

## Editor's Choice

# Stripe fractionalization: the quantum spin nematic and the Abrikosov lattice

J. Zaanen\* and Z. Nussinov

Instituut-Lorentz for Theoretical Physics, Leiden University  
P.O. Box 9506, 2300 RA Leiden, The Netherlands

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In part (I) of this two paper series on stripe fractionalization [J. Phys. IV (France) **12**, Prg-245 (2002)], we argued that in principle the “domain wall-ness” of the stripe phase could persist in the spin and charge disordered superconductors, and we demonstrated how this physics is in one-to-one correspondence with Ising gauge theory. Here we focus on yet another type of order suggested by the gauge theory: the quantum spin nematic. Although it is not easy to measure this order directly, we argue that the superconducting vortices act as perturbations destroying the gauge symmetry locally. This turns out to give rise to a simple example of a gauge-theoretical phenomenon known as topological interaction. As a consequence, at any finite vortex density a globally ordered antiferromagnet emerges. This offers a potential explanation for recent observations in the underdoped 214 system.

**1. Introduction** Among others, stripe order means that the charge stripes are domain walls in the stripe antiferromagnet. In part I of this series of two papers [1] we explained that the physics of this domain wall-ness in the case that the stripes form a quantum liquid is formalized in terms of the most elementary field theory controlled by local symmetry: the Ising gauge theory. We showed that the gauge fields have a geometrical meaning. These parametrize the fluctuations of sublattice parity, the property that a bipartite space can be subdivided in two ways in two sublattices:  $\dots - A - B - \dots$  or  $\dots - B - A - \dots$ . In stripe language, the ordered (deconfining) state of the gauge theory corresponds with the stripes being intact as domain walls, implying that space is either  $\dots - A - B - \dots$  or  $\dots - B - A - \dots$ . The theory predicts a phase transition corresponding with the destruction of the stripe domain wall-ness, such that space turns non-bipartite (confinement). Remarkably, the gauge theory insists that this is a garden-variety quantum phase transition, which could be behind the quantum criticality of the optimally doped cuprate superconductors.

We concluded part I with the observation that this topological (dis)order can only be probed directly by topological means: non-local, multipoint correlation functions (Wilson loops) which seem to be out of reach of even the most fanciful experimental machine. At the same time, direct experimental evidence is required because theoretically one can only argue that it can happen. If it happens is a matter of microscopic details, which cannot be analyzed in general terms. This part II is dedicated to a potential way out of this problem. According to the theory there is yet another state of matter to be

\* Corresponding author: e-mail: [jan@lorentz.leidenuniv.nl](mailto:jan@lorentz.leidenuniv.nl)

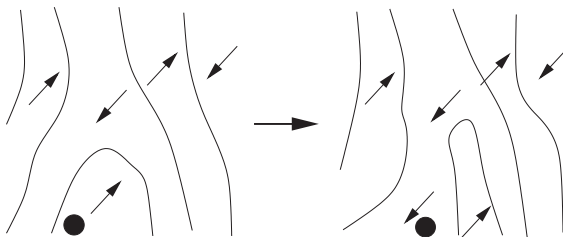
expected: the quantum spin nematic. This corresponds with a superconductor carrying a special type of anti-ferromagnetism characterized by an staggered order parameter which is minus itself (Section 2). Although such an order cannot be observed by the standard probes of anti-ferromagnetism (like neutron scattering and magnetic resonance) it is not as hidden as the pure topological order of part I.

By principle, superconducting order is required to protect the local Ising symmetry. In the type II state of the superconductor, the superconducting order is destroyed locally, in the vicinity of the vortices. Accordingly, the vortices correspond with “gauge defects” where the local Ising symmetry turns into a global one in isolated regions in space. These gauge defects are quite interesting theoretically: they correspond with an elementary example of the principle of “topological interaction”, non-dynamical influences mediating information over infinite distances (Section 3). In the stripe interpretation this just means that at the moment that vortices appear a piece of the spin-nematic turns into a long range ordered anti-ferromagnet. In the final section we give a recipe to study experimentally the spin nematic, making the case that it might well be that the recently observed magnetic field induced antiferromagnet in the  $\text{La}_{1.9}\text{Sr}_{0.1}\text{CuO}_4$  [2] is of this kind.

