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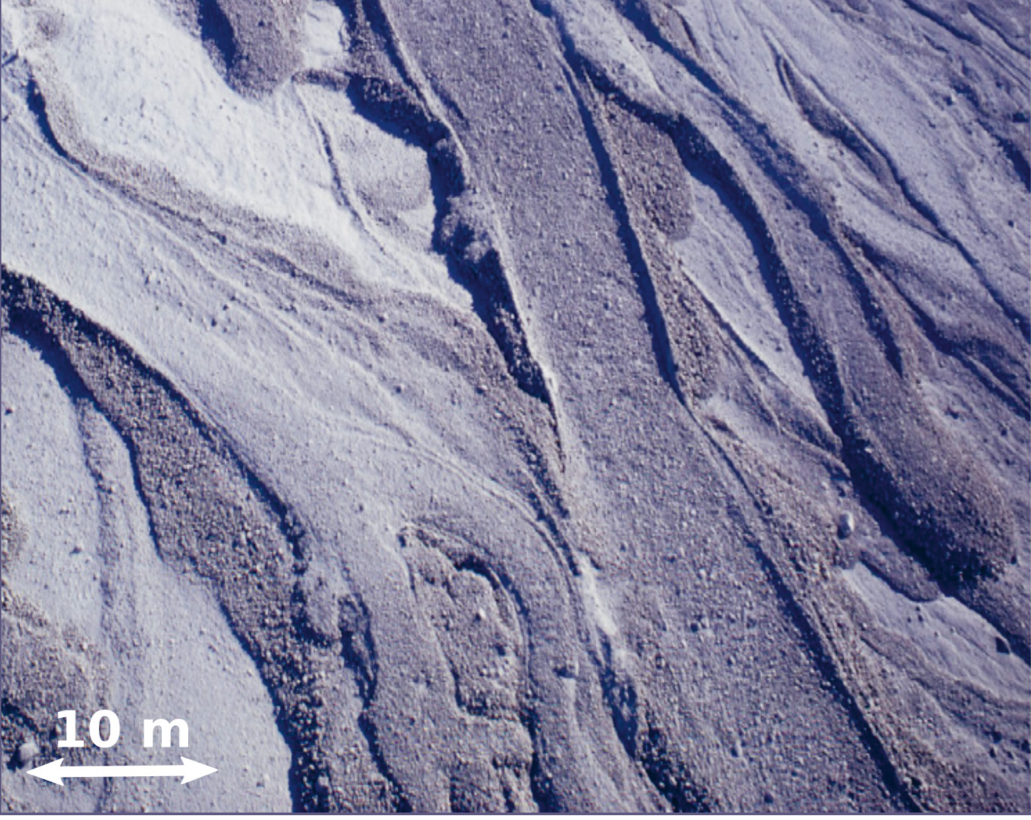

MODELLING EROSION AND DEPOSITION IN GEOPHYSICAL GRANULAR MASS FLOWS

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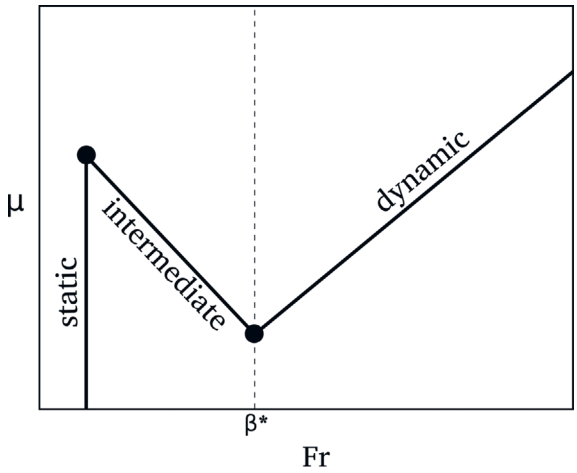
During hazardous geophysical mass flows, such as rock or snow avalanches, debris flows and volcanic pyroclastic flows, a continuous exchange of material can occur between the slide and the bed. The net balance between erosion and deposition of particles can drastically influence the behaviour of these flows. Recent advances in describing the non-monotonic effective basal friction and the internal granular rheology in depth averaged theories have enabled small scale laboratory experiments (see fig. 1) to be quantitatively reproduced and can also be implemented in large scale models to improve hazard mitigation.

