Effect of a voltage probe on the phase-coherent conductance of a ballistic chaotic cavity

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The effect of an invasive voltage probe on the phase-coherent conduction through a ballistic chaotic cavity is investigated by random-matrix theory. The entire distribution $P(G)$ of the conductance G is computed for the case that the cavity is coupled to source and drain by two point contacts with a quantized conductance of $2e^2/h$, both in the presence ($\beta = 1$) and absence ($\beta = 2$) of time-reversal symmetry. The loss of phase coherence induced by the voltage probe causes a crossover from $P(G) \propto G^{-1+\beta/2}$ to a Gaussian centered at $G = e^2/h$ with a β-dependent width.

I. INTRODUCTION

A basic notion in mesoscopic physics is that the measurement of a voltage at some point in the sample is an invasive act, which may destroy the phase coherence throughout the whole sample. Büttiker introduced a simple but realistic model for a voltage probe,[1] and used it to investigate the transition from coherent to sequential tunneling through a double-barrier junction, induced by the coupling to a voltage lead of the region between the barriers. The mechanism by which the measurement of a voltage destroys phase coherence is that electrons which enter the voltage lead are reinjected into the system without any phase relationship. Büttiker's model has been applied successfully to a variety of physical situations,[2–9] including diffusive transport in a disordered wire, ballistic transport through quantum point contacts, and edge-channel transport in the quantum Hall effect. In order to analyze their experimental data, Marcus *et al.*[10] proposed to use Büttiker's model to describe inelastic processes in ballistic and chaotic cavities ("quantum dots"). Here, we present a detailed analysis of the effect of a voltage probe on the entire conductance distribution of such a system.

Several recent theoretical papers dealt with the phase-coherent conduction through a ballistic chaotic cavity, either by means of a semiclassical approach,[11] or by means of the supersymmetry method,[12–14] or by random-matrix theory.[15–17] Quantum interference has a striking effect on the conductance G of the quantum dot if it is coupled to source and drain reservoirs by means of two ballistic point contacts with a quantized conductance of $2e^2/h$. Classically, one would expect a conductance distribution $P(G)$, which is peaked at $G = e^2/h$, since half of the electrons injected by the source are transmitted on average to the drain. Instead, $P(G)$ was found to be[15,16]

$$P(G) \propto G^{-1+\beta/2}, \quad 0 \leq G \leq 2e^2/h, \tag{1.1}$$

where $\beta \in \{1, 2, 4\}$ is the symmetry index of the ensemble of scattering matrices ($\beta = 1$ or 2 in the absence or presence of a time-reversal-symmetry breaking magnetic field; $\beta = 4$ in zero magnetic field with strong spin-orbit scattering). Depending on β, the conductance distribution is either uniform, peaked at zero or peaked at $2e^2/h$. As we will show, strong coupling of the quantum dot to a voltage lead causes a crossover from Eq. (1.1) to a Gaussian, peaked at e^2/h. A small displacement of the peak of the Gaussian for $\beta = 1$, and a β-dependent width of the peak are the remnants of the weak localization and mesoscopic fluctuation effects which are so pronounced in the case of complete phase coherence.[15,16]

A strong coupling of the voltage probe is achieved by means of a wide ballistic lead with many scattering channels (Sec. IV). If the voltage lead contains a single channel, we may reduce the coupling to zero by means of a tunnel barrier in this lead (Sec. III). Together, these two sections cover the full range of coupling strengths. In the next section we first formulate the problem in some more detail, and discuss the random-matrix method used to compute the conductance distribution.

II. FORMULATION OF THE PROBLEM

We consider a ballistic and chaotic cavity (quantum dot) coupled by two leads to source and drain reservoirs at voltages V_1 and V_2. A current $I = I_1 = -I_2$ is passed from source to drain via leads 1 and 2. A third lead is attached to the quantum dot and connected to a third reservoir at voltage V_3. This third lead is a voltage probe, which means that V_3 is adjusted in such a way that no current is drawn ($I_3 = 0$). The coupling strength of the voltage probe is determined by the number N of scattering channels (propagating transverse modes at the Fermi level) in lead 3 and by the transparency of a tunnel barrier in this lead. We assume that each of the N modes has the same transmission probability Γ through the tunnel barrier. We restrict ourselves to the case that the current-carrying leads 1 and 2 are ideal (no tunnel barrier) and single-channel (a single propagating transverse mode). This case maximizes the quantum-interference effects on the conductance. We assume that the capacitance of the quantum dot is sufficiently large that we

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may neglect the Coulomb blockade, and we will regard the electrons to be noninteracting.

