Resonant Josephson Current Through a Quantum Dot

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Abstract. We calculate the DC Josephson current through a semiconducting quantum dot which is weakly coupled by tunnel barriers to two superconducting reservoirs. A Breit-Wigner resonance in the conductance corresponds to a resonance in the critical current, but with a different (non-lorentzian) lineshape. For equal tunnel rates $\Gamma_0/\hbar$ through the two barriers, the zero-temperature critical current on resonance is given by $(e/\hbar)\Delta_0\Gamma_0/(\Delta_0+\Gamma_0)$ (with Δ_0 the superconducting energy gap).

Superconductor–two-dimensional electron gas–superconductor (S-2DEG-S) junctions derive much of their recent interest from potential applications in a three-terminal transistor based on the Josephson effect. The ability of an electric field to penetrate into a 2DEG would allow one to modulate the critical supercurrent of the Josephson junction by means of the voltage on a gate electrode, in much the same way as one can modulate the conductance of a field-effect transistor in the normal state. Of particular interest are junctions which show a quantum-size effect on the conductance, since one would then expect a relatively large field-effect on the critical current [1]. Quantum-size effects on the conductance have been studied extensively in nanostructures such as quantum point contacts and quantum dots [2]. The corresponding effects on the Josephson current have received less attention.

Quantum-size effects on the Josephson current through a quantum point contact have been the subject of two recent theoretical investigations [3,4]. It was shown in Ref. [3] that, provided the point contact is short compared to the superconducting coherence length ξ_0, the critical current increases *stepwise* as a function of the contact width or Fermi energy, with step height $e\Delta_0/\hbar$ *independent* of the parameters of the junction—but only dependent on the energy gap Δ_0 in the bulk superconductor. This discretization of the critical current is analogous to the quantized conductance in the normal state. The present paper addresses the superconducting analogue of another familiar phenomenon in quantum transport: conductance resonances in a quantum dot.

Consider a small confined region (of dimensions comparable to the Fermi wavelength), which is weakly coupled by tunnel barriers to two electron reservoirs. At low temperatures and small applied voltages (small compared to the spacing ΔE of the bound states), conduction through this quantum dot occurs via resonant tunneling through a single bound state. Let ϵ_R be the energy of the resonant level, relative to the Fermi energy E_F in the reservoirs, and let

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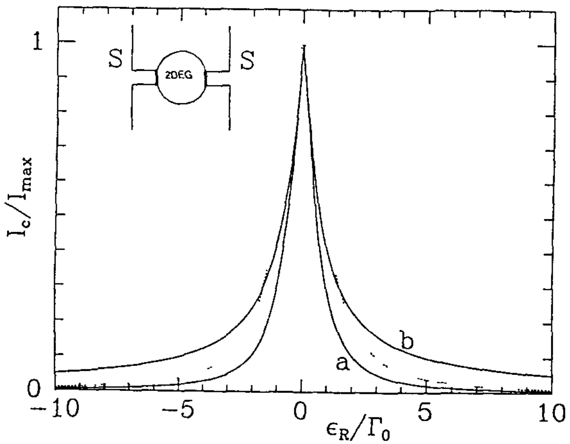

Figure 1: Normalized critical current versus energy of the resonant level at zero temperature and for equal tunnel barriers ($\Gamma_1 = \Gamma_2 \equiv \Gamma_0$). The two solid curves are the results (6) and (7) for the two regimes $\Gamma_0 \gg \Delta_0$ (curve a) and $\Gamma_0, \epsilon_R \ll \Delta_0$ (curve b). The dotted curve is the Breit-Wigner transmission probability (1). The inset shows schematically the S-2DEG-S junction.

$\Gamma_1/\hbar$ and $\Gamma_2/\hbar$ be the tunnel rates through the left and right barriers. We denote $\Gamma \equiv \Gamma_1 + \Gamma_2$. If $\Gamma \ll \Delta E$, the conductance G in the case of non-interacting electrons has the form

$$G = 2\frac{e^2}{h}\frac{\Gamma_1\Gamma_2}{\epsilon_R^2 + \frac{1}{4}\Gamma^2} \equiv 2\frac{e^2}{h}T_{BW}, \qquad (1)$$

where T_{BW} is the Breit-Wigner transmission probability at the Fermi level. The prefactor of 2 accounts for a two-fold spin-degeneracy of the level. Eq. (1) holds for temperatures $T \ll \Gamma/k_B$. (At larger temperatures a convolution with the derivative of the Fermi function is required.) The question answered below is: What does the Breit-Wigner lineshape for the conductance imply for the lineshape of the critical current? That problem has not been considered in earlier related work on Josephson tunnel-junctions containing resonant impurity levels in the tunnel barrier [5,6].

