Charge- Versus Spin-Driven Stripe Order: Role of Transversal Spin Fluctuations

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(Received 18 July 1997)

The separation of the charge- and spin-ordering temperatures of the stripe phase in cuprate superconductors has been used to argue that the striped phase is charge driven. Scaling analysis of a nonlinear sigma model shows that the effect of spatial anisotropy on the transversal spin fluctuations is much more drastic at finite temperatures than at zero temperature. These results suggest that the spin fluctuations prohibit the spin system to condense at the charge-ordering temperature, despite a possible dominance of charge-spin coupling in the longitudinal channel. [S0031-9007(97)05247-2]

PACS numbers: 74.20.Mn, 71.27.+a, 74.72.–h

The observation of a novel type of electronic order in cuprate superconductors and other doped antiferromagnets has attracted considerable attention recently. In this stripe phase, the carriers are confined to lines which are at the same time Ising domain walls in the Néel background [1]. Substantial evidence exists that dynamical stripe correlations persist in the normal- and superconducting states of the cuprates [2].

A further characterization of the fluctuation modes of the stripe phase is needed. In this regard, the finite temperature evolution of the static stripe phase might offer a clue. Both in cuprates [1] and in nickelates [3], the charge orders at a higher temperature than the spin, and both transitions appear to be of second order. Zachar, Emery, and Kivelson [4] argue on the basis of a Landau free energy that the stripe instability is *charge driven:* if the coupling between the charge and longitudinal spin mode would dominate, charge and spin would order simultaneously in a first order transition. This is a mean-field analysis, and fluctuations can change the picture drastically. For instance, at length scales larger than the interstripe distance the spin system remaining after the charge has ordered is just a quantum Heisenberg antiferromagnet in 2 + 1 dimensions which cannot order at finite temperatures according to the Mermin-Wagner theorem. Zachar *et al.* argue that the orientational ("transversal") fluctuations of the spin system can be neglected at the temperatures of interest, because it appears that the spin system left behind after the charge has ordered is not radically different from the antiferromagnet in the half-filled cuprates, exhibiting a Néel temperature of order 300 K, an order of magnitude larger than that in the stripe phase.

An important constraint is that the $T = 0$ staggered magnetization in the stripe phase appears to be comparable to that at half filling [2]. If the transversal fluctuations are responsible for the charge and spin transitions, it has to be demonstrated that the additional thermal fluctuations due to the presence of stripes have a much greater effect on the Néel state than the $T = 0$ quantum fluctuations. To investigate this, we consider the simplest possible source of stripe induced spin disorder. Following Castro Neto and Hone (CH) [5] we assume that the exchange coupling between spins separated by a charge stripe is weaker than the interdomain exchange, so that the collective spin fluctuations are described by a spatially anisotropic $O(3)$ quantum nonlinear sigma (AQNLS) model. From our scaling analysis we find that a moderate anisotropy (a factor of ~4 difference in spin wave velocities) can explain a reduction of the Néel temperature by an order of magnitude, while the $T = 0$ staggered magnetization is reduced only by a factor of 2 from its isotropic value. The reason can be inferred from the crossover diagram (Fig. 1). As a function of increasing anisotropy, the $T = 0$ transition between the renormalized classical (RC) and quantum disordered (QD) states scales to smaller coupling constant, but the dimensionless temperature associated with the crossover renormalized