The scattering-matrix S of the system has dimension $M = N + 2$ and can be written as

$$S = \begin{pmatrix} r_{11} & t_{12} & t_{13} \\ t_{21} & r_{22} & t_{23} \\ t_{31} & t_{32} & r_{33} \end{pmatrix}, \tag{2.1}$$

in terms of reflection and transmission matrices r_{ii} and t_{ij}. The currents and voltages satisfy Büttiker's relations[18]

$$\frac{h}{2e^2} I_k = (N_k - R_{kk}) V_k - \sum_{l \neq k} T_{kl} V_l, \quad k = 1, 2, 3, \tag{2.2}$$

where $R_{kk} = \operatorname{tr} r_{kk} r_{kk}^\dagger$, $T_{kl} = \operatorname{tr} t_{kl} t_{kl}^\dagger$, and N_k is the number of modes in lead k. The two-terminal conductance $G = I/(V_1 - V_2)$ follows from Eq. (2.2) with $I_1 = -I_2 = I$, $I_3 = 0$:

$$G = \frac{2e^2}{h} \left(T_{12} + \frac{T_{13} T_{32}}{T_{31} + T_{32}} \right). \tag{2.3}$$

From now on, we will measure G in units of $2e^2/h$.

An ensemble of quantum dots is constructed by considering small variations in shape or Fermi energy. To compute the probability distribution $P(G)$ of the conductance in this ensemble, we need to know the distribution of the elements of the scattering matrix. Our basic assumption, following Refs. 15 and 16, is that for ideal leads the scattering matrix is uniformly distributed in the space of unitary $M \times M$ matrices. This is the circular ensemble of random-matrix theory.[19,20] The distribution $P_0(S)$ for the case $\Gamma = 1$ is therefore simply

$$P_0(S) = \frac{1}{V}, \tag{2.4}$$

where $V = \int d\mu$ is the volume of the matrix space with respect to the invariant measure $d\mu$. Both V and $d\mu$ depend on the symmetry index $\beta \in \{1, 2, 4\}$, which specifies whether S is unitary ($\beta = 2$), unitary symmetric ($\beta = 1$), or unitary self-dual ($\beta = 4$).

A characteristic feature of the circular ensemble is that the average $\bar{S}$ of the scattering matrix vanishes. For nonideal leads this is no longer the case, and Eq. (2.4), therefore, has to be modified if $\Gamma \neq 1$. In Ref. 17 we showed, for a quantum dot with two nonideal leads, how the probability distribution $P(S)$ of the scattering matrix can be computed by expressing the elements of the full scattering matrix S (quantum dot plus tunnel barriers) in terms of the scattering matrix S_0 of the quantum dot alone (with ideal leads). A more general analysis along these lines[21] shows that for an arbitrary number of leads the distribution takes the form of a Poisson kernel,[22,23]

$$P(S) = c \, |\det(1 - \bar{S}^\dagger S)|^{-\beta M - 2 + \beta}, \tag{2.5a}$$

with normalization constant

$$c = \frac{1}{V} [\det(1 - \bar{S}^\dagger \bar{S})]^{\frac{1}{2}\beta M + 1 - \frac{1}{2}\beta}. \tag{2.5b}$$

In the present case of two single-channel ideal leads and one nonideal lead, the average $\bar{S} = \int d\mu \, S P(S)$ of the scattering matrix is given by

$$\bar{S}_{nm} = \begin{cases} \sqrt{1 - \Gamma} & \text{if } 3 \leq n = m \leq M, \\ 0 & \text{otherwise.} \end{cases} \tag{2.5c}$$

One verifies that for $\Gamma = 1$, $P(S)$ reduces to the distribution (2.4) of the circular ensemble.

Equation (2.5) holds for any $\beta \in \{1, 2, 4\}$. In what follows, however, we will only consider the cases $\beta = 1, 2$ of unitary or unitary symmetric matrices, appropriate for systems without spin-orbit scattering. The case $\beta = 4$ of unitary self-dual matrices is computationally much more involved, and also less relevant from a physical point of view.

As indicated by Büttiker,[1] the cases $N = 1$ and $N > 1$ of a single-channel and multichannel voltage lead are essentially different. Current conservation (i.e., unitarity of S) poses two restrictions on T_{31} and T_{32}: (i) $T_{31} \leq 1$, $T_{32} \leq 1$; and (ii) $T_{31} + T_{32} \leq N$. The second restriction is effective for $N = 1$ only. So for $N = 1$, current conservation imposes a restriction on the coupling strength of the voltage lead to the quantum dot, which is not present for $N > 1$. We treat the cases $N = 1$ and $N > 1$ separately, in Secs. III and IV. For $N = 1$ we treat the case of arbitrary Γ, but for $N > 1$ we restrict ourselves for simplicity to $\Gamma = 1$.

