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Creating Kinematics-dependent Pedestrian Crossing Willingness Model When Interacting with Approaching Vehicle

Kai Tian¹, Gustav Markkula¹, Chongfeng Wei¹ and Richard Romano¹

Abstract—The interaction between automated vehicles (AVs) and vulnerable road users is increasingly important since the adoption of AVs is closer to reality. Particularly, the pedestrians' crossing behaviour are extremely complex, and it is difficult for AVs to predict pedestrians' decisions and motion behaviour. One of the important problems is how to characterize pedestrians crossing willingness (PCW), which is important for AV systems. Currently, few models have been proposed to characterize PCW. The most relevant models, pedestrian gap acceptance models, are mostly pure statistical approaches which are difficult to apply to a wide range of scenarios. In this paper, to avoid these drawbacks, we developed a novel PCW model by employing a continuously changing psychophysical stimulus, looming, which characterizes the visual information of approaching vehicles through the kinematics model of crossing scenario. In addition, a perception threshold is introduced to constrain the model. Results in this study showed that the PCW model can accurately capture the effects of the vehicle speed, distance and size on pedestrians' behaviour pattern. It was also found that pedestrians have maximum willingness to cross the street when this stimulus is beyond the perception threshold. We found that the model fit well with data collected from previous gap acceptance studies.

I. INTRODUCTION

For over a decade, AVs have been expected to be the most promising solution to protect vulnerable road users from traffic accidents. However, as research continues, some critical problems have emerged. How to interact with pedestrians is one of the challenges that needs to be solved. As predicted, there will be situations of mixed traffic with both pedestrians and AVs on the road, giving rise to uncertainty about safety in the near future [1]. Hence, the interaction process plays an essential role in these problems. Before the wide adoption of AVs, research on interaction between pedestrians and AVs is required.

Generally, the studies on the interaction between pedestrians and vehicles in the case of pedestrian crossing can be explored broadly as behavioural psychology research and modelling research. For behavioural psychology studies, researchers investigated the social aspects of crossing behaviour and identified various factors that affect the pedestrian's decision-making process through questionnaires, field tests or traffic videos. Those factors can be considered to revolve around external and internal factors. External factors include dynamic factors (e.g. gap size, time-to-collision (TTC), vehicle distance, vehicle speed, etc.), traffic factors (e.g. vehicle size, vehicle type, etc.) and environmental

factors (e.g. lane quantity, length of the lane, etc.) [2]. Internal factors consider humans' individual characteristics, including demographics (e.g. gender, age, etc.), social factors (e.g. group size, social status, culture, etc.) and psychological factors (e.g. temperament, etc.) [2]–[4]. For the research on modelling pedestrian's crossing behaviour, especially the crossing willingness, to the best of the author's knowledge, very few studies proposed to solve this problem. Some similar models which model pedestrians' acceptance of the available gap between vehicles were most relevant to this topic. Gap-acceptance models intend to capture the critical gap judgement of pedestrians, which can be defined as the time gap that half of the people would accept and others would reject [5], [6], in the interaction process. In the following section, several common gap-acceptance models will be introduced.

A number of models calculated the critical gap directly as a constant, called fixed critical gap methods. For instance, Raff's method [7] defined the critical gap as the value where the probability of accepted shorter gaps equals the probability of longer gap rejection. Those methods were easy to perform, but the obtained fixed rough approximations were difficult to apply in practice. Second, different from the fixed critical gap method, the maximum likelihood method supposed the gap-acceptance data as a random variable obeying a certain distribution [8]. Those models estimated the critical gap by assuming that the pedestrian's critical gap is between their largest rejected gap and accepted gap. However, all aforementioned methods have assumed that the pedestrians are homogenous. In other words, most of the factors of the pedestrian (e.g. gender, age, walking speed, etc.) were considered to be the same, which led to a poor generalization performance [8]. Since the crossing decision only has two alternatives, it is suitable to use a binary logistic regression (LR) model to deal with this problem [9], [10]. From Wang et al.'s study [10], the LR model took into account factors such as gap size, pedestrian number, age and gender to predict crossing decisions. Recently, with the rapid development of machine learning theory, artificial neural networks (ANN) has proven to have an excellent ability to fit complex non-linear relationships between multiple input features and predictions. Kadali et al.'s ANN model involved 14 factors to predict the behaviour of pedestrians [11].