**2. The quantum spin nematic** In part I, we assumed implicitly that both the antiferromagnetic order and the charge order of the stripes were both fully destroyed and we discussed the fluctuating domain wall-ness in isolation. However, there is yet another state possible [3–5]. As long as the stripe dislocations do not proliferate, the spin system is not frustrated in essential ways; it can be argued that the domain wall-ness of the static stripes has everything to do with organizing the motions of the holes in such a way that the frustrating effect of the isolated hole motions are avoided. This unfrustrating influence of the stripes stays intact even when the stripes are completely delocalized, as long as they form connected domain walls. Hence, a state can exist in principle where the charge is disordered while next to the sublattice parity also the spin system maintains its antiferromagnetic order. However, due to the stripe fluctuations this is not a normal antiferromagnetic but instead a spin-nematic.

The nature of this state is easy to understand. Take a snapshot on a timescale short as compared to the charge fluctuations and we would see an ordered antiferromagnet except for the fact that the staggered order parameter flips every time a domain wall is crossed (Fig. 1). At some later time it will look similar except that all domain walls will have moved. At long times, we cannot say where the domain walls are with the ramification that the staggered order parameter becomes minus itself:  $\langle \mathbf{M}(\mathbf{r}) \rangle \equiv \langle (-1)^r \mathbf{S}(\mathbf{r}) \rangle \equiv -\langle \mathbf{M}(\mathbf{r}) \rangle$ . Hence, the order parameter is no longer a  $O(3)$  vector but instead an object pointing on the sphere having no head or tail: this is the director (or “projective plane”) order parameter well known from nematic liquid crystals, and it is therefore called a spin nematic [6].

This can be easily formalized in terms of a gauge theory [7]. The (fluctuating) antiferromagnetic order can be described in terms of (coarse) grained  $O(3)$  quantum rotors  $\mathbf{n}$ , quantized by an angular momentum  $\mathbf{L}$ , such that  $[L^\alpha, n^\beta] = i\varepsilon^{\alpha\beta\gamma} n^\gamma$ . As compared to the usual quantum non-linear sigma model



**Fig. 1** The physical nature of the quantum spin-nematic (or Higgs phase of the  $O(3)/Z_2$  theory) in terms of fluctuating stripes. As long as the domain walls are fully connected, the spin system is protected against frustration and one would see an antiferromagnetic order (arrows) upon taking a snapshot on a time scale short compared to the charge fluctuation scale. However, at long times the domain walls are delocalized with the effect that the fluctuating stripes turn the staggered order parameter into minus itself: watch what happens at the black dot.

description, the only difference is that the rotors are now minimally coupled to the  $Z_2$  gauge fields. We remind the reader of the Hamiltonian of the pure Ising gauge theory [8], parametrizing the dynamics of the domain wall-ness (see part I),

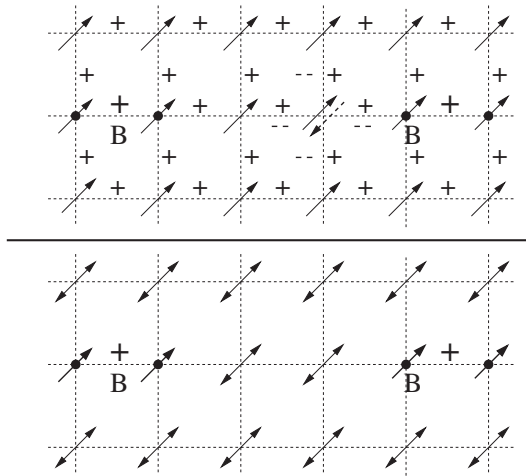
$$H_{\text{gauge}} = -K \sum_{\square} \sigma^3 \sigma^3 \sigma^3 \sigma^3 - \sum_{\langle ij \rangle} \sigma_{ij}^1 \quad (1)$$