Granular material is everywhere in our everyday life, from foodstuffs, industrial bulk materials and pharmaceuticals to the rings of Saturn and the surfaces of other planetary bodies. On Earth, geophysical mass flows are spectacular natural phenomena that pose a significant hazard to communities living in mountainous regions and on the flanks of volcanoes. They are composed of numerous grains of rock or ice of differing sizes and shapes, and which may be mixed with interstitial water or hot air. As these complex mixtures flow down a mountainside they behave in a liquid-like way. While the relationship between stress and strain (the rheology) in a Newtonian liquid such as water is well understood, much about this relationship is still unknown for granular materials, making modelling of geophysical mass flows a challenging task. Moreover, when an avalanche flows over an erodible substrate, a continuous exchange of granular material

▲ **FIG. 1:** Deposits of the July 22nd 1980 eruption of Mount St Helens, USA (left) (from Kokelaar *et al.* 2014, Photo courtesy of Dan Miller and USGS). Erosion-deposition wave in the laboratory (right) (from Edwards *et al.*, 2017)



▸ **FIG. 2:** Evolution of the friction μ as a function of the Froude number Fr for a constant flow thickness. The vertical dashed line represents the transition between dynamic and intermediate regimes which occurs at Fr = β* (Edwards *et al.* 2019).

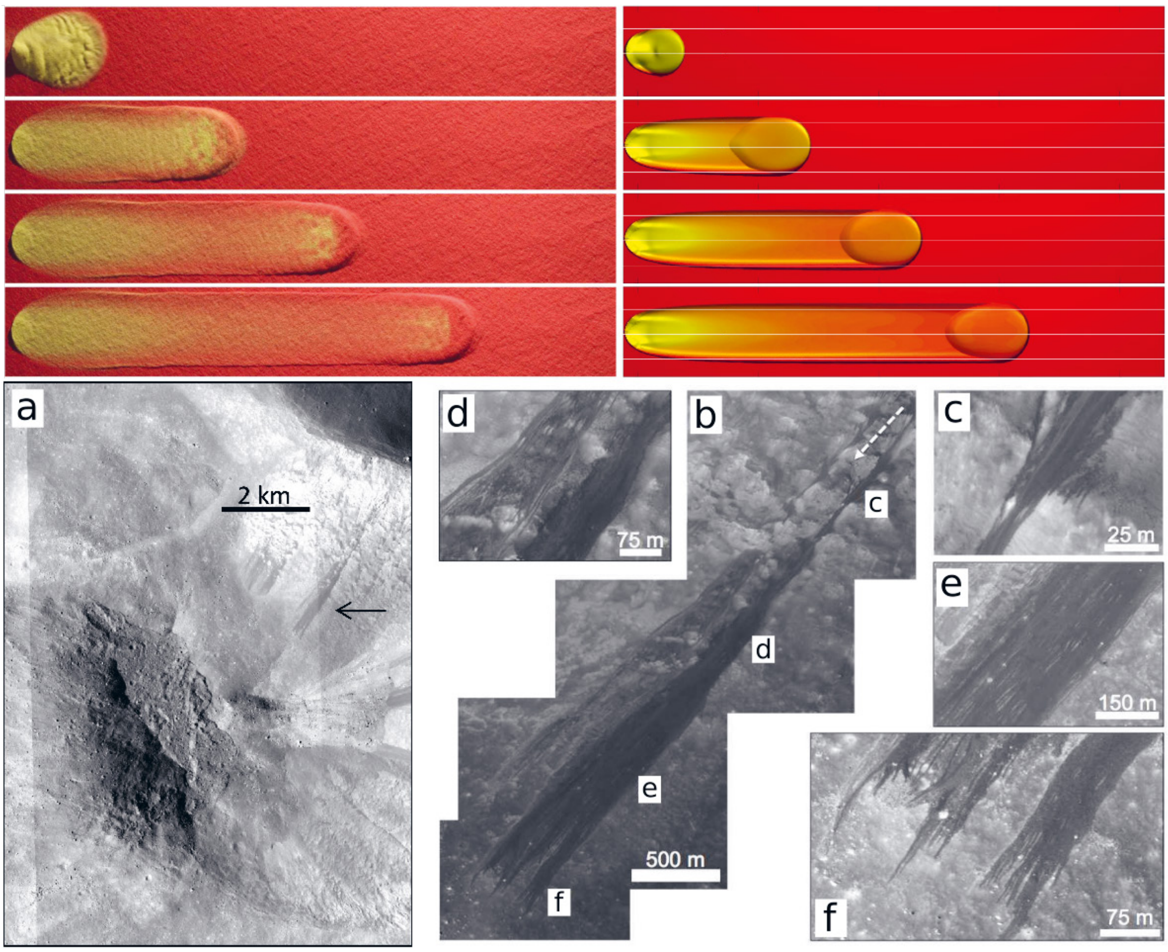
occurs between the avalanche and the bed as grains are eroded and deposited. When the avalanche erodes grains from the bed, its mass and run-out distance can be increased considerably, increasing the hazard posed.

