Proposal for Production and Detection of Entangled Electron-Hole Pairs in a Degenerate Electron Gas

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We demonstrate theoretically that the shot noise produced by a tunnel barrier in a two-channel conductor violates a Bell inequality. The nonlocality is shown to originate from entangled electron-hole pairs created by tunneling events—without requiring electron-electron interactions. The degree of entanglement (concurrence) equals $2(T_1T_2)^{1/2}(T_1 + T_2)^{-1}$, with $T_1, T_2 \ll 1$ the transmission eigenvalues. A pair of edge channels in the quantum Hall effect is proposed as an experimental realization.

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The controlled production and detection of entangled particles is the first step on the road towards quantum information processing [1]. In optics this step was taken long ago [2], but in the solid state it remains an experimental challenge. A variety of methods to entangle electrons have been proposed, based on quite different physical mechanisms [3]. A common starting point is a spin-singlet electron pair produced by interactions, such as the Coulomb interaction in a quantum dot [4–6], the pairing interaction in a superconductor [7–10], or Kondo scattering by a magnetic impurity [11]. A very recent proposal based on orbital entanglement also makes use of the superconducting pairing interaction [12].

It is known that photons can be entangled by means of linear optics using a beam splitter [13–15]. The electronic analog would be an entangler that is based entirely on single-electron physics, without requiring interactions. But a direct analogy with optics fails. Electron reservoirs are in local thermal equilibrium, while in optics a beam splitter is incapable of entangling photons from a thermal source [16]. That is why previous proposals [11,17] to entangle electrons by means of a beam splitter start from a two-electron Fock state, rather than a many-electron thermal state. To control the extraction of a single pair of electrons from an electron reservoir requires strong Coulomb interaction in a tightly confined area, such as a semiconductor quantum dot or carbon nanotube [3]. Indeed, it has been argued [18] that one cannot entangle a spatially separated current of electrons from a normal (not-superconducting) source without recourse to interactions.

What we propose here is an altogether different, interaction-free source of entangled quasiparticles in the solid state. The entanglement is not between electron pairs but between electron-hole pairs in a degenerate electron gas. The entanglement and spatial separation are realized purely by elastic scattering at a tunnel barrier in a two-channel conductor. We quantify the degree of entanglement by calculating how much the current fluctuations violate a Bell inequality.

Any two-channel conductor containing a tunnel barrier could be used in principle for our purpose, and the analysis which follows applies generally. The particular implementation described in Fig. 1 uses edge channel transport in the integer quantum Hall effect [19]. It has the advantage that the individual building blocks have already been realized experimentally for different purposes. If the two edge channels lie in the same Landau level, then the entanglement is between the spin degrees

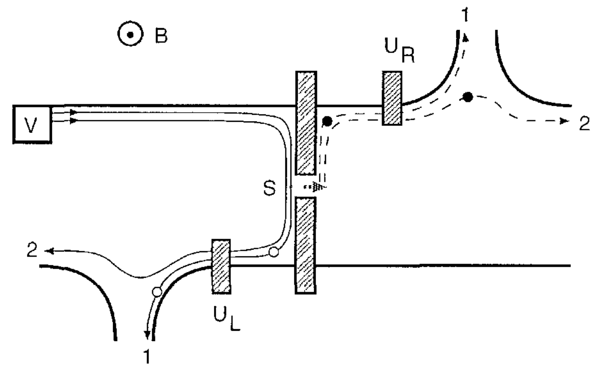

FIG. 1. Schematic description of the method to produce and detect entangled edge channels in the quantum Hall effect. The thick black lines indicate the boundaries of a two-dimensional electron gas. A strong perpendicular magnetic field B ensures that the transport near the Fermi level E_F takes place in two edge channels, extended along a pair of equipotentials (thin solid and dashed lines, with arrows that give the direction of propagation). A split gate electrode (shaded rectangles at the center) divides the conductor into two halves, coupled by tunneling through a narrow opening (dashed arrow, scattering matrix S). If a voltage V is applied between the two halves, then there is a narrow energy range eV above E_F in which the edge channels are predominantly filled in the left half (solid lines) and predominantly empty in the right half (dashed lines). Tunneling events introduce filled states in the right half (black dots) and empty states in the left half (open circles). The entanglement of these particle-hole excitations is detected by the violation of a Bell inequality. This requires two gate electrodes to locally mix the edge channels (scattering matrices U_L, U_R) and two pairs of contacts 1, 2 to separately measure the current in each transmitted and reflected edge channel.

