Spontaneous Emission in Chaotic Cavities

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The spontaneous emission rate Γ of a two-level atom inside a chaotic cavity fluctuates strongly from one point to another because of fluctuations in the local density of modes For a cavity with perfectly conducting walls and an opening containing N wave channels, the distribution of Γ is given by $P(\Gamma) \propto \Gamma^{N/2-1}(\Gamma + \Gamma_0)^{-N-1}$, where Γ_0 is the free-space rate For small N the most probable value of Γ is much smaller than the mean value Γ_0 [S0031-9007(97)04001-5]

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The modification of the rate of spontaneous emission in a cavity has been a subject of extensive research [1–8] It was shown that the cavity can both enhance and inhibit the spontaneous emission at microwave and optical frequencies The effect is due to a modification by the environment of the local density of modes at the position of the radiating atom The efforts were concentrated on the fabrication of cavities of prescribed regular shape, the atoms being kept close to nodes or antinodes of the field patterns of the cavity modes

What can be said if the shape of the cavity is not regular and the exact position of the atom is unknown? Irregular cavities have a complicated "chaotic" field pattern, and it becomes difficult to state whether the spontaneous emission rate Γ of a particular atom is increased or decreased with respect to the free-space rate $\Gamma_0 = d^2\omega_0^3/3\pi\epsilon_0\hbar c^3$ (corresponding to an electric dipole transition with moment d, frequency ω_0) Nevertheless, a precise statement can be made about the statistical distribution of Γ The distribution is universal, i e , independent of the shape or size of the cavity, provided it is chaotic

A chaotic cavity is large compared to the wavelength $\lambda_0 = 2\pi c/\omega_0$, and has a shape such that the light is scattered uniformly in phase space (In a circular or cubic cavity, chaotic behavior may still occur because of diffuse boundary scattering or due to randomly placed scattering centers) The only parameter which enters the distribution of Γ/Γ_0 is the strength of the coupling of the cavity modes to the outside world We assume that the coupling is via a hole that is small compared to the size of the cavity and transmits a total of N wave channels (For a hole of area A, $N \approx 2\pi A/\lambda_0^2$) Our result for the distribution of Γ takes the universal form

$$P(\Gamma) \propto \frac{\Gamma^{N/2-1}}{(\Gamma + \Gamma_0)^{N+1}}, \tag{1}$$

shown in Fig 1 for several values of N The distribution eventually becomes narrow and Gaussian for $N \gg 1$, while it is still broad and strongly non-Gaussian for N as large as 10 The mean value of Γ equals Γ_0, but the most probable value is smaller than Γ_0

As a possible experimental setup, one can imagine an array of cavities, each containing a few excited atoms, or a single cavity containing many excited atoms (widely separated so that they decay independently) The array of cavities might occur naturally in a porous material Let $n(t)$ be the number of atoms that has not decayed by the time t The fraction $n(t)/n(0)$ is the Laplace transform $\int_0^\infty d\Gamma\, P(\Gamma)\exp(-\Gamma t)$ of the distribution (1), which is a confluent hypergeometric function A time-resolved measurement of the emitted intensity yields $n(t)$ and thereby the probability distribution $P(\Gamma)$ Fluctuations of the spontaneous emission rate give rise to an algebraic decay $n(t) \propto t^{-N/2}$ for large t, instead of the usual exponential decay $\propto \exp(-\Gamma_0 t)$

We proceed with the derivation of Eq (1) We assume that the system is in the perturbative regime [9], so that the rate of spontaneous emission is given by the Fermi

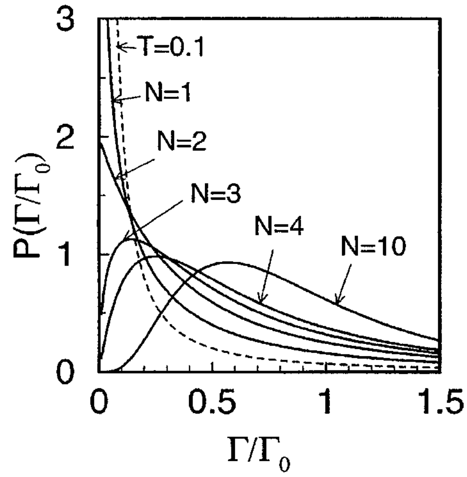

FIG 1 Probability distribution of the spontaneous emission rate Γ (normalized by the free-space rate Γ_0), as given by Eq (1) for several values of the number N of wave channels transmitted by the hole in the cavity For $N \geq 2$ the distribution reaches its maximum at a rate $\Gamma = \Gamma_0(N-2)/(N+4)$ that is smaller than the mean value Γ_0 The variance of Γ diverges for $N \leq 2$ and equals $4\Gamma_0^2/(N-2)$ for larger N The dashed curve is the result (15) for a hole much smaller than a wavelength (transmittance $T = 0$ 1)

We first compute the distribution of the vector $\vec{v} = B^{-1}\vec{u}$, which is given by Eq. (13) with the delta function replaced by $\delta(\vec{v} - B^{-1}\vec{u})$. The result is $P(\vec{v}) \propto (1 + |\vec{v}|^2)^{-N-1}$. Because of rotational invariance of the Gaussian distribution for $\vec{u}$, the distributions of x and $|\vec{v}|^2$ are the same. Hence $P(x) = \int d\vec{v}\, P(\vec{v})\delta(x - |\vec{v}|^2) \propto x^{N/2-1}(1 + x)^{-N-1}$. This is the result (1) announced in the introduction and plotted in Fig. 1. It decreases monotonically for $N \leq 2$, and has a maximum at nonzero Γ for larger N.