Modelling interaction between pedestrians and vehicles is, of course, very challenging. The gap acceptance models mentioned above attempted to predict the pedestrian's behaviour by using pure statistics methods. These models ignored human perception processes, vehicle kinematics,

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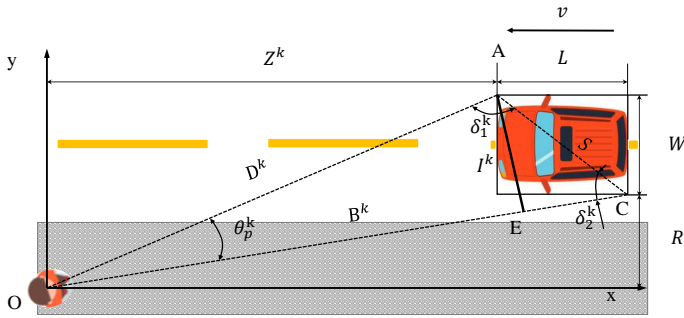


Fig. 1. The model schematics and the variables of the unmarked crossing scenario

lack of interpretability, and were not suitable for fine-grained interaction research. For instance, when pedestrians face a decelerating vehicle, the decision-making process is no longer a simple binary choice model, so LR method cannot deal with these situations. Moreover, since the ANN model is poor interpretable, it is difficult to analyze the individual effect of the single factor in the model. But recent literature suggests some possible directions: a continuous visual stimulus, looming, characterizes the information of an approaching vehicle to observers based on the spatial relationship between observers and vehicles [12]–[14]. The model has been used to control a driver's braking behaviour [12]. Moreover, Markkula et al. used looming as evidence to model the pedestrian crossing decision-making process and obtained reasonable results [13].

According to the literature review and description above, four main objectives of this paper will be pursued: (i) propose a novel PCW model which includes a corrected looming stimulus to characterize the pedestrian perceived visual information of approaching vehicles; (ii) illustrate that this model can qualitatively reproduce pedestrian crossing behaviour patterns (i.e. speed effects, distance dependence, and vehicle size effects) reported in the literature; (iii) illustrate this model can quantitatively fit to pedestrian behaviour data from previous study; (iv) illustrate that the potential applications of the model to pedestrian and AVs interactions.

II. METHODOLOGY

A. Model Framework

Since pedestrians crossing the street from both sides are similar, the following model only considers the situation with a one-way lane. To simplify the model, there is only one pedestrian interacting with one vehicle. As in Fig. 1, the position of the pedestrian is set at the origin of the coordinate axis. The vehicle is moving forward with speed v , while the pedestrian stands at the curb and observes the state of the vehicle. A set of variables constrain the spatial relationship between pedestrian and car. W and L refer to the width and length of the vehicle. S is the length of the diagonal of the vehicle. Z^k is the distance between the pedestrian and the vehicle or object. θ_p^k is the visual angle subtended by the approaching vehicle. R is the lateral distance from the car to the pedestrian. I^k is the length of the projection of the

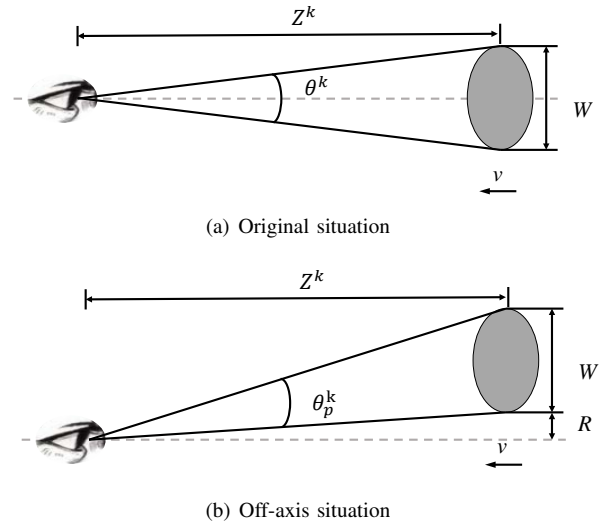


Fig. 2. The simple optic geometrics for looming [15]

vehicle in the direction of line of sight. The length of the OA line and OC line are D^k and B^k . The value of $\angle OAC$ and $\angle OCA$ are δ_1^k and δ_2^k . k are the time steps.