of freedom. Alternatively, if the spin degeneracy is not resolved by the Zeeman energy and the two edge channels lie in different Landau levels, then the entanglement is between the orbital degrees of freedom. The beam splitter is formed by a split gate electrode, as in Ref. [20]. In Fig. 1 we show the case that the beam splitter is weakly transmitting and strongly reflecting, but it could also be the other way around. To analyze the Bell inequality an extra pair of gates mixes the orbital degrees of freedom of the outgoing states independently of the incoming states. (Alternatively, one could apply a local inhomogeneity in the magnetic field to mix the spin degrees of freedom.) Finally, the current in each edge channel can be measured separately by using their spatial separation, as in Ref. [21]. (Alternatively, one could use the ferromagnetic method to measure spin current as described in Refs. [3,22].)

It is easiest to understand what happens if the beam splitter does not mix the edge channels. An electron can tunnel from either Landau level into the empty right half of the system, leaving behind a hole in the filled left half with the same Landau level index. This correlation entangles the electron-hole pair. Let us assume, for the simplest example, that each edge channel tunnels with the same probability T. The resulting state is a superposition of the vacuum state $|0\rangle$ (all states filled at the left and empty at the right) with weight $\sqrt{1-T}$ and the maximally entangled Bell pair $(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$ with weight $\sqrt{T}$. The role of the spin indices $\uparrow, \downarrow$ is played by the Landau level indices $i = 1, 2$. The first index in the ket $|\uparrow\uparrow\rangle$ refers to the hole at the left and the second index to the electron at the right. We now generalize this elementary example to an arbitrary scattering matrix, including channel mixing and unequal transmission probabilities.

Electrons are incident on the beam splitter from the left in a range eV above the Fermi energy E_F. (The states below E_F are all occupied at low temperatures, so they do not contribute to transport properties.) The incident state has the form

$$|\Psi_{\rm in}\rangle = \prod_{0<\varepsilon<eV} a^\dagger_{{\rm in},1}(\varepsilon) a^\dagger_{{\rm in},2}(\varepsilon)|0\rangle. \tag{1}$$

The fermion creation operator $a^\dagger_{{\rm in},i}(\varepsilon)$ excites the ith channel incident from the left at energy ε above the Fermi level. Similarly, $b^\dagger_{{\rm in},i}(\varepsilon)$ excites a channel incident from the right. Each excitation is normalized such that it carries unit current. It is convenient to collect the creation operators in two vectors $a^\dagger_{\rm in}$, $b^\dagger_{\rm in}$ and to use a matrix notation,

$$|\Psi_{\rm in}\rangle = \prod_\varepsilon \begin{pmatrix} a^\dagger_{\rm in} \\ b^\dagger_{\rm in} \end{pmatrix} \begin{pmatrix} \frac{1}{2}\sigma_y & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a^\dagger_{\rm in} \\ b^\dagger_{\rm in} \end{pmatrix} |0\rangle, \tag{2}$$

with σ_y a Pauli matrix.

The input-output relation of the beam splitter is

$$\begin{pmatrix} a_{\rm out} \\ b_{\rm out} \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{\rm in} \\ b_{\rm in} \end{pmatrix}. \tag{3}$$

The 4×4 unitary scattering matrix S has 2×2 submatrices r, r', t, t' that describe reflection and transmission of states incident from the left or from the right. Substitution of Eq. (3) into Eq. (2) gives the outgoing state

$$|\Psi_{\rm out}\rangle = \prod_\varepsilon (a^\dagger_{\rm out} r \sigma_y t^{\rm T} b^\dagger_{\rm out} + [r\sigma_y r^{\rm T}]_{12} a^\dagger_{{\rm out},1} a^\dagger_{{\rm out},2} + [t\sigma_y t^{\rm T}]_{12} b^\dagger_{{\rm out},1} b^\dagger_{{\rm out},2})|0\rangle. \tag{4}$$

The superscript "T" indicates the transpose of a matrix.

