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Momentum noise in a quantum point contact

A Tajic, M Kindermann, and C W J Beenakker
Instituut Lorentz, Universiteit Leiden, P O Box 9506, 2300 RA Leiden, The Netherlands
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Ballistic electrons flowing through a constriction can transfer momentum to the lattice and excite a vibration of a free-standing conductor. We show (both numerically and analytically) that the electromechanical noise power P does not vanish on the plateaus of quantized conductance — in contrast to the current noise. The dependence of P on the constriction width can be oscillatory or stepwise, depending on the geometry. The stepwise increase amounts to an approximate quantization of momentum noise.

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Not long after the discovery of conductance quantization in a ballistic constriction[1] it was predicted that the quantization is noiseless.[2] The time dependent current fluctuations should vanish at low temperatures on the plateaus of quantized conductance and they should peak in the transition from one plateau to the next. The conclusive experimental verification of this prediction followed many years later,[3] delayed by the difficulty of eliminating extraneous sources of noise. The notion of noiseless quantum ballistic transport is now well established.[4]

The origin of noiseless transport lies in the fact that the eigenvalues T_n of the transmission matrix product $tt^\dagger$ take only the values 0 or 1 on a conductance plateau. The current noise power at zero temperature $P_I \propto \Sigma_n T_n(1-T_n)$ then vanishes.[5] In other words, current fluctuations require partially filled scattering channels, which are incompatible with a quantized conductance.

In this paper we point out that the notion of noiseless quantum ballistic transport does not apply if one considers momentum transfer instead of charge transfer. Momentum noise created by an electrical current (socalled electromechanical noise) has been studied in the tunneling regime[6] and in a diffusive conductor,[7] but not yet in connection with ballistic transport. Our analysis is based on a recent scattering matrix representation of the momentum noise power P, according to which P depends not only on the transmission eigenvalues but also on the eigenvectors.[8] This makes it possible for the electrons to generate noise even in the absence of partially filled scattering channels.

The geometry is shown schematically in Fig. 1. We consider a two-dimensional electron gas channel in the x-y plane. The width of the channel in the y direction is W and the length in the x direction is L. The channel contains a narrow constriction of length $\delta L \ll L$ and width $\delta W \ll W$ located at a distance L' from the left end. (We choose $x=0$ at the middle of the constriction, so that the channel extends from $-L'<x<L-L'$.) A voltage V drives a current through the constriction, exciting a vibration of the channel. We seek the low-frequency noise power

$$P=2\int_{-\infty}^{\infty} dt\ \overline{\delta F(0)\,\delta F(t)}=\lim_{t\to\infty}\frac{2}{t}\overline{\Delta\mathcal{P}(t)^2} \qquad (1)$$

of the fluctuating force $\delta F(t)\equiv F(t)-\bar{F}$ that drives the vibration. The noise power is proportional to the variance of the momentum $\Delta\mathcal{P}(t)$ transferred by the electrons to the channel in a long time t.

We assume that the electron gas is deposited on top of a doubly clamped beam extended along the x axis and free to vibrate in the y direction. The solution of the wave equation is $\mathbf{u}(\mathbf{r},t)=\hat{y}u(x)\cos\omega t$, with ω the mode frequency and $u(x)$ the mode profile. Both u and du/dx vanish at the ends of the beam and $u(x)$ is normalized such that it equals 1 at the point x_0 at which the amplitude is measured.[7] We choose $x_0=0$ so that F corresponds to a point force at the location of the constriction.

The wave functions are represented by scattering states. The incident wave has the form $\phi_n(\mathbf{r})=|\hbar k_n/m^*|^{-1/2}\exp(ik_n x)\Phi_n(y)$, where m^* is the effective mass, $n=1,2,\ldots,N$ is the mode index, Φ_n the transverse wave function, and $k_n=\pm(2m^*/\hbar^2)^{1/2}(E_F-E_n)^{1/2}$ the longitudinal wave vector (at Fermi energy E_F larger than the cutoff energy E_n). We take k_n positive to the left and negative to the right of the constriction. Incident and outgoing waves are related by the $2N\times 2N$ unitary scattering matrix

$$S=\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \qquad (2)$$

containing the $N\times N$ transmission and reflection matrices t,t',r,r'. We assume time reversal symmetry, so that Φ_n is real and S is symmetric.