where  $\sigma^{1,3}$  are Pauli-matrices acting on Ising bond variables.  $\sum_{\square} \sigma^3 \sigma^3 \sigma^3 \sigma^3$  is the plaquette interaction, such that Eq. (1) commutes with the generator of gauge transformations  $P_i = \prod_j \sigma_{ij}^1$ . To couple in the matter fields, put the rotors on the sites of the lattice of the gauge theory, and define

$$H_{O(3)/Z_2} = H_{\text{gauge}} - J \sum_{\langle ij \rangle} \sigma_{ij}^3 \mathbf{n}_i \cdot \mathbf{n}_j - \sum_i L_i^2. \quad (2)$$

Hence, the gauge fields determine the sign (“ferro” or “antiferromagnetic”) of the “exchange” interactions between the rotors on neighboring sites. Consider the case that both  $K$  and  $J$  are large. The gauge sector will be deconfining and the unitary gauge fix (all bonds +1) is representative [8]. Since  $J$  is also large the  $O(3)$  symmetry is also spontaneously broken and all rotors will point in the same direction (Fig. 2). Apply now a gauge transformation at some site  $i$ ; all bonds emerging from this site will turn from ferromagnetic in antiferromagnet and when one multiplies simultaneously  $\mathbf{n}_i$  by  $-1$  the energy will stay invariant. Hence, the gauge transformations take care of changing the (unphysical, non gauge invariant) antiferromagnet into the physical (gauge invariant) spin-nematic, characterized by a staggered order parameter “having no head or tail” (actually, the projective plane). Equation (2) is just the quantum interpretation of the classical  $O(3)/Z_2$  model studied in a great detail Lammert, Rokhsar and Toner [7]. The phase diagram is completely known, and the spin disordered deconfining and confining phase discussed in part I share a second order 3D Heisenberg transition and a first order quantum phase transition with the spin nematic, respectively.

Could there be such a spin nematic phase around in the context of cuprate superconductors? An obvious place to look for it would be the underdoped 214 system with its strong tendency towards antiferromagnetic order. In highly doped 2212 and 123 there are good reasons to believe that for other reasons the spin system is strongly quantum disordered. The spin nematic shares the attitude with the domain wall gauge fields to hide itself from detection in standard experiments. However, it is not as successful in this hiding game as the pure gauge fields are. Antiferromagnets can be directly probed using neutron scattering, NMR and  $\mu$ SR, because these experiments measure in one or the other way the two point (staggered) spin correlator  $S(|\mathbf{r} - \mathbf{r}'|) = \langle \mathbf{M}(\mathbf{r}) \mathbf{M}(\mathbf{r}') \rangle$ . Because in the spin nematic  $\mathbf{M}(\mathbf{r}) \equiv -\mathbf{M}(\mathbf{r})$ , independently at every  $\mathbf{r}$ , it follows that  $S = -S$ , meaning that it has to vanish:  $S$  is not gauge invariant. Employing again the “stripe detectors” of part I ( $\tilde{\sigma}^3(\mathbf{r})$  acquiring values  $-1, +1$



**Fig. 2** Construction of the spin nematic state, and the topological interactions between the spatially disconnected gauge defects. The unitary gauge (all bonds +) is representative and for large  $J$  the rotor degrees of freedom living on the sites will also order (upper panel). By performing gauge transformations (the dashed bonds and spins) the rotors turn into directors, which are like vectors except that their heads and tails are the same (lower panel). By applying an external field  $B$  giving a definite sense to the sign of isolated bonds the gauge symmetry is broken at the 4 sites labeled by dots in the figure. Remarkably, one finds following the same procedure as for in the absence of the gauge symmetry breaking that the heads of the  $O(3)$  vectors at the gauge defects all point in the same direction.

