

Giant backscattering peak in angle-resolved Andreev reflection

C. W. J. Beenakker, J. A. Melsen, and P. W. Brouwer

Instituut-Lorentz, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

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It is shown analytically and by numerical simulation that the angular distribution of Andreev reflection by a disordered normal-metal–superconductor junction has a narrow peak at the angle of incidence. The peak is higher than the well-known coherent backscattering peak in the normal state, by a large factor G/G_0 (where G is the conductance of the junction and $G_0=2e^2/h$). The enhanced backscattering can be detected by means of ballistic point contacts.

Coherent backscattering is a fundamental effect of time-reversal symmetry on the reflection of electrons by a disordered metal.^{1,2} The angular reflection distribution has a narrow peak at the angle of incidence, due to the constructive interference of time-reversed sequences of multiple scattering events. At zero temperature, the peak is twice as high as the background. Coherent backscattering manifests itself in a transport experiment as a small negative correction of order $G_0=2e^2/h$ to the average conductance G of the metal (*weak localization*³). Here we report the theoretical prediction, supported by numerical simulations, of a giant enhancement of the backscattering peak if the normal metal (N) is in contact with a superconductor (S). At the NS interface an electron incident from N is reflected either as an electron (normal reflection) or as a hole (Andreev reflection). Both scattering processes contribute to the backscattering peak. Normal reflection contributes a factor of 2. In contrast, we find that Andreev reflection contributes a factor G/G_0 , which is $\gg 1$.

If the backscattering peak in an NS junction is so large, why has it not been noticed before in a transport experiment? The reason is a cancellation in the integrated angular reflection distribution which effectively eliminates the contribution from enhanced backscattering to the conductance of the NS junction. However, this cancellation does not occur if one uses a ballistic point contact to inject the current into the junction. We discuss two configurations, both of which show an excess conductance due to enhanced backscattering which is a factor G/G_0 greater than the weak-localization correction.

We consider a disordered normal-metal conductor (length L , width W , mean free path l , with N propagating transverse modes at the Fermi energy E_F) which is connected at one end to a superconductor (see inset of Fig. 1). An electron (energy E_F) incident from the opposite end in mode m is reflected into some other mode n , either as an electron or as a hole, with probability amplitudes r_{nm}^{ee} and r_{nm}^{he} , respectively. The $N \times N$ matrices r^{ee} and r^{he} are given by⁴

$$r^{ee} = s_{11} - s_{12}s_{22}^*(1 + s_{22}s_{22}^*)^{-1}s_{21}, \quad (1a)$$

$$r^{he} = -is_{12}^*(1 + s_{22}s_{22}^*)^{-1}s_{21}. \quad (1b)$$

The s_{ij} 's are submatrices of the scattering matrix S of the disordered normal region,

$$S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix} = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \begin{pmatrix} -\sqrt{\mathcal{R}} & \sqrt{\mathcal{T}} \\ \sqrt{\mathcal{T}} & \sqrt{\mathcal{R}} \end{pmatrix} \begin{pmatrix} u' & 0 \\ 0 & v' \end{pmatrix},$$

where u, v, u', v' are $N \times N$ unitary matrices, $\mathcal{R} = 1 - \mathcal{T}$, and \mathcal{T} is a diagonal matrix with the transmission eigenvalues T_1, T_2, \dots, T_N on the diagonal.

We first consider zero magnetic field ($B=0$). Time-reversal symmetry then requires that S is a symmetric matrix, hence $u' = u^T$, $v' = v^T$. Equation (1) simplifies to

$$r^{ee} = -2u \frac{\sqrt{1-\mathcal{T}}}{2-\mathcal{T}} u^T, \quad r^{he} = -iu^* \frac{\mathcal{T}}{2-\mathcal{T}} u^T. \quad (2)$$

We seek the average reflection probabilities $\langle |r_{nm}|^2 \rangle$, where $\langle \dots \rangle$ denotes an average over impurity configurations. Following Mello, Akkermans, and Shapiro,⁵ we assume that u is uniformly distributed over the unitary group. This ‘‘isotropy assumption’’ is an approximation which ignores the finite