B. Looming Theory

Looming refers to the rapid expansion in the size of the images on the observer's retina. It is usually defined as the changing rate of the visual angle subtended by the object or the lead vehicle [16]. The basic optic geometrics of looming is presented in Fig. 2(a). An axisymmetric spherical object of width W approaches the observer's eye with a constant velocity v . The object subtends a visual angle θ^k and the derivative of θ^k refers to looming $\dot{\theta}^k$, which can be derived from the following functions. For small values of θ^k :

$$\dot{\theta}^k = \frac{\theta^k \cdot v}{Z^k} \quad (1)$$

where $v = -\dot{Z}^k$; $\theta^k \simeq \frac{W}{Z^k}$ [16]

When humans hunt prey or avoid danger, their movements or decisions need a precise spatial and temporal cue so that they can judge the position and motion of the target, and looming provides that kind of information. According to [16], [17], the $\dot{\theta}^k$ is visual available and simple to calculate.

C. Making Looming Fit for the Crossing Scenario

In the original looming theory [16], [17], the target is assumed to be an image which is symmetrical with respect to the line of sight. Although this assumption reasonably simplifies the model, it only works on a relatively small class of objects. For instance, in the crossing scenario, the moving vehicle is not an axisymmetric image, as in Fig. 1, so the original looming theory is not applicable to the crossing scenario. The case of pedestrian crossing is more likely an off-axis case, as in Fig. 2(b), where the object deviates R to one side of the axis and bypasses the observer. For more detailed information about the looming in the off-axis situation, please refer to [15]. To fit the original

looming theory to the crossing scenario, we revised the original looming $\dot{\theta}^k$ and proposed the $\dot{\theta}_p^k$ approach. In Fig. 1, when a small car approaches the pedestrian, the actual visual angle θ_p^k would be subtended by I^k . It is easy to find that the length I^k changes from W and L . To simplify the model, the W is considered as the maximum width of the vehicle front profile. Assuming W , L and R are given, we had the following equations:

$$S = \sqrt{W^2 + L^2} \quad (2)$$

$$D^k = \sqrt{(Z^k)^2 + (R + W)^2} \quad (3)$$

$$B^k = \sqrt{(Z^k + L)^2 + R^2} \quad (4)$$

$$\delta_1^k = \arctan\left(\frac{Z^k}{R + W}\right) + \arctan\left(\frac{L}{W}\right) \quad (5)$$

After that, according to the sines rule and the cosines rule, the actual visual angle θ_p^k and the projection width I^k are formulated as:

$$\theta_p^k = \arcsin\left(\frac{S \cdot \sin(\delta_1^k)}{B^k}\right) \quad (6)$$

$$\delta_2^k = \arcsin\left(\frac{D^k \cdot \sin(\delta_1^k)}{B^k}\right) \quad (7)$$

$$I^k = \sqrt{S^2 + (B^k - D^k)^2 - 2S(B^k - D^k) \cos(\delta_2^k)} \quad (8)$$

Finally, taking the temporal derivative of θ_p^k :

$$\dot{\theta}_p^k(Z^k, v, W, L, R) = -F_1(k)[F_2(k)F_5(k) \frac{1}{R+W} - F_3(k)F_7(k)]v \quad (9)$$

where

$$F_1(k) = \frac{1}{\sqrt{1 - F_6^2(k)}}; F_2(k) = \frac{S \cos(\delta_1^k)}{B^k}; F_3(k) = \frac{S \sin \delta_1^k}{(B^k)^2};$$

$$F_4(k) = \frac{Z^k}{R + W}; F_5(k) = \frac{1}{1 + F_4^2(k)}; F_6(k) = \frac{S \sin(\delta_1^k)}{B^k};$$

$$F_7(k) = [(Z^k + L)^2 + R^2]^{-1/2} \cdot (Z^k + L);$$

D. Capturing the Crossing Willingness of a Pedestrian

In the previous sections, we used the $\dot{\theta}_p^k$ approach to represent the pedestrian's sensory signals in the crossing scenario. In this section, A PCW model is developed based on the $\dot{\theta}_p^k$ approach. Generally, the probability of gap acceptance is modelled by the logit method [10]. Therefore, the probability of gap rejection is defined by the following equation:

$$p = \frac{e^{(\beta x + \omega)}}{1 + e^{(\beta x + \omega)}} \quad (10)$$

where x is the certain factor, β and ω are the coefficients controlling the relationship between x and p . To connect looming stimulus to crossing willingness, a proper model should be selected. Tarko depicted the driver's risk to speed by form of v^n , where n is a constant number within 2-4 [18]. Gupta et al. used $1/gap$ to describe pedestrian perceived-risk towards the approaching vehicle [19]. Moreover, Zhuang

et al. characterized pedestrian perceived-risk by assuming that risk is inversely proportional to the probability of gap acceptance [20]. Based on this, we assume that the crossing willingness is inversely proportional to the probability of rejection. Therefore, the crossing willingness should have the following form [20]:

$$PCW \propto \frac{1}{p}; \frac{1}{p} \propto e^{-(\beta x + \omega)} \rightarrow PCW \propto e^{-(\beta x + \omega)} \quad (11)$$

Adding $\dot{\theta}_p^k$ to (11), the PCW model is formulated as:

$$PCW(k) = e^{-(\beta \dot{\theta}_p^k + \omega)} \quad (12)$$

In (12), the coefficients β and ω are redefined as the sensitivity coefficients, because the humans' internal factors (e.g. age, gender, temperament, etc.) would affect their sensitivity to the stimuli from approaching vehicles [2]–[4]. It would be possible to use these coefficients to link the PCW model to those internal factors. Based on this idea, in the following section, the model will be improved and simplified by another perception theory.

E. The Looming Threshold

Although humans have an advanced perceptual system, it has been found that the capability of looming perception could be limited. Hoffmann et al. proposed the looming threshold theory, and found that the threshold value for adults may be between 0.002-0.003 rad/s [21]. Hence, we applied the threshold theory to the PCW model. Since pedestrians cannot perceive looming when it is equal and below the threshold, the crossing willingness should be maximum in these situations. Therefore, when $\dot{\theta}_p^k = \dot{\theta}_{thresh}$, $e^{-(\beta \dot{\theta}_p^k + \omega)}$ should equal to one and the final formulation of the model is:

$$PCW(k) = \begin{cases} e^{-\beta(\dot{\theta}_p^k - \dot{\theta}_{thresh})} & \dot{\theta}_p^k > \dot{\theta}_{thresh} \\ 1 & \dot{\theta}_p^k \leq \dot{\theta}_{thresh} \end{cases} \quad (13)$$

where $\dot{\theta}_{thresh}$ is the looming perception threshold. Usually, $\dot{\theta}_{thresh}$ for the adults (excluding the elderly) is 0.003 rad/s [21]. In (13), the unknown coefficients are reduced to one (i.e. β), where the ω is replaced by the product of $\dot{\theta}_{thresh}$ and β . In addition, several studies indicated that the threshold theory can be connected to some space after period. For instance, Hoffmann found TTC judgment performance is strongly dependent on the age of the observers. He indicted that the 5-6-year-old, 7-8-year-old, 9-10-year-old children and adults have different thresholds, about 0.04, 0.04, 0.008 and 0.002 rad/s [21]. Recent work by Wann et al. also showed that looming thresholds have strong developmental trends in sensitivity [22]. Based on the current research, it would be possible to make the PCW model suitable for pedestrians of different ages by adjusting the thresholds.

