

Phenomenology of the superconducting state of a marginal Fermi liquid with BCS model interaction

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We study the effect of an additional simple BCS model interaction in a system in which the *dominant* scattering causes marginal-Fermi-liquid (MFL) behavior. It leads to a superconductor that shows a number of unusual features as compared to weak-coupling BCS superconductors. The superconducting transition temperature is lowered due to the MFL pair-breaking effects. If the bosonic mode that causes the MFL behavior, and which presumably is of electronic origin, is largely suppressed below T_c due to the gap opening up, $\Delta(0)$ remains practically unaffected by the MFL behavior. As a result, the ratio $2\Delta(0)/k_B T_c$ is enhanced, and the gap opens up much more rapidly than in the standard BCS approximation; even a gap discontinuity at T_c is possible. Furthermore, we find a suppression of the coherence peak in the nuclear-spin-relaxation rate and a more dramatic decrease below T_c of the ultrasonic attenuation rate and the penetration depth.

I. INTRODUCTION

The unconventional normal-state behavior of the high-temperature superconductors, as compared to normal metals, is well described by the marginal-Fermi-liquid (MFL) hypothesis.¹ Most of the anomalous normal-state properties can, according to this hypothesis, be understood from a single assumption about the polarizability of the (strongly interacting) system, namely,

$$\text{Im}P(\mathbf{q}, \omega) \propto \tanh(\omega/T).$$

It leads for instance to an electronic scattering rate

$$1/\tau = (\lambda\pi/2) \max(T, |\omega|),$$

where $\lambda = g^2 N(0)^2$, with g being the coupling of the electrons with the unidentified excitations that lead to the unusual polarizability and $N(0)$ the density of states at the Fermi energy.

Also the superconducting state of the high-temperature superconductors shows a number of unusual features in comparison with weak-coupling BCS superconductors. Most prominent are the absence of coherence peaks in the nuclear-spin-relaxation rate $1/T_1 T$ (Ref. 2) and in the conductivity $\sigma_1(\omega)$ (Ref. 3) (though in the latter quantity peaks arising from lifetime effects or thermal fluctuations can appear),⁴ and an unusually large ratio $2\Delta(0)/k_B T_c$, for which values up to 10 have been reported.⁵

The assumption that the dominant scattering occurs by exchange of the excitations that give rise to the unusual polarizability immediately leads to the observation that below T_c , as the gap opens up, this scattering process will be suppressed considerably (most dramatically if the coupling of the electrons to the polarizability is such that it leads to type-I coherence factors). As a consequence the quasiparticle lifetime will rapidly increase as the temperature is lowered. Measurements of the surface resistance of $\text{YBa}_2\text{Cu}_3\text{O}_7$ (Ref. 6) do indeed appear to give evidence

for a suppression of the quasiparticle scattering rate, and hence support the assumption about the dominant role of an electronic scattering process involving a nontrivial renormalization of the polarizability.

It is aesthetically most attractive to assume that the (yet unknown) physics underlying the MFL hypothesis is also the origin of the superconductivity.⁷ Kuroda and Varma⁸ and Littlewood and Varma⁹ have pursued this idea in an Eliashberg strong-coupling approach, in which the MFL polarizability plays the role of the bosonic mode that is responsible for the superconductivity, i.e., where the MFL polarizability replaces the usual electron-phonon coupling $\alpha^2 F(\omega)$. They find *s*-wave superconductivity with an enhanced ratio $2\Delta(0)/k_B T_c$ and a sharply decreasing $1/T_1 T$ below T_c rather than a coherence peak.

In this paper we shall show, without referring to any specific theory about the origin of the superconductivity, that the superconducting state is very much affected by the normal-state MFL behavior. We demonstrate this by analyzing the effect of an additional attractive BCS-type potential on a system in which the *dominant* scattering causes MFL behavior. The transition temperature T_c for this case can be determined from the ladder instability of the normal state. Below T_c , when the gap opens up, the suppression of the (electronic) bosonic mode is modeled by a scattering rate that decreases with decreasing temperature. The gap at zero temperature, $\Delta(0)$, remains practically unaffected by the MFL behavior above T_c if the MFL scattering is sufficiently suppressed at $T=0$. Consequently, the ratio $2\Delta(0)/k_B T_c$ is enhanced even for a weak-coupling superconductor, and the gap opens up rapidly; even a discontinuous first-order transition at T_c is possible. Furthermore, we find that this behavior of the gap together with the smearing of the singularity in the BCS density of states due to the MFL scattering suppresses the coherence peaks.