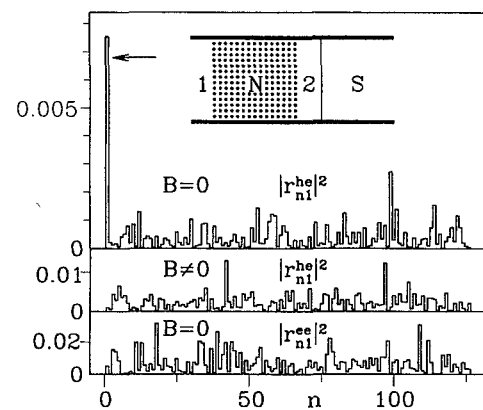


FIG. 1. Numerical simulation of a 300×300 tight-binding model for a disordered normal metal ($L=9.5l$), in series with a superconductor (inset). The histograms give the modal distribution for reflection of an electron at normal incidence (mode number 1). The top two panels give the distribution of reflected holes (for $B=0$ and $B=10 h/eL^2$), the bottom panel of reflected electrons (for $B=0$). The arrow indicates the ensemble-averaged height of the backscattering peak for Andreev reflection, predicted from Eq. (7).

time scale of transverse diffusion. The reflection probabilities contain a product of four u 's, which can be averaged by means of the formula⁶

$$\langle u_{ni} u_{mj} u_{nk}^* u_{ml}^* \rangle = (N^2 - 1)^{-1} (\delta_{ik} \delta_{jl} + \delta_{nm} \delta_{il} \delta_{jk}) - (N^3 - N)^{-1} (\delta_{il} \delta_{jk} + \delta_{nm} \delta_{ik} \delta_{jl}). \quad (3)$$

The result is [with the definition $\tau_k \equiv T_k(2 - T_k)^{-1}$]

$$\langle |r_{nm}^{ee}|^2 \rangle = \frac{\delta_{nm} + 1}{N^2 + N} \left(N - \left\langle \sum_k \tau_k^2 \right\rangle \right), \quad (4a)$$

$$\langle |r_{nm}^{he}|^2 \rangle = \frac{\delta_{nm} + 1}{N^2 + N} \left\langle \sum_k \tau_k^2 \right\rangle + \frac{N \delta_{nm} - 1}{N^3 - N} \left\langle \sum_{k \neq k'} \tau_k \tau_{k'} \right\rangle. \quad (4b)$$

In the metallic regime $N \gg L/l \gg 1$. In this large- N limit we may factorize $\langle \sum_{k \neq k'} \tau_k \tau_{k'} \rangle$ into $\langle \sum_k \tau_k \rangle^2$, which can be evaluated using

$$\left\langle \sum_k f(T_k) \right\rangle = (NI/L) \int_0^\infty dx f(1/\cosh^2 x). \quad (5)$$

The result for normal reflection is

$$\langle |r_{nm}^{ee}|^2 \rangle = (1 + \delta_{nm}) N^{-1} (1 - \frac{1}{2} l/L). \quad (6)$$

Off-diagonal ($n \neq m$) and diagonal ($n = m$) reflection differ by precisely a factor of 2, just as in the normal state.⁵ In contrast, for Andreev reflection we find

$$\langle |r_{nm}^{he}|^2 \rangle = \frac{1}{2} l/NL \quad (n \neq m), \quad \langle |r_{nn}^{he}|^2 \rangle = (\pi l/4L)^2. \quad (7)$$

Off-diagonal and diagonal reflection now differ by an order of magnitude $NI/L \approx G/G_0 \gg 1$.

Equations (6) and (7) hold for $B=0$. If time-reversal symmetry is broken (by a magnetic field $B \gtrsim B_c \equiv h/eLW$), then the matrices u, u', v, v' are all independent.⁷ Carrying out the average in the large- N limit, we find

$$\langle |r_{nm}^{ee}|^2 \rangle = N^{-1} (1 - \frac{1}{2} l/L), \quad \langle |r_{nm}^{he}|^2 \rangle = \frac{1}{2} l/NL. \quad (8)$$

Diagonal and off-diagonal reflection now occur with the same probability.