As derived in Ref. 8, the noise power P and the mean force $\bar{F}$ for a localized scatterer can be expressed in terms of the matrix S and a Hermitian matrix $\mathcal{M}_{nn'}$

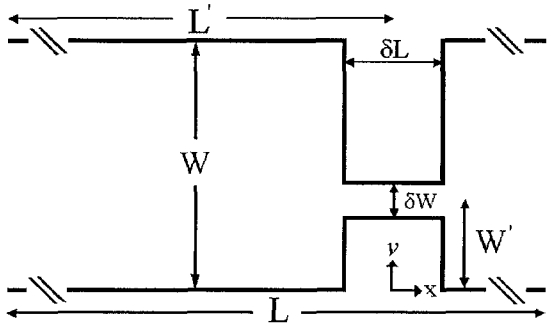

FIG. 1. Schematic diagram of a two-dimensional channel containing a constriction.

$=m^{*-1}\Sigma_{\alpha\beta}\langle n|p_\alpha u_{\alpha\beta}p_\beta|n'\rangle$ of expectation values in the basis of incident modes. The expectation value is taken of the electron momentum flux $m^{*-1}p_\alpha p_\beta$, weighted by the strain tensor $u_{\alpha\beta}=\frac{1}{2}(\partial u_\alpha/\partial x_\beta+\partial u_\beta/\partial x_\alpha)$. The matrix $\mathcal{M}$ is block diagonal,

$$\mathcal{M}=\begin{pmatrix} M_{\rm L} & 0 \\ 0 & M_{\rm R}\end{pmatrix}. \tag{3}$$

At zero temperature and to first order in the voltage one has, for a twofold spin degeneracy,

$$P=\frac{4eV}{h}\mathrm{Tr}(rr^\dagger M_{\rm L}^* t't'^\dagger M_{\rm L}^* + r'r'^\dagger M_{\rm R}^* tt^\dagger M_{\rm R}^* s - 2tr^{\rm T}M_{\rm L}^* rt^\dagger M_{\rm R}^*), \tag{4}$$

$$\bar{F}=\frac{2eV}{h}\mathrm{Tr}(M_{\rm L}+rr^\dagger M_{\rm L}^*+tt^\dagger M_{\rm R}^*). \tag{5}$$

In Eq. (5) we have not included the equilibrium contribution to the mean force (at $V=0$). Electron-electron interactions (screening) are not accounted for, since we do not expect any appreciable charge accumulation in a ballistic system.

For a transverse vibration the blocks $M_{\rm L}, M_{\rm R}$ have elements

$$(M_{\rm L,R})_{nm}=\frac{\hbar(k_n+k_m)}{2i|k_nk_m|^{1/2}}\int dy\ \Phi_n(y)\Phi_m'(y)\int_{\rm L,R}dx u'(x) \times\exp[ix(k_m-k_n)]. \tag{6}$$

The integral over x extends over the region $(-L',-\delta L/2)$ to the left of the constriction for $M_{\rm L}$ and over the region $(\delta L/2, L-L')$ to the right of the constriction for $M_{\rm R}$. We abbreviate $q_{nm}=k_m-k_n$. For $|n-m|$ of order unity one has q_{nm} of order $1/W$, so that the range of x that contributes to the integral is of order W. [Contributions from inside the constriction are smaller by a factor $\min(\delta W,\delta L)/W$.] Since W is much greater than the Fermi wave length λ_F, we are justified in using the asymptotic plane wave form of the scattering states to calculate $\mathcal{M}$.

We take hard-wall boundary conditions at $y=0,W$, hence $\Phi_n(y)=(2/W)^{1/2}\sin(n\pi y/W)$, $E_n=(\hbar^2/2m^*)(n\pi/W)^2$, $N=[2W/\lambda_F]$, and $(k_n+k_m)q_{nm}=(\pi/W)^2(n^2-m^2)$. The overlap $\int_0^W dy\ \Phi_n\Phi_m'$ is evaluated straightforwardly, but the integration over x requires more care. The derivative $u'(x)=du/dx$ of the mode profile vanishes at the two clamped ends of the beam, as well as at its center. We assume that the constriction is off-center, therefore $u'(\pm\delta L/2)\approx u'(0)\neq 0$. We write $u'(0)=u_0/L$, with u_0 a number of order unity. Upon partial integration we find, to first order in W/L,

$$\int_{\rm L,R}dx u'(x)\exp(\imath x q_{nm})=\pm\frac{u_0}{iq_{nm}L}\exp(\mp\imath q_{nm}\delta L/2)+\mathcal{O}(W/L)^2. \tag{7}$$