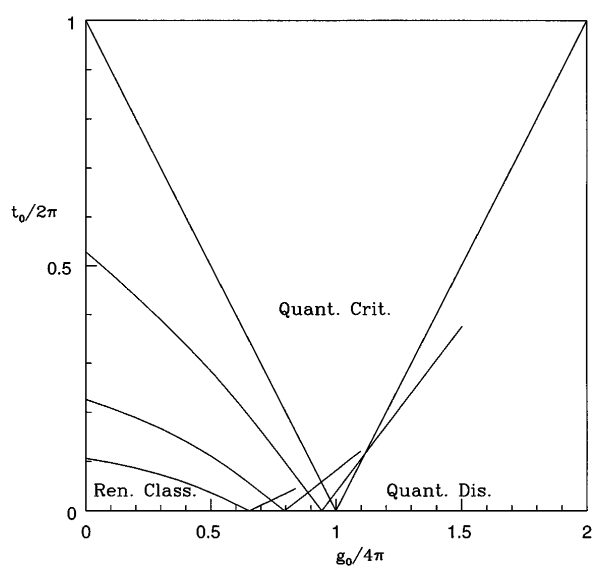

FIG. 1. Crossover diagram for the anisotropic QNLS. The lines are for $\alpha = 1$, 0.4, 0.1, and 0.025 from top to bottom. The end points of the quantum-critical to quantum-disordered lines map onto $(g_1, t_1) = (8\pi, 2\pi)$. Notice that when t_0 becomes larger than the crossover temperature from renormalized classical to quantum critical at $g_0 = 0$ one dimensional fluctuations are dominating for all values of g_0.

classical to quantum critical scales down much faster. Alternatively, we find that the behavior found by Chakravarty, Nelson, and Halperin (CNH) [6] for the correlation length in the RC regime of the isotropic model can be directly generalized to the anisotropic case: the expression for the classical *anisotropic* model remains valid when the bare stiffness is replaced by the renormalized stiffness. It is suspected that this holds more generally. If so, the strong disordering influence of temperature as compared to the quantum fluctuation might be generic: whatever the disordering influence of the stripes is, it exerts it in an effectively three dimensional classical system at zero temperature and in a two dimensional system in the finite temperature renormalized classical regime.

It is assumed that the Néel order parameter fluctuations in the charge ordered stripe phase are governed by an AQNLS model [5,7],

$$S_{\mathrm{AQNLS}} = \frac{1}{2g_0}\int_0^u d\tau \int d^2x \times \left(\alpha(\partial_x \hat{n})^2 + (\partial_y \hat{n})^2 + \frac{2}{1+\alpha}(\partial_\tau \hat{n})^2\right). \tag{1}$$

where the bare coupling constant g_0 and the spin-wave velocity c are those of the isotropic system, while α parametrizes the anisotropy. In the classical limit, this describes spin waves with velocity $c_y(\alpha) = c\sqrt{(1+\alpha)/2}$ and $c_x(\alpha) = \sqrt{\alpha}\, c_y(\alpha)$ in the y and x directions, respectively. The slab thickness in the imaginary time direction u is given by $\beta\hbar c\Lambda$, where Λ is the cutoff of our spherical Brillouin zone. This model is derived by taking the naive continuum limit of a Heisenberg model with exchange couplings J and αJ in the y and x directions, respectively.

The renormalization of this model has received some attention recently [5,8]. We adopt here a variation on the procedure as proposed by Affleck [8]. The central observation is that this model contains two ultraviolet cutoffs. As a ramification of the anisotropy, the highest momentum states in the x direction will have an energy $E_x^{\max}$ which is a factor of $\sqrt{\alpha}$ smaller than that of the highest momentum states in the y direction. Therefore, the initial renormalization flow from $E_y^{\max}$ down to $E_x^{\max}$ is governed by one dimensional fluctuations. At $E_x^{\max}$ the resulting model can be rescaled to become isotropic, albeit with "bare" parameters which are dressed up by the one dimensional high energy fluctuations.

Keeping the full model Eq. (1), the one dimensional fluctuations are integrated out (using momentum-shell renormalization [6]) by neglecting the dispersions in the x direction entirely. This causes the anisotropy parameter α to become a running variable as well, which is always relevant. When the renormalized $\alpha = 1$, the model has become isotropic, albeit with renormalized bare coupling constants.

Writing $\hat{n} = (\vec{\pi}, \sigma)$, where σ is the component of $\hat{n}$ in the direction of ordering, we expand to one-loop order in $\vec{\pi}$. Subsequently, we Fourier transform the $\vec{\pi}$ fields according to

$$\vec{\pi}(\vec{x},\tau) = \sum_{n=-\infty}^{\infty}\int \frac{d^2k}{(2\pi)^2}\,\vec{\pi}(\vec{k},n)e^{\imath\vec{k}\cdot\vec{x}-\imath\omega_n\tau}, \tag{2}$$

where $\omega_n = 2\pi n/u$ are the Matsubara frequencies. The momenta k are rescaled with Λ to become dimensionless. Separating the fields according to

$$\vec{\pi}(\vec{k},n) = \begin{cases} \vec{\pi}_>(\vec{k},n); & e^{-l} < |k_y| < 1, \\ \vec{\pi}_<(\vec{k},n); & 0 < |k_y| < e^{-l}, \end{cases} \tag{3}$$