In small scale experiments, when a granular material flowing steadily down a rough inclined plane is brought to rest by stopping the supply, it leaves a constant thickness static deposit on the plane. The deposited layer must, however, be inclined to a steeper angle before it begins to flow again. This exemplifies a key property of granular materials, namely hysteresis, where a layer of grains of a given thickness can exist in either a static or flowing equilibrium state. The origin of hysteresis is still not yet fully understood (Perrin *et al.* 2019), but it plays a key role in the transition between static and flowing regions of grains. Thus, an understanding of hysteresis is essential for modelling the exchange of mass between a flowing avalanche and its underlying substrate.

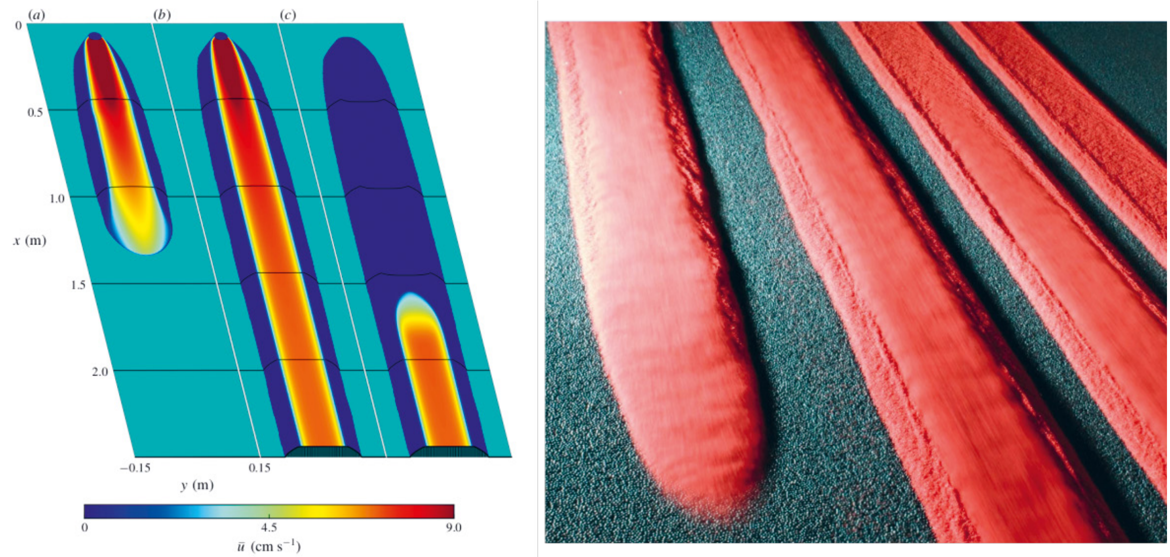
Is it a fluid, a solid or a gas?