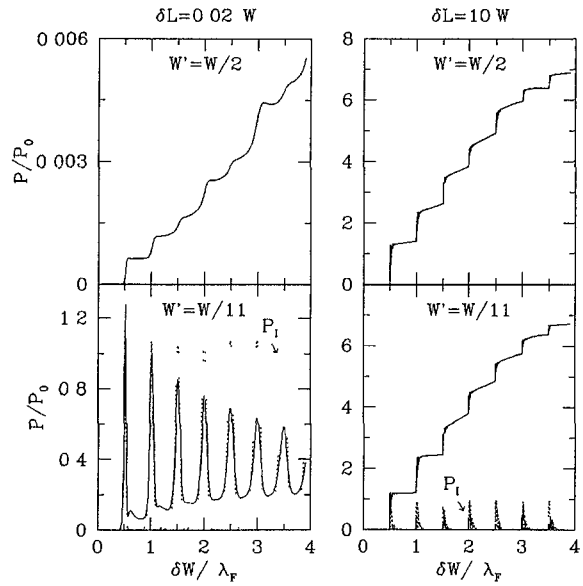

FIG. 2 Solid lines: noise power P for transverse vibration versus width of constriction δW, at fixed width $W=49.9\,\lambda_F$ of the wide channel. The left panels are for a short constriction with and without axial symmetry. The right panels are the corresponding results for a long constriction. The dotted line is the current noise P_I in units of e^3V/h (which is nearly the same with and without axial symmetry).

(The upper sign is for region L, the lower sign for region R.) We thus arrive at $M_{\rm L}=-M_{\rm R}\equiv M$, with

$$M_{nm}=\frac{2\hbar W u_0}{\pi^2 L}(\sigma_{nm}-1)\frac{nm(k_n+k_m)^2}{(n^2-m^2)^2|k_nk_m|^{1/2}} \times\exp[i\,(\,|k_n|-|k_m|\,)\,\delta L/2]. \tag{8}$$

The symbol $\sigma_{nm}=\frac{1}{2}[1+(-1)^{n+m}]$ selects indices of the same parity, so that $M_{nm}=0$ if n and m are both even or both odd.

Our constriction has left-right symmetry, so $r=r'$ and $t=t'$. We contrast the case $W'=\frac{1}{2}W$ of axial symmetry with the case $W'\ll\frac{1}{2}W$ of a constriction placed highly off-axis. We also contrast the short-constriction case $\delta L\ll W$ (point contact geometry) with the long-constriction case $\delta L\gg W$ (microbridge geometry). The reflection and transmission matrices are calculated by matching wave-function modes at $x=\pm\delta L/2$, cf. Ref. 9.

In Fig. 2 we show the dependence of the transverse noise power P [in units of $P_0=(4eV/h)(Nu_0\hbar/L)^2$] on the width δW of the constriction, at fixed width W of the wide channel. (We choose $W=49.9\,\lambda_F$, so $N=99$.) The average transverse force $\bar{F}$ is shown in Fig. 3, normalized by $F_0=(2eV/h)(Nu_0\hbar/L)$. (Note that $\bar{F}=0$ for the axially symmetric case.) The conductance $G=(2e^2/h)\mathrm{Tr}\,tt^\dagger$ and the current noise $P_I=(4e^3V/h)\mathrm{Tr}\,tt^\dagger(1-tt^\dagger)$ are included in these plots for comparison.