III. SINGLE-CHANNEL VOLTAGE LEAD

In the case $N = 1$, Eq. (2.5) reduces to

$$P(S) = \frac{1}{V} \Gamma^{\beta+1} \left[1 + (1 - \Gamma) |S_{33}|^2 - 2(1 - \Gamma)^{1/2} \operatorname{Re} S_{33} \right]^{-\beta - 1}. \tag{3.1}$$

In order to calculate $P(G)$, we need to know the invariant measure $d\mu$ in terms of a parametrization of S, which contains the transmission coefficients explicitly. The matrix elements of S, in the case $N = 1$, are related to R_{kk} and T_{kl} by $S_{kk} = \sqrt{R_{kk}} e^{i\phi_{kk}}$, $S_{kl} = \sqrt{T_{kl}} e^{i\phi_{kl}}$, where ϕ_{kl} are real phase shifts. When time-reversal symmetry is broken ($\beta = 2$), we choose R_{11}, R_{22}, T_{12}, T_{21}, ϕ_{13}, ϕ_{23}, ϕ_{33}, ϕ_{32}, and ϕ_{31} as independent variables, and the other variables then follow from unitarity of S. In the presence of time-reversal symmetry ($\beta = 1$), the symmetry $S_{kl} = S_{lk}$ reduces the set of independent variables to R_{11}, R_{22}, T_{12}, ϕ_{13}, ϕ_{23}, and ϕ_{33}.

We compute the invariant measure $d\mu$ in the same way as in Ref. 15. Denoting the independent variables in the parametrization of S by x_i, we consider the change dS in S associated with an infinitesimal change dx_i in the independent variables. The invariant arclength $\operatorname{tr} dS^\dagger dS$ defines the metric tensor g_{ij} according to

$$\operatorname{tr} dS^\dagger dS = \sum_{i,j} g_{ij} dx_i dx_j. \tag{3.2}$$

The determinant $\det g$ then yields the invariant measure,

$$d\mu = |\det g|^{1/2} \prod_i dx_i. \tag{3.3}$$

The result turns out to be independent of the phases ϕ_{kl} and to have the same form for $\beta = 1$ and 2,

$$d\mu = (\beta J)^{-1/2} \Theta(J) \prod_i dx_i. \tag{3.4a}$$

The quantity J is defined by

$$J = \begin{cases} 0 \text{ if } R_{11} + T_{12} > 1 \text{ or } R_{22} + T_{21} > 1 \\ 4R_{22}T_{12}T_{13}T_{23} - (R_{22}T_{12} + T_{13}T_{23} - R_{11}T_{21})^2 \text{ otherwise,} \end{cases} \tag{3.4b}$$

and $\Theta(J) = 1$ if $J > 0$ and $\Theta(J) = 0$ if $J \leq 0$. The independent variables x_i are different, however, for $\beta = 1$ and $\beta = 2$ — as indicated above.

We have calculated the probability distribution of the conductance from Eqs. (2.3), (3.1), and (3.4). The results are shown in Fig. 1, for several values of Γ. For $\Gamma = 0$ (uncoupled voltage lead), $P(G)$ is given by[15,16]

$$P(G) = \begin{cases} \frac{1}{2} G^{-1/2} & \text{if } \beta = 1 \\ 1 & \text{if } \beta = 2. \end{cases} \tag{3.5}$$

For $\Gamma = 1$ (maximally coupled single-channel voltage lead), we find

$$P(G) = \begin{cases} 2 - 2G & \text{if } \beta = 1 \\ \frac{4}{3}\left[2G - 2G^2 - (3G^2 - 2G^3)\ln G - (1 - 3G^2 + 2G^3)\ln(1 - G)\right] & \text{if } \beta = 2. \end{cases} \tag{3.6}$$

The average $\langle G \rangle$ and variance $\operatorname{var} G$ of the conductance can be calculated in closed form for all Γ. We find that $\langle G \rangle$ is independent of Γ,

$$\langle G \rangle = \begin{cases} \frac{1}{3} & \text{if } \beta = 1 \\ \frac{1}{2} & \text{if } \beta = 2. \end{cases} \tag{3.7}$$

The variance does depend on Γ,

$$\operatorname{var} G = \begin{cases} \frac{1}{45}(1 - \Gamma)^{-2}\left(4 - 11\Gamma + 7\Gamma^2 - 3\Gamma^2 \ln \Gamma\right) & \text{if } \beta = 1 \\ \frac{1}{36}(1 - \Gamma)^{-3}\left(3 - 11\Gamma + 17\Gamma^2 - 9\Gamma^3 + 4\Gamma^3 \ln \Gamma\right) & \text{if } \beta = 2. \end{cases} \tag{3.8}$$

