Charge Detection Enables Free-Electron Quantum Computation

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It is known that a quantum computer operating on electron-spin qubits with single-electron Hamiltonians and assisted by single-spin measurements can be simulated efficiently on a classical computer. We show that the exponential speedup of quantum algorithms is restored if single-charge measurements are added. These enable the construction of a CNOT (controlled NOT) gate for free fermions, using only beam splitters and spin rotations. The gate is nearly deterministic if the charge detector counts the number of electrons in a mode, and fully deterministic if it only measures the parity of that number.

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Flying qubits transport quantum information between distant memory nodes and form an essential ingredient of a scalable quantum computer [1]. Flying qubits could be photons [2], but using conduction electrons in the solid state for this purpose removes the need to convert material qubits to radiation. Since the Coulomb interaction between free electrons is strongly screened, an interaction-free mechanism for logical operations on electronic flying qubits could be desirable. The search for such a mechanism is strongly constrained by a no-go theorem [3,4], which states that the exponential speedup of quantum over classical algorithms cannot be reached with single-electron Hamiltonians assisted by single-spin measurements. Here we show that the full power of quantum computation is restored if single-charge measurements are added. These enable the construction of a CNOT (controlled NOT) gate for free fermions, using only beam splitters and spin rotations.

The no-go theorem [3,4] applies only to fermions, not to bosons. Indeed, in an influential paper [2], Knill, Laflamme, and Milburn showed that the exponential speedup over a classical algorithm afforded by quantum mechanics can be reached using only linear optics with single-photon detectors. The detectors interact with the qubits, providing the nonlinearity needed for the computation, but qubit-qubit interactions (e.g., nonlinear optical elements) are not required in the bosonic case. This difference between bosons and fermions explains why the topic of "free-electron quantum computation" (FEQC) is absent in the literature, in contrast to the active topic of "linear optics quantum computation" (LOQC) [5–12]. Here we would like to open up the former topic, by demonstrating how the constraint on the efficiency of quantum algorithms for free fermions can be removed. We accomplish this by using the fact that the electron carrying the qubit in its spin degree of freedom has also a charge degree of freedom. Spin and charge commute, so a measurement of the charge leaves the spin qubit unaffected. To measure the charge the qubit should interact with a detector, but no qubit-qubit interactions are needed.

Charge detectors play a prominent role in a variety of contexts: as which-path detectors they control the visibility of Aharonov-Bohm oscillations [13], in combination with a beam splitter they provide a way to entangle two noninteracting particles [14], in combination with spin-dependent tunneling they enable the readout of a spin qubit [15,16]. The experimental realization uses the effect of the electric field of the charge on the conductance of a nearby point contact [17]. The effect is weak, because of screening, but measurable if the point contact is near enough. Such a device functions as an *electrometer*. It can count the occupation number of a spatial mode (0, 1, or 2 electrons with opposite spin). If the point contact is replaced by a quantum dot with a resonant conductance, then it is possible to operate the device as a *parity meter*. It can distinguish occupation number one (when it is on resonance) from occupation number 0 or two (when it is off resonance)—but it cannot distinguish between 0 and 2. We will consider both types of charge detectors in what follows.

The general formulation of fermionic quantum computation [18] is in terms of local modes which can be either empty or occupied. The annihilation operator of a local mode is a_{is}, with spatial mode index $i = 1, 2, 3, \ldots$ and spin index $s = \uparrow, \downarrow$. For noninteracting fermions the Hamiltonian is bilinear in the creation and annihilation operators. A local measurement in the computational basis has projection operators $n_{is} = a_{is}^{\dagger} a_{is}$ and $1 - n_{is} = a_{is} a_{is}^{\dagger}$. Terhal and one of the authors [3] showed that the probability of the outcome of any set of such local measurements is the square root of a determinant. Since a determinant of order N can be evaluated in a time which scales polynomially with N, the quantum algorithm can be simulated efficiently on a classical computer. This is the no-go theorem mentioned in the introduction.