The most visible aspect of powder snow avalanches and pyroclastic flows is a large cloud of dust and grains suspended by turbulent mixing in the surrounding air. Beneath this cloud, a rapid shallow dense granular flow moves in a liquid-like manner over a layer of static, but potentially erodible grains. It follows that grains in a single flow may behave as a gas, as a liquid, or as a solid, which complicates modelling.

Theoretical modelling of granular flow can be done via different methods depending on the size of the simulation and in a sense, how closely we look at the flow. When looking closely, each individual grain can be modelled as a solid sphere, where its motion is determined by Newton's laws and a contact model that describes the forces exerted by particles on one another. This discrete method is expensive in CPU time and consequently limited to a few million particles (about the number in half a cup of sand). A more macroscopic viewpoint treats granular material as a continuum, modelled as a fluid by solving the Navier-Stokes equations with a specific non-Newtonian granular rheology. Finally, at a large geophysical scale, a discrete



▸ **FIG. 3:** Steady travelling wave propagating on an erodible layer of constant thickness. Experiments (top left panel), corresponding numerical simulation via a depth-averaged model with viscous terms and a non-monotonic friction law (top right panel). Debris-flow deposit in the Virtanen F crater interpreted to result from the propagation of erosion-deposition waves (bottom panel) (from Viroulet *et al.* 2019).



◂ FIG. 4: (left) Numerical simulation of a granular flow from a constant inflow mass flux propagating steadily downslope and spontaneously forming static levees on either side of the central flowing channel. (right) Corresponding self-channelization experiment using monodisperse red sand on a non-erodible bed (Rocha *et al.* 2019).

simulation of every particle is far beyond today's computational resources, and even solving the Navier-Stokes equations for these complex three-dimensional flows would be a tremendous task. For these reasons, a depth-averaged approach was developed in the early 1990's (Savage & Hutter 1989). The shallowness of the flow enables the mass and momentum equations to be integrated through the flow depth assuming that the material is incompressible and the pressure is lithostatic. In doing this, the equations are reduced from three to two dimensions, which drastically simplifies their numerical solution.

This depth-averaged approach is widely exploited for modelling geophysical granular flows and is consequently used to calculate hazard maps and other operational tools for avalanche mitigation (Christen *et al.* 2010). Being able to include an accurate friction law in these models is of central importance to the predictions.

Rheology and granular hysteresis

Underneath the visible cloud of ash or snow in a geophysical event, a dense liquid-like granular flow rapidly propagates downslope. It is this part that is most destructive and therefore the most important to predict. The depth-averaged models still need to know about the effective basal friction and the depth-averaged viscosity, which are both dependent on the assumed rheology. In the early 2000's, a local rheology called the $\mu(I)$-rheology was developed (Jop *et al.* 2006) which directly relates the shear-stress τ to the pressure p by the friction μ, *i.e.* $\tau = \mu p$. Unlike the classical Mohr-Coulomb relation the friction is not constant, but is a function of the non-dimensional inertial number $I = \dot{\gamma} d / \sqrt{p/\rho}$ where $\dot{\gamma}$ is the shear-rate, and d and ρ are the particle size and density, respectively. Using velocity and pressure profiles derived from a steady-uniform solution of the $\mu(I)$-rheology, a depth-averaged viscous-like term can be constructed and included in the depth-averaged momentum equation (Gray & Edwards 2014; Baker *et al.* 2016). This new term introduces lateral variation in the downslope velocity across a flow in a channel and provides an important mechanism for wavelength selection (Rocha *et al.* 2019).