To identify the entangled electron-hole excitations we transform from particle to hole operators at the left of the beam splitter: $c_{{\rm out},i} = a^\dagger_{{\rm out},i}$. The new vacuum state is $a^\dagger_{{\rm out},1} a^\dagger_{{\rm out},2}|0\rangle$. To leading order in the transmission matrix the outgoing state becomes

$$|\Psi_{\rm out}\rangle = \prod_\varepsilon (\sqrt{w}|\Phi\rangle + \sqrt{1-w}|0\rangle), \tag{5}$$

$$|\Phi\rangle = w^{-1/2} c^\dagger_{\rm out} \gamma b^\dagger_{\rm out}|0\rangle, \qquad \gamma = \sigma_y r \sigma_y t^{\rm T}. \tag{6}$$

It represents a superposition of the vacuum state and a particle-hole state Φ with weight $w = {\rm Tr}\,\gamma\gamma^\dagger$.

The degree of entanglement of Φ is quantified by the concurrence [23,24],

$$C = 2\sqrt{{\rm Det}\,\gamma\gamma^\dagger}/{\rm Tr}\,\gamma\gamma^\dagger, \tag{7}$$

which ranges from 0 (no entanglement) to 1 (maximal entanglement). Substituting Eq. (6) and using the unitarity of the scattering matrix we find after some algebra that

$$C = \frac{2\sqrt{(1-T_1)(1-T_2)T_1T_2}}{T_1 + T_2 - 2T_1T_2} \approx 2\sqrt{T_1T_2}/(T_1 + T_2) \quad \text{if } T_1, T_2 \ll 1. \tag{8}$$

The concurrence is entirely determined by the eigenvalues $T_1, T_2 \in (0, 1)$ of the transmission matrix product $t^\dagger t = \mathbb{1} - r^\dagger r$. The eigenvectors do not contribute. This means, in particular, that channel mixing does not degrade the entanglement as long as the transmission eigenvalues remain unaffected. Maximal entanglement is achieved if the two transmission eigenvalues are equal: $C = 1$ if $T_1 = T_2$.

The particle-hole entanglement is a nonlocal correlation that can be detected through the violation of a Bell inequality [25,26]. We follow the formulation in terms of irreducible current correlators in the frequency domain of Samuelsson, Sukhorukov, and Büttiker [12], which in the tunneling limit $T_1, T_2 \ll 1$ is equivalent to a more general formulation in the time domain [18]. We will demonstrate explicitly later on that we need the tunneling assumption.

The quantity $C_{ij} = \int_{-\infty}^{\infty} dt\, \overline{\delta I_{L,i}(t)\delta I_{R,j}(0)}$ correlates the time-dependent current fluctuations $\delta I_{L,i}$ in channel $i = 1, 2$ at the left with the current fluctuations $\delta I_{R,j}$ in channel $j = 1, 2$ at the right. It can be measured directly in the frequency domain as the covariance of the low-frequency component of the current fluctuations At low temperatures ($kT \ll eV$) the correlator has the general expression [27]

$$C_{ij} = -(e^3V/h)|(rt^\dagger)_{ij}|^2 \tag{9}$$

We need the following rational function of correlators

$$E = \frac{C_{11} + C_{22} - C_{12} - C_{21}}{C_{11} + C_{22} + C_{12} + C_{21}} = \frac{\mathrm{Tr}\,\sigma_z r t^\dagger \sigma_z t r^\dagger}{\mathrm{Tr}\, r^\dagger r t^\dagger t} \tag{10}$$