The plots show a remarkably varied behavior: For the short constriction without axial symmetry the noise power P of the transverse force oscillates as a function of the constric-

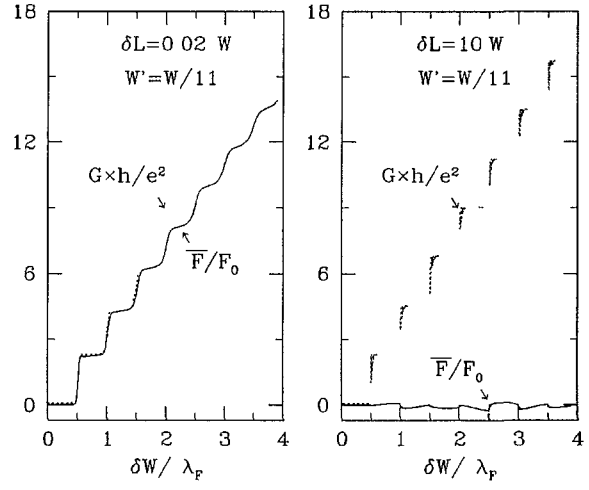

FIG 3 Solid lines average transverse force as a function of constriction width, in the absence of axial symmetry (positive values point in the positive y direction in the geometry of Fig 1, for a current flowing in the positive x direction) The left and right panels are for a short and long constriction, respectively The conductance of the constriction is shown as a dotted line. The average transverse force is identically zero for the axially symmetric geometry ($W' = W/2$)

tion width δW, in much the same way as the current noise power P_I oscillates.[2,5] However, the minima in P do not go to zero like the minima in P_I, demonstrating nonzero momentum noise on the plateaus of quantized conductance. If the short constriction is precisely at the center of the channel, P increases nearly monotonically with δW. For a long constriction P increases nearly monotonically regardless of whether there is axial symmetry or not. The increase of the noise power is stepwise, reminiscent of the conductance. (The current noise in the long constriction remains oscillatory.) The mean transverse force behaves similarly to the conductance for the short constriction, but fluctuates around zero for the long constriction.

In order to explain the approximate quantization of momentum noise in analytical terms it is convenient to decompose the (symmetric) transmission matrix as $t_{nm} = \Sigma_{n'} U_{nn'} U_{mn'} \sqrt{T_{n'}}$, where U is an $N \times N$ unitary matrix and $T_n \in [0,1]$ is the transmission eigenvalue (eigenvalue of $tt^\dagger$). Similarly, the reflection matrix is decomposed as $r_{nm} = i\Sigma_{n'} U_{nn'} U_{mn'} \sqrt{1-T_{n'}}$. In this representation Eq. (4) takes the form

$$P = \frac{8eV}{h} \sum_{n,m} |X_{nm}|^2 \{(1-T_n)T_m + [T_n(1-T_n)T_m \times (1-T_m)]^{1/2}\}, \quad X = U^\dagger M^* U. \tag{9}$$

The matrix M couples only mode indices of different parity, cf. Eq. (8). The presence or absence of axial symmetry manifests itself in the matrix U, which couples only indices of the same parity if $W' = W/2$. In this axially symmetric case $X_{nm} = 0$ if n,m have the same parity.

In a simple model[10] of a long and narrow ballistic microbridge, U is a random matrix while the transmission eigenvalues take on only two values: $T_n = 1$ for $1 \leq n \leq \delta N$ and $T_n = 0$ for $\delta N < n \leq N$. The number $\delta N = [2\delta W/\lambda_F]$ is the quantized conductance of the constriction (in units of $2e^2/h$). Averages of U over the unitary group introduce Kronecker delta's (cf. App. B in Ref. 10). We need the average $\Omega_{pp'qq'nm} \equiv \langle U^*_{pn} U^*_{qm} U_{p'm} U_{q'n} \rangle$, given by

$$\Omega_{pp'qq'nm} = \frac{1}{N^2-1}\left(\delta_{pq'}\delta_{qp'} - \frac{1}{N}\delta_{pp'}\delta_{qq'}\right) \quad \text{if} \quad n \neq m, \tag{10}$$

in the case of broken axial symmetry and

$$\Omega_{pp'qq'nm} = \frac{4}{N^2-\sigma_{N1}}\delta_{pq'}\delta_{qp'}\sigma_{pn}\sigma_{qm} \quad \text{if} \quad \sigma_{nm} = 0, \tag{11}$$

in the axially symmetric case.

