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When, and why, did Frege read Bolzano?*

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1. Lack of Evidence?

Michael Dummett wrote:

The only nineteenth-century philosopher of whom it would be reasonable to guess, just from the content of his writings and those of Frege, that he had influenced Frege, is Bernhard Bolzano, who died in the year Frege was born; but there is no evidence whatever that Frege ever read Bolzano.¹

Subsequently he was taken to task by Wolfgang Kühne for having made the 'grave mistake' of misspelling 'Bernard', the first name of Bolzano.² However, in my opinion, this is not the only mistake in the quote from Dummett. In the present note I wish to dispute that 'there is no evidence whatever that Frege ever read Bolzano'. On the contrary, by combining two well-known sets of facts, I shall argue, one obtains strong evidence that Frege *did* read Bolzano late 1905 or early 1906.³

2. The Missing Link: Alwin Korselt

It has been noted in the literature that Frege's partners (victims?) in scholarly discussion drew his attention to the works of Bolzano at least three times.⁴ First,

* My lecture at LOGICA '99 dealt with the dating of Frege's distinction between *Sinn* and *Be-deutung*, and will appear in the *History and Philosophy of Logic*. However, given the Bolzano connection, what follows might not be out of place in a LOGICA Yearbook published in Prague. The material was presented in 1998 at workshops in Leyden and Helsinki. I am indebted to participants for helpful discussion. Kai Wehmeier and Helge Rückert, presently both at Leyden University, offered detailed comments on the penultimate draft.

¹ [1991, p. vii]. Dummett is not alone in his view. See, for instance, Mancosu [1996, p. 117, fn. 69]: 'As is well known there is no evidence that Frege ever read Bolzano, since he never quotes him', and Kühne [1997, p. 203]: 'Husserl, Kerry and Korselt were critical of Frege, and Frege in turn was very critical of them. Perhaps that's why he never bothered to read [Bolzano] an author they praised, - who knows ...'.

² [1997, p. 203].

³ William Boos in his pioneering [1985, pp. 156-7] suggests *en passant* that Frege's work on independence was not independent from Bolzano, but refrains from working out his suggestion further.

⁴ For instance, in Picardi [1994] and Kühne [1997a].

Benno Kerry made ample use of the *Wissenschaftslehre* in several installments of his lengthy series of articles.⁵ However, Frege does not appear to have taken the hint, since careful search has yielded no traces of direct influence from Bolzano in his writings from the *Hochleistungsperiode* 1890-95. Thus, as far as mediation through Benno Kerry is concerned, I am prone to agree with Dummett: there is no supporting evidence, in the form of even remotely Bolzanian passages, in Frege's writings prior to, say, *Grundgesetze*, Vol. II from 1903, or at least, none has been found.⁶

The relevance of Bolzano's *Wissenschaftslehre* for his logical concerns was pointed out to Frege yet again – and this time quite forcefully so – in 1903 and 1905/6. First Alwin Korselt firmly pointed Frege in the direction of the *Wissenschaftslehre*: Frege inaugurated his acrimonious debate with David Hilbert on the foundations of geometry in private letters, but when Hilbert did not agree to publication of their correspondence, Frege, true to his polemical habits, brought the matter into public view through a two-part article entitled *Über die Grundlagen der Geometrie*. In [1903] Korselt, who had corresponded with Frege concerning Russell's paradox, intervened in the debate with Hilbert and attempted to take an intermediate stand between Frege and Hilbert.⁷ He also published (an essay on the foundations of mathematics that amounts to) a critical notice [1905] of Frege's recent Gg II. In virtually all his writings on the foundations of mathematics Korselt refers to Bolzano's *Wissenschaftslehre* in the most enthusiastic terms. In particular, in the two early pieces aimed directly at Frege, Korselt informs him, with singular lack of tact, that he would have avoided many mistakes by taking the trouble to study Bolzano:

Die modernen Mathematiker wären nicht in Widersprüche oder Verworfenheiten ... gefallen, wenn sie Bolzanos "Wissenschaftslehre" ... studiert hätten. Bolzano, der große Gegner Kants, ist seit Leibniz der erste philosophische Mathematiker und mathematische Philosoph.⁸

⁵ See Peckhaus [1994] and Picardi [1994] for bibliographical references and further information concerning Kerry.

