímur Pálsson, Icelandic 1946-58, *Vélskólinn* / School of Mechanics.
Gestur Magnússon, Icelandic 1958-69, *Gagnfræðaskóli verknáms* lower secondary school.
Ágúst Sigurðsson, Danish 1946-67, Teacher Training College.
Jón Magnússon, English 1946-67, Head Reporter, National Radio Station.
Ólafur Hansson, history, 1947-74, Reykjavík High School.
Ástvaldur Eydal, geography 1950-58, *Gagnfræðaskóli Vesturbæjar* lower secondary school, University of Iceland.
Einar Magnússon, geography, 1959-65, Reykjavík High School.
Guðmundur Þorláksson, geography 1966-72, Teacher Training College.
Guðmundur Kjartansson, natural science 1946-72, Flensborg Lower Secondary School, University of Iceland.
Guðmundur Arnlaugsson, physics 1948-69, Reykjavík High School.
Steinþór Guðmundsson, mathematics 1946-62, *Gagnfræðaskóli Vesturbæjar*, *Gagnfræðaskólinn við Vonarstræti* lower secondary school.
Björn Bjarnason, mathematics 1963-71, Reykjavík High School.

The National Examination in Practice

The numbers of schools and pupils offering the national examination in three periods are shown in tables D1–D3. The number of pupils undergoing the examination and the number of pupils reaching the entrance level are given separately for the Reykjavík area, for the area outside Reykjavík and for the whole country, together with the percentage of the 16-year age cohort taking the examination and reaching the entrance level. Some pupils might be older, as they were repeating the national examination or for other reasons.[1062] Hafnarfjörður is counted together with Reykjavík, as is Kópavogur in table D3.

[1061] Ólafur Þ. Kristjánsson and Sigrún Harðardóttir (1958–1988). Archives of the Ministry of Education: Skýrslur (reports)
[1062] Bjarni Vilhjálmsson (1952): 12–22

Table D1. National Examination 1946–1951

Year	Reykjavík area: Schools	Total no. of pupils	Entrance	Outside Reykjavík: Schools	Total no. of pupils	Entrance	The Whole Country: Total schools	Total no. of pupils	% of population	Total entrance	% of population[1063]
1946	**3**	166	87	**9**	60	24	**12**	226	8.3	111	4.1
1947	**3**	188	73	**9**	62	43	**12**	250	9.2	116	4.3
1948	**3**	195	86	**13**	88	59	**16**	283	10.9	145	5.6
1949	**4**	227	125	**14**	107	77	**18**	333	13.7	202	8.3
1950	**4**	209	135	**20**	237	172	**24**	446	17.9	307	12.3
1951	**3**	234	124	**21**	263	202	**24**	497	20.3	326	13.3

Table D2 National examination 1958–1960

Year	Reykjavík: Schools	Total no. of pupils	Entrance	Outside Reykjavík: Schools	Total no. of pupils	Entrance	The Whole Country: Total schools	Total no. of pupils	% of population	Total entrance	% of population
1958	**2**	271	172	**25**	255	192	**27**	526	18.4	364	12.7
1959	**4**	319	204	**24**	242	189	**28**	561	18.9	393	13.2
1960	**4**	333	236	**24**	290	203	**28**	623	20.8	439	14.6

Table D3 National examination 1967–1969

Year	Reykjavík: Schools	Total no. of pupils	Entrance	Outside Reykjavík: Schools	Total no. of pupils	Entrance	The Whole Country: Total schools	Total no. of pupils	% of population	Total entrance	% of population
1967	**8**	571	375	**30**	392	299	**38**	963	24.4	674	17.1
1968	**8**	679	460	**29**	496	375	**37**	1175	29.8	835	21.2
1969	**8**	807	470	**31**	601	391	**39**	1408	33.9	861	20.7

[1063] Jónas Pálsson and Hjálmar Ólafsson (1961)

Appendix E – Halldór Elíasson's Article in Menntamál 39(2) 1966

1. Í námsskrá okkar er „leikni í almennum reikningi" sett sem markmið með kennslunni. Þessi leikni er ekki lengur nauðsynleg, vegna þess að við erum farin að reikna með vélum. Jafnvel þótt við náum leikni í reikningi í barnaskóla, þá er hún dæmd til að fara forgörðum, þar sem við flest þurfum ekki á henni að halda. Þekking í almennum reikningi er nægileg krafa, og í stað leikni skal koma skilningur.
2. Námsefnið einkennist um of af því að veita nemendum leikni í því að framkvæma vissar reikningslistir, í stað þess að leggja höfuðáherzlu á þekkingu á grundvallarhugtökum og hugsun í almennum reikningi. ... Námskerfi okkar og prófttilhögun gerir það freistandi fyrir kennara að leggja höfuðáherzlu á einfalda handavinnu, en með því bregðast þeir skyldu sinni. ...
3. Í núverandi námsbókum er nokkuð mikið um slæmar reikningslistir. ... Hlutfallareikningurinn er bein móðgun við heilbrigða skynsemi í því formi, sem hann er kenndur. ... Hins vegar tryggir notkun þríliðunnar svo til, að nemendur hafa ekki hugmynd um, hvað þeir eru að gera og hafa enga aðstöðu til að dæma um, hvort raunverulega sé rétt að nota þríliðu. ... Það þarf alltaf einhverja þekkingu til að geta reiknað dæmi, þekkingu á þeim hugtökum, sem þar koma fyrir. Það, sem hér vantar, er kennsla í meðferð og notkun hugtaka. Til dæmis hugtök eins og verð, lengd, vextir o.s.frv. á að kenna *að nota* í reikningi. Með því er hægt að losna við þriliðuna og komast að kjarna þess, sem felst í hugtakinu hlutfallareikningur.
4. Það eru tíðkaðar nokkrar mismunandi kennsluaðferðir í skólum, og oft er um það deilt, hver sé sú heppilegasta. Ég hef ekki aðstöðu til að leggja neinn dóm á það, en vil þó benda á eitt, sem mér finnst vera vanrækt. Það er, að kennarinn haldi fyrirlestur um námsefnið. Tilgangurinn með því er ekki einungis að útskýra námsefnið fyrir nemendum, heldur einnig og ekki síður að sýna nemendum, hvernig þeir eigi að hugsa. ... Það að breyta raunhæfu viðfangsefni yfir í stærðfræðilegt, krefst ætíð einhvers snefils af stærðfræðilegri hugsun. Hins vegar má ekki vanrækja að veita nemandanum nokkra þjálfun í því að tala skýrt og skilmerkilega um námsefnið, tjá hugsun sína, en það fæst ekki með því að toga út úr honum orð og orð á stangli.