We now add measurements of the local charge $Q_i = n_{i\uparrow} + n_{i\downarrow}$ to the algorithm. The eigenvalues of Q_i are 0, 1, 2. The probability that charge one is measured is given by the expectation value of the projection operator

$$P_i = 1 - (1 - Q_i)^2 = a_{i\uparrow}^\dagger a_{i\uparrow} a_{i\downarrow} a_{i\downarrow}^\dagger + a_{i\downarrow}^\dagger a_{i\downarrow} a_{i\uparrow} a_{i\uparrow}^\dagger. \quad (1)$$

The operator P_i is the sum of two local operators in the computational basis. The probability that M spatial modes are singly occupied therefore consists of a sum of an exponentially large number (2^M) of determinants, so now a classical simulation need no longer scale polynomially with the number of modes. Notice that a measurement of Q_i contains less information about the state than separate measurements of $n_{i\uparrow}$ and $n_{i\downarrow}$. The fact that partial measurements can add computational power is a basic principle of quantum algorithms [1].

Let us now see how these formal considerations could be implemented, by constructing a CNOT gate using only beam splitters, spin rotations, and charge detectors. To construct the gate we need one of two new building blocks that are enabled by charge detectors. The first building block is the Bell-state analyzer shown in Fig. 1. For this device it does not matter whether the charge detector operates as an electrometer or as a parity meter. The second building block, shown in Fig. 2, converts a charge parity measurement to a spin parity measurement. We present each device in turn and then show how to construct the CNOT gate.

The Bell-state analyzer makes it possible to teleport [19] the spin state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ of electron A to another electron A', using a third electron B that is entangled with A'. The teleportation is performed by measuring the joint state of A and B in the Bell basis

$$|\Psi_0\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}, \quad (2)$$

$$|\Psi_1\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, \quad (3)$$

$$|\Psi_2\rangle = (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}, \quad (4)$$

$$|\Psi_3\rangle = (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)/\sqrt{2}. \quad (5)$$

A no-go theorem [20,21] says that such a Bell measurement cannot be done deterministically (meaning with 100% success probability) without using interactions between the qubits. However, it has been noted that this theorem does not apply to qubits that possess an additional degree of freedom [22], and that is how we will work around it.

In Fig. 1 we show how a deterministic Bell measurement for fermions can be performed using three 50/50 beam splitters, three charge detectors, and two local spin rotations (represented by Pauli matrices σ_x and σ_z). The beam splitter scatters two electrons into the same arm (bunching) if they are in the singlet state (2), and into two different arms (antibunching) if they are in one of the triplet states (3)–(5). (This can be easily understood [23] from the antisymmetry of the wave function under particle exchange, demanded by the Pauli principle. The singlet state is antisymmetric in the spin degree of freedom, so the spatial part of the wave function should be

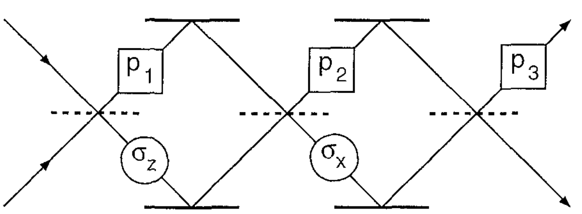

FIG. 1. Bell state analyzer for noninteracting electrons, consisting of three 50/50 beam splitters (dashed horizontal lines), four mirrors (solid horizonal lines), two local spin rotations (Pauli matrices σ_x and σ_z), and three charge detectors (squares). The charge detectors may operate either as electrometers (counting the occupation $q_i = 0, 1, 2$ in an arm) or as parity meters (measuring $p_i = q_i$ mod2). The first charge detector can identify the spin singlet state $|\Psi_0\rangle$, which is the only one of the four Bell states (2)–(4) to show ($p_1 = 0$). Since $(\mathbb{1} \otimes \sigma_z)|\Psi_1\rangle = -|\Psi_0\rangle$ the second charge detector can identify $|\Psi_1\rangle$ when $p_2 = 0$. Finally since $(\mathbb{1} \otimes \sigma_x\sigma_z)|\Psi_2\rangle = |\Psi_0\rangle$ the third charge detector can identify the two remaining states $|\Psi_2\rangle$ (when $p_3 = 0$) and $|\Psi_3\rangle$ (when $p_3 = 1$).