Although the *dominant* scattering leads to MFL behavior a weaker interaction can be the origin of the su-

perconductivity if retardation effects play a role, i.e., if the different interactions involve different time scales. It is therefore quite possible that the anomalous properties of the superconducting state of the cuprates are mainly due to the fact that the pairing occurs in a strongly correlated system (with MFL behavior), and that they are only weakly related to the origin of the pairing. Indeed, our results demonstrate a large degree of insensitivity of the unusual superconducting properties to the pairing mechanism. This becomes especially apparent by comparing our results to those of Littlewood and Varma,⁹ which are very similar, but obtained with the MFL boson taken as the origin of the superconductivity.

II. $2\Delta(0)/k_B T_c$ AND SUPPRESSION OF T_c

With the simple model form for the attractive BCS interaction (up to an energy ω_0) T_c can be determined from the ladder instability of the normal state. Here we neglect differences between the pair-formation temperature and the actual transition temperature. We take the normal-state propagators with the MFL self-energy,

$$\Sigma(\mathbf{k}, \omega) = \lambda [\omega \ln(x/\omega_c) - i(\pi x/2)].$$

Here $x = \max(|\omega|, T)$ and ω_c is a large cutoff scale, which is estimated to be of the order of 0.5 eV from Raman scattering;¹ also the infrared measurements of Rotter *et al.*,¹⁰ which show a quasiparticle scattering rate, which is linear in ω up to frequencies of 3000 cm^{-1} (0.37 eV) indicate a cutoff of this order. The value of λ can be determined from resistivity measurements. Taking from Ref. 11 the result of resistivity measurements up to 600 K on $\text{Bi}_2\text{Sr}_{2.2}\text{Ca}_{0.8}\text{Cu}_2\text{O}_8$ that

$$d\rho(T)/dT = 0.46 \mu\Omega \text{ cm/K},$$

we find $\lambda = 0.23$, assuming a plasma frequency of 1 eV. Though the value of λ is certainly not the same for all high- T_c materials, it is of this order of magnitude.

The temperature Green function in the normal state then is

$$G(\mathbf{k}, i\omega_n) = [i\omega_n - \epsilon(\mathbf{k}) - \Sigma(i\omega_n)]^{-1}, \quad (1)$$

where from analytic continuation of the marginal Fermi liquid $\Sigma(\mathbf{k}, \omega)$ it follows that

$$\Sigma(i\omega_n) = -i\lambda T \arctan\left[\frac{T}{\omega_n}\right] - \frac{1}{2}i\lambda\omega_n \ln\left[\frac{\omega_n^2 + \omega_c^2}{\omega_n^2 + T^2}\right]. \quad (2)$$

The pairing-instability occurs at the temperature where the particle-particle scattering amplitude

$$\Gamma[\mathbf{k}, \omega, \mathbf{k}, \omega; -\mathbf{k}, -\omega, -\mathbf{k}, -\omega]$$

diverges (Fig. 1),¹²

$$1 = N(0)V T \sum_{|\omega_n| < \omega_0} \int d\epsilon(\mathbf{k}) G(-\mathbf{k}, -i\omega_n) G(\mathbf{k}, i\omega_n), \quad (3)$$

where $N(0)$ is the density of states at the Fermi energy and V is the strength of the BCS-type interaction with a cutoff energy ω_0 . The $\epsilon(\mathbf{k})$ integration is straightforward, leading to

$$1 = 2N(0)V \sum_{n=0}^{\omega_0/2\pi T - (1/2)} \left\{ (2n+1) \left[1 + \frac{\lambda}{2} \ln\left[\frac{(2n+1)^2\pi^2 T^2 + \omega_c^2}{(2n+1)^2\pi^2 T^2 + T^2}\right] + \frac{\lambda}{(2n+1)\pi} \arctan\left[\frac{1}{(2n+1)\pi}\right] \right] \right\}^{-1}. \quad (4)$$

