Operators in the Lexicon

On the Negative Logic of Natural Language

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To my parents Maria and Jules Jaspers-Forier
To my teacher Wim de Geest

Acknowledgements

The trouble began when I got hooked on a linguistic puzzle two and a half years ago: why is not there a word *nand* in everyday speech to express the combined meanings of *not* and *and*? It would have been a plausible and useful word, given the existence of *nor* for the combination of *not* and *or*. In slightly more technical terms: why do we need two separate words in everyday natural language to express the combination of a negation and a conjunction?

The terminology used – words, negation, conjunction – indicates that an answer had to be sought in an area where *linguistics* and *logic* meet. But that is a sizeable area. Linguistics and logic are among the oldest branches of systematic inquiry and their connection much analyzed. Moreover, both are probably more highly formalized than any other field in the humanities. Small wonder then that it turned out to be much easier to state the problem than to come up with a viable solution, let alone find a way to present it in readable format.

While I took great pleasure in trying to figure out what the lexical gap was about, it was an arduous task to streamline my findings, and that is an understatement. There will doubtless remain imperfections and I deplore my state of ignorance more than ever. Yet I entertain some hope that the result of the energy invested offers a useful perspective on an algebraic aspect of our shared cognitive capacities.

In a way, the topic of this study is that of one of the earliest sciences in history – logic, so that the material that bears on my concerns is virtually endless. I have therefore made a selection purely in terms of the problem I sought to solve and the theoretical hypotheses I wished to develop. In view of that, it is quite well possible that the line taken deviates in a number of ways from what is standard. Thus I believe Neo-Gricean pragmatic analyses pull too many aspects of “hard” semantics into the rather soft and weakly formalized realm of pragmatics. Yet, it is no less true that much of what I try to do here is highly influenced by Laurence Horn's (1989) work.

When Willy Vandeweghe (to whom I am very grateful for the reference) pointed out the centrality of Löbner’s “Wahr neben Falsch” (1990) to my topic, I noticed that a number of ideas I had been working on – a two-relational asymmetry hypothesis and the emphasis on negative trafficking between operators – are central to his study. But there are several interesting differences between the analyses. Thus, the reduction of the Boethian square to two relations is initially executed in Aristotelian terms rather than by means of negation. More importantly, I trace all negative approaches to standard logical operator relations (including Sheffer’s stroke, Löbner’s duality squares, Seuren’s (2002, 20-21) bitriangular “improved square” and my own proposal) back to C.S. Peirce’s (1989 [1880]) original negative approach in “A Boolean algebra with one constant”. An enriched and cognitively more realistic version thereof not only lends support to the asymmetry first postulated by Löbner but provides a plausible further theoretical anchoring for it. In addition, it leads to a changed perspective on other parts of his duality approach. It is in these respects that the present proposal

intends to lend support to Löbner's seminal work, but also to build on it and suggest modifications.

I am convinced that if there is originality and value to the cocktail served, it is in no small measure because the giants on whose shoulders I have chosen to stand – including George Boole, Sylvain Bromberger, Noam Chomsky, Jeffrey S. Gruber, Jerry Fodor, Sebastian Löbner, Charles S. Peirce, Bertrand Russell, Pieter A.M. Seuren – have provided me with the best of its ingredients. I am thinking of Boole's algebra and his assertion that logic is not just a formal system, but an empirical inquiry into "the constitution of the human intellect", Bromberger's emphasis on "what we know we don't know", Chomsky's minimalism, internalism and view of language as both a formal system and a branch of psychology, Fodor's language of thought, Löbner's duality squares and phasal quantification, Gruber's concept of configurationality, Peirce's emphasis on the irritation of doubt and its stimulation "to action until it is destroyed", and Bertrand Russell's link between disjunction and cognitive perplexity or doubt.

I take great pleasure in giving special mention to Jeffrey S. Gruber and Pieter A.M. Seuren, whose influence on my thinking has been enormous. Over the years, Jeffrey Gruber has been a constant source of fresh thinking and spiritual depth, a real teacher who patiently gave me new ideas, inspiration and encouragement. Pieter Seuren's work on valuation space analysis, discourse semantics and cognitive logic has shaped my own ideas and influenced me more than anything else. I first encountered the **nand*-puzzle in his work, and as it turns out also several of the tools I needed to solve the riddle. Pieter has given very generously of his time and was always ready to comment on (often very rough) draughts. His keen eye was sure to detect errors or omissions, his learnedness and wisdom the source of many pieces of good advice. I have benefited from his generosity and friendship over many years in more ways than words can express.

The first draft of the first fragment of this thesis contained the mystically sounding claim that *cognitive logic is an entirely negative affair* underlyingly and that this should somehow provide a solution to the **nand* problem. I have tried to think that initial idea through to the end and have attempted to develop new concepts and visual representations of my own making where needed. My feeling was that the empirical pattern which showed itself was so systematic it would have been lacking in courage not to follow the facts directly to where they led me. My main hope is that the triadic logic of the mind that has come out of those explorations will shine through.

The second chapter is relatively conservative in its approach. Its main claim is that the Boethian Square of Oppositions can be taken to pieces more radically than often thought: it is not a square, nor a triangle, but no more than two primitive relations which constitute a Cartesian coordinate system. Though the model will be given the outlook of such a coordinate system from the outset, it especially in chapters 4 to 6 that the full meaning of this choice will transpire.

In chapter three the two basic logical relations that remain – namely contradictoriness (CD) and entailment (ENT) – are tied to lexical items rather than to propositions. If there is some back and forth trafficking in my argumentation from the lexical to the propositional level, it is simply because properties of the latter are well-explored and hence easier to work with.

Chapter four raises the question why logical calculi were traditionally defined in terms of Aristotelian logical entailment and not in terms of the other foundational relation of the system, namely contradictoriness and its associated operation of negation. A proposal is made to decompose the lexical meaning of logical operators in negative terms along lines first proposed by Charles Saunders Peirce. This results in a system of three basic operators per calculus, all of which are complex compositions on the basis of a single negative operator. It is this Peircean-style novelty which will provide a theoretical explanation as to why the pivot is *necessarily* more basic than the other operators. A comparison will be drawn with Löbner's duality approach.

Chapter five provides further evidence for the pivotal status of logical operators in the Boethian I-corner (*or, some*, etc.) and the ways in which such pivots differ from the corresponding non-pivotal pairs. Both the core semantic content of pivots and modification thereof by contextual factors will be addressed. The most striking difference that will emerge is the chameleonic semantic variability of I-corner elements.

Chapter six describes the modifying effects of context and stress on the comparatively 'light' lexical semantic structure of pivots. It introduces a distinction between two lexically different types of pivots and illustrates the differences between them by means of a comparison of the indefinite article *a* and the pronoun *any*.

The conclusion will begin with a brief summary of the trajectory traversed, after which a solution will be proposed for the *nand-puzzle.

To my knowledge, the psychological conception of set demarcation in chapter 3, the relation which is established between Boole and Peirce and the details of the specific decomposition of chapter 4 break virgin ground and are consequently highly exploratory. On the whole, however, there is one general line that will keep popping up: attention to language provides strong evidence for viewing the foundations of logic as a cognitive system internally represented in the human mind. It will be clear that an attempt to describe the core lexical meaning of propositional and predicate calculus operators in terms of such patterns of thought and language imposes strong limitations on the hypothesis space. There being strong evidence in favour of the mentalist stance, however, the limitations were not artificial hurdles, but rather indispensable constraints. Awareness of them forced me to distill what I sincerely hope to be a useful way of looking at old questions. The core conclusion of the study is that *a single negative operator* makes its presence felt in several areas of the computational system of human language, including the lexical semantics of logical operators.

A stylistic consequence of the omnipresence of one operator and one small algorithm is that everything in this narrative hangs very tightly together. Though it was a tall order to translate a complex web of interrelated ideas into a single linearized string, the numerous revisions have hopefully made the end-result accessible, structured and sufficiently formalised. In this latter respect, I have adopted Jackendoff's (1997: 4) position that "the proper formalization of a theory [is] a delicate balance between rigor and lucidity: enough to spell out carefully what the theory claims, but not too much to become forbidding." As much as is needed to see clearly, but nothing beyond that.

As I have reached the end of my explorations and anachronistically write these opening lines, I wish to thank all those who have been supportive along the way. I will first of all single out those whose help and/or direct influence or reactions have been crucial: Sjef Barbiers, Hans Bennis, Sylvain Bromberger, Noam Chomsky, Jan Ceuppens, Norbert Corver, Wim de Geest, Marcel den Dikken, Johan de Schryver, Berty Goudriaan, Jeffrey Gruber, Dirk Ghysels, Anja Jacobs, Lysbeth Jans, Jan Koster, Jan-Frans Lindemans, Danny Masschelein, Filip Noë, Isabelle Peeters, Joël Rooms, Drea Maier, Pieter Seuren, Colette Storms, Tom Toremans, Jeroen van Craenenbroeck, Viviane van Dessel, Ignace Vandewoestyne, Martine Van Goubergen, Mark van Hoecke, Henk Van Riemsdijk, Katrin Naert, Philip Vermoortel, Walter Verschueren, Emma Vorlat, Jan-Wouter Zwart.

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May the ideas contained in these pages give to others as much pleasure and, more importantly, inspiration and new ideas as I feel they have given me. Only then will the proverbial pain not have been in vain.

Dany Jaspers *Brussels, 31 July 2005.*

Table of Contents

1 NEITHER *NAND NOR *NALL – ON THE DIFFERENCE BETWEEN EX-LOGIC AND IN-LOGIC

1.1 An empirical problem and an internalist perspective

What sparked off this study is surprise at a peculiar hole in the lexicalization of logical operators in natural language, observed by numerous scholars (Horn 1972, 1989: 252-267; Levinson 2000: 69-71; Seuren 2002: 21, n. 6). In probably all languages of the world there are single-item lexicalizations for the standard universal and existential quantifiers, in English the words *all* and *some*, respectively. Single lexicalizations for the negation of the existential quantifier, *no* or *none* in English, are also attested in many, though by far not all languages of the world (Horn 1972 ch. 4; 1989: 254). Yet, there never is a single-item lexicalization for the negation of the universal quantifier, for which the nonce-word **nall* could be coined, the putative lexical counterpart of the syntactically complex phrase *not all*.

(1)

all	*<u>nall</u>
some	no(ne)

The question is: what is it about language that militates against single-item lexicalization of this logical operator? Put differently: why cannot the grammatical syntactically complex constituent *not all* be expressed as the morphologically complex lexical item **nall* or as a monomorphemic word?

Lest it might be thought that this lexicalisation gap is purely accidental, another paradigm can be provided proving that the problem is systematic (cf. Horn 1972, chapter 4; 1989: 252-266; Levinson 2000: 69-71). The set I have in mind concerns the propositional operators of conjunction and disjunction. In probably all languages of the world these are lexicalized as single lexical items, in English the words *and* and *or*, respectively. In addition, many languages – though not a majority – have a lexicalization for the negation of the disjunction ‘not (P or Q)’, in English: *(neither)…nor*. No language, however, appears to have a lexicalized expression for the negation of the conjunction ‘not (P and Q)’. For usage in special contexts, a nonce word has been devised to refer to this case: **nand*.

(2)

and	*<u>nand</u>
or	(neither…) nor

The parallellism between this paradigm and that of the quantifiers above is straightforward. The question then becomes: what do these two paradigms have in common and why is it that one of the four logically possible lexicalisations is blocked?

To find a solution to the conundrum, we shall try to describe in detail the lexical meaning properties of a number of logical operators. Our interest is, first of all, in these words as *lexical items with their own lexical meanings*, but in addition also in the *semantic relations* between them.

The lexical items under analysis are all so-called logical constants, i.e. elements that allow for the automatic, formal derivation of logical consequences when used in sentences.

Of the following pair of sentences, for instance,

(3) a. P: *All* flags are green
b. Q: *Some* flags are green

the first logically entails the second (or, in formal notation $P \vdash Q$): whenever P is true, Q must of necessity also be true, on account of the meanings of the logical constants *all* and *some* involved in P and Q. A caveat is in order however. The claim that there is logical entailment between P and Q can only be upheld when the logical constants have existential import, i.c. when the set of flags is nonnull. The problem empty F-sets cause for entailment can be illustrated by means of the following pair, where the set of trespassers may remain empty forever, in which case a. does not entail b:

(4) a. All trespassers will be prosecuted.
b. Some trespassers will be prosecuted.[1]

For the time being, this problem will be shoved under the carpet, but only for reasons of gradual build-up. The issue will be addressed in chapter 4 and a solution will be shown to be available which saves entailment.

The term logical constants expresses the view that within the logical system the status of *all* and *some* is different from that of the other elements in the sentences (*flags, green*), which play no immediate role in the formal computation of entailments and can therefore be replaced by predicate letters: whether the sentences above are about flags being green or tomatoes being red is immaterial. It is not on the latter predicates, but on the logical constants and their meanings that the formal computation turns. Since the nature of this system of computation will be crucial to our solution of the problem posed above, I shall first of all introduce the logical notions I need. This will involve a look at (i) the workings and architecture of Aristotle's *Predicate Calculus* and the way it treats certain logical properties of *simple sentences*, and (ii) an excursion into the Stoics' *propositional*

[1] Actually, the problems caused by trivial operators for classical APC are more serious than just the collapse of these entailments. Of all the logical oppositions only the contradictoriness relationships *all flags are green* vs. *not all flags are green* and *some flags are green* vs. *no flags are green* remain valid as these pairs of propositions cannot be jointly true or jointly false.

calculus and its account for logical properties of *compound sentences*, i.e. sentences consisting of "two or more coordinated main clauses" (Quirk et. al. 1985: 987)[2]. As their features are introduced, they will be looked at with a minimalist eye. The primitive relations and categories will be sized down to a very sparse set. A parallelism between the two calculi will be exploited to show that the **nand*- and the **nall*-problem are two manifestations of the same lexical gap. In the process, it will be illustrated that the lexicalisation puzzle is much more general and widespread than illustrated so far in this chapter (cf. Horn 1989, Löbner 1990). A solution to the problem will therefore have to be stated at a level that spans the different calculi.

The logical, lexical and epistemological properties of the operators discussed in this study lead to the conclusion that logical intuition as expressed in natural language is grounded in properties of the mind, a position also defended in Macnamara (1986), Seuren (1998), Ludlow (in prep.). This makes our analysis a mentalist one, which takes *natural logic* to be a psychological phenomenon, a component of natural language. This mentalist perspective subscribes to Chomsky's (1986: 21-24) general outlook on language. The basis for language in human beings is the existence in the brain of a component called the "language faculty", a component which is "dedicated to language and its use" (Chomsky 2000: 77). The initial state of that component is "determined by biological endowment" and "so similar across the species that we can reasonably abstract to *the* initial state of the language faculty, a common human possession" (Chomsky 2000: 78) labelled Universal Grammar (UG). Under the partly triggering, partly shaping influence of the environment, UG grows until a fairly stable state is reached "at about puberty". That mature state "is a computational (generative) system" which Chomsky calls an I-language, where I is chosen "to suggest that the conception is internal, individual, and intensional (in the technical sense; that is, the characterization of a function in intension)." (Chomsky 1995: 6). In other words, I-Language is the steady state of a person's (= *individual*) mentally represented (= *internal*) linguistic knowledge (= *intensional*), a computational system which characterizes "an infinite class of linguistic expressions, each a certain array of phonetic, structural, and semantic properties" (Chomsky 2000: 78)

This technical concept of language as an instance of I-language stands in contrast with E-language, which is a construct "understood independently of the properties of the mind/brain" (Chomsky 1986: 20): external(ized), social and extensional[3]. Descriptive linguistics and behavioural psychology, which reigned supreme in the first half of the twentieth century, have operated with such concepts of EX-language (Chomsky 1986: 19; Botha 1989: 69). So has structural linguistics, witness Bloomfield's definition of language as "the totality of utterances that can be made in a speech community". And many leading analytic philosophers (Burge, Dummett, Putnam) take the *externalist*, *anti-individualist* position that language is a social object existing "independently of any

[2] The lexical items that function as propositional calculus operators (*and, or, (neither)…nor*) are of course not restricted to compound sentences, e.g. *John is in Paris and Mary is in Spain*, but figure also in all kinds of other complex phrases (NP: *Mary and John*; AP: *bright and beautiful*; VP: *sing and dance*, etc.) in which their semantics can hardly be completely disjoint from that in the propositional calculus.

[3] Since the labels I and E will be needed for another contrast later on, I will use IN-Language and EX-Language for the Chomskyan contrast from here onwards.

particular speakers", an object of which individual speakers only have a "partial, and partially erroneous, grasp". (Dummett 1986). For the purpose of naturalistic inquiry into the nature of language, EX-notions are, however, in many ways problematic: there is no possible delimitation of languages, regiolects and dialects as natural categories or idealisations independent of choice and interests.

In terms of Chomsky's definition of language, it will be clear that the logic envisaged here is a psychological *IN-logic* rather than a mind-external EX-Logic. Historically, this psychological perspective on logic is not unprecedented. Upon Hegel's death in 1831, for instance, idealistic philosophy lost ground and the success of the natural sciences gave rise to a positivistic, naturalistic shift in philosophy. This fostered "the viewpoint that the ideal of the knowledge and the justification of the empirical sciences holds for philosophy as well." (Kusch 1995:2) Accordingly, the study of the Kantian a priori "was (…) taken to be an enquiry into what is psychologically or physiologically prior to whatever humans obtain as material knowledge, and thus a topic for the physiologists (like von Helmholtz), or the psychologists (like Wundt)." (Kusch 1995:2) And logic, as seen from this new naturalistic perspective in logic, came to be treated as a branch of psychology. Thus, Mill (1979: 359) wrote that logic is

> "not a science distinct from, and co-ordinate with Psychology. So far as it is a science at all, it is a part, or branch, of Psychology; differing from it, on the one hand as the part differs from the whole, and on the other, as an Art differs from a Science."
>
> Mill (1979: 359)

But the internalist approach to logic of Mill, Erdmann, Lipps, Sigwart, etc. was gradually pushed to the background not long after George Boole (1815-1864) had proved in his seminal essay *The Mathematical Analysis of Logic* (1847) and in *The Laws of Thought* (1854) that classical logic could be treated purely with algebraic terminology and operations. Boole himself was still very explicit in mentioning the link between logic and the mind: in the first sentence of *The Calculus of Logic* (Boole 1848: 183) his stated aim is "the application of a new and peculiar form of Mathematics to the expression of the operations of the mind in reasoning"[4]. But his success[5] in discovering those mathematical laws may have been instrumental in deflecting attention from the

[4] Cf. Grattan-Guinness (1982) and Devlin (1997: 72), who states: "Were he alive today, he would undoubtedly refer to himself as a cognitive scientist, a term that was first used in the early 1950s". Though Boole doubtless holds views with a "psychologistic air", Richards (1980) argues that his position is not subject to "the criticisms of psychologism" and thereby differs from Mill's. While both agree "that the general Laws of Nature are inductive inferences" (Richards 1980: 29), Boole (pace Mill) contends that this is not the case with the Laws of thought, the knowledge of which "does not require as its basis any extensive collection of observations", but "clear apprehension of a single instance" (Boole 1854 [1958]: 4). This is because of "the ability inherent in our nature to appreciate Order; and the concurrent presumption, however founded, that the phenomena of Nature are connected by a principle of Order' (Boole 1854 [1958]: 403; from Richards 1980: 29). In sum, while Boole is a cognitive scientist in search of the laws of thought, he rejects psychologism: "the central feature of psychologism – the limitation to the subjective nature of our knowing – is not present in the work of Boole." (Richards 1980: 30)

[5] Bertrand Russell said "Pure mathematics was discovered by George Boole in a work which he called the *Laws of Thought* (1854)" (Russell 1918 [2004]: 57).

individual human being's intellect. It was his algebraic *Laws*, more than the underlying features of *Thought* which commanded attention. Boole himself never stopped believing the mathematical laws were determined by the structure of the mind. But in spite of his argumentation, some of his contemporaries and logicians after him gradually traded in the complexities of thought and everyday language for the purity of mathematics. And since it was primarily mathematicians and mathematically oriented philosophers like Gottlob Frege (1848-1925) and Bertrand Russell (1872-1970) who took symbolic logic further after Boole, the trend towards mathematicisation and depsychologisation deepened (Seuren 2002). A thorough understanding of the reasons for the shift requires a more detailed description than I can here provide of the so-called psychologism dispute, which stretched from the 1880s to the 1920s and as a consequence of which the antipsychologism of Frege and Husserl came to hold sway (Kusch 1995). But the following summarizes the core of the matter. First and foremost, there was the judgment that attempts to tie the laws of logic to the mind destroys the revered objectivity of logic and hence endangers all knowledge claims. To counter the danger Frege worked out arguments to establish that the laws of logic are not temporal and are hence different from psycho-physical events. This led to the theory that *Gedanke* (thoughts) belong to a "dritte Reich"[6] outside the physical and the mental, a realm of non-physical entities which it would be ill-taken to construe as empirical. This brings us to a second argument: the naturalist turn that had begotten the view of logic as a branch of psychology was outspokenly empiricist, so that arguments against empiricism were taken to affect the validity of the former as well. And no doubt partly because psychology was still in its infancy, what caught the eye about the workings of the psyche was not its systematicity, but rather that it was often illogical, full of false starts, error-prone. And natural language seemed to reflect that volatile character. Thus, Frege "started from the assumption that natural language is a defective instrument, and that what the logician needs is not a theory of the working of natural language but a theory of the working of an improved language which could ideally replace it, and, for the most rigorous scientific purposes, may actually do so." (Dummett 1981: 585). In other words, Frege's interest was only in a "logically perfect language" useful for science and he considered natural language" too "imperfect" to merit much attention" (Chomsky 1996: 46). Actually, "not only imperfect, but even "in principle incoherent," Dummett argues" (Chomsky 1993a: 27). On the whole, logic was to be kept far from the irregularities of what Russell referred to as *common* language and cognitive questions[7], even if most of its central problems had originally been raised at some point in the development of classical natural-language-based logic (cf. Kneale & Kneale 1962). Natural language analysis and logic got separated and were for a long time seen as completely divergent scientific endeavours.

Boole, it should be stressed, never gave up his conviction that the basis of logic is ultimately to be found in "the constitution of the human intellect". Towards the end of his life he actually expressed some dissatisfaction with his *Laws of Thought* and intended to revise it:

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[6] Frege's name for this Platonic realm is obviously rather unfortunate in view of later political developments. Ironically, it so happened that when Karl Popper basically adopted a similar realm in his own exercise of ontological pluralism, he called it "Third World", a name which for its connotation he later traded in for "World 3" (at the suggestion of John Eccles).

[7] Russell spoke of "the fundamental principles of logic known under the *quaint* name of 'laws of thought'" (Russell 1908 [1956]: 63; italics mine).

> "what he had in mind was a development of his epistemological views rather than any alteration of the formal side of his work. In these papers he mentions in particular a distinction between the Logic of Class (i.e. his calculus of logic) and a higher, more comprehensive logic that cannot be reduced to a calculus but may be said to be 'the Philosophy of *all* thought which is expressible in signs, whatever the object of that thought'.
>
> (Kneale & Kneale 1962: 406)

While Kneale & Kneale still deplore this move in 1962 and contend Boole was on a wrong track, it seems more correct to conclude that Boole was never blinded by the undeniable elegance and truth of his formal calculus. He realised and rightly concluded that depsychologized mathematical logic, though doubtless a valuable stepping stone, was only part of the story; that a logic which permanently drops any reference to the processes of the mind and remains restricted to the logical operator part of the lexicon is bound to be limp. It leaves far too many possible thoughts and mind-internal categories – however shady – unaccounted for, including "Homeric gods, relations, chimeras and four-dimensional spaces" (Russell 1903 [37]: 449).

> "Both these studies [of Logic and Probabilities, DJ] have also an interest of another kind, derived from the light which they shed upon the intellectual powers. They instruct us concerning the mode in which language and number serve as instrumental aids to the processes of reasoning; they reveal to us in some degree the connexion between different powers of our common intellect; they set before us what, in the two domains of demonstrative and probable knowledge, are the essential standards of truth and correctness, - standards not derived from without, but deeply founded in the constitution of the human faculties."
>
> (Boole 1854 [1958]: 2)

The importance and value of such an internalist perspective are defended against those who believe the main relevance of the system is its practical value:

> "These ends of speculation yield neither in interest nor in dignity, nor yet, it may be added, in importance, to the practical objects, with the pursuit of which they have been historically associated. To unfold the secret laws and relations of those high faculties of thought by which all beyond the merely perceptive knowledge of the world and of ourselves is attained or matured, is an object which does not stand in need of commendation to a rational mind."
>
> (Boole 1854 [1958]: 2-3)

To be fair to Russell, it should be said that he too made a "psychologistic" turn after having been a staunch antipsychologist for years. According to Monk (1996: 518-519), the change of opinion occurred sometime in 1918:

> "Russell says: 'A proposition is just a symbol', and, as the theory of symbolism is fundamentally psychological, it follows that the nature of logic cannot be fully understood outside a study of psychology. The implications

> of this 'psychologistic' line of thought were to become increasingly manifest to Russell over the coming months and would loom ever larger in his philosophical work of the next few years, but, in essence, it was already there in 'Lectures on Logical Atomism'"
>
> (Monk 1996: 518-519)

A description of logical constants in his later work testifies to this change and provides a good anchor for our own analysis:

> "It is obvious that 'the book is *somewhere* in the room' cannot be a judgement of perception; you cannot perceive *somewhere*, you can only perceive *there*. But a judgement of memory is different. You may remember 'I saw the book when I was in this room', or something of that kind. You may remember saying 'Oh there's that book' while you were in the room. Or you may have a purely verbal memory of saying 'I see I did put that book on a shelf'. These, however, are only the grounds for your judgement, they are not an analysis of it.
>
> The analysis of such a judgement must be essentially similar to that of a disjunction. There is a state of mind in which you perceive 'the book is in this place', another in which you perceive ' the book is in that place', and so on. The state of mind when you judge 'the book is somewhere in the room' contains what all these have in common, together with perplexity."
>
> (Russell 1940 [1969]: 86)

I fully agree where he concludes that "in the case of a judgement about *some*, as in disjunction, we cannot interpret the words except in reference to a state of mind." (Russell 1940 [1969]: 86). In what follows, this internalist stance with crucial dependence on "states of mind" will be adopted throughout, and its scope will be argued to include not only the logical operators themselves, but also the core logical relations of contradictoriness and entailment that connect them into a single paradigm of semantically cognate lexical items.

A major consequence of viewing logic as a psychological phenomenon – a *natural logic*[8] – is that its study is empirical science. Chomsky's (1980: 211) and Lenneberg's (1967) conclusion about mentalistic linguistic theories carries over seamlessly:

> "Lenneberg was quite right to take the trouble to emphasize that 'the discovery and description of innate mechanisms is a thoroughly empirical procedure and is an integral part of modern scientific inquiry' and to insist that there is no room here for dogmatism or *a priori* doctrine."
>
> Chomsky (1980: 211)

Natural logic is not a given thing, but has to be discovered. Consequently, existing logic should not be accepted as it stands if it does not fit the empirical evidence provided by language and cognition. This may be an uncommon position, but I see no alternative: theories have to be tailored to empirical data and not vice versa. So wherever it looks as

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[8] This term is used by Sanchez Valencia (1991) and in Ludlow (in prep.).

if language is too messy for logic, I proceed on the assumption that it is more accurate to say that the architecture of existing logic is too much of a Procrustean bed for the facts of language. I hope that will not be interpreted as lack of reverence or interest in existing theories, but as an attempt to contribute something of value to empirical linguistic and cognitive science.

In Aristotle's logical system, logical constants were considered to be different in nature from the other elements in the sentences and were introduced syncategorematically, i.e. by stipulation and without categorial status. For most of the time since, this did not change. Thus, we find such a syncategorematic introduction of the logical constants in Montague's "The Proper Treatment of Quantification in Ordinary English" and also (p. 57) still in Dowty et al.'s (1981: 56-61) *Introduction to Montague Semantics*, for instance, with separate definitions (p. 60) for the attainment of truth in a model. As it is our intention to analyse the logical constants as regular words, i.e. as *natural language lexical items*, which after all they are, a different line will be taken.

In essence, I shall defend the view that logical constants are best regarded as semantic predicates, a proposal which is not new (cf. McCawley 1967, Barwise and Cooper 1981). For the operators *all* and *some*, for the negation operator *not* and for quantifiers more generally (including *most, more than half,* etc.), such an approach was provided and formalized in the theory of Generalized Quantifiers, whose study was initiated by the Polish logician Andrej Mostowski (1957) and came to fruition in Barwise and Cooper (1981) and a lot of linguistic and logical work since.[9] I shall analyse *and, or* and *(neither…)nor* along the same lines.

In a nutshell, an empirical puzzle and a theoretical hypothesis define the contents and structure of this study. The empirical challenge is to provide an account why <u>*neither *Nand nor *Nall*</u> are possible as ordinary lexical items in natural language. To meet that challenge, the theoretical position developed is that logic as operational in natural language is in its basics an innate part of our mental make-up, i.e. it constitutes an <u>*IN-logic*</u>. To underpin the latter view, two main theses are defended and worked out:

There is an isomorphism between the structure of logical calculi, the lexical-morphological properties of logical operators and a number of their epistemological properties.
This isomorphism can be accounted for in terms of the semantic properties of logical operators in the mental natural language lexicon.

1.2 Nand and *nand are two: EX-Logic versus IN-logic

Before setting out on the first leg of our journey through the realms of propositional and predicate logic in chapter 2, a few words to sharpen the abovementioned notion IN-logic and to set it off against the concept of EX-Logic. The difference between them can once more be introduced by considering the lexicalisation gap. One aspect of the latter is that while **nall* does not exist at all, the story for **nand* is different. Though it is not attested

[9] For a recent overview, see Keenan (2002)

as an *ordinary* or *natural* word, it *is* listed in dictionaries[10] as a *specialised* term in the context of logic (the NAND function – also called Sheffer stroke) and electrical engineering (digital circuits: the NAND-gate). The hypothesis worked out in this dissertation is that the non-existence of a natural language item **nand* reflects a constraint of the IN-logic inherent in natural language. The existence of the specialised term *nand* must then be taken to mean that it belongs to a different realm, provisionally called EX-Logic, a branch of consciously crafted science.

The crucial difference between the two is in the mode of acquisition: IN-logic, like natural language, has an innate basis and its expressions grow or mature naturally in the acquisition process, without conscious interference or literal learning. EX-Logical concepts, like specialised terms more generally, are artefacts created by conscious processes of reflection and artificial learning which transcend the bounds of what is innate and acquired naturally. It is in terms of the distinction between natural, non-monitored mental processes in natural language acquisition on the one hand and artificial, monitored and consciously controlled mental activity in science – Chomsky's (1993a: 33) "Science Forming Capacity" – that the difference between IN-logical **nand* and EX-Logical *nand* can be made sense of. The two elements belong to different realms of lexical knowledge, with the natural realm more restricted than that of artificially created extensions.

In sum, there is a difference between *IN-Logic* (*natural* (Ludlow (in prep.)) or *informal logic*) and scientific *EX-Logic* (or *formal, symbolic logic*). The former is part of natural language and does not permit **nand.* Nor does it permit self-entailments such as *If Mary is ill then Mary is ill* for instance, which indeed sound unnatural. EX-Logic, for its part, sheds such concerns with naturalness and intuitive common sense by putting certain cognitive constraints of IN-language aside, a standard case of idealization for scientific purposes. This is why in EX-Logic – both in the context of the hardware transistor implementations which are called logic gates and in the context of Sheffer's stroke – *nand* can function as a perfectly fine technical term and there is no ban on unnatural entailments[11].

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[10] E.g. The Free On-line Dictionary of Computing (http://smac.ucsd.edu/cgi-bin/http_webster?nand): NAND Not AND. The {Boolean} function which is true unless both its arguments are true, the {logical complement} of {AND}: A NAND B = NOT (A AND B) = (NOT A) OR (NOT B).

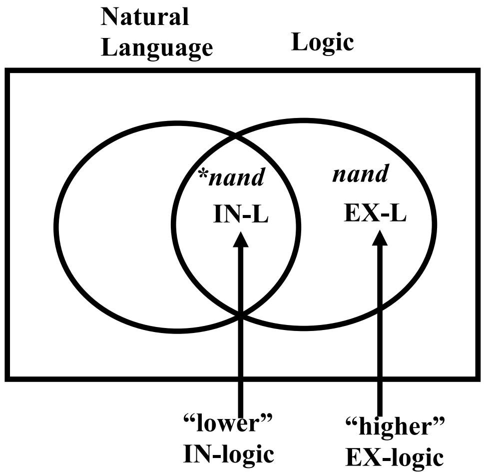
[11] Note that that there is a corresponding distinction between

IN-mathematics, overlapping with ordinary language (*lower, natural* (cf. Honda & O'Neill 1993; Devlin 2000) or *informal mathematics* (Ginsburg 1977), including Dehaene's *number sense* (Dehaene 1997; Butterworth 1999), and

EX-mathematics, which departs from common sense and certainly looks rather different from ordinary language, with its esoteric symbols, axioms, theorems, etc. (*higher, scientific, advanced, formal mathematics*).

Devlin (2000) distinguishes between natural mathematics and formal mathematics: the former he calls "formalized common sense", while the latter is characterised by formal definitions which tend to go against common sense.

(5)



Our concern is clearly IN-logical. In order to explain why **nand* is impossible as a *natural* lexical item and as part of IN-logic, the nature of the latter will have to be determined first. This issue will be addressed in chapter 2 already, in whose first part the **nall/*nand*-puzzle will be placed against the background of Aristotle's *Predicate Calculus* and the Stoics' *propositional calculus*. In the second part, these traditional calculi will be reduced to their IN-logical foundations: a two-dimensional structure with two basic relations of *contradictoriness* and *entailment* and three operators: *and, or, nor* for the propositional calculus; *all, some, no(ne)* for the predicate calculus. The third chapter zooms in on the two relations. They will be looked at not only from a logical angle, but also from a set-theoretic and a Boolean algebraic perspective. In chapter four, contradictoriness and entailment will each be linked to a 'dynamic' operation, namely *negation* and *conjunction* respectively, of whose workings they are arguably the 'static' end-result. These operations will then be restated as complex notions built from a *single negative operator*. The latter will then be called upon to set up a lexical decomposition of the three lexicalised operators.

The most important consequence of this decomposition is that one of the lexicalised operators in each calculus, namely *or* in the propositional calculus, *some* in the predicate calculus is *necessarily* semantically less complex than the two others. Chapters 5 and 6 will adduce as much empirical evidence as possible for this "anchoring" of an asymmetry hypothesis first proposed in Löbner (1990). In that context, an epistemological perspective will be sketched which highlights the role of the different logical operators in the acquisition and expansion of world knowledge. This will secure the conclusion that the only viable logic for natural language is a triadic cognitive IN-logic.

2 FROM THE BOETHIAN SQUARE TO A TWO-DIMENSIONAL CARTESIAN COORDINATE SYSTEM

2.1 The Predicate Calculus and the Propositional Calculus

2.1.1 Aristotle's Predicate Calculus (APC)

The quantifier words *all, some* and *no* are probably among the best-studied lexical items in western history ever since the invention of logic, when they were introduced as logical constants in Aristotle's predicate calculus (APC). On the basis of these items, a conception of truth and falsity, the distinction between subject and predicate and some additional axioms, Aristotle established entailment relations between sentences like the following, and logicians after him elaborated on his findings. The table below makes a distinction between logic and natural language. This should not be taken to imply that they are disjoint systems. This work is full of indications they are not. The distinction is drawn because some combinations of semantic logical operators, namely NOT-SOME in (9) and (13), are not realized as *not some* at the linguistic surface, but come out as the surface word *no*:

(6)

Logic	**Natural language**
(7) [$_{su}$ ALL flags] are[$_{pr}$ green]	[$_{su}$ All flags] are[$_{pr}$ green]
(8) [$_{su}$ SOME flags] are [$_{pr}$ green]	[$_{su}$ Some flags] are [$_{pr}$ green]
(9) NOT [$_{su}$ SOME flags] are [$_{pr}$ green]	[$_{su}$ No flags] are [$_{pr}$ green]
(10) NOT [$_{su}$ ALL flags] are [$_{pr}$ green]	[$_{su}$ Not all flags] are [$_{pr}$ green]
(11) [$_{su}$ ALL flags] are NOT [$_{pr}$ green]	[$_{su}$ All flags] are not [$_{pr}$ green]
(12) [$_{su}$ SOME flags] are NOT [$_{pr}$ green]	[$_{su}$ Some flags] are not [$_{pr}$ green]
(13) NOT [$_{su}$ SOME flags] are NOT [$_{pr}$ green]	? [$_{su}$ No flags] are not [$_{pr}$ green]
(14) NOT [$_{su}$ ALL flags] are NOT [$_{pr}$ green]	? [$_{su}$ Not all flags] are not [$_{pr}$ green]

All of the propositions above consist of a *subject* and a *predicate*. Both in logic and linguistics these notions are hybrids, so a few definitions are indispensable. In Aristotelian usage, subjects are reference objects, i.e. the entities referred to in the real world, rather than linguistic expressions. That is no longer common: subjects are now viewed as syntactic constituents which are used to *select* reference objects (rather than *are* them). So by subject, I mean *syntactic subject.* The bracketed predicate in the examples above (*green*) does not include functional categories and their syntactic features (here *are*), which is why it is often referred to as a *lexical predicate*. This use of the term predicate is the one we adhere to, but it is different from most versions of traditional grammar, where the predicate is a grammatical function defined as the sum of all those parts of a clause which are not included in the subject. Let us refer to this latter predicate as the *syntactic predicate*. In our examples the syntactic predicate is *[$_{pr}$ are* *green]* rather than just *[$_{pr}$ green]*[12]. The lexical category contained in the subject term (i.c. *flags*) will also be called a lexical predicate, which means that lexical predicates can be used (a) in a syntactic predicate to select a property which will be assigned "to a given entity in such a way that the assignment is true or false" (Seuren 1998, 304) or (b) in a syntactic subject to select a correct reference object. Seuren (1998, 304) calls the former a lexical predicate in propositional use, the latter a lexical predicate in referential use or term predicate.

(15)

all **X**	are **Y**
term predicate	predicate in propositional use

Looking at the structure of the propositions in (6), we note that all have at least an affirmative quantifier in the subject term[13]. In addition, negation can be added to the subject terms ((9), (10), (13), (14)) or to the lexical predicates in propositional use ((11)-(14)). The former type of negation is called *external* sentence negation or *outer negation* and results in a statement contradicting the affirmative one (for instance, (10) vs. (7)): the truth of the one entails the falsity of the other. Negation of the lexical predicate in propositional use, on the other hand, is called *internal* predicate negation or *inner negation* and leads to contrary statements. Such statements cannot, for Aristotle, both be true at the same time, but they can both be false at the same time. While it is impossible that all flags are green and not green at the same time, it is possible for both statements to be false at the same time, namely in a state of affairs when some (or not-all) flags are green.

Examples (7)-(14) illustrate the well-known fact that there are a number of discrepancies between natural language and logic as traditionally conceived. First of all, on the logical side *not* is placed outside the subject term in (9), (10), (13) and (14), though constituency tests clearly show that in natural language *not* is included in the syntactic subject constituent. Aristotle's choice is probably due to the sentential scope of this kind of negation and to the fact that it is independently possible in his logic to negate the lexical predicate inside a subject (*all not*-flags). In no example is it clearer that the negation is part of the syntactic subject in natural language than in (9) and (13), where NOT SOME is spelled out as a single word *no* (cf. above). This indicates that the surface realisation and the underlying logical representation of a lexical item need not be identical. While *no* is not morphologically complex, its meaning structure does contain two elements.
A second surface difference between natural language and logic concerns internal negation ((11)-(14)). Horn (1989: 226) makes the following observation, which he

[12] At the surface, it is not always possible in language to disentangle the lexical predicate and functional features. In *runs*, for instance, the third person singular present tense ending is attached to *run-*, and it is only the latter element which can be classified under the heading lexical predicate. But since functional features are introduced into the derivation as separate heads, it is possible to preserve the definition of lexical predicate as given.

[13] Aristotelian logic does not deal with singular statements, i.e. statements with subject terms without quantification, whether definite descriptions (*the queen*) or proper names (*Mary*). Nor is it able to deal with syntactic predicates containing other quantifier words than negation, like *read many books*, for instance.

attributes to Jespersen (1917: 86 ff.): “there is a tendency – often disparaged as ‘illogical’ – for an apparent universal negation (*{all/every}…not*) to be read as a negated universal (*not…{all/every}*)”, as in:

(16) All that glisters is not gold
(Shakespeare, *The Merchant of Venice*; Horn 1989: 226)

Though this phenomenon is a telling example of the discrepancy between logical representations and linguistic expressions at the surface, it is tangential to our present discussion. The interpretation of internal negation that we have in mind for the present discussion is the one in which *not* does not have scope over *all*.

Since the relations Aristotle observed hold independent of the choice of concrete lexical predicates, predicate letters can be used in what follows: ALL F are G.
Aristotle noticed entailment relations between sentences (9) and (11) and sentences (8) and (14). As a matter of fact, (11)-(14) are each equivalent to one of the statements in (7)-(10):

(17)

(7) ≡ (13)	[$_{su}$ ALL flags] are [$_{pr}$ green]	≡	NOT [$_{su}$ SOME flags] are NOT [$_{pr}$ green]
(8) ≡ (14)	[$_{su}$ SOME flags] are [$_{pr}$ green]	≡	NOT [$_{su}$ ALL flags] are NOT [$_{pr}$ green]
(9) ≡ (11)	NOT [$_{su}$ SOME flags] are [$_{pr}$ green]	≡	[$_{su}$ ALL flags] are NOT [$_{pr}$ green]
(10) ≡ (12)	NOT [$_{su}$ ALL flags] are [$_{pr}$ green]	≡	[$_{su}$ SOME flags] are NOT [$_{pr}$ green]

To state the equivalences between these expressions formally, modern notational devices will be used for the quantifiers (∀ for ALL, ∃ for SOME , ¬ for NOT) and an additional symbol ! to spell out the implicit AFFirmative complement of NOT. What is irrelevant will be suppressed.

(18)

Example numbers	Formulas
(7) (13)	!∀! ≡ ¬ ∃ ¬
(8) (14)	!∃! ≡ ¬ ∀ ¬

Example numbers	Formulas
(9) (11)	¬∃! ≡ ! ∀ ¬
(10) (12)	¬∀! ≡ ! ∃ ¬

To calculate the equivalences or *conversions*, it suffices to switch the logical predicates: ¬ becomes ! and vice versa, ∀ becomes ∃ and vice versa. The relevance of these conversion tables to this chapter will become clear further on, when the so-called NAND-function is discussed.

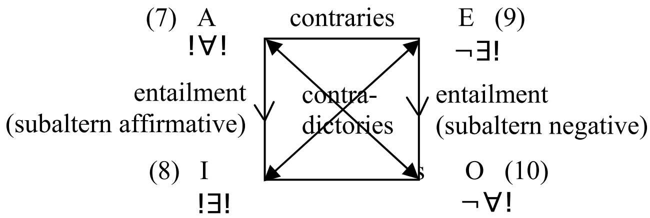
In view of these equivalences, the conclusion must be that there are in essence only four proposition types, given that we end up with *four* pairs of each time two equivalent propositions. Furthermore, the expressions in boldface are more common in natural language, i.e. there seems to be a preference for (i) fewer negatives over more negatives; (ii) external negation over internal negation.