when a domain wall is detected or not, respectively), the gauge invariant correlation function which can “see” the spin nematic order is  $S_{ZZ}(|\mathbf{r} - \mathbf{r}'|) = \langle \mathbf{M}(\mathbf{r}) \Pi'_{\mathbf{r}-\mathbf{r}'} \tilde{\sigma}^3(\mathbf{l}) \mathbf{M}(\mathbf{r}') \rangle$ , i.e. the “matter correlator with the Wilson line inserted”. Relative to the Wilson loops of part I, this does not seem to add much to the comfort of the experimental physicist.

However, with the matter fields present there is more to look for. In the coarse grained  $O(3)$  language, although  $\mathbf{n}$  is not gauge invariant the traceless tensor  $Q_{\alpha\beta} = n^\alpha n^\beta - 1/3 \delta_{\alpha\beta}$  [6, 7] is a gauge singlet because it transforms like  $n^2$ . This tensor is actually measured in two magnon Raman scattering [9]. There is unfortunately a practical problem. Imagine that a spin nematic would be realized in, say,  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . The 5 meV gap observed in the superconducting state in the spectrum of incommensurate spin fluctuations would then be interpreted as the charge fluctuation scale. At energies below the gap the structure factor vanishes because the spin nematic sets in. However, at energies above the gap the antiferromagnetism becomes visible because the neutrons are just “taking the snapshots” as in Fig. 1. On a side, this interpretation actually offers a simple interpretation for the observation that this gap disappears above the superconducting  $T_c$ : when the phase order disappears the charge fluctuations become relaxational and there is no longer a characteristic charge fluctuation scale protecting the gauge invariance dynamically [10, 11], although it might be still around in the statics [12]. In order to nail down the spin nematic one would like to see the characteristic behavior associated with spin waves in the Raman response (intensity  $\sim \omega^3$ ) at energies less than 5 meV where the neutrons seem to indicate there is nothing. Unfortunately it seems impossible to isolate the two magnon scattering from the Raman signal at these low energies [13].

**3. Vortices as gauge defects** Fortunately, there is a much less subtle way to look for the spin nematic. As we explained in part I, the emergence of the gauge invariance requires the presence of the superconducting order. Hence, when superconductivity is destroyed the gauge invariance is destroyed and the local  $Z_2$  symmetry turns global. Upon applying a magnetic field to the superconductor, the Abrikosov vortex lattice is created where the superconductivity is locally destroyed in the vicinity of the vortices. This suggests that we have to consider the general problem of what happens with the gauge theory when the gauge invariance turns into global  $Z_2$  invariance at isolated regions in space: the “gauge defects”. Let us first consider this problem on an abstract level, using the lattice gauge theory, to continue thereafter with a consideration what this all means for stripes.

Breaking the gauge symmetry, even in isolated spots in space, is a brutal operation. In first instance it does not matter how one breaks it. Let us therefore take the Hamiltonian Eqs. (1), (2) and add the simplest “impurity” term breaking the local symmetry,