The remaining sum over the Matsubara frequencies can, under the condition that $\omega_0 < \omega_c$ or that λ is sufficiently small, be approximated by

$$2N(0)V \sum_{n=0}^{\omega_0/2\pi T - (1/2)} \frac{1}{(2n+1)[1 - \lambda \ln((2n+1)\pi T/\omega_c)]} \quad (5)$$

$$\approx 2N(0)V \sum_{n=0}^{\omega_0/2\pi T - (1/2)} \frac{1}{(2n+1)[1 + \lambda \ln(\omega_c/\pi T)]} \left[1 + \frac{\lambda \ln(2n+1)}{1 + \lambda \ln(\omega_c/\pi T)} \right]. \quad (6)$$

Performing the sum in the first term and writing the second sum as an integral gives the instability condition, which can be written as a quadratic equation for $\ln(\omega_0/\pi T_c) \equiv X$, $AX^2 + BX + C = 0$, with the coefficients

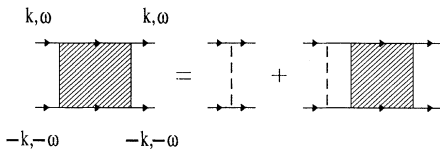


FIG. 1. The particle-particle ladder. The dashed lines denote simple BCS interactions, the propagators have a marginal Fermi liquid self-energy.

$$\begin{aligned} A &\equiv \frac{5\lambda}{4} - \frac{\lambda^2}{N(0)V}, \\ B &\equiv -\frac{2\lambda}{N(0)V}(1 - \lambda \ln \alpha) \\ &\quad - \lambda \left[\ln 2 + \Psi\left[\frac{1}{2}\right] \right] + (1 - \lambda \ln \alpha), \\ C &\equiv -\frac{1}{N(0)V}(1 - \lambda \ln \alpha)^2 \\ &\quad - \left[\ln 2 + \Psi\left[\frac{1}{2}\right] \right] (1 - \lambda \ln \alpha). \end{aligned} \quad (7)$$

Here $\Psi(x)$ is the digamma function and $\alpha \equiv \omega_0/\omega_c < 1$. The critical temperature then is found to be

$$T_c(\lambda) = \frac{\omega_0}{\pi} \exp \left[\frac{B - \sqrt{B^2 - 4AC}}{2A} \right]. \quad (8)$$

Taking for instance $\lambda=0.23$, $\omega_0=0.2$ eV, $\omega_c=0.5$ eV, and $N(0)V=0.5$ yields $T_c=91$ K. In the limit $\lambda \rightarrow 0$ one recovers the BCS critical temperature, $T_c^{\text{BCS}} = 1.13\omega_0 \exp[-1/N(0)V]$, which is higher than $T_c(\lambda > 0)$. In Fig. 2 we have plotted $T_c(\lambda=0)/T_c(\lambda_{ns})$ as a function of λ_{ns} for different values of $N(0)V$ and with $\alpha=0.4$. One sees that the stronger the BCS coupling is, the smaller the relative suppression of T_c due to the MFL scattering. Thus the critical temperature is suppressed as compared to the BCS value by the MFL behavior of the normal state. This is a natural consequence of the pair breaking caused by the inelastic MFL scattering.

An immediate consequence of the electronic origin of the MFL mode is that at low temperatures where the gap develops, the MFL mode itself will also develop a gap, and the low-energy scattering will be suppressed.⁹ The suppression of the quasiparticle scattering rate below T_c can be described phenomenologically by $1/\tau(T) = [\lambda(T)\pi/2]T$, where λ is constant for $T > T_c$, indicated by λ_{ns} . As explained above we estimate $\lambda_{ns}=0.23$ from the experiment of Ref. 11. Upon entering the superconducting state $\lambda(T)$ decreases.