The breaking of phase coherence caused by a single-channel voltage lead is not strong enough to have any effect on the average conductance, which for $\beta = 1$ remains below the classical value of $\frac{1}{2}$. The variance of the conductance is reduced somewhat when Γ is increased from 0 to 1, but remains finite. (For $\beta = 1$ the reduction is with a factor $\frac{5}{8}$, for $\beta = 2$ with a factor $\frac{5}{9}$.) We will see in the next section that the complete suppression of quantum-interference effects requires a voltage lead with $N \gg 1$. Then $\langle G \rangle \to \frac{1}{2}$ and $\operatorname{var} G \to 0$.

IV. MULTICHANNEL VOLTAGE LEAD

Now we turn to the case of a multichannel ideal voltage lead ($N > 1$, $\Gamma = 1$). Current conservation yields,

$$\begin{aligned} T_{13} &= 1 - R_{11} - T_{12} = 1 - |S_{11}|^2 - |S_{12}|^2, \\ T_{31} &= 1 - R_{11} - T_{21} = 1 - |S_{11}|^2 - |S_{21}|^2, \\ T_{32} &= 1 - R_{22} - T_{12} = 1 - |S_{12}|^2 - |S_{22}|^2. \end{aligned} \tag{4.1}$$

To determine $P(G)$ it is thus sufficient to know the distribution $\tilde{P}(S_{11}, S_{12}, S_{21}, S_{22})$ of the matrix elements S_{kl} with $k, l \leq 2$. This marginal probability distribution has been calculated by Mello and co-workers[24] for arbitrary dimension $M \geq 4$ of S. As in Sec. III, we parametrize $S_{kl} = \sqrt{T_{kl}} e^{i\phi_{kl}}$ if $k \neq l$ and $S_{kk} = \sqrt{R_{kk}} e^{i\phi_{kk}}$ $(k, l \leq 2)$. We abbreviate $\prod_i dy_i \equiv dR_{11} dR_{22} dT_{12} dT_{22} \prod_{k,l=1}^{2} d\phi_{kl}$. For the cases $\beta = 1, 2$ one then has[24]

$$d\tilde{P} = \begin{cases} c_1 \delta(T_{12} - T_{21}) \delta(\phi_{12} - \phi_{21}) F^{(M-5)/2} \Theta(F) \prod_i dy_i & \text{if } \beta = 1 \\ c_2 F^{M-4} \Theta(F) \prod_i dy_i & \text{if } \beta = 2, \end{cases} \tag{4.2a}$$

where F is defined by

$$F = \begin{cases} 0 \text{ if } R_{11} + T_{12} > 1 \text{ or } R_{22} + T_{21} > 1 \\ (1 - R_{11})(1 - R_{22}) + (1 - T_{12})(1 - T_{21}) - 1 - 2(R_{11}R_{22}T_{12}T_{21})^{1/2} \cos(\phi_{11} + \phi_{22} - \phi_{12} - \phi_{21}) \text{ otherwise.} \end{cases} \tag{4.2b}$$

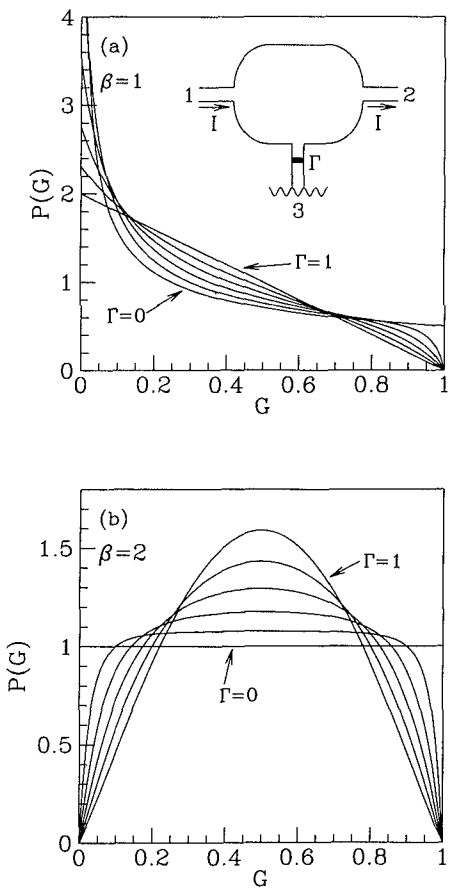

FIG. 1. Distribution of the conductance G (in units of $2e^2/h$) for a single-channel voltage lead ($N = 1$). The voltage lead contains a tunnel barrier with transmission probability Γ, which varies from 0 to 1 with increments of 0.2. (a) Time-reversal symmetry ($\beta = 1$), (b) broken time-reversal symmetry ($\beta = 2$). The quantum dot is shown schematically in the inset.