(19)

	Affirmative statements		Negative statements	
Universal statements	(7)	[$_{su}$ ALL flags] are[$_{pr}$ green]	(9)	[$_{su}$ NO flags] are [$_{pr}$ green]
Particular statements	(8)	[$_{su}$ SOME flags] are [$_{pr}$ green]	(10)	[$_{su}$ NOT ALL flags] are [$_{pr}$ green]

Centuries after Aristotle, Boethius (480-524) formalised this insight in his Square of Oppositions. He gave labels to each of the four basic sentence types: (1) = A, (2) = I (from **A**ff**I**rmo, "I affirm") on the affirmative side of the square; (3) = E and (4) = O on the negative side (from n**E**g**O**, "I deny"), and indicated the entailments:

(20)



For two sentences P and Q, the relations in the square can be described as follows[14]:

(21)

Relation	Description	Example
Contradictories	P and Q can neither be both true nor be both false at the same time	P: All flags are green – Q: Not all flags are green P: Some flags are green – Q: No flags are green
Entailment	Whenever P is true, Q must also be true	Subaltern affirmative: P: All flags are green ⊢[15] Q: Some flags are green

[14] The doctrine stems from Aristotle (*De Interpretatione* 6-7, 17b.17-26 and *Prior Analytics* I.2, 25a.1-25) (Stanford Encyclopedia of Knowledge: http://plato.stanford.edu/ entries/square/). The first diagram is from the second century BC and attributed to Lucius Apuleius (c.124–c.170), author of ***Asinus aureus*** (*The Golden Ass* or *The metamorphoses*). (Londey & Johanson 1984, Franklin 1999).

		Subaltern negative: P: No flags are green ├ Q: Not all flags are green
Contraries	P and Q cannot both be true at the same time, but may be both false at the same time	P: All flags are green – Q: No flags are green
Subcontraries	P and Q cannot both be false at the same time though they may be both true at the same time	P: Some flags are green – Q: Not all flags are green

2.1.2 Thomas Aquinas and the O-corner

In comments on Aristotelian logic, Thomas Aquinas (1224/25-1274) mentioned the special feature of the O-corner hinted at in the introductory paragraph of the present study:

(22)

> "In negativis autem non est aliqua dictio posita, sed possumus accipere, *non omnis*; ut sicut, *nullus*, universaliter removet, eo quod significat quasi diceretur, *non ullus*, idest, *non aliquis*, ita etiam, *non omnis*, particulariter removeat, in quantum excludit universalem affirmationem."
>
> [80398] Expositio Peryermeneias, lib. 1 l. 10 n. 13
> http://www.corpusthomisticum.org/cpe.html
>
> For the particular negative (the O-corner), 'there is no designated word, but "not all" [*non omnis*] can be used. Just as "no" removes universally, for it signifies the same thing as if we were to say "not any", [i.e. 'not some'], so also "not all" removes particularly inasmuch as it excludes universal affirmation.'
>
> (Aquinas, In Arist. De Int., lesson 10, in Oesterle 1962 : 82-83; also in Horn 1989: 253)

Horn (1972, chapter 4; 1989: 252-262) and Levinson (2000: 69-71) – like Saint Thomas – observe that for some reason the O-corner (cf. example (10) in (19)) seems to resist simplex lexicalisation by means of a single word. In (19), for instance, the O-corner has *[not all flags] are [$_{pr}$ green]*. One might try to attribute this gap to the composite nature of the underlying logical representation $\neg\forall$. However, that cannot be the right answer since the logical representation of the E-corner is complex too ($\neg\exists$) and yet can be realised as a single word (*no*) in a number of languages, albeit often a bimorphemic one (*no-thing, no-body, n-ever,* etc.) (Löbner 1990: 95). So the question is: why cannot the derivational morphological process yielding such E-corner forms as *none, nothing,* etc. generate similar bimorphic forms for the O-corner? We would expect O-corner forms such as **nall, *neverything* on a par with *none, some,* but the fact is that there are none

[15] ├ is the symbol for logical entailment, ENT an alternative abbreviation: P ├ Q = P ENT Q = P entails Q. As indicated earlier, we temporarily restrict our attention to the non-trivial use of operators (with existential import for affirmative quantifiers, which was the usual classical opinion). The empty F-class problem will be tackled in chapter 4.

available in natural language. And if a reason can be found for the absence of that kind of derivational morphological process, why is suppletion not a valid alternative? That the gap is not accidental but systematic is shown in the following list of different types of quantifiers (23) supplemented with corresponding examples from Dutch (24):

(23)

	affirmo			nego
A	∀		E	¬∃
		1a. all		1c. none
	THING	2a. everything, all		2c. nothing
	BODY	3a. everyone, everybody		3c. noone, nobody
	TIME	4a. always, (*for once and for*) all		4c. never
	PLACE	5a. everywhere		5c. nowhere
	TWO THINGs	6a. both		6c. neither
I	∃		O	¬∀
		1b. one, some, any		**1d. *n-all**
	THING	2b. something, anything		**2d. *n-everything**
	BODY	3b. someone, anyone, somebody, anybody		**3d. *n-everyone**
	TIME	4b. sometime(s), ever		**4d. *n-always**
	PLACE	5b. somewhere, anywhere)		**5d. *n-everywhere**
	TWO THINGs	6b. either[16]		**6d. *n-both**

(24)

	affirmo			nego
A	∀		E	¬∃
		1a'. al, allemaal		1c'. geen[17]
	THING	2a'. alles		2c'. niets

[16] In *on either side of the road*, the disjunctive meaning of *either* is restricted to the inclusive part of the disjunction, thus making the meaning similar to that of *both*, which is also distributive.
[17] Etymologically, this form is a combination of the negative particle *neg- (*cf. Du. *niks ('nothing')*, German *nicht*) and the indefinite article *-een* ('*one'*)

	BODY	3a'. alleman, iedereen		3c'. niemand
	TIME	4a'. altijd		4c'. nooit
	PLACE	5a'. overal		5c'. nergens
	TWO THINGs	6a'. beide		6c'. (geen van beide)
	WAY	7a'. alleszins[18] (completely, fully, in all respects in every way)		7c'. geenszins (by no means, not at all, in no way)
		8a'. allerlei (all sorts/kinds of)		8c. generlei (no…what(so)ever, at all)
I	$\exists$		O	$\neg\forall$
		1b'. een, sommige(n)		**1d'. *n-al, *n-allemaal**
	THING	2b'. iets		**2d'. *n-alles**
	BODY	3b'. iemand		**3 d'. *n-alleman, *n-iedereen**
	TIME	4b'. soms, somtijds, ooit		**4 d'. *n-altijd**
	PLACE	5b'. ergens		**5 d'. *n-overal**
	TWO THINGs	6b'. (een van beide)		**6 d'. *n-beide**
	WAY	7b'. enigszins (somewhat, a little, rather, slightly, in a way)		**7 d'. *n-alleszins**
		8b'. enigerlei (any…(what(so)ever))		**8d'. *n-allerlei**

The problem at hand cannot be that the semantic content of the O-corner is inexpressible, given that syntactic combinations such as *not all / niet alle, not everyone / niet iedereen, not always / niet altijd* capture the intended meaning unproblematically. So the real question is: why can the combination of a negation and a universal quantifier never be *realised* as a *single* lexical item? But before we have a stab at that problem, let us mention a few similar data which over the years have been added to the pile (cf. Horn 1989, Löbner 1990).

2.1.3 The Propositional Calculus

There is a well-known relationship between the logical operators $\forall$and $\exists$ of Aristotle's predicate calculus (APC) and the operators $\wedge$ (*and*) and $\vee$ (*or*) in the Stoics' propositional calculus (e.g., Reichenbach 1947: 92). Take the universe of objects to consist of two entities a and b and let G be a lexical predicate in propositional use. With

[18] There is also the elative *veelszins* which, according to Van Dale means: 1."veelal" ("often"), "in vele gevallen of omstandigheden" ("in many cases or circumstances"); 2. "op meer dan een wijze" ("in more than one way"), "in meer dan een opzicht" ("in more than one respect").

x a lexical variable ranging over the two entities, the following parallel emerges between an APC with a lexical variable ranging over two entities (APC_2) and the propositional calculus (PROPC).

(25)

	APC_2	PROPC
(i)	$\forall x G(x) \equiv$	$G(a) \wedge G(b)$
(ii)	$\exists x G(x) \equiv$	$G(a) \vee G(b)$
(iii)	$\neg \exists x G(x) \equiv$	$\neg(G(a)) \wedge \neg(G(b))$

With a and b flags and G the property green, (i) all flags are green iff flag a and flag b are both green; (ii) some flag is green iff flag a is green, or flag b is green, or flag a and b are both green, and (iii) no flag is green iff it is not the case for a or b that they are green, in other words: it is not the case that a is green and it is not the case that b is green either.

Similar results can be obtained when the lexical variable ranges over three entities (or more), but working with three entities (or more) would merely require recursive use of $\wedge$ and $\vee$ and consequently would not add anything new to conclusions arrived at on the basis of a single use of the propositional operators.

The existence of a close relationship between APC and PROPC is also illustrated by the fact that the conversions of the Predicate Calculus noted in (18) and repeated here are matched by analogous equivalences in the PROPC, which go by the name of *De Morgan's Law*. It suffices (i) to replace the universal quantifier $\forall$ by the conjunction $\wedge$ and the existential quantifier $\exists$ by the disjunction $\vee$ and (ii) to realize that in PROPC, internal negation amounts to narrow scope negation of each of the arguments of the binary propositional operator. Thus, De Morgan's Law of b is easily obtained.

(26) a. APC conversions:

$! \wedge !$ $\equiv$ $\neg \vee \neg$	$! \vee !$ $\equiv$ $\neg \wedge \neg$	$\neg \vee !$ $\equiv$ $! \wedge \neg$	$\neg \wedge !$ $\equiv$ $! \vee \neg$

b. De Morgan's Law (abbreviated like the APC conversions and in more usual format):

Abbreviated	$! \wedge !$ $\equiv$ $\neg \vee \neg$[19]	$! \vee !$ $\equiv$ $\neg \wedge \neg$	$\neg \vee !$ $\equiv$ $! \wedge \neg$	$\neg \wedge !$ $\equiv$ $! \vee \neg$
Usual format	$P \wedge Q$ $\equiv$ $\neg(\neg P \vee \neg Q)$	$P \vee Q$ $\equiv$ $\neg(\neg P \wedge \neg Q)$	$\neg(P \vee Q)$ $\equiv$ $\neg P \wedge \neg Q$	$\neg(P \wedge Q)$ $\equiv$ $\neg P \vee \neg Q$

[19] $\neg \vee \neg$, with external and internal negation, is short for $\neg(\neg p \vee \neg q)$

In view of the parallellism just described between the predicate calculus and propositional calculus, it is not surprising that the lexical gap noted earlier with respect to predicate calculus operators carries over to the new set of operators, where the O-corner once again cannot be lexicalised as a single word. While “some languages (albeit a minority) provide a simple lexicalisation for the **E** value of joint denial” (Horn 1989: 256), “no language seems to have a conjunctive root *nub*, with the property that A nub B means “not both A and B”” (Zwicky 1973: 477):

(27)

affirmo		**nego**	
A	$\wedge$ and	**E**	$\neg\vee$ nor
I	$\vee$ or	**O**	$\neg\wedge$ ***nand / *nub**

This observation is valid only for everyday natural speech, where nobody uses *nand* to express the combination of ideas that flag a is not green, or flag b is not green, or neither is green. As mentioned in §1.2, however, the term *nand* did arise in twentieth century logic and came to be used in the context of efforts to reduce truth-functional connectives to a short list of primitive connectives. It was originally named for the American logician Henry M. Sheffer as the Sheffer stroke or, in modern terminology, NAND function[20]. It is now common coin in digital logic, more specifically in the context of the hardware transistor implementations which are called logic gates, one of which is the NAND-gate. It will be clear that this usage is restricted to a specialised field and has no direct bearing on the question why the word is absent in everyday language. Yet, its existence does serve to prove that the gap in everyday natural language is not to be resolved in terms of complete inaccessibility. Our view is that a solution can be found in terms of the relation between cognitive logic and lexicalisation. As the term from digital logic shows, Boëthius’ O-corner with all its entailment relations is real in the sense that its logical properties are perfectly expressible (where that is useful for practical purposes) by predicating the atomic logical predicate *not* over *and* and coining a single lexical item like Zwicky’s *nub* or Sheffer’s *nand* if desirable for that complex predicate. So, the question remains, why can the O-corner not be lexicalised in a way similar to the other corners? To close in on the problem by exclusion, we shall first have a look at the properties of the lexical items in the other three corners.

2.2 Relations

2.2.1 Three hubs and two relations

Why are the sets of lexicalisable items studied here – namely, {all, some, no} and {and, or, (neither…)nor} – triads? The hypothesis defended here is that this is not accidental, but attributable to a deep fact: *logic as the formal calculus of entailments is built on one*

[20] Actually, C.S. Peirce (1989 [1880]) had already been aware in 1880 (in a paper titled by Peirce’s editors *A Boolean Algebra with One Constant*) that a single connective sufficed for the expression of all truth-functional connectives, an analysis which will be taken up in chapter 4.

pivotal lexical item and two basic relations, whose effects at the level of entire propositions are entailment and contradictoriness respectively[21]. The former relationship is illustrated in (28), the latter in (29), where R contradicts Q.

(28) P: *All* flags are green
Q: ***Some*** flags are green

(29) Q: ***Some*** flags are green
R: *No* (= NOT-SOME) flags are green

The two relations are both applied to a single item X, which we shall call the ***pivot*** – in the predicate calculus this item is *some*. Since the relations are different from one another, application of them to a single pivot means that two more items Y (*all*) and Z (*no*) are needed to establish the complete basic set-up of logical constants and logical relations. At the level of propositions containing the logical constants, the corresponding effects are that (i) the Y-proposition logically entails the X-proposition (or Y-prop$\vdash$X-prop) and (ii) the Z-proposition contradicts the X-proposition (or Z-prop CD X-prop).

There is much to be said for the view that this essentially two-dimensional set-up is already implicit in Aristotle's calculus inasmuch as he claimed that categorical propositions can be characterized by their *quantity* and by their *quality*. These are clearly two dimensions, and they match the abovementioned two relations: the entailment-leg of the logical fork indeed represents a distinction in *quantity* (*universal* P vs. *particular* Q) whereas the contradictoriness-leg represents a distinction in *quality* (*affirmative* Q vs. *negative* R). In an optimally economic conception, the least that is required is all that is available, which results in a system with no more than these two dimensions and hence three focal points: (1) a *pivot*, (2) an *entailer*, i.e. an item with which the pivot is related by the lexical counterpart of entailment, and (3) the *contradictor*, i.e. an item to which the pivot is related by the lexical counterpart of contradictoriness.

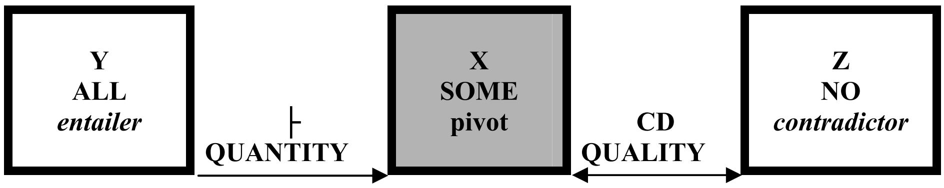
(30) a. abbreviations[22]:

All F is G	is written as	ALL	(or Y)
Some F is G	is written as	SOME	(or X)
No F is G	is written as	NO	(or Z)

[21] The first study to propose an asymmetry-model based on a pivotal "Type 1"-operator, was Löbner (1990: 105). Differences between the present proposal and Löbner's analysis will surface in § 2.2.5.2 as well as in chapter 4, where the relations and operator meanings are spelled out in detail. These differences concern (i) the relationship between the *some* and *all*, which I will argue is more complex than Löbner assumes; (ii) the lexical structure of *all*; (iii) the status of O-corner lexicalisations, which are considered marked by Löbner, impossible by me; (iv) the status of negation, (v) the ultimate reason why *some* is basic. In sum, while the model proposed here has the basic architecture of Löbner's and additional evidence will be sought for it, there are several interesting differences.

[22] Since the intrinsic meanings of the lexical items in question have not yet been described, we here illustrate their meanings and the relations between them indirectly, namely in terms of sentences in which they are used, with all other constituents kept constant. That is the rationale behind the abbreviations. Further on, we shall see that the relations obtain between (the meanings of) the lexical items themselves as much as between the propositions in which they are used.

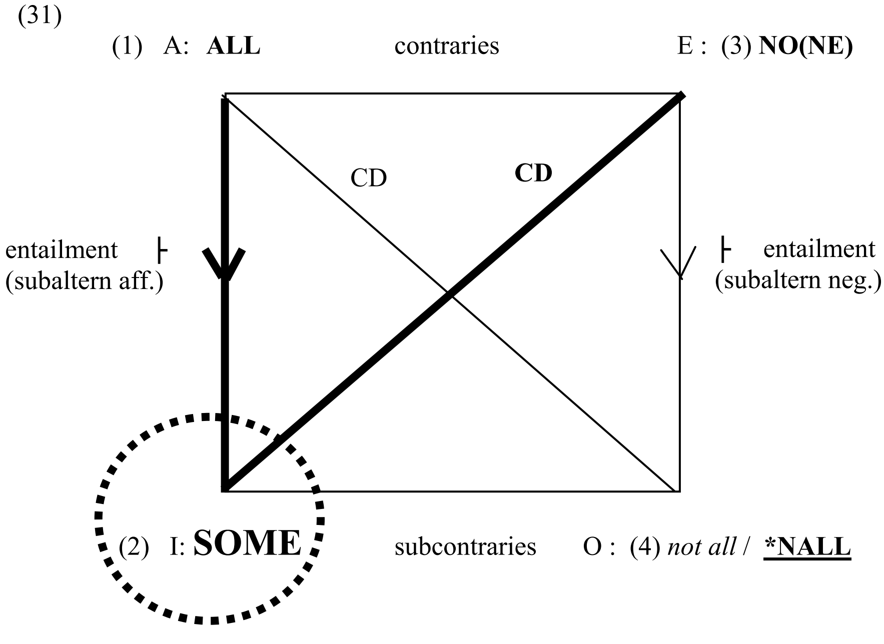
b. calculus relations



This would of course immediately exclude *NALL from the set of focal points of the logical system, as can be seen in (31) below when we superimpose the triad and its relations on Boethius' Square of Oppositions, with the pivot encircled and the two other hubs and the basic quantity and quality relation in boldface.

Some further assumptions about the nature of lexicalisation will of course be required to link non-focality in the logical system to non-lexicalisability as a single word in natural language. But as a preparatory step before the issue of lexicalisation itself can be addressed, it is useful first to make the argument about the triadic basis of the logical system stick. This means that it has to be shown that all types of logical relations in APC other than those between the focal points, namely CD between *all* and *not all*, ENT (entailment) between *no(ne)* and *not all*, contraries and subcontraries, are defined in terms of a combination of the basic entailment and contradiction relation operating on *some.* If that can be done, it will follow that those other relations plausibly are not primitives, not part of the minimal logical toolkit.

(31)



2.2.2 Primitive and derived elements of the Square of Oppositions

The present reduction of the Square of Oppositions starts from the informal characterisations of the relations in the square given above for two sentences P and Q, with those we take as basic for the present derivation in boldface:

(32)

Relation	Description	Example
1. Contradictories (i)	**P and Q can neither be both true nor be both false at the same time**	P: Some flags are green – Q: No (=NOT SOME) flags are green
2. Entailment (i)	**Whenever P is true, Q must also be true**	*Subaltern affirmative*: P: All flags are green $\vdash$ Q: Some flags are green
3. Contradictories (ii)	P and Q can neither be both true nor be both false at the same time	P: All flags are green – Q: Not all flags are green
4. Entailment (ii)	Whenever P is true, Q must also be true	*Subaltern negative*: P: No flags are green $\vdash$ Q: Not all flags are green
5. Contraries	P and Q cannot both be true at the same time, but may be both false at the same time	P: All flags are green – Q: No flags are green
6. Subcontraries	P and Q cannot both be false at the same time though they may be both true at the same time	P: Some flags are green – Q: Not all flags are green

Since Aristotle devised his logic as a formal calculus of *entailments*, we shall here first describe and derive all relations in terms of that concept. Its definition was given above:

(33) logical entailment:
P $\vdash$ Q : whenever P is true, Q must of necessity also be true.

The first of the two foundational relations of the predicate calculus is CD between *some* and *no(ne)*. In terms of entailment, CD can be formalised as follows and applied to the relation between SOME and NO(NE):

(34)

Definition	In terms of entailment:
For every sentence X, ¬X is the contradictory of X	X $\vdash$ ¬¬X
	SOME $\vdash$ ¬¬SOME (i.e. ¬NO(NE))

One additional law that has be assumed is the *Law of Contraposition*. The contents of this law is that an entailment relation may be inverted, on condition that both terms of the relation are negated: if $X \vdash Y$, then also $\neg Y \vdash \neg X$. Seuren (1998: 308) explains it as follows:

> "Intuitively, this is easily understood: $X \vdash Y$ means 'in all cases where X is true, Y is also true'. Now suppose Y is false, and $\neg Y$ therefore true. In such a case X cannot be true, since if it were true, Y would also be, in virtue of $X \vdash Y$. Given that there are only two possibilities, 'true' or 'false', for both X and Y, it follows that, if $X \vdash Y$, in all cases where $\neg Y$ is true, $\neg X$ is also true, hence $\neg Y \vdash \neg X$." (Seuren 1998: 308)

This results in the following full formal statement of CD in terms of entailment and its application to the relationship between SOME and NONE:

(35)

<table>
<tr><td>Definition</td><td>In terms of entailment:</td><td>Law of Contraposition</td></tr>
<tr><td rowspan="2">For every sentence X, $\neg X$ is the contradictory of X</td><td>$X \vdash \neg\neg X$</td><td>$\neg\neg\neg X \vdash \neg X$
or $\neg X \vdash \neg X$</td></tr>
<tr><td>SOME $\vdash \neg$NO(NE)

($\neg$NO(NE) ≡ $\neg\neg$SOME)</td><td>NO(NE) $\vdash \neg$SOME)

(NO(NE) ≡ $\neg\neg$NO(NE) ≡ $\neg\neg\neg$SOME)</td></tr>
</table>

Admittedly, a certain degree of complexity (namely, an accumulation of negatives) is caused by the choice to state such definitions as (35) in terms of Aristotle's entailment relation. There is no other option, however, since only deductive reasoning, i.e. reasoning in terms of entailment, *guarantees* that the truth of a conclusion follows necessarily from the truth of the premises. (As has been well-known since Hume (1748 [1993]), induction cannot attain to that level.)

The second foundational relation of the system is the subaltern affirmative entailment relation between ALL and SOME, as in:

(36) P: All flags are green $\vdash$ Q: Some flags are green

This relationship has the following definition – with the effect of the Law of Contraposition added in an additional column:

(37)

Definition	In terms of entailment:	Law of Contraposition
For all sentences ALL and SOME, ALL entails SOME	ALL $\vdash$ SOME	$\neg$SOME $\vdash \neg$ALL or NONE $\vdash \neg$ALL

2.2.3 Deriving the rest of the Square[23]

The four primitives detailed above – (i) the pivot operator SOME, (ii) the relation CD between the pivot SOME and NO(NE), (iii) the entailment relation (ENT = $\vdash$) from ALL to SOME, and (iv) the Law of Contraposition – suffice to derive all relations in the Square.
With ALL and entailment from ALL to SOME introduced into the system, application of the primitive operation negation to ALL introduces a new element ¬ALL into the system and yields the relation CD between the two, thus deriving relation 3. of (32).
As already indicated, application of the Law of Contraposition to the entailment relation between ALL and SOME, derives the subaltern negative between NO(NE) and the newly introduced item ¬ALL, i.e. relation 4. in (32):

(38)

Basic relation: Subaltern affirmative	Law of Contraposition	Derived relation: Subaltern negative
ALL $\vdash$ SOME	$\longrightarrow$	¬SOME (= NO(NE)) $\vdash$ ¬ALL

Relation 5. of (32), the relation between ALL and NO(NE) is derived by defining *contraries* as:

(39) For all sentences X and Y: X and Y are contraries iff X $\vdash$ ¬Y and hence, by contraposition Y $\vdash$ ¬X

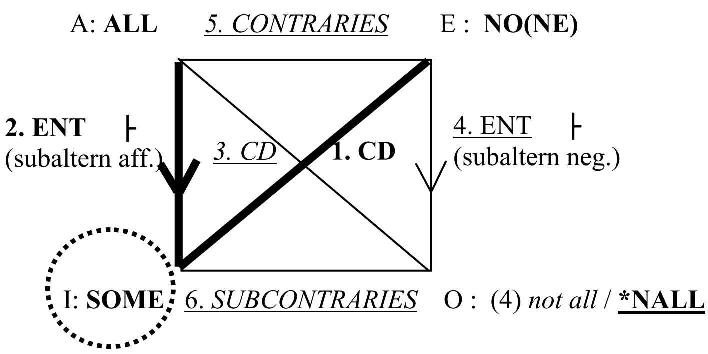
ALL and NO(NE) are thus contraries since by primitive (iii) above ALL (X) indeed entails ¬NO(NE) (¬Y; = SOME) and by contraposition NO(NE) (Y) entails ¬ALL (¬X) (cf. relation 4).

Relation 6. of (32), finally, the relation *subcontraries* between SOME and ¬ALL is derived as follows:

(40) For all sentences X and Y: X and Y are subcontraries iff ¬X $\vdash$ Y and hence, by contraposition ¬Y $\vdash$ X

This makes SOME and ¬ALL subcontraries, given that by the derived relation 4. NO(NE) (¬X; = NOT-SOME) indeed entails ¬ALL and by primitive (iii) ¬¬ALL (¬Y; = ALL) entails SOME (X).

[23] Thanks to Pieter Seuren for crucial help with this section.



One of the consequences of the present reduction of the logical system, the most important one in terms of the puzzle that started this quest, is that the O-corner is the only one which cannot be defined *directly* in terms of a single operation on the pivot (cf. Löbner (1990: 106)). According to Löbner (1990: 106) it is this need for two derivational steps to the O-corner which makes for increased processing complexity and resulting semantic markedness. This status is at least consonant with O-corner operators' non-accessibility for lexicalisation as a single word. In other words, the net is clearly closing. Yet, I believe it would be wrong to assume that this is all there is to the account. Processing complexity wil be argued to be a necessary, but not a sufficient feature to explain the non-existence of **nand* and **nall*. The ambition of this study is to prove that such lexical items are blocked as a matter of semantic incongruity, not complexity: they are arguably not just semantically marked, but semantically impossible lexical items[24]. Still, as a step in the right direction, the idea that it is impossible to get from the pivot to O in one fell swoop is crucial: the semantic incongruity of **nand* and **nall* in natural language will indeed be shown to be due to an *intermediate* hub between the pivot and the O-corner.

2.2.4 Extension of the results to the propositional calculus

The triad {*and, or, (neither…)nor*}, can be dealt with in the same vein as the predicate calculus operators.

(41) a. John is in the garden *and* Peter is in the garden ($P \wedge Q$)
b. John is in the garden *or* Peter is in the garden ($P \vee Q$)
c. Neither John is in the garden, *nor* Peter is in the garden ($\neg(P \vee Q)$)

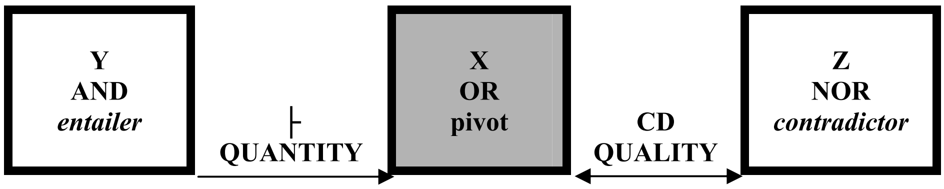
Sentence (41) a. logically entails sentence (41) b.: whenever a. is true, b. must of necessity also be true, on account of the meanings of the logical constants *and* and *or* involved in a. and b.. Sentence c., for its part, is the contradictory (or negation) of sentence b.

[24] Recall that I restrict my attention to the predicate calculus, the propositional calculus and number, where O-corner lexicalization is completely barred (Horn 1972). Löbner (1990: 89) argues that there are lexicalised Type 4 (=O-corner) expressions in other domains. If so – and I have at present no reason to doubt the correctness of the assumption – I do not see how that can be harmonized with my findings for the operators studied here.

(42) a. abbreviations:

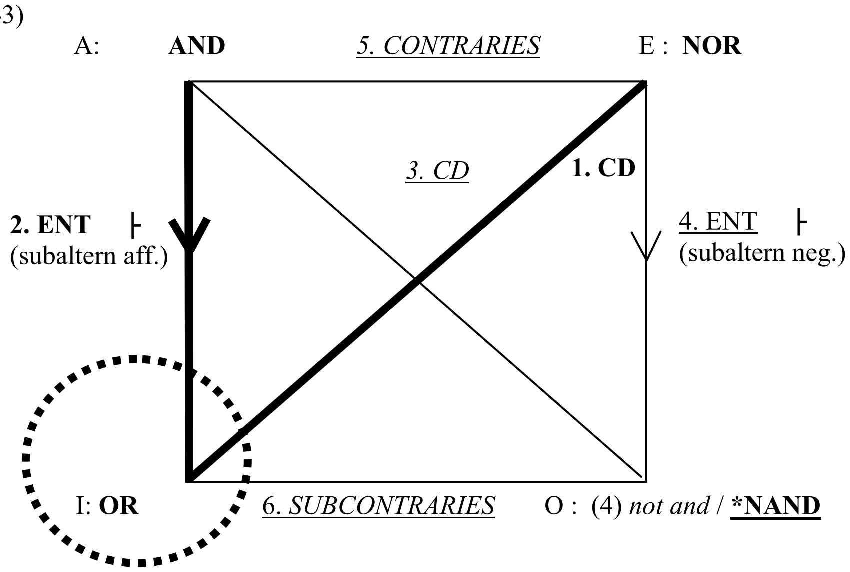
$P \wedge Q$	is written as	AND	(or Y)
$P \vee Q$	is written as	OR	(or X)
$\neg(P \vee Q)$	is written as	NOR	(or Z)

b. calculus relations



Given this parallellism, all relations in the corresponding Square can once again be derived from the primitive pivot OR, CD between OR and NOR, the entailment relation from AND to OR, and the Law of Contraposition, exactly as in the previous section, same primitive relations, definitions and all.

(43)



(44)

Relation	Description	Example
1. Contradictories (i)	**P and Q can neither be both true nor be both false at the same time**	P: John is in the garden *or* Peter is in the garden – Q: Neither John is in the garden, *nor* Peter is in the garden

2. Entailment (i)	**Whenever P is true, Q must also be true**	*Subaltern affirmative*: P: John is in the garden *and* Peter is in the garden ├ Q: John is in the garden *or* Peter is in the garden
3. Contradictories (ii)	P and Q can neither be both true nor be both false at the same time	P: John is in the garden *and* Peter is in the garden – Q: It is not the case that John is in the garden *and* Peter is in the garden
4. Entailment (ii)	Whenever P is true, Q must also be true	*Subaltern negative*: P: Neither John is in the garden, *nor* Peter is in the garden ├ Q: It is not the case that John is in the garden *and* Peter is in the garden
5. Contraries	P and Q cannot both be true at the same time, but may be both false at the same time	P: John is in the garden *and* Peter is in the garden – Q: Neither John is in the garden, *nor* Peter is in the garden
6. Subcontraries	P and Q cannot both be false at the same time though they may be both true at the same time	P: John is in the garden *or* Peter is in the garden – Q: It is not the case that John is in the garden *and* Peter is in the garden

The definitions of the basic relations applied to the propositional system are:

(45) contradictoriness (CD):

Definition	In terms of entailment:	Law of Contraposition
For every sentence X, ¬X is the contradictory of X	OR ├ ¬NOR (¬NOR ≡ ¬¬OR)	NOR ├ ¬OR) (NOR ≡ ¬¬NOR ≡ ¬¬¬OR)

(46) entailment (├ , ENT):

Definition	In terms of entailment:	Law of Contraposition
For all sentences AND and OR, AND entails OR	AND ├ OR	¬OR ├ ¬AND or NONE ├ ¬AND

With AND and entailment from AND to OR introduced, application of negation to AND introduces ¬AND and yields the relation CD between the two, thus deriving CD relation 3.

(47)

3. Contradictories (ii)	P and Q can neither be both true nor be both false at the same time	P: John is in the garden and Peter is in the garden – Q: It is not the case that [John is in the garden and Peter is in the garden]

As already indicated, application of the Law of Contraposition to the entailment relation between AND and OR, derives the subaltern negative entailment relation between NOR and the newly introduced item ¬AND, i.e. relation 4. :

(48)

Basic relation: Subaltern affirmative		Law of Contraposition	Derived relation: Subaltern negative
AND	├ OR	⟶	¬OR (= NOR) ├ ¬AND

Relation 5., the relation between AND and NOR is derived by defining *contraries* as in (39) above (for all sentences X and Y: X and Y are contraries iff X ├ ¬Y and hence by contraposition Y ├ ¬X). AND and NOR are thus contraries since by the primitive subaltern affirmative entailment relation between AND and OR, AND (X) indeed entails ¬NOR (¬Y; = OR) and by contraposition NOR (Y) entails ¬AND (¬X) (cf. relation 4).

Relation 6. of (32), finally, the relation *subcontraries* is the same as in (40), namely: for all sentences X and Y: X and Y are subcontraries iff ¬X ├ Y and hence, by contraposition ¬Y ├ X. This makes proposition calculus operators OR and ¬AND subcontraries: by the derived relation 4., NOR (¬X; = NOT-OR) indeed entails ¬AND and by the primitive subaltern affirmative entailment relation (between AND and OR) ¬¬AND (¬Y; = AND) entails OR (X).

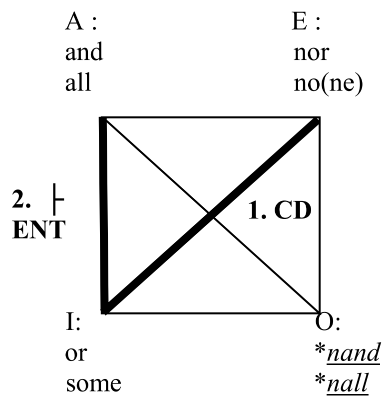
While the parallellism between the sentences in (28) and (29) on the one hand and those in (41) on the other was already observed by the ancient Greek inventors of logic, the present analysis adds to their insight the claim that the predicate calculus and the propositional calculus can be reduced to two primitive relations (CD and ENT). Since these relations operate on a single pivotal operator, the core calculus ends up having as primitives:

a. the two axes (the relations);
b. the primitive operator we have called the pivot (such as *or* c.q. *some*);
c. two more focal points: the entailer (*and*, c.q. *all*) and the contradictor (*nor,* c.q. *no(ne)*)

All the rest is derived: contraries, subcontraries, etc. This turns the Boethian Square into a much leaner birelational system, in effect a 2D Cartesian Coordinate System, with the pivot in the I-corner as the postulated "origin" of the calculus[25].

(49)

Logic as a 2D Cartesian Coordinate System
Two relations : the contradictoriness axis and the entailment axis



2.2.5 Comparison with alternative models

2.2.5.1 Triangular models

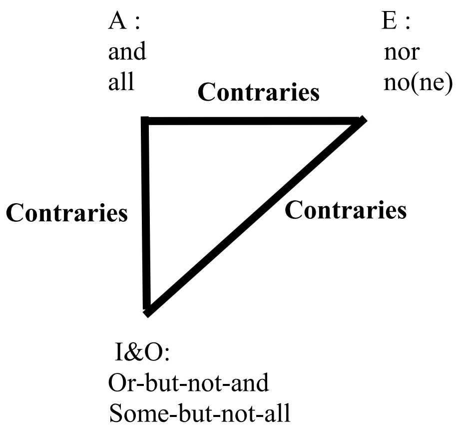
The present system is not only different from Boethius' Square of Oppositions, but also from a number of proposals in the literature in which the square is reduced to a triangular system of relations instead of to a 2D Cartesian Coordinate System: De Morgan's (1858: 121) "trichotomy"; Jespersen's (1924: 324-325) "tripartitions"; Horn's (1989: 253) "three-cornered square" and Seuren's (2002, 20-21) bitriangular "improved square".

De Morgan and Jespersen achieve their reduction from a Square to a Triangle not by eliminating O, but rather by collapsing I and O into a single complex operator I&O: exclusive *or* (= *or-but-not-and*) c.q. strong *some* (= *some-but-not-all*). In the words of Jespersen (1924: 324): "It should be noted that *some (something,* etc.) is here taken in the ordinary meaning it has in natural speech, and not in the meaning logicians sometimes give it, in which it is the positive counterpart of *no (nothing)*, and thus includes the possibility of *all*"

[25] The reference to the Cartesian notion of "origin" is intended. The origin of a two-dimensional Cartesian coordinate system is the point of intersection, where the x axis (abscissa or horizontal axis **o**x) and the y-axis (ordinate or vertical axis **o**y) meet. (Descartes 2001 [1637], Mazur 2003: 77 ff; Aczel 2000: 122-129). In chapters 4 and 6 (esp. § 6.3.1.3) other reasons for calling the present system a Cartesian coordinate system (or *Cartesian plane*) than the fact that it has two axes will be fleshed out.

(50)

Logic as a Trichotomy/Tripartition Triangle
One relation : contraries



This proposal is characterized by the illusion of theoretical good news and the reality of empirical bad news. Let us consider them in turn.

On the theoretical side, it certainly looks advantageous to be able to cut the relations down to a single one, namely contrariness. To illustrate that the three corners are indeed thus related, recall the informal and formal definition of contraries given above and applied here to the corners A, E and I&O:

(51) Informal definition: P and Q cannot both be true at the same time, but may be both false at the same time

What should be and is ruled in by the definition of the relation "contraries": cases where P and Q are not both true at the same time, but may be both false at the same time		What should be and is correctly ruled out.	
A, ¬I&O, ¬E	It is not the case that there is nobody in the garden, nor that some-but-not-all are in the garden, but it is the case that all of them are in the garden	*¬A, I&O, E	*Not all of them, but both none of them and some-but-not-all are the garden.
¬A, I&O, ¬E	Neither all of them, nor none of them is in the garden: some-but-not-all are.	*A, ¬I&O, E	*All of them and none of them, but not some-but-not-all of them are in the garden.

¬A, ¬I&O, E	Neither all of them, nor some-but-not-all of them are in the garden: none are.	*A, I&O, ¬E	*Not none of them, but both all and some-but-not-all of them are in the garden.
		*¬A, ¬I&O, ¬E	*Not all of them, not nobody, nor some-but-not-all are in the garden.
		*A, I&O, E	All of them, none of them and some-but-not-all of them are in the garden

(52) Formal definition: For all sentences P and Q: P and Q are contraries iff P $\vdash \neg$Q (and hence, by contraposition Q $\vdash \neg$P)

Applied to the three corners, this yields – correctly given the nature of the focal points – the following entailments.

(53)

A $\vdash \neg$ I&O	I&O $\vdash \neg$ A
A $\vdash \neg$E	E $\vdash \neg$A
I&O $\vdash \neg$E	E $\vdash \neg$I&O

But the neatness of this result should not cover up an immediate theoretical problem: all the above definitions are stated using the symbols $\vdash$ and ¬, i.e. this version of Aristotle's calculus is stated in terms of truth and falsity (the relation between which is CD, realized by the operation negation ¬) and entailment ($\vdash$). These are of course precisely the correlates of the two relations of the 2D Cartesian Coordinate System. So in the trichotomy analysis either these relations are still presupposed – but no longer represented in the triangle – or a way has to be found to define contrariness without recourse to them. As far as I can tell, the latter is an impossible task. But if the former obtains and CD and ENT indeed still belong to the primitives of the system, then a 2D Cartesian Coordinate System, which does not have contrariness in its stock of primitives is to be preferred on grounds of economy.

But let us suppose that a way could be found to define contraries without recourse to CD or ENT. Then there is still an insurmountable *empirical* problem for the Trichotomy/Tripartition proposal: it cannot accommodate simplex I operators such as inclusive *or* and weak *some*, though – contrary to what Jespersen seems to think – these are clearly attested in "natural speech", if only the context is properly chosen:

(54) a. Have you talked to John or Bill? (=inclusive or: "John, or Bill, or both") vs.
b. Mary talked to John or Bill (= exclusive: "but not to John *and* Bill")

(55) a. There are some linguists sick (= weak, cardinal)
b. Some linguists are tall (= strong, quantificational: "but not all")

Specifically, in contexts where *knowledge is incomplete* simplex I is needed and cannot be eliminated in favour of I&O. Thus, in interrogative (54) a. there is doubt as to how many talking events have taken place. And in stage-level predication (55) a. there is uncertainty which proportion the set of sick linguists represents relative to the set of all linguists. Jespersen is apparently not aware of this systematicity. De Morgan, for his part, is:

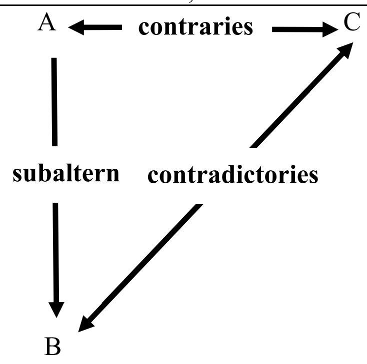
> "This point is clearly recognized by De Morgan. Writing sixty years before Jespersen, he warns that the TRICHOTOMY apparently possible to one with **complete knowledge** must yield, in a logic based on the **imperfectly epistemic human condition**, to the classical four-way opposition (with qualitative and quantitative axes) mapped out in the Square [of Boethius, DJ]"
>
> Horn (1989: 219; boldface added, DJ)

A problem that besets all remaining triangular systems is that they invariably have at least three primitive relations, namely one per side of the triangle, rather than being fully slimmed down to the minimum of two relations. That holds true for Horn's three-cornered square:

(56)

Logic as a Three-Cornered Square (Horn 1989: 253)

three relations : subaltern, contradictories and contraries



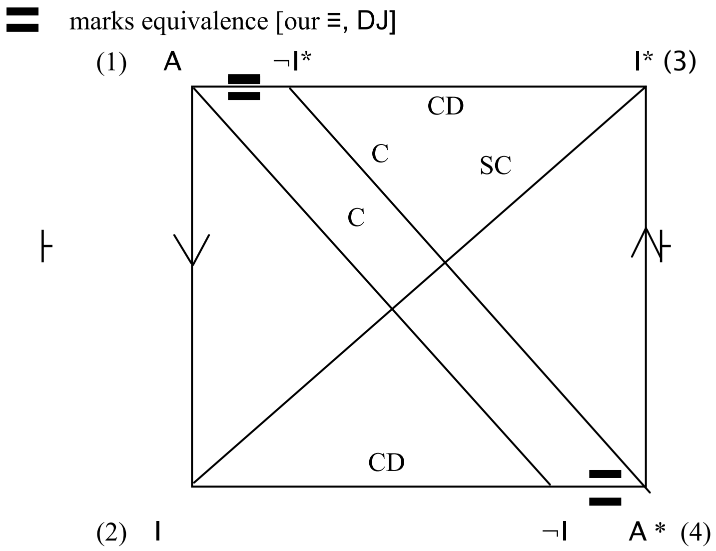
A= A-corner, B = I-corner, C = E-corner

Consider also the bitriangular representation below, which is Seuren's (2002, 20-21) *Improved Square* for the Predicate Calculus. His revision is based on the linguistic consideration that the Boethian Square fails to represent the role of negation in the system properly (Seuren's notation is ¬ for external negation, * for internal negation). Instead of a single Square, his revision of the Square results in a combination of two logically analogous *triangles* (with ¬I = E).