   Að lokum vil ég taka eftirfarandi fram í sambandi við prófin margumtöluðu. Ég tel, að ekki sé rétt, að nemendur séu prófaðir í námsefni, sem engin ástæða er til að þeir kunni. Ef þetta sjónarmið væri viðurkennt, þá væru t.d. próf í lesnum dæmum ekki tíðkuð. Það er algjörlega einskis virði að nemendur læri þessi dæmi utan að. Lakari nemendur freistast einmitt til þess, en ættu að verja tíma sínum á skynsamlegri hátt. Önnur ástæða til þess, að slík próf eru óæskileg, er sú, að þau hafa mjög neikvæð áhrif á kennsluaðferðir. Landsprófskennslan hefur verið átakanlegt dæmi um þetta.

… Mér virðist sem full þörf sé á að gerbreyta kennslubókum fyrir nemendur á fræðsluskyldualdrinum … Nýjar kennslubækur þurfa helzt að vera skrifaðar af kennurum, sem hafa nokkra þekkingu í æðri stærðfræði, og sem hafa tekið þátt í grundvallarrannsóknum á námsefninu. ...

Það, sem koma þarf, er fyrst og fremst þetta:

1. Háskóladeild, sem m.a. útskrifar kennara í menntaskóla, kennaraskóla og gagnfræðaskóla, og heldur „seminör" með starfandi kennurum til rannsókna á námsefni téðra skóla og til fræðslu.
2. Kennaraskóladeild, sem m.a. sér barnaskólum fyrir nokkrum kennurum, hæfum til að þróa reikningskennsluna, og sér um rannsóknir á námsefni barnaskólanna.
3. Frjálsleg fræðslulöggjöf, sem gefur kennurum svigrúm til að þróa kennsluna smám saman.
4. Hæfum kennurum sé gefinn kostur á, og þeir hvattir til að stunda rannsóknir á námsefni og kennsluháttum (minni kennsluskylda, fjárhagslegur stuðningur).

1. In our curriculum "skills in general arithmetic" is set as a goal of the teaching. These skills are no longer necessary as we have begun to compute with machines. Even if we developed skills in arithmetic in the primary schools, they are destined to deteriorate as most of us do not have any use for it. Knowledge of general arithmetic is a sufficient requirement, and understanding should replace skills.
2. The syllabus is much characterized by training the pupils in exercising certain arithmetic skills, instead of placing the main emphasis on knowledge of fundamental concepts and thinking in general arithmetic. … Our educational system and examination arrangements make it tempting for the teachers to lay the main emphasis on simple manual tasks, but thereby they fail in their duties. …
3. In the present textbooks there are good many examples of bad computation tricks. ... The calculation of proportions and ratios is an absolute insult to common sense in the form it is taught. … the use of *regula-de-tri* ensures that the pupils have no idea about what they are doing and are in no position to judge if it is really correct to use *regula de tri*. ... Some knowledge is always needed to be able to solve a problem, knowledge of the concepts that occur. What is needed is education in the use and handling of concepts. For example, one should teach the handling and use of concepts such as price, length, interest, etc. In that way one can get rid of the *regula-de-tri* and come closer to the core of the concept of proportional calculation.
4. There are different teaching methods in use in the schools … I think is neglected … that the teacher gives a lecture about the syllabus. The purpose is not only to explain the syllabus for the pupils, but … to show the pupils how they should think. … To transform a practical problem into a mathematical problem always requires at least a minimum of mathematical thinking. … it must not be neglected to supply the pupil with some training in talking clearly and understandably about a mathematical subject, to express his/her thinking, but this will not be reached by pulling single, scattered words out of his mouth.

   Finally I would like to express myself about the much-discussed examinations. I do not think that it is right to examine pupils in a syllabus which there is no reason for them to know. If this view were accepted, then for example one would not run examinations in seen problems. It is worth absolutely nothing for the pupils to learn these problems by heart. The less able students are tempted to do exactly so, while they ought to spend their time in a more sensible way. Another reason why such examinations are undesirable is that they have very negative influences on teaching methods. The teaching for the national examination is a sad example of this.

… It seems to me that there is a great need to change completely the textbooks for children in the compulsory school system … New textbooks should preferably be written by teachers who have some knowledge in higher mathematics, and who have participated in a basic research of the syllabus.