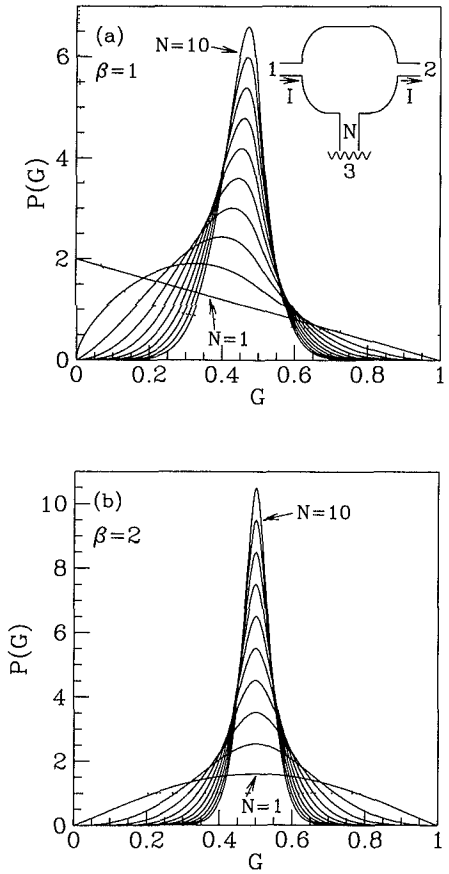

FIG. 2. Conductance distribution for a multichannel ideal voltage lead ($\Gamma = 1$). The number N of transverse modes in the lead varies from 1 to 10 with increments of 1 (solid curves). The dotted curve is the distribution in the absence of a voltage lead. The cases $\beta = 1$ and 2 are shown in (a) and (b), respectively.

The coefficients c_1 and c_2 are normalization constants. Calculation of the probability distribution of the conductance is now a matter of quadrature.

Results are shown in Fig. 2, for N up to 10. As N increases, $P(G)$ becomes more and more sharply peaked around $G = \frac{1}{2}$. In the limit $N \to \infty$, $P(G)$ approaches a Gaussian, with mean and variance given by

$$\langle G\rangle = \begin{cases} \frac{1}{2} - \frac{1}{2}N^{-1} + O(N^{-2}) & \text{if } \beta = 1 \\ \frac{1}{2} & \text{if } \beta = 2, \end{cases} \tag{4.3}$$

$$\operatorname{var} G = \begin{cases} \frac{3}{4}N^{-2} + O(N^{-3}) & \text{if } \beta = 1 \\ \frac{1}{4}N^{-2} + O(N^{-3}) & \text{if } \beta = 2. \end{cases} \tag{4.4}$$

The variance of G is reduced by a factor 3 when time-reversal symmetry is broken in the limit $N \to \infty$. The offset of $\langle G\rangle$ from $\frac{1}{2}$, when $\beta = 1$ is a remnant of the weak localization effect.

V. CONCLUSION

We have calculated the entire probability distribution of the conductance of a quantum dot in the presence of a voltage probe, for single-channel point contacts to source and drain, in the presence and absence of time-reversal symmetry (no spin-orbit scattering). The average conductance is not changed if a single-channel voltage lead containing a tunnel barrier is attached, but the shape of the distribution changes considerably. A strikingly simple result is obtained for a single-channel ballistic voltage lead in zero magnetic field ($N = 1$, $\Gamma = 1$, $\beta = 1$), when $P(G) = 2 - 2G$, to be compared with $P(G) = \frac{1}{2}G^{-1/2}$ without the voltage probe.[15,16] (In both cases $G \in [0, 1]$ is measured in units of $2e^2/h$.) When the number N of channels in the voltage lead is increased, the probability distribution becomes sharply peaked around $G = \frac{1}{2}$. Both the width of the peak and the deviation of its center from $\frac{1}{2}$ scale as $1/N$ for $N \gg 1$. The width is reduced by a factor $\sqrt{3}$ upon breaking the time-reversal symmetry.

The loss of phase coherence induced by a voltage probe can be investigated experimentally by fabricating a cavity with three leads attached to it. Furthermore, as emphasized by Marcus *et al.*,[10] the inelastic scattering which occurs at finite temperatures in a quantum dot might well be modeled effectively by an imaginary voltage lead.

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