In case the MFL interaction is completely frozen out at zero temperature the zero temperature gap $\Delta(0)$ will not be affected by the MFL scattering and thus assumes the BCS value. Therefore, the ratio $2\Delta(0)/k_B T_c$ increases entirely due to the effect of the MFL pair-breaking scattering on the transition temperature,

$$\frac{2\Delta(0)}{k_B T_c} = 3.53 \frac{T_c(\lambda=0)}{T_c(\lambda_{ns})}. \quad (9)$$

In general, if the MFL mode is not entirely suppressed at zero temperature, a smaller ratio is obtained. When a possible small effect of a temperature-independent inelas-

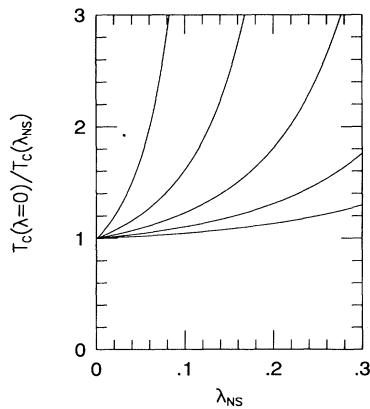


FIG. 2. $T_c(\lambda=0)/T_c(\lambda_{ns})$ vs λ_{ns} . The different curves correspond, from top to bottom, with values of $N(0)V=0.3, 0.45, 0.6, 0.75$, and 0.9 .

tic scattering on the ratio $2\Delta(0)/k_B T_c$ is neglected, one has

$$\frac{2\Delta(0)}{k_B T_c} = 3.53 \frac{T_c(\lambda(T=0))}{T_c(\lambda_{ns})}. \quad (10)$$

Thus with separate origins of the anomalous normal-state properties (arising from the MFL polarizability) and the superconductivity (originating from the BCS model interaction) a situation arises that resembles the strong-coupling situation with an enhanced value of $2\Delta(0)/k_B T_c$. The reason is that the inclusion of the MFL self-energy in the propagators of the particle-particle ladder describes the pair-breaking effects, which are suppressed at low temperatures. Such a suppression also occurs in the strong-coupling phonon situation, where the pair breaking due to thermal phonons has a relatively large effect on the transition temperature but only a small effect on the gap at zero temperature, where no thermal phonons are present.

In the extreme limit in which the quasiparticle scattering rate becomes negligible as soon as the gap opens up, the present model reduces to the BCS problem for all $T < T_c$. Then, $\Delta(T)$ follows a BCS dependence $\Delta_{\text{BCS}}(\lambda=0, T)$ for all $T < T_c(\lambda_{ns})$. The behavior of $\Delta_{\text{BCS}}(\lambda=0, T)$ is indicated by the uppermost dashed curve in Fig. 3. Clearly, in this extreme limit, Δ jumps to a finite value at $T_c(\lambda_{ns})$. In the general case in which the quasiparticle scattering rate remains nonvanishing below T_c , the opening up of the gap is less drastic, but still more

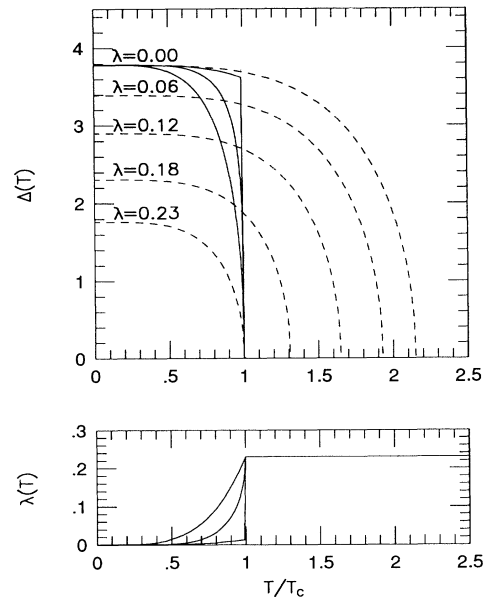


FIG. 3. The gap as a function of temperature (upper figure) with a temperature-dependent λ (lower figure) with $2\Delta(0)/k_B T_c \approx 7.8$. The curves in the lower figure correspond to curves in the upper figure: the lower the $\lambda(T)$ curve, the higher the $\Delta(T)$ curve. The dashed lines are BCS curves (thus with $2\Delta(0)/k_B T_c = 3.53$) through $T_c(\lambda)$, with the values of λ given in the figure. We used $N(0)V=0.6$, $\omega_c=0.5$ eV and $\omega_0=0.2$ eV.