The geometry which we have studied is shown schematically in the inset of Fig. 1. It consists of two superconducting reservoirs [with pair potential $\Delta = \Delta_0 \exp(i\phi_{1,2})$], separated via tunnel barriers from a 2DEG quantum dot (with $\Delta = 0$). Instead of using the conventional tunnel-Hamiltonian approach, we treat the DC Josephson effect by means of a scattering formalism (which is more generally applicable also to non-tunneling types of junctions). In order to have a well-defined scattering problem in the 2DEG, we have inserted 2DEG leads, of width W and length L both much smaller than ξ_0, between the tunnel barriers and the reservoirs. Since $W \ll \xi_0$, the leads do not significantly perturb the uniform pair potential in the reservoirs. To obtain a simplified one-

dimensional problem, we assume that the leads support only one propagating mode and are coupled adiabatically to the reservoirs. From the Bogoliubov-De Gennes equation, and using the Breit-Wigner formula, we calculate the 4×4 scattering matrix $S(\epsilon)$ for quasiparticles of energy $E_F + \epsilon$. (The dimension of the scattering matrix is 4, rather than 2, because for each lead we have to consider both an electron and a hole channel, coupled by Andreev reflection at the 2DEG-S interface.) The scattering matrix yields the quasiparticle excitation spectrum, hence the free energy, and finally the Josephson current [7,8].

The discrete spectrum (obtained from the poles of S) consists of a single non-degenerate state at energy $\epsilon_0 \in (0, \Delta_0)$, satisfying

$$\Omega(\epsilon_0) + \Gamma\epsilon_0^2(\Delta_0^2 - \epsilon_0^2)^{1/2} = 0. \qquad (2)$$

The function $\Omega(\epsilon)$ is defined by

$$\Omega(\epsilon) = (\Delta_0^2 - \epsilon^2)(\epsilon^2 - \epsilon_R^2 - \tfrac{1}{4}\Gamma^2) + \Delta_0^2\Gamma_1\Gamma_2\sin^2(\delta\phi/2), \qquad (3)$$

where $\delta\phi \equiv \phi_1 - \phi_2$ is the phase difference between the superconducting reservoirs [9]. The continuous spectrum extends from Δ_0 to ∞ with density of states

$$\rho(\epsilon) = \text{constant} + \frac{1}{2\pi i}\frac{d}{d\epsilon}\ln\left(\frac{\Omega(\epsilon) + i\Gamma\epsilon^2(\epsilon^2 - \Delta_0^2)^{1/2}}{\Omega(\epsilon) - i\Gamma\epsilon^2(\epsilon^2 - \Delta_0^2)^{1/2}}\right), \qquad (4)$$

as follows from the relation $\rho = \text{constant} + (1/2\pi i)(d/d\epsilon)\ln(\text{Det}\, S)$ between the density of states and the scattering matrix [10]. The first "constant" term is independent of $\delta\phi$. The Josephson current–phase difference relationship $I(\delta\phi)$ is obtained from the excitation spectrum by means of the formula

$$I = -\frac{2e}{\hbar}\tanh\left(\frac{\epsilon_0}{2k_BT}\right)\frac{d\epsilon_0}{d\delta\phi} - \frac{2e}{\hbar}2k_BT\int_{\Delta_0}^{\infty} d\epsilon\, \ln\left[2\cosh\left(\frac{\epsilon}{2k_BT}\right)\right]\frac{d\rho}{d\delta\phi}, \qquad (5)$$

valid for a $\delta\phi$-independent $|\Delta|$ [7]. To evaluate Eq. (5), one has to compute the root of Eq. (2) and to carry out an integration. These two computations are easily done numerically, for arbitrary parameter values. Analytical expressions can be obtained in various asymptotic regimes. Here we only state results for the critical current $I_c \equiv \max I(\delta\phi)$, at $T = 0$.