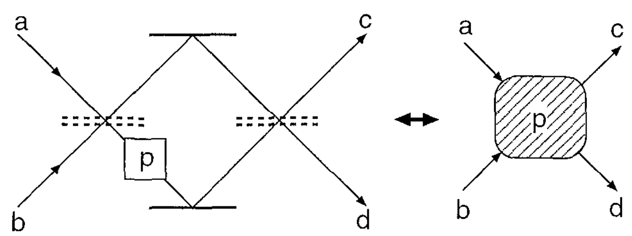

FIG. 2. Gate that converts a charge parity measurement to a spin parity measurement. The shaded box at the right represents the circuit shown at the left. A pair of electrons is incident in arms a and b. A polarizing beam splitter (double dashed line) transmits spin up and reflects spin down. A charge detector records bunching ($p = 0$) or antibunching ($p = 1$) and passes the electrons on to a second polarizing beam splitter. If each electron at the input is in a spin eigenstate $|\uparrow\rangle$ or $|\downarrow\rangle$, then output equals input and p measures the spin parity ($p = 1$ if the two spins are aligned, and $p = 0$ if they are opposite). The gate can be used to encode a qubit $|\uparrow\rangle$ as the two particle state $|\uparrow\rangle|\uparrow\rangle$ and $|\downarrow\rangle$ as $|\downarrow\rangle|\downarrow\rangle$. For that purpose the input consists of the qubit to be encoded in arm a plus an ancilla in arm b in the state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. The output is the required two-particle state in arms c and d for $p = 1$. For $p = 0$ it becomes the required state after a spin-flip (σ_x) operation on the electron in arm d.

symmetric, and vice versa for the triplet state.) Let p_i be the charge q_i measured by detector i, mod2. So $p_i = 0$ means bunching and $p_i = 1$ means antibunching after beam splitter i. The quantity

$$\mathcal{B} = p_1 + p_1 p_2 + p_1 p_2 p_3 \qquad (6)$$

takes on the value 0, 1, 2, or 3 depending on whether the incident state is $|\Psi_0\rangle$, $|\Psi_1\rangle$, $|\Psi_2\rangle$, or $|\Psi_3\rangle$, respectively. The measurement of $\mathcal{B}$ is therefore the required projective measurement in the Bell basis. It is a destructive measurement, so it does not matter whether the charge detector operates as an electrometer (measuring q_i) or as a parity meter (measuring p_i).

In Fig. 2 we show how a charge detector operating as a parity meter can be used to measure in a nondestructive way whether two spins are the same or opposite. "Nondestructive" means without measuring whether the spin is up or down. The device consists of two polarizing beam splitters in series, with the charge detector in between. (A polarizing beam splitter fully transmits $\uparrow$ and fully reflects $\downarrow$.) At the input two electrons are incident in different arms. Input equals output if each electron is in a spin eigenstate. The measured charge parity then records whether the two spins are the same or opposite. We will refer to this device as an *encoder*, because it can deterministically entangle a qubit in the arbitrary state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ and an ancilla in the fixed state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$ into the two-particle entangled state $\alpha|\uparrow\rangle|\uparrow\rangle + \beta|\downarrow\rangle|\downarrow\rangle$.

To construct a CNOT gate using the Bell-state analyzer we follow Ref. [2], where it was shown that teleportation can be used to convert a probabilistic logical gate into a nearly deterministic one. It is well known that a probabilistic CNOT gate can be constructed from beam splitters and single-qubit operations. The design of Pittman *et al.* [7] has success probability $\frac{1}{4}$ and works for fermions as well as bosons. It consumes an entangled pair of ancillas, which can be created probabilistically using a beam splitter and charge detector [14]. Because the gate is not deterministic, it cannot be used in a scalable way inside the computation. However, the CNOT gate can be repeatedly executed off-line, independent of the progress of the quantum algorithm, until it has succeeded. Two Bell measurements teleport the CNOT operation into the computation [24], when needed. In this way a quantum algorithm can be executed using only single-particle Hamiltonians and single-particle measurements.