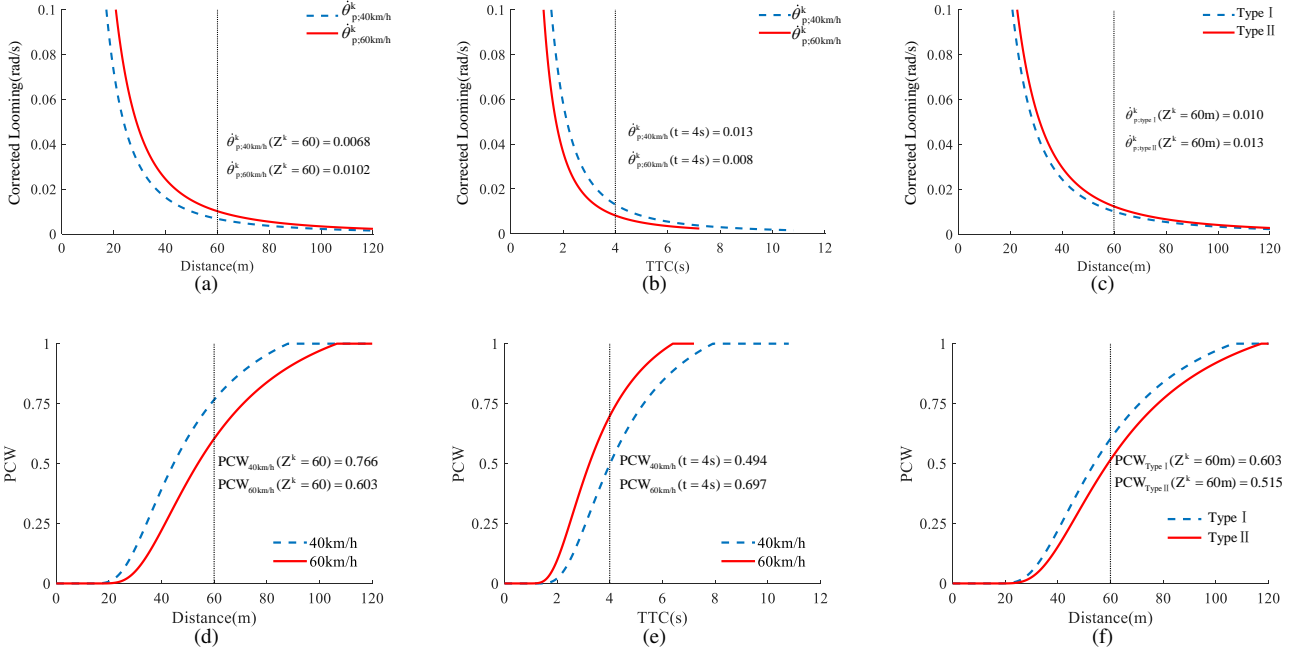


Fig. 3. The results of the numerical simulation experiments. The black dotted lines refer to the gaps between the pedestrian and the vehicle when pedestrian observes the vehicle. (a) The looming values in the speed effects simulation; (b) The looming values in the distance dependence simulation; (c) The looming values in the vehicle size effects simulation; (d) The results of the PCW model in the speed effects simulation; (e) The results of the PCW model in the distance dependence simulation; (f) The results of the PCW model in the vehicle size effects simulation.

III. NUMERICAL SIMULATION

In this section, three numerical simulations were carried out to analyze the PCW model. Based on distinct typical pedestrian crossing scenarios, we used the model to interpret pedestrian behaviour patterns (i.e. speed effects, distance dependence, and vehicle size effects). In these simulations, the risk sensitivity coefficient β and the looming threshold θ_{thresh} are selected appropriately and fixed. According to the experience and references, $\beta = 70$ and $\theta_{thresh} = 0.003$ rad/s were selected. Other parameters are shown in Table I.

A. The Effects of the Vehicle Speed

Assuming two vehicles of the same type were approaching the pedestrian at the same distance, 60m, with different speeds, 40km/h and 60km/h. The perceived $\hat{\theta}_p^k$ and PCW are shown in Fig. 3(a) and Fig. 3(d). Since the starting distance is the same, the change rate of the visual angle subtended by the vehicle with higher speed is bigger than the other one. Therefore, the relatively bigger sensory stimulus would increase the perceived-risk of pedestrian and inhibit PCW. As in Fig. 3(d), pedestrians are more likely to cross when facing a vehicle at slower speed. The results align with Oxley et al.'s and Lobjois et al.'s studies [23], [24], where it was shown that when the car came to 110m, the pedestrian's positive response to the vehicle at 40km/h was 100%, while the positive response to the vehicle at 60km/s was about 95%.