where l is small, we integrate out the fields $\pi_>$, using a square Brillouin zone for convenience. Rescaling k_y, $\pi_<$, u, g, and α, we find that the model scales to larger α (smaller anisotropy). We obtain the following flow equations:

$$\alpha = \alpha_0 e^{2l}, \tag{4}$$

$$\frac{\partial g}{\partial l} = -\frac{\alpha}{1+\alpha}g + g^2 I, \tag{5}$$

$$\frac{\partial t}{\partial l} = t + tgI, \tag{6}$$

where

$$I = \frac{\sqrt{1+\alpha}}{4\sqrt{2}\,\pi^2}\int_{-1}^{1} dk_x \frac{\coth(\frac{u}{2}\sqrt{\frac{1+\alpha}{2}}\sqrt{\alpha k_x^2+1})}{\sqrt{\alpha k_x^2+1}}, \tag{7}$$

and where t is the dimensionless temperature, $t_0 = k_B T/\rho_s^0$. From Eqs. (5) and (6), we find for the slab thickness $u = g/t$

$$u = u_0\sqrt{\frac{1+\alpha_0}{1+\alpha}}\,e^{-l}. \tag{8}$$

From Eq. (4) it follows that $\alpha = 1$ corresponds with $l = l_1 = -\ln\sqrt{\alpha_0}$. The bare coupling constant (at $T = 0$) and bare slab thickness of the effective isotropic model follow by integrating Eqs. (5) and (6) down to l_1 [$g_1 = g(l_1), t_1 = t(l_1)$],

$$g_1 = g_0 \Bigg/ \left[\sqrt{\frac{2}{1+\alpha_0}} - \frac{g_0}{2\pi^2}\{\mathrm{arsinh}(\sqrt{\alpha_0})/\sqrt{\alpha_0} + \ln(1+\sqrt{1+\alpha_0}) - \ln[\sqrt{\alpha_0}(1+\sqrt{2})^2]\}\right]. \tag{9}$$

$$g_1/t_1 = (g_0/t_0)\sqrt{\alpha_0(1+\alpha_0)/2}. \tag{10}$$

Except for these altered bare quantities, the isotropic model is analyzed in the standard way [6].

Putting $g_1 = g_c = 4\pi$ and solving g_0, we find the critical bare coupling for the anisotropic model

$$g_c(\alpha_0) = 4\pi\sqrt{\frac{2}{1 + \alpha_0}} \Big/ \left[1 + \frac{2}{\pi}\{\text{arsinh}(\sqrt{\alpha_0})/\sqrt{\alpha_0} + \ln(1 + \sqrt{1 + \alpha_0}) - \ln[\sqrt{\alpha_0}(1 + \sqrt{2})^2]\}\right]. \quad (11)$$

We find this result to be the same within a couple of percents as the outcome of large-N mean-field theory [5], while the difference originates in an inaccuracy in our calculation related to the switch from the square (at $E > E^x_{\max}$) to the spherical Brillouin zone of the effectively isotropic model.

For $\alpha_0 = 1$, the one-loop crossover lines between the quantum-critical (QC) and the RC/QD regime are given by $t = \pm 2\pi(1 - g/4\pi)$. Taking (g_1, t_1) to lie on these lines and iterating the flow equations backwards, we obtain the crossover diagram for the anisotropic model, shown in Fig. 1. Note that the anisotropy has a stronger effect on the t dependence of the RC to QC line than on its g dependence. This already indicates that the $T = 0$ properties will be less affected by the anisotropy than those at finite temperatures.

The one-loop mapping to an isotropic QNLS provides a simple way of calculating the correlation length in the anisotropic model. Noting that the correlation length in the y direction scales as $\xi = \xi_0 e^{-l}$ under Eq. (3), it immediately follows that $\xi(g_0, t_0) = e^{l_1}\xi_{\text{isotr}}(g_1, t_1)$. Inserting the one-loop expression for ξ_{isotr} in the RC regime [6] and using Eqs. (9) and (10) [the use of the $T = 0$ expression for g_1 Eq. (9) is a good approximation if $g_1/t_1 \gg 1$],