(57)

Symbols	**Natural language**
A	[$_{su}$ All flags] are[$_{pr}$ green]
I	[$_{su}$ Some flags] are [$_{pr}$ green]
¬I	[$_{su}$ No flags] are [$_{pr}$ green]
	[$_{su}$ Not all flags] are [$_{pr}$ green]
A *	[$_{su}$ All flags] are not [$_{pr}$ green]
I*	[$_{su}$ Some flags] are not [$_{pr}$ green]
¬I*	? [$_{su}$ No flags] are not [$_{pr}$ green]
	? [$_{su}$ Not all flags] are not [$_{pr}$ green]

(58)

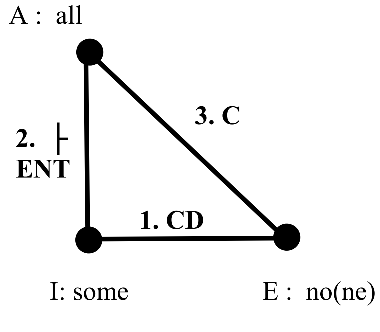
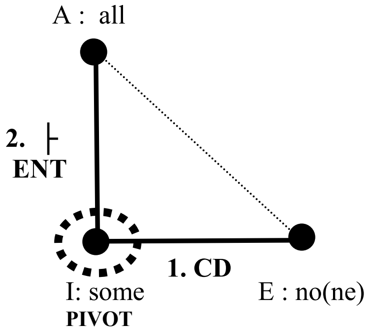


This diagram is already much more in line with the linguistic facts than Boethius' Square in that like a 2D Cartesian Coordinate System it is based on three focal points (A, I, ¬I (= E-corner)). A first difference, however, is that they are used twice to improve the square. Moreover, the set of basic relations has not been reduced to ENT(A, I) and CD(I, E). More specifically, the set of primitives still includes the hypothenuse relation of each triangle, namely C (contraries), as well as SC (subcontraries). This also means that in a triangle of relations no corner is more prominent than any other[26]. A 2D Cartesian

[26] In Seuren's bitriangular Square, one might attribute the pivotal role of I to the fact that it enters into 3 relations (ENT, CD, SC), which is one more than the two other corners of each triangle. A problem with that solution, however, is that there is something strange about the O-corner (I*) in the "negative triangle" of Seuren's system if you look at it from language's end: why can we use *NOT ALL people are in the garden* for that corner at all? The system would predict it to be blocked in favour of *SOME people are NOT in the garden*, at least on the assumption that there is an isomorphism between the lexicalised forms and the symbolisations of the corners of the *improved square*.

Coordinate System is different. It straightforwardly assigns a pivotal role to one of the corners (i.c. the I-corner) as the only focal point on which *both* of its two basic relations operate.

(59)

a. Logical Triangle	b. 2D Cartesian Coordinate System
Three relations: contradictoriness, entailment, contraries	Two relations: contradictoriness and entailment
Three focal points: A, I, E	Three focal points: A, I, E, One pivot: I
	

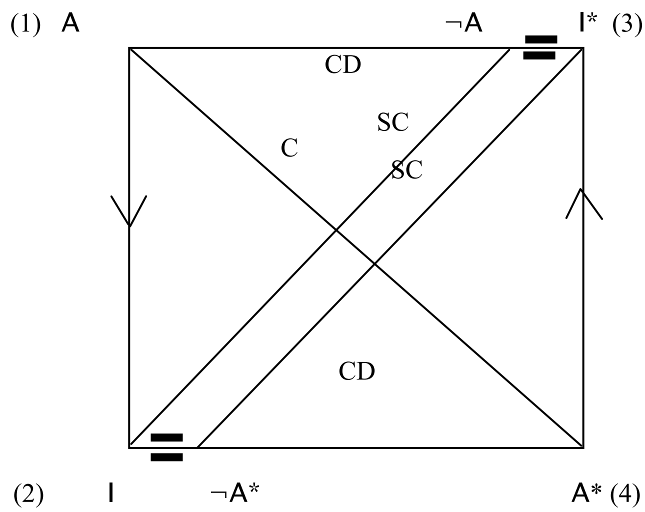
So if independent linguistic facts and further logical and lexical analysis can be shown to lend support to the hypothesis that the I-corner is pivotal, that will at the same time be evidence favoring a 2D Cartesian Coordinate System over any triangular approach.

As regards the O-corner lexicalisation puzzle, Seuren claims that his “Improved Square also solves a (quasi-)problem raised by Horn (1972, 1989: 252-67) and Levinson (2000: 69-71)”, namely the **nall*-problem. He notes that “although their observation appears to be correct, the Improved Square shows that the problem is imaginary: it is merely an artifact of the deficient way Boethius formalized APC. The Boethian O-corner has lost its place to the three quantifier expressions *all, some* and *no*” (Seuren 2002: 21-22, fn. 6).

While triangular and 2D Cartesian Coordinate Systems certainly stand a greater chance of bringing us closer to a solution to the O-corner problem, Seuren’s conclusion is too rash, however. Though his bitriangular scheme is a linguistically better representation of the facts than Boethius’ Square, the same basic triad of lexicalisatons with which he sets his bitriangle up, namely *all (*=A*), some (*=I*)* and negation (=¬), could in principle have given rise equally as well to an alternative bitriangular scheme similar to the one in (57). The corresponding triad of lexicalisations would then be *all, some, nall*, while *none* would be turned into the eliminated corner of the original Square. Instead of an O-corner problem, this set-up – which no principle forbids – would predict an E-corner problem instead of an O-corner problem. It would make *none* lose its place among the lexicalisable quantifier expressions, contrary to linguistic fact (see (60)).

Of course, the fact that we use the basic triad of lexical items *all, some/one, no(ne)* rather than *all, some, nall*, indicates that Seuren’s (57) is superior to (60), but that empirical fact does not answer the question why. The inadequacy of the theoretical alternative (60) from the same basic stock of primary lexicalisations (*all, some, n-*) still has to be established. We shall call this problem the "pivot anchoring" problem. The strategy adopted in the present study fares better than Seuren's proposal in this respect: by reducing the set of basic relations to two and providing not just empirical evidence but also a theoretical explanation for the pivotal status of the I-corner item among the hubs, the I-corner will get "anchored" as the pivot or Cartesian origin of the whole system. The two remaining focal points are then introduced by the basic relations. In the context of such an anchored system, fewer questions have to be answered to establish that the lexicalisable corners, which relate directly to the fixed pivotal one, are categorially different from the non-lexicalisable one, which does not.

(60)



2.2.5.2 The First Asymmetry Model

Löbner (1990) was the first to propose an asymmetry model which takes the I-corner operator (across several calculi) as the most unmarked and semantically basic operator of a square. This is expressed in his "These zur Typenasymmetrie":

(61) These zur Typenasymmetrie

> "Als der unmarkierte Fall ist Typ 1 [the pivot, DJ] relativ zu den anderen Typen semantisch elementar. Die bedeutung der übrigen ergibt sich aus der des Typs 1 durch konzeptuelle Operationen, die den Effekt von N[egation], S[ubnegation] oder D[ualnegation] haben, und ist damit komplexer als die Bedeutung des Typs 1."
>
> (Löbner 1990: 105)

To construct the other corners from the pivot, he appealed to different forms of negation, essentially exploiting all the possibilities of outer and inner negation[27] (cf. the conversions of (18)).

(62)

	Type of Negation	External negation	Internal negation	Quantifier (e.g. of Predicate logic, but extendable to other calculi)
Type 1 – pivot (e.g. *some*)		-	-	$\exists x\ fx$
Type 2 – entailer (e.g. *all*)	Dual negation[28] (=Dualnegation)	+	+	$\neg\ \exists x\ \neg fx$ ($\equiv \forall$ by (18))
Type 3 – contradictor (e.g. *no*)	Outer negation (= Negation)	+	-	$\neg\ \exists x\ fx$
Type 4 – O-corner operator (e.g. **nall*)	Inner negation (=Subnegation)	-	+	$\exists x\ \neg fx$ ($\equiv\neg\forall$ by (18))

Moreover, since "any random order combination of two kinds of negation is equivalent to the third kind" (Smessaert 1991: 264), Löbner postulates that inner negation is a combination of dual and outer negation, hence derived.

(63)

Inner negation	≡	Outer negation of dual negation
$\exists x\ \neg fx$	≡	$\neg\ (\neg\ \exists x\ \neg fx)$

By doing this, he creates relational asymmetry in his duality square by reducing the primitive relations to Dual Negation and Outer Negation. On the further assumption that (outer) negation is conceptually more complex than dual negation, a markedness hierarchy is established among the quantifiers: Type 1 – the pivot – is the least marked, Type 2 (entailer) is derived from it by dual negation and is therefore less marked than Type 3 (contradictor), which is derived by outer negation. Type 4, finally, the combination of dual and outer negation (or vice versa) applied to the pivot, is the most marked type of the "duality square", as it is the only operator which can only be derived in two steps from the pivot (cf. (64) below).

[27] The parallel definition of the four truth-functions of the propositional square in terms of two primitive ones (in casu I-corner disjunction and negation) was provided in Russell and Whitehead (1910-13)'s *Principia Mathematica* and in Russell (1919, [2000]: 148) and can also be culled from the definitions of *or* ($\vee$), *and* ($\neg\vee\neg$), *nor* ($\neg\vee$), **nand* ($\vee\ \neg$) in (26) above. Löbner's proposal (like ours) generalizes over different types of quantifiers; those of predicate logic are used for exemplification of the duality square relations in this context.

[28] The dual of an operator is "the outer negation of its inner negation (or vice versa)" (Westerståhl 2005: 5)

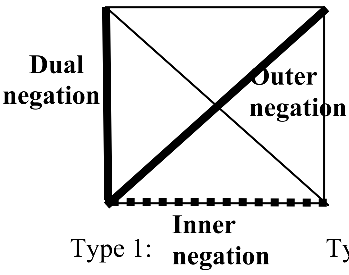
Now, if two independent proposals converge on the same architecture, the question is whether the later one takes the issues further. I believe that in a number of ways it does.

(64)

Two primitive relations :
Dual negation and outer negation

Type 2: $\neg\ \exists\ \neg$ all

Type 3: $\neg\ \exists$ no(ne)



Type 1: $\exists$ some

Type 4: $\exists\ \neg$ *<u>nall</u>

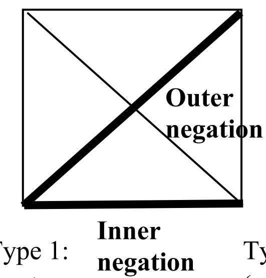
First of all, beyond the arguments provided by Löbner (in terms of aspectual particles such as *already, still, no longer, not yet*) there is a stronger and more immediate reason why inner negation cannot belong to the class of primitives if outer negation does. Just try to take inner and outer negation as the two primitive relations and consider the square of oppositions for the propositional calculus.

(65)

Alternative primitive relations
inner negation and outer negation?

Type 2: $\neg(\neg P \vee \neg P)$ Pand P

Type 3: $\neg(P \vee P)$ P nor P



Type 1: $(P \vee P)$ P or P

Type 4: $(\neg P \vee \neg P)$ P *nand* P

It is easily seen that with an identical choice of propositions, inner and outer negation can no longer be held to be two separate categories: they coincide since Type 3 $\neg(P \vee P)$ and Type 4 $(\neg P \vee \neg P)$ are both equivalent to $\neg P$, so that the relation between Type 1 and Type 3 and the relation between Type 1 and Type 4 become indistinguishable.

No such problem befalls the pair dual negation – outer negation if these are selected as primitives. They remain separate relations under the same identical proposition conditions: Type 1 and Type 2 remain each others duals, as do Type 3 and Type 4: applying inner and outer negation to the one still results in the other. Type 1 and Type 3, for their part, remain each other's outer negations, as do Type 2 and Type 4. In sum, dual and outer negation (like their counterparts entailment and contradictoriness in our proposal) are separate categories throughout, as desired if they are real primitives.

Aside from additions like the above argument, the most important differences between Löbner's approach and mine are that outer negation, dual negation and the pivot itself will be subjected to decomposition with the same means as everything else in the Square and will thereby all be identified as non-primitives built from a single operator.

2.3 Conclusion: Main Questions

Aside from solving the **nand*-riddle, our aim is to bring to light an isomorphism between the relational structure of logical calculi, the semantic-cognitive properties of logical operators and their lexical-morphological realizations. To reach that goal, Boethius' Square has been reduced to a 2D Cartesian coordinate system in this chapter. A detailed analysis of each element of the resulting system will be undertaken in stepwise fashion in the chapters to come.

Chapter 3, first of all, will focus on the two basic *relations.* The following questions will be answered:

> 1. What are the properties of the relation of *contradictoriness* (CD) in a formal calculus of entailments in natural language?
> 2. What are the properties of the relation of *entailment* (ENT) in a formal calculus of entailments in natural language?

Since these two primitive relations are arguably predicated on a pivotal operator, the next question that imposes itself, the central one of this study, is:

> 3. What are the main semantic features of the element which functions as the *pivot* of a formal calculus of entailments in natural language? How does it differ from the other two hubs? How does it acquire pivot status?

An exhaustive answer to this question, based on an analysis of several pivots (*or, either, some, any, a*, etc.), will be undertaken in chapters 4, 5 and 6. But there is one characteristic which must be emphasized here, since it will provide scaffolding for the analysis of the two relations (CD and ENT) in chapter 3. Specifically, the most distinctive feature of pivots is that they invariably signal indeterminacy[29] and tentativeness: when you say that *there are some men in the garden* they are in Russell's terms still "ambiguous", some men *or* other. In the propositional calculus, the pivot *or* also introduces a new, second possibility and by widening the hypothesis space it too introduces an element of uncertainty: *John is in the garden or Peter is in the garden* indicates that the speaker is still in doubt. Note that there is no absolute lack of knowledge, but rather partial knowledge: the men may be ambiguously some *or* other men, but they are nonetheless men; and it may be John *or* it may be Peter who is in the garden, but the speaker is convinced that there is at least someone in that garden. "Every disjunction which is not logically exhaustive (i.e., not such as 'A or not-A') gives *some* information about the world, if it is true; but the information may leave us so hesitant as to what to do that it is *felt* as ignorance" (Russell 1940 [1969]: 82), a sense which Bromberger (1992) describes as "what we know we don't know". If there were no such hub of partial ignorance in the logical system and if that plausibly entailed that we could not experience or know we are ignoramuses, we neither could nor would feel the urge to do something about it. Nor could we draw a mind-internal distinction between worse and better knowledge. There could be no drive to cure my ignorance if I were not somehow aware of it, had no way of being perplexed by it[30].

These observations about the pivot I-corner square well with epistemological assumptions expressed by Charles Saunders Peirce's (1839-1914) on "The Fixation of Belief" (Peirce 1877) and the irritation of doubt: "We generally know when we wish to ask a question and when we wish to pronounce a judgment, for there is a dissimilarity between the sensation of doubting and that of believing." "Belief guides our desires and shapes our actions"; consequently, "doubt is an uneasy and dissatisfied state from which we struggle to free ourselves". The only modification I would propose is that this feeling of dissatisfaction is only as acute as Peirce's explanation suggests in contexts where truth is the prime concern, as in science. There are many other contexts – both very general everyday pragmatic situations as more specific ones such as when one is reading a novel, for instance – where the search for truth and general knowledge is not relevant.

This notion of partial knowledge with a remaining element of indeterminacy makes pivots different from the two other corners, where no such doubt remains: *Peter is in the garden and Quentin is in the garden* expresses (positive) certainty that they both are (twice value 1: 1 1). From the equivalence of the propositional calculus PROPC and the two-object universe predicate calculus APC_2 explained in section 2.1.3, it follows that *all men are in the garden* expresses positive certainty (universal) in APC_2. The same conclusion of expressed certainty – though this time negative – holds for *neither Peter is in the garden nor Quentin is in the garden* (value 0 for both: 0 0) and its $APC_{(2)}$ counterpart *no man is in the garden* respectively. The difference is represented in the following integrated truth table for *and* ($\wedge$), inclusive *or* ($\vee$) , and *nor* ($\neg\vee$)

[29] Cf. the observation about "*knowledge is incomplete*"-indeterminacy and De Morgan's reference to the "*imperfectly epistemic human condition*" made in the context of the pivotal operators in (54) and (55) above.

[30] What's true for me, is equally true for others, not only in daily life but also in the context of scientific research: thus "…ignorance can also define and determine the value of scientific contributions" (Bromberger 1992: 112).

(66)

PROPC	Truth table P Q	mind			
∧ (P Q)	1 1		+	certainty	A-corner
∨ (P Q)	1 0 0 1	?		doubt	I-corner
¬∨ (P Q)	0 0		-	certainty	E-corner

Our guiding common sense assumption about actual reality is that something is the case (+) or is not the case (-). What is deemed impossible is that something is simultaneously the case and not the case at the same time (the Law of Contradiction), or in an unstable state of indecision (?)[31]. Consequently, the I-corner indeterminate state of mind (?) is plausibly conceived of as a Peircean dissatisfied state "from which we struggle to free ourselves". There is an epistemological drive away from the I-corner. I shall return to these semantic differences and their epistemological consequences in chapter 4 and sketch the role of indeterminacy as one of the triggers for searches for certainty and knowledge of the type expressed by entailers and contradictors. The essence of the special position of pivots in natural logic and epistemology will however be sufficiently clear from this brief exposé to move on.

The perspective sketched here makes sense only if natural logic is an internalist affair. Direct evidence in favour of that position is that *uniquely* cognitive, mind-internal aspects – such as the experience of partial ignorance (?) – play a pivotal role in the basic architecture of the logical system. But the radical internalist will go further and maintain that it is only, by formal mind-internal contrast with such partial ignorance (?) that A-corner and E-corner propositions, which affirm what is (+) or is not (-) the case respectively, manage to *appear* to be independent of individual psychology and mind-external. This effect might well be the equivalent in language and thought of the suppression of the sense that one is looking at a photograph when one says "that is my mother" instead of "that is a picture of my mother" (cf. Jackendoff 1983). It is likely that such suspension of attention to representationality is easier to get with a photograph than with, say, a painting in which the idiosyncratic individuality of the painter is more dominant.

In view of its mentalist linguistic aspirations, our analysis cannot afford to sidestep a detailed description of the internal semantic features of the focal points of the system as *lexical items* in the mental natural language lexicon, each with their own properties (cf. (59)b.). This will be the sum and substance of chapter 4.

[31] The quantum world is not common sense.

In chapter 7, finally, the discussion will bring us back to the fourth and final question, which rephrases the O-corner problem that started the whole quest in terms of the design and constraints of the natural language lexicon.

> 4. What is the precise nature of the condition on *lexicalisation* which excludes **nand* and **nall* and what is its relation to the calculus?

The latter question, a more precise formulation of the original puzzle, will be kept on the backburner till the final sections of this study. That is not done for rhetorical reasons, but simply because no answer is within reach as long as the first three questions have not been solved. Moreover, while the empirical puzzle is easy to grasp and so general that the pattern jumps out, its theoretical import is comparatively smaller than that of the answers to the other questions raised above. The latter provide the theoretical substance of the present study. It is therefore advisable not to let the final destination become the sole focus of interest or the source of impatience or irritation along the way. While a solution to question 4. may well be the icing on the cake, most of the tasty ingredients have gone into the cake itself. So let’s have that cake and eat it too.

3 THE TWO RELATIONS OF THE 2D CARTESIAN COORDINATE SYSTEM

3.1 Introduction

In the previous chapter, Boethius' Square of Oppositions was reduced to two foundational relations: contradictoriness and entailment. In the history of linguistics and philosophy, the study of these two notions has filled many tomes. Not only is it far beyond my abilities to detail the history of those discussions and reconstruct the past, it is also far beyond the scope of this work to address more than a modest bit of the issues brought up. My aim will be primarily to supplement the *logical perspective* on contradictoriness and entailment with a *set-theoretic perspective* and an analysis in terms of *Boolean algebra*. This will bring to light that Boole's views on set-demarcation are very helpful tools for a theory which seeks to describe logical operators as 'normal' lexical items in the natural language lexicon. It will also help determine that contradictoriness and entailment are properties of an *internalist* nature, whose cognitive foundations it is hard if not impossible to deny.

3.2 Contradictoriness

The foundation for a good understanding of the relation of contradictoriness is the Law of Contradiction (LC)[32], Aristotle's rule that 'the same thing cannot at the same time both belong and not belong to the same object and in the same respect' (*Met*. 1005b19-23). He claims it is 'the most certain principle of all' of his logic of opposition, which has the status of a primitive or axiom for which no proof can be provided and none should be asked. In this section, I study the nature and position of this principle within the set of axioms of Aristotle's *logic* and represent the *set-theoretic conception* of its contents. In line with my internalist goals, I shall then try to underpin the claim that LC is a *cognitive* constraint.

3.2.1 The Law of Contradiction in Logic

The most commonly cited form of the principle is the propositional calculus version from Russell and Whitehead (1910-13)'s *Principia Mathematica*:

(67) $\neg (p \wedge \neg p)$

It expresses that nothing can be the case and not be the case at the same time, hence also that no sentence P can be true and false simultaneously.

(68) a. Not both: all men are mortal and not all men are mortal
b. Not both: snow is white and snow is not white
c. Not both: Adam begat Seth and Adam did not beget Seth
(Mates 1965 [1972]: 87-88)

[32] Also referred to as the Law of Noncontradiction or the Principle of Contradiction

A term-based version more faithful to the original phrasing of the principle, is provided by Rescher (1969: 149) and Geach (1972 [1980]: 74-75) and mentioned in Horn (1989: 20): $\neg\exists x\,(Px \wedge \neg Px)$, or its equivalent $\forall x\,\neg\,(Px \wedge \neg Px)$. Yet another version (Barnes 1969; Lear 1980; Horn 1989: 20) as a statement of second-order predicate logic, is: $(\forall P)\,(\forall x)\,\neg\,(Px \wedge \neg Px)$.

3.2.2 Contradiction, Sets and Predicates

According to George Boole, the Law of Contradiction and much of logic more generally are in an important respect about classes and set theory, a point which will be clarified and elaborated in this section. Boole considered this finding the core of his insights when he developed what he called his Logic of *Classes* and it points to an issue whose consideration will be helpful in discovering the nature of lexical items, including the predicates studied here, namely the precise nature of a *set*. Spending time on such a basic notion may sound like idle pedantry, but that is a mistaken impression for a number of reasons. First, as Russell wrote: "to distinguish this notion from all the notions to which it is allied, is one of the most difficult and important problems of mathematical philosophy." (Russell 1903 [1937]: 67) Mathematical philosophy not being our concern, some of those problems can be avoided, but some aspects of the definition of a set are nonetheless relevant. Second, a central element of the present study is a proposal about set demarcation as a *cognitive* procedure. In particular, a set-theoretic perspective on what occurs when a lexical item is dug up from our lexical competence and activated for use will be worked out (see esp. chapter 4). Consequently, it is important to be clear about a number of set-theoretical preliminaries. To that end, the next sections will be devoted to (i) what sets and their members are (3.2.2.1), and (ii) how the notion of a set enters into the natural language concept of a lexical predicate (3.2.2.2).

3.2.2.1 Aspects of Sets and their Members

A core property of sets or classes is that their existence is fundamentally cognitively mediated. To appreciate this point, consider the following situation. Imagine you want to travel abroad by plane and pass the customs officer who asks you to prove who you are. You cannot just tell him to have a good look at you, 'because this is who I am'. This direct, object-oriented, single-level approach will not wash. Instead, what you have to do is go about it in what looks like a roundabout way: you have to produce your identity card, in essence a description of the unique properties that characterize you. For the customs officer, you will have proved your identity if you are the unit set, i.c. the person, matching the description on that card. The description on the card is clearly not a person itself, it is an object at a different level than the object it serves to identify. It is the search procedure the customs officer uses to identify you, if all is well it is a series of conditions which singles you out as the sole element of the set it denotes.

Generally speaking, a set may be defined as (a) a description or concept and (b) a collection of all those and only those entities to which the description or concept applies. The collection is the "field of applicability of a concept" (Langer 1967: 116) known as the *extension* of that concept. The class-concept or "range of applicability of a concept" which is needed to determine the extension is sometimes referred to as the class in *intension*. Though one can rightly maintain that the substance of the class is its extension, it is nonetheless equally true that the latter owes its demarcation and hence its existence as a class to the intensional cognitive filter. The latter is called cognitive, because as is clear from the identity card example, the description is in terms of words and thoughts (and probably also a pictorial representation).

> "We cannot take classes in the *pure* extensional way as simply heaps or conglomerations. If we were to attempt to do that, we should find it impossible to understand how there can be such a class as the null-class, which has no members at all and cannot be regarded as a "heap"; we should also find it very hard to understand how it comes about that a class which has only one member is not identical with that one member. I do not mean to assert, or to deny, that there are such entities as "heaps". As a mathematical logician, I am not called upon to have an opinion on this point. All that I am maintaining is that, if there are such things as heaps, we cannot identify them with the classes composed of their constituents."
>
> (Russell 1919, [2000]: 183)

With the intension-extension pair in one's armoury, the concepts of a null-set and a unit set make sense: the null set is the class whose intensional range is defined in such a way that the field of applicability it selects is empty, nothing at all is left. Suppose the identity card above had stated that its bearer has red hair and is bald. These conditions being contradictory, not a single person can match the description and the extension set defined by the intensional description is consequently the null-set. The unit set, for its part, is the class whose intension yields a field of applicability consisting of a single element, as is normally the case with an ID.[33] Sets in this sense, this much is clear, are radically different from the "heaps" or conglomerations that collectors are interested in.[34]

Observe that the demarcation procedure sketched above is crucially a matter of *exclusion*: each of the descriptive statements (the intension) on the identity card (to continue with that example) excludes a large section of possible elements, which is why it is possible to end up with an empty set as the extension. The notion *exclusion* is a *negative* one, an insight which will prove highly relevant when we turn to a description of the intensional meaning of the logical operators. The extension itself, is what remains – if anything – after the intension has done its exclusionary work. The extension itself is consequently not itself negative.

An important feature of the information on the card which the customs officer uses to check my identity, is that it consists of a series of so-called *propositional (or 'characteristic') functions*. Eyes: blue; nationality: Belgian, etc. are taken by the customs officer to contain a substitutional variable: x has blue eyes; x is Belgian, etc. It is by

[33] The two-level nature of a set also explains why class-membership (the relation 'is an element of': $\in$) is different from the relation of part to whole, an insight due to the Italian mathematician and logician Giuseppe Peano (1858 - 1932). Part-whole is a single-level transitive relation: a piece of a wedge of cake is a piece of the cake. Not so for $\in$: "Socrates is a man, and men are a class, but not Socrates is a class" (Russell 1903, [1937]: 19).

[34] One would question the sanity of someone who stated that he wanted to become a collector of empty sets.

rotating the variable over its range, in this case by replacing the variable by me – in the form of a percept of me – that the officer determines whether I am indeed the unit set of elements whose substitution for the x's, invariably results in a true proposition. The reference to 'percept' in this description points at a problem with extensions. The 'real' extension 'an sich' is unattainable in principle (Kant (1787 [1933])) and can therefore only indirectly – if at all – be the extension the set is about. But even what the customs officer sees when he looks at me, the percept, is not the extension called up by the intensional description. The latter is rather itself still in the mind of the customs officer, which is why it gets the label IN(ternal/intensional)-extension in the scheme below[35]. It is when this IN-extension has the experiential property 'out-thereness'[36] expressing a match with a percept, that it is not felt as purely intensional[37]. This analysis creates a natural place for virtual, nonexisting intensional objects such as Pegasus or Sherlock Holmes or the nonexistent cat that I have just painted (cf. Seuren 2001a, 227). Though there is felt to be no EX-extension and hence no percept in those cases[38], there is still the feeling that when talking about Sherlock Holmes, we are not just referring to a name, but to an entity (person), even if in a fictitious referential framework. The notion of IN-extension provides the category that is needed to accommodate such virtual entities, which according to Meinong (1904) have "being" but not "existence". While we are confident that the difference between real and virtual entities is an ontological difference, the only way we have of being aware of that difference is by experiencing the former as having a reference value in the world as perceived, while the latter are felt to lack such a value. What is the case in the real world an sich is inaccessible.

(69)

DJ	Customs officer		
Real/actual EX-extension	Percept	IN-extension	Intension
Real or actual field of applicability	Perceptual representation/ experience of the actual field of applicability	Field of applicability called up by the set description	Range of applicability of the set description

3.2.2.2 Of Sets and Predicates

As is well-known from model-theoretic semantics, another name for a characteristic function is '*predicate'*. It is not surprising, therefore, that the abovementioned distinction extension-intension, which has a very long pedigree in the philosophical tradition, first

35 For the same conclusion, but for different reasons, cf. Seuren 2001, 227.

36 "That is, "out-thereness" is as much a mentally supplied attribute as, say, squareness." (Jackendoff 1983: 26)

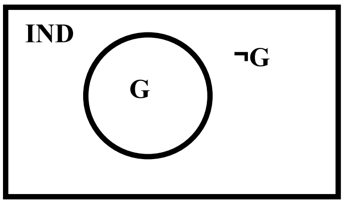
37 Pure "out-thereness" characterizes the monsters that haunt someone's dream, which are experienced as 'real' for as long as the dream lasts.

38 It is of course possible to erect a statue for the virtual entitity Sherlock Holmes, in which case the abstract virtual entity is given a material form. But even in that case, what is perceived as being there in EX-extension is experienced as a representation of a virtual person, not as an actual person.

arose in the context of *predicates* rather than in that of sets, which are a relatively recent invention. But the convergence is auspicious, in that the above description of the demarcation of sets carries over seamlessly to predicates and the way they '(s)elect'[39] the individuals, relations and situations in the real world that they are about. This connection, which ties the set-theoretic notions elaborated above to natural language predicates, is the topic of this section. It brings the notion of a set and the undeniably linguistic-cognitive notion of a predicate together, thus paving the way for a fully elaborated IN-language perspective on set demarcation, the Law of Contradiction (LC and contradictoriness.

Adoption of John Venn's (1834-1923) set-diagrams leads to the following representation of how a *predicate* G "(s)elects" a set of individuals, thus dividing a universe of discourse in two, into what is G and what is not G. Suppose G = *Greenlander*; then the universe of possible individuals IND - divides into two classes: the set of individuals who are Greenlanders **G** and its complement set $\overline{G}$ (here equivalently represented as **¬G** (= not-G)), the set of individuals who are not Greenlanders:

(70)



These two sets are complementary: they have no members in common – their intersection is the empty set ø: $G \cap \neg G = \varnothing$ –, but divide the universe between them – their union is the universe class IND: $G \cup \neg G = IND$. And this is where the Law of Contradiction kicks in again. The property of having no members in common is none other than the set-theoretic version of the term-based version of LC, $\neg\exists x\ (Gx \wedge \neg Gx)$: there is no individual who is both a Greenlander and a non-Greenlander. The opposition between the two classes G and ¬G is one of *quality*: the members of G all have the quality *Greenlander*; the members of ¬G lack it. From this perspective, what LC expresses is that an individual cannot both have a quality and not have it at the same time. The function of each predicate (hence, as I would claim, each lexical item) is thus to regulate set membership by isolating members from nonmembers; and what LC adds to that is the stipulation that there can be no individual that straddles the fence by belonging both to G and to ¬G. Each actual individual i_a either is an element of the class of individuals described by the predicate G, in which case i_a has value 1 for G: it is a Greenlander; if i_a is not an element of G and is consequently a member of ¬G, it has value 0 for G, it is not a Greenlander.

To arrive at a precise definition of the class ¬G, Augustus de Morgan (1847) added a distinction between the total universe of individuals IND and a subset thereof, the *universe of discourse*. Kneale & Kneale (1962: 408) describe the latter as follows: "not the totality of all conceivable objects of any kind whatsoever, but rather the whole of

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[39] This refers to Boole's (1847) use of the term "elective symbol" for this operation.

some definite category of things which are under discussion.” From a lexical perspective, this distinction is motivated by the insight that it is unrealistic to assume that the substitutional variable of a characteristic function *Greenlander (x)* rotates over all possible individuals of the entire universe IND of individuals (including such individuals as *shoe, painting, potty…*). A more realistic conception takes the function to rotate over a designated subset of IND. But then the question arises how the nature of that subset is determined in given cases and how that knowledge is represented. A first part of the answer to these questions is that the notion *universe of discourse* has to be stripped of its association with discourse and that which is “under discussion”. Instead, it should be tied to lexical knowledge, more specifically to lexical presuppositions.

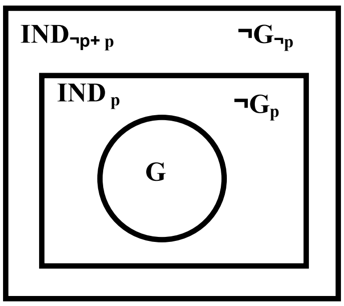
To see this, consider the following sentences:

(71) a. x is a Greenlander
b. x is not a Greenlander

Our expectation with respect to (71) is that if x is not a Greenlander, s/he may be a Belgian or an Englishman, etc. but normally not a *shoe.* This is captured if it is assumed that the lexical item Greenlander carries the lexical presupposition that its subject term is human. As is well-known, presuppositions remain constant under negation[40], so that not just (71)a, but also (71)b presupposes that x is human. Put differently, one of the *selection restrictions* of *Greenlander* is that its argument is +HUMAN. It is this feature of the predicate *Greenlander* which determines the limited universe over whose individuals the substitutional variable of the characteristic function *Greenlander (x)* rotates. In other words, selection restrictions are responsible for the fact that what De Morgan calls the universe of discourse is a subset of the entire universe of individuals IND, in this case the set characterized by the propositional function *human (x)*.

On the whole, this means that in the *Greenlander*-case $\neg G$ (= $\overline{G}$, the complement of G; or in Boolean algebra 1-G, where 1 stands for the universe), has two parts: the presuppositional inner complement of non-Greenlandic humans $\neg G_p$ (= De Morgan’s universe of discourse), and the non-presuppositional outer complement $\neg G_{\neg p}$, comprising the set of individuals which are neither Greenlandic, nor even human.[41]

(72)



[40] At least under “non-metalinguistic” (Horn 1989) or “minimal” (Seuren 2002) negation.
[41] This analysis is a transfer of Seuren’s valuation space interpretation of negation from propositions/situations to predicates (Seuren 2002: 30).

The role of the non-presuppositional outer complement shows in sentences involving the cancellation of a well-known presuppositional category. Take Russell's well-known example:

(73) The present king of France is not bald

This sentence exemplifies an existential presupposition[42] (*there is a presnt king of France*) due to the lexical predicate *bald* and the most straightforward interpretation is one that preserves that presupposition, hence which locates the present king of France in the presuppositional inner complement of *bald*. But sentence (74) illustrates that the present king of France being or not being a member of the set denoted by *bald* does not exhaust the set of possible situations compatible with the sentence *The present king of France is not bald*. It is also possible that there simply is no king of France, which would locate him in the non-presuppositional outer complement of *bald*. This is evidenced by the coherence of asserting the second part of the sentence alongside the first.

(74) The present king of France is NOT bald – there is no king of France

Analogously, if one said: *my shoe is NOT a Greenlander,* that would only make sense as an instance of the radical kind of falsity illustrated in (74) and referred to as presupposition-cancelling by Seuren (2001b: 338). Horn (1989) analyses these examples as cases of metalinguistic negation.

Note that our leading assumption is that the set-theoretic properties involved are associated with the lexical items of natural language as selection restrictions and are hence part of their IN-meaning. That this is the proper perspective is not hard to prove: with a designated change of lexical predicate, the presupposition of existence disappears, which proves the link lexical item-presupposition. Witness the following sentence pair:

(75) a. Unicorns aren't bald
b. Unicorns do not exist

While one of the readings of (75) a. presupposes existence of the subject *unicorns* due to the selection restrictions of *bald*, replacing the latter predicate by *exist* has the consequence that the presupposition of existence is no longer available. (The fact that *exist* does not presuppose existence is not strange: if existence were already presupposed, there could be no point in asserting it anymore.)

Since under the present conception presuppositions are generated by a *predicate's* preconditions, e.g. the semantic predicate *bald*, or the abovementioned *Greenlander*, it is preferable to define the inner complement $\neg G_p$ and the outer complement $\neg G_{\neg p}$ on the latter rather than on propositions. This decision reflects our general strategy to remove from syntactic levels properties which are more likely tied to lexical items. Their effect will automatically percolate to the propositions composed from them anyway. And from

[42] That *the present king of France isn't bald* has an existential presupposition is maintained by Frege and Strawson, but not by Russell himself.

an internalist viewpoint too, it is better to encode presuppositions as part of the lexical knowledge stored for a predicate in long-term memory than as features of online constructed propositions/sentences only.

3.2.3 From predicates to propositions

A set-theoretic approach is not only possible for sets of individuals in IND elected by a term predicate, but has further relevance for natural language: it is also applicable to the sets of situations in SIT in which a given proposition is true, a perspective that is the focus of this section.

(76)

Language	Set-theory
term predicate	sets of individuals in IND
proposition	sets of situations in SIT

Take the proposition P: *John is in the garden.* Analogous to the manner in which the predicate *king* (s)elects a set of individuals, this proposition does not directly present, but rather represents or describes a set of possible situations as its IN-extension[43]. If the actual situation is an element of that set because it matches the description given, the sentence is true, i.e. an accurate description of the actual situation in the outside world I wanted to refer to. The IN-extension of P, the set of possible situations in which P is true, is called the *valuation space* of P, represented as /P/ (Van Fraassen 1971, Seuren 1998: 331, Seuren et al. 2001).

(77) $||P|| = /P/$

The main difference with our identity card is that while a good identity card is designed to single out a person as the extension of its description, i.e. is a description of a unit set of individuals, sentences single out situations as their extension, mostly a larger set than the unit set. There are many Johns and many gardens in the world, and the description is equally correct for all of them if the containment relation expressed by *in* applies to them in the right way. This is why the notion 'element of' is crucially required for the calculus of truth: if the actual situation is an element of the class of possible situations in which the proposition is true, the sentence is true; if not, it is false.

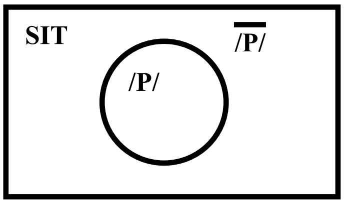
(78) a. When P is true, the actual situation $s_a \in /P/$

b. When P is false, the actual situation $s_a \notin /P/$, i.e. $s_a \in \overline{/P/}$, the complement of /P/

[43] The Fregean assumption that the extension of a proposition is a truth value is not followed. Note that the Fregean conception drives a strange wedge between extensions in the universe IND of individuals and the universe SIT of situations anyway. If the IN-extension of *book* is a set of possible books and not a reference value, then the IN-extension of a phrase *two books* is most naturally characterized as a set of possible pairs of books, and the IN-extension of the still larger constituent P: *John bought a book* as a set of possible P-situations, rather than as a truth value.

Here too, it is possible to represent by means of Venn diagrams how the sentence P "(s)elects" a set of situations as its IN-extension, thus dividing the universe of discourse – here the universe SIT of all possible situations – in two, into what is /P/ and what is /not-P/. Suppose /P/ = */John is in the garden/*; then the universe SIT of all possible situations - divides into two classes: /P/, the set of situations where P is true; and /¬P/ (= /not-P/), the set of situations in which ¬P is true, which equals /$\overline{P}$/, the set of situations where P is false, i.e. the complement of /P/ in SIT:

(79)



In this case, the property of having no members in common (the intersection is the empty set ø: /P/ ∩ /$\overline{P}$/= ø) represents the *propositional* version of the Law of Contradiction: ¬ (p ∧ ¬p).
The function of propositions is consequently analogous to that of each predicate, viz. to regulate set membership by isolating member situations from nonmember-situations. Each actual situation s_a either is an element of the class of situations described by the proposition P, in which case s_a has truth value 1 for P; if s_a is not an element of /P/ and is consequently a member of /P/ , it has truth value 0 for the proposition P, it is not a situation belonging to */John was in the garden/.*

3.2.4 A set-theoretic definition of contradictoriness

Having availed ourselves of Boole's and Van Fraassen's mode of representing sentences or propositions and the situations they denote in set-theoretic terms, that tool can now be applied to the types of sentences which are the real focus of our interest and are repeated here[44].

(80)

Entailment	Contradictoriness
P: *All* flags are green Q: *Some* flags are green	Q: *Some* flags are green R: *No* (= NOT-SOME) flags are green

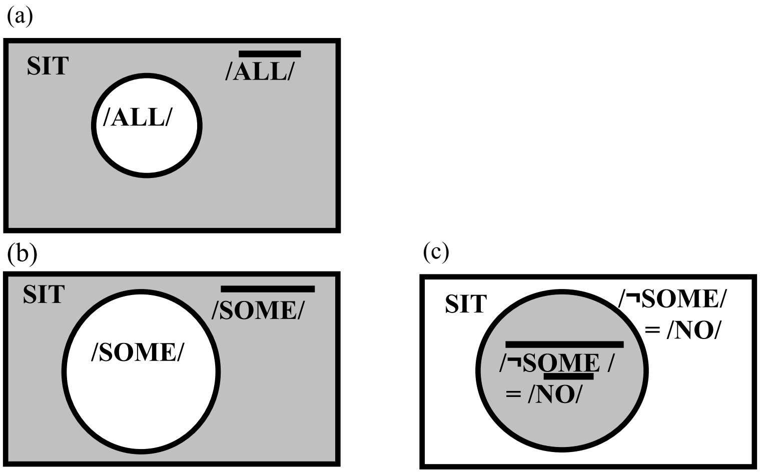
To represent an individual proposition such as P: *All flags are green*, we can simply adopt the notation /P/ to refer to its valuation space and then just set up a scheme as in (79). Similar valuation schemes can be set up for Q and R. Going one step further, nothing now prevents us from generalizing over all sentences with the same logical

[44] The section that follows is based on Seuren et al. (2001) up to the extension of the analysis to the propositional calculus.

affirmative-universal format as P and set up a valuation space representation with predicate letters instead of *flags* and *green*, i.e. for *all F is G*. The same can be done for Q: *some F is G* and R: *no F is G.* To do this, the same abbreviations will be used which were introduced earlier, but this time enriched with the slashes indicating that what is intended is valuation spaces, i.e. sets of all possible situations in which the proposition (type) in question is true.

(81)[45]

/All F is G/	is written as	/ALL/
/Some F is G/	is written as	/SOME/
/No (=not-some) F is G/	is written as	/NO/



Note that SIT is not to be interpreted as the total universe of situations, but only those where F and G are defined. In other words, /ALL/ is defined in terms of parameters[46].
Valuation diagram (81) (a) subdivides the universe SIT into /ALL/, the set of situations where a sentence *All F is G* (whatever the concrete content of F and of G) is true – and its complement /ALL/ , the set of situations in which *All F is G* is false.
Diagram (81) (b) does the same for /SOME/ and its complement /SOME/. For any sentences X and Y, the relation CD, i.e. X contradicts Y or X is a contradictory of Y, can therefore be defined as in Seuren (2002: 27):

[45] As before, the bars over the labels in the shaded areas represent the complement-set-symbol.
[46] So F and G are not predicate variables, since that would in all cases yield trivial sets, namely:

(a) $\exists x\ \exists y,\ \forall z,\ z \in X \rightarrow z \in Y$
(b) $\forall x\ \forall y,\ \forall z,\ z \in X \rightarrow z \in Y$
(c) $\forall x\ \exists y,\ \forall z,\ z \in X \rightarrow z \in Y$
(d) $\exists x\ \forall y,\ \forall z,\ z \in X \rightarrow z \in Y$

(82) Contradictoriness or CD:

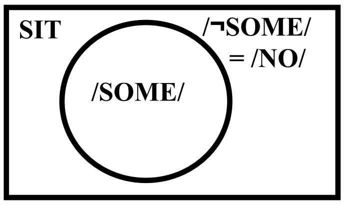
For all sentences X and Y:

CD(X,Y) iff /X/ is the complement of /Y/, or: $/X/ = \overline{/Y/}$

Since the set of situations in which *Some F is G* is false is the same as the set of situations in which *No (= not-some) F is G* is true, (b) and (c) can be integrated. Thus, /NO/ being the contradictor (= NOT-SOME) of /SOME/, (b) and (c) can be collapses and the shaded complement-backgrounds can be dropped. Each of the spaces in the diagram is now stated as the set of situations in which a proposition (type) is true, not in terms of being a space that is a complement of something else: the inner space is the area containing precisely the situations where /SOME/ is true, the outer space the area containing precisely the situations where a sentence /NO/ (= NOT-SOME) is true

(83)

(b) and (c)



The same valuation space analysis extends to the propositional calculus in view of the proposition types and relations below.