⁶ This passage might need revision. When the present paper was essentially complete Professor Ettore Casari (Scuola Normale Superiore, Pisa) presented me with copies of a number of his writings on Bolzano. In particular, if I have deciphered the Italian correctly, Casari [1989] strongly suggests that Bolzano's theory of meaning is a "forgotten" source for Frege's theory of *Sinn* and *Bedeutung*. An evaluation of this intriguing possibility will have to wait for another occasion. Künne [1997] spells out similarities between the theories of Frege and Bolzano in considerable detail.

⁷ For the letters, see WB, pp. 140-4. Relevant articles by Korselt are listed in the references. I am indebted to Dr. Volker Peckhaus (Erlangen) for giving me access to his unpublished 'Alwin Rheinhold Korselt' that, apart from biographical material, contains convenient bibliographical summaries.

⁸ Korselt [1903, p. 495]. The WB letters show that Korselt knew of Russell's paradox by June 1903.

Frege's scholarly temper was notoriously short. Being publicly reminded, at this particular juncture (1903!), of the fact that "modern mathematicians" have fallen into contradiction, and that they could have avoided these pitfalls by studying Bolzano, is not something that would greatly endear the author of that remark to Frege. Also, he would not have liked to see Bolzano being put so firmly in the place of the first (or *foremost*) mathematical philosopher and philosophical mathematician after Leibniz; that place, I suspect, Frege reserved for himself. Korselt is equally tactless in his "critical notice" of Gg II:

Und doch erstrebte schon Leibniz, ein Begründer der modernen Mathematik, die Erforschung ihrer Grundlagen. Bolzano, sein geistiger Nachfolger, hat zwar einigen Einfluß gewonnen, aber seine ganze mathematisch-logisch-erkenntnistheoretische Bedeutung, die sich in seiner "Wissenschaftslehre" offenbart, ist noch lange nicht ausgenutzt. Kein wunder, da nicht einmal die der Gegenwart näher liegenden Schriften von Frege ... Beachtung gefunden haben. ...

Der Verfasser ... möchte nun diejenigen Bemerkungen Freges widerlegen, die ihm unrichtig oder übertrieben erscheinen.⁹

Frege was a master polemicist, and he, of all philosophers, was most certainly not prepared to be hectored by his inferiors. Thus, when Frege joins battle again to fire the three shots of his second salvo, also entitled *Über die Grundlagen der Geometrie*, his tone has been harshened considerably and may with some justice be called unpleasant.¹⁰

The third time that Frege's attention was drawn to Bolzano can be found in the later part of the correspondence with Husserl. In 1906, Husserl's Vth and last survey of the German publications on logic during the period 1895 to 1899 dealt with the final two articles of Anton Marty's series on subjectless sentences. The article in question formed the occasion for the resumption of the correspondence between Frege and Husserl. In Husserl's article Bolzano figures more or less prominently, and the same is true also for his letter (presumably no longer extant, but see Wehmeier – Schmidt am Busch [2000]) to Frege of 10.11.1906, as we know from Scholz's gloss on the content.¹¹

⁹ Korselt [1905, p. 364].

¹⁰ [1906, I-III]. In these articles, Hilbert having been dismissed already in the first series, Korselt serves as Frege's main target. Examples of Frege's sharp use of invective are collected by William Boos [1985].

¹¹ Husserl [1904, p. 103, fn.* and p. 112]. The letter in question from Husserl to Frege carries the code XIX/4 in WB, p. 104.

3. Frege on the (In)Dependence of Geometrical Axioms

If Kerry's mention of Bolzano did provoke Frege into reading the *Wissenschaftslehre*, the study left no readily visible impact on his *oeuvre*. Matters are different with respect to this second (Korselt/Husserl) round of Bolzano-pointers. In the very works that Frege published in *direct response* to Korselt's prompting we find passages unmistakably reminiscent of Bolzano. To be specific, in ÜGG2:III we read:

(I)

Es sei nun Ω eine Gruppe von wahren Gedanken. Aus einem oder einigen Gedanken dieser Gruppe möge durch einen logischen Schluß ein Gedanke G folgen, so daß dabei außer logischen Gesetzen kein nicht zur Gruppe Ω gehörender Satz gebraucht wird. Wir bilden nun eine neue Gruppe von Gedanken, indem wir der Gruppe Ω den Gedanken G hinzufügen. Was wir so getan haben, mag ein logischer Schritt heißen. Wenn wir nun durch eine Folge von solchen Schritten, bei der jeder Schritt das Ergebnis des vorangehenden zum Ausgang nimmt, eine Gruppe von Gedanken erreichen können, die den Gedanken A enthält, so nennen wir A abhängig von der Gruppe Ω . Wenn dies nicht möglich ist, so nennen wir A unabhängig von Ω . Dies wird immer stattfinden wenn A falsch ist.¹²

Frege's notion of dependence holds among true propositions only: when a Thought (proposition) A is dependent on a group Ω of Thoughts the latter all have to be true. For Frege this must be so, since when A is dependent on Ω there is a chain of logical inferences from Ω to A , and, according to Frege, one can only infer from truths. Considering the one-premiss case only:

(*) The true Thought A is dependent on the true Thought B when there is a chain of valid inferences from B to A .