… What is needed is first and foremost this:

1. A university department, which among other things educates teachers for high schools, teacher training colleges and lower secondary schools, and holds seminars for working teachers to research the syllabus of the schools in question and for their own education.
2. A teacher training department, which … will supply the primary schools with several teachers who are competent to develop arithmetic teaching and be responsible for research on the primary school syllabus.
3. A liberal educational legislation, which gives the teachers scope to gradually develop their teaching.
4. Qualified teachers be given the opportunity and encouragement to work on research on curriculum and teaching (less teaching duty, a financial support).[1064]

[1064] Halldór Elíasson (1966): 95–99

Appendix F – 1968 Preliminary Mathematics Curriculum

Fyrir allmörgum árum síðan var námsefni framhaldsskóla í stærðfræði tekið til endurskoðunar undir forystu O.E.C.D., Efnahags- og framfarastofnunarinnar í París. Þessi endurskoðun hefur leitt til þess, að víðast hvar hafa á undanförnum árum orðið róttækar breytingar á námsefni og kennsluháttum í stærðfræði.

Stefnt er að því

1. að grundvalla skólastærðfræðina á frumhugtökum mengjafræðinnar, sem í senn eru einföld og almenns eðlis
2. að leggja meiri áherzlu á inntak og eðli talna og talnareiknings en tíðkast hefur.

Fyrir fjórum árum var hafizt handa um að breyta námsefni í menntaskólunum hér á landi í samræmi við þessa nýju stefnu. Þá þegar varð ljóst, að þessar breytingar gætu ekki til fulls náð tilætluðum árangri, nema breytt yrði um stefnu í stærðfræðikennslu í landsprófsbekkjum, þar sem grundvöllur er lagður að algebru, einnar veigamestu undirstöðugreinar menntaskólastærðfræðinnar. En með því að ekki fundust á íslenzku kennslubækur með viðeigandi sniði og kennarar höfðu fæstir átt þess kost að kynna sér hin nýju viðhorf, reyndist alls kostar ókleift að hrinda slíkum breytingum í framkvæmd. Með útkomu bókar Guðmundar Arnlaugssonar, *Tölur og mengi*, og námskeiðum fyrir stærðfræðikennara breyttist viðhorfið svo til betri vegar, að unnt reyndist fyrir tveimur árum að sveigja námsefnið að nokkru inn á hinar nýju brautir, og var þá skólastjórnum gefinn kostur á að taka upp nýtt námsefni, en ekki fyrirskipað það. Hér var að sjálfsögðu ekki um framtíðarlausn að ræða, enda aðeins gerð í því skyni að veita stærðfræðikennurum tíma til að átta sig á breyttum viðhorfum. – Í fyrra vetur naut meginþorri (rúmlega 2/3) landsprófsnemenda uppfræðslu í nýja efninu, og með því að nú má ætla, að stærðfræðikennurum hafi gefizt tóm til að semja sig að breyttum siðum, verður nú frá því horfið að hafa námsefni af tvennu tagi, en tekið upp sama námsefni fyrir alla.

Ekki þótti samt annað kleift en að sníða námsefnið að svo til öllu leyti eftir þeim kennslubókum, sem þegar eru til á íslensku, þótt sumar hverjar séu komnar til ára sinna og henti ekki þeim kröfum sem gera þyrfti.

Námsefnið miðast við að kennt sé:

1. Tölur og mengi eftir Guðmund Arnlaugsson, öll bókin.
2. Kennslubók í algebru eftir Ólaf Daníelsson, aftur að lesmáli framan við æf. XI, þó að undanskildum flóknari dæmum í æf. VI (eins og t.d. dæmin 29-44).
3. Nokkuð um rétt hlutfall og skiptireikning í Reikningsbók eftir Ólaf Daníelsson eða Reikningsbók fyrir framhaldsskóla, II. hefti A, eftir Jón Á. Gissurarson og Steinþór Guðmundsson eða Reikningbók handa framhaldsskólum, II. og III. hefti, eftir Kristin Gíslason og Gunngeir Pétursson.

Markmið

Stærðfræðikennslu í landsprófsbekk skal miða við, að nemendur, sem þar stunda nám, hafi í huga framhaldsnám í menntaskólum og öðrum þeim skólum, sem landspróf veitir rétt til að setjast í.

Stefnt skal að því, að nemendur öðlist

1. aukið öryggi í almennum talnareikningi,
2. skilning á talnaritun vorri og eðli reikningslegra aðgerða,
3. traustar hugmyndir um grundvöll algebru, skilning á táknmáli og leikreglum,
4. leikni í umskriftum á fullyrðingum, sem skráðar eru á táknmáli algebrunnar,
5. leikni í að leysa fyrsta stigs jöfnur með einni, tveimur eða þremur óþekktum,
6. leikni í að umrita setningar úr venjulegu máli á táknmál algebrunnar.

Námsefni

1. Rétt hlutfall og skiptireikningur.
2. Frumhugtök mengjafræðinnar, mengi, íbúi, vennmyndir, jafnvægi mengja, fjöldatala, hlut-, sam- og sniðmengi, tómamengi, mengjamismunur.
3. Yrðingar, almengi og fyllimengi, opnar yrðingar og lausnamengi þeirra.
4. N_0, mengi náttúrlegra talna og núlls. Mynztur í N_0, skipting N_0 í leifaflokka, prímtölur, talnaritun í tugakerfi og öðrum sætiskerfum. Framkvæmd samlagningar, frádráttar, margföldunar og deilingar í öðrum sætiskerfum en tugakerfi, einkum tvíundarkerfi.
5. Reikningsaðgerðirnar fjórar: Samlagning, frádráttur, margföldun og deiling í mengi ræðra talna, grundvallarlög aðgerðanna og reiknireglur, sem af þeim má leiða.
6. Þáttun (breyting liðastærðar í pródúkt), deiling í mengi liðastærða.
7. Fyrsta stigs jöfnur með einni, tveimur eða þremur óþekktum og leysing slíkra.
8. Óuppsett dæmi, sem leiða til jafna af sama tagi og um getur í 7. lið.
9. Stærðarstig og tíuveldatáknun.