Substituting these values of T_n into Eq. (9) and averaging over U with the help of Eqs. (10) and (11), we find

$$P = \frac{8eV}{h} \sum_{n=\delta N+1}^{N} \sum_{m=1}^{\delta N} \sum_{p,p',q,q'=1}^{N} \Omega_{pp'qq'nm} M^*_{pp'} M_{q'q} = \frac{8eV}{h}\frac{\delta N}{N}\left(1 - \frac{\delta N}{N}\right) \mathrm{Tr}\, M^2 = \frac{\pi^2}{9}\delta N P_0, \quad N \gg \delta N, \tag{12}$$

regardless of whether axial symmetry is present or not. We thus obtain a stepwise increase of P as a function of δW with step height $\Delta P = (\pi^2/9)P_0$. The numerically obtained step height in Fig. 2 agrees within 10% with the analytical estimate for the first step. For subsequent steps the agreement becomes worse, presumably because the approximation of a uniform distribution of U breaks down as δW increases. We can also calculate the mean transverse force in the same way,

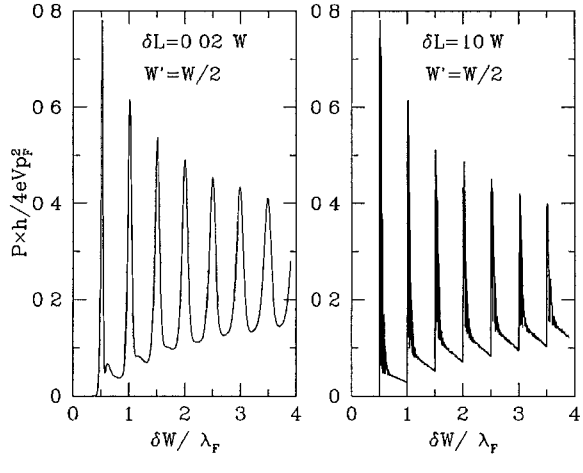

FIG 4. Noise power for longitudinal vibration of a short constriction (left panel) and a long constriction (right panel) These plots are for $W' = W/2$, but there is no noticeable dependence on the ratio W'/W The mean longitudinal force (not shown) decreases stepwise as a function of δW in both the short and long constriction.

starting from Eq (5), and find $\bar{F} \propto \mathrm{Tr}\, M = 0$, in accordance with the numerical result that $\bar{F} \ll F_0$ for a long constriction

In the short-constriction case $\delta L \ll W$ we may not treat U as uniformly distributed in the unitary group, and this has prevented us from finding a simple analytical representation of the numerical data

This rich geometry dependence of the noise power is characteristic for a transverse vibration For comparison we discuss the case of a longitudinal vibration, corresponding to a mode profile $\hat{x}u(x)$ oriented along the direction of the current through the constriction (instead of perpendicular to it) Such a longitudinal vibration corresponds to a compression mode of the beam, which is at a higher frequency than the *bending* mode excited by a transverse vibration For a longitudinal vibration the matrices M_{L}, M_{R} are diagonal $(M_{\mathrm{L}})_{nm} = -(M_{\mathrm{R}})_{nm} = \delta_{nm}\, \hbar\, |k_n|\, u(0)$ We take $u(0) = 1$

The noise power is plotted in Fig 4 for both a long and a short constriction It does not depend on the presence or absence of axial symmetry and is also rather insensitive to the length of the constriction The order of magnitude of the longitudinal noise power is $(4eV/h)p_F^2$, with $p_F = \hbar k_F$ the Fermi momentum This is larger than the typical transverse noise power P_0 by a factor of order $(k_F L/N)^2 \simeq (L/W)^2$ Inserting parameters $V = 1$ mV, $k_F = 10^8$ m^{-1}, typical for a two-dimensional electron gas, one estimates $(4eV/h)p_F^2 \simeq 10^{-40}$ N^2/Hz This is below the force sensitivity of present day nanomechanical oscillators, but is hoped to be reached in future generations of these devices [11]

In summary, we have demonstrated that the momentum noise of ballistic electrons does not vanish on the plateaus of quantized conductance Conductance quantization requires absence of backscattering in the constriction, but it does not preclude inter-mode scattering Momentum noise makes this inter-mode scattering visible in a way that current noise can not The dependence of the momentum noise on the constriction width was found to be remarkably varied, ranging from oscillatory to stepwise, depending on the direction of the vibration (longitudinal or transverse to the constriction), the presence or absence of axial symmetry, and the length of the constriction The stepwise increase amounts to a quantum of momentum noise that might be observable with an ultrasensitive oscillator

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