B. The Phenomenon of the Distance Dependence

We use the same type of vehicles at different speeds. Instead of assuming the same spatial gap, we set the two

vehicles approaching pedestrians at the equivalent temporal gap (i.e. 4s). This situation was studied by many researchers, and a pedestrian crossing behaviour pattern against common sense was observed, called distance dependence [23]–[25], i.e. at the same time gap, more pedestrians were willing to cross the road when the vehicle approaches at a higher speed. Since the time gap was the same, vehicles with relatively higher speed have bigger spatial gap sizes. Therefore, researchers proposed that, compared to the time gap or the velocity, pedestrians rely on the distance to make crossing decisions. However, in our work, we proposed an alternative explanation. First, the mainstream perception theories supposed that human perception systems use multiple clues to make a judgement (e.g. external factors, built-in functions or prior knowledge) [26]. Second, the PCW model captured the so-called distance dependence phenomenon by involving several cues (i.e. speed, distance, car size and relative position). In Fig. 3(b), $\hat{\theta}_{p;60km/h}^k$ is smaller than $\hat{\theta}_{p;40km/h}^k$ at 4s, which means the faster car has smaller looming stimulus to the pedestrians. The results of the PCW model show that pedestrian would be more likely to cross at a higher speed condition, as in Fig. 3(e).

C. The Effects of the Vehicle Size

Assuming two different types of vehicles, I (size: $1.8m \times 4.8m$) and II (size: $2.2m \times 6m$), drive to the pedestrian at the same speed, 60 km/h, from the same distance, 60m. From previous studies, although the two conditions had the same distance and time gap, different pedestrian behaviour patterns were observed. In [27], there was a clearly positive

relationship between the accepted gap sizes and the length of the vehicle. From the results of PCW model, as in Fig. 3(c) and Fig. 3(f), the pedestrian perceives a higher value of looming at the vehicle II condition, $\hat{\theta}_{p;I}^k(Z^k = 60) = 0.010 < \hat{\theta}_{p;II}^k(Z^k = 60) = 0.013$. It can be interpreted as the bigger car's approaching risk is higher than the small car, so the pedestrian would choose a longer gap size and not be willing to cross the street, $PCW_I = 0.603 < PCW_{II} = 0.515$.

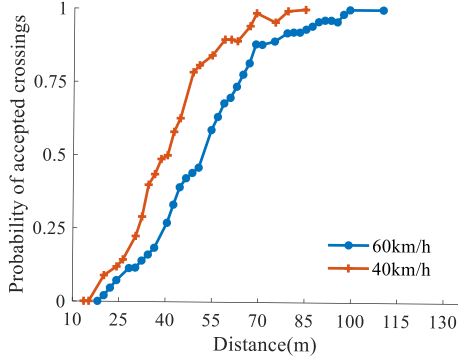
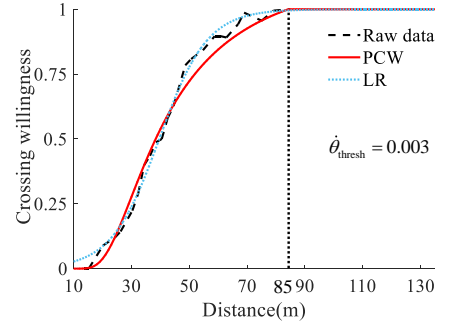


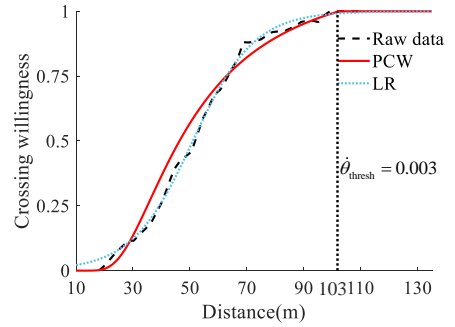
Fig. 4. The data of the percentage of crossing acceptance in spatial gaps for 20-30-year-old group [24].