(84)

Entailment	Contradictoriness
$P \wedge Q$: John is in the garden *and* Peter is in the garden	$P \vee Q$: John is in the garden *or* Peter is in the garden
$P \vee Q$: John is in the garden *or* Peter is in the garden	$\neg(P \vee Q)$: Neither John is in the garden, *nor* Peter is in the garden

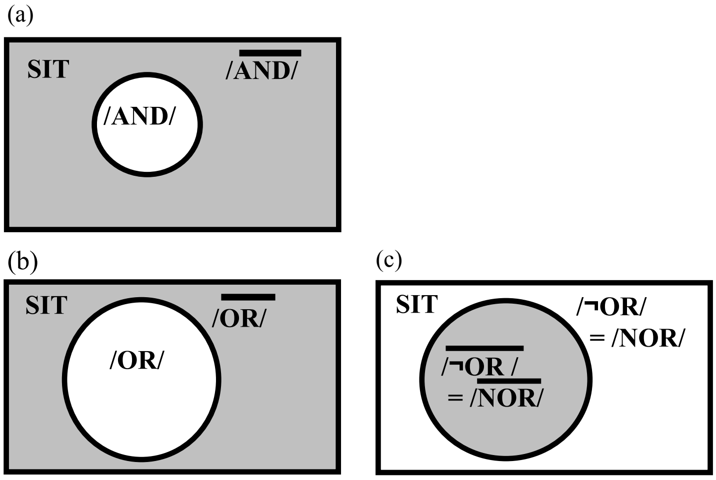
Enriching the abbreviations introduced earlier with a valuation space interpretation by means of the formal slashes-device and taking P and Q as variables rather than the concrete sentences of (84), we get:

(85)

$/P \wedge Q/$	is written as	/AND/
$/P \vee Q/$	is written as	/OR/
$/\neg(P \vee Q)/$	is written as	/NOR/

yielding the following diagrams:

(86)

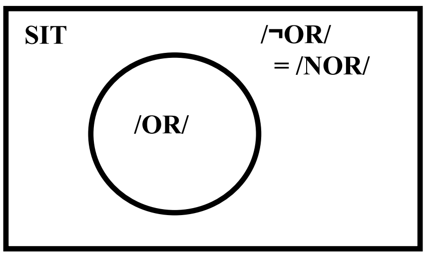


Here (b) and (c) give us a clear indication of the existence of a relation between operations and morphemic realization: /¬OR/ is configurationally more complex than its contradictory /OR/, which is matched by bimorphemic vs. monomorphemic lexicalisation on the morphological side (Löbner 1990: 95). The fact that language occasionally wears semantic complexity on its morphological sleeves like this bodes well for our hope to solve the lexicalisation puzzle why **nand* (and **nall*) is impossible. If indeed there exists a link between underlying semantic structure and material morphological realization as suggested by (b) and (c), looking for an abstract logical or semantic solution to the material absence of a particular conceivable lexicalisation is a wise road to take. The isomorphism of the logical structure, meaning and morphology of operators postulated in this study has the advantage that the three can serve as mutual correctives and checks.

And once again (b) and (c) can be integrated to yield a valuation space diagram with only areas defined positively in terms of truth, not falsity. /¬OR/ (= /NOR/) comes out of this as the contradictor of /OR/ (and vice versa), as required:

(87)

(b) and (c)



Having arrived at an adequate representation of CD in terms of valuation space diagrams and an integration of (b) and (c), we conclude this section with a remaining question: is it possible to relate the (a) and (b) diagrams of (81) and (85) and the sets of situations contained in them as well? The answer is yes, but that brings us to the section on the second relation on which logic revolves: entailment.

3.3 Entailment

This section is devoted to the second basic relation of the calculus alongside CD, namely entailment. First of all, in 3.3.1. the existing notion will be illustrated and a brief overview will be given of some definitions proposed in the literature. In 3.3.2., the concept will be looked at from the perspective of Van Fraassen's (1971) and Seuren's (1998: 331, Seuren et al. 2001) valuation space analysis of predicate logical operators and an extension of their proposals to the propositional calculus will be proposed. Third, in 3.3.3. entailment as a set-theoretic property will be looked at from an internalist cognitive perspective, tying it in with the internalist view on set demarcation and CD developed above. First, it will be illustrated that it is a relation which does not only affect propositions with logical constants (which could in principle still be maintained to constitute an EX-logic) but propositions with more ordinary lexical items from the natural language lexicon as well. This I take to confirm the general idea that in the study of natural language and its natural logic, logical constants are to be treated as regular lexical items like the rest of the natural language lexicon. The fact that entailment extends to the latter is then not unexpected. A second internalism issue – to be treated in 3.3.4. – concerns the concept of entailment itself, namely the existence of "unnatural" entailment relations such as (88), which the standard definition cannot exclude:

(88) a. P: Some rhinoceros is not a rhinoceros entails
b. Q: Some senile professor has pink stockings

P being a necessary falsehood (contradiction), it entails any sentence (e.g. Q), since whenever P is true, Q is also true. This leniency of standard entailment is felt as a defect from the viewpoint of cognitive reality and natural language intuitions, which brand the entailments in question as unnatural. So, somehow the ill has to be cured in IN-logic. Evidence will indicate that the solution is to be sought in an informativeness constraint on natural language.

3.3.1 Definitions of entailment

Contradictoriness (CD) as defined above is a matter of set demarcation. It represents Aristotle's *fundamentum divisionis*, dividing a universe into what is /G/ and what is /not-G/, thus delineating a set.[47] The second leg of the twin foundations of logic, the relation known as *entailment*, does not involve one set, but two (Zwarts 1986).

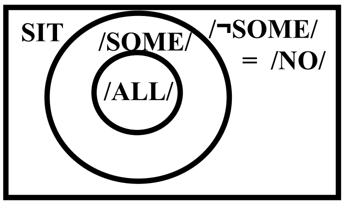
The core example of entailment introduced before was the following pair:

[47] CD and entailment being defined as relations between sentences/propositions, we keep working with valuation spaces.

(89) a. P: *All* flags are green
b. Q: *Some* flags are green

Leaving aside the cases relevant to the existential import problem for the time being (i.e. the empty F-class case to be dealt with in chapter 4), the first logically entails the second: whenever P is true, Q must of necessity also be true. In set-theoretic terms, this can only mean that the valuation space /P/, which equals /ALL/ (cf. (81) above) is a subset of /Q/, i.e. /SOME/. This yields the following diagram, which shows very clearly that logical entailment is a *quantitative* relation between two sets:

(90)



This is a variant of the diagram in Seuren (2002: 26), but there are a number of differences in perspective: on the basis of the architecture of the Cartesian Coordinate System outlined earlier and the operations perspective there is a logically hierarchical, procedural structure to the diagram proposed here: its outlook is the result of applying operations to the basic pivot which generate the relation CD between pivot and contradictor and entailment from entailer to pivot at the propositional level. That stratified conception, with set demarcation of the valuation space of /SOME/ conceived of as cognitively less complex than that of its contradictory and entailer, is the new perspective suggested by our valuation space version of Löbner's asymmetry hypothesis. A further novelty is that I take this diagram to define the relationships on the logical operators qua *lexical items*, i.e. on what will be defined as lexical *proposition types* with parameters F, G rather than on propositions with lexically filled argument positions, such as *all flags are green*. Specifically, the lexical items in question will take the form of two-place higher order predicates, i.e. as predicates over pairs of sets (here F and G), which is the standard Generalized Quantifier Theory (GQT) conception (cf. Barwise & Cooper 1981, Keenan and Stavi 1986, Zwarts 1983, Van Benthem 1986). A final difference with Seuren (2002) is that the approach will be proved to extend to the logical constants of the proposition calculus.

In set-theoretic terms, the quantity relation expressed by entailment is set *inclusion* (/ALL/ $\subseteq$ /SOME/). What it does, is regulate inferential transitions between sets and subsets (Ladusaw 1980, Zwarts 1986). In this respect a caveat is in order. Sets are complex constructs, i.e. intension-extension pairings, and not non-cognitive purely external physical things. Consequently, set-inclusion "does not mean just the same thing as "inclusion" of (say) one box in another" (Langer 1967: 135). Rather, it is a relation between two cases of set-membership, and its definition is therefore in terms of the IN-extensional elements of the two sets:

(91) a. $/P/ \subseteq /Q/ \equiv_{def} \forall x: x \in /P/ \rightarrow x \in /Q/$,
or stated more generally with variables instead of valuation spaces:
b. $X \subseteq Y \equiv_{def} \forall x: x \in X \rightarrow x \in Y$

Set X is included in set Y (or: set X is a subset of set Y) is equivalent by definition to the proposition that all members of set X are also members of set Y.

Some alternative names for entailment have been in use (*necessitation*, for instance), and the concept has over the years been expressed with other formal notation symbols than those given above. A staple:

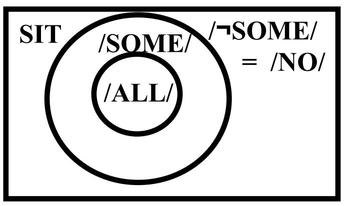
(92) a. A necessitates B if and only if whenever [in any situation or possible world in which] A is true, B is also true. (Van Fraassen 1968: 138)
b. $(p \Rightarrow q) \equiv \sim$ poss $(p \ \& \sim q)$
"if p entails q, then it is not logically possible for both *p* to be true and not-*q* to be true and conversely" (Lyons 1977a: 165)

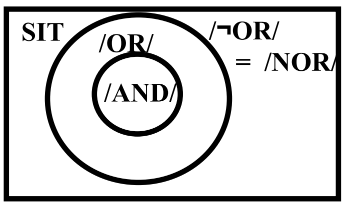
These definitions are, however, not fundamentally different and a definition in set-theoretic terms seems an adequate formal statement.

3.3.2 Valuation spaces of logical operators and entailment

Entailment was visualised earlier by means of a valuation space diagram for predicate calculus operators – cf. (93) a. The analysis will now be extended in that the same pattern will be shown to characterize the valuation space diagrams of propositional operators, as in (93) b.

(93)

a. 

b. 

That (93) is an accurate representation of the valuation spaces of the propositional operators *and* ($\wedge$)*, or* ($\vee$) and *n-or* ($\neg\vee$), can be proved in terms of the truth-functional properties of the operators in question. These properties are well-understood and have not changed (notational devices aside) since they were first invented twenty-three centuries ago[48]:

[48] Polish notation is used, with the operators at the beginning. This will make it easier to conflate the three columns below.

(94)

	$\wedge$ (P Q)	$\vee$ (P Q)	$\neg$ $\vee$ (P Q)
(a)	**1** 1 1	**1** 1 1	**0** 1 1 1
(b)	**0** 1 0	**1** 1 0	**0** 1 1 0
(c)	**0** 0 1	**1** 0 1	**0** 1 0 1
(d)	**0** 0 0	**0** 0 0	**1** 0 0 0

To bring the tables and the set-inclusions obtaining between the different columns closer to the valuation space diagram (93), we modify representation (94). To begin with, observe that in (94) the possible situations have been repeated three times in the truth tables given. On the assumption that they form a single paradigm, a more economical, single-column representation can be set up, from which the repetitiveness is removed. In that representation, all shaded cells, whose truth value is invariably **0**, have to be gone. The reason is that we are setting up a valuation space diagram, which by definition contains nothing but situations with value **1**, i.e. situations in which given expressions are *true* (the IN-extension or valuation space of P is the set of possible situations in which P is true). This leaves two further steps to be taken, namely the integration of the white parts of the leftmost and the rightmost column into the middle column. The truth value of these white parts is invariably 1, so they cannot be left out from the diagram. As far as the rightmost cell is concerned, the operation is very simple. With the shaded cell of the middle colum removed, the white *nor*-area $\neg$(P$\vee$ Q) can be straightforwardly mapped onto the emptied area of the middle column (= 95 a). The final step consists in superimposing the white area of P $\wedge$ Q onto that of P $\vee$ Q, as indicated in (95 b).

(95)

b

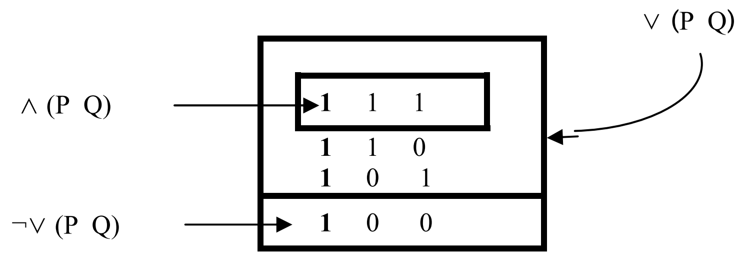
$\wedge$ (P Q)	$\vee$ (P Q)	$\neg$ $\vee$ (P Q)
1 1 1	**1** 1 1	**0** 1 1 1
0 1 0	**1** 1 0	**0** 1 1 0
0 0 1	**1** 0 1	**0** 1 0 1
0 0 0	**0** 0 0	**1** 0 0 0

a

Taking away the heading row of the representation and representing the meaning of the operators by means of the white areas only, we get (96) [49]

[49] Note that $\neg$ $\vee$ is taken together as a single operator in (96), so that of the third subtable in (95) – the one for nor ($\neg$ $\vee$) – only the first column of values (in boldface) is relevant for the construction of (96), not the one under the disjunction symbol $\vee$.

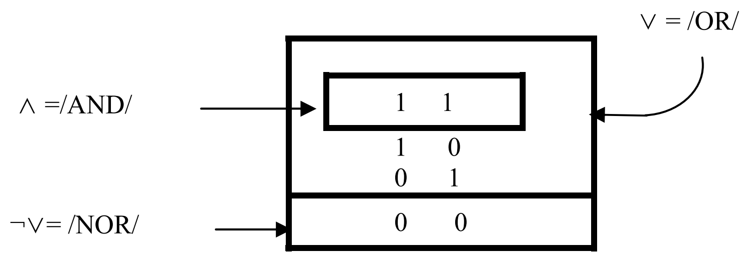
(96)



Observe that step b in (95) was different from step a in that it resulted in overlap. As (96) shows, the P ∧ Q rectangle indeed blocks off the inclusive part of the P ∨ Q placemat. Yet that effect of step b is auspicious. The resulting overlay squares well with the fact that there are two ways to view the meaning of disjunction ∨: there is the inclusive interpretation (P or Q or both) which you literally get underlyingly, namely when the small overlay of conjunction is ignored. And then there is the exclusive interpretation (P or Q, but not both), which is what results when the overlay conjunction blocks off the 'both' part of inclusive ∨, leaving only the exclusive part of the disjunction accessible.

The presence of two arguments P and Q which *and, or* and *nor* invariably require is already evident from the two rightmost columns of truth values in the table itself, so that the paradigm can be further simplified. The boldface values 1 can be left out too, since a valuation diagram by definition contains only situations for which given expressions are true, i.e. have value 1. Repetition of the boldface values is therefore unnecessary. Adding the usual valuation space slashes and interpretation, we get:

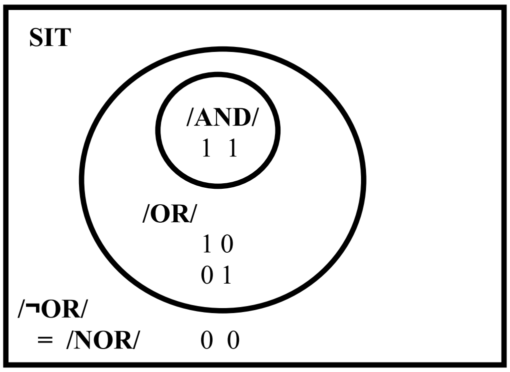
(97)



The result is now a kind of "placemat" arrangement, a set-theoretic construct with rectangular Venn-diagrams, which shows that the IN-extension set of /AND/ is entirely included in that of /OR/ (meaning that /AND/ entails /OR/) – it lies on top of /OR/ and covers part of the placemat of /OR/.

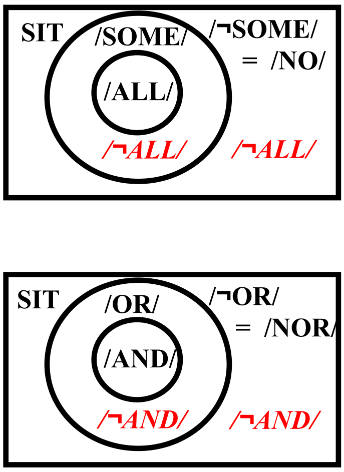
Conflating the truth table representation and the valuation space diagram is now easily done.

(98)



Given the definition of CD, is is possible to add two more labels in the two diagrams, namely those of the categories **/NALL/* and **/NAND/*, which do not exist as lexical labels:

(99)

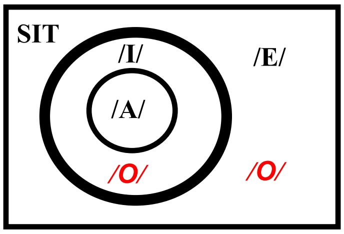


If the I-corner spaces /SOME/ and /OR/ are indeed pivotal, then these diagrams suggest two possible ways to look at the absence of **nall* and **nand* in natural language.

One way is to assume with Löbner (1990: 106) that the O-corner item (his "Typ 4") can only be reached in two procedural steps ("zwei ableitungsschritte") away from the pivot ("Typ 1"), namely via /AND/ (c.q. /ALL/) and then to /NAND/ (c.q. /NALL/). Entailers (/AND/, /ALL/) and contradictors (/NOR/, /NONE/), for their part, are only one step away from the pivot. This makes Typ 4 the most marked option ("am markiertesten").

The second way to address the O-corner problem accepts the two-step hypothesis, but claims that the second step is not just marked, but impossible. This is due to the way in which the pivot is basic in the system: specifically, its meaning will be argued to be a *presuppositional* subpart of the meaning of the A-corner entailer. The effect is that the boundary between the pivot (/OR/, /SOME/) and its E-corner contradictor (/NOR/, /NONE/) – bold in Venn-diagram (100) – remains an inviolable substrate, whatever operator one tries to compose.

(100)



/I/ = pivot (/OR/, /SOME/)
/A/ = entailer (/AND/, /ALL/)
/E/ = contradictor /NOR/, /NONE/
/O/= /NAND/, /NALL/

If so, O-corner items, which denote the complement of /A/ and hence break through the /I/-/E/ boundary, are not just marked, but impossible as natural lexical items. It is the latter conception that I have been led to by the facts that will be detailed in the rest of this study[50].

3.3.3 Natural entailment

Our interest is in natural language and hence also in a natural rather than EX-logical conception of entailment. On the one hand, this means that the definition given so far will have to be *constrained* to bar entailments which from an ordinary language viewpoint are considered unnatural, such as P ├ P, e.g. *from Mary is ill it follows that Mary is ill*. This issue will be addressed in 3.3.4. From a different perspective, however, the definition given so far is too *narrow* and will have to be *relaxed*. That is what will be done in the present section.

3.3.3.1 Logical entailment and semantic entailment

The pair of sentences *All flags are green – some flags are green* involves propositions with logical constants (*all, some*). The distinguishing feature of propositions with logical constants is that a formal calculus is available for them, allowing for the computational derivation of the entailment relations. The type of entailment involved is therefore called *logical entailment* and was defined above as a set-inclusion relationship between valuation spaces (see (91)):

(101) Logical entailment

For all sentences X and Y, X ├ Y iff $/X/ \subseteq /Y/$

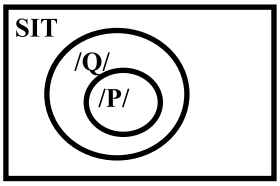
[50] The impossibility of O-corner items – though not in terms of the presupposition hypothesis – is also defended by Barwise and Cooper (1981).

But *logical entailment* is not all there is to entailment in natural language. Entailment also affects propositions with ordinary lexical items other than logical constants, e.g. *father* and *male* in the following pair of sentences:

(102) a. P: John is a father
b. Q: John a male

P entails Q in the sense that whenever P is true in a situation, Q is also true in that situation. To represent this formally, valuation space modelling in the universe of possible situations SIT can once again be used, with /P/ the set of possible situations in which the proposition *John is a father* is true, and /Q/ the set of possible situations in which the proposition *John is a male* is true. Venn diagram (103) adequately represents the inclusion relation, since there are Johns who have no children, for whom assertion Q is true, but P is not.

(103)



I adopt Seuren's (1998: 302) term *semantic entailment* for these entailment relations "based on one's intuitive understanding of the meanings concerned" only, thus keeping them terminologically separate from logical entailment in "a formal calculus enabling one to derive the entailment automatically in virtue of the logical constants and the structure of the entailing statement or statements" (Seuren 1998: 302).

Semantic entailment can easily be defined in set-theoretic terms and valuation spaces since aside from the extension from logical constants to other vocabulary nothing changes:

(104) Semantic entailment[51]
For all sentences X and Y, $X \models Y$ iff $/X/ \subseteq /Y/$

[51] The systematicity of extra-logical semantic set-inclusion relationships and hierarchies has been recognized ever since the birth of logic. It is also central to the discussion of monotonicity in the medieval part of logic called *topics,* including such topics as 'what is predicated of the species is also predicated of the genus', which 'can explain the validity of the inference *a man walked* ergo *an animal walked'*. (Sánchez Valencia 1994: 2). What is involved here is clearly an entailment relation induced by extra-logical vocabulary (*man, animal*): the set of possible situations in which A:*a man walked* is true is a subset of the set of possible situations in which B: *an animal walked* is, hence A semantically entails B.
For a more recent plea to broaden the discussion from logical particles to extra-logical vocabulary, cf. Katz (1972: 186).

The symbol used in definition (104) ($\models$) is different from the logical entailment-symbol ($\vdash$) used earlier in the text. This is to mark that semantic entailment ($\models$) is a more general notion than its subspecies logical entailment ($\vdash$).

3.3.3.2 Lexical "entailment" between predicates

As stated, in (102) the valuation space /P: John is a father/ is a subset of /Q: John is a male/. This set-inclusion at the level of the valuation spaces of propositions correlates with a set inclusion relationship between the IN-extensions of the lexical items *father* and *male.* Thus, the set of possible individuals denoted by the "superordinate" lexical predicate *father* (= "MALE PARENT") is a subset of the set of individuals denoted by the "subordinate" lexical predicate *male* (the terms are taken from Katz 1972: 192). This pattern indicates that in many cases the source of semantic entailment at the propositional level is a corresponding meaning inclusion relationship at the lexical level. In the case at hand, the lexical item *father* is conceptually more complex (a conjunction of MALE and PARENT), hence imposes more conditions on possible individuals than *male*, and hence its extension set is a subset of that of *male.*

Since set-inclusion works no different in the universe of individuals IND than in the universe of situations SIT[52], I see no principled reason not to try to extend the concept of entailment to set inclusion at predicate level, starting from the following concept of "IN-extension inclusion"[53].

(105) IN-extension inclusion
For all predicates X and Y, $X \models Y$ iff
the IN-extension of X $\subseteq$ the IN-extension of Y

The term "IN-extension" is used because valuation space analysis is not available for predicates[54]. But let us try to find a solution to the problem. Valuation space modelling

[52] Take A to be the set of inhabitants of Antwerp: $\{x:A(x)\}$ and B the set of inhabitants of Belgium: $\{x:B(x)\}$. Since all inhabitants of Antwerp are inhabitants of Belgium, but not vice versa, the relation between the two sets is one of inclusion. Each actual individual i_a either is an element of the class of individuals described by the predicate A, in which case i_a has value 1 for the function A: it is a person living in Antwerp; if i_a is not an element of A and is consequently a member of $\neg$A, it has value 0 for A, it is not a person living in Antwerp. Second, each actual individual i_a either is an element of the class of individuals described by the predicate B, in which case i_a has value 1 for B: it is person living in Belgium; if i_a is not an element of B and is consequently a member of $\neg$B, it has value 0 for B, it is not a person living in Antwerp. The result is exactly the same kind of diagram as those representing set-inclusion/entailment relations between propositions with logical constants in SIT.

[53] The problem with Katz's (1972: 192) term "meaning inclusion" is that not all cases of lexical entailment are really due to meaning inclusion. *Wednesday* lexically entails *weekday* (*If it's Wednesday it's a weekday (but not the other way round)*), yet there is no meaning inclusion: the concept WEDNESDAY does not include a subconcept WEEKDAY. The cases that interest us in this study are however all cases where I argue there *is* meaning inclusion, so the Katzian choice of terminology would be harmless.

[54] The same problem arises if one wants to define the CD relation of (82) on lexical items rather than full propositions. Though the attempted transfer from the propositional level to the lexicon is not crucial to make the decompositions of chapter 4 go through, it is briefly taken up here, mainly

was introduced by van Fraassen (1971), Seuren (1998: 331), Seuren et al. (2001) as a theory for propositional languages and possible situations in the universe SIT. Attempts to transfer it to lexical predicates, for instance the lexical item *bald*, face the difficulty that these are propositional functions of type <e, t>, which cannot get a valuation space analysis, since the latter is about the set of possible situations in which a proposition is true. Since the function involved contains a variable of type e, however, it still has to rotate over the range of the variable. Only after replacement of the variable by the name of a member of its range is there a proposition for which the set of possible situations in which it is true can be determined. A possible avenue would be to consider the valuation space of the predicate *bald* to be the union of all sets of possible situations in which a proposition with *bald* is true when the variable is replaced by the name of a member of its range. This is a rather convoluted way of putting it however. Instead of multiple substitution and taking the union of the resulting spaces for each substitution, a proposal which has the same effect is to assume that the lexical representation of *bald* does not contain a variable of type e, but rather a generalized quantifier[55]. It is a variable of this type (<e,t>,t>) which is required to make a valuation space analysis possible. The question then is which generalized quantifier? Montague employed numerically subscripted pronouns he_0, he_1, … for this purpose, but it seems much more logical to choose an indefinite pronoun, namely *(at least) one*$_{<<e,t>,t>}$ (not accidentally the *pivot* of IN-logic). Depending on the selection restrictions imposed by a given predicate (+PERSON or + PHYSICAL OBJECT, for instance), the meaning of the generalized quantifier amounts to AT LEAST ONE PERSON or AT LEAST ONE THING. This yields the following representations for *bald*:

(106) a. BALD as a concept: $\lambda \boldsymbol{x}[\mathbf{BALD}(\mathbf{x})]$ type <e,t>
$\lambda \boldsymbol{x}_{<e>}[\mathbf{BALD}(\mathbf{x})]$ type <e,t>
b. *Bald* as a word: $\lambda \boldsymbol{Q}[\boldsymbol{Q}(\underline{\mathbf{BALD}})]$ type <<e,t>,t>, t>
$\lambda \boldsymbol{Q}_{<<e,t>,t>}[\boldsymbol{Q}_{<<e,t>,t>}(\underline{\mathbf{BALD}})]$ type <<e,t>,t>, t>
$\lambda \boldsymbol{Q}_{<<e,t>,t>}[\boldsymbol{Q}_{<<e,t>,t>}(\lambda \boldsymbol{x}_{<e>}[\mathbf{BALD}(\mathbf{x})])]$ type <<e,t>,t>, t>

The advantage of this perspective is that the word *bald* need not be considered a function from individuals to propositions, but is rather a function from generalized quantifiers to propositions, for which valuation is possible.

(107) /(ONE) BALD/$_{<<e,t>,t>,\,t>}$

The valuation space of this ***proposition type*** is then the set of possible situations in which *(at least some-) one (or other) be bald* is true[56].

Observe that the relationship between the valuation space of such a *proposition type* and that of a full proposition in which the lexical predicate is used, is one of set-inclusion or

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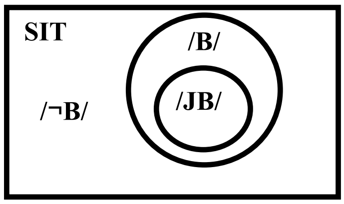
to stress that logic should be primarily stated at the lexical level and even deeper at the level of the Language of Thought rather than at the clausal or sentential level only.

55 Cf. also Montague's (1973) view that language has no basic expressions of type <e>. Note that in type theory, if you have an expression of type <e>, by lifting you also have <<e,t>,t>– a theorem in the Lambek calculus.

56 The infinitive is used not to let tense constrain the determination of the relevant space.

entailment: the set of situations in which the proposition *John is bald*, for instance, is true (= /JB/), is a subset of the set of situations in which the proposition type /(ONE) BALD/, with the generalized quantifier *(at least) one*, is true (= /B/).

(108)	**/(ONE) BALD/**	**:**	**/B/**
	/JOHN IS BALD/	**:**	**/JB/**



The fact that /(ONE) BALD/ is a *proposition type* warrants the use of valuation space analysis. Importantly, it constitutes "stable" meaning stored in the lexicon, while /JB/ does not. As opposed to lexical items, whose properties are stored in long-term memory, /JB/ and complete propositions in general are the creative result of the operation of free combination rules on lexical information[57].

Lexical entailment can now be defined as:

(109) Lexical "entailment"
For all predicates X and Y, $X \models Y$ iff /(ONE) X/ $\subseteq$ / (ONE) Y/

If valuation space modelling can thus be exploited in the lexicon, it yields a taxonomic categorization, with not only relations between superordinate or "entailed" and subordinate or "entailing" sets (e.g. *male, boy*; Katz 1972: 192), but also between sets and their "contradictories" (*human*, *non-human*), sets and their "contraries" (*human, inhuman* : with many values in between), etc.

All in all, the conclusion of the present section is that natural entailment is a broader concept than just logical entailment, namely *semantic entailment*. Moreover, the most natural place to encode the inferential set-subset relations that trigger entailment at the level of the proposition is the level of the lexical predicate (*lexical "entailment"*).

[57] For this view on syntax, cf. the whole generative syntax tradition and for an antecedent in traditional grammar, cf. Otto Jespersen's *Philosophy of Grammar* (1924), in which he states that besides those things in language which are formulaic (and often irregular) in character, there are also "free expressions", built on the basis of a "notion of (...) structure" (Jespersen 1924:19) which guides the speaker in "framing sentences of his own". These expressions always "show a regular formation." (Jespersen 1924:24). This idea of rule-governed freedom, of creativity of language use requires not only this "notion of structure", the syntactic computational system, but also a resource of stored information for the syntactic computational system to work on, the lexicon. As far as the latter is concerned, "(…) apart from the phonetic features that are accessed by articulatory-perceptual systems, the properties of an expression that enter into language use are completely drawn from the lexicon: the computation organizes these in very restricted ways, but adds no further features; that is a considerable simplification of earlier assumptions, which would, if correct, require considerable rethinking of the "interface" between the language faculty and other systems of the mind." (Chomsky 2000: 123).

3.3.4 Unnatural entailment

The present section further illustrates the usefulness of valuation spaces for the description of the relations of IN-logic. At the same time, however, it introduces a notion of discourse informativeness, whose effect is to filter out a set of linguistically unnatural entailments which an unconstrained application of semantic entailment still fails to eliminate. Next, it will be argued that a generalized version of informativeness is active at a much earlier level than that of discourse: it arguably constrains syntactic concatenation and the composition of complex lexical items, a point which will prove relevant in the next chapter.

Recall that the set-theoretic definitions of entailment in (104) and (105) – here repeated as (110) and (111), respectively – identify the concept as a matter of set-inclusion:

(110) Semantic entailment
For all sentences X and Y, X $\models$ Y iff /X/ $\subseteq$ /Y/

(111) Lexical entailment / IN-extension inclusion
For all predicates X and Y, X $\models$ Y iff
the IN-extension of X $\subseteq$ the IN-extension of Y

However valuable they may be, these definitions – focusing on semantic entailment to begin with – still look insufficiently restrictive from the viewpoint of *natural* language and *natural* IN-logic: (110) posits entailment-relations between certain sentences for which it strains natural linguistic intuition to claim that the one sentence "follows" logically from the other. Consider the following entailments:

(112) *Ex necessarie falso sequitur quodlibet*:
a necessary falsehood (contradiction) entails every sentence
P: Some rhinoceros is not a rhinoceros $\vdash$ Q: Some senile professor has pink stockings

(113) *Identical propositions*
P: John was in the garden $\vdash$ P: John was in the garden

(114) *Verum sequitur ad quodlibet:*
a necessary truth (tautology) is entailed by every sentence.
P: Julius Caesar was a (wo)man $\vdash$ Q: A rose is a rose

The example in (112) illustrates a first kind of cognitively unnatural entailment: a contradiction entails everything. Since the contradiction is never true, the entailment relation holds trivially.
The "self-entailment" or improper entailment found in (113) fits the definition of entailment given in (110) above, since whenever the first P is true, the second P is also true, which is all the definition is interested in to establish entailment. Yet, spelling out such an entailment sounds unnatural in natural language: what could be the point of saying that it follows from P that P? In (114), it does not matter to the entailment relationship whether the entailer-sentence is true or not. The entailed sentence being a necessary truth, i.e. a sentence that is true in all possible situations, the status of the entailer-sentence cannot affect the entailment-relation, which obtains in any case.

In EX-logic, such unnatural entailments are little cause for concern. As it is our goal, however, to contribute to a model of a cognitively realistic logic, i.e. one which is not at variance with natural language intuitions about the nature of logical relations, a way has to be found to explain the unnaturalness of such entailments.

3.3.4.1 Informativeness

For (112) and (113) this solution will be sought by generalizing a condition for pragmatic well-formedness for discourse, which was formulated by Scharten (1997: 65) in an incremental semantic framework. For concreteness' sake, I shall first introduce some discourse data which the condition is designed to handle. Then I shall introduce Scharten's definitions and propose an improved version thereof. That version will be shown to solve the unnatural entailments problem of (112) and (113).

The following pair of examples illustrates that discourse is conjunctive (or, to state it in algebraic terms: multiplicative):

(115) a. John was in the garden (= A). Mary came in (= B).
b.John was in the garden and Mary came in.

The set of possible situations in which $A \wedge B$ is true is a proper subset of the set of possible situations in which A alone is true: $/A/ \cap /B/$ is a proper subset of /A/.

Now compare the previous pair with the next one:

(116) a. ??John was in the garden. John was in the garden.
b. ??John was in the garden and John was in the garden.

This pair illustrates that in multiplicative systems like discourse, there is a requirement at work which demands what might be called *information increase*. Each new sentence should add information:

(117) **Information Increase Requirement**:
An utterance P uttered in discourse context C must be informative in C

Scharten's version of the requirement is the following principle:

(118) **pragmatic principle**
A discourse is well-formed if every successive utterance is informative with respect to its context

She gives the principle – inspired by Van der Sandt's (1982: 185) discourse acceptability conditions and Seuren's (1985: 274) notion of informativeness – further flesh by means of the following formal definition of the notion *informative* (Scharten 1997: 64).

(119) Definition
An utterance P is informative in the context C iff

/C+P/ is a proper subset of /C/ and /C+P/ ≠ the empty set ø

In this definition, the valuation space of P is symbolized as /P/, and the result of updating a context C with P is written as C+P. This latter convention might lead to confusion, however. In contrast with what Scharten's use of the +-sign suggests, updating C with P has the effect of intersecting /C/ and /P/, which in Boolean terms is a matter of conjunction, i.e. multiplication (= x) in his algebra, rather than addition (= +).[58]

An improved set-theoretic version of the definition of informativeness is consequently:

(120) **Informativeness**
An utterance P is informative in the context C iff
CP, i.e. /C/∩/P/ is a proper subset of /C/ and /C/∩/P/ ≠ the empty set ø[59]

The exclusion of the empty set as the extension of the conjunction of /C/ and /P/ "expresses the condition that in any discourse inconsistencies should be avoided" (Scharten 1997: 64).

To illustrate how the notion *informative* works, let /C/ be /*John is dead/.* When asserting this sentence, one asserts that the actual situation s_a is a member of the extensional set /C/ of possible situations in which C is true. Next we add the utterance P:*John was murdered*, whose valuation space is /P/. This new utterance is informative if the class of situations to which the combination /C/∩/P/ is simultaneously applicable is a proper subset of the set of possible situations to which /C/ is applicable. In the example given that is the case, since not everyone who is dead was murdered. If, however, upon uttering P the set /C/ of situations in which C is true is unchanged, P is not informative in C. Such a context can be created by turning the examples above around, i.e. by letting C be *John was murdered* and P *John is dead*. This time P is not informative in discourse context C, since /C/∩/P/ = /C/, hence violates the proper subset condition.

But note that a sentence spelling out an entailment relation *John was murdered, hence John is dead* has precisely the structure in which the last uttered clause is not informative relative to the first. Since the sentence is perfectly fine, however, spelling out an entailment relation is better looked upon as the identification and spelling out of a context presupposed for an utterance P, so that informativeness applies as always – P is informative in C:

(121) P: *John was murdered, hence* C: *John is dead*

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[58] The symbol **+**, for its part, is the sign of the operation of aggregation or addition, corresponding to disjunction, the operation for which language uses the expression (*either)…or.* Thus the class of things which are either x or y, the union of x and y, is represented by x **+** y.

[59] Note the similarity between this *discourse* notion and the (*lexical/propositional*) notion of entailment (similarity, *not* identity): /C/∩/P/ is a proper subset of /C/ means that /C/∩/P/ entails /C/. Entailment is a bit looser than informativeness in that it requires inclusion (⊆), but not proper inclusion (⊂).

The pragmatic requirement of contextual information increase provides a solution to the unnaturalness of the entailment in (112). P being a contradiction and the valuation space of P being empty, the intersection of /P/ and the valuation space of any proposition one chooses (quodlibet) as context C will always yield the null set. In terms of informativeness, the result is always Ø (the valuation space of a contradiction: true in no situation at all; the empty valuation space), hence noninformative, which I take to be the source of the unnaturalness of the entailment:

(122)

Ex (necessarie) falso sequitur quodlibet		
P :Some rhinoceros is not a rhinoceros ⊢ C: Some senile professor has pink stockings	/P/= Ø	/C/∩Ø = Ø, hence P is not informative in context C

The same informativeness approach correctly marks (113) (=(123)) as unnatural:

(123)

Identical propositions		
P: John was in the garden ⊢ C: John was in the garden	/C/ = /P/	/C/∩/P/ = /C/, hence P is not informative in context C

This time, the intersection /C/∩/P/ fails to be a proper subset of /C/, hence informativeness is once again violated. The fact that this type of "self-entailment" or improper entailment is as EX-logically impeccable as ex necessarie falso sequitur quodlibet entailments, but ruled out as unnatural in natural language by the same informativeness principle, indicates that the latter is a most useful tool.

Finally, an account has to be found for the unnaturalness of *verum sequitur ad quodlibet* entailments. In such cases, it does not matter to the entailment relationship whether the entailer-sentence is true or not. The entailed sentence being a necessary truth, i.e. a sentence that is true in all possible situations, the status of the entailer-sentence cannot affect the entailment-relation, which obtains in any case. That is a problem, however, since the entailment is felt to be unnatural in natural language. Moreover, the account developed above for *ex (necessarie) falso sequitur quodlibet* and *identical propositions* does not work straighforwardly for these *verum sequitur ad quodlibet* cases, since /C/, the extension of the necessary truth, is 1 (= the universe SIT), of which any P (except another necessary truth or contradiction) is a nonnull proper subset. This means that the pattern satisfies (120).

(124)

Verum sequitur ad quodlibet	P: John is in the garden ⊢ C: A rose is a rose

Verum sequitur ad quodlibet	$/C/ = SIT$	$/C/\cap/P/ = /P/$, hence P is informative in /C/

However, with a minor modification this case can be dealt with. Assume that for any discourse there is an initial "default" context Ci, whose extension is SIT. Before anything is said, no possible situation has been excluded yet – note the negative perspective again – and hence the set of possible situations in which the initial zero discourse context Ci is true, is the universe SIT. On that basic assumption, a rose is a rose in verum sequitur ad quodlibet entailments is the first P uttered within the initial context Ci. However, the intersection of /P/ and /Ci/ is not a subset of /Ci/ and consequently the tautology is not informative.

(125)

P: A rose is a rose	$\vdash$ Ci
$/Ci/ = SIT$	$/Ci/\cap/P/ = /Ci/$, hence P is not informative in /Ci/

By economy, the context C in verum sequitur ad quodlibet sentences is superfluous: uttering P: John is in the garden in the context of C: a rose is a rose is no different in terms of informativeness to uttering P directly in context Ci, which is the more economical, hence doubly more natural option. The overall pattern turns out to be that natural entailments are those involving propositions which can be informative. Necessary truths and contradictions are the two types of propositions that cannot.

On the whole, it turns out to be possible to make cognitive sense of unnatural entailment relations and explain the nature of the unnaturalness involved if entailment is dealt with in function of properties of valuation spaces and discourse informativeness. The latter provides means to rule out unnatural entailments and its connection with the language user's cognition is obvious.

3.3.4.2 Informativeness generalized

There are reasons to believe that there is a well-formedness or economy principle like the following which constrains concatenation at other levels than discourse alone.

(126) **well-formedness principle**
A constituent is well-formed in an existing context if it is informative with respect to that context

What I have in mind is a generalized notion of informativeness, for which it suffices to replace the word "utterance" of (120) by "constituent":

(127) **Constituent Informativeness**
A [consequent] constituent P is informative in the [precedent] context C iff CP, i.e. $/C/\cap/P/$ is a proper subset of /C/ and $/C/\cap /P/ \neq$ the empty set ø

A first reason to believe that this concept of incremental informativeness is well-taken, is that the precedent-consequent asymmetry it introduces is instantiated in the head-nonhead difference in syntactic phrases. In modifier-head constructions, such as *skilful violinist* for instance, the extension of the head *violinist* is the precedent context C whose intersection with the extension of the consequent modifier *skilful*, i.e. P, has to result in a proper subset of C for P to be informative in the complex constituent.

But more importantly, as the decomposition of the entailers *all* and *and* in the next chapter will show, there too a precedent-consequent asymmetry internal to their meaning can be observed: a basic pivotal meaning will function as the presuppositional context C and a further semantic specification will narrow down the extension to that of a more restrictive, hence more informative fully-fledged entailer. This generalized concept of constituent informativeness will be further motivated in § 4.2.5 and 4.4.5.1[60].

3.4 Conclusion

The general conclusion of this chapter is that the two relations of the 2D Cartesian Coordinate System are core features of IN-logic. The latter is far more comprehensive than just an analysis of the features of logical constants at the expense of other lexical items. The two relations are best stated at the level of such lexical items, from where their properties percolate and play a role at the higher levels of phrases, clauses, sentences, discourse. An attempt has been made in this chapter to define the two relations in logical, set-theoretical and algebraic terms. While CD is at the root of set demarcation, entailment concerns inferential relationships between two sets. In a linguistic setting, entailment is subject to a further condition of procedural economy called informativeness:

(128) a. A naturally entails B iff A is informative in the context B
b. A naturally entails B iff the IN-extension of A is a proper subset of the IN-extension of B and the intersection of A and B is not the null set

The procedural-dynamic aspect of the informativeness requirement is the source of asymmetry. New constituents have to be informative relative to old ones. In that sense, informativeness imposes a constraint which has a narrowing effect on the set of natural entailments. While the definition of entailment need not be modified and remains stated in terms of set inclusion, the effect of the added constraint of informativeness is to rule out the unnatural cases, namely those involving the null set, the universe set and improper set inclusion.

At this point, however, a new question arises. The classical logical calculi were stated in terms of only one of the two relations described in this chapter, namely as calculi of *entailments*. But why? Given the at least equally foundational role in our 2D Cartesian Coordinate System of the relation of contradictoriness, I shall now explore whether a

[60] An important caveat is that constituent informativeness as formulated here only pertains to the entailment (set-subset) leg of the logical system. It is not valid for the quality leg (i.e. contradictoriness; set-complement). For instance: if consequent sentence negation takes a precedent (non-referential) clause as its argument, then the intersection of /S/ and /NOT (x)/ is clearly not a nonnull subset of S.

shift from the classical approach in terms of entailment to an approach in terms of CD and more specifically its linguistic expression *negation* would not be a promising option from the perspective of natural logic. The next chapter is devoted to showing that such a move indeed opens new perspectives and brings the format of the calculus closer to the reality of natural language expressions.