In the limits of wide or narrow resonances we have, respectively,

$$I_c = \frac{e}{\hbar}\Delta_0[1 - (1 - T_{BW})^{1/2}], \text{ if } \Gamma \gg \Delta_0, \qquad (6)$$

$$I_c = \frac{e}{\hbar}(\epsilon_R^2 + \tfrac{1}{4}\Gamma^2)^{1/2}[1 - (1 - T_{BW})^{1/2}], \text{ if } \Gamma, \epsilon_R \ll \Delta_0. \qquad (7)$$

In both these asymptotic regimes only the discrete spectrum contributes to the Josephson current. As shown in Fig. 1, the lineshapes (6) and (7) of a resonance in the critical current (solid curves) differ substantially from the lorentzian lineshape (1) of a conductance resonance (dotted curve). For $\Gamma_1 = \Gamma_2$, I_c has a *cusp* at $\epsilon_R = 0$ (which is rounded at finite temperatures). The cusp in I_c reflects the cusp in the dependence of ϵ_0 on $\delta\phi$ at $|\delta\phi| = \pi$, following from Eq. (2). On resonance, the maximum critical current $I_{\max}$ equals $(2e\Delta_0/\hbar\Gamma)\min(\Gamma_1,\Gamma_2)$ and $(e/\hbar)\min(\Gamma_1,\Gamma_2)$ for a wide and narrow resonance, respectively. An analytical formula for the crossover between these two regimes can be obtained for the case of equal tunnel rates, when we find that

$$I_{\max} = \frac{e}{\hbar}\frac{\Delta_0\Gamma_0}{\Delta_0+\Gamma_0}, \text{ if } \Gamma_1 = \Gamma_2 \equiv \Gamma_0. \tag{8}$$

The characteristic temperature for decay of $I_{\max}$ is $\min(\Gamma,\Delta_0)/k_B$. Off-resonance, I_c has the lorentzian decay $\propto 1/\epsilon_R^2$ in the case $\Gamma \gg \Delta_0$ of a wide resonance, but a slower decay $\propto 1/\epsilon_R$ in the case $\Gamma, \epsilon_R \ll \Delta_0$. Near $\epsilon_R \simeq \Delta_0$ this linear decay of the narrow resonance crosses over to a quadratic decay (not shown in Fig. 1).

Since we have assumed non-interacting quasiparticles, the above results apply to a quantum dot with a small charging energy U for double occupancy of the resonant state. Glazman and Matveev have studied the influence of Coulomb repulsion on the resonant Josephson current [6]. The influence is most pronounced in the case of a narrow resonance, when the critical current is suppressed by a factor Γ/Δ_0 (for $U, \Delta_0 \gg \Gamma$). In the case of a wide resonance, the Coulomb repulsion does not suppress the Josephson current, but slightly broadens the resonance by a factor $\ln(\Gamma/\Delta_0)$ (for $U, \Gamma \gg \Delta_0$). The broadening is a consequence of the Kondo effect, and occurs only for $\epsilon_R < 0$, so that the resonance peak becomes somewhat asymmetric [6].

The scattering formulation of the DC Josephson effect presented here is sufficiently general to allow a study of more complicated systems than the two-lead geometry of Fig. 1. We are currently extending our results to include the influence on the resonant supercurrent of a third lead connecting the quantum dot to a 2DEG reservoir, motivated by Büttiker's treatment of the influence of inelastic scattering on conductance resonances [11].

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References

[1] H. van Houten, Appl. Phys. Lett. **58**, 1326 (1991).

[2] For a review of quantum transport in semiconductor nanostructures, see: C. W. J. Beenakker and H. van Houten, Solid State Physics **44**, 1 (1991).

[3] C. W. J. Beenakker and H. van Houten, Phys. Rev. Lett. **66**, 3056 (1991).

[4] A. Furusaki, H. Takayanagi, and M. Tsukada, Phys. Rev. Lett. **67**, 132 (1991).

[5] L. G. Aslamazov and M. V. Fistul', Sov. Phys. JETP **56**, 666 (1982); A. V. Tartakovskiĭ and M. V. Fistul', Sov. Phys. JETP **67**, 1935 (1988).

[6] L. I. Glazman and K. A. Matveev, JETP Lett. **49**, 659 (1989).

[7] C. W. J. Beenakker and H. van Houten, in: *Proc. Int. Symp. on Nanostructures and Mesoscopic Systems*, ed. by W. P. Kirk (Academic, New York, to be published).

[8] Alternative scattering approaches to the DC Josephson effect have been formulated by A. Furusaki and M. Tsukada, Solid State Comm. **78**, 299 (1991), and by B. J. van Wees, K.-M. H. Lenssen, and C. J. P. M. Harmans, Phys. Rev. B **44**, 470 (1991).

[9] In the case $\delta\phi = 0$, our Eq. (2) for the discrete spectrum agrees with P. I. Arseyev and B. A. Volkov, Solid State Comm. **78**, 373 (1991).

[10] E. Akkermans, A. Auerbach, J. E. Avron, and B. Shapiro, Phys. Rev. Lett. **66**, 76 (1991).

[11] M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988).