In Fig. 3 we show how to construct a CNOT gate using the encoder. Our design was inspired by that of Pittman *et al.* [7], but rather than being probabilistic it is exactly deterministic. We take two encoders in series, with a change of basis on going from the first to the second encoder. The change of basis is the Hadamard transformation

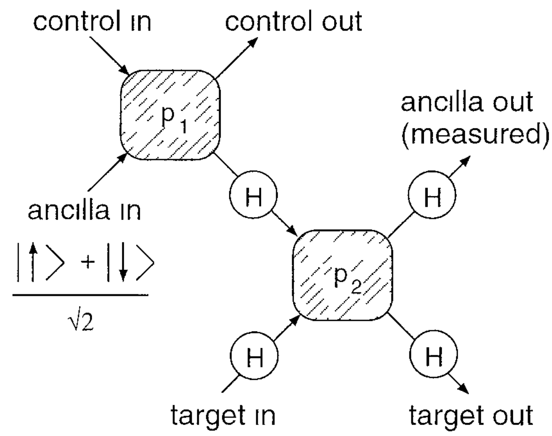

FIG. 3. Deterministic CNOT gate for noninteracting electrons. Each shaded box contains a pair of polarizing beam splitters and a charge detector, as described in Fig. 2. The four Hadamard gates $H = (\sigma_x + \sigma_z)/\sqrt{2}$ rotate the spins entering and leaving the second box. The input of the CNOT gate consists of the control and target qubits plus an ancilla in the state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. The spin of the ancilla is measured at the output. The outcome of that measurement together with the two parities p_1, p_2 measured by the charge detectors determine which operations σ_c, σ_t one has to apply to control and target at the output in order to complete the CNOT operation. For the control, $\sigma_c = \sigma_z$ if $p_2 = 0$ while $\sigma_c = \mathbb{1}$ if $p_2 = 1$. For the target, $\sigma_t = \sigma_x$ if the ancilla is down and $p_1 = 1$, or if the ancilla is up and $p_1 = 0$. Otherwise, $\sigma_t = \mathbb{1}$. The calculation is given in Ref. [30].

$$|\uparrow\rangle \to (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}, \qquad |\downarrow\rangle \to (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} \qquad (7)$$

The CNOT operation flips the spin of the target qubit if the spin of the control qubit is $\downarrow$. Control and target are input into separate encoders. The ancilla of the encoder for the control is fed back into the encoder for the target. At the output, the spin of the ancilla is measured. Conditioned on the outcome of that measurement and on the two parities measured by the encoders, a Pauli matrix has to be applied to control and target to complete the CNOT operation.

The computational power of the parity detectors is remarkable. The CNOT gate of Fig. 3 requires a single ancilla to achieve a 100% success probability, while the optimal design of LOQC needs n ancillas in a specially prepared entangled state for a $1 - 1/n^2$ success probability [8]. In this respect it would seem that FEQC is computationally more powerful than LOQC, but we emphasize that Fig. 3 applies to bosons as well as fermions. If parity detectors could be realized for photons (and there exist proposals in the literature [6]), then the design of Fig. 3 would dramatically simplify existing schemes for LOQC.

In conclusion, we have shown that free-electron quantum computation (FEQC) is possible in principle, either nearly deterministically (using a Bell-state analyzer with a charge detector operating as an electrometer) or exactly deterministically (using an encoder with a charge detector operating as a parity meter). Unlike photons, electrons interact strongly if brought close together, so there is no need to rely exclusively on single-particle Hamiltonians. We expect that FEQC would ultimately be used for flying qubits [25], while other gate designs based on short-range interactions [15,26] would be preferred for stationary qubits.

The two ingredients of the circuits considered here, beam splitters [27,28] and charge detectors [13,16,17], have both been realized by means of point contacts in a two-dimensional electron gas. The time-resolved detection required for the operation as a logical gate has not yet been realized. The currently achievable time resolution for charge detection is μs [16], while the resolution required for ballistic electrons in a semiconductor is in the ps range. That time scale is not inaccessible [29], but it might not be possible to reach the required single-electron sensitivity due to the unavoidable shot noise in the charge detector. In the light of this, is could be more practical to start with isolated electrons in an array of quantum dots, rather than with flying qubits, in order to investigate the potential and limitations of our theoretical concept on a presently accessible time scale.

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