$$H_{\text{imp}} = -B \sum_{\langle kl \rangle} \sigma_{kl}^3, \quad (3)$$

where we pick some bonds  $kl$  as the “impurity sites”. This term “removes” the gauge from the bond, and the gauge invariance is destroyed on the two sites connected by the  $kl$  bond when  $B \neq 0$ . For a single impurity, the symmetry turns locally into a global  $O(3)$  symmetry. Consider now the case that spin nematic order is present and insert two gauge defects with  $B > 0$ , separated by some large distance (Fig. 2). Take the unitary gauge: all bonds +1 including the impurity bonds. Obviously, when  $K$  and  $J$  are both large this is a representative gauge, regardless the presence of the two +1 global bonds, and in this gauge all rotors point in the same direction. In a next step, perform gauge transformations everywhere except for the four sites where the gauge symmetry broken. This will turn the medium into a spin nematic (Fig. 2). What has happened? Although the two impurity sites are separated by a medium which seem to have no knowledge about where the heads and the tails of the rotors at the impurity site are, there seems to be a remarkable “action at a distance”: although the two impurity sites can be infinitely apart the spins know that they have to stick their heads in the same direction! It is easily checked that the unitary gauge stays representative also in the presence of virtual vison pairs and it is only when the visons proliferate, destroying the spin nematic order, that this “action at a distance” is destroyed. The conclusion is that a local breaking of the gauge invariance suffices to cause an global  $Z_2$  “headness” *long range order* of the rotors, so that they together break

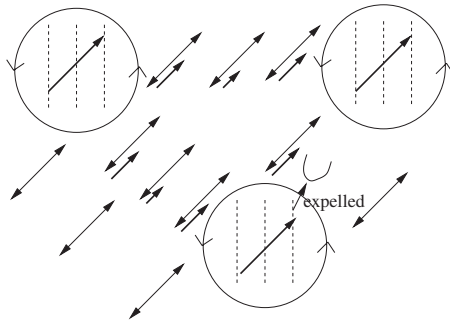
the ungauged  $O(3)$  symmetry. In this sense the local symmetry is infinitely fragile with regard to global violations. It is noticed that the above is an elementary example of a topological interaction, i.e. an information carrying influence which is entirely non-dynamical and not mediated by propagating excitations. These are known to occur in much less trivial theories, like for instance  $2 + 1$  dimensional gravity [14].

In fact, the above is not quite representative yet for the stripe case, because we have to build in communication with the translational symmetry. All we have in the gauge theory is the simple “auxiliary” lattice on which the theory is defined, and the minimal way to let the spin system know about this lattice is by incorporating a sense of antiferromagnetism. Upon breaking the gauge this is easily achieved by taking for the gauge defects a negative “exchange”  $B < 0$ . The “action at a distance” for this case can be constructed in a similar way as for the “ferromagnetic” case. Start again with unitary gauge (everywhere  $+1$  bonds) and perform gauge transformations producing a negative bond at the impurity bonds, to subsequently restore the gauge invariance away from the impurity sites. One now encounters an ambiguity. One can perform the gauge transformation on the site to the “left” or the “right” of a impurity bond, and one finds that pending this choice the orientation of the staggered order reverses relative to a reference impurity. At first sight it seems that for staggered configurations the “action at a distance” fails, because the heads and the tails of the local staggered order parameters point in arbitrary directions. However, this is not the case: this indeterminedness has nothing to do with the “topological gauge force” but instead with a left-over translational invariance. The generators of gauge transformations live on the sites and by breaking the gauge invariance on a single bond, the gauge invariance is broken on the two sites connected by this bond which remain therefore translationally equivalent. This translation is responsible for the flipping of the staggered order. One should instead center the gauge symmetry breaking on a site. Apply for instance the symmetry breaking  $-B\Pi_l\sigma_{kl}^3$ , fixing all bonds coming out of the site  $k$ , to find that in this case the gauge action-at-a-distance acts in exactly the same way for the staggered order parameter as it does for the uniform case.

Summarizing, using an elementary argument, we identified a ghostly, non-dynamical action at a distance ordering the rotors at spatially disconnected “gauge impurities” which requires nothing more than spin nematic order. As a caveat, we found that in order to find the same global order for staggered spin we have to add as an extra requirement also the translational symmetry breaking by the impurities. We will now argue that these general features of the gauge theory acquire a quite mundane interpretation in terms of the stripes.