4 THE 2D CARTESIAN COORDINATE SYSTEM AND TWO OPERATIONS IN PRELEXICAL SYNTAX

4.1 From Propositional and Lexical Relations to Prelexical Operations

The relations of the Square of Oppositions were reduced to two foundational ones in chapter 2, namely contradictoriness (CD) between the I-corner and the E-corner and entailment (ENT) from the A-corner to the I-corner respectively. The basic relation of contradictoriness CD (I, E) was shown to generalize over CD (OR, NOR) and CD (SOME, NONE); the relation of entailment ENT (A, I) over ENT (AND, OR) and ENT (ALL, SOME)[61]. In the third chapter, it was argued that the relations in question can be stated on lexical items as proposition types.

Within the system of relations, the definition of CD turned out to be more complex than that of entailment, with an accumulation of negatives:

(129)

For every sentence X, ¬X is the contradictory of X	OR ⊢ ¬NOR (¬NOR ≡ ¬¬OR)

The second foundational relation, namely the subaltern affirmative entailment relation between AND and OR, was simpler to state.

(130)

For all sentences AND and OR, AND entails OR	AND ⊢ OR

The reason behind this difference is simple: the definition of the two basic relations as well as the derivation of all other relations in the Boethian Square has always been carried out in terms of entailment, not in terms of CD. Since the relation between AND and OR is itself an instance of entailment, it is not surprising that it was easier to state in terms of entailment than CD.

But why was that choice made? The main answer is that the analysis has always been carried out in Aristotelian terms. His system is one of deductive reasoning, which is the method for arriving at particular (I-corner) from universal (A-corner) truths, in other words, in terms of entailment. History has shown that compared to inductive reasoning – which generalizes from particular to universal truths, i.e. from I to A –, deduction is the only way to guarantee that the truth of a conclusion follows *necessarily* from the truth of the premises. Yet, setting up all definitions in terms of the relation of entailment at the

[61] Actually, as has long been recognized for the Boethian Square, ENT (A, I) and CD (I, E) are not restricted to the operators of the predicate calculus and the propositional calculus but have more instantiations, e.g. ENT (2, 1) and CD (1, 0); ENT (necessity, possibility) and CD (possibility, impossibility); ENT (affirmative, question) and CD (question, negation). Cp. Löbner 's (1990: 78-89) duality groups, which generalize over the standard operators of predicate and propositional logic, epistemic and deontic modality, aspectual adverbs (*already, still, yet, no longer*) and aspectual verbs (*begin, continue, cease)*.

level of full sentences turns out not to do justice to a number of detectable internal properties of the lexical items that figure in logical systems. Since the third chapter led to the conclusion that CD and ENT are best stated on lexical items in the natural language lexicon, detectable morphological and semantic features of lexicalised operators cannot be ignored. Thus the CD relation between propositions with *(either…) or* and (*neither…) nor*

(131) a. *Either* Mary is in Paris *or* Jill is in Paris
b. *Neither* Mary is in Paris *nor* Jill is in Paris

is in essence based on a single morphological feature, namely the negation morpheme *-n*. In this chapter, it will be argued that it is not accidental that the negative operator $\neg$ is the only symbol beyond $\vdash$ which was used in the definitions of (129) and (130). Specifically, a cognitively realistic description of logical primitives has to be carried out in negative terms entirely, more specifically in terms of negative operations internal to the conceptual structure of logical operators, and only derivatively in terms of entailment relations at sentence level. This does not mean that the latter become any less real. Their continued – but derived – existence guarantees that the traditional results are not lost.

The proposal that will be made is to postulate that the relations CD and ENT are not primitives, but *effects* at the lexical, syntactic and discourse level of deeper operations in prelexical syntax. More specifically, the processes that generate those effects are two operations in the internal, prelexical syntax of logical operators as lexical items, namely *negation* (NON) and *conjunction* (ET[62]) respectively. One of the arguments for postulating the former as the operation behind CD is the abovementioned presence of negation in the internal structure of *nor*, which is at the root of a CD relation between OR- and NOR-propositions (cf. Löbner 1990: 95; further arguments will be provided in 4.5 and in chapter 5). Logical and semantic arguments will lead to the postulation of the operation conjunction as the prelexical correlate of ENT.

These two operations themselves, however, are still not primitives yet. They are variants of a single negative operator (as originally proved by C.S. Peirce (1989 [1880])), which implies that it is possible in principle to account for the internal semantic composition of the propositional operators *and, or, nor* and *not* in terms of a single underlying negative operator. The precise nature of the operator in question will be presented in § 4.4, as will the formal structure of its variants, the two *operations* NON and ET. It is the latter which are used to *compose* the semantic *concepts* of A-corner entailers and E-corner contradictors, all lexical items.

The required shift in perspective from the propositional and lexical relations CD and ENT to the operations NON and ET in the Language of Thought and prelexical syntax will be set out in six parts. Since our analysis is a set-theoretic approach to the extension of propositional operators in terms of valuation spaces, 4.2 will be devoted to the way in which sets in general are demarcated according to George Boole. In particular, a law proposed by him to derive the logical Law of Contradiction will be described. It will be

[62] Latin labels are used to stress the status of the operators as universals. The underline is used to mark that they are abstract, i.e. non-lexicalisable and below conscious awareness.

argued that Boole's algebraic approach to sets is cognitively real and can be stated at the lexical concept level. It thereby becomes a feature of IN-logic. A core property of Boole's approach to set demarcation is its invariably binary nature, even for simple sets. In 4.3, that property of strict binarity will be shown to be a core feature of C.S. Peirce's derivation of all propositional truth functions from a single binary, negative truth function as well, hardly an accident. While Peirce's analysis is again stated at the level of propositions, 4.4 will illustrate that a modification of his analysis turns his derivation into a psychologically realistic description of the internal semantic structure of propositional operators as lexical items. The proposal will then be extended to predicate calculus operators. In 4.5 and 4.6, empirical evidence for the decomposition analysis will be gathered. The data given will provide evidence that the two relations CD and ENT are indeed correctly viewed as the products of the prelexical operations of *negation* (NON; 4.5) and *conjunction* (ET; 4.6) respectively.

An important feature of the present analysis, which it shares with generalized quantifier theory, Löbner's duality approach and Seuren's logic of thinking, is that logical relations between operators are lexically encoded. Consequently, the laws of logic somehow have to be transferred from the level of sentences in which operators are used to the level of the Language of Thought and the conceptual semantic structure inside the lexical items that operators represent. That this transfer is possible is due to the fact that the lexical items in question are proposition types (cf. 3.3.1. and 3.3.3.2)[63]. Thus the lexical item *or*, for instance, has two variable positions, say P and Q, in the lexicon, which are substituted in syntax by propositional constants to yield the complex propositions on which the propositional calculus is traditionally defined. It is for this reason that laws whose effect can be observed at sentence level, can be traced back to the level of the internal conceptual structure of the operators themselves. It is in this respect that the reader is asked to constantly keep in mind that where theories are invoked that were originally stated at the level of full sentences or propositions, the aim is invariably to translate their content into generalizations about lexical items. The motivation for this transfer is always the same: the entailment relation at the propositional level between, say, *All F are G* and *Some F are G* can only be due to the difference in semantic contribution between the words all and some, all other things being equal in the propositions at hand. Since lexical items, not creative sentences, are the linguistic elements which are stored in long-term memory anyway, most properties are most naturally stated at the lexical level. Predicating the generalisations on entire sentences is not a valid alternative: the latter involve free choice of lexical items and are therefore novel and mostly fleeting creations (stereotyped expressions aside). This word of caution is intended to make sure that if I should not have sufficiently stressed this general line in one place or another, it is nonetheless constantly intended.

Since Löbner's duality approach is the closest relative of my revised Peircean approach, an overview of the main differences between the two will be given in 4.8. Their effect is to radicalize his asymmetry hypothesis, but to reject the idea that the number of steps in the derivation of an operator can explain the **nand/nall* gap.

[63] If they were not propositional, a valuation space analysis could not be adopted.

4.2 Sets in Boolean Algebra

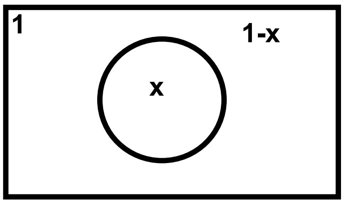
4.2.1 The Boolean Logic of Classes

In this section, a cognitive perspective on set demarcation needed for a decompositional approach to lexical operators will be elaborated. Its main focus is a law discovered by George Boole, namely his so-called Law of Duality. This law enabled him to derive the Law of Contradiction (LC) and the set-theoretic fact that it is impossible for an individual to be both a member and a non-member of a set. As Aristotle and the whole tradition after him had proclaimed that LC is an axiom for which no proof can be provided, Boole's analysis and his claim that LC can be *derived* is of great importance for the relation of contradictoriness of our 2D Cartesian Coordinate System. Moreover, it wil help us identify a single *binary* operator behind all set demarcation, including that of the sets denoted by logical operators as lexical items in the natural language lexicon.

Boole combined a set-theoretic conception of "election" by means of a predicative *elective symbol* G with the idea that successive operations of election are analogous to multiplication in an algebra of the numbers 0 and 1. This is the basis of the Boolean Logic of Classes, some aspects of which will now be set out.

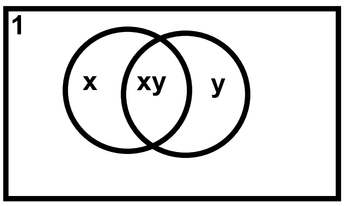
Boolean logic works with a number of variables x, y, z and two constants (1 and 0). The symbol 0 represents the null set. The symbol 1, or unity, is taken "to represent the Universe [in casu, the universe of individuals IND, DJ] (…) comprehending every conceivable class of objects whether actually existing or not, it being premised that the same individual may be found in more than one class, inasmuch as it may possess more than one quality in common with other individuals." (Boole 1847: 15). "The symbol x operating on any subject comprehending individuals or classes" selects "from that subject all the X's which it contains" (Boole 1847: 15). The Universe 1 is thereby divided in two subsets: x and (1-x).

(132)



If from the result x of this first operation we select the y's, the outcome of these two operations in succession, represented by x x y, or xy, will be the class of things which are both x's *and* y's. In Boolean algebra, successive selection is a case of multiplication, in set-theory the equivalent of this operation is the intersection of x and y. From a logical perspective, successive selection of two sets is a case of conjunction. Such successive selection will play a crucial role in the system to be developed (cf. 4.2.2.) and can be represented as follows:

(133) a.



b.

	Logic	set-theory	algebra
Successive selection: x and y	conjunction	intersection	multiplication
	$x \wedge y$	$x \cap y$	$x \times y$, or xy

4.2.2 Double Selection of the Same Set: a Cognitive Double-check?

The relevance of this system to the Law of Contradiction is that Boole uses his algebra to prove that "what has been commonly regarded as the fundamental axiom of metaphysics is but the consequence of a law of thought, mathematical in its form" (Boole 1854: 50). He does this by first observing that repetition of the same operation of election does not alter the result: selecting the x's and then from the result of that selecting again all the x's merely gives the class of x's. Thus

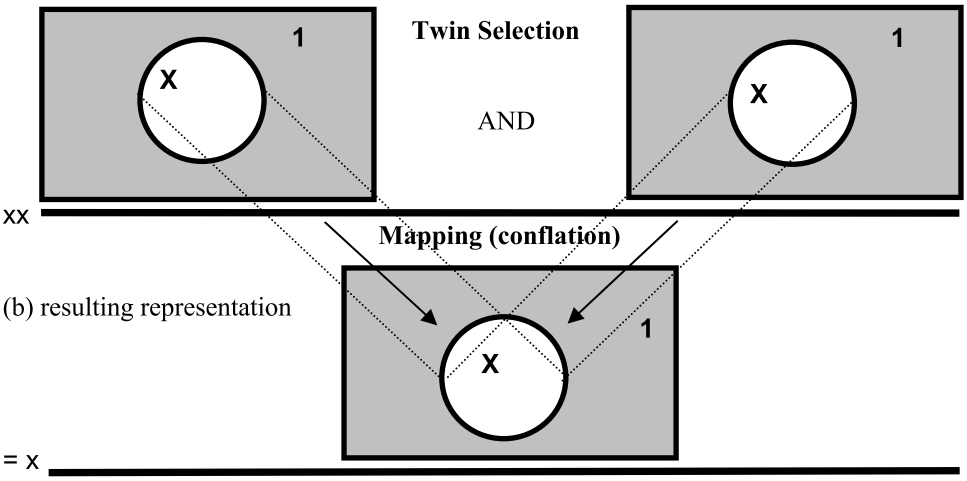
$x\, x = x$, or $\quad x^2 = x$

This algebraic equation is called Boole's "Law of Duality", one of his fundamental laws of thought. He shows how the Law of Contradiction is a consequence of this law in the following terms:

> "Let us write this equation in the form
> $x^2 - x = 0$,
> whence we have
> $x\,(1 - x) = 0$;
> both these transformations being justified by the axiomatic laws of combination and transposition (II.13).
> [Stepwise, we get the derivation: (a) $x^2 - x = 0$; (b) $x^2 = x$; (c) $0 = x - x^2$; $0 = x\,(1 - x)$, which is of course identical to $x\,(1 - x) = 0$ as in Boole's text, DJ]
> Let us, for simplicity of conception, give to the symbol *x* the particular interpretation of *men*, then $1 - x$ will represent the class of "not-men" (Prop. III.) Now the formal product of the expressions of two classes represents that class of individuals which is common to them both (II.6) Hence $x\,(1 - x)$ will represent the class whose members are at once "men," and "not men," and the equation (1) thus expresses the principle, *that a class whose members are at the same time men and not men does not exist.* In other words, that *it is impossible for the same individual to be at the same time a man and not a man.* Now let the meaning of the symbol *x* be extended from the representing of "men," to that of any class of beings characterized by the possession of any quality whatever; and the equation (1) will then express that it is impossible for a being to possess a quality and not to possess that quality at the same time. But this is identically that "principle of contradiction" which Aristotle has described as the fundamental axiom of all philosophy."
> (Boole 1854: 49)

Since each and every set obeys this set-theoretic version of the Law of Contradiction, we adopt Boole's insight and interpret it in cognitive terms as meaning that *whenever* a set x is demarcated at the prelexical concept level, this is done by (s)electing x twice at the same time, a double procedure (134) (a) that will be called *twin selection* and which, given the identity of the two selections, results in a single overlay representation (134) (b) in the mind if all is well. That is the only way to guarantee or check that no element is at the same time inside and outside a set, i.e. the set-theoretic version of the Law of Contradiction. Twin-selection is represented by means of Venn diagrams in (134).

(134)
(a) operations



(b) resulting representation

Selection of x takes place twice in (134) a : xx. Since the second selection yields precisely the same result as the first, its result cannot differ from the application of the first selection. The intersection of the first and second selection of x consequently results in x itself, as in (134) b. Boole's equivalence of xx and x above was represented by letting the two results of the selections in the twin selection operation map into a single set x, thus visualising that the intersection xx after twin selection, equals x itself. In sum, selecting the same x twice, results in a cognitive representation (b) in which the products of the two selections are conflated by intersection.

The numbering of the two selections as *selection 1* and *selection 2* crucially does not imply temporal or logical ordering. The fact that the two selections are presented in sequence is purely a matter of convenience and the numbering is added merely for ease of reference. In actual fact, twin selection as operational in the Language of Thought is simultaneous and unordered[64]. The two selections not being temporally or logically ordered, the resulting representation we actually get is the output of the twin selection procedure, i.e. the single, integrated representation of (134) (b).

An important distinction to be drawn at this point concerns the level at which LC is observed. When Boole claims that a class NMAN whose members are at the same time men and not men does not exist, this is to be taken as an expression of the law of duality and LC at the level of the internal semantic structure of concepts. At this level, there is

[64] This claim will be further motivated in 4.2.5. below.

no conscious control or awareness, so that the law is inviolable and hence makes it totally impossible to construct such a set. It therefore also precludes the formation of a natural lexical predicate *nman* with such conceptual content. In syntax, however, it is possible to construct a sentence which as a whole expresses a violation of LC:

(135) $John_i$ is a man and $John_i$ is not a man.

Though this sentence is still experienced as a contradiction, it is nonetheless possible to construct it and interpret it. The difference between LC at the concept level and at the level of full propositions then is that at the former it is the inviolable law described by Boole above, while at the latter it is a violable law that is here honoured in the breach.

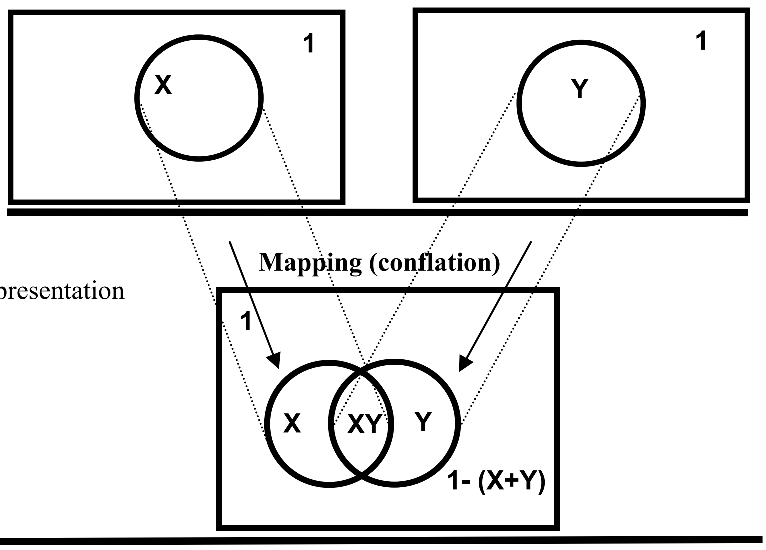
4.2.3 Extending Twin Selection to Selection of Different Sets

The above argument in favour of twin selection of the same set X to demarcate a single set was indirect, namely the fact that it enabled Boole to derive the Law of Contradiction as a law of thought rather than to have to postulate it as a metaphysical axiom. That was a major result and one may wonder why it has not had a greater influence. The likely answer is that skeptical readers may have questioned the validity of adopting this hypothesis because the mapping leading to the integrated representation of (134) (b) involves two instances of the *same* set, resulting in *conflation* and a representation constituting just a single set. Why select the same set twice to end up with a single one? Well, for those for whom the derivation of LC is not convincing enough as a validation for this kind of cognitive double-check procedure, we are now at a juncture where an additional argument can be provided in favour of twin selection for the demarcation of sets. This is achieved by broadening the perspective from successive selection of the *same* set (= *same-selection*) to successive selection of two different sets X and Y (= *different-selection*). The latter involves exactly the same binary selection procedure used earlier, the only difference being that it takes two *different* sets as input.

(136) **Twin Selection**

(a) operations

(b) resulting representation



To understand what 1-(X+Y) denotes in this diagram, it is to be noted that Boolean algebra has *two* binary operations, namely multiplication and addition, defined as follows (and explained below the box):

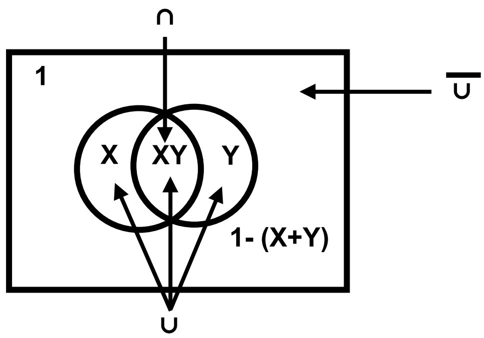
(137)

addition:	(1) a + 0 = a (2) a + 1 = 1 (3) a + a = a	multiplication:	(1') a x 0 = 0 (2') a x 1 = a (3') a x a = a

That multiplication in Boolean algebra corresponds with intersection in set-theory will already be clear from the derivation of the Law Of Contradiction in terms of the Law of Duality (a x a = a) above. Addition, for its part, corresponds with union in set theory: the union of set a with the universe 1, for instance, equals the universe 1 (cf. (137)-(2): a + 1 = 1); the union of a and a is the set a itself (cf. (137)-(3): a + a = a). In view of this, X+Y denotes the union of X and Y and 1-(X+Y) the complement thereof, i.e. everything that is neither in X nor in Y in (136) (b).

The effect of generalizing twin selection from same-selection to different-selection and considering not just multiplication/intersection but also addition/union of the two selections is that the resulting diagram can represent different types of sets and the relationships between them as a single integrated whole, as follows.

(138)



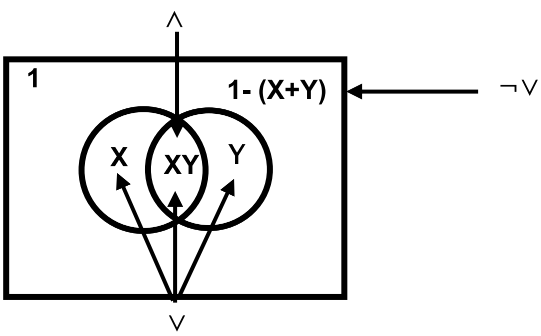
The reason why this is advantageous is that these three types of complex sets reintroduce the extension sets of the lexical items forming the three hubs of propositional logic, namely *and* (conjunction ∧), *or* (disjunction ∨) and *nor* (the negation of a disjunction ¬∨), this time in set-theoretic guise.

(139)

	logic	set-theory	algebra
OR	disjunction	Union	Addition
	$X \vee Y$	$X \cup Y$	$X+Y$
AND	conjunction	intersection	Multiplication
	$X \wedge Y$	$X \cap Y$	$X x Y$
NOR	negation of disjunction	complement of union	universe minus addition
	$\neg (X \vee Y)$	$\overline{X \cup Y}$	$1-(X+Y)$

The assumption of twin selection with different sets and the resulting diagram thus yield the triadic logical structure outlined earlier. The integrated diagram expresses the independent intuition that the three focal propositional operators form a *single paradigm* of semantically related items.

(140)



Furthermore, it also identifies disjunction (∨) as less restrictive (hence arguably semantically less complex) than the two other operators: three cells are compatible with its description, whereas only one cell is compatible with conjunction (∧) and one with joint falsehood (¬∨).

It would have been possible in principle to add 1-(XxY), i.e. the complement of the intersection of X and Y in (138) and the corresponding negation of the conjunction in (140). At this point, where no lexicalisation constraints have been introduced yet, it is not yet clear why such O-corner items are not possible lexical items in everyday natural language. It will be necessary to develop the linguistic and cognitive side of the analysis further to gain ground.

Note finally that in correspondence with the commutative laws of addition and multiplication in algebra (a+b = b+a; axb = bxa), symmetry has to be assumed to characterize the basic triad of operators ($X\vee Y \equiv Y\vee X$; $X\wedge Y \equiv Y\wedge X$; $\neg(X\vee Y) \equiv \neg(Y\vee X)$). This means that twin selection does not care about the order of selection (a point that will be further elaborated in § 4.2.5). This in turn excludes implication from the basic kit of operators, as it is antisymmetric and therefore does care about order. In view of our triadic perspective on natural logic, that is a welcome result. Moreover, implication is easily convertible into a combination of the basic operators: $X \rightarrow Y \equiv \neg X\vee Y$.

4.2.4 The Law Of Contradiction

On the whole, it turns out that there are two cases of twin selection which can be ruled out by the informativeness condition (127). First of all, there is the contradiction x(1-x). In this case, mapping leaves *nothing at all* after integration by means of intersection of the information provided by the two selections.

(141) $X\,(1\text{-}X) = 0 \qquad X \cap \overline{X} = \emptyset$

This situation represents the effect of contradictory sets on twin selection at the level of the internal structure of concepts. The latter module is arguably below the level of conscious control and maybe even awareness (except in the artificial context of scientific study when we try to approach them). It cannot therefore give rise to conscious toying and constitute an inviolable substrate. I take this to mean that at this level no violation of informativeness is possible, so that no contradiction is allowed or possible either. This in turn predicts that there cannot be *natural* lexical items whose conceptual structure is internally contradictory, such as **nman* or **nwhite*, with the contradictory internal meaning structures [[MAN]&[NON-MAN]] and [[WHITE]&[NON-WHITE]] respectively. This seems to be correct.

The other case that is barred as a natural predicate is the universal predicate, which is the result of twin-selection of the universe 1. Once again informativeness, which imposes a nonnull *proper subset* condition on the output of twin selection relative to the input universe 1, rules these cases out as natural predicates.[65]

Regarding the difference between inviolable application of constraints in certain modules as opposed to others, consider the following. Since lexical selection and concrete use of syntactic concatenation rules (e.g. the fact that Merge is appealed to, say, 137 times in a particular derivation) are bound to involve conscious choice, laws of thought which cannot be violated in the Language of Thought module can if so desired be contravened in syntax. Consequently, syntactic phrases in violation of informativeness and with a contradictory meaning - e.g. *non-white white* and *Peter*$_i$ *is not Peter*$_i$ – can be produced (even though similar violations at the level of the internal structure of concepts are not possible). Such constructions are still clearly felt to be contradictory and thus testify to the reality of the abovementioned law of contradiction in the Language of Thought. Yet, they can nonetheless be produced, in my view precisely because from the lexicon up into syntax we are in the realm of conscious *choice* of lexical material, so that IN-logic is from here on observed in the breach as much as in the observation[66]. The importance of recognizing this difference between violability of rules from the lexicon up into syntax and inviolabity of rules in the Language of Thought module is considerable in that it solves a problem that has beset psychological IN-logic for a very long time. At least since Kant it was recognized that there is a problem with viewing logic as a set of hard and fast rules in the mind. The reason is that such a conception would leave no room for the undeniable fact that logical rules are so often broken in reasoning. In the modular conception developed here, however, these two features (namely "hard and fast rules" and "broken in reasoning") are no longer incompatible. If logical rules are hard and fast and inviolable in the underground realm of the Language of Thought, that suffices to be able to conclude that logic *is* in the mind. And if conscious, free choice is indeed involved in lexical selection, syntactic concatenation and discourse, then lexical selection

[65] The present observation that all trivial predicates (the empty predicate and the universal predicate) are barred broadens Zwarts' (1983: 38-39) (and Westerståhl's (1985)) condition that trivial determiners which make any sentence in which they are used true (*the universal determiner*) or false (*the empty determiner*) cannot exist.

[66] Note that the dividing line is not claimed to be between word and phrase, but between lexicon and syntax. This is because of the existence of formulaic stereotyped phrases. The existence of collocations also indicates that the boundary is rather fuzzy.

restrictions and logical laws can be broken in those realms and ungrammaticality and contradictions can be produced. They are still recognized as such because of the hard and fast underground Laws of Thought, yet conscious choice and free concatenation make the construction of such violations of logical laws possible. In sum, in a modular system as here envisaged, the Kantian paradox for psychological IN-logic evaporates.

4.2.5 Twin Selection is Unordered Simultaneous Selection

An important specification has to be added to Boole's axiomatic Law of Duality, namely a requirement of simultaneity of selection within twin selection. The Law then states more accurately that (s)electing the same set twice *at the same time* systematically yields the same result as a single selection of that set, from which it follows algebraically that $x(1\text{-}x) = 0$, which in turn entails that it is impossible for an element to be at the same time inside and outside a set. The qualification 'at the same time' is crucial due to the fact that a set is invariably a pairing intension–extension and extensions may change over time: *kings of the Belgians* had a different field of applicability in 1990 than it has since Albert II became king on August 9, 1993. The extension involved, it should be noted, is once again IN-extension: for someone who only learned about Albert II's accession to the throne one week after it happened, the IN-extension during that week was still the same as before – and even then it may have been incomplete for lack of knowledge of all Belgian kings before Albert II. Since for many uses of the selection, that difference has no implications, specific knowledge about all the actual individuals comprised in the extension is not a precondition for use of the symbol. Actually, if such strictness were imposed, absolute knowledge would be a precondition for speech and thought and silence would be the predictable result: the meaning of 'book' would change with every new publication, a conclusion which is patently false. In this sense, the existence of IN-extensions, which are conceptual, idealized, hypothetical and fallible (but corrigible), is crucial and inescapable. But their corrigibility and changeability over time does of course make the instantaneous conception of twin selection crucial in Boole's derivation of the Law of Contradiction. Note that the claim that twin selection is simultaneous entails that it is necessarily an operation without *internal* sequential order (there is no "successive" selection (as Boole had it), but simultaneous selection). Specifically, internal to twin selection the informativeness requirement (§ 3.3.4.1), which is the principle that imposes order, does not hold. The reason is that the definition of informativeness (see (120) in § 3.3.4.1 and (127) in § 3.3.4.2) is predicated on a difference between a *precedent* context and a *consequent* constituent:

(142) **Informativeness**
A [consequent] constituent P is informative in the [precedent] context C iff
CP, i.e. /C/∩/P/ is a proper subset of /C/ and /C/∩/P/ ≠ the empty set ø

Given that the two sets selected in Boole's twin selection are necessarily selected simultaneously, no difference between precedent and consequent can be established and hence informativeness cannot apply.

The properties of simultaneous selection and lack of order extend to all the variants of twin selection to be described below (including negation NON and conjunction ET). As mentioned earlier, all these notions are part of the Language of Thought and hence operate below conscious awareness and choice.

4.2.6 Why Twin Selection? Why Exclusion?

One might wonder with Boole why his deduction of the Law Of Contradiction is so crucially dependent on *twin* selection of the same set, and not on triple or quadruple selection, for instance. Boole senses this is a fundamental question about our cognition, but has no real answer to it other than the observation that only twin selection yields the right results algebraically. Our hypothesis runs as follows: set demarcation is – as I argued § 3.2.2.1 – a matter of *exclusion* of all that does not belong to it. As stated there, exclusion crucially involves negation. Now, C.S. Peirce (1989 [1880]) proved that all propositional truth functions, including the unary truth function negation, can be derived from a single underlying *negative* *binary* truth function, namely the so-called joint falsehood function (*neither...) nor* (= ¬∨). This is the hypothesis that will be embraced in § 4.3. I will argue that the latter claim is not only correct in EX-logic, but that cognition employs the same negative, binary operator to demarcate sets and hence also to generate the extension sets of surface operators of the lexicon. This is once again a transfer of insights at the level of propositions to the lexical level of propositional functions and their internal semantic structure. If correct, the necessity of Boolean twin selection identified above is a corollary of the binary nature of the basic Peircean operator which governs all set demarcation – arguably the only primitive propositional operator our cognition works with. The fact that the Law Of Contradiction as a Law of Thought can only be derived on the basis of twin selection thus for the first time finds a natural explanation. This issue will be returned to when the reduction of all propositional truth functions to the underlying joint falsehood function has been established.

On the whole, the Boolean concept of set-demarcation developed so far, namely (a) by *twin* selection and (b) by *exclusion* is a wide-ranging and important notion in IN-logic. First of all, it is too close to the Peircean hypothesis that all propositional operators can be derived from a single a *binary, negative* operator for the relationship to be accidental. Actually, the double link – twin ≈ binary and exclusion ≈ negative – between the two theoretical proposals has a mutually reinforcing effect. And another effect of linking them is that the properties of set demarcation (twin selection, negative exclusion) as introduced above need not be postulated as primitives of xx = x, but can be argued to derive from the negative, binary nature of the underlying primitive operator itself. Furthermore, since Boole proved that LC can be derived if twin selection and his Law of Duality are adopted, his derivation of the Law Of Contradiction via twin selection can ultimately be tied to the Peircean underlying operator as well. This derivation of a whole cluster of properties from those of a single axiomatic underlying truth function is the core of the present chapter.

4.3 Peirce's negative disjunctive truth function

In this section, a basic negative truth function with the value of *nor,* i.e. $\neg(P \vee Q)$, will initially be used as the basic ingredient of a purely EX-logical decomposition analysis of the binary propositional truth functions (*and, or, nor*). The analysis in question was proposed by C.S. Peirce, and all that will be added to it in this section is (i) as didactic as possible a presentation of its contents and (ii) a notation system that makes that content more accessible. Once that is done, the stage is set for the next section, in which Peirce's EX-logical analysis will be modified to yield a cognitively realistic IN-logical system at the prelexical level rather than a purely EX-logical system at sentence level.

4.3.1 Getting the NEC of it – A single primitive truth function

The present attempt to decompose propositional operators does not have to start from scratch. In EX-logic, it has been known for over a century that the truth-functions *and, or, nor* and *not* are not all independent, but can be related to one another by reduction to elements that underly all of them. "There is no great difficulty in reducing the number to two", wrote Russell (1919, [2000]: 148). In his and A.N. Whitehead's *Principia Mathematica* (1910-1913) the two chosen were negation and disjunction.

(143)[67]

A	AND	$\neg(\neg P \vee \neg Q)$	$\neg(P \vee Q)$	NOR	E
I	OR	$P \vee Q$ (equivalent to $\neg\neg(P \vee Q)$)	$\neg P \vee \neg Q$	*NAND	O
	NOT	$\neg P$			

Taking into account the equivalence of the two formulas for OR, it can be upheld that all operators can be reduced to a formula involving negation and disjunction.
A reduction to negation and conjunction (rather than disjunction) is equally as feasible:

(144)

A	AND	$P \wedge Q$ (equivalent to $\neg\neg(P \wedge Q)$)	$\neg P \wedge \neg Q$	NOR	E
I	OR	$\neg(\neg P \wedge \neg Q)$	$\neg(P \wedge Q)$	*NAND	O
	NOT	$\neg P$			

Comparing this reduction to the previous one, we note that the common denominator is negation, from which it is plausible to conclude that if further reduction of propositional operators should be possible, it would probably also involve a negative operator. This prominence of negation is not accidental. First of all, it is common coin in logic that two negatives make a positive, a toggle which is crucial if one wishes to be able to somehow switch from the negative operators NOR and NOT (and NAND) to the positive

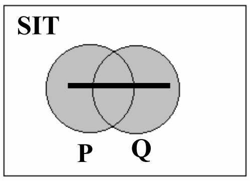
[67] The labels from Boethius' Square of Oppositions (cf. chapter 2) are added: A = universal affirmative, I = particular affirmative, E = universal negative and O = particular negative (from n**E**g**O**, "I deny"). I use the same labels for the truth functions as defined in § 2.2.4: (i) $P \wedge Q$ is written as AND, (ii) $P \vee Q$ is written as OR, (iii) $\neg(P \vee Q)$ is written as NOR, etc. In § 4.4, valuation space bars will be used around the operators to shift to my own lexical-item based cognitive analysis: from there on, the operators are therefore to be viewed as lexical items (i.e. propositional functions) again: /AND (P,Q)/, abbreviated into /AND/.

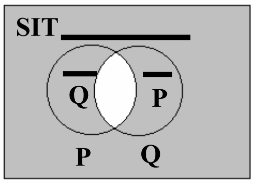
operators AND and OR, and then back by means of a single toggle. No similar results can be achieved if one starts from a positive corner: two positives do not make a negative. Secondly, there is a close association between the idea that there is a basic negative operator and our earlier observation that set demarcation (CD) and inferential relations between sets (entailment) involve set *exclusion*, which is itself a negative notion. The link can be shown by means of Venn-diagrams. Take the formulas for NOR and AND in (143), repeated here as (145)a., and compare them with the equivalent set-theoretic diagrams in (145)b. To bring out the parallellism between the two, one auxiliary device has been added in (145)b., namely a number of bars to mark the subsets of the rectangular universe SIT that are excluded (shaded).

(145) a.

NOR	AND
$\neg(P \vee Q)$	$\neg(\neg P \vee \neg Q)$

b.





Note how the number and scope of the bars in the excluded areas of the Venn-diagram match precisely the number and scope of the negatives in the logical formulas: NOR has a single negation spanning its two terms which are correspondingly shaded in the Venn-diagram, AND has a negation for each of its terms and the wide scope negation over the two negated terms is an instruction to exclude everything that is not-P or not-Q. That means excluding anything that is in the complement of P or the complement of Q, in all resulting in the shading of three cells in the diagram.

Since logic and set-theory are plausibly alternative modes of looking at the same objects, this one-on-one relationship between the application of a negative operator and the exclusion of cells is no coincidence. In what follows, I shall try to prove for all propositional operators that the exclusion of subsets (cells) from the Universe SIT of possible situations that the rectangular Venn-diagram represents, is none other than the set-theoretic equivalent of the application of a *single* negative operator in logic.

Since Peirce (1989 [1880]), Sheffer (1913) and Nicod (1917), it has indeed been appreciated that the logical vocabulary can be made even more economical than achieved in (143) and (144) above: all operators can be generated from a *single* basic truth function. As Boethius' square has two negative corners, there are in principle two viable negative candidates for the status of single basic truth function: (i) Peirce's *joint* *falsehood*[68] function, represented as P↓ Q, read as 'neither P nor Q', 'NOR (P,Q)'; and (ii) *incompatibility*, i.e. the Sheffer (or Nicod) stroke, represented as P | Q, read as 'not both P and Q' or NAND (P,Q). As Peirce and Sheffer proved, both can be made to work and derive the whole set of propositional operators if the matter is looked at from a purely logical viewpoint[69].

The claim worked out here is that logical operators in natural language are based on a single, underlying operator too and that of the two candidates which 'divested' EX-logic, i.e. logic unconstrained by internalist mentalist considerations, provides, Peirce's joint falsehood operator is the one which turns out to be cognitively and linguistically real. This is why I shall now focus on Peirce's system and present the way he derives all the operators from 'joint falsehood' NOR.

4.3.2 The joint falsehood operator *nor* and the Peirce's decomposition of propositional operators

In Peirce's *nor*-system, the joint falsehood operator ↓ is a single binary operator whose meaning is identical to that of the E-corner operator complex ¬∨ (i.e. "nor"; in algebraic terms: (1-(P+Q))). But to be able to translate and decompose the whole operator system of (143) into Peirce's system, a few more tools are needed over and beyond *merger* of ¬∨ into ↓. First of all, there is a problem with negation (as in ¬P) to be solved. Given that negation is a single-term operator while ↓ is binary, the single-term expression has to be somehow converted into a two-term expression. Peirce's solution is to exploit the Law of Tautology, according to which P and P∨P are equivalent expressions. This allowed him to expand ¬P into the equivalent expression ¬(P∨P), so that he could consequently translate ¬(P∨P) into P ↓ P by application of merger. Second, to translate the standard formula for disjunction P∨Q, which contains a disjunction symbol but cannot be input to merger in the absence of negation, Peirce appealed to the Law of Double Negation, according to which every proposition is the complement of its complement: P∨Q is thus equivalent to ¬¬(P∨Q). This brings in negatives so that merger becomes possible and the translation procedure can proceed (as detailed below). Finally, to convert P∧Q, which lacks both negation and disjunction, De Morgan's Law was appealed to, yielding the equivalent expression ¬(¬P∨¬Q)), ensuring the presence of both negation and disjunction.

With these four tools – *merger* of ¬∨ into ↓, *the Law of Tautology*, the *Law of Double Negation* and *De Morgan's Law* – Peirce arrived at the following translation of the two-operator negation-and-disjunction system into the single-operator NOR-system, a most remarkable achievement:

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[68] Horn (1989: 256) uses *joint denial*, but in order not to mix categories, we shall not use the speech act category *denial.*

[69] For the historical record it is worth emphasizing that Sheffer's (1913) reduction of the propositional logic to a single connective was much later than Peirce's (1880). Whitehead and Russell apparently were not aware of Peirce's achievement, since they stated in the second edition of *Principia Mathematica* that Sheffer's stroke was the most important development in logic since the first edition of their work.

(146)

	Tools used	Conversion steps
NOR	 Merger	$\neg(P \vee Q)$ $\mathbf{P \downarrow Q}$
NOT	 Law of Tautology Merger	$\neg P$ $\neg(P \vee P)$ $\mathbf{P \downarrow P}$
OR	 Law of Double Negation Merger Law of Tautology Merger	$P \vee Q$ $\neg\neg(P \vee Q)$ $\neg (P \downarrow Q)$ $\neg ((P \downarrow Q) \vee (P \downarrow Q))$ $\mathbf{(P \downarrow Q) \downarrow (P \downarrow Q)}$
AND	 De Morgan's Law Merger Law of Tautology (twice) Merger	$P \wedge Q$ $\neg(\neg P \vee \neg Q))$ $\neg P \downarrow \neg Q$ $\neg (P \vee P) \downarrow \neg (Q \vee Q)$ $\mathbf{(P \downarrow P) \downarrow (Q \downarrow Q)}$
NAND	 De Morgan Merger Tautology Merger Tautology (4 times) Merger (4 times)	$\neg(P \wedge Q)$ $\neg(\neg(\neg P \vee \neg Q)))$ $\neg(\neg P \downarrow \neg Q)$ $\neg(\neg P \downarrow \neg Q) \vee (\neg P \downarrow \neg Q)$ $(\neg P \downarrow \neg Q) \downarrow (\neg P \downarrow \neg Q)$ $(\neg(P \vee P) \downarrow \neg(Q \vee Q)) \downarrow (\neg(P \vee P) \downarrow \neg(Q \vee Q))$ $\mathbf{((P \downarrow P) \downarrow (Q \downarrow Q)) \downarrow ((P \downarrow P) \downarrow (Q \downarrow Q))}$

Admittedly, these much more elaborate formulas look rather cumbersome. After explaining why such elaborateness is not really a problem and actually even to be expected in underground computation I will argue that there is nonetheless a way of improving on Peirce's notation which makes the formulas more palatable and transparent.

Cumbersome elaborateness need not be a problem for a computational system. This is because elaborateness of output expressions and complexity of the rule set should not be confounded. If by making expressions more elaborate the rule set (algorithm) can be made simpler (one operator instead of two), that may well be a desirable result under a modular conception of the linguistic system. Recall that it is our goal to recast the Peircean analysis into a proposal about the internal lexical structure of operators. From that perspective, a simpler rule set can be conceived of as the minimalist computational system active below conscious awareness. The forms (PF-features) in the lexicon will then have a crucial function of their own, namely to link the elaborate output products of the underlying computational system with a less elaborate and fast retrievable formal representation. This encapsulates the underlyingly elaborate representation and gives the whole a separate address and set of PF-features in the lexicon[70]. Many lexical items can thus be seen as a trade-off: the concepts they embody are underlyingly elaborate but computationally simpler. In the lexicon, however, the elaborate semantic routines are stored as a whole and matched with a compact PF-feature set which makes direct, single-address retrievability of the lexical item and its complex conceptual content possible for the purpose of lexical selection.

Independent of these considerations, Peirce's notation can be made a bit more palatable and transparent. To get rid of all the burdening brackets, it is possible to use a bar to represent the NOR-operator[71].

(147)

Two-operator notation	Peirce's single-operator notation	Single-operator bar notation
NOR	NOR	NOR
$\neg (P \vee Q)$	$(P \downarrow Q)$	$\overline{P\ Q}$

Extending over the two terms in their scope, bars do not only express scope more clearly, they also automatically bring out hierarchical relationships in complex expressions with more than one bar, as the table below reveals. They thus have the same advantage over Peirce's notation that tree-diagrams, with their visualisation of vertical hierarchy relationships, have over labelled bracketing in syntax[72]:

(148)

	Peirce's notation	bar notation
NOR	$P \downarrow Q$	$\overline{P\ Q}$
NOT (N(E)-)	$P \downarrow P$	$\overline{P\ P}$
OR	$(P \downarrow Q) \downarrow (P \downarrow Q)$	$\overline{\overline{P\ Q}\ \ \overline{P\ Q}}$

[70] Thus a lexical item such as *kill* can be viewed as a lexical encapsulation (with a separate address) of computationally more elaborate prelexical *cause -become-not-alive*.

[71] This brings back the bars used in the Venn-diagrams of (145)b., though this time they (more appropriately) represent ↓ rather than its subconstituent ¬ alone. The bar-notation introduced here, which I was taught by Jeffrey Gruber, will become a crucial help when a link will be established between logical expressions and syntactic trees.

[72] Thanks to Jeffrey Gruber for long discussions and e-mailinteraction, in which he tended towards *nand* as the basic operator, whereas I have chosen to defend and work out the claim that it is *nor* which is cognitively basic. The different sides we took have forced me to state my arguments more clearly and have therefore been an enormous help.