Ábendingar til kennara

1. … Á skyldustigi hafa nemendur kynnst fjölmörgum dæmum þess, að venzlum sé þannig háttað milli tveggja breytistærða, að önnur er í réttu hlutfalli við hina. Þótt rétt hlutfall sé ekki nefnt á nafn í dæmum þessum, hafa þeir samt fundið, t.d., verð í réttu hlutfalli við þunga eða stykkjatölu, þunga í réttu hlutfalli við rúmmál, vexti á einu vaxtatímabili í réttu hlutfalli við höfuðstól, vegalengd, sem farin er með jöfnum hraða, í réttu hlutfalli við tíma. Sennilega hafa fæstir þeirra þó komið auga á hið sameiginlega í slíkum venzlum, hin eiginlegu stærðfræðivenzl, sem lýst er með orðasamstæðunni „í réttu hlutfalli við" og síðar á námsbraut þeirra verða tjáð með opinni yrðingu í forminu y = ax (a:tala).

   Með því að taka „rétt hlutfall" í námsefnið er ætlast til, að reynt sé að fá nemendur til þess að skynja, hvað í þessum orðum felst um talnavenzl. ...

2. Miklu varðar, að nemendur öðlist skýrar hugmyndir um frumhugtök mengjafræðinnar og venzl þeirra, og nái góðu valdi á táknmálinu.

   Frumhugtök þessi koma fram í grundvelli hverrar stærðfræðigreinar, og verður því oft til þeirra að vísa og beita þeim. Táknmálið gefur kost á að tjá hugmyndir og hugmyndatengsl með nákvæmum og augljósum hætti. Æskilegt er, að ekki sé byrjað á algebru, sem miðast við talnamengi (þ.e. hinni venjulegu algebru), fyrr en nemendur hafa náð góðum tökum á venzlum mengja og þeim vísi að mengjaalgebru, sem finnst ... í kennslubókinni.

   Að því er mengjaalgebru viðvíkur er gott að skýrt komi fram eftirfarandi:

   1. Að mynda sammengi og sniðmengi má skoða sem <u>tvenndaraðgerðir</u> í mengi mengja, þ.e. sammengi [og] sniðmengi eru hverju sinni mynduð af <u>tveimur</u> mengjum (sem þó geta verið eitt og sama mengið).
   2. Að um þessar aðgerðir gildi reglurnar [listi yfir víxlreglur, tengireglur og dreifireglur mengja] ...

3. Hafi nemendur eitthvað fengist við að leysa jöfnur er rétt að vekja athygli þeirra á, að hver slík jafna er opin yrðing við almengi, sem í flestum tilvikum er mengi ræðra talna.

4. Talnaritun og talnareikningur í öðrum sætiskerfum en tugakerfi á m.a. að vekja nemendur til íhugunar um gildi talnaritunar vorrar og kenna honum að gera greinarmun á hugtakinu tala og þeim táknum, sem unnt er að tjá hana með. ... hver náttúrleg tala lýsi[r] einkenni sem er sameiginlegt með innbyrðis jafnvægum mengjum. Þannig segir talan fimm, hvernig svo sem hún er rituð, frá því sameiginlega með öllum mengjum, sem jafnvæg eru mengi fingranna á réttskapaðri mannshönd. ...

5. Algebrur eru til af ýmsu tagi. Allar eiga þær sér þó það sameiginlegt, að þær fjalla um eðli einhvers konar „reiknings". Til grundvallar er lagt ákveðið mengi, sem ekki er tómamengi. Úr hverjum tveimur íbúum þessa mengis er með „aðgerð" búinn til nýr íbúi þessa sama mengis. ...

Appendix G – Ratio of Word Problems in 1957–1975

In 1946–1965 the national examination was divided into problems seen before by the examination candidates, the "seen problems," and unseen problems. In the following table over the ratio of word problems to total number of problems, the "seen problems" are labelled I and the new problems labelled II.

Table G1: Ratio of Word Problems to Total Number of Problems 1957– 1965.

Year	I	II	Average
1957	3 out of 6 – 50%	5 out of 7 – 71%	61%
1958	2 out of 6 – 33%	4.5 out of 7 – 64%	49%
1959	3.5 out of 6 – 58%	5 out of 7 – 71%	67%
1960	3 out of 6 – 50%	4.5 out of 7 – 64%	57%
1961	5 out of 8 – 62.5%	6 out of 10 – 60%	61%
1962	5 out of 7 – 71%	7 out of 10 – 70%	71%
1963	5 out of 7 – 71%	5 out of 8 – 62.5%	67%
1964			
1965	5 out of 7 – 71%	4 out of 8 – 50%	61%

In 1966 there were no "seen problems". Part I was composed of short problems and part II contained composite problems. For the first time, the relative weight was now indicated.

Table G2: Ratio of Word Problems to Total Number of Problems 1966.

Year	I	II	Average
1966	7 out of 13 – 54%	9 out of 12 – 75%	65%

In 1967 there were two kinds of syllabuses for the first time, the "old" syllabus and the "new" one, with "modern" mathematics. The first part of the examination, the arithmetic, was separate, while the second part, the algebraic, was in common. The arithmetic part was a fill-in type test, where the answers were either right or wrong.

In 1968 the arrangement was similar, except that one problem out of five in part II was different. In part I, 16 out of 25 problems were multiple-choice problems.