This calculation holds for the so-called orthogonal symmetry class (symmetry index $\beta = 1$), relevant for optical systems with time-reversal symmetry. The local density of states for systems with broken time-reversal symmetry (unitary class, $\beta = 2$) or with broken spin-rotational symmetry (symplectic class, $\beta = 4$) is relevant in condensed matter physics. We have repeated our calculations for $\beta = 2, 4$ and found $P_N^{(\beta)} = P_{\beta N}^{(1)}(x)$, with $P^{(1)}(x)$ given by Eq. (1).

So far we have assumed that the hole in the cavity fully transmits at least one wave channel, so that the transmittance T of the hole (the ratio of the transmitted and incident power) is ≥ 1. If the hole is smaller than a wavelength, then T becomes <1. The scattering matrix $S(T)$ of the cavity coupled by a hole with transmittance $T < 1$ can be expressed in terms of the scattering matrix $S|_{T=1}$:

$$S(T) = \frac{S|_{T=1} + \sqrt{1 - T}}{1 + S|_{T=1}\sqrt{1 - T}}. \tag{14}$$

To find the distribution of the local density of modes, we start from Eq. (8) with S replaced by $S(T)$, repeat similar steps, and average over $S|_{T=1} = e^{i\phi}$ at the end. The result is

$$P(x) = \frac{2}{\pi^2\sqrt{xT}} \int_0^{\pi} d\phi \times \frac{\sqrt{2 - T + 2\sqrt{1 - T}\cos\phi}}{[1 + x(2 - T + 2\sqrt{1 - T}\cos\phi)/T]^2}, \tag{15}$$

plotted also in Fig. 1 (dashed line, for $T = 0.1$). It decreases monotonically for any $T < 1$.

The variance $\langle(\Gamma - \Gamma_0)^2\rangle$ diverges if $N \leq 2$ but the divergency is removed when we take into account the condition of applicability of the Fermi golden rule (2). The perturbative treatment is valid as long as the decay rate Γ of the excited atom remains smaller than the width γ_μ of the cavity modes contributing to the decay. Estimating the width of the main contributing mode as $1/\rho\mathcal{V} = \Gamma_0/\Gamma\rho_0\mathcal{V}$, we get a condition $\Gamma \ll (\Gamma_0/\rho_0\mathcal{V})^{1/2}$. Therefore, any divergent contribution of the large Γ tail should be cut off at $\Gamma \simeq (\Gamma_0/\rho_0\mathcal{V})^{1/2}$. The weight of the tail is negligibly small provided $(\Gamma_0/\rho_0\mathcal{V})^{1/2} \gg \Gamma_0$, hence if $\Gamma_0\rho_0\mathcal{V} = d^2\omega_0^5\mathcal{V}/9\pi^3\epsilon_0\hbar c^6 \ll 1$. To estimate this parameter, we write $d = zea_B$ (a_B is the Bohr radius), $\omega_0 = 2\pi c/\lambda_0$, $\mathcal{V} = L^3$. Then $\Gamma_0\rho_0\mathcal{V} \approx 3.21 z^2 a_B^2 L^3/\lambda_0^5$ is close to 1 for $z = 0.17$, $L = 0.53$ mm, $\lambda_0 = 530$ nm. We can get large room for applicability of Eqs. (1) and (15) by going to weaker (possibly magnetic) dipoles, smaller cavities, or larger (possibly microwave) wavelengths.

We conclude with a comparison with previous work on the local density of states in chaotic cavities [11–13]. That work was motivated by different physical applications (Knight shift in NMR or optical absorption). Our application is in a sense dual to that of Ref. [13], where complicated electronic states interact with simple radiation states. Instead, we have the simplest possible electronic system—a two level atom—and a complicated structure of radiation modes. In Refs. [11–13] it was assumed that the cavity was coupled to the outside via a tunnel barrier of large area. In this case statistical fluctuations in the broadening of the levels γ_μ (from level to level and from cavity to cavity) can be ignored. In the case of a relatively small opening, considered here, fluctuations of the γ_μ's are essential. The resulting distribution (1) of the local density of modes turns out to be very simple, compared with the result of Ref. [13] (involving a fivefold integral in the case of unbroken time-reversal symmetry). We obtained our result within the framework of random-matrix theory. It would be interesting to see if it can be reproduced using the supersymmetry technique of Refs. [11,13].

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