By mixing the channels locally in the left and right arm of the beam splitter, the transmission and reflection matrices are transformed as $r \to U_L r$, $t \to U_R t$, with unitary 2×2 matrices U_L, U_R The correlator transforms as

$$E(U_L, U_R) = \frac{\mathrm{Tr}\,U_L^\dagger \sigma_z U_L r t^\dagger U_R^\dagger \sigma_z U_R t r^\dagger}{\mathrm{Tr}\, r^\dagger r t^\dagger t} \tag{11}$$

The Bell-CHSH (Clauser-Horne-Shimony-Holt) parameter is [25,28]

$$\mathcal{E} = E(U_L, U_R) + E(U_L', U_R) + E(U_L, U_R') - E(U_L', U_R') \tag{12}$$

The state is entangled if $|\mathcal{E}| > 2$ for some set of unitary matrices U_L, U_R, U_L', U_R' By repeating the calculation of Ref [29] we find the maximum [30]

$$\mathcal{E}_{\max} = 2\left[1 + \frac{4(1-T_1)(1-T_2)T_1T_2}{(T_1+T_2-T_1^2-T_2^2)^2}\right]^{1/2}$$
$$\approx 2[1 + 4T_1T_2(T_1+T_2)^{-2}]^{1/2} \quad \text{if } T_1, T_2 \ll 1 \tag{13}$$

Comparison with Eq (8) confirms the expected relation $\mathcal{E}_{\max} = 2(1 + C^2)^{1/2}$ between the concurrence and the maximal violation of the CHSH inequality [31] As mentioned above, we need the tunneling limit If T_1 and T_2 are not $\ll 1$ there is no one-to-one relation between $\mathcal{E}_{\max}$ in Eq (13) and C in Eq (8)

As a final consistency check we consider the effect of dephasing [32] Dephasing is modeled by introducing random phase factors in each edge channel, which amounts to the substitutions

$$U_L \to U_L \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}, \qquad U_R \to U_R \begin{pmatrix} e^{i\psi_1} & 0 \\ 0 & e^{i\psi_2} \end{pmatrix} \tag{14}$$

We average $E(U_L, U_R)$ over the random phases, uniformly in $(0, 2\pi)$, and find

$$\mathcal{E}_{\max} = \frac{2|\mathrm{Tr}\,\sigma_z r t^\dagger \sigma_z t r^\dagger|}{\mathrm{Tr}\, r^\dagger r t^\dagger t} \le 2 \tag{15}$$

So for strong dephasing there is no violation of the Bell inequality $|\mathcal{E}| \le 2$ The intermediate regime between weak and strong dephasing is more complex There exists a range of dephasing strengths for which $\mathcal{E} \le 2$ but the electron-hole state is still entangled [33] All of this is as expected for entanglement of a mixed state [26]

In conclusion, we have demonstrated theoretically that a tunnel barrier creates spatially separated currents of entangled electron-hole pairs in a degenerate electron gas Because no Coulomb or pairing interaction is involved, this is an attractive alternative to existing proposals for the interaction-mediated production of entanglement in the solid state We have described a possible realization using edge channel transport in the quantum Hall effect There is a remarkable contrast with quantum optics, where a beam splitter cannot create entanglement if the source is in local thermal equilibrium This might well explain why the elementary mechanism for entanglement production described here was not noticed before

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[24] The concurrence C quantifies the entanglement of a two-qubit state It is meaningful in the tunneling limit, when the full state $\Psi_{\rm out}$ in Eq (4) can be reduced to the two qubit state Φ superimposed on the vacuum We will show that in this limit C may be directly measured by a current correlator For arbitrary transmission, the degree of entanglement of $\Psi_{\rm out}$ can be quantified by the entanglement of formation $\mathcal{F}$ (measured in bits per second) We find $\mathcal{F} = -(eV/h)[T_1 \log T_1(1-T_2) + T_2 \log T_2(1-T_1) + (1-T_1-T_2)\log(1-T_1)(1-T_2)]$ Although the full state is entangled for arbitrary transmission, we know how to measure this entanglement only in the tunneling limit

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