$$\xi(g_0, t_0) = \frac{0.9}{\sqrt{\alpha_0}}\frac{g_1}{2t_1}\exp\left[\left(1 - \frac{g_1}{4\pi}\right) \Big/ t_1\right]$$
$$\simeq 0.9\frac{g_0}{2t_0}\sqrt{\frac{1 + \alpha_0}{2}}\exp[\sqrt{\alpha_0}\,\rho_s(0)/k_B T], \quad (12)$$

where the renormalized $T = 0$ stiffness is given by

$$\rho_s(0) = \rho_s^0\left(1 - \frac{g_0}{g_c(\alpha_0)}\right). \quad (13)$$

Equations (12) and (13) are our central result. It shows that the correlation length in the renormalized classical regime has a twofold *exponential* dependence on the anisotropy, both originating in the high frequency one dimensional fluctuations. As already pointed out by CH [5], the anisotropy causes g_c to decrease (e.g., Fig. 1), leading to a reduction of ξ at a given temperature. However, we find an additional $\sqrt{\alpha}$ in the exponent which has been overlooked by CH, although it is included in the paper of Wang [9]. This is the specific way in which the greater effect of the thermal fluctuations, which we noted earlier, shows up in the renormalized classical regime. In fact, it shows that the basic invention of CNH [6] is straightforwardly extended to the anisotropic case. The correlation length is given by the expression for the classical system, and quantum mechanics enters only in the form of a redefinition of the stiffness. However, for the classical correlation length expression one should use the one for the *anisotropic classical model*. Using the same procedure as for the quantum model, it is easy to demonstrate that the correlation length of the anisotropic classical $O(3)$ model in 2D behaves as $\xi \sim \exp(\sqrt{\alpha_0}\,\rho_s^0/k_B T)$, and this explains the occurrence of the additional $\sqrt{\alpha_0}$ factor [10].

The finiteness of the Néel temperature is caused by small intraplanar spin anisotropies and interplanar couplings. Keimer *et al.* [11] have shown that in La_2CuO_4 the former dominate, and these can be lumped together in a single term α_{eff} which plays the role of an effective staggered field. The Néel temperature can be estimated by comparing the thermal energy $k_B T_N$ to the energy cost of flipping all spins in a region the size of the correlation length in the presence of the effective staggered field.

$$k_B T_N(\alpha) \simeq J\alpha_{\text{eff}}\left(\frac{\xi(T_N, \alpha)}{a}\frac{M_s}{M_0}\right)^2. \quad (14)$$

Because it is not expected that stripes will influence the spin anisotropies strongly, we can use the estimate for α_{eff} as determined for the half-filled system: $\alpha_{\text{eff}} = 6.5 \times 10^{-4}$ [11]. For our estimate of T_N, we will use spin-wave results for the renormalized stiffness, susceptibility, and spin-wave velocity [12]. For $S = 1/2$, they are $\hbar c = 0.5897\sqrt{8}Ja$, $\chi_\perp(0) = 0.514\hbar^2/8Ja^2$, and $\rho_s = c^2\chi_\perp(0)$. The bare coupling constant is obtained from $g_0/4\pi = 1/(1 + 4\pi\chi_\perp c/\hbar\Lambda)$ [6], which yields $g_0 = 9.107$ for $\Lambda a = 2\sqrt{\pi}$. We notice that the one-loop result for the prefactor is not correct, but this factor is not very important as far as the reduction of the Néel temperature is concerned.

Since our $T = 0$ results coincide with those obtained by CH [5], we use their expression for the zero temperature staggered magnetization [13],

$$\frac{M_s(\alpha)}{M_s(1)} = \sqrt{\frac{1 - g_0/g_c(\alpha)}{1 - g_0/4\pi}}, \quad (15)$$

and its anisotropy dependence is shown together with the results for the Néel temperature in the inset in Fig. 2. To illustrate the effects of a different α_{eff} in the stripe phase (e.g., $J_\perp$ may be much reduced due to frustration) we have also plotted the results for $\alpha_{\text{eff}}(\alpha < 1) = 10\alpha_{\text{eff}}$ $(\alpha = 1)$ (upper dashed line) and for $\alpha_{\text{eff}}(\alpha < 1) = 0.1\alpha_{\text{eff}}$ $(\alpha = 1)$ (lower dashed line). In Fig. 2 T_N is plotted versus M_s. As expected, the dependence of T_N on anisotropy is considerably stronger than that of M_s. A reduction of M_s by a factor of 2 due to a spin-wave anisotropy of $\sim\sqrt{\alpha} \sim 1/4$ order is accompanied by a suppression of T_N by roughly an order of magnitude.