AND	(P ↓ P) ↓ (Q ↓ Q)	$\overline{\overline{P\;P}\quad\overline{Q\;Q}}$
NAND	((P ↓ P) ↓ (Q ↓ Q)) ↓ ((P ↓ P) ↓ (Q ↓ Q))	$\overline{\overline{\overline{P\;P}\quad\overline{Q\;Q}}\quad\overline{\overline{P\;P}\quad\overline{Q\;Q}}}$

4.4 Modification of Peirce's NOR: the need for the cognitive set-operator NEC

What will be proposed is that the internal semantic structure of lexicalised propositional operators is determined by the prelexical equivalent of the negative operator in question. I will from this point on use the Latin label <u>*NEC*</u> (instead of NOR) for the basic cognitive operator. The Latin is once again used to stress the status of the operator as a universal in the Language of Thought; the underline to mark its non-lexicalisable status and the fact that it is below conscious awareness.

4.4.1 Why modify Peirce's analysis?

Peirce's derivation of all propositional operators from the basic joint falsehood operator NOR is an EX-logical one and is not intended as a representation of operations in the IN-logic of cognition. To turn his system into a cognitively realistic one, a number of changes will be made.

First of all, the basic operator Peirce used will be taken to represent a *cognitive* one, whose activity is to be located, however, at the cognitive underground level of conceptual structure (or prelexical syntax) in the Language of Thought[73].

Correspondingly, Peirce's derivation will be transferred from the level of full sentences to that of the internal semantic structure of lexical items. In view of this, the bar-code patterns developed in § 4.3.2 will be taken to constitute the core part of the internal semantic make-up of propositional operators as lexical items. A consequence of this is that the means by which the bar-code patterns are created, namely the cognitive operator <u>NEC</u> and its two variants <u>NON</u> and <u>ET</u>, are not to be identified with surface operators in the lexicon. Multiple applications of the cognitive operators are required to yield surface operators. In other words, the lexical items expressing negation (*n(e)-, not*) an conjunction (*and*) will be shown to be more complex than the primitive operators <u>NON</u> and <u>ET</u> that enter into them.

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[73] In scientific study and hence in scientific language, however, such computation can be made explicit. This is precisely what is attempted here. Since natural language and scientific language are qualitatively different in that the latter invokes the science forming faculty SFF, the attempt made here to make operations and representations in prelexical syntax explicit does not undermine or contradict the claim they are below awareness in natural language (cf. chapter 2). That prelexical syntax is considered compatible with the Language of Thought hypothesis may sound strange, since they are usually pitted against each other. The claim they are nonetheless compatible is because the operations that constitute prelexical syntax remain below awareness, so that its products – the concepts – are, as Fodor claims with regard to his Language of Thought hypothesis, atomic for computation in the more strictly linguistic modules of grammar.

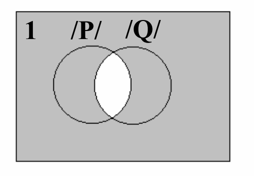
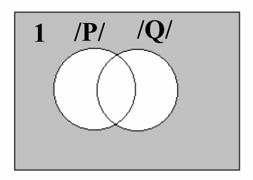
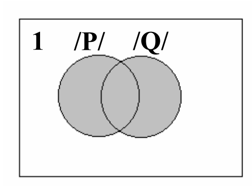
Arguably, the complexity of the logical calculus hubs that are composed with <u>*NEC*</u> and a few other basic rules belongs to our cognitive biological endowment, which would explain the automatic nature and inaccessibility to conscious modification in natural language acquisition.
The changes that a cognitive IN-logical reinterpretation of Peirce's system will necessitate, will be based both on detectable *semantic* properties of existing logical operators in IN-language and the structure and format of linguistic *lexicalisations* thereof. The upshot of the whole analysis will be that the triad of binary operators of the propositional calculus are composite items built from a single negative disjunctive operator, as is the fourth lexical operator *not*. For each propositional operator there will be argued to be a parallel operator in the predicate calculus.

4.4.2 The meaning of lexical operators

So let us analyse which operations occur at the prelexical level and how the logical triads are derived. The first operator that will be decomposed is the propositional calculus pivot *or*. The result obtained will then be applied to the predicate calculus pivots (*any/some, either, at least one*). Next, the contradictors and entailers of each of these two calculi will be treated. To see what has to be arrived at, recall the proposition calculus, set-theoretic and algebraic operations which the three corners of the 2D Cartesian Coordinate System are traditionally held to represent and the corresponding Venn-diagrams.

(149)

	logic	**set-theory**	**Algebra**
A-corner (*and*)	conjunction	intersection	multiplication
	$X \wedge Y$	$X \cap Y$	XxY
I-corner (*or*)	disjunction	union	Addition
	$X \vee Y$	$X \cup Y$	$X+Y$
E-corner (*nor*)	negation of disjunction	complement of union	universe minus addition
	$\neg (X \vee Y)$	$\overline{X \cup Y}$	$1-(X+Y)$

entailer intersection *and* $/P/ \cap /Q/$	pivot union *or* $/P/ \cup /Q/$	contradictor complement of union *nor* $\overline{/P/ \cup /Q/}$
		

4.4.3 Pivots

This section is in two parts. The first part concerns the propositional calculus pivot *or*, while the second is devoted to predicate calculus operators.

4.4.3.1 The Propositional Calculus Pivot $\mathrm{OR}_{\mathrm{incl}}$ (P,Q)

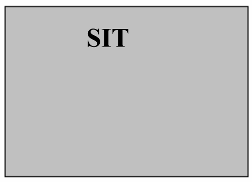
As illustrated by Venn diagram (149), the propositional pivot that will be described is the inclusive variant, which is taken to represent the basic lexical meaning – a standard position in logic.

4.4.3.1.1 The universe SIT of possible situations

In the lexical meaning $\mathrm{OR}_{\mathrm{incl}}$ (P,Q), P and Q are to be seen as variables for any propositional constants which the lexical item *or* could take as argument. So, the extension of *or*, i.e. /OR (P,Q)/, is the set of all possible situations in which *P or Q* is true, with P and Q any choice of propositional constants. But how is such a disjunctive set demarcated in human cognition?

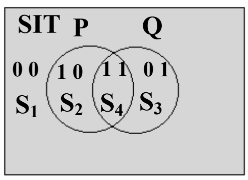
Set demarcation always happens within and hence starts from a universe of possible members (in Boolean algebraic terms = 1). In the case of the propositional calculus pivot $\mathrm{OR}_{\mathrm{incl}}$ (P,Q), the universe in question is the universe SIT of possible situations.

(150)



Given the twin-selection hypothesis, this universe is structured in 4 cells. Indeed, recall that the valuation space diagram for the propositional operators was related to truth tables by means of a placemat-construction in chapter 2 and in § 3.3.2. Given the two-argument nature of the operators (*P and Q, P or Q, P nor Q*) and the two values true and false, the number of possible value combinations for any P and Q (assuming they are different) is $2^n = 2^2 = 4$.

(151)



It is this Boolean algebra structure which is the domain on which the negative operator NEC will carry out its operations. It is therefore also against the background of this universe that the extension of OR_{incl} (P,Q) is delineated. From this universe of possible situations (151) – the *domain* of the set demarcation function – no situations have been excluded yet at the initial stage, nor have any been activated yet: everything is selectable, nothing selected yet. The fact that no cells have been excluded is represented by means of the underlined value *1?* for those cells, as in the following diagram. The question mark merely serves to indicate that the 1-value is still hypothetical, that the elements of the relevant cell are selectable, but have not been selected as actual members yet.

(152)

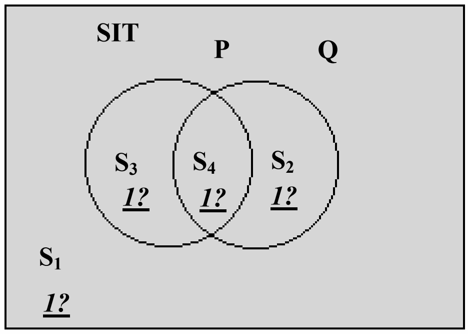
		cell structure of the universe SIT	***Stage 0.*** Universe SIT = 1
P	Q		*value*
1	1	S4	*1?*
1	0	S3	*1?*
0	1	S2	*1?*
0	0	S1	*1?*

Activation of the extension set of OR_{incl} is executed by means of an exclusion procedure operating on stage ***0.*** in (152). Since set demarcation starts from the entire universe (in Boolean algebraic terms = 1) all of whose situations are in principle selectable (*1?*), set delineation is *necessarily a negative procedure*, a matter of downsizing the domain in question to the proper subset of actual members of the target set. The present description makes it clear how crucial the negative conception is. The only way to reduce the set of possible members SIT to the intended subset of actual members is by eliminating – a negative procedure – the possible members that do not fit the requirements. In algebraic terms this amounts to subtraction (1 -…). Subtraction is the only viable option, since starting from 1, a positive operation such as addition would be pointless: it would result in 1 + (P+Q) = 1, given that the union of the universe and a subset always still equals the universe and hence constitutes a non-productive or noninformative operation: nothing gets eliminated[74].

As indicated, the underlined positive values *1?* in diagram (152) signal that the elements of the cell in question belong to the set of *possible members* of OR_{incl}: they are the elements under consideration for set demarcation. But though they are part of the *domain*, the elements of *1?* cells do not yet represent the ultimately activated range of *actual members* of the set, i.e. the set of *selectees (=Sel)*. As long as no exclusion occurs and we are still at the initial (non-activated competence) stage (***0.***) of the system, the boxes in the diagram still have a light shading, indicating that actual selection still has to get started. This stage can also be represented by means of the following Venn-diagram.

[74] Cf. two negatives make a positive, but two positives don't make a negative.

(153)



4.4.3.1.2 The basic operator NEC

The sole operator of the whole system is the foundational binary, twin-exclusion operator NEC, which is used to set the cell exclusion-procedure in motion. It represents the basic subtractive (=negative-disjunctive) operation. Initial application of NEC is the first step of a two-step exclusion procedure to determine the complete valuation space of $P \vee Q$, i.e. OR_{incl}. The definition of the operation is:

(154) Step **1. :** application of **NEC**, whose definition and properties are the following:

NEC (X,Y)	
Logic	$X , Y \rightarrow \overline{X \quad Y}$
Set-theory	$/X/, /Y/ \rightarrow \overline{/X/ \cup /Y/}$
Algebra	$X, Y \rightarrow (1 - (X+Y))$

As soon as NEC becomes active and the exclusion procedure (1-…) gets going, we use a bold non-underlined **0** as the exclusion value of the affected cells, denoting *actual* exclusion. The latter is represented in diagram by darker shading of the relevant cell(s). What is not excluded remains selectable as before, which is what the question mark in cell S1 at stage **1.** and in the corresponding description (***z ∈ Sel?***) expresses.

(155)

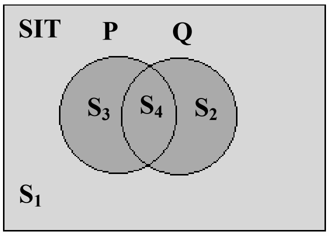
		cell structure of the universe	***0.*** Universe SIT = 1	***1.*** (1?-(P+Q))[75]
P	Q		*value*	*Value*
1	1	S4	1?	***0***
1	0	S3	1?	***0***
0	1	S2	1?	***0***
0	0	S1	1?	1?

z ∈ S2-S4 → z ∉ Sel (rows S4, S3, S2)

z ∈ S1 → *z ∈ Sel?* (row S1)

As the diagram shows, application of NEC results in stage ***1.*** and has the effect of excluding all cells with a 1 value for P or Q, i.e. cells S2-S4. The relevant operation subtracts the initial value of the relevant cells from the universe of discourse: 1- 1?, which leaves exclusion value **0** as the exclusion value of the relevant cells at ***stage 1.***[76] The meaning that would be expressed if this stage were expressible is that any situation z that belongs to S2-S4 does not belong to the set of selectees, i.e. **z ∈ S2-S4 → z ∉ Sel.**[77] In the Venn-diagram, this exclusion results in darker shading of the corresponding cells.

(156)



The status of S1, for its part, has not been affected by the exclusion operation and hence remains lightly shaded. In other words, at this stage S1 is still as before a set of *possible* but not *actual* members of the set that is being demarcated (value 1? in the table, as before). For any x in S1, it is not clear yet whether it does or does not belong to the set of selectees: if x is an element of S1, is it an element of Sel? Formally: **z ∈ S1 → *z ∈ Sel?***

---

[75] The question mark has no influence on the nature of subtraction. While it will always be used in the cells S1-S4 of further diagrams, it will be suppressed in the heading descriptions of the stages, which will in all diagrams to follow be shortened to the simpler and more traditional format (1 - (P+Q)).

[76] There is of course a difference between the two numbers in this subtraction 1-1. The first represents the whole domain of discourse, i.e. Universe SIT, while the second is a positive value for a subset of that Universe only. Still, carrying out the algebraic subtraction as indicated is harmless, since the output value **0** is also restricted to the relevant subset: 1-1=**0**.

[77] The variable z is used to refer to situations in SIT.

Due to (i) the undetermined nature of S1 and (ii) the fact that no elements have been positively identified as selectees, no set has been demarcated yet and hence this level cannot be lexicalised as a lexical predicate. Since the point of the set demarcation procedure is to determine the set of selectees for OR_{incl} positively, i.e. the z's that *are* elements of Sel, this can therefore only be an intermediate stage in the procedure. But it is a necessary stage nonetheless, if exclusion is, as I claim, all that is available qua operation.

4.4.3.1.3 Same-selection NEC: NON

For the next step (step 2.) in the demarcation of OR_{incl}, NEC has to apply once more. However, the variant that is needed is a specific one, namely 'same-selection' NEC, i.e. the form of NEC which takes twice the same argument. First, the general definition and format of this variant of NEC will be given, after which it will be applied to the stage 1. output in (155) to yield the second and final step in the demarcation of the extension of OR_{incl}.

As said, the type of NEC that has to apply at this point is the one that takes the same input argument twice. This operation, same-selection NEC, is none other than the operation of negation, which will be referred to as NON (in set-theoretic terms: the complement set rule).

(157)

NON	
Logic	$X \rightarrow \overline{X \quad X}$
Set-theory	$/X/ \rightarrow \overline{/X/ \cup /X/}$
Algebra	$X \rightarrow (1- (X + X))$

In derivational operation mode, this rule can be described as an application of NEC to an input by copying that input and taking the original and the copy as its two identical arguments, resulting in a binary same-selection configuration. Though this operation gets a separate name (NON) for the sake of terminological differentiation, it is just one of the cases that NEC covers, so it does not represent a new axiomatic entity or primitive in the system.

An important aspect of this rule which still has to be added, is that because of the identity of the two arguments, they can be **conflated** into a single one. There are two ways of representing conflation: either one of the arguments is bracketed in the formula (and indicated by means of boldface square brackets **[…]**) or it is deleted.

(158)

Conflation	Algebra	NEC bar-code
	(1-(X+X)) = (1-X)	$\overline{X\ \ X} \equiv \overline{X\ [X]} \equiv \overline{X}$

Note that conflation as described here corresponds with the negative version of the Law of Tautology for Disjunction as expressed in the logical equivalence $\neg(P \vee P) \equiv \neg P$ in (146). An alternative name for *conflation* would consequently be *Negative Law of Tautology for Disjunction*. As opposed to NEC (and its variant NON) *it is not an operation*, not part of the procedural side of the system, but a property of the resulting representation whenever same-selection occurs.

When enriched with this notion of conflation, NON comes out as:

(159)

NON		
	Description of the rule	
Logic	$X \rightarrow \overline{X\ \ [X]}$	$X \rightarrow \overline{X}$
Set-theory	$/X/ \rightarrow \overline{/X/\ [\cup\ /X/]}$	$/X/ \rightarrow \overline{/X/}$
Algebra	X → (1- (X [+ X]))	X → (1- (X))

In sum, the operation of negation NON can be described as application of NEC to an input by copying the input, placing the resulting two identical arguments under the NEC-bar which has scope over both of them and resulting in conflation of the two arguments, indicated by bracketing or deletion of one of them.

Let us now apply this rule (with conflation included) to the output of step 1. in the demarcation of the extension of OR_{incl}:

(160)

Step 2.: application of **NON** to stage 1.	
	Description of the rule
Logic	$\overline{P\ \ Q} \rightarrow \overline{\overline{P\ \ Q}\ \ [\overline{P\ \ Q}]}$
Set-theory	$\overline{/P/ \cup /Q/} \rightarrow \overline{/P/ \cup /Q/\ [\cup\ /P/ \cup /Q/]}$
Algebra	(1 – (P+Q)) → (1- ((1 – (P+Q)) [+ (1 – (P+Q))])

This results in $/OR_{incl/}$ as the union of /P/ and /Q/, algebraically (1-(1-(P+Q)), cf. (161)

(161)

OR_{incl}	*algebra*		*Bar-code*	
conflation	bracketed	deleted	bracketed	deleted
	(1-(1-(P+Q)) [+ (1-(P+Q))])	(1-(1-(P+Q)))	$\overline{\overline{P\ \ Q}\ \ \overline{[P\ \ Q]}}$	$\overline{\overline{P\ \ Q}}$

As is indicated by these formulas, the end-result is derived in two negative-disjunctive steps, as represented in flowchart (162) and the corresponding Venn-diagram sequence (163):

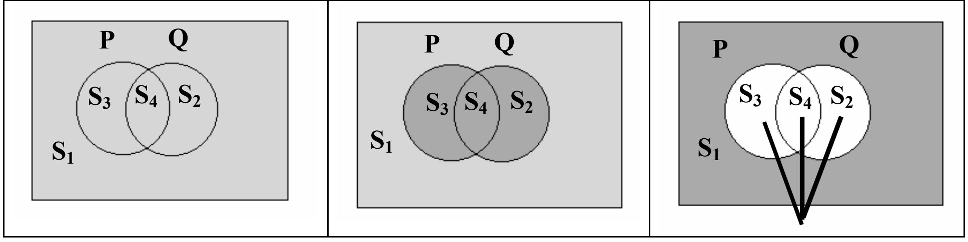
(162)

		cell structure of the universe	***Stage 0.*** Universe SIT = 1	***Stage 1.*** (1-(P+Q))	***Stage 2.*** (1-(1-(P+Q)) [+ (1-(P+Q)))]
P	Q		*value*	*Value*	*value*
1	1	S4	<u>*1?*</u>	***0***	***1***
1	0	S3	<u>*1?*</u>	***0***	***1***
0	1	S2	<u>*1?*</u>	***0***	***1***
0	0	S1	<u>*1?*</u>	<u>*1?*</u>	***0***

[*or*]

(163)

Stage 0.	Stage 1.	Stage 2.



[*or*]

Stage 0.	***Stage 1.***	***Stage 2.***
z ∈ S1-S4 → <u>z ∈ Sel?</u>	**z ∈ S2-S4 → z ∉ Sel** **z ∈ S1 → <u>z ∈ Sel?</u>**	**z ∈ S2-S4 → z ∈ Sel** **z ∈ S1 → z ∉ Sel**

At ***Stage 2.***, the members of all cells of the universe are positively specified as being elements of Sel (1) or not being elements of Sel (0), so that the set has been demarcated by being both set off against its complement (0 value: $\mathbf{z} \in \mathbf{S1} \rightarrow \mathbf{z} \notin \mathbf{Sel}$) and selected (1 values: $\mathbf{z} \in \mathbf{S2\text{-}S4} \rightarrow \mathbf{z} \in \mathbf{Sel}$). No indeterminacy – in the form of underlined values – remains and the PF- features of the union of the 1-cells are drawn in, i.e. lexicalisation of the set as [*or*] (square brackets indicate lexicalisation).[78] The set boundary between the selectees and the non-selectees, i.e. Aristotle's fundamentum divisionis, has thus been established. The non-selectees are those possible situations for which direct expression by non-negative propositions is not available. The actual selectees are the elements of S2-S4[79]. At this point, the lexical predicate or propositional function can be spelled out as ***or***, a lexical item with two variable arguments P and Q which have to be instantiated in syntax, i.e. filled with a choice of two concrete propositional constants to yield a regular *or*-proposition.

Let us look at the bracket part in (164), in which the lexical item *or* is used, and see how its interpretation draws on the abovementioned lexical properties of *or*:

(164) If [P: John is in the garden or Q: Bill is in the garden], I'm satisfied.

The (inclusive) lexical meaning of *or* determines that the set of selectees is the union of set of possible situations in which P is true (S3), the set of possible situations in which Q is true (S2) and the set of possible situations in which both P and Q are true (S4). It is to be noted in this respect that what is singled out by the disjunction P or Q is the <u>*set*</u> of selectees S2-S4, not a particular situation. There is not one situation which is singled out as the one being referred to, as being actual. Rather than actually selecting a situation, the two-stage process of set-demarcation cuts away all situations that are <u>not</u> compatible with the description *P or Q*. All the possible situations which <u>are</u> compatible with *P or Q* are therefore selected, not just a single referent situation. Depending on the context, the meaning will remain that way or be narrowed down from all possible situations compatible with *P or Q* to a single referent. In (164), the conditional context is one in which the bracketed constituent remains non-referential ("P or Q or both") and

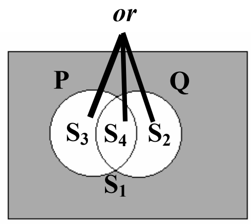
---

[78] For didactic purposes, this set demarcation procedure can be likened to the negative-positive process in photography. The Universe SIT is like the photosensitive film before use, on which any picture can in principle be produced, but none has been yet. <u>NEC</u> results in a negative image, which is comparable to exposure of a film to light which makes the areas exposed opaque on the negative. Reapplication of <u>NEC</u> (i.e. the same selection variant <u>NON</u>), finally, returns a positive copy. In photography, this second step means that the dark areas of the negative translate into light areas on the final representation, while the areas which have not been darkened on the negative, become dark in the negative-to-positive stage. In set-demarcation, the excluded areas become the selected ones at this stage, and those that had been neither selected nor deselected in the first step, become excluded, yielding the fully demarcated set.

[79] A point which will not be worked out or explained in detail in the text deserves mentioning because it is likely to be of use to formal semanticists. The two-step set demarcation procedure systematically cuts a (selectable) Boolean algebra down to a (selected) *join semilattice* (with the bottom S1 cut off). (For definitions and a very clear account of lattice theory, consult Szabolcsi (1987)). Szabolcsi (1987: 5) points out that: "Mathematically, meet semilattices and join semilattices are the same thing, only the relation is inverted. Linguistically, it may be interesting to note that while there are many applications for join semilattices, I do not know of applications of meet semilattices." If the set demarcation procedure outlined in the text is as general a procedure as I claim, the fact that join semi-lattices are so common constructs in semantics, but meet semi-lattices apparently play no role, finds a natural explanation.

represents the basic inclusive meaning of *or* described by the set-demarcation process outlined above.

(165)



It is only in the act of referring in referential contexts such as affirmatives that this lexical meaning with a complete set of selectees is narrowed down to a single, referential selectee, the actual situation:

(166) John is in the garden or Bill is in the garden

Though it is not clear yet to the speaker which of the two propositions P and Q is the true one, the affirmative context does impose referentiality, i.e. there is a *single*, referential selectee. Consequently, the inclusive part of the original reading ("both P and Q") is no longer available. There is a single, actual situation that obtains, and it is either P, or Q, but not both. In other words, the affirmative context imposes the exclusive disjunctive reading for *or*. Viewed this way, the negative-subtractive set demarcation device always selects a number of candidate referents by excluding all the rest, but whether any of the candidates left will be selected as the actual referent is determined by other factors than the lexical meaning of *or.*

4.4.3.1.4 Set-demarcation and characteristic functions

One of the advantages of the two-step negative operation based on a single operator outlined in 4.4.3.1.1-4.4.3.1.3 is that it decomposes the set-demarcating concept *characteristic function* (cf. § 3.2.2.1 and § 3.2.2.2) into a composite two-step function and hence eliminates it as a primitive, a valuable reduction that supports the analysis. Let us clarify this point. A description of the notion characteristic function (as a primitive) is provided by Seuren (1998: 341), who uses the expression '$x < 5$' to illustrate it:

> "The extension of the expression '$x<5$', or $\|x<5\|$, is the set of those numbers whose names, when substituted for x, result in a true proposition.
>
> For $x<5$ this is the set $\{0,1,2,3,4\}$"
>
> "…defaultwise, '$x<5$' expresses a *propositional function*, a function from the variable range, in this case the set of natural numbers, to propositions with a truth value, since for each natural number name put in the position of the substitutional variable there is a truth value. *Any function from a set of objects X to truth values is a propositional function*. If we use the symbol '**1**' for truth and '**0**' for falsity, $\|x<5\|$ constitutes the following infinite propositional function, expresses by '$x<5$':

(40) ‖x<5‖ = {<0,**1**>, <1,**1**>, <2,**1**>, <3,**1**>, <4,**1**>, <5,**0**>, <6,**0**>, <7,**0**>, <8,**0**>,…}

> Other names for a propositional function are *characteristic function* and *predicate*. The former name is used since the function delivers the value '**1**' for a subset Y of X, hence characterizes Y. In the context of linguistic expressions, our context, the preferred name for "a function from a set of objects X to truth values" is *predicate*" (Seuren 1998: 342).

If our negative deselection perspective on set-demarcation (as introduced on the basis of the lexical predicate *or*) is correct, two negative subtractive steps and hence stages are invariably required to identify a set positively. If so, this must be taken to apply to x<5 and its extension as well, with stage 0. and stage 1. once again abstract and underground due to their unselected *1?* values.

(167)

Stage 0.	Universe = variable range = set of natural numbers	= {<0,**1?**>, <1,**1?**>, <2,**1?**>, <3,**1?**>, <4,**1?**>, <5,**1?**>, <6,**1?**>, <7,**1?**>, <8,**1?**>,…}
Stage1.	1-{x | x<5}	= {<0,**0**>, <1,**0**>, <2,**0**>, <3,**0**>, <4,**0**>, <5,**1?**>, <6,**1?**>, <7,**1?**>, <8,**1?**>,…}
Stage2.	1-1-{x | x<5}	= {<0,**1**>, <1,**1**>, <2,**1**>, <3,**1**>, <4,**1**>, <5,**0**>, <6,**0**>, <7,**0**>, <8,**0**>,…}

On the whole, this must be taken to mean that the set-demarcating concept *characteristic function* is really the same composite two-step function based on a single operator that was employed for *or* and can hence be eliminated as a primitive.

Notice also the parallel between the two-step subtraction process described here to demarcate a set and the equivalence expressed by the logical law of double negation : $\neg\neg G \equiv G$. The former part of this equivalence is the logical expression of the two subtractive stages in the process of set demarcation, the latter the set thus demarcated. The fact that this law thus immediately falls out if the negative-subtractive version of set demarcation proposed in this chapter is adopted, is a strong argument in the latter's favour.

4.4.3.2 The predicate calculus pivots EITHER, ANY and AT LEAST ONE

The purpose of this section is to show how the bar-code pattern behind the meaning of the propositional calculus pivot plays a role in the realm of the predicate calculus as well. It can there be used to characterize the shared aspects of the meaning of the predicate calculus pivots EITHER, ANY and AT LEAST ONE[80]. The analysis proposed here exploits a similarity between different calculi. In particular, the parallelism between PROPC and APC_2 that has been mentioned a number of times suggests that a semantic description of *or* should not be completely dissociated from its APC_2 counterpart *either*. Consider the following set of sentences, the first with the propositional pivot *or* and the second and third with the corresponding APC_2 operator *either* and with F the <u>restrictor</u> term and G the <u>matrix</u> term:

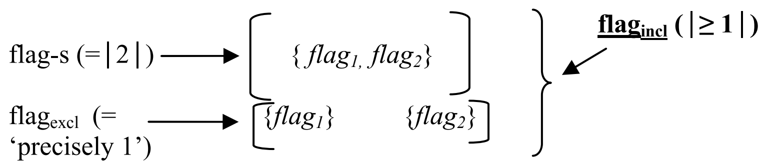
(168) a. I do not think that [[*flag₁*] is [*green*]] or [*flag₂*] is [*green*]] –
b. I do not think that [[*either flag*] is [*green*]]
c. I do not think that [[*either F*] is [*G*]]

A crucial assumption is that the denotation of bare semantic predicates such as *flag* does not just include all individual flags but also all the possible groups made up of those individuals[81]. Consider the following examples:

(169) a. I haven't not seen either *flag*$_{\mathbf{incl}}$. (neither the one, nor the other, <u>*nor both*</u>)
b. I haven't seen [[*flag*$_1$] or$_{\mathbf{incl}}$ [*flag*$_2$]]. (neither flag_1, nor flag_2, <u>*nor both*</u>)
c. Have you seen either *flag*$_{\mathbf{incl}}$? Yes, I have seen <u>*both*</u>/one of them.
d. Have you seen [[*flag*$_1$] or$_{\mathbf{incl}}$ [*flag*$_2$]]? Yes, I have seen <u>*both*</u>/one of them.

In the non-referential negative and interrogative contexts (169) a. and c., the singular bare semantic predicate *flag*$_{\mathbf{incl}}$ does not apply to individuals only, but also to pluralities (*both*). In other words, the lexical meaning of *flag* is not exclusively singular, but "inclusive" in that it denotes subsets of all possible cardinalities of the set of flags, except the null set, i.e. it denotes sets of cardinality $|\geq 1|$ ("at least one"). The universe being restricted to two individuals in APC_2, this inclusive interpretation brings in only one possible group with a cardinality beyond that of the individuals in the case at hand, namely $\{flag_1, flag_2\}$.

(170)



In sum, in (169) a. and c. *either* ranges not only over the individual flags but also over the pair of individuals which is part of the denotation of its restrictor term *F*.

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[80] The reading of *either* and *any* dealt with here is its particular existential reading in negative and interrogative contexts: *Have you met either/any of my colleagues?* Quasi-universal interpretations, as in *either man can tell you that* (~"both") and *any man can tell you that* (~"every") will be treated in chapter 6.

[81] This conception is inspired by Chierchia (2003), but differs from it in that Chierchia restricts the group interpretation to the morphological plural (i.c. *flags*) and does not view singularities as singleton sets. These differences are not crucial to the point made here.

A description of the underlying similarity between PROPC and APC_2 can then be brought out if (i) the Universe SIT of possible situations is replaced by the universe IND of individuals (comprising no more than two individuals of the restrictor type in the case of APC_2) and (ii) the two arguments of the APC_2-operator are $Flag_1$ and $Flag_2$, rather than the propositional constants P and Q of the PROPC. This makes EITHER ($Flag_1$, $Flag_2$) structurally analogous to OR_{incl} (P, Q) and results in the following two-step derivation[82].

(171)

		cell structure of the universe	***0.*** universe IND = 1	***1.*** (1-(F1+F2))	***2.*** (1-(1-(F1+F2)) **[**+ (1-(F1+F2)))**]**
F1	F2		*value*	*Value*	*value*
1	1	S4	*1?*	***0***	***1***
1	0	S3	*1?*	***0***	***1***
0	1	S2	*1?*	***0***	***1***
0	0	S1	*1?*	*1?*	***0***

[***either***]

Under the present conception, the meaning of *either flag* is characterized (both provisionally and informally) as "flag 1 (S3), flag 2 (S2) or flags 1 and 2 (S4) (with the cardinality of the extension of *flag* equal to 2)".

Next, the difference between APC_2 and APC in general is a matter of the extending the domain of quantification from two entities (F1, F2) to larger domains of quantification (Fx, Fy) by introduction of variables: I don't think that *any flag (x) or other (y)* is green, i.e. neither a random flag x nor any other flag y.

(172) a. I do not think that [[*flag*$_x$] is [*green*]] or [*flag*$_y$] is [*green*]] –
b. I do not think that [[*any flag*] is [*green*]]
c. I do not think that [[*any (Fx, Fy)*] is [*G*]]

Correspondingly, the meaning of *any idea* (with *idea* the restrictor term) in:

(173) Do you have *any idea* how this problem could be solved?

can be characterized as: "idea x (S3), idea y (S2) or any plurality of ideas (S4)". Here too, the meaning of the NP incorporates the feature "at least one" and the denotation of

[82] The fact that EITHER (x,y) is binary, does not make it a transitive predicate with subject-object difference like HATE (x,y). Note that it is like OR_{incl} (P, Q) in this respect (as it should be): the binary nature of OR_{incl} (P, Q) does not imply that its arguments are in a transitive subject-object relationship either.

the restrictor predicate *idea* is inclusive, fitting the cardinality description $|\geq 1|$. In a universe with three ideas a, b and c, for instance, this yields:

(174)

					idea$_{incl}$ ($|\geq 1|$)
		{abc}			
idea-s (= $|\geq 2|$)	{ab}	{ac}	{bc}	(s4)	
idea$_{excl}$ (= 'precisely 1')	{a}	{b}	{c}	(s2-s3)	

A corresponding example with *(at least) one* and its cardinality description $|\geq 1|$ is the following conversation:

(175) (about bagles) Have you eaten *one* yet? – Yes, I've even eaten two/three/four/…

Given (171), the pivotal meanings characterized informally in this section – *either, any*, and (*at least) one* – are suggested to have the same binary negative-disjunctive substrate as the propositional pivot *or* (cp. (171) with (162)). In bar-notation, this means:

(176)

EITHER, ANY, ONE			
Algebra		*Bar-code*	
bracketed	deleted	Bracketed	deleted
(1-(1-(Fx+Fy)) [+ (1-(Fx+Fy))])	(1-(1-(Fx+Fy)))	$\overline{\overline{Fx\ Fy}\ \ \overline{(Fx\ Fy)}}$	$\overline{\overline{Fx\ Fy}}$

The analysis proposed here can be improved to fit the relational perspective (Zwarts 1983, Van Benthem 1986, De Hoop 1992: 4) on Generalized Quantifiers. The latter views quantifying determiners D (i.e. *any,none, all,both,*etc ...) as two-place higher order predicates, i.e. as predicates over pairs of sets R(estrictor) and M(atrix predicate):

(177) a. Any F are G
b. ANY (F, G)
c. D (R, M).

Thus, the extension of *at least one flag* is the set of all sets of individuals X such that the cardinality of the intersection of the extension of flag and X is equal to or larger than 1.

(178) a. $\|\text{at least one flag}\| = \{X \subseteq I : |\ \|\text{flag}\| \cap X\ | \geq 1\}$

b. $\|$either flag$\|$ = $\|$at least one flag$\|$ iff $|\ \|$flag$\|\ |$ = 2, otherwise undefined[83].

Using lambda notation and the predicate labels R(estrictor) and M(atrix) – which are variables for elements in the universe of properties such as *flag, green,…* – to write the denotation of such generalized quantifiers, the formulas for *at least one* and *at least one flag* are:

(179) *at least one*

a. $\lambda R \lambda M\ [\,|\ R \cap M\ |\ \geq 1]$

b. $\lambda R \lambda M\ \exists x\ [\ R(x) \wedge M(x)]$

(180) *at least one flag*

a. $\lambda R \lambda M\ [\,|\ R \cap M\ |\ \geq 1]\ (\|\text{flag}\|)$ >

b. $\lambda M\ [\,|\ \|\text{flag}\|\ \cap M\ |\ \geq 1]$

c. $\lambda R \lambda M\ \exists x\ [\ R(x) \wedge M(x)]\ (F)$ >

d. $\lambda M\ \exists x\ [\ F(x) \wedge M(x)]$

Using the negative-disjunctive binary bar-notation instead, this perspective yields the following denotations for *either flag, any flag, at least one flag*[84]:

[83] These formalisations are notational variants of those in De Hoop (1992: 4). When a quantifier phrase can only be interpreted under certain conditions (in casu the restriction of the cardinality of the extension of F to 2) the interpretation function is partial (De Hoop 1992: 4). De Hoop's (1992: 4) description of the meaning of an *either F* phrase and ours share the feature that both postulate a close relationship between *either F* and *at least one F*. The only difference between them is that in the case of *either F* the cardinality of the extension of *F* equals two.

[84] This representation incorporates the claim that an existential $\exists x\ F(x)$ is really created from a binary distributive disjunction structure (*Fx or Fy*). My formalization is probably imperfect, since x and y occur as free variables in the bar-code and I am not sure which operator I should insert to bind them. In any case, the formalisation that is required would be of the type that also takes care of *one or other* in *one* or *other flag*, which also expresses existential quantification over flags by means of a distributive disjunction. My best guess is that the binder is the set of flags itself: compare *They talked to one another*, in which *they* is the binder set of its token-distinguished but type-identical members as denoted by the distributive structure *one another*. Similarly, in *one or other flag*, the words *one* and *other* are token-distinguished, but instantiate the same type *flag*.
In any case, the main claim behind the proposal should be clear from the text: the negative-disjunctive means used for the propositional calculus operators is at the root of the formalisation of the predicate calculus operators as well. This hypothesis will be further supported by the analysis of PROPC and APC contradictors and entailers, where the parallellisms are too striking to be accidental.

(181)

EITHER flag , ANY flag, AT LEAST ONE flag	
bracketed	deleted
$\overline{\overline{\lambda M\ \ Fx\,M\ \ Fy\,M}\quad \overline{(Fx\,M\ \ Fy\,M)}}$	$\overline{\overline{\lambda M\ \ Fx\,M\ \ Fy\,M}}$

That the bar-notation can be transferred from the propositional calculus (and the situations denoted by the propositional constants) to the predicate calculus (and the individuals that are quantified over in restrictor phrases) evinces the generality of the underlying system in terms of <u>NEC</u> and of the Boolean algebra within which it operates. It will be argued in chapter 5 that the algorithm can be generalized further and underlies the singular-plural distinction and the natural number scale.

The descriptions given for the internal semantic structure of propositional calculus and predicate calculus pivots have all been "inclusive" cases, i.e. those for which S4 is part of the interpretation. As was pointed out with respect to the exclusive, referential reading of (166), exclusive *or (= or but not and)* and hence also the "exclusive" reading of the corresponding predicate calculus pivot *some (but not all)* need special provisions if the set of selectees is to be narrowed down from S2-S4 to the exclusive denotation S2-S3. These additional constraints will be gone into in more detail and given a formal characterization in chapter 5. For the time being, the attention will remain focused on the inclusive interpretation of pivots, which is the most basic one. It is the latter meaning which enters into the meanings of contradictors, taken up in the next section.

4.4.4 Contradictors

In this section, the bar-codes for contradictors and the way they derive from pivotal meanings will be outlined. The descriptions shed light on the nature of negation and on the concept of a characteristic function, which is a complex rather than a primitive notion.

4.4.4.1 The proposition calculus contradictor NOR_{incl} (P,Q)

As its morphology betrays, the proposition calculus contradictor *nor* is derived from *or*. The compositional semantics of *nor* will therefore be based on that of the pivotal semantic predicate OR_{incl} (P,Q). Since OR_{incl} (P,Q) is a stage 2. item (in bar code it has two bars), we take spelled out (182) (represented by square brackets) as input

(182)

OR (P,Q) [***or***]	*algebra*	*bar-code*
	(1-(1-(P+Q))	$\overline{\overline{P\ \ Q}}$

and apply the rule <u>NON</u> (i.e. same-selection <u>NEC</u>, cf. (159)), yielding the stage 3. (hence triple-bar) lexical item *nor*:

(183)

N-OR [***nor***]	*algebra*		*bar-code*	
	bracketed	deleted	bracketed	deleted
	(1-(1-(1-(P+Q))) [+ (1-(1-(P+Q))))]	(1-(1-(1-(P+Q)))	$\overline{\overline{\overline{P\ \ Q}}\ \ \overline{\overline{[P\ \ Q]}}}$	$\overline{\overline{\overline{P\ \ Q}}}$

The diagram in (184) shows that the effect of <u>NON</u> is to reverse all the values of stage 2. OR_{incl}:

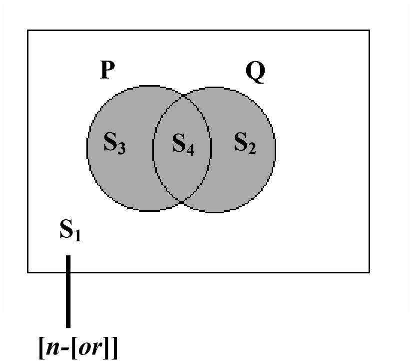
(184)

P	Q	cell structure of the universe	***0.*** Universe SIT = 1	***1.*** (1-(P+Q))	***2.*** (1-(1-(P+Q)) [+ (1-(P+Q)))]	***3.*** (1-(1-(1-(P+Q))) [+ (1-(1-(P+Q))))]
			value	*value*	*value*	
1	1	S4	*<u>1?</u>*	***0***	***1***	***0***
1	0	S3	*<u>1?</u>*	***0***	***1***	***0***
0	1	S2	*<u>1?</u>*	***0***	***1***	***0***
0	0	S1	*<u>1?</u>*	*<u>1?</u>*	***0***	***1***

[or] ***[n-[or]]***

As in the case of OR_{incl} at ***stage 2.***, the members of *all* the cells of the universe are positively specified at ***stage 3.*** as being elements of Sel (=1) or not being elements of Sel (=0), so that the set has been demarcated by being both set off against its complement (0 values: **z ∈ S2-S4 → z ∉ Sel**) and selected (1 values: **z ∈ S1 → z ∈ Sel**). Consequently, lexicalisation as **[*n*-[*or*]]** can occur. The resulting Venn-diagram is:

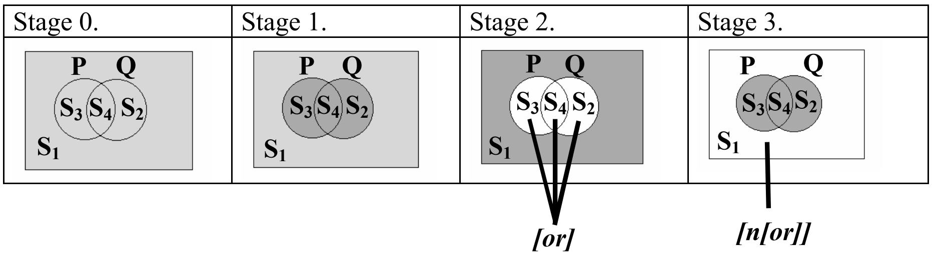
(185)



From the perspective drawn above, a number of features of natural language negation fall in place.

First of all, linguistic negation in the form of a negative lexical item (morpheme or word) invariably takes a lexicalised argument at ***stage 2.*** to apply to. This is due to ***stage 1.*** being indeterminate and non-expressible ($\mathbf{z} \in \mathbf{S1} \rightarrow \underline{\boldsymbol{z \in Sel?}}$), so that two further steps are required to attain a lexicalizable stage for the negative.

(186)



Stage 0.	***Stage 1.***	***Stage 2.***	***Stage 3.***
$z \in S1\text{-}S4 \rightarrow$ $\underline{z \in Sel?}$	$z \in S2\text{-}S4 \rightarrow$ $z \notin Sel$ $z \in S1 \rightarrow$ $\underline{z \in Sel?}$	$z \in S2\text{-}S4 \rightarrow$ $z \in Sel$ $z \in S1 \rightarrow$ $z \notin Sel$	$z \in S2\text{-}S4 \rightarrow$ $z \notin Sel$ $z \in S1 \rightarrow$ $z \in Sel$

That lexicalised negative forms are only attainable at ***stage 3.*** in our model confirms the general intuition that negatives are literally marked as opposed to affirmatives and that to negate, there has to be something to be negated.

> "With respect to the vocal sound, affirmative enunciation is prior to negative because it is simpler, for the negative enunciation adds a negative particle to the affirmative."
>
> (Thomas Aquinas, commentary on Aristotle, *De Interpretatione*, in Oesterle 1962: 62; cited from Horn 1989: 154)

> "The fact that affirmatives are unmarked and negatives are linguistically marked is completely correlated with the finding in this study and previous studies that affirmatives are psychologically less complex than negatives."
> (Just and Carpenter 1971: 248-249; cited from Horn 1989: 154)

This is entirely in agreement with our view that a third 1-…-step is required to get beyond ***stage 2.*** (a lexicalised argument) to a lexicalisable complement *not-X* of lexicalised X (i.c. *or*).

Secondly, the fact that negation has the semantic force it has, namely 1-…, ties in well with the fact that 1-… (exclusion) is the only operation on values available in the whole system anyway, which is turned to both to demarcate an expressible set X itself (in two steps) and its expressible complement (by means of a further step).