rapid than in the BCS case: the quicker $\lambda(T)$ decreases below T_c , the quicker the gap opens up, and vice versa. Although, in principle, the full complex gap equations have to be solved below T_c to study this behavior, the general trend can be understood as follows. Consider a temperature-independent λ ; for a given value of λ the transition temperature $T_c(\lambda)$ is known from (8) to increase with decreasing λ . In a rough approximation, we can take for the temperature dependence of $\Delta(\lambda, T)$ for fixed λ a BCS curve through $T_c(\lambda)$. Curves of this type are indicated by dashed lines in Fig. 3. (The ratios of the T_c 's for $\lambda=0, 0.06, 0.12, 0.18$, and 0.23 are taken from the $N(0)V=0.6$ curve in Fig. 2.) If we now consider a temperature-dependent $\lambda(T)$ as the upper two curves drawn in the lower part of Fig. 3, we obtain a gap that "scans" the constant- λ curves between the $\lambda=0.23$ curve at T_c and the $\lambda=0$ curve at $T=0$. Evidently this leads to a relatively rapid opening up of the gap and a larger ratio $2\Delta(0)/k_B T_c$, which for the curve we have shown is about 7.8. In fact, if we take the scattering rate as a function of $\Delta(T)$ and T , rather than of T only, the opening up of the gap and the decrease of the scattering rate even show a discontinuity at T_c , very much like the behavior discussed above in the limit $\lambda \rightarrow 0$ for all $T < T_c$. This discontinuity is found to be very large when we use

$$\lambda(T < T_c)/\lambda_{ns} = 2/\{1 + \exp[\Delta(T)/T]\}.$$

(This ratio is the low frequency limit of the ultrasonic attenuation in BCS theory, i.e., a coherence-type-I suppression). Of course, it also leads to a discontinuity in λ at T_c . Finite frequencies (and presumably also correlation effects leading to a vertex renormalization) might weaken this drastic behavior, leading to a smaller jump of the gap at T_c and a remaining amount of inelastic scattering in the superconductive state. Experiments seem to agree that the temperature dependence of $\Delta(T)$ is very steep, if not discontinuous.⁴

III. COHERENCE PEAKS

BCS theory predicts coherence peaks in the nuclear-spin-relaxation rate and the microwave conductivity as a function of temperature. These peaks appear at about $0.85T_c$ and have a width of typically $0.3T_c$. They reflect the properties of the quasiparticles and the singular nature of the density of states just above the gap, $N_s(\omega) = N_n(0)\omega/\sqrt{\omega^2 - |\Delta|^2}$. In the high-temperature superconductors these coherence peaks are absent.² In Eliashberg strong-coupling theory (with an $\alpha^2F(\omega)$ from the MFL mode^{8,9} or from phonons¹³⁻¹⁵) it is found that they may disappear for sufficiently strong coupling.

With $2\Delta(0)/k_B T_c$ large, the coherence peaks become narrower and are located closer to T_c than in the BCS case. This is due to the same effect that leads to the steep behavior of the gap: As the parameter λ changes upon entering the superconductive state a scan of the BCS curves between the $\lambda=0.23$ curve at T_c and the $\lambda=0$ curve at $T=0$ (as described above for the gap) is made, as shown in Fig. 4. This yields a narrow coherence peak.

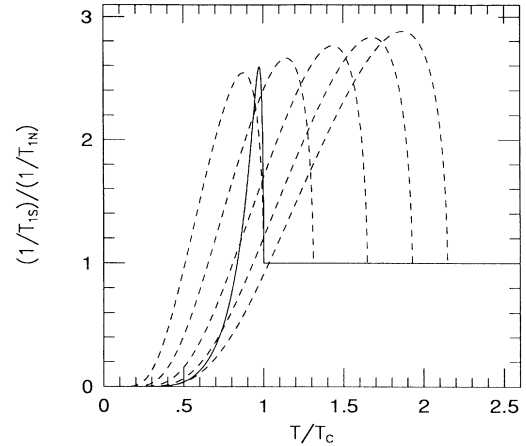


FIG. 4. Coherence peak (solid curve) with a temperature-dependent quasiparticle lifetime $1/\tau = [\lambda(T)\pi/2]T$, $\lambda(T)$ taken as the middle $\lambda(T)$ curve in Fig. 3, but without broadening of the density of states taken into account. The dashed curves are BCS coherence peaks, T_c determined by (8) with, from right to left, $\lambda=0, 0.06, 0.12, 0.18$, and 0.23 .