It became increasing difficult to decide if a problem was a "word problem" or not. The "modern" mathematics problems are of theoretical type. Even if the problems were in words, they were less and less concerned with real life objects, but with the properties of numbers.

Table G3: Ratio of Word Problems to Total Number of Problems 1967–1968.

Year	I	II	Average
1967 – "Old"	16 out of 20 – 80%	3.5 out of 5 – 70%	75%
1967 – "New"	8 out of 20 – 40%	3.5 out of 5 – 70%	55%
1968 – "Old"	16 out of 25 – 64%	3 out of 5 – 60%	62%
1968 – "New"	10 out of 25 – 40%	2.5 out of 5 – 50%	45%

In the period 1969–1972 there was only one type of examination with "modern" mathematics, and the examination was in one part, taken in a three hours' session.

Table G4: Ratio of Word Problems to Total Number of Problems 1969–1972.		
Year	Ratio	In %
1969	15 out of 50	30%
1970	18 out of 50	36%
1971	20 out of 50	40%
1972	16 out of 50	32%

The ratio of word problems, which had been around 60% is now below 40%. In 1973, there were choices of "old" syllabus with Guðmundur Arnlaugsson's version of "modern" mathematics together with Ólafur Daníelsson's *Algebra* on one side and a translated Swedish version of "modern" mathematics on the other side. In 1975, the third choice was added, Hörður Lárusson's version of "modern" mathematics.

Table G5: Ratio of Word Problems to Total Number of Problems 1973–1975.		
Year	Ratio	In %
1973 – GA/ÓD	24 out of 70	34%
1973 – Swedish Series	17 out of 70	24%
1974 – GA/ÓD	34 out of 100	34%
1974 – Swedish Series	20 out of 100	20%
1975 – GA/ÓD	28 out of 100	28%
1975 – Swedish Series	24 out of 100	24%
1975 – HL	30 out of 100	30%

During the conventional syllabus period up to 1966 the "word problems", reasonably connected with real life, were about 60%. In 1967 "modern" mathematics was introduced. At that time, the number of pupils attempting the examination had become so large that some measures had to be taken for one person to be able to examine the papers in the whole capital area. Fill-in sheets, and right/wrong answers were introduced, and multiple-choice technique for one year. In the 1970s, only the solutions of pupils with grades in the interval 5.50–6.50 were marked by an external examiner. In order to ensure equality in grading, the grading had to be incontestable.

With the introduction of "modern" mathematics the word problems became fewer, and more abstract. They might be in words, but there was no story. They were increasingly short, and the number of problems increased inversely with the shortness of the problems. Simultaneously, the ratio of the word problems to the total number of items in the examination decreased markedly, i.e. from up to two-thirds of the examination to less than one-third, even as little as one-fourth.

Possibly, the committee member found that lower-ability pupils, an increasingly large proportion of the examination candidates, when a larger proportion of the year cohort attempted the examination, did better on the single item problems without stories. The needs of those pupils had previously been met by generous grading for all first attempts at a problem, with increased demand when the scale came closer to full credit.[1065] The question remains if this new trend of shorter problems with less concrete content affected the pupils' attitude towards mathematics for the better or the worse.

[1065] Haraldur Steinþórsson, January 22, 2003

Appendix H – Founder Members of the Mathematical Society

In 1947 *Íslenska Stærðfræðafélagið*, the Icelandic Mathematical Society, was founded on Dr. Ólafur Daníelsson's 70th years' birthday. The following is a list of founder-members. Five of them were mathematicians:[1066]

Ólafur Dan Daníelsson (1877–1957). Mag. Scient. 1904 and Dr. Phil. in mathematics in 1909, both at the University of Copenhagen. Teacher at Teacher Training College 1908–20, Reykjavík High School 1919–41. Actuary 1935–53.

Sigurkarl Stefánsson (1902–1995). Cand. Mag. in mathematics, University of Copenhagen 1928. Teacher at Reykjavík High School 1928–75.

Leifur Ásgeirsson (1903–1990). Doctoral degree in mathematics at Göttingen University in 1933. Headmaster at Laugar District School 1933–43. Professor at University of Iceland 1943–73.

Guðmundur Arnlaugsson (1913–1996). Cand. Mag. in mathematics at the University of Copenhagen 1942. Teacher at Akureyri High School 1936–39 and 1945–46, Reykjavík High School 1946–65. Headmaster at Hamrahlíð High School 1965–80.

Björn Bjarnason (1919–1999). Cand. Mag. in mathematics at the University of Copenhagen 1945. Teacher at Akureyri High School 1946–48, Reykjavík High School 1948–69. Headmaster at Sund High School 1969–89.

Three founder-members were actuaries:
Brynjólfur Stefánsson (1896–1960),
Árni Björnsson (1898–1978) and
Kr. Guðmundur Guðmundsson (1908–1993).

Five founder-members were physicists or astronomers, who taught mathematics:

Þorkell Þorkelsson (1876–1961). A degree in physics in 1903 at the University of Copenhagen. Teacher at Akureyri Lower Sec. School 1908–18, Reykjavík High School 1920–28. Director of Meteorological Institute 1920–46.

Steinþór Sigurðsson (1904–1947). Degree in astronomy, chemistry, physics and mathematics at the University of Copenhagen. Teacher at Akureyri High School 1929–35, Reykjavík High School 1935–39. Headmaster of Iceland Commercial College 1938–41, teacher at the University of Iceland 1940–47.

Trausti Einarsson (1907–1984). Dr. Phil. in astronomy at Göttingen University 1934. Teacher at Akureyri High School 1935–44, Professor at the University of Iceland 1944–77.