We have checked this theoretical prediction of a giant backscattering peak by a numerical simulation along the lines of Ref. 9. The disordered normal region was modeled by a tight-binding Hamiltonian on a two-dimensional square lattice (dimensions 300×300 , $N = 126$), with a random impurity potential at each site ($L/l = 9.5$). The scattering matrix S was computed numerically and then substituted into Eq. (1) to yield r^{ee} and r^{he} . Results are shown in Fig. 1. This is raw data from a single sample. For normal reflection (bottom panel) the backscattering peak is not visible due to statistical fluctuations in the reflection probabilities (speckle noise). The backscattering peak for Andreev reflection is much larger than the fluctuations and is clearly visible (top panel). A magnetic flux of $10 h/e$ through the disordered region completely destroys the peak (middle panel). The arrow in the top panel indicates the ensemble-averaged peak height from Eq. (7), consistent with the simulation within the statistical fluctuations. The peak is just one mode wide, as pre-

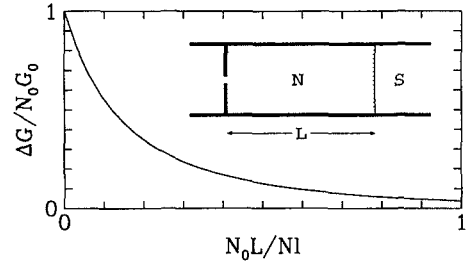


FIG. 2. Excess conductance $\Delta G = \langle G(B=0) \rangle - \langle G(B \gtrsim B_c) \rangle$ of a ballistic point contact in series with a disordered NS junction (inset), computed from Eqs. (11) and (12). At $B=0$ the contact conductance is twice the Sharvin conductance $N_0 G_0$, provided $N_0 L/Nl \ll 1$.

dicted by Eq. (7). If $W > L$ the isotropy assumption breaks down⁵ and we expect the peak to broaden over W/L modes. Figure 1 tells us that for $L=W$ the isotropy assumption is still reasonably accurate in this problem.

Coherent backscattering in the normal state is intimately related to the weak-localization correction to the average conductance. We have found that the backscattering peak for Andreev reflection is increased by a factor G/G_0 . However, the weak-localization correction in an NS junction remains of order G_0 .^{4,8} The reason is that the conductance

$$G = 2G_0 \sum_{n,m} |r_{nm}^{he}|^2 \quad (9)$$

contains the sum over all Andreev reflection probabilities,¹⁰ so that the backscattering peak is averaged out. Indeed, Eqs. (7) and (8) give the same G , up to corrections smaller by factors $1/N$ and l/L . In order to observe the enhanced backscattering in a transport experiment one has to increase the sensitivity to Andreev reflection at the angle of incidence. This can be done by injecting the electrons through a ballistic¹¹ point contact (width $\ll l$, number of transmitted modes N_0). For $B=0$, one can compute the average conductance from⁴

$$\langle G \rangle = 2G_0 \int_0^1 dT \rho(T) T^2 (2-T)^{-2}. \quad (10)$$

The density of transmission eigenvalues $\rho(T)$ is known,^{12,13} in the regime $N_0 \gg 1$, $N \gg L/l$. One finds

$$\langle G \rangle = G_0 \left[\frac{1}{2} (1 + \sin \vartheta) / N_0 + L/Nl \right]^{-1}, \quad (11a)$$

$$\frac{1}{2} \vartheta (1 + \sin \vartheta) = (N_0 L/Nl) \cos \vartheta, \quad \vartheta \in (0, \pi/2). \quad (11b)$$

In the absence of time-reversal symmetry ($B \gtrsim B_c$) we find from the large- N limit of Eqs. (1) and (9) that

$$\langle G \rangle = G_0 (1/N_0 + L/Nl)^{-1}. \quad (12)$$

This is just the classical addition in series of the Sharvin conductance $N_0 G_0$ of the point contact and the Drude conductance $(NI/L) G_0$ of the disordered region.