In the above we relate different experimentally accessible quantities (spatial and spin anisotropies, Néel temperature, $T = 0$ staggered order, correlation length) and

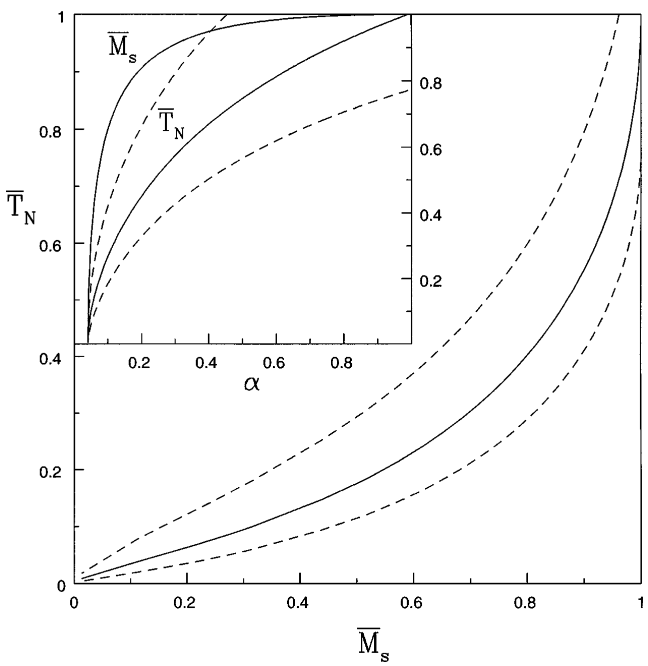

FIG. 2. The Néel temperature versus the zero temperature staggered magnetization, with the anisotropy as implicit parameter. Both quantities are normalized with respect to their value in the isotropic system. The upper/lower dashed line gives $\bar{T}_N$ for $\alpha_{\rm eff}$ (Néel stabilizing field) a factor of 10 larger/smaller than in the isotropic system. Inset: $\bar{T}_N$ and $\bar{M}_s$ as a function of the anisotropy parameter α.

further [14,15] experimentation is needed to unambiguously demonstrate that spatial anisotropy is the cause of the low spin-ordering temperature. If the fluctuation behavior in the RC regime is indeed as general as suggested by the present analysis, other sources of stripe induced spin order could have similar consequences. For instance, local charge deficiencies in the stripes caused by quenched disorder would give rise to unscreened (by charge) pieces of domain walls. Such stripe defects are like the dipolar defects discussed by Aharony *et al.* [16], and their frustrating effect is expected to be disproportionally stronger at finite temperature than at zero temperature.

Above all, the present analysis shows that a Landau mean-field analysis falls short as a description for the thermodynamic behavior of the stripe phase because of the importance of fluctuations. Furthermore, thermodynamics does not offer an unambiguous guidance regarding the microscopy (frustrated phase separation [17] versus "holon" type mechanisms [18]). Here we have focused on the transversal spin fluctuations, and given that there is ample evidence for a pronounced slowing down of the spin dynamics at the charge-ordering temperature, these undoubtedly play an important role. It is noted that recent results point at a similarly important role of fluctuations in the charge sector [19].

We thank V. J. Emery, B. I. Halperin, S. A. Kivelson, and W. van Saarloos for stimulating discussions. Financial support was provided by the Foundation of Fundamental Research on Matter (FOM), which is sponsored by The Netherlands Organization of Pure Research (NWO). J. Z. acknowledges support by the Dutch Academy of Sciences (KNAW).

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$$\frac{1}{t_0} = \frac{1}{k_B T}\left[\rho_s(0) + \frac{k_B T}{2\pi\sqrt{\alpha_0}}\ln\!\left(\frac{\hbar c\Lambda}{k_B T}\sqrt{\frac{1+\alpha_0}{2}}\right) + \mathcal{O}(T^2)\right],$$

where $g_c(\alpha_0)$ is now precisely the critical coupling obtained by CH. Momentum-shell renormalization of this 2D model again yields Eq. (12).
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