4.4.4.2 The contradictor NOT (P,P)

For the derivation of the propositional operator *not*, exactly the same three stage-analysis is required as for *n(e)-*. This time, the argument of the negative is a single proposition however. The two lower bars are therefore supplied by a propositional constant P:

(187)

NOT	$\overline{\overline{\overline{P}}\ \ \overline{\overline{P}}}$	≡	$\overline{\overline{\overline{P}}}$

(188)

		cell structure of the universe	***0.*** Universe SIT = 1	***1.*** (1-(P+P))	***2.*** (1-(1-(P+P)) [+ (1-(P+P)))]	***3.*** (1-(1-(1-(P+P))) [+ (1-(1-(P+P))))]
P	P		*value*	*value*	*value*	
1	1	S2	*1?*	***0***	*1*	*0*
0	0	S1	*1?*	*1?*	***0***	***1***

[P] **[NOT (P)]**

As mentioned above, all lexicalised forms of negation (word-internal, phrasal or sentential) need a *linguistic* argument (morpheme, phrase, clause), which is semantically a fully demarcated set (i.e. a ***stage 2.*** representation) to operate on.

(189) a. word-internal negation: [*n*-[or]], [*n*-[ever]]
b. phrasal negation: [*not* [a single book]] was sold

c. clausal negation: Mary is *not* in Spain ([NOT [P: MARY BE IN SPAIN]])

In this respect, lexicalised negations are more complex than the single bar rule of negation NON of (159), which operates at the prelexical level already (to turn ***stage 1.*** into ***stage 2.***). NON was shown to be crucially able to take non-lexicalised representations as input. In sum, negation as a lexical item is lexicalised at ***stage 3.*** only and is hence more complex than its underground operational counterpart NON (“1-…”) at the prelexical level. Though extensionally equivalent in terms of what they exclude, the lexical item has positively identified selectees set off against non-selectees, whereas NON is purely exclusionary. It remains indeterminate about any positive values and hence non-lexicalisable.

Horn (1989: 257) calls any Peircean type analysis of negation “as unnatural as it is elegant, as emerges clearly from the attempt to think intuitively of *not p* as a shorthand for ‘neither **p** nor **p**’ or for ‘not **p** and **p**’.” But as is often the case in scientific analysis, feelings of unnaturalness and elegance are highly deceptive and moreover highly dependent on the theoretical framework adopted. Negation being a propositional operator, the fact that it is the only propositional operator with a single argument among a series of binary ones cries for an explanation in anybody’s theory. In a system with twin selection due to a single underlying binary operator, not only the Law of Contradiction can be derived as Boole did, but the derivation of negation in terms of same-selection and conflation gives a principled *explanation* for the fact that it is the sole *one-term* operator of the calculus. It seems to me that same-selection and conflation, which are independently needed, thus provide a plausible (though maybe not commonsensically intuitive) account[85].

Identifying the propositional operators as a unified set – all binary underlyingly – and being able to relate them all and derive them from a single category is an instance of unification. This means that it is a valuable move to embrace the idea that the joint falsehood operator (i.e. NEC) is conceptually primitive (pace Gale 1976: 6; Horn 1989: 257). Since its adoption gives a principled reason for the one-term nature of negation, the cohesion of all propositional operators, and to top it all reduces the number of primitives considerably, barring it as a primitive on grounds of purely subjective counterintuitivity is unwarranted.

4.4.4.3 The predicate calculus contradictors NEITHER and NO(NE)

To transfer the analysis for NOR (P,Q) to the predicate calculus contradictors, (i) the universe SIT of possible situations is once more traded in for the universe IND of

[85] An argument for such a Peircean type of account for negation, comes from Wittgenstein (1975: 11, section 5.1311), who argues that the symbols (“Bezeichnungsweise”) standardly used for disjunction and negation conceal the underlying relationship between them:

> “Wenn wir von $p\vee q$ und ~p auf q schliessen, so ist hier duch die Bezeichnungsweise die Beziehung der Satzformen von ‘$p\vee q$’ und ‘~p’ verhüllt. Schreiben wir aber z.B. statt ‘$p\vee q$’ ‘p ∣ q. ∣ .p ∣ q’ und statt ‘~p’ ‘p ∣ p’ (p ∣ q = weder p, noch q), so wird der innere Zusammenhang offenbar.” (Wittgenstein 1975: 11, section 5.1311)

individuals (comprising no more than two individuals in the case of APC_2) and (ii) variables over propositional constants P and Q are replaced by the variables x and y over sets of individuals denoted by the restrictor term F, e.g. table-set_x or table-set_y.

By applying <u>NON</u> to (171) and (176), *neither table* and *no table* are derived as the complements of the sets denoted by *either table* and *at least one table* respectively.

(190)

		cell structure of the universe	***0.*** universe IND = 1	***1.*** (1-(x+y))	***2.*** (1-(1-(x+y)) [+ (1-(x+y)))]	***3.*** (1-(1-(1-(x+y))) [+ (1-(1-(x+y))))]
x	y		*value*	*value*	*value*	
1	1	S4	*<u>1?</u>*	***0***	*1*	***0***
1	0	S3	*<u>1?</u>*	***0***	*1*	***0***
0	1	S2	*<u>1?</u>*	***0***	*1*	***0***
0	0	S1	*<u>1?</u>*	*<u>1?</u>*	***0***	*1*

[*either*]/[-at least one] [*n*-[*either*]]/[*n*-[*o(ne)*]]

4.4.5 Entailers

The derivation of the semantic content of entailers represents a more radical departure from existing analyses than that of pivots. In this section the compositional semantics in bar-codes will be provided for the propositional calculus operator *and* and the predicate calculus operator *all*. The codes proposed are the most complex ones in the system in terms of elaborateness, i.e. the number of times <u>NEC</u> is applied. They do not add any further algorithmic complexity, however, since the only tool used throughout is the single operator <u>NEC</u> and multiple use of the same rule only adds to elaborateness, not to complexity of the rule system (algorithm). Here I will limit myself to presenting the proposal for the propositional and the predicate calculus entailers and provide an argument in its favour involving what is known in the predicate calculus as the notion of existential import. This concept will be explained and illustrated and the problem it poses will be shown to be solved if the composite structure of entailers as proposed in this study is accepted.

4.4.5.1 The propositional calculus entailer AND (P,Q)

The idea guiding our analysis of the 2D Cartesian Coordinate System has been that in extensional set-theoretic terms each step in the composition of the meaning of an expressible operator must differ from the previous one by a more comprehensive exclusion of cells due to the informativeness constraint (§ 3.3.4). In intensional terms, this is achieved by adding further specifications in terms of the basic <u>NEC</u>-operator. More precisely, I contend that the intensional meaning of AND (P,Q) is more highly specified and hence more complex than that of OR_{incl} (P,Q) : it consists of the meaning of OR_{incl} (P,Q) as specified above in terms of NEC and an additional semantic feature to be identified. The meaning of OR_{incl} (P,Q) that was identified in our NEC-flowchart (162) above, is the inclusive one (‘*at least one of P or Q has value 1*’). That this property *can* be viewed as part of AND (P,Q) is straightforwardly true: if *P and Q* is true, i.e. both propositions have value 1, then it is necessarily the case that *‘at least one of P or Q has value 1’*, which makes the following equivalence legitimate.

(191) $P \wedge Q \equiv [P \vee Q] \wedge [\neg [\neg P \vee \neg Q]]$

$P \wedge Q$	$\equiv$	$[P \vee Q]$	$\wedge$	$[\neg [\neg P \vee \neg Q]]$
		‘at least one of P or Q has value 1’, i.e. *‘P or Q or both have value 1’*	and	*‘neither P nor Q have value 0’*

The first part of the longer expression is OR_{incl} (P,Q) with its *at least one* requirement. The added second part boils down to the requirement that neither P nor Q have value 0.

Standard analyses have held that the second part alone suffices to get the truth table for *and*. So why include the meaning of OR_{incl} (P,Q) if AND (P,Q) can apparently be reached in a much simpler, single step $[\neg [\neg P \vee \neg Q]]$?

The answer is that if the cognitively realistic negative subtractive derivation on the basis of NEC is on the right track, the item AND (P,Q) *cannot* be reached in one step, contrary to what is built into most logical systems. Let us spell this out: assume first of all that in the Language of Thought there is a variant of NEC with two negative arguments $\neg X$ and $\neg Y$ as input. The format of NEC specified in (154) does not preclude this possibility. This variant of NEC is the general rule of conjunction ET, since its formal structure is identical to the traditional way of defining conjunction in negative-disjunctive logical terms: $[\neg [\neg X \vee \neg Y]] \equiv X \wedge Y$.

(192)

ET (P, Q) = NEC ($\overline{X}$, $\overline{Y}$)	
Logic	$\overline{X}, \overline{Y} \rightarrow \overline{\overline{X}\ \overline{Y}}$
Set-theory	$\overline{/X/}, \overline{/Y/} \rightarrow \overline{\overline{/X/} \cup \overline{/Y/}}$
Algebra	(1-X), (1-Y) $\rightarrow$ (1 – ((1-X)+(1-Y)))

Why cannot direct application of this operation to the Universe SIT (i.e., without OR_{incl} (P,Q) as prior input) yield the desired result in a single step? The theoretical answer I propose and will try to further motivate on empirical linguistic grounds throughout this study is that *exclusion (1-...) can only apply successfully to an initial cell if there is something that can be excluded*, i.e. if there is a 1 value for P or Q. This squares well with the view that negation needs something positive to operate on. Let us adopt this hypothesis and try to apply ET to the universe SIT.

(193)

		cell structure of the universe	***0.*** SIT = 1	***1.*** *(1-((1-P)+(1-Q))
P	Q		*value*	*Value*
1	**1**	S4	*1?*	***1?***
1	0	S3	*1?*	*(1? –* **1** *=)* ***0***
0	**1**	S2	*1?*	*(1? –* **1** *=)* ***0***
0	0	S1	*1?*	**(1? – 0 =) 1?*

The result that has to be obtained to get the meaning of AND(P,Q) is (a) exclusion of cells S1-S3 and (b) selection of S4. On both counts, however, immediate application of ET fails. As the boldface values for P and Q indicate, only S2-S4 contain positive values. As there is no positively valued P or Q in S1 for subtraction to yield 1-**1**=0, sequence (193) fails to result in complete exclusion of S1-S3. Specifically, S1 cannot end up with a zero value, even though it should to represent the content of the formula (1-((1-P)+(1-Q)). As far as selection of S4 is concerned: on this count immediate application of ET to SIT fails as well: the value of S4 at stage 1. is that of a selectable, not that of a selectee yet[86].

ET as described here is as said a conjunctive variant of NEC. If the above hypothesis will prove to be correct, ET is different from the lexicalised conjunction *and*, just as the semantic structure (159) of NON is not to be identified with that of the lexical item *not* of (187) in terms of internal semantic complexity (cf. § 4.4.4.2). This is because ET – like NON – is a variant of NEC which is (a) purely exclusionary and hence non-expressible and (b) impossible to apply directly to the universe SIT because the algebra behind NEC-operations (including ET) can only affect initial positive values.

[86] The force of this theoretical assumption will depend on how many empirical arguments can be provided that the meaning of AND(P,Q) indeed necessarily contains that of OR_{incl} (P,Q). The logician will recognize in the latter hypothesis the propositional calculus counterpart of the so-called property of existential import in the predicate calculus. The more arguments that can be provided for some version of the latter property (cf. § 4.4.5.2), the more solid the present proposal for the propositional calculus.

Initial plausibility for the "affect initial values **1** only" assumption is provided by the fact that this is precisely what an ordinary NEC sequence does (cf. (194)): S2-S4 all have at least one positively specified (boldface) item P or Q, so that *1?* – **1** will yield 0 as required. Note that one might even conclude that NEC is what it is precisely because all 1-… can do is *switch all cells with an initial 1 value to 0*, which could only be NEC, (algebraically: (1-(P+Q))).

(194)

		cell structure of the universe	***0.*** SIT = 1	***1.*** (1-(P+Q))
P	Q		*value*	*Value*
1	**1**	S4	*1?*	*(1? –* **1** *=) 0*
1	0	S3	*1?*	*(1? –* **1** *=)* ***0***
0	**1**	S2	*1?*	*(1? –* **1** *=)* ***0***
0	0	S1	*1?*	*(1? –* 0 *=) 1?*

The problem with the conjunction cell S4 of (193) is solved and the need for positive identification of selectees is met if the pivotal meaning OR (P,Q) is taken to be part of the meaning of AND (P,Q), basically as specified in (191). One thing has to be changed in formula (191), however. If the cognitively real system is the negative NEC-system, as I am trying to establish, and it turns on the joint falsehood operator NEC (= $\neg\vee$), then we have to apply De Morgan to (191) – repeated here as (195)a. – to turn it into the NEC-compatible formula (195)b.:

(195) a. $P \wedge Q \equiv [P \vee Q] \wedge [\neg [\neg P \vee \neg Q]]$
b. $P \wedge Q \equiv \neg[\neg[P \vee Q] \vee [\neg P \vee \neg Q]]$

Converted to the bar-notation, we then get:

(196)

$$\overline{\underset{\textbf{(i)}}{\overline{P\ \ Q}}\quad \underset{\textbf{(ii)}}{\overline{P}\ \ \overline{Q}}}$$

Since this formula evidently contains two twin selections of P and Q, I shall initially consider each of them separately and then determine how they are connected. Performing the double-bar exclusion instruction of the first half **(i)** of the above complex formula

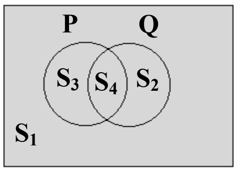
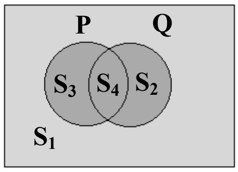
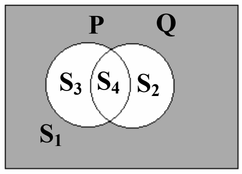
(197)

$$\underset{\textbf{(i)}}{\overline{\overline{P\ \ Q}}}$$

we get the diagram for OR_{incl} (P,Q) with positive identification of selectees in the usual two stages.

(198)

		cell structure of the universe	***0.*** Universe SIT = 1	***1.*** (1-(P+Q))	***2.*** (1-(1-(P+Q)) [+ (1-(P+Q)))]
P	Q		*value*	*Value*	*value*
1	1	S4	*1?*	***0***	***1***
1	0	S3	*1?*	***0***	***1***
0	1	S2	*1?*	***0***	***1***
0	0	S1	*1?*	*1?*	***0***

		
Stage 0.	***Stage 1.***	***Stage 2.***
z ∈ S1-S4 → ***z ∈ Sel?***	**z ∈ S2-S4 → z ∉ Sel** **z ∈ S1 →** ***z ∈ Sel?***	**z ∈ S2-S4 → z ∈ Sel** **z ∈ S1 → z ∉ Sel**

In the sense that it positively identifies selectees, this can be called the existential part Or_{incl}, guaranteeing the existence of selectees, which is the first stepping stone in the construction of the meaning of AND (P,Q).
As opposed to what happened in the transition from OR_{incl} (P,Q) to its contradictor NOR_{incl} (P,Q), the meaning at ***stage 2.*** is not lexicalised this time. The effect of lexicalisation in the case of the *or-nor* transition was to make the meaning of *or* available for operations which could *change* that meaning and in effect had it turned into its contradictory. In the case of the transition from OR_{incl} (P,Q) to AND (P,Q), however, the meaning of OR_{incl} (P,Q) does not get lexicalised and is thereby kept immune from such modification: OR_{incl} (P,Q) remains active as it is and contributes its meaning unmodified to the more complex construct AND (P,Q).

This brings us to the second part **(ii)** of formula (196)

(196)

$$\overline{\overline{P}\ \overline{Q}}$$

(ii)

which is the application of rule ET of (192) to P and Q, with the purely exclusionary effect that is typical of ET. This yields the following stage 3. addition to the diagram:

(199)

		cell structure of the universe	***0.*** Universe SIT = 1	***1.*** (1-(P+Q))	***2.*** (1-(1-(P+Q)) **[**+ (1-(P+Q)))**]**	***3.*** (i) (1-(1-(P+Q))) **[**+ (1-(1-(P+Q))))**]** (ii) (1-(1-P) + (1-Q))
P	Q		*value*	*value*	*value*	
1	1	S4	*1?*	***0***	***1***	***1***
1	0	S3	*1?*	***0***	***1***	***0***
0	1	S2	*1?*	***0***	***1***	***0***
0	0	S1	*1?*	*1?*	***0***	***0***

/OR (P,Q)/ **[*and*]**

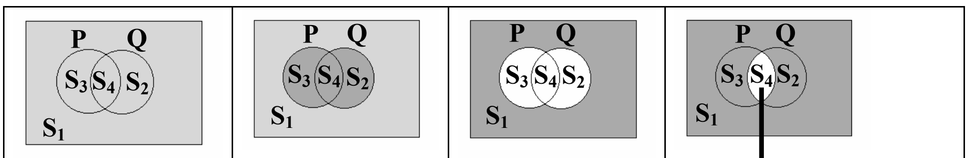
All in all, to get to AND (P, Q) from OR_{incl} (P,Q):

(i) the meaning of OR_{incl} (P,Q) is contributed as is and

(ii) the additional, second instruction is ET (P,Q), i.e. an instruction to exclude what is not P, i.c. the cell S2 of Q that is outside P, and to exclude what is not Q, i.c. the cell S3 of P (and vacuously also the cell S1 outside both P and Q, which is also excluded by **(i)**).

While **(i)** positively selects the members of the union of P and Q (S2-S4), **(ii)** cuts that positive selection down to the members of the intersection of P and Q (S4). This results in the following Venn-diagrammatic derivation for the lexical item *and*, which positively identifies the selectees in the intersection of P and B: both P and Q have value 1. It does so asymmetrically: **(i)** is prior to **(ii)** in that the latter is purely exclusionary and therefore cannot denote a set without the former (cf. (193)).

(200)



/OR (P,Q)/ ***and***

Stage 0.	***Stage 1.***	***Stage 2.***	***Stage 3.***
z ∈ S1-S4 → ***z ∈ Sel?***	**z ∈ S2-S4 →** **z ∉ Sel** **z ∈ S1 →** ***z ∈ Sel?***	**z ∈ S2-S4 →** **z ∈ Sel** **z ∈ S1 →** **z ∉ Sel**	**(i) the meaning of OR_{incl} (P,Q):** **z ∈ S2-S4 →** **z ∈ Sel** **z ∈ S1 →** **z ∉ Sel** **(ii) ET (P,Q)** **z ∈ S1-S3 →** **z ∉ Sel** **ergo:** **z ∈ S4 → z ∈ Sel**

The crucial difference between the conjunctive cell S4 of ***stage 3.*** in the above derivation (199) and the conjunctive cell S4 of ***stage 1.*** in (193) is that the intersection cell S4 is now positively identified as representing a set of identified selectees, whereas S1 remained lightly shaded in (193). That difference constitutes the contribution of postulating that OR_{incl} (P,Q) in (199) is part of the meaning AND (P,Q) of the lexical item *and*: at least one of P and Q has value 1 and hence there are selectees. Due to the further constraint imposed by ET (P,Q) the set of actual selectees is narrowed down to elements in the intersection of P and Q.

A final point concerns the nature of the compositional structure

(201)

$$\overline{\underset{\textbf{(i)}}{\overline{P\ \ Q}}\quad \underset{\textbf{(ii)}}{\overline{P}\ \ \overline{Q}}}$$

As formulated here in bar-code, it looks as if all is obtained in a single operation since there are only two levels of bars. Yet, the double occurrence of the pair P Q and the appeal to **(i)** and **(ii)** already indicate that something more complex than a single twin selection is involved. If we look at the segmented ***stage 3.*** description,

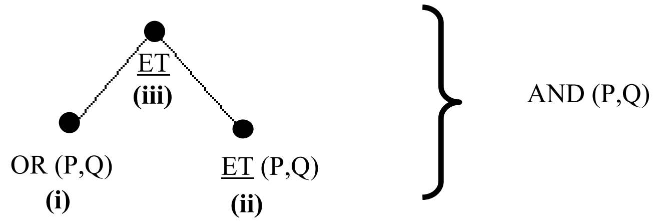
(202)

Stage 3.
(i) (contribution of stage 2., i.e. the meaning of ORincl (P,Q)) **z ∈ S2-S4 → z ∈ Sel** **z ∈ S1 → z ∉ Sel** **(ii) (contribution of ET (P,Q)**

$z \in$ S1-S3 $\rightarrow z \notin$ Sel
ergo:
$z \in$ S4 $\rightarrow z \in$ Sel

it is clear that the latter consists of two separate parts **(i)** and **(ii)**, the first the contribution of OR_{incl} (P,Q), the second the contribution of the exclusionary rule ET (P,Q). But how are these two linked in prelexical syntax? How is the *ergo*-conclusion arrived at? To begin with, the relationship between **(i)** and **(ii)** cannot be the same as that between ***stage 2.*** *or* and ***stage 3.*** *nor*. The latter was the result of the application of NON to ***stage2.*** The relationship between **(i)** and **(ii)**, however, is not a set-complement relationship. Rather, the two parts **(i)** and **(ii)** have to be structurally *conjoined* (set-theoretically: intersected) to yield the final interpretation that **$z \in$ S4 $\rightarrow z \in$ Sel**. Since conjunction was most minimally defined as ET above, it is the latter operation by which I claim **(i)** and **(ii)** are conjoined. Using **(iii)** to denote this third component, the complete description of the semantic structure AND (P,Q) of the lexical item *and* is: the conjunction **(iii)** (= ET) of **(i)** OR_{incl} (P,Q) and **(ii)** ET (P,Q).

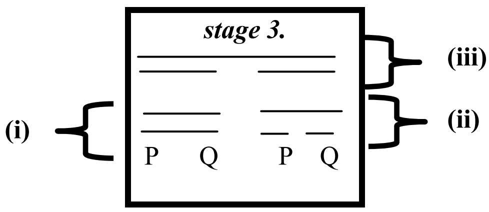
(203)



(i) OR (P,Q)	**(iii)** ET	**(ii)** ET (P,Q)	= AND (P,Q)
at least one of P and Q has truth value 1	ET	neither P nor Q have truth value 0	= AND (P,Q)

Note that **(i)** is the *positive* contribution to the set of selectees, its effect being that P or Q *have* a truth value 1, while **(ii)** is a *negative* contribution in the sense of being purely exclusionary. In negative-disjunctive bar-notation, we get the following elaborate but algorithmically optimally simple formula:

(204)



The upper bar and the two shorter ones immediately below it are the contribution of the highest conjunction ET **(iii)** (cf. the bar-structure in (192)); the lowest two equal bars **(i)** over the first instance of P Q are the contribution of OR_{incl} (P,Q) (cf. the bar-code in

(182)); the lowest two levels **(ii)** over the second instance of P Q represent ET (P,Q) (cf. the bar-structure in (192)). Though elaborate, the resulting structure is very simple, since all that is used is NEC and its subvarieties NON and ET. As said before, the latter are nothing but instances of NEC with a particular choice of arguments, namely twice the same nonnegative argument combined with conflation in the case of NON (cf. (159)), and two different negative arguments in the case of ET (cf. (192)).

The elaborate formula in (204) is logically equivalent to the simpler formula given in (201) (as required). This is not hard to establish, given that vertically, two consecutive bars with the same scope can be erased whenever they occur (given the law of double negation). By erasing the two middle bars over each P Q pair in (204), the complex formula of (204) is reduced to the simpler one of (201). This reduction due to extensional equivalence should however not be read as implying that cognition actually needs to apply such a reduction. Since it needs to set up the more elaborate formula anyway, applying the reduction is additional work that does not change anything semantically and is hence superfluous. The morale of this tale is important: mental computation at the level of the Language of Thought is just as automatic and cheap as repetitive computer computation. The Language of Thought is good at the same type of computation that astounds us in a computer: it has no trouble whatsoever with elaborate repetitive patterns as long as the set of rules and primitives is minimal, the rules are inviolable and hence their application non-creative. It is from the natural language lexicon upwards, where syntactic on-line computation begins, that there is free choice in the use of resources from the lexicon, awareness of rule application and hence ability to break the rules. Consequently, computation in the syntactic module is a radical departure from computer computation: it is no longer the cheap, non-creative, boringly repetitive, minimalist and elaborate computation of the Language of Thought. From lexical selection onwards, creativity, reliance on short term and mid-term memory and the attendant need for compactness of expression play an important role. This, I believe, is why each lexical item has the structure of a signe: as it is the interface between the Language of Thought and syntactic computation, it needs to match a complex semantic configuration with a usable label. It is this lexicon with its compactly packed complex semantic structures, combined with the syntactic ability to apply the rules of Merge and Move to such lexical material which I believe constitute the basis for Chomsky's Creative Aspect of Language Use (CALU)[87]. They make the crucial difference between natural language on the one hand and algorithmically less complex but configurationally unwieldy cognitive systems such as nonlinguistic Language of Thought.

[87] Chomsky himself keeps CALU out of syntax, as he considers the latter to be in its entirety abstract and part of the competence system. Yet, lexical selection clearly involves free choice. Moreover, I fail to understand why the particular (necessarily finite) number of times one *uses* Merge and Move in a certain derivation would be entirely abstract. It seems more natural to assume that *use* of these rules represents concrete action in time (with possibly a lot of that action parallel activity of course), hence part of performance. The lexical items used as well as the templates of Merge and Move themselves, however, are part of the competence resources in long term memory and called up in the course of the creation of a derivation. It seems natural to distinguish between such knowledge "at rest" (competence) and actual use of that knowledge in a creatively formed, hence concrete linguistic structure (performance) in this fashion.

4.4.5.2 The predicate calculus entailer ALL

In this section, the bar-code developed for the PROPC entailer *and* will be transferred to the predicate calculus entailer *all*. This will shed new light on the position I am driven to regarding one of the distinguishing features of the predicate calculus entailer *all*, namely that it poses the problem of existential import[88]. Consider once more the sentence:

(205) All flags are green - All F are G

Imagine a situation to which this sentence applies, but in which there are no flags. The question then is: is sentence (205) true or false? These are the only two options in Aristotle's predicate calculus (APC) but whichever is chosen, the calculus collapses. If the sentence is taken to be true, it follows by subaltern entailment that some flags are green, which in turn entails that there are flags, which is in conflict with the situation involved. If taken to be false, it would follow that some flags are not green, which once again entails by existential import that there are flags, counter to fact in the situation at hand. "In fact, the whole of APC turns out to have existential import, as it requires the non-emptiness of the F-class. Only non-empty predicates are allowed in the F-position." (Seuren 2002: 22)

Seuren (2002: 22) concludes that this

> "is fatal for any logic. For if a logic is, as we have agreed it is, a method for the computation of entailments merely on grounds of the semantic definition of the logical constants, then it must be irrelevant whether or not there exist, right now, any instances of *F*, be they gnomes or one-hundred-year-old Scots, or black swans."

How has this problem been dealt with? The choices that the history of the discipline provides are on the one hand Russell's Modern Predicate Calculus, on the other hand Strawson's (1952) presupposition approach. Russell's solution is a typical EX-logical solution: he exploits the set-theoretical fact that the null set is a subset of any set and consequently considers examples like (206) true in this world since the empty F-class (*unicorns*) is a subset of the G-class (*live in Georgia*).

(206) (There being no unicorns, it is set-theoretically accurate to affirm that)
All unicorns live in Georgia

If given the part between brackets, ordinary speakers would have to acknowledge the scientific correctness of what is said, but would probably consider the speaker a bit of a "smart aleck" (Abusch & Rooth (2004)) using a clever science trick to be put in the right. The feeling is typical of "unnatural" EX-scientific solutions that are surprisingly accurate but go against natural intuitions (cf. the discussion about the difference between

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[88] For a revised version of Aristotelian Predicate Calculus in which the existential import problem is solved in the way outlined here, cf. Seuren 2002. The point of invoking Seuren's theory, is that it can be shown to confirm our bar-code decomposition. Moreover, the bar-code analysis suggests an expansion of the proposal to the propositional calculus entailer *and*.

EX-scientific 'language' and natural language in chapter 2). While most valuable for scientific purposes and hence of course not to be discarded in that respect, as a description of the meaning of natural language *all*-sentences Russell's solution is problematic. In some cases, such as the following set of examples from Seuren (2002: 24) it is radically in conflict with natural judgments. If Russell's approach were right, all of these examples would be true in any situation in this world without gnomes (but with visitors in c. and d.). The intuitive judgments of ordinary speakers are different (marked as True or False between brackets behind the examples).

(207) a. All gnomes are fictitious (T)
b. No gnome is fictitious (F)
c. Some visitors talked with all gnomes (F)
d. Some visitors didn't talk with any gnome (trivially T) (Seuren 2002: 24)

Aside from this empirical trouble, Russell's solution demolishes most of the Square of Oppositions. As is well-known, it makes nearly all relations in it invalid: subaltern entailments, contraries, subcontraries. Since so many aspects of the semantics of natural language and IN-logic seem to find a natural place in the Square (and have for the larger part of two millennia), and since it turns out to be possible to set up a very minimalist version of the calculus in terms of a single negative-disjunctive operator, giving nearly all of that up because a single problem *appears* insoluble within its confines, seems unwise.

The more so since another option is suggested by the experienced effect of sentences like (206) on natural language users. If its bracketed part is not provided and the rest is used as a serious assertion, people are likely to react to it as they would to sentences like

(208) The present king of France is bald

whose use has been argued (Strawson 1950; 1952) to involve an assertive part which is neither true nor false but "spurious" (or on Seuren's analysis *radically* false), the reason being that it contains a *presupposition* that there currently is a king in France, which is not realized. Since the presupposition is not part of what is asserted in the sentence, but rather presented as given knowledge on whose foundations the assertive part of the sentence builds new information, a gullible listener could easily be taken in by a false presupposition. But the attentive listener will realize that there is something wrong with what is presupposed, protest and say "but, there is no present king of France". In the case of the universally quantified sentence, he would analogously react "but there are no unicorns".

One implementation of the presupposition analysis for *all* is to adopt the assumption that its semantic definition incorporates the presupposition. In Strawson's (1952: 174-176) case, this took the form of the proposal that the universal quantifier (and also *the*) carries an existential presupposition. Seuren (2002: 31) modifies this into the proposal to "associate with the universal and the existential quantifier in natural language the presupposition that the *F*-class is non-empty." The advantage of that move (as I interpret it on the basis of several of Seuren's other articles and on other people's criticism of Strawson's existential presupposition) is that non-emptiness of a class has the advantage of being agnostic about real existence. In this sense, existential import is really is misnomer. Take the following sentence:

(209) All gnomes are fictitious

The F-class (*gnomes*) and the G-class (*fictitious*) are both sets of intensional, nonexistent elements (the use of term elements rather than entities is inspired by the assumption in the Russellian tradition that only what has extensional existence can be called an entity). The non-emptiness condition here does not require real existence, but merely non-emptiness of F. The Generalized Quantifier Theory (GQT) conception of quantifiers like *all* looks to me to be operative in the same way here as in sentences with extensional predicates and F-classes (e.g. *all men are mortal*). In both cases, *all* functions as a two-place higher order predicate, i.e. as a predicate over pairs of sets.

Only in specific G-predicate settings does the nonemptiness presupposition amount to a presupposition of existence, to literal existential import, namely when the G-predicate imposes an extensional reading (Seuren 2002):

(210) All unicorns graze in my father's meadow

Grazing in my father's meadow being an extensional predicate G (i.e. a G-class whose elements are existing entities, cf. Seuren 2002), the presupposition that F is non-empty can now only amount to the requirement that the elements of the subject set be *existing* entities as well, otherwise they could not possibly be elements of G. Since *unicorns* represents a set of nonexistent elements and the intersection of F and G can only contain existing entities as elements due to the extensional nature of G, the sentence is in breach of the presupposition of non-emptiness, which is here a presupposition of existence because of the extensional matrix predicate.

A qualification is in order, however. Take the following well-known example:

(211) All trespassers will be prosecuted

Assume that this warning pertains to a given, particular infraction. It is possible that there have been no trespassers so far and also that this warning is effective enough to keep the F-class empty forever. Yet, there is nothing wrong with this apparent empty F-class sentence. Note, however, that the nature of the G-predicate is crucial. In (211) there is no actual prosecution event but only virtual ones in the future, which means that the G-predicate is still intensional, hence does not impose actual existence on *all trespassers*. Compare:

(212) All trespassers were prosecuted

If the F-class is empty in this case where the G-predicate denotes an actual past, we get the Russellian "smart aleck" effect again ("there having been no trespassers, it is set-theoretically correct to say that all trespassers were prosecuted"), betraying a breach of presupposition.

In light of the crucial role of the extensional vs. intensional meaning of the G-predicate in all the cases discussed above, I believe the most accurate approach is to associate with the universal and the existential quantifier in natural language the presupposition that the *intersection* of the *F*-class and the *G*-class is non-empty, whence non-emptiness of the F-class follows automatically, even though the nature thereof is then conditional upon the nature of the G-class. When the nature of the G-predicate is such that the proposition is a future contingent one (cf. (211)), non-emptiness of the F-set is also non-actual[89].

When the G-predicate denotes actual rather than potential, future contingent events, actual nonemptiness of the F-class follows automatically from the presupposition that the intersection of the *F*-class and the *G*-class is non-empty. And when a G-predicate is extensional, there is a presupposition of existence on top of non-emptiness of the F-class, as in (212). When the G-predicate is intensional, there is no presupposition of existence (209). In sum, "existential import" is too narrow a notion and therefore a misnomer for the first part of the meaning of entailers. The presupposition involved is more accurately called "F-class non-emptiness" or the "non-empty intersection presupposition".

Auspiciously, the lexical decomposition analysis proposed here in terms of NEC precisely accommodates the presupposition analysis outlined above, without however having to stipulate the non-emptiness presupposition separately for predicate calculus operators. Rather, it is a consequence of the more general assumption that the meaning of a pivot enters into that of its entailer, more specifically – as we can now add – as its presuppositional part. In other words, analogous to the way in which part **(i)** of the meaning of *and* guarantees the non-emptiness of the set of selectees, the first part **(i)** of the meaning of *all* embodies the equivalent non-emptiness condition requiring that there be at least one F (which is G). To see the parallel, the template AND (P, Q) (204) for *and* has to be converted into the bar-code for *all*, whose meaning then comes out as follows:

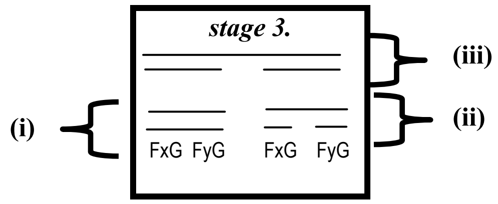
(213)

"**(i)** Flagx is Green or_{incl} Flagy is Green, i.e. at least one of Fx or Fy is G
(iii) ET
(ii) neither [Flagx is not Green] nor [Flag y is not Green], i.e. there isn't any Flagx or Flag y that is not Green: no Fx or Fy is not G.

[89] Note the parallelism with the non-actual nature of the NP *the candidate who gets the most votes* in Russell's well-known example of knowledge by description. Compare his sentence with an "actual event" past tense correlate.

(i) the candidate who gets the most votes will be elected
(ii) the candidate who got the most votes was elected

In (i) the actual election winner is not yet known (maybe forever unknown if no election ever takes place), in (ii) there is an actual person who answers to the description if the sentence is true.

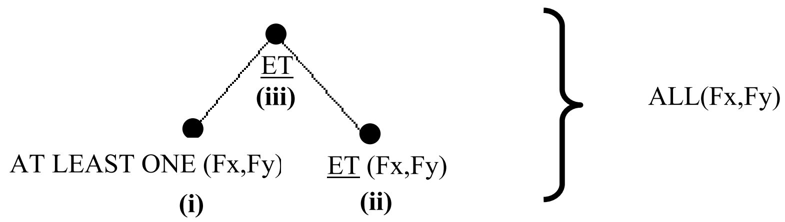


(F_xG = Flag$_x$ is Green (with x a variable))

Though elaborate, this meaning description contains both the specification of non-emptiness **(i)** needed for a Strawsonian approach to the "existential import" problem, and the universal specification that there is not a single F that is not a member of G **(ii)**. Because of the non-emptiness condition, saying that *all F are G* when there are no F violates **(i)**, the presupposition imposed as part of the lexical meaning of *all*.

In another imaginable case, namely when the F-set is not empty but infinitely large, part **(ii)** of the meaning of *all* represents a leap of induction: although not all F can actually be checked and found to be members of G, the result up to a certain point may be so systematic that the leap of induction as expressed by meaning element **(ii)** is taken: there are no flags that are not green. Finally, **(i)** and **(ii)** are joined by means of the conjunction <u>ET</u> **(iii)**.

(214)



In this representation, the meaning involved is rendered informally in terms of the restrictor DP, but it can be given in propositional terms too.

(215)

In terms of the restrictor DP	(i) AT LEAST ONE (Fx,Fy)	**(iii)** <u>ET</u>	(ii) <u>ET</u> (Fx,Fy)	= ALL (Fx, Fy)
	AT-LEAST-ONE OF Fx or Fy	<u>ET</u>	NO Fx or Fy NOT	= ALL (Fx,Fy)
In propositional terms	*at least one of (Fx is G) or (F y is G) has truth value 1*	<u>ET</u>	*Neither any(Fx is G) nor any (F y is G) has truth value 0*	= ALL (Fx,Fy)

4.4.6 Summary of the lexical decomposition analysis

The derivation of logical operator triads from a single negative disjunctive operator has brought to light a systematic parallelism between the proposition calculus and the predicate calculus, with APC_2 the calculus which most clearly evinced the close relationship between the two types.

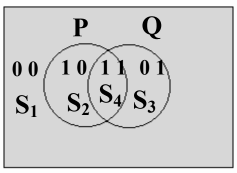
The only tool needed to derive the three hubs of the 2D Cartesian Coordinate System, is the nonlexicalisable, binary, negative-disjunctive operator <u>NEC</u>, which comes in three varieties:

1. Different selection <u>NEC</u> (X,Y) > <u>NEC</u>
2. Same selection <u>NEC</u> (X,X) > <u>NON</u>
3. Negative input <u>NEC</u> (¬X, ¬Y) > <u>ET</u>

1. <u>NEC</u> (X,Y)	
$X, Y \rightarrow \overline{X\ Y}$	
2. <u>NEC</u> (X,X) = <u>NON</u> (conflation added)	
$X \rightarrow \overline{X\ \ (X)}$	$X \rightarrow \overline{X}$
3. <u>NEC</u> ($\overline{X}, \overline{Y}$) = <u>ET</u>	
$\overline{X}, \overline{Y} \rightarrow \overline{\overline{X}\ \overline{Y}}$	

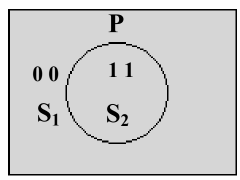
The starting point for different selection <u>NEC</u> (P, Q) is a Universe of selectable entities which – given two truth values and the binarity of <u>NEC</u> – has the structure of a **dyadic** Boolean Algebra, set-theoretically representable with four cells:

(216)



For same-selection <u>NEC</u> (P, P) i.e. <u>NON</u> (cf. Boole's Law of Duality), the starting point is a more minimal **monadic** Boolean algebra with only a bottom (00) and a top (11) and hence a set-theoretic representation with two cells only.

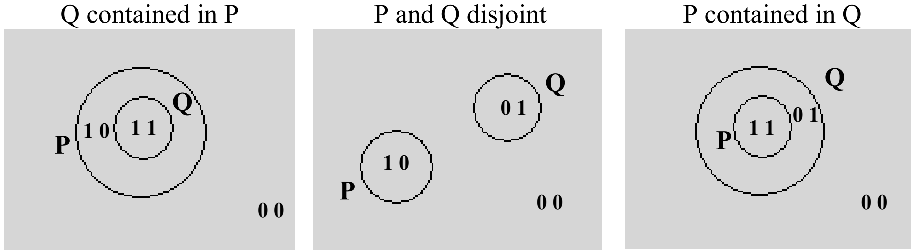
(217)



In a twin-selection framework (with two selections, namely different ones P and Q or twice the same, P and P) these are the only two possible Boolean algebras to start from[90]. In all other logically possible twin-selection configurations, namely when Q is contained in P, P and Q are disjoint, or P is contained in Q, one of the four possible truth value combinations for different selection goes missing, so that the starting point for set-demarcation would fail to be a Boolean algebra:

(218)

Q contained in P | P and Q disjoint | P contained in Q



no cell 01 | no cell 11 | no cell 10

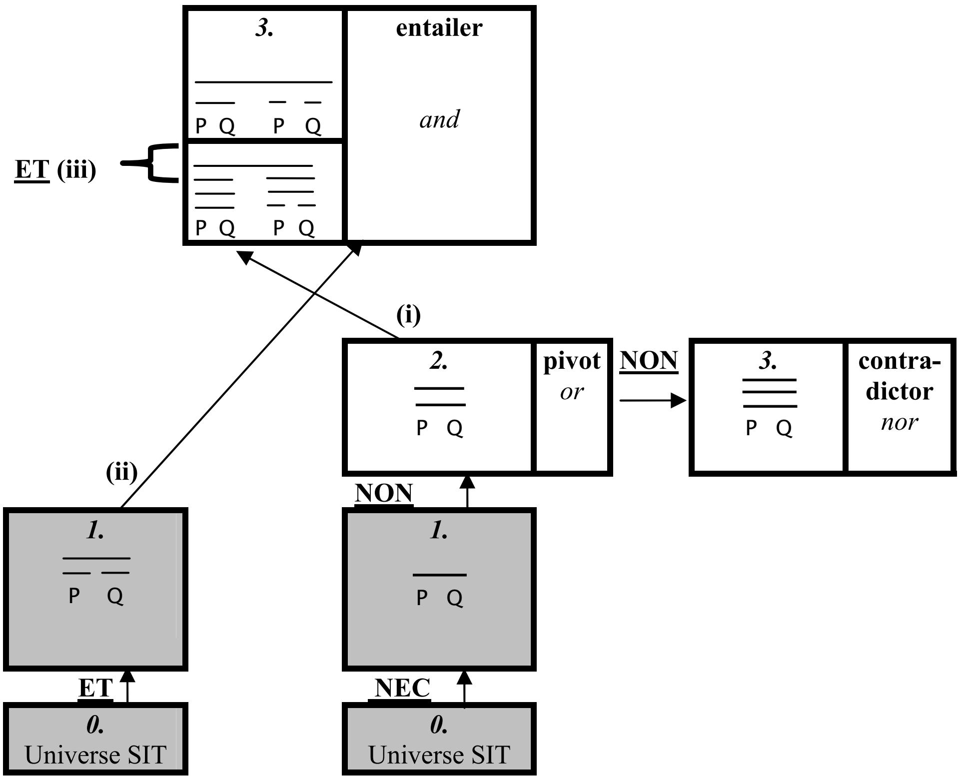
Stage 1. is reached by a first application of <u>NEC</u> to (216) or (217), whose effect is to exclude a number of cells. At this stage, no set is demarcated yet since no cells are actually selected, only deselected. The input (***stage 0.***) and the output (***stage 1.***) of this first application of <u>NEC</u> therefore remain underground.

It is after the second subtractive step, namely at ***stage 2.*** of a derivation that the first sets are demarcated and expressible meanings surface, namely those of the pivots. When they are lexicalised and no further morphological operations occur, they represent lexical items with pivotal meaning (e.g. *or, some/any, either*). Alternatively, they can be lexicalised, after which <u>NON</u> turns them into a contradictor lexical item (e.g. *or* > *n-or;*

[90] There is an uncanny resemblance between the two configurations produced by same selection and different selection and the distinction in the later Plato (Theophrastus, Metaphysics vi) between two principles, namely the Monad (or: the One) and 'the Indefinite Dyad' respectively, the latter of which was taken to generate plurality. For arguments that a four-cell different selection Boolean Algebra indeed generates plurality, cf. § 6.4.2. Plato's theory prefigures the two basic kinds of Boolean algebras that twin selection operates with (cf. http://plato.stanford.edu/entries/xenocrates/). The resemblance is probably no coincidence: there is evidence (Grattan-Guinness 1982: 39) that Boole's conception of his universe as 1 was inspired by the idea of 'the One', not only in Plato's version, but also in later Neo-Platonic and more religious versions: "The Monism on which he was then instructed […] re-appears in his interpretation of his universe 1 in terms of religious thought (1854, 411-416)." (Grattan-Guinness 1982: 39)

one > *n-one*), whose extension is the complement of that of the corresponding pivot (cf. Löbner 1990: 95). A final possibility is that there is no lexicalisation at ***stage 2.*** and the meaning expressed enters into a larger construct with an ET-conjunction to produce an entailer. Analogous to how an earlier proposition P conjoined with a later proposition Q is presuppositional for the interpretation of the latter, the pivotal meaning **(i)** in the entailer is presuppositional for the second segment **(ii)** of its meaning. This is represented in the following bottom-up flow-chart for PROPC, which summarizes how (Peirce-inspired) multiple application of a single operation NEC suffices to generate the cognate complex meanings of the lexical items of the propositional calculus operator triad.