Despite the major breakthrough of the μ(I)-rheology, this rheology alone cannot model the simultaneous presence of static and flowing layers on an incline, and therefore cannot be used to study erosion/deposition or the formation of static levees in geophysical flows. To cope with this issue, a non-monotonic effective basal friction law, first suggested by Pouliquen & Forterre (2002), has been modified and extended by Edwards *et al.* (2019). Depending on the thickness of the flow and the value of the Froude number, which represents the ratio of the depth-averaged velocity to the gravity wave speed, dynamic, static and intermediate regimes are defined each with an associated friction. The dynamic and static regimes can be interpreted as the friction when the material is flowing or at rest respectively. The static to flowing transition is modelled via the intermediate regime. A plot of the effective basal friction μ as a function of the Froude number Fr for a constant flow thickness is shown on figure 2. It consists of a multivalued static friction, a velocity decreasing intermediate friction and a velocity increasing dynamic friction.

Geophysical granular flows in the laboratory

Direct observations of geophysical granular flows are extremely difficult because of their inherent unpredictability and the risks of staying near the area during the event. Moreover, for snow avalanches or pyroclastic flows there is often no direct observation of the dense granular part of the flow due to the cloud of snow or ash in the air. The scale invariance of the underlying theory suggests, however, that small scale analogue experiments can be performed to shed light into the physical processes at work. Figure 3 shows a comparison between small scale experiment, depth-averaged numerical simulations and deposits of granular erosion-deposition waves on the Moon.

The experiment was performed by releasing a small amount of yellow sand on a static erodible layer of identical red sand. Although the avalanche propagates steadily downslope, maintaining its shape and velocity, a continuous exchange occurs between the flow and the erodible layer. Yellow particles are progressively deposited along the flow path and the wave ends up completely composed of red particles (eroded from the bed) by the end of the chute. The exact balance between eroded and deposited particles allows the avalanche to propagate indefinitely, provided the slope angle and the erodible layer thickness is maintained. Depth-averaged numerical simulations that include both the viscous-like terms derived from the μ(I)-rheology and a non-monotonic friction law are in good qualitative and quantitative agreement with the experiments.

Dense granular flows also have the tendency to spontaneously self-channelize by forming static levees on each side of the flow (see fig. 1). This phenomenon appears regardless of whether an erodible layer is present and whether or not the flow is composed of a single grain size or particles of many different sizes. The ability of flows to self-channelize prevents lateral spreading and maintains flow depths for longer (Kokelaar *et al.* 2014) allowing avalanches to dramatically increase their overall runout. Figure 4 shows depth-averaged numerical simulations and small scale self-channelization experiments, which are in very good quantitative agreement with each other, and with an exact solution for the height, width and velocity profile across the central channel (Rocha *et al.* 2019). The theory also predicts the transition to unsteady pulsing flow below a critical mass flux. These are just two examples of how depth-averaged models can very efficiently and effectively reproduce small scale experiments of dry granular flow with application to geophysical events.

Upscaling the models to a geophysical scale

Although the new depth-averaged model with non-monotonic friction and viscous dissipation produces very good agreement with small scale experiments, quantitative prediction of flows at the geophysical scale remains a challenging issue. This is partly because the frictional parameters are more difficult to determine without being able to perform controlled experiments, but also because the depth-averaged theory implicitly assumes that the grains are either static or mobile throughout their entire depth. The theory is therefore much easier to apply to snow avalanches (where the maximum amount of entrainable snow may be defined by a clear horizon) than for a debris flow or rockfall. There are also other complications such as variable topography (Gray *et al.* 1999; Christen *et al.* 2010) and highly polydisperse materials whose grains size distribution evolves as part of the flow (Gray 2018). Nevertheless there are very strong similarities between leveed flow deposits observed in the field (Felix & Thomas 2004) as well as observations of regularly pulsing debris-flows at Illgraben in Switzerland (McArdell 2016), which suggest that the theory will be able to make useful predictions in situations of practical interest. ■

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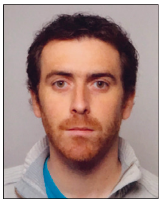

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