In Fig. 2 we have plotted the difference $\Delta G = \langle G(B=0) \rangle - \langle G(B \gtrsim B_c) \rangle$ of Eqs. (11) and (12). If $N_0/N \ll l/L \ll 1$ the conductance drops from $2N_0 G_0$ to

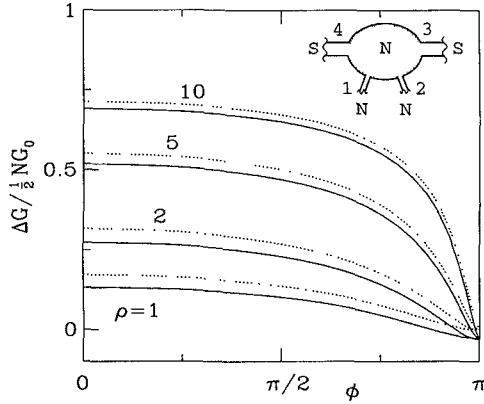


FIG. 3. Solid curves: excess conductance $\Delta G = \langle G(\phi) \rangle_{\text{COE}} - \langle G \rangle_{\text{CUE}}$ of a four-terminal Josephson junction (inset), computed (Ref. 20) from Eqs. (13) and (14) for $N_1 = N_2 \equiv N$, $N_3 = N_4 \equiv \rho N$, with $N = 10$. The dotted curves are the large- N limit (Ref. 21). The excess conductance at $\phi=0$ is a factor $G/G_0 = O(N)$ larger than the negative weak-localization correction at $\phi = \pi$.

$N_0 G_0$ upon breaking time-reversal symmetry. A doubling of the contact conductance at $B=0$ is well known¹⁴ in *ballistic* NS junctions ($l \gg L$). There it has a simple classical origin: An electron injected towards the NS interface is reflected back as a hole, doubling the current through the point contact. Here we find that the conductance doubling can survive multiple scattering by a disordered region ($l \ll L$), as a result of enhanced backscattering at the angle of incidence.

As a second example we discuss how enhanced backscattering manifests itself when electrons are injected into a Josephson junction. The system considered is shown schematically in Fig. 3. A disordered metal grain is contacted by four ballistic point contacts (with N_i modes transmitted through contact $i = 1, 2, 3, 4$). The scattering matrix S has submatrices s_{ij} , the matrix element $s_{ij, nm}$ being the scattering amplitude from mode m in contact j to mode n in contact i . The grain forms a Josephson junction in a superconducting ring. Coupling to the two superconducting banks is via point contacts 3 and 4 (phase difference ϕ , same electrostatic potential). Contacts 1 and 2 are connected to normal metals (potential difference V). A current I is passed between contacts 1 and 2 and one measures the conductance $G = I/V$ as a function of ϕ . Spivak and Khmel'nitskiĭ computed $\langle G(\phi) \rangle$ at temperatures higher than the Thouless energy.¹⁵ They found a periodic modulation of the weak-localization correction, with amplitude of order G_0 . Zaitsev and Kadigrobov *et al.* have discovered that at lower temperatures the amplitude increases to become much greater than G_0 .^{16,17} Here we identify enhanced backscattering as the origin of this increase.

The reflection matrices r^{ee} and r^{he} (with elements $r_{ij, nm}$) contain the combined effect of scattering in the normal grain (described by the matrix S) and Andreev reflection at the two contacts with the superconductor. By summing a series of multiple Andreev reflections we obtain expressions analogous to Eq. (1),

$$r^{ee} = a - b \Omega c^* \Omega^* (1 + c \Omega c^* \Omega^*)^{-1} d, \quad (13a)$$

$$r^{he} = -ib^* \Omega^* (1 + c \Omega c^* \Omega^*)^{-1} d, \quad (13b)$$

where we have abbreviated

$$a = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}, \quad b = \begin{pmatrix} s_{13} & s_{14} \\ s_{23} & s_{24} \end{pmatrix}, \quad c = \begin{pmatrix} s_{33} & s_{34} \\ s_{43} & s_{44} \end{pmatrix},$$

$$d = \begin{pmatrix} s_{31} & s_{32} \\ s_{41} & s_{42} \end{pmatrix}, \quad \Omega = \begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix}.$$