By using directly the steep gap, one also finds this narrow coherence peak. With λ being a function of $\Delta(T)$, as was considered above, the coherence peak disappears completely when the discontinuity of the gap, and thus of λ at T_c is sufficiently large. If λ changes continuously at T_c , always a narrow peak remains. The same mechanism leads to a more rapid suppression of the ultrasonic attenuation rate below T_c , as shown in Fig. 5.

Close to T_c , in the region of the narrow peak, the inelastic scattering is still active. This scattering, in addition to the BCS interaction, has the effect of smearing the square-root singularity in the BCS density of states. With a quasiparticle lifetime of the order of 0.2×10^{-13} sec close to T_c , as given by Bonn *et al.*,⁶ the spectral

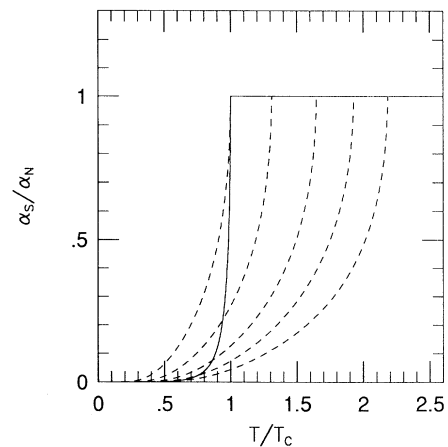


FIG. 5. The ultrasonic attenuation rate in the superconducting state normalized to the normal-state rate. The same values of λ as in Fig. 4 are used.

broadening near T_c is of the order of the zero-temperature gap in the cuprates. At lower temperatures the broadening is strongly reduced due to the suppression of the inelastic scattering.

The smearing of the BCS density of states due to the MFL scattering is shown in Fig. 6. It is calculated from the spectral function of a BCS superconductor, with an imaginary part of the self-energy $1/\tau = [\lambda(T)\pi/2]T$, i.e.,

$$\frac{N_s(\omega)}{N_n(0)} = \frac{1}{2\pi i} \int d\epsilon(\mathbf{k}) [G(\mathbf{k}, z \rightarrow \omega - i/\tau) - G(\mathbf{k}, z \rightarrow \omega + i/\tau)], \quad (11)$$

where

$$G(\mathbf{k}, z) = \frac{z + \epsilon(\mathbf{k})}{z^2 - E(\mathbf{k})^2} \quad (12)$$

and $E(\mathbf{k}) = \sqrt{\epsilon(\mathbf{k})^2 + |\Delta|^2}$. The integral can be evaluated

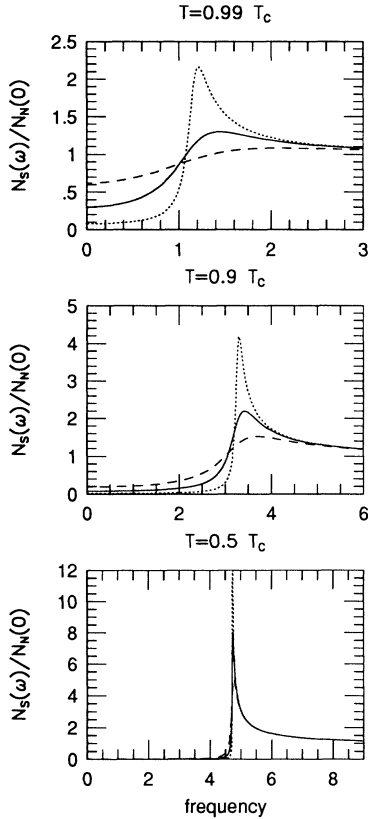


FIG. 6. The density of states in the superconducting state with the MFL interaction at different temperatures. At low temperatures the MFL interaction is suppressed and the BCS square-root singularity is recovered. At higher temperatures the MFL interaction is stronger, and the smearing of the singularity stronger. The dotted curve is obtained with a scattering rate of $0.02 \times \Delta(0)$ at T_c , the solid line with a scattering rate of $0.05 \times \Delta(0)$ at T_c and the dashed line with a scattering rate of $0.2 \times \Delta(0)$ at T_c . We used a decrease of the scattering rate as indicated by the solid curve in Fig. 3, i.e., $\lambda(T) = (T/T_c)^4 \lambda_{ns}$ below T_c . Even our largest scattering rate is modest compared to the one given by Bonn *et al.* (Ref. 6).