Sveinn Þórðarson (1913–). Dr. Phil. at Jena University, Germany, (physics, mathematics, chemistry). Teacher at Akureyri High School 1939–52. Headmaster at Laugarvatn High School 1952–59.

Þorbjörn Sigurgeirsson (1917–1988). Cand Scient. in physics from University of Copenhagen 1943. Professor at the University of Iceland 1957–84.

Two founder-members were engineers, but taught mathematics:
Bolli Thoroddsen (1901–1974), Reykjavík City Engineer 1944–61.
Gunnar Böðvarsson (1916–1989), professor in mathematics and geophysics at Oregon State University 1964–89.

[1066] Björn Birnir et al. (1998): 97–104. Ólafur Þ. Kristjánsson and Sigrún Harðardóttir (1958–1988)

Appendix I – List of Books on Mathematics

Guðmundur Arnlaugsson listed in his 1971 article 28 books which he recommended as reasonably easy reading for mathematics teachers, admitting that the choice was coloured by his own knowledge and taste. The first few were his references, including books on general pedagogy. They were:

Jerome S. Bruner: *The Process of Education*. Random House. New York.
Jerome S. Bruner: *On Knowing – Lessons for the Left Hand*. Belknap Press of Harward University 1966.
Philipp Franck: *Einstein – His Life and Times*. Jonathan Cape. London 1948.
Mathematical Reflections. Edited by the Members of the Association of Teachers of Mathematics. Cambridge University Press 1970.
W. W. Sawyer: *Vision in Elementary Mathematics*. Penguin Books 1964.
Alexander Wittenberg: *Bildung und Mathematik.* Klett Verlag. Stuttgart 1963.
Z. P. Dienes and E. W. Golding: *Learning Logic, Logical Games*. The Educational Supply Association Ltd. Pinnacles, Harlow, Essex, England.

The list included only two books in Icelandic:

A.N. Whitehead: *Stærðfræðin / Introduction to Mathematics,* translated in 1932 by Guðmundur Finnbogason.
Stærðfræðin / Mathematics in the series Life Science Library, published by Time Inc., New York, in 1963, translated into Icelandic in 1966 by Björn Bjarnason.

In other Nordic languages (At this time people read them more easily than English):

Poul la Cour: *Historisk Matematik*, 4th ed. 1942.
T. Danzig: *Tallet, videnskabens sprog*. Gyldendals Uglebøger 1965.
Irving Adler: *Den nye matematikken*. J. W. Cappelens forlag, Oslo 1965.
Tord Ganelius: *Introduktion till matematiken*. Natur och Kultur, Stockholm 1966.
Bent Christiansen et al.: *Almene begreber fra logik, mængdelære og algebra*. Munksgaard, København 1964.
Bent Christiansen et al.: *Matematik 65*. Munksgaard, København 1965.

Some of the books were relatively new, published in connection with the “modern” mathematics wave. Bent Christiansen’s books were used in the teachers’ in-service courses and for the B.A. students in mathematics at the University.

In English:

Courant and Robbins: *What is Mathematics?* Oxford University Press, 12th ed. 1963.
Lucienne Felix: *Modern Mathematics and the Teacher*. Cambridge Univ. Press 1966.
Lancelot Hogben: *Mathematics for the Million*. 1st ed. 1936.
Morris Kline: *Mathematics in Western Culture*. Oxford University Press 1964.
James Newman: *The World of Mathematics I-IV*. Simon and Schuster (paperback) 1966.
G. Polya: *How to Solve It. A New Aspect of Mathematical Method*. Princeton 1948.
W. W. Sawyer: *Mathematician’s Delight*. Penguin Books.
W. W. Sawyer: *Prelude to Mathematics*. Penguin Books.
W. W. Sawyer: *The Search for Pattern*. Penguin Books.
W. Schaaf: *Basic Concepts of Elementary Mathematics*. John Wiley & Sons 1965.
The *NCTM Yearbooks*, especially from 1957 (*Insight into Modern Mathematics*), 1964 (*Topics in Mathematics for Elementary School Teachers*) and 1969 (*Historical Topics in the Classroom*).

Appendix J – Content of the Bundgaard Material

A list of the content of the *Stærðfræði – Reikningur* / *Arithmetic – Mathematics* series by Agnete Bundgaard and her collaborator in the first two years, Eeva Kyttä.

1 Introduction to counting and computing.

2a Addition by the associative law, one-digit numbers and two-digit numbers. Subtraction, two-digit numbers without borrowing. Introduction to multiplication with 2, 3, 4 and 5. Commutative law in multiplication. Odd and even numbers. Multiplication with zero.

2b Addition, two-digit numbers. Subtraction, two-digit numbers, borrowing. Three-digit numbers, introduction, addition, subtraction. Sums of products. Multiplication tables. Multiplying with the aid of the distributive law. Ordering by size. Filling into arithmetic sequences. Pairing numbers by a given function. Finding the function.

3a Decimal number notation. Roman numerals. The transverse sum. The nine-times table and the sum of the digits. The number 0 in multiplication. Multiplication of two- digits numbers. Introduction to division.

3b Prime numbers. Permutation of three digits. The associative and distributive laws. Multiplication of two-digit numbers by two-digit numbers. The decimal place value system. Place value notation to the base 5.

4a Long division. Sets. Set symbolic notation. Pairing. Subsets. Intersection. Divisors. Composite numbers and prime factors. Zero in division.

4b Union of sets. Associative and commutative law of positive integers and zero. Place value number notation to various bases. Geometry: points, lines in a set-theoretical frame. Length of a segment. The decimal / metric system. Decimal fractions. Addition and subtraction of decimal fractions. Multiplication of decimal fractions with the aid of the associative law. Division of decimal fractions by an integer.