The four-terminal generalization of Eq. (9) is¹⁸

$$G/G_0 = R_{21}^{ee} + R_{21}^{he} + \frac{2(R_{11}^{he} R_{22}^{he} - R_{12}^{he} R_{21}^{he})}{R_{11}^{he} + R_{22}^{he} + R_{12}^{he} + R_{21}^{he}}, \quad (14a)$$

$$R_{ij}^{ee} = \sum_{n,m} |r_{ij, nm}^{ee}|^2, \quad R_{ij}^{he} = \sum_{n,m} |r_{ij, nm}^{he}|^2. \quad (14b)$$

Following Ref. 19, we evaluate $\langle G \rangle$ by averaging S over the circular ensemble. At $B=0$ this means that $S = UU^T$ with U uniformly distributed in the group $\mathcal{U}(M)$ of $M \times M$ unitary matrices ($M = \sum_{i=1}^4 N_i$). This is the circular orthogonal ensemble (COE). If time-reversal symmetry is broken, then S itself is uniformly distributed in $\mathcal{U}(M)$. This is the circular unitary ensemble (CUE). In the CUE we can do the average analytically for any N_i and ϕ . The result is

$$\langle G \rangle_{\text{CUE}} = G_0 N_1 N_2 / (N_1 + N_2), \quad (15)$$

independent of ϕ . In the COE we can do the average analytically for $N_i \gg 1$ and $\phi=0$, and numerically²⁰ for any N_i and ϕ . We find that the difference $\Delta G(\phi) = \langle G(\phi) \rangle_{\text{COE}} - \langle G \rangle_{\text{CUE}}$ is positive for $\phi=0$,

$$\frac{\Delta G(0)}{\langle G \rangle_{\text{CUE}}} = \rho + \frac{1}{2}(1 + \rho)^2 - (1 + \rho) \sqrt{\rho + \frac{1}{4}(1 + \rho)^2}, \quad (16)$$

with $\rho \equiv (N_3 + N_4) / (N_1 + N_2)$. The excess conductance (16) is a factor G/G_0 greater than the negative weak-localization correction, which is observable in Fig. 3 at $\phi = \pi$. For $N_i \gg 10$ the finite- N curves (solid) are close to the large- N limit²¹ (dotted) which we have obtained using the Green's function formulation of Refs. 13 and 16.

The excess conductance is a direct consequence of enhanced backscattering. This is easiest to see for the symmetric case $N_1 = N_2 \equiv N$, when $\langle R_{12}^{he} \rangle = \langle R_{21}^{he} \rangle$, $\langle R_{11}^{he} \rangle = \langle R_{22}^{he} \rangle$. Current conservation requires $R_{11}^{he} + R_{21}^{he} + R_{11}^{ee} + R_{21}^{ee} = N$. For $N \gg 1$ we may replace $\langle f(R_{ij}) \rangle$ by $f(\langle R_{ij} \rangle)$. The average of Eq. (14) then becomes

$$\langle G/G_0 \rangle = \frac{1}{2}N - \frac{1}{2}\langle R_{11}^{ee} - R_{21}^{ee} \rangle + \frac{1}{2}\langle R_{11}^{he} - R_{21}^{he} \rangle. \quad (17)$$

The first term $\frac{1}{2}N$ is the classical series conductance. The second term is the weak-localization correction due to enhanced backscattering for normal reflection. Since $\langle R_{11}^{ee} - R_{21}^{ee} \rangle = O(1)$ this negative correction to $\frac{1}{2}N$ can be neglected if $N \gg 1$. The third term gives the excess conductance due to enhanced backscattering for Andreev reflection. Since

$\langle R_{11}^{he} - R_{21}^{he} \rangle = O(N)$ this positive contribution is a factor $G/G_0 = O(N)$ greater than the negative weak-localization correction.

In conclusion, we have predicted (and verified by numerical simulation) an order G/G_0 enhancement of coherent backscattering by a disordered metal connected to a superconductor. The enhancement can be observed as an excess conductance which is a factor G/G_0 greater than the weak-

localization correction, provided ballistic point contacts are used to inject the current into the junction. The junction should be sufficiently small that phase coherence is maintained throughout. Several recent experiments²² are close to this size regime, and might well be equipped with ballistic point contacts.

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