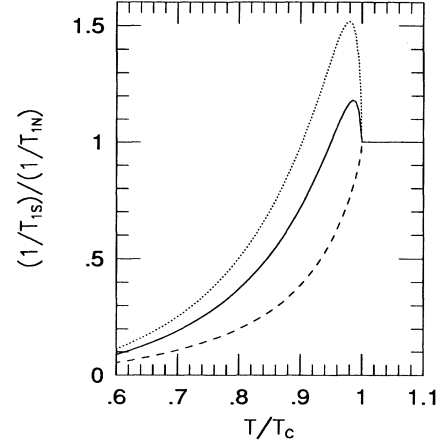


FIG. 7. The nuclear spin relaxation rate $1/T_1$ normalized to the normal-state value with broadening of the density of states taken into account. The scattering rates are the same as in Fig. 6.

exactly by contour integration:

$$\frac{N_s(\omega)}{N_n(0)} = \frac{1}{2\tau\gamma\sqrt{\beta^2 + \gamma^2}} [(\beta + \alpha^2)(2\sqrt{\beta^2 + \gamma^2} + 2\beta)^{1/2} + \gamma(2\sqrt{\beta^2 + \gamma^2} - 2\beta)^{1/2}] \quad (13)$$

where $\alpha^2 = \omega^2 + \Delta^2 + (1/\tau)^2$, $\beta = \omega^2 - \Delta^2 - (1/\tau)^2$, and $\gamma = 2\omega/\tau$. The quasiparticle spectrum obtained in this way is always gapless in case of nonvanishing MFL scattering, with the zero frequency value

$$N_s(\omega \rightarrow 0, T)/N_n(0) = 1/\sqrt{1 + \Delta(T)^2\tau(T)^2}.$$

In Fig. 6 we have shown $N_s(\omega)/N_n(0)$ for different temperatures and different scattering rates at T_c . The temperature dependence of the MFL scattering rate shows up in the decreased amount of broadening at low temperatures.

Such a broadening of the square root singularity in the density of states, which occurs especially near T_c , where the narrow coherence peak is located, suppresses the peak largely or totally, as is shown in Fig. 7. This suppression even occurs for scattering rates that are smaller by a factor of two than quoted above.

IV. CONCLUSIONS

We have considered a system of electrons with an (unidentified) dominant scattering mechanism that leads to marginal Fermi-liquid behavior and a BCS model interaction, that causes superconductivity. In absence of the MFL scattering the system would be a BCS superconductor. The MFL scattering is a pair-breaking process that lowers the critical temperature, which we calculated from the divergence of the particle-particle scattering amplitude. If the MFL mode is sufficiently suppressed below T_c it has little effect on the zero-temperature gap. Consequently, $2\Delta(0)/k_B T_c$ is enhanced. The gap opens up relatively rapidly, possibly even with a discontinuity at T_c . The inelastic MFL scattering also broadens the

square-root singularity in the density of states which, together with the enhanced value of $2\Delta(0)/k_B T_c$, leads to a suppression of the coherence peaks. Type-I-coherence behavior as followed by the ultrasonic attenuation is more rapidly suppressed below T_c than in the BCS case. This also applies for the penetration depth. Our simple description embodies the relevant features of the Eliashberg strong-coupling approach: pair breaking and broadening of the density of states due to inelastic scattering, and therefore indeed recovers many of the results obtained by Littlewood and Varma.⁹ However, in

our case the interaction that causes superconductivity is of BCS type, rather than the MFL mode. Our results also are similar to those obtained in numerical calculations by Bandte, Hertel, and Appel.¹⁶

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