**4. Magnetic field induced antiferromagnetism** Anything in the gauge theory should be in one-to-one correspondence to something in stripe physics. This is also true for the gauge defects and in fact it becomes so simple in the stripe interpretation that the latter is an ideal tool to convince the gauge theory student that the ghostly “action at a distance” is actually not a big deal.

Given that the spin nematic exist, it has to be that the competitor of the superconductor is a fully ordered stripe phase. As we will discuss in more detail, it is reasonable to expect that in the proximity of the vortex cores the charge density order of the stripe phase will re-emerge, and it might be that this is already observed in the form of the stripy “halo’s” surrounding the vortex cores as seen by Hoffman and coworkers by STM [15]. Charge is bound to the domain wall-ness and when charge orders the domain walls come to rest, and the spin-nematic turns into a stripe antiferromagnet which can be seen by conventional means like neutron scattering, see Fig. 3. The charge order is the gauge defect, making the magnetic order visible which already pre-existed in the superconductor. The amount of antiferromagnetic order is expected to be proportional to the volume taken by the charge-ordered halo’s, because this corresponds to the volume of the system where the local symmetry turned global. What determines the correlation length of this antiferromagnet? We remind the reader of the translational symmetry breaking required for staggered order as discussed in the previous section. In the stripe context it has the following meaning. Although the spins always become static, a full stripe antiferromagnetic order also needs a full translational order of the sublattice parity which is the same



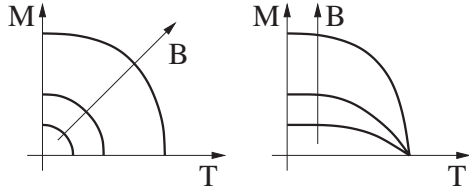
**Fig. 3** A cartoon picture of the charge coming to rest at the vortex cores, with the effect that the sublattice parity stops fluctuating locally. This automatically turns the spin nematic into a normal anti-ferromagnet.

as translational order in the charge sector: the antiferromagnetic correlation length is identical to the charge-order correlation length.

With regard to the thermal phase transition one expects that the spin nematic behaves similar to the antiferromagnet. In strictly 2D one cannot have true spin-nematic long range order (LRO) [12]. However, due to weak 3D couplings, etcetera, one expects nevertheless a true LRO at low temperatures. A difficult question is if the spin nematic completely disorders at this finite temperature transition or that a topologically ordered phase can be realized. In the first case, the transition has to be first order but it is likely so weakly first order that it is hard to distinguish from a second order transition. Our most striking prediction is that when an external magnetic field is applied, the temperature where this thermal phase transition occurs should at least initially be field independent. The reason is simple. In the absence of the field the spin-nematic order is already well developed, protected by a large cohesive energy of order of the observed transition temperature  $\sim 40$  K. Since the external field couples in through its energy, and since the field energy (a few Tesla's) is small compared to the spin-nematic cohesive energy, the field cannot change the transition significantly. Hence, the specialty of the quantum spin nematic, which we believe is unique to this form of matter, is that it causes an apparent dissimilarity between the sensitivity of the zero-temperature antiferromagnetic order as induced by the magnetic field and the insensitivity of the thermal phase transition temperature to the same field. The magnetic order is already strongly developed at zero field but it cannot be measured by neutrons, etcetera. Upon applying the field, the spin nematic turns in part into an antiferromagnet, becoming visible in magnetic experiments with a magnitude determined by the induced charge order. This is to be strongly contrasted with the "conventional" interpretation that the magnetic field *creates* the antiferromagnetic order.