IV. MODEL VALIDATION

To verify whether the PCW model can be used to analyze the experimental data, the pedestrian crossing data collected from Lobjois et al.'s work was used to calibrate the model [24]. The experimental setup meets our requirements and the fixed parameters used in the model are shown in Table. I. For more data information, please refer to the reference [24]. In this study, we adopted the acceptance probability of adults aged 20-30 years. Since the PCW model has one free parameter, it would be easy to calibrate the model through a non-linear least square estimation approach (NLLSE). Moreover, a logistic regression (LR) method was used for comparison. Figure 4 shows the discrete raw data, and the results of two models are illustrated in Fig. 5. In Table. II, both the PCW model and the LR method successfully fit the data with good R^2 . The overall SSE of the PCW model is 0.041 and 0.091 for 40km/h and 60km/h. The $RMSE$ of the PCW model is 0.041 and 0.050 for different speed conditions. Although the LR method can better fit the data within the distances from 55 to 70m in Fig. 5(a) and from 30 to 70m in Fig. 5(b), it cannot reach zero value when the distance is small enough. From the statistical point of view, we cannot be certain that no one will cross the street at such a close distance, but the actual data is that it is impossible for any normal person to cross the street within such a small distance, 10m-20m. The PCW model has more reasonable results at this distance, because it generates the intensive looming stimulus and extremely inhibits PCW. Moreover, the spatial gap at which the PCW values reach the $\hat{\theta}_{thresh}$ is in good agreement with the spatial gap of the raw data. That is, the maximum willingness to cross the street is reached when the pedestrian cannot perceive the looming. This may



(a) 40km/h



(b) 60km/h

Fig. 5. The fitting results of PCW method and logistic regression method as the function of speed ((a) 40 km/h; (b) 60km/h) and spatial gap. The black dotted lines refer to the spatial gap when looming reaches threshold.

TABLE I
THE LIST OF THE FIXED PARAMETERS USED IN THE SIMULATION AND THE MODEL FITTING [24]

Fixed parameters	$L(m)$	$W(m)$	$R(m)$	$\hat{\theta}_{thresh}(rad/s)$	$Z^k(m)$	$v(km/h)$	β
Simulation	4.8;6	1.8;2.2	3	0.003	0-120	40;60	70
Model fitting	4.42	1.72	2.09	0.003	10-135	40;60	-

imply that there exists a threshold in the human perception system that prevents them from judging the movements of the approaching objects from a long distance. This aligns with the looming threshold theory. Overall, the PCW model can fit the data as well as the LR method and has more reasonable results in the maximum and minimum willingness than the LR method.

V. CONCLUSION AND FUTURE WORK

In this study, we have modelled pedestrian crossing willingness at an unmarked roadway. It used an interpretable way to predict the willingness of pedestrians by employing looming theory and a kinematics model. After that, a threshold theory was introduced to constrain the model. Moreover, to verify and calibrate the PCW model, the model was tested and analyzed using numerical simulation and experimental data. Compared with previous related models, the PCW model has the following features. First, compared to the typical gap acceptance models (e.g. LR), the PCW model based on psychology theories is interpretable and has psychophysical

TABLE II
THE ESTIMATED COEFFICIENTS OF THE PCW MODEL AND THE
LOGISTIC REGRESSION MODEL

	Speed(km/h)	ω	β	SSE	$RMSE$	R^2
PCW model	40	-	54.17	0.041	0.041	0.985
	60	-	54.17	0.091	0.050	0.980
LR	40	-4.780	0.121	0.018	0.027	0.994
	60	-4.759	0.093	0.014	0.019	0.997

significance rather than a black box. Secondly, the model includes the kinematics of which vehicle and pedestrian, which enables information to pass between them. Finally, the PCW model is continuous in time, so it would be possible to develop a real-time model. In practice, the proposed model has two potential applications. First, since the looming stimulus is an important evidence reflecting the psychological state of pedestrians, it can be used to design a more realistic pedestrian simulation model and improve the driving or the traffic simulators. Secondly, As mentioned above, the model can use the trajectory data of the vehicle to calculate the crossing willingness of the pedestrian, so it has the potential to improve the development of AVs' decision-making strategies or control systems.

Before adapting the PCW model to other studies, several limitations still need to be addressed in the future. The experimental data used to analyze the model are not enough. For example, although the performance of the age 20-30 data seems good enough, the model might not work well with older people or children. Because of the relatively poor perceptual, cognitive, and motor abilities, their behaviour patterns may be significantly different from the young adults. Therefore, we need to use more data to calibrate the PCW model to fit the behaviour patterns of different groups. In addition, the scenario used in the model is too simplified, for instance, it only considered interactions involving a vehicle at constant speed. It is hoped that more detailed external information could be included in our final model.

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