(II)

Indem wir einen logischen Schritt von der Gedankengruppe Ω aus machen, wenden wir ein logisches Gesetz an. Dieses ist nicht zu den Prämissen zu rechnen, braucht also in Ω nicht vorzukommen. Es gibt also gewisse Gedanken, nämlich die logischen Gesetze, die bei der Frage nach der Abhängigkeit nicht mitzurechnen sind.¹³

¹² [1906, pp. 423-4].

¹³ [1906, P. 424].

Here Frege hit on a point that has become familiar through Lewis Carroll [1895]: on pain of an infinite regress, the rule of inference according to which a certain inference is drawn, is not to be counted among the premises for the inference in question. Frege does not spell out the regress, though.

But for the above explanation of the notion of dependence, Frege also gave another, partial, criterion for independence, which is formulated in terms of changing "vocabularies". In recent secondary literature this has been seen as an exercise anticipating contemporary ("Tarskian") logical theory, thereby proving how farsighted Frege must have been.¹⁴ The crucial text runs:

(III)

Es handele sich nun darum, ob ein Gedanke G von einer Gruppe Ω von Gedanken abhängig sei. Wir können diese Frage verneinen, wenn mittels unseres Vokabulars den Gedanken der Gruppe Ω die Gedanken einer Gruppe Ω' entsprechen die wahr sind, während dem Gedanken G ein Gedanke G' entspricht, der falsch ist; denn wenn G von Ω abhängig wäre, so müsste, da die Gedanken von Ω' wahr sind, auch G' von Ω' abhängig sein, und dann wäre G' wahr.¹⁵

According to this characterisation in terms of independence, the thought A is independent from B if there is a "vocabulary" V for the non-logical parts of A and B , such that B' , that is, the result of translating B according to the vocabulary V , is true, whereas A' , that is A under the same vocabulary V , is false. The first, direct characterisation of dependence in terms of inference is applicable to true thoughts only, whereas this second characterisation in terms of vocabularies makes sense also for arbitrary (groups of) Thoughts, irrespective of their truth.

Frege formulates his second (partial) criterion as a sufficient condition only: in the presence of a "counter-vocabulary", dependence cannot hold. If we regard this sufficient condition also as necessary, a second characterisation of dependence – in terms of vocabularies – is readily forthcoming. Thus:

(#) A is not independent from B , if A is not false under any vocabulary which makes B true.

¹⁴ See Steiner [1964-5], Kreiser [1973], Resnik [1974], Kambartel [1975], Boos [1985], Demopolous [1985], Blanchette [1996], Wehmeier [1997], Ricketts [1997], Tappenden [1997]. Resnik, Boos and Demopolous mention Bolzano. Demopolous even suggests that Frege deserves credit for anticipating Tarski's treatment of logical consequence. Such credit might be his due, if the work is independent of Bolzano. On balance, it seems more likely that Frege knew Bolzano's work when he wrote the final part of ÜGG2, than that he did not.

¹⁵ [1906, p. 428].

From the classical, non-constructive point of view that was shared by Frege and Bolzano, this condition (#) is equivalent to:

(**) *A* is dependent on *B* if *A* is true under any vocabulary which makes *B* true.

For Frege, the condition (*) clearly entails (**), since the inferences of the chain have to preserve truth from premisses to conclusions. In the absence of a completeness-theorem for vocabularies, the opposite direction is unclear. Anything which can be refuted by a counter-vocabulary is certainly independent, but does it also hold that everything which cannot be obtained by logical inference from certain premisses can also be refuted under a suitable "counter-vocabulary"?