Zhang, Demler and Sachdev [4] have developed a general phenomenological theory, dealing with the case that the antiferromagnetic order is created by the field, arriving at a number of strong predictions. Their starting point is a soft-spin, Ginzburg–Landau–Wilson description of the antiferromagnetic order parameter field  $\phi$  and the superconducting field  $\Psi$ . The lowest order coupling between the two fields is  $B|\Psi|^2|\phi|^2$ . They arrive at the counter-intuitive result that, starting with a quantum disordered antiferromagnet, one has to exceed a critical strength of the magnetic field before LRO antiferromagnetism sets in which is delocalized over the system. The reason is the self-interaction of the antiferromagnetic order parameter field preventing it from localizing itself in the vicinity of the vortex cores. Comparing it to the data by Lake et al. [2], they argue that  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  shows already antiferromagnetic order in zero-field implying that this superconductor coexists with an antiferromagnet. A worry is that this zero field antiferromagnetism has a completely different temperature dependence (not showing signs of a finite temperature phase transition) while it is apparently varying strongly from sample to sample, suggesting that it is a dirt effect. At the same time, the field induced antiferromagnetism seems to come up smoothly with the field and there is no sign of a critical threshold. Even more worrisome is the fact that the temperature where the field induced antiferromagnetism appears is rather independent of the applied field and this is very hard to understand in this competing order framework. Since the antiferromagnetic order is created by the field, it is very weak when the field is small and accordingly one would expect that initially  $T_N$  is very small, increasing rapidly with the





**Fig. 4** Left panel: the expected magnetic field dependence of the induced magnetism as function of temperature in the competing order framework. The magnetic transition temperature is expected to be strongly dependent on the applied field. Right panel: assuming that a spin-nematic is present, the transition temperature should barely depend on the field, because the order is already present in the absence of the field, to become just observable in the neutron scattering in the presence of the field. The data of Lake et al. for  $\text{La}_{1.9}\text{Sr}_{0.1}\text{CuO}_4$  look like the right panel.

increase of the zero temperature staggered order parameter. In fact, assuming that  $T_N$  is due to 3 dimensional couplings and spin anisotropies, one expects  $T_N$  to be linearly proportional to  $M_0$  [16], the zero temperature staggered magnetization for small  $M_0$ . Instead,  $T_N$  is in the Lake experiments rather field independent and we take this as strong evidence in favor of the spin nematic (Fig. 4).

Can the fraction of the spin nematic turning into antiferromagnetic order as function of the magnetic field be quantified? In fact, this is possible although the solution is only available right now in numerical form. The problem of the pinning of the charge density wave by the vortex lattice is also addressed in some detail by Zhang et al. [4]. The crucial difference with the antiferromagnet is that the charge density wave communicates directly with the vortex lattice because both fields break translational invariance. As a result, the vortex-lattice acts as a spatially varying potential on the charge-ordering field (Eq. (1.12) in Ref. [4]) with the consequence that charge order directly accumulates in the vicinity of the vortex cores at any value of the external field. Zhang et al. present some numerical results on the behavior of the charge order in the magnetic field (Figs. 15, 16 in Ref. [4]). A caveat is that these are calculated in the presence of a low lying magnetic exciton and it is not immediately clear if these results are directly applicable to the spin nematic case. A related issue is to what extent the commensuration effects associated with the stripe charge order versus the vortex lattice can give rise to strong charge order correlations between the “halo’s” centered at different vortices. As we discussed, such correlations are a necessary condition to find correlations in the spin system exceeding the vortex distance. Notice, however, that these theoretical difficulties can be circumvented using experimental information: when the spin nematic is present, the antiferromagnet order should closely follow the charge order, in strong contrast with the expectations following from the competing order ideas.

In conclusion, we have presented the hypothesis that in underdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  a new state of quantum matter might be present: a superconductor which is at the same time showing spin nematic order. We have argued that it should be possible to proof or disproof the presence of such a state using conventional experimental means, while existing experiments already strongly argue in favor of this possibility. What really matters is that, if the spin nematic is indeed realized, the proof of principle is delivered that the domain wall-ness of the ordered stripe phase can persist in the quantum fluid. This would add credibility to the possibility that the stripe topological order could even persist in the absence of any spin order, which in turn could be responsible for the anomalies of the best superconductors.

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