5a The metric system. The money system. Geometry, measurement. Division by multi-digital integers. Multiplication modulo 9. Division of a decimal fraction by a decimal fraction.

5b Average. Linear functions of proportions. Geometry in a set-theoretical frame: angles, rectangles, polygons. Approximation, errors. Sets, set difference.

6 Division, common fractions. Size and order of common fractions. Cancelling. Addition and subtraction of common fractions. Mixed numbers (whole numbers and fractions). Multiplication of common fractions. Division of common fractions.

Appendix K – Science Teachers Graduates

Table K1: Graduates from the BA programme at the University of Iceland – 1953–1972[1067]

	Year	Major	Minor	Main Institute in 1960s-1970s
Sigurður Sigfússon	1953	Physics	English	Lower sec. school
Hreinn Bernharðsson	1954	Math.	Physics	Lower sec. school
Sigurður Óli Brynjólfsson	1954	Math.	Physics	Lower sec. school
Guðmundur G. Magnússon	1955	Math.	Physics	Lower sec. school
Karl Stefánsson	1957	Math.	Physics	Lower sec. school
Helgi Þorsteinsson	1959	Math.	Danish	Lower sec. school
Haukur Melax	1959	Math.	Physics	
Hörður Lárusson	1960	Math.	Physics	Upper sec. school
Guðmundur P. Sigmundsson	1961	Math.	Chemistry	Lower sec. school
Þórir Ólafsson	1961	Chemistry	Physics	Upper sec. school
Þórarinn Guðmundsson	1962	Math.	Physics	Upper sec. school
Þórarinn G. Andrewsson	1962	Math.	Physics	Lower sec. school
Anna Kristjánsdóttir	1967	Math.	History	Lower sec. school
Valdimar Valdimarsson	1967	Math.	Physics	Upper sec. school
Ingvar Ásmundsson	1968	Math.	Physics	Upper sec. school
Hildigunnur Halldórsdóttir[1068]		Math.		Upper sec. school
Kristín Bjarnadóttir	1968	Physics	Math.	Lower sec. school
Einar Kristinsson	1968	Physics	Math.	Upper sec. school
Margrét Ó. Björnsdóttir	1968	Physics	Math., Chem.	Upper sec. school
Birna Ólafsdóttir	1970	Physics	Math., Chem.	
Már Ársælsson	1970	Math.	Chemistry	Upper sec. school
Baldur Sveinsson	1970	Math.		Upper sec. school
Kristín Halla Jónsdóttir	1971	Math.	Physics	Upper sec. school
Örn Arnar Ingólfsson	1971	Math.		Upper sec. school
Eygló Guðmundsdóttir	1972	Math.	Physics, Chem.	Upper sec. school
Björn Búi Jónsson	1972	Physics	Math., Chem.	Upper sec. school
Hallgrímur Hróðmarsson	1972	Physics	Math., Chem.	Upper sec. school

Table K2: Total number of mathematics graduates from a programme at the University of Iceland until 1988.

1952-1988	Total	Lower and upper sec. level	Whereof permanently	University teaching	Other Educational occupation	Other occupation
1952-1955	4	4	4		0	0
1956-1960	4	3	1		2	1
1961-1965	4	4	3	1	0	0
1966-1970	8	6	5	1	0	2
1971-1972	6	5	4	1		1
1972-1975	15	6	5	2	0	8
1976-1980	27	6	2	9	0	16
1981-1985	15	6	5	1	0	9
1986-1988	9	3	1	0	?	?

[1067] Archives of the University of Iceland: Protocols of the Faculty of Arts (heimspekideild). *Fréttabréf Félags raungreinakennara* 5(2) (1988): 26–27. Ólafur Þ. Kristjánsson et al. (1958–1988)
[1068] Graduated from University of Washington in Seattle with a B.Sc. degree in 1966, M.Sc. in 1966

Appendix L – SRD Workgroups on Mathematics Textbooks

Members of a Work-group on Primary Level Mathematics Textbooks[1069]

The SRD workgroup, established in 1971 in order to create new material for the primary level, published an experimental textbook for the first grade by Anton Sigurðsson and Hörður Zóphaníasson in 1972. Textbooks for the second grade by Anton Sigurðsson, Hörður Lárusson, Ragnhildur Bjarnadóttir, Örn Arnar Ingólfsson and Ingibjörg Þorkelsdóttir, and revised versions of the first-grade books were published in 1973. In 1974, textbooks up to the third grade had been published in an experimental edition and for the first grade in a final edition. In 1975 Hanna Kristín Stefánsdóttir joined the group to write for the fourth grade and upwards, and in 1976 Rúnar Þorvaldsson joined in writing for the fifth grade. In 1978 a textbook for the sixth grade was made. Rúnar Þorvaldsson withdrew soon.

From 1977 a new edition of the SRD textbook series for the primary level was published, where Anna Kristjánsdóttir had joined the group of authors. Kolbrún Hjaltadóttir joined in from the fourth grade.

Members of a Work-group on Lower Secondary Level Mathematics Textbooks

In 1978 a work-group chaired by Anna Kristjánsdóttir began to prepare a new series of textbooks, turning away from the set theoretical approach. Some starting points were used to introduce classical topics by an investigational approach, where applicable. The group of authors included Anna Kristjánsdóttir, Baldvin Bjarnason, Ingólfur Ármannsson, Kristín Bjarnadóttir and Rúnar Þorvaldsson. Later the group was joined by Guðmundur Arnlaugsson, Hildigunnur Halldórsdóttir, Ásgerður Magnúsdóttir and Björg Birgisdóttir, while Baldvin Bjarnason and Rúnar Þorvaldsson withdrew.