4. Bolzano, *Ableitbarkeit* and *formale Abfolge*

A comparison of the three passages (I) – (III) with Bolzano's *Wissenschaftslehre* reveals striking similarities. The fragment (I) presents Bolzano's notion of *Abfolge* between true propositions, as is shown by inspection of WL §§ 162, 198, 199, and 220.¹⁶ In fact, the sequence of propositions $\langle A_1, \dots, A_k \rangle$, where $A_1 \in \Omega$ and $A_k = A$, is nothing but a branch in the tree which serves as Bolzano's pictorial representation of *das Geschäft des Aufsteigens von der Folge zu ihrem Grunde* – the process of ascending from consequence to ground – with respect to the *Wahrheit an sich A* (§ 220).

Frege's rider in (II), concerning the role of logical laws was not original with him, nor for that matter, was Carroll's Tortoise the first to run the regress. Bolzano had already considered the matter fully in § 199 of WL, which bears the tell-all title *Ob auch die Schlußregel mit zu den Teilgründen einer Schlußwahrheit gezählt werden könne* – Whether also the rule of inference could be counted among the grounds for a true conclusion. As we would expect, with the benefit of hindsight, Bolzano gave a negative answer, precisely because of that very *regressus ad infinitum*, that is familiar from Carroll's amusing presentation.

Bolzano, however, did not only consider the notion of an *Abfolge* among truths (Frege's *Abhängigkeit*). He also made use of the notion of an *Ableitbarkeit*, which corresponds closely enough to our modern notion of consequence among propositions, be it logical or not. *Ableitbarkeit* is a three-place

¹⁶ Korselt [1903, p. 405] explains (prior to mentioning Bolzano):

Ein Grundsatz *a* heißt abhängig von den Grundsätzen *b, c, ...* wenn *a, b, c, ...* derselben formalen Theorie angehören und *a* nur scheinbar unmittelbarer Satz ist, vielmehr selbst oder seine Verneinung zu den Sätzen *b, c, ...* im Verhältnis der *Abfolge* steht. (my emphasis)

Passage I reads as, and if I am right is, Frege's attempt to spell out this Korselt sentence.

relation between a proposition C , a sequence of propositions A_1, \dots, A_k , and a collection Γ of *Vorstellungen an sich* which indicates the places at which variation takes place.¹⁷ Frege's dual criterion for independence in fragment (III) coincides with the *Unabhängigkeit* that Bolzano formulates in § 158.1, (with variation regarding all places not occupied by logical constants). In fact, Korselt drew explicit attention to Bolzano's explanation of independence among axioms:

Nur die *Unabhängigkeit* der Grundsätze bliebe fraglich, die Theorie hätte möglicherweise noch nicht ihre einfachste Form erhalten. Dieser schon Bolzano bekannte Begriff der "Unabhängigkeit und Verträglichkeit" von Sätzen wird wohl nicht mehr in Vergessenheit geraten, nachdem Hilbert ihn so glänzend verwertet hat.¹⁸

For Frege, however, the *direct criterion is applied only among truths*. Thus his notion of *Abhängigkeit*, which is characterised directly in terms of inference, and indirectly in terms of preservation of truth under variation of vocabularies, strongly resembles (or is but a variant of) Bolzano's notion of *formale Abfolge*.

5. The Missing Link (Part ii); Confirmation

Paolo Mancosu observes that the *Wissenschaftslehre* § 530 contains a treatment, *contra* Kant, which shows how to eliminate assumptions of false propositions from indirect proofs.¹⁹ Frege, in his 1914 lectures on *Logik in der Mathematik*, offers exactly the same treatment, even down to the fine details of the identical geometrical example.²⁰ Mancosu concludes:

[Frege] did this by employing Bolzano's strategy either by hitting on it independently or by borrowing it directly from the *Wissenschaftslehre*. Of course, there is also the possibility that Frege was influenced by some other work containing Bolzano's reduction or one similar to it. But until I am shown such a text, I will opt for a direct influence of Bolzano on Frege.²¹

¹⁷ Contrary to the modern notion of consequence, *Ableitbarkeit* demands also that the antecedent propositions are compatible (*verträglich*) (WL §155).

¹⁸ [1905, p. 387]. Thus, Korselt considers exactly those two (Bolzanian) criteria, direct and indirect, that are later discussed by Frege.

¹⁹ Mancosu [1996, pp. 110-17]. I am indebted to Paolo Mancosu for drawing my attention to this passage in discussion after my Helsinki lecture.

²⁰ NS, pp. 264-6.

²¹ [1996, p.117, fn. 69]

I entirely concur.