[1069] Menntamálaráðuneytið, skólarannsóknadeild (April 1979): 63–67

Appendix M – Swedish NKMM Material

Translated Swedish NKMM Material for Lower Secondary Schools

In 1970 or 1971, Anna Kristjánsdóttir, Hörður Lárusson and Helga Björnsdóttir translated a series of mathematics textbooks from Swedish. Three volumes have been found. Only the authors of the first volume on geometry are known. The authors of the other volumes are not mentioned in the textbooks and have not been identified. A textbook named *Algebra I* is mentioned in a SRD report in April 1979. It is missing from libraries and could be the same book as the *Geometry*. It is also possible that the Algebra I and II are translations from a series by Bent Christiansen, Copenhagen, Matts Håstad, Stockholm and Ragnar Solvang, Oslo, revised in 1965 by Ove Hemer and Nils Sander, Malmö.[1070]

Rúmfræði / Geometry, by G. Bergendal, O. Hemer and N. Sander, published in 1970, translated by Anna Kristjánsdóttir, 80 pages including answers to exercises, contains an introduction to set theory (15 pages). The remaining text contains the basic concepts of plane geometry, such as points, lines, line segments, planes, angles and curves (10 pages), measuring length, angles and curves (18 pages), parallel lines, the theorem about the sum of the angles in a triangle and polygons (14 pages), measuring area (15 pages).

Algebra unglingaskóla II / Algebra II is 114 pages, and was probably taught in the second grade of lower secondary school. Here, negative integers are introduced (13 pages), factoring numbers, prime factors and exponents (14 pages), rational numbers and equations (34 pages), the place value number systems to the bases ten, two and eight (10 pages), decimal fractions (13 pages), approximation and scientific notation (14 pages) and percentages (15 pages).

Stærðfræði handa gagnfræðaskólum / Mathematics for lower secondary schools, translated by Helga Björnsdóttir, published in 1972, c. 160 pages, contains negative exponents, numerical values, unions and intersections and Cartesian products (28 pages), rational numbers and polynomials (44 pages), real numbers and the co-ordinate system (32 pages), linear equations and inequalities (53 pages).

[1070] Göran Emanuelsson, email May 21, 2006

Appendix N – High School Textbooks in 1964–1972

a. First Year

In 1967–1969, algebra was omitted in the Reykjavík High School in order to introduce sets, Cartesian products, graphs and polynomials from home-made study material. At Hamrahlíð High School, Guðmundur Arnlaugsson's *Algebra og hornafræði fyrir menntaskóla / Algebra and Trigonometry for High Schools* was abandoned in 1968 and replaced by home-made study material on algebra by Hildigunnur Halldórsdóttir. In 1969 both schools introduced *School Mathematics Project (SMP) Book T4* in the second semester of the first year.

In 1969–1971 Reykjavík High School returned to Jul. Petersen's *Rúmfræði / Geometry* and Guðmundur Arnlaugsson's *Algebra and Trigonometry*. Andersen et al.'s *Geometri / Geometry* was taught for one year in 1971–1972, together with a new book, *Stærðfræði handa 1. bekk menntaskóla / Mathematics for the First Grade of High School* by Hildigunnur Halldórsdóttir, in the second semester[1071]. From 1972 a new geometry textbook appeared, *Rúmfræði handa 1. bekk menntaskóla / Geometry for the 1st grade of High School* by Hörður Lárusson.[1072] It was taught in both schools.

b. Mathematics Stream

The *Principles of Mathematics* was taught at the mathematics stream of Reykjavík High School to four year-cohorts from 1964, for three years each. In 1968 Björn Bjarnason switched to the English textbook series *The School Mathematics Project* (SMP) *Advanced Mathematics I*, containing "modern" mathematics material, more oriented towards applied mathematics than the American material. When the pupils studying this new Anglo-Saxon syllabus reached the University, Guðmundur Arnlaugsson no longer taught mathematical analysis there. Soon these textbooks were regarded as providing too little calculus preparation for the mathematical analysis courses at the university.[1073] The geometry was also lacking.[1074]

From 1969, the mathematics stream was divided into a natural and physical science streams, and the physical science stream took a little more mathematics. The SMP series was taught to three year-groups. In 1971 a Swedish series, *Matematik för gymnasiet, NaTe*, by Bergendal, Håstad and Råde[1075] was introduced for the three-year science departments. This series was continued for several years.

When it was Guðmundur Arnlaugsson's turn to choose textbooks for the mathematics stream at his new Hamrahlíð High School in 1967, he reverted to the Danish set theoretical series by Kristensen and Rindung.[1076] It was used through the three-year course for two year-groups in 1967 and 1968. In 1969 *SMP Advanced Mathematics, Book 1* was taught to one year-group, while in 1970 the *NaTe* series by Bergendal, Håstad and Råde was introduced, to remain in use for several years.

[1071] Hildigunnur Halldórsdóttir (1972)
[1072] Hörður Lárusson (1972b)
[1073] Jón Ragnar Stefánsson, July 15, 2003
[1074] Jón Hafsteinn Jónsson, August 21, 2003
[1075] Bergendal, Håstad, Råde (1970) årskurs 1 for the first year. Same series, årskurs 2 and årskurs 3 for the two following years.
[1076] Kristensen, E. and Rindung, O. (1962)

Index

A

B

C

D

E

F

G

H

I

J

K

L

N

O

P

Q

R

S

T, Þ