Remarkably enough, Alwin Korselt also drew attention to this treatment in another article published in the Yearbook of the Society of German Mathematicians in 1911:

Ein indirekter Beweis ist ein Umweg, eine Unvollkommenheit, die sich aber wegschaffen läßt wie Bolzano in W{issenschaftslehre} § 530 zeigt.²²

That Frege read Korselt's articles of 1903 and 1905 we need not doubt. They are published in a Yearbook that Frege took and both dealt directly with his work. Did Frege read also this [1911] article? Without doubt he did; the opening lines of the aborted Schoenfliess-reply show that Korselt and Schoenfliess were both authors that he followed.²³ Korselt [1911], however, is a reply to an earlier paper of Schoenfliess in the Yearbook from the same year. In his reply Korselt defends Frege's views on "*Wortdefinitionen*". This paper is larded with an unusually high proportion – even for Korselt – of references to Bolzano; it ends with the peroration:

Schröder läßt manchmal die Bolzanosche Schärfe vermissen.

Die Kantischen Antinomien dürfen uns von der Philosophie nicht abschrecken, sie sind schon oft, insbesondere in W{issenschaftslehre} § 315 als Schein aufgedeckt worden.

Wenn es mir gelungen sein sollte, einige Dunkelheiten aufzuklären, verdanke ich das nur der (häufig wörtlich angeführten) Wissenschaftslehre von Bolzano. Ich bitte den Leser, sie seiner Aufmerksamkeit zu würdigen.

In my opinion Frege had already followed that piece of sound advice, and he was to heed it yet again, as shown by Mancosu.

6. *Post hoc, propter hoc*

As I already stressed, both sets of circumstances – that Korselt and Husserl prompt, or perhaps better, *provoke*, Frege with respect to Bolzano in 1905/6, and that ÜGG2:III contains passages that strongly resemble Bolzano's treatment in the *Wissenschaftslehre* – are well-known in the literature. My only claim to novelty lies in the suggestion that in this case the temporal nexus is also a causal one; *post hoc* really becomes *propter hoc* by interposing a reading on Frege's part of the *Wissenschaftslehre* in late 1905 or early 1906. This abduction provides the explanation of why Frege suddenly – otherwise,

²² Korselt [1911, p. 366].

²³ NS, pp. 191-9.

more or less out the blue – should turn to something that so very strongly resembles (model-theoretic) consequence between Thoughts (propositions, that is, judgeable contents), contrary to his lifelong insistence on the absoluteness of logical matters.²⁴ Inference, on the other hand, in terms of which Frege's treatments are invariably cast, is an act of passage from (known) judgement(s) to a novel judgement, the conclusion, which gets known in the act of inference, that is, the mediate act of judgement.²⁵

On the strength of internal evidence I have argued that Frege did read Bolzano. Was it in fact possible for him to do so? It certainly was, as Dr. Uwe Dathe, of the Philosophical Institute at Jena University, has been kind enough to check.²⁶ The University Library at Jena owns a set of Bolzano's collected works from 1882. The acquisition is not dated, but from the library stamp and binding it is clear that the set must have been obtained shortly after its appearance. Unfortunately, the library ledgers for the years 1821-1899, which have miraculously been retained, are in too bad a state to allow for any conclusion whether Frege actually borrowed the work during that period.²⁷

Finally, if, as I aver, Frege did read Bolzano, why does he not simply say so? The answer here surely lies in his character: throughout his career Frege never acknowledges, but always disagrees.²⁸ His spirit seems to have been essentially adversarial. He is the typical *Gegner* who only attacks, but who cannot be bothered to agree.

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²⁴ Fairness bids me to remark that Frege [1903, p. 272] does discuss independence of axioms prior to Korselt [1903], in terms that, with the benefit of hindsight and much good will, can be seen as anticipatory of his [1906] treatment, where the use of "vocabularies" accommodates points that were made in terms of various "geometries" – "A-geometry", "B-geometry", etc. .

²⁵ Consequence is not an epistemic notion but preserves truth from proposition(s) to proposition, whereas inference is epistemic and preserves knowability from premiss judgement(s) to conclusion judgement. My LOGICA '97 lecture, that is, Sundholm [1998], spell this out in some detail.

²⁶ Private letter, November 26, 1998.

²⁷ Of course, if I am right, a later loan, in 1905 or 1906, outside the period of the ledgers, would be more likely.

²⁸ Frege's treatment of Lotze provides a good case in point. For details and references, see Hovens [1997].

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