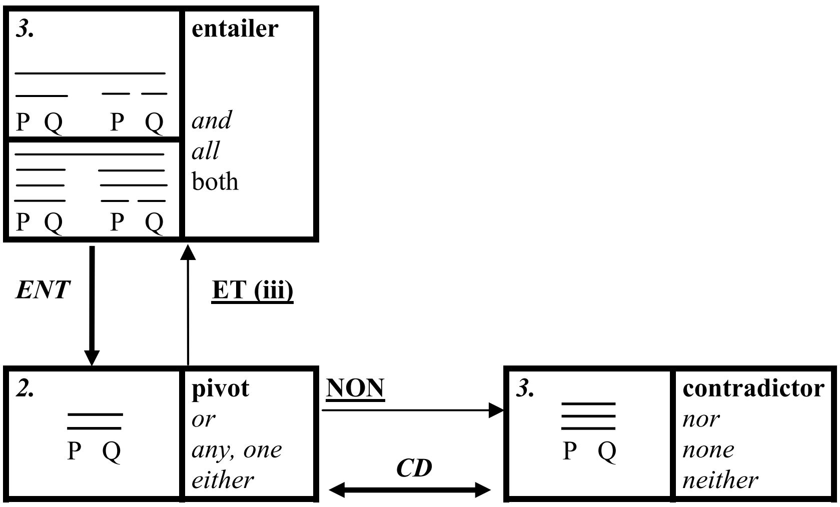
(219)



When stripped of the underground stages before ***stage 2.***, the pattern that emerges is precisely that of the 2D Cartesian Coordinate System with its two relations CD and ENT and the operations NON and ET to match respectively. Observe that these two operations both take the pivot meaning as their input to yield the other hubs: the contradictor by NON, the entailer by ET.

In the rest of this chapter, as much evidence as possible will be accumulated to prove that the relation CD is a natural correlate of the underlying operation of NON (§ 4.5), while ENT from entailer to pivot is a relationship resulting from the application of ET to a pivotal meaning (§ 4.6).

(220)



4.4.7 Prelexical operations and lexical relations

The distinction between the derivational *operations* perspective (NON and ET) and the representational *relations* perspective (CD and ENT) is in essence a distinction between Language of Thought (LOT) *process* and lexical *product*, i.e. a difference between derivational, dynamic operations in the prelexical language of thought and resulting static relations among created representations in the lexicon and the propositions in which they are used[91]. Each of these two is as cognitively real as the other: LOT operations on prelexical semantic primitives are *processes*, and they inevitably lead to *products*, in casu, lexical items. The latter can therefore be screened to find residual features of the primitives and processes that entered into their formation during language acquisition (the *procedural or derivational* perspective) or they can be looked at without further attention to their genesis during acquisition, purely to identify the resulting static relationships that exist between them in the lexicon and between propositions in which they are used (the *relations or representational* perspective). Surprisingly – to me at least – this difference between prelexical process (= operation) and lexical product (= representation) correlates with the difference between an inductive and a deductive orientation, as can be seen from the directionality of the arrows in the diagrams of (221).

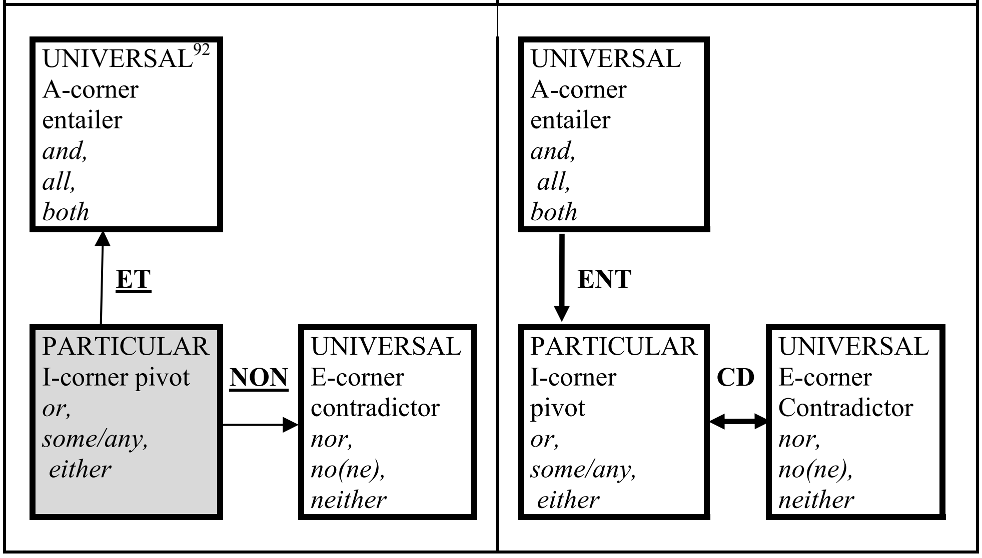
Induction generalizes from particular knowledge to general knowledge in the form of a universal statement, while deduction goes in the other direction and arrives at particular from universal truths. Now, in the operations perspective on logical operators, the non-pivotal *universal* hubs of the system are set up on the basis of the *particular* pivot, hence

[91] See also Löbner (1990: 75), who in a discussion of Horn's (1973) analysis of the Square draws a distinction between the "Operation" of negation and four "Bedeutungsrelationen", including the two mentioned in the text.

inductively oriented. The direction is from particular to universal operators by negation and conjunction.

(221)

DERIVATIONAL LOT operations <u>NON</u> and <u>ET</u> inductive orientation	REPRESENTATIONAL Lexical/propositional relations CD and ENT deductive orientation



But terminological caution is in order. Saying that the *orientation* is the same as that of induction should not be mistaken for the claim that each universal actually involves induction from particular knowledge (which is of course false). Only in certain contexts do actual leaps of induction from the particular to the universal occur, more particularly when a particular I-corner hypothesis *some F are G* is checked for a subset[93] of all possible Fs and the result is so systematically in the affirmative that the speaker leaps to the conclusion that the property of being green can be extrapolated from the cases of F checked to the the remaining unchecked cases: *all F are G*. A leap of induction to an E-corner universal, for its part, occurs when a provisional particular I-corner hypothesis,

[92] Literally speaking, the label "universal" pertains to the quantifiers only, not to the propositional calculus operators. Thus, [P:*Mary is in the garden*] *and* [Q: *Elisabeth is in the garden*] obviously does not express that everybody is. The label can be taken to carry over to *and*, however, in the sense that in the case of P and Q *both (= universal)* arguments have truth value 1; and in the case of E-corner *nor*, P and Q *both (=universal)* have truth value 0. In the case of *or*, on the other hand, there is *at least one (= existential, particular)* of P and Q which has truth value 1.

[93] If the checked subset of F is the null set, the inductive leap is trivial and the checking procedure is frustrated from the beginning. "Whenever the assessment of a sentence must start with a scan of an N' set of a given noun phrase [the F-set, DJ], assessment is stalled if the set is empty. In this case, the sentence is marked as anomalous, empirically irrelevant, or undefined, regardless of its semantic interpretation." (Lappin & Reinhart 1988: 1031)

e.g. *some F are G*, is systematically refuted for each F that is checked for property G and a decision is made to stop further checking for remaining Fs whether the particular null hypothesis is correct. Instead, an inductive leap is made to the negative universal *no F are G*, given that the outcome of all checking up to that point has been so systematically in the negative. As is commonly known, in induction cases the conclusion is not deductively valid.

It is with this qualification in mind that it can be claimed that the direction in the logical 2D Cartesian Coordinate System is from particular to universal operators by negation and conjunction. In affirmative predicate calculus contexts, the most basic element is the particular truth operator in the I-corner, i.e. *some/any*. To derive the universal truth operator, conjunction <u>ET</u> has to be applied to the I-corner operator and brings in the entailer, i.e. the affirmative universal A-corner (*all*)). Analogous to the inductive direction from I-to-A, one also starts from the particular existential I-corner in negative contexts[94]. On that particular I-corner, which is the locus not only of *some*, but also of existential *one*, negation operates and introduces the universal operator of the negative E-corner, in casu *n-o(ne)*). The meanings of these hubs define a lexical set of entailment relations enabling *deduction* at the propositional level, namely entailment ENT from the A-corner *(all)* propositions to the pivot *(some)* propositions and CD between I-corner *(some)* and E-corner propositions and vice versa.

The system of relations (ENT, CD) is less homogenous than the system of operations (<u>ET</u>, <u>NON</u>). The operations both take the pivot as their input and asymmetrically lead away from that pivot. The relations are different in that they do not uniformly take the pivot as their input and moreover, while entailment is asymmetrical, CD is symmetrical. While this more hybrid nature of the representational system is not problematic for a calculus in terms of entailment, the stricter homogeneity and algorithmic simplicity of the operations system in terms of <u>NEC</u> does indicate that it is the most likely candidate for constituting the algorithmically simple, binary basis of IN-logic. The asymmetric pivot-based set-up of hubs is reflected in the greater complexity of the non-pivotal hubs. It is these assumptions for which further arguments and evidence will be sought in § 4.5 and § 4.6.

Similar observations regarding directionality can be made with respect to the propositional calculus. Earlier it was observed that the notion of partial knowledge with a remaining element of indeterminacy makes pivots different from the two other corners, where no such doubt remains. *John is in the garden or Peter is in the garden* leaves one in the dark as to who precisely is in the garden. *John is in the garden and Peter is in the garden*, however, expresses certainty that they both are, and the same conclusion of expressed certainty holds for *John nor Peter is in the garden*. An epistemological correlate of this difference is that we apparently seek truth or fixed belief of the A-corner and E-corner type when the doubt and indeterminacy of pivotal I-corner knowledge bugs us. Thus, upon hearing that John is in the garden or Bill is in the garden, we want to find out who exactly is. In other words, I-corner indeterminacy generates the irritation that triggers searches for certainty and causal, general knowledge of the type expressed by entailers and contradictors (cf. chapter 2). Thus conceived, I-corner indeterminacy is

[94] Negation always needs a prior category that can then be turned into its contradictory.

epistemologically prior as the required generator of the irritation and drive to secure better knowledge. Analogously, I-corner pivotal meanings are semantically prior and less complex than those of entailers and contradictors.

In the system as conceived, *process* and *product* are not in conflict: due to modularity, the pivot-fleeing orientation in LOT and the deductive orientation of entailment in the lexicon and in syntax are both indispensable and part of the same linguistic architecture. Even if the pivot meaning enters into those of the two other hubs in prelexical syntax – an asymmetry – the three resulting operators in the lexicon are 'static' lexical items, i.e. stored in long-term memory and extracted from the lexicon in their entirety for the purpose of syntactic concatenation. It will be clear that among such lexical elements (all of which are proposition types and hence amenable to a valuation space treatment) entailment relations can obtain. Derivatively, entailment relations also obtain between larger representations – sentences, discourse – which are created dynamically and uniquely in use contexts. Take discourse contexts such as that of (222) by way of example, where the symbol > indicates progression in discourse from one stage to the next. Each newly introduced proposition (in boldface) represents the new information of a new stage in the discourse process. Old information, that is information introduced at an earlier stage of the same discourse, is preserved at later stages.[95]

(222)

Progression in discourse		**Entailment relation**
Stage 1: P:John was in the garden > Stage 2: P: John was in the garden. (*And*) Q: **Mary came in.**	$P > P \wedge Q$	$P \wedge Q$ entails P

The choice of P and Q is clearly the creative result of the operation of free combination rules on lexical information, i.e. of online construction. Once these representations have come into being and the information they introduce stored in a mid-term discourse memory – a mental discourse domain D – entailment relations automatically obtain between the resulting representations as indicated.

In sum, deduction in terms of entailment relations among proposition types in the lexicon and among entire propositions or stretches of discourse does not contradict the recognition that the prelexical operations of the logical system turn out to be differently oriented. To further substantiate the postulated relationship between CD and <u>NON</u> on the one hand and between ENT and <u>ET</u> on the other, additional semantic, epistemological and morphological evidence will be provided in § 4.5 and esp. in § 4.6.

4.5 Contradictoriness and Negation <u>NON</u>

Regarding the contradictoriness leg of natural logic, it is empirically obvious that this *static bidrectional relation* between the I-corner and the E-corner needs to be looked upon as the end result of a *dynamic, asymmetric operation* of negation on the I-corner (cf. Löbner 1990: 95), witness the following table:

[95] This picture of discourse is doubtless an idealisation, but nonetheless useful as it stands.

(223)

English		Dutch	
Dynamic rule of negation		Dynamic rule of negation	
Input $\longrightarrow$ output		Input $\longrightarrow$ output	
One	*n*-o(ne)	Een	(ne)geen[96]
Ever	*n*-ever	Iets	*n*-iets
Either	*n*-either	Iemand	*n*-iemand
		Ooit	*n*-ooit
		Ergens	*n*-ergens
		Enerlei	*(ne)*generlei
$\longleftrightarrow$ Bidirectional static relation: CD		$\longleftrightarrow$ Bidirectional static relation: CD	

The main consequence of accepting an asymmetric rule of negation as the linguistically and cognitively real source of CD is that the affirmative input (SOME) is *prior to and less complex semantically* than its negative contradictory (NOT-SOME, i.e. NO(NE)).

4.6 Entailment and Conjunction ET

In this section, the attention will be shifted from negation NEG to conjunction ET. First of all, it will be shown that there exists a very general connection between entailment and conjunction which exceeds the bounds of logical calculi. Next, semantic reasons will be adduced for the postulated relationship between entailment and ET in the propositional calculus and the predicate calculus.

4.6.1 Evidence for the connection between entailment and conjunction

That there exists a link between conjunction as an operation and a resulting static entailment relation is easy to establish by stepping outside of the domain of logical calculi for a moment and reconsidering the discourse example (222) above. Thus, the incremental progression in discourse from stage 1: *P* to stage 2: *P (and) Q* in example (222) consists in a *conjunction* (set-theoretically: *intersection*) of their meanings at the point when *Q* is introduced, an operation which establishes an entailment relation between the more complex construct *P and Q* and the input *P* of the conjunction operation which adds *Q* : whenever $P \wedge Q$ is true, *P* must also be true. Note the different orientations: the dynamic operation *conjunction* operates on the less complex "old" input expression (*P*) and results in a more complex conjoined expression (*P and Q*), and the entailment relation is thereby created in the opposite direction, from stage 2: *P and Q* to stage 1: *P*.

[96] According to De Vries and de Tollenaere (1991: 141), Middle Dutch (1150-1500) has *geen* (1253) and *gein* (c. 1220-40) alongside *engeen* (c.1265-70) and *negeen* (c. 1237). Cp. Old Saxon *nigên*, Old High German *nihhein*, from **nih* (= Gothic *nih* and Latin *neque*, 'and not', 'not even', 'neither…nor') and **aina*- ('one').

Similar conclusions about the workings of conjunction/intersection can be drawn from the entailment relationship obtaining between Q and P (Q entails P) due to a phrasal difference in the following example.

(224) a. P: Bill is [$_{DP}$ a doctor].
b. Q: Bill is [$_{DP}$ a young doctor]

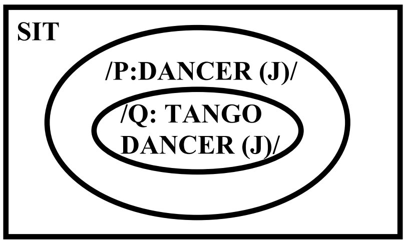
While it has been standard since Aristotle to state entailment at the level of the propositions, it is clear that the source is to be found at a smaller level, namely the NP level, where the NP of the predicate nominal DP of the entailer Q is once again the result of a *conjunction* – an intersection set-theoretically –, this time of the denotations of *young (x)* and *doctor(x)*. Once again, this results in an asymmetric entailment relation at the propositional level in the opposite direction, i.e. from the proposition with the more complex DP (*young doctor*) to that with the less complex DP (*doctor*): if someone is a young doctor, this entails that s/he is a doctor, but the opposite is not necessarily true.

Entailment can also result from a conjunction of information at word-internal level:

(225) a. P: John is a dancer
b. Q: John is a tango dancer

The predicate *dancer* being less highly specified intensionally than *tango dancer*, there is a set-inclusion relationship at the subsentential level – the set of tango dancers is a subset of the set of dancers – and correlating entailment from Q to P.

(226)



The point made here about the relationship between semantic entailment at the propositional level and meaning inclusion at the level of smaller constituents, was also made by Katz (1972: 172):

> "The relation of *meaning inclusion* is the counterpart at the level of subsentential constituents of the relation of semantic entailment at the level of full sentences. The including term, *superordinate*, is the counterpart of the entailing sentence, and the included term, *subordinate*, the counterpart of the entailed sentence." (Katz 1972: 192)

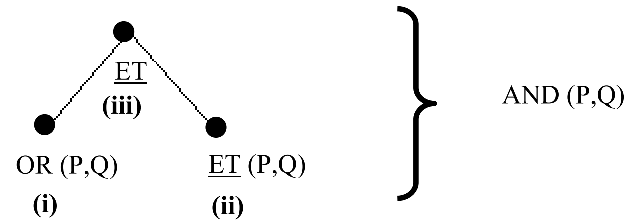
Katz gives a series of subordinate-superordinate pairs by way of example: *human-dilettante; dwelling-cottage, male-boy, digit-index finger, stone-pebble.* (Katz 1972: 192)

These observations result in the hypothesis that entailment in logical calculi should also be linked to the internal semantics of the subsentential lexical items responsible for the relationship at the propositional level. Thus, the predicate calculus entailment relation P: *All flags are green* $\vdash$ Q: *Some flags are green* is due to a semantic difference at the lexical level between *all* and *some*, all other things being equal in the sentences. Similarly, there is just a single difference between the entailing proposition *P and Q* and the entailed proposition *P or Q*, namely the lexical difference between *or* and *and*. In view of the discourse, phrasal and lexical evidence so far, the hypothesis I have adopted is that the difference between the lexical items *or* and *and*, as well as between *all* and *some* amounts to a different degree of complexity, with the meaning of the entailer the more complex one. More specifically, the entailer is a *conjunction/intersection* of the meaning of the pivot and an additional semantic specification. That would explain the entailment relations and their direction at the propositional level.

4.6.2 Semantic Evidence for Conjunction ET from the Propositional Calculus

The key question of this section is: is there evidence strong enough to conclude that the meaning of the entailer *and* is indeed more complex than that of *or* and, more specifically, is it the result of a *conjunction* (ET **(iii)**) of the meaning of pivot **(i)** OR (P,Q) and **(ii)** ET (P,Q) as claimed in (203), repeated here?

(227)



(i) OR (P,Q)	**(iii)** ET	**(ii)** ET (P,Q)	= AND (P,Q)
at least one of P and Q has truth value 1	ET	*neither P nor Q have truth value 0*	= AND (P,Q)

First of all, two theoretical arguments in favour of the complexity hypothesis for *and* will be provided; next, empirical linguistic data will show that complex structure (227) is an empirically accurate description of the internal semantic structure of *and*.

4.6.2.1 Is *and* more complex than *or?*

Acquisition facts provide an indication that *and* may well be more complex than *or*. Chierchia & McConnell-Ginet (1990: 352) note a fact that may sound counterintuitive when first heard: semantically more complex words are quite generally acquired before less complex ones:

> "Children acquire words like *kill* and *die* long before they learn words like *cause* and *become*. Of course, the IPC[97] CAUSE and BECOME[98] predicates need not be equated with *cause* and *become*; nonetheless, it is striking that what we have analyzed as the relatively more complex items semantically are apparently more directly salient for children. It is often said that young children, while being attuned to causal relations, lack *explicit* knowledge of abstract notions, like causation, that serve to cross-classify many diverse types of events."
>
> (Chierchia & McConnell-Ginet 1990: 352; italics+underline mine)

This order of acquisition can be made sense of. To begin with, the fact that *kill* is acquired before less complex *become* even though *kill* contains the concept BECOME as part of its semantics must mean that concept formation in the Language of Thought and lexical labelling are two different things. Concept formation happens in the Language of Thought-component, which as said throughout is inaccessible to introspection. Consequently, complex concepts are constructed from the simple ones automatically and without conscious control[99]. Acquisition of lexical items in the sense of labelling of concepts, on the other hand, takes concepts as its input material. It is a matter of attaching local, language-particular labels to concepts constructed in the universal Language of Thought. Its language-particular form has to mean that labelling occurs under the influence of triggering experience. Now, in one's early experience, the particular, direct and concrete (FATHER, KILL) is plausibly more immediate and salient than the more general or abstract (MALE, PARENT, BECOME). That is why "Children know about fathers long before they know about males and parents." (Fodor 1987: 161). It is the semantically less complex concepts which are less particularized and hence harder to discover in (or recover from) our Language of Thought. In view of all this, it is only natural that the complex concepts, for which there is environmental triggering early on, are the first to get a lexical label[100].

---

[97] IPC= Intensional Predicate Calculus.

[98] As in our own analysis, English words in capital letters are used to designate a simple or complex concept. A simple concept belongs to "a stock of universal semantic components from which all languages draw in constructing the concepts their lexicons label" (Chierchia & McConnell-Ginet 1990: 352); a complex concept is such a composite construct made from simple concepts.

[99] Possibly, the nature of the complexity which complex concepts represent is somehow "received from without through the medium of the senses", but then only in a triggering sense, "because certain exterior things contain or express more particularly the causes which determine us to certain thoughts" (Leibniz 1902, § XXVII). In any case, all the material needed to have the knowledge, i.e. the primitive concepts and the composition principles must already be in the head. Without innate inner resources nothing from outside could cause any concept to coagulate.

[100] Note that once concepts are labelled, they become objects which are consciously controlled and freely selected for syntactic concatenation. They are even open to introspection for the purpose of linguistic science (as in the present lexical decomposition context).

The relevance of this acquisition fact to *and* and *or* is that there are a number of studies (Braine & Rumain 1981, Braine & Rumain 1983, Sacco et.al. 2001, Garcia-Madruga et al. 2001, Yang & Johnson-Laird www.ccm.ua.edu/pdfs/238.pdf) reporting that acquisition of disjunction is later and/or more difficult than that of conjunction.

> "our results show the following trend of difficulty among connectives in both the Abstract and the Pragmatic Protocol: conjunction is easier than disjunction and biconditional, and the latter are easier than conditional."
> (Sacco et.al. 2001: 879)
>
> "With particular reference to disjunction, the findings from several studies are interpreted as showing that only the truth conditions associated with exclusive-*or* are available to young children (e.g., Beilin and Lust 1975; Braine and Rumain 1981, 1983; Paris 1973). There is a related claim, that even when children respond as if they have access to a broader range of truth conditions, namely those associated with inclusive-*or*, children's adult-like responses are the result of a failure to distinguish *or* from *and* (Paris 1973)."
> (Gualmini, Meroni & Crain, 1999)

To my knowledge, the opposite claim, i.e. that *and* is harder to acquire than *or*, has never been made. If (a) disjunctive lexical operators are harder to acquire than conjunction and (b) acquisition of conceptually less complex lexical items is harder, the two add up in support of our general line: *or* is lexically less complex and more general than *and*.

Recently, however, the claim that disjunction is acquired later has come under attack. Gualmini et al. (1999: 247) report that Chierchia et al. (1998) ran a series of experiments "which revealed children's adult-like knowledge of logical connectives, including disjunction.". Note, however, that under our conception, with a difference between LOT and labelling, the two results are not incompatible and it need not be surprising that children can assign the inclusive-*or* interpretation early on. It actually confirms our conviction that the relevant concepts are available and active in LOT long before children speak, probably innate. That does not, however, preclude the possibility that acquisition of the *labelled* lexical item *or* and *conscious* access may still settle later than use of *and*. Bearing in mind the distinction between concepts and full lexical items, the reported findings need not be in conflict with one another.
A further important point is that if difficulty of acquisition and conceptual complexity are not proportional (there is even a tendency towards inverse proportionality), it follows that tying the non-lexicalisability of the O-corner operator purely to a higher number of steps in its composition is not likely to be the right approach.

4.6.2.2 Reconstructing the meaning of *or* and *and* on empirical grounds

As a starting point in the present reconstruction of the meaning of *or* and *and*, the basic meaning of *or* is taken to be inclusive: AT LEAST ONE OF P AND Q HAS TRUTH VALUE 1 (cf. § 4.4.3.1.3, esp. (162)). This is a standard conception, and it suffices to have a look at column 2 in truth table (162) to see that *at least one* value has to be 1 for a set of situations to be in the shaded area of *or*.

So then let us focus on differences between *or* and *and* to identify on empirical grounds what differentiates the latter from the former semantically:

(228)

a. The Russians, who used to buy the skins, are broke **and so** are the Chinese, whose demand for wool has dried up. (The Times, Dec. 5 1998)	b. * The Russians, who used to buy the skins, are broke **or so** are the Chinese, whose demand for wool has dried up.

The *so* in the second conjoin of (228) a. expresses that the predicate which asserts that the first conjoin is a true statement is taken to apply to the subject of the second conjoin as well. It expresses that the subjects of the two conjoins are alike relative to the predicate, in other words, the two conjoins are asserted to have *equivalent truth values*. This is a semantic feature that *or* does not possess, witness the ungrammaticality of (228)b.

This empirical observation squares well with the hypothesis that the meaning of *and* is that of *or* with an additional element of meaning to make it more highly specified. Informally put, the intensional meaning of *and* comes out as:

(229) AT LEAST ONE OF P AND Q HAS VALUE 1 & P AND Q HAVE THE SAME VALUE.

The second specification, which makes *and* more specific than *or*, is in fact an *equivalence requirement* imposed on the two conjoins. The two conjoins have to yield the same truth value at the level of the proposition when predicated over. Given the AT LEAST ONE OF P AND Q HAS VALUE 1 specification, the equivalence requirement can only amount to imposing that both P and Q have truth value 1. Or stated in the negative terms of our decomposition analysis (227): **(ii)** NEITHER P NOR Q HAS TRUTH VALUE 0.

Further examples supporting the postulation of an equivalence requirement for *and* but not for *or* are the following.

(230)

(a) John bought a book *and* Bill did *the same.*	(a') * John bought a book *or* Bill did *the same.*
(b) John bought a book *and* Bill bought one *too*.	(b') * John bought a book *or* Bill bought one *too*.

The well-formed cases contain *and* in combination with a word or phrase expressing equivalence of the second conjoin to the first, namely *the same* and the so-called adverb of addition *too*, respectively. The latter are overt manifestations of the “same value” requirement imposed by *and* on its two arguments. Once again, such sentences with overt manifestation of the requirement of truth value equivalence do not allow counterparts with *or*[101].

The &-symbol of (229), finally, has to be a case of conjunction as the two specifications on either side of it have to be fulfilled simultaneously for the meaning of the complex construct to be that of *and.* In negative terms, this means that neither of the two specifications is false. This is of course the conjunctive semantics of <u>ET</u> (in casu **(iii)**). On the whole, this makes (229) identical to (227) and confirms the decomposition analysis (cf. (203), (204) above) – cf. (231) below.

On the whole, the claim that the meaning of *and* is the result of a conjunction of the meaning of *or* and an equivalence specification is supported by factual evidence. This in turn renders support to the earlier hypothesis that a lexical decomposition approach to non-pivotal hubs in general (not only the visibly complex hubs *n-or* and *n-one*, but also *and* and *all*) makes sense. It also supports the hypothesis that the complexity of the hubs increases in the direction away from the pivot: in the case of the morphologically complex ones the complexity is achieved by means of negation of the pivot and in the case of the monomorphemic complex hub *and* by means of the operation of conjunction <u>ET</u> applied to the pivot.

(231)

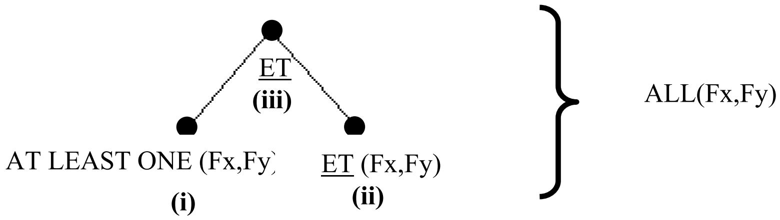
(i): OR (P,Q)	**(iii):** <u>ET</u> (**(i), (ii)**)	**(ii):** <u>ET</u> (P,Q)	= AND (P,Q)
at least one of P and Q has truth value 1	<u>ET</u>	*neither P nor Q have truth value 0*	= AND (P,Q)
$\overline{\overline{P\ Q}}$	$\overline{\overline{\text{(i)}}\ \overline{\text{(ii)}}}$	$\overline{\overline{P}\ \overline{Q}}$	$\overline{\overline{\overline{\overline{P\ Q}}}\ \ \overline{\overline{\overline{P}\ \overline{Q}}}}$
			$\overline{\overline{P\ Q}\ \ \overline{P}\ \overline{Q}}$

4.6.3 Semantic Evidence for Conjunction <u>ET</u> from the Predicate Calculus

The aim of this section is to provide further evidence that the meaning of the entailer *all* is more complex than that of its pivotal counterparts (*any/some*; *either, (at least) one*) and, more specifically, that it is the result of a *conjunction* (<u>ET</u> **(iii)**) of the meaning of the pivot **(i)** AT LEAST ONE (Fx, Fy) and **(ii)** <u>ET</u> (Fx,Fy) as claimed in (214) and (215), repeated here. So, while pivots consist of a ***<u>single</u>*** semantic specification (i), entailers are dyadic: a conjunction of ***<u>two</u>*** semantic specifications (i) and (ii).

[101] As a final remark about the equivalence specification in the meaning of *and* we note that identity of value relative to the predicate crucially does not imply semantic identity of the conjoins themselves but only of their value. Identity of conjoins is even excluded in principle: *Jan en Jan gingen weg* always involves two different Johns, in compliance with Binding Condition B, in essence a Principle of Disjoint Denotation.

(232)



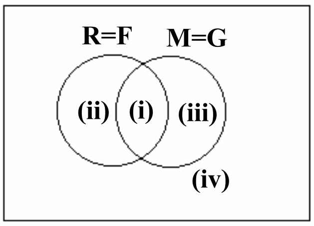
(233)

In terms of the restrictor DP	**(i)** AT LEAST ONE (Fx,Fy)	**(iii)** <u>ET</u>	**(ii)** <u>ET</u> (Fx,Fy)	= ALL (Fx, Fy)
	AT-LEAST-ONE OF Fx or Fy	<u>ET</u>	NO Fx or Fy NOT	= ALL (Fx,Fy)
In propositional terms	*at least one of (Fx is G) or (F y is G) has truth value 1*	<u>ET</u>	*Neither any(Fx is G) nor any (F y is G) has truth value 0*	= ALL (Fx,Fy)
Bar-codes	**(i) =** Fx G Fy G	**(iii) =** (i) (ii)	**(ii) =** Fx G Fy G	Fx G Fy G Fx G Fy G
				Fx G Fy G Fx G Fy G

To find fresh support for this hypothesis, we start from the Generalized Quantifier Theory (GQT) conception of quantifiers as two-place higher order predicates, i.e. as predicates over pairs of sets R(estrictor) and M(atrix predicate). To represent this perspective on the meaning of quantifiers the following generalized Venn diagram will be used. It contains four cells which can in principle be relevant to determine the denotation of a particular determiner D, i.e. to determine whether particular instances of D (R, M) such as ALL (F,G): *all flags are green*, AT LEAST ONE (F,G): *at least one flag is green*, etc. are true or false.

(234)

(i)	F ∩ G
(ii)	F - G
(iii)	G - F
(iv)	*1 – (F + G)*



4.6.3.1 Restrictions on the denotation of GQs

What has been established in GQT is that there are considerable restrictions on the range of admissible denotations of quantifiers and that "natural language determiners (at least "simple" or "normal" ones) (…) do not require the checking of all four areas" (Szabolcsi 1997: 10) of (234). Consider sample (235) and the description of the determiner denotations in the set-theoretic terms of the four-cell Venn diagram above[102].

(235)

Boethian corner	*Sentences D (F,G)*		*Size of Domain*	*(i) $F \cap G$*		(ii) *F - G*
I-corner Pivot	*a. At least one F G*	is true iff		$\vert(i)\vert \geq 1$		
I-corner Pivot	*b. Either F G* (in negatives and interrogatives)	is true iff	$\vert F\vert=2$;	$\vert(i)\vert \geq 1$		
I-corner Pivot	*c. Any F G* (in negatives and interrogatives)	is true iff		$\vert(i)\vert \geq 1$		
A-corner Entailer	*d. {Every/all} F G*	is true iff		$\vert(i)\vert \geq 1$	and	$\vert(ii)\vert = 0$
A-corner Entailer	*e. Both F G*	is true iff	$\vert F\vert=2$;	$\vert(i)\vert \geq 1$	and	$\vert(ii)\vert = 0$
E-corner Contradictor	*f. No F G = Not one/any F is G*	is true iff		$\vert(i)\vert = 0$, i.e, it is not the case that $\vert(i)\vert \geq 1$		
	g. Most F G	is true iff		$\vert(i)\vert > \vert(ii)\vert$		

These descriptions show that the largest part of the Venn diagram that matters for checking (for these and the overwhelming majority of quantifiers) is the restrictor set F,

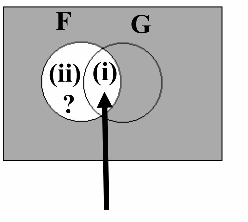
[102] GQ definitions have been provided by many: Barwise and Cooper (1981), Zwarts (1983), Van Benthem (1986), Keenan & Stavi (1986),, De Hoop (1992), Szabolcsi (1997), Seuren (1998), among many others.

i.e. cell (i) and sometimes also cell (ii)[103]. This is the property of *conservativity* (Van Benthem 1986, Keenan and Stavi 1986, Szabolcsi 1997: 11).

(236) D is conservative iff D (F) (G) = D (F) (F ∩ G)

What is even more surprising, however, is the fact that there is a perfect match between the definitions of (235) and our decomposition analysis in terms of complexity. Thus, for the pivots *at least one, either* and *any* i.e. (235) a.-c., just a *single* cell is relevant, namely the *intersective* cell (i) *F ∩ G*. What the definition of the pivots in question stipulates is that there is at least one element in the intersection of F and G: some (=x) or other (=y) element that belongs to the set of flags is also a member of the set of green entities, precisely what is also expressed by the bar-codes. Note that this description leaves it open whether there are or aren't elements in (ii), i.e. flags which are not green.

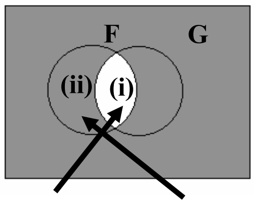
(237)

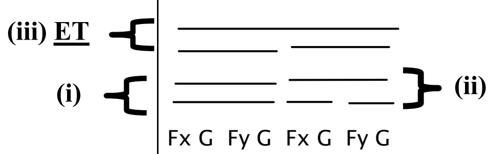
Boethian corner	Sentence	(i) = F ∩ G; (ii) = F - G	*Link with bar-codes*
I-corner Pivot	*a. At least one F G* *b. Either F G* (|R|=2) *c. Any F G*	 |(i)| ≥ 1	(i) = ≥ 1: Fx G Fy G *at least one of (Fx is G) or (F y is G) has truth value 1*

Entailers, i.e. (235) d.-e., for their part, are more complex in that the *two* cells of F have conditions imposed on them, i.e. both cell (i) *F ∩ G* and cell (ii) *F - G*. Moreover, these stipulations have to be complied with conjunctively (cp. **(iii)** ET in (233)). Note once more the surprising parallellism between the dyadic structure of the GQT description and the dyadic structure (namely, with two Fx G - Fy G selections) of the bar-codes, which can hardly be accidental.

[103] Possible exceptions involving *many* and *few* were brought up by Westerståhl (1985).

(238)

Boethian corner	*Sentence*	*(i) = F ∩ G; (ii) = F - G*	*Link with bar-codes*
A-corner Entailer	*d. {Every/all} F G* *e. Both F G* (|F|=2)	 |(i)| ≥ 1 and |(ii)| = 0	(i) = ≥ 1: Fx G Fy G *at least one of (Fx is G) or (F y is G) has truth value 1* (ii) = 0: Fx G Fy G *Neither any (Fx is G) nor any (F y is G) has truth value 0 =* *Neither any (Fx is not G) nor any (F y is not G) has truth value 1*



Fx G Fy G Fx G Fy G

While the pivots discussed in (237) (*either, any, at least one*) are intersective, there are certain pivots which are ambiguous between a simple intersective and a dyadic reading, more commonly referred to as a weak or cardinal and a strong or quantificational reading respectively (Milsark 1977):

(239) Some F_{plural} G : Some florists entered the garden

The strong, stressed reading (also called proportional or partitive) can be paraphrased as “some but not all”: some florists entered the garden, others did not. The reading is quantificational, i.e. involves reference not only to cell (i) of (234) but also to cell (ii): there is a set of at least one element fitting the predicate *florist* but not the predicate *entered the garden*. The weak, unstressed reading, on the other hand, is purely intersective and cardinal: only the cardinality of florists who entered the garden, i.e. cell (i) matters to determine the truth of the sentence. This distinction can be accommodated both in GQT and in the bar-system as developed so far.

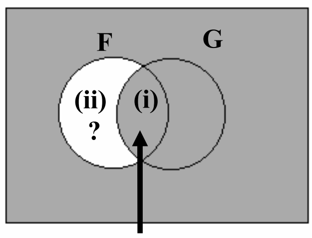
(240)

Boethian corner	*Sentences D (F,G)*		*Reading*	*(i) F ∩ G*		*(ii) F - G*
I-corner Pivot	*Some$_{pl}$* F G	is true iff	cardinal	$\mid$(i)$\mid$ ≥ 2		
			quantificational	$\mid$(i)$\mid$ ≥ 2	<u>ET</u>	$\mid$(ii)$\mid$ ≥ 1

However the question whether this ambiguity is lexical or contextual is resolved (an issue which will be taken up in chapter 5), the means (bar-codes) to represent the two readings are available.

To further illustrate the parallellism between the GQT definitions and the lexical bar-code analysis, observe that contradictors such as (235) f. negate a pivotal interpretation, i.e. they deny the presence of any element in the intersective cell (i) *F ∩ G*.

(241)

Boethian corner	Sentence	(i) = F ∩ G; (ii) = F - G	*Link with bar-codes*
E-corner Contradictor	f. No F G	 $\mid$(i)$\mid$ = 0, i.e. it is not the case that $\mid$(i)$\mid$ ≥ 1	$\mid$(i)$\mid$ = 0: Fx G Fy G *Neither any (Fx is G) nor any (F y is G) has truth value 1*

Example (235)g. illustrates that even though quantifier theory has to be generalized to other operators than those in the three corners of the Boethian Square (whence the term Generalized Quantifier Theory), checking tends to remain restricted to the restrictor set F throughout. In the case of determiner *most*, the operator is *proportional* within F in the sense that the truth of the assertion is dependent on the proportion of green flags relative to the whole set of flags.

(242) D is proportional iff D (F) (G) depends on (F ∩ G) / F[104]

On the whole, this section has provided evidence for the decomposition approach to predicate calculus operators by showing the striking parallellism between the structure of GQT definitions of the denotation of these quantifiers and their semantic

[104] For a clear exposition and definitions of this and further restrictions (intersectivity, extensionality and quantity), cf. Szabolcsi (1997: 10) and De Hoop (1992: 5-7).

compositionality in the NEC-system. From the GQT perspective as much as from the NEC-perspective, entailers such as *all* and *every* are semantically more complex than pivots such as *at least one, any* and *either.*

4.6.4 A difference between *and* and *all*

There exists a difference in lexical meaning between *and* and *all* which, though superficially seeming to disconfirm the parallel bar-codes proposed in this study, is on closer analysis predicted by the lexical structures introduced above and hence supports the decomposition. Consider the following examples with *and.*

(243) a. P:John is in the garden and Q:Peter is in the garden.
b. P: Flag1 is green and Q: Flag2 is green.

Saying that John and Peter are in the garden does not in principle bar the possibility that there are more people in the garden, but upon hearing (243) a. the hearer will often pragmatically conclude that the speaker did not mention more people because the whole set of persons in the garden was exhausted or the speaker had no knowledge of the presence of more people in the garden (cf. one of Grice's submaxims of quantity: "make your contribution as informative as is required (for the current purposes of the exchange)" (Grice 1989: 27)). So, while strictly speaking (243) a. only asserts that *at least* John and Peter are in the garden, under normal falling intonation there is the implicature that nobody else is. Similarly, (243) b. asserts that *at least* Flag1 and Flag2 are green. But though its asserted content remains agnostic about Flag3, etc. (if any), the standard implicature under normal falling intonation is that no other flag is green. In both examples, however, the implicature is not part of the lexical meaning of *and*[105].

That what is involved is indeed an implicature, can be illustrated by applying three tests devised by Horn (1972: 39-40) to establish the presence of an implicatum:

(244) a. Asserting the implicature:
Just John and Peter are in the garden, not John and Peter and Henry[106]
b. Suspending the implicature:
John and Peter are in the garden, {or even/if not} John and Peter and Henry.
c. Contradicting the implicature:
John is in the garden and Peter is in the garden. In fact, John is in the garden (and) Peter is in the garden and Henry is in the garden.

---

[105] The notion *at least* in the description of the lexical meaning of *and* is the overt expression of what Horn (1989) refers to as the property of "lower boundedness" (no less than one of P and Q is true) and "upward compatibility" (possibly more than one of P and Q is true) in his discussion of the lexical meaning of disjunction (*P or Q*). The difference with Horn is my claim that these properties are not restricted to disjunction, but (as illustrated in the text) apply to the conjunctive logical predicate equally as well.

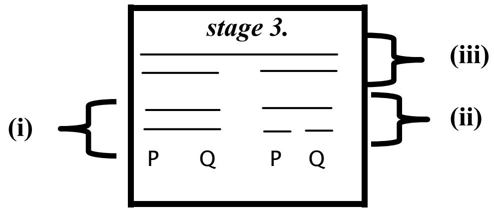
[106] Examples with full propositions instead of NP conjunctions are hard to get in (244) a. and b., but the fact that propositions are fine for test c. and that NP conjunction works for the other implicature tests, is sufficient proof.

Now consider the predicate calculus correlate of *and*, namely *all.*

(245) All flags are green

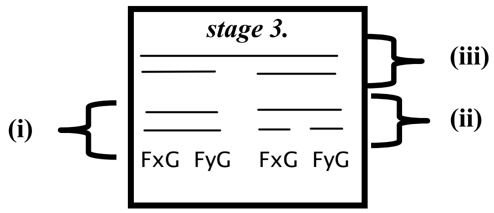
In this case, there is no upward compatibility: when all flags are green, it is impossible for there to be any other flags, let alone more green ones. While this difference between and and all superficially appears to cast doubt on the identical bar-codes proposed for the two operators, it actually confirms their correctness. This is because the semantic difference is not generated by the bar-codes themselves, but is due to the different types of arguments the operators take. Specifically, it is the effect of the choice of arguments on meaning component **(ii)** of the bar-code which generates the semantic divergence:

(246) *and*



(i) P: Flag1 is Green or_{incl} Q: Flag2 is Green, i.e. at least one of F1 or F2 is G
(iii) ET
(ii) *neither [P: Flag1 is not Green] nor [Q: Flag 2 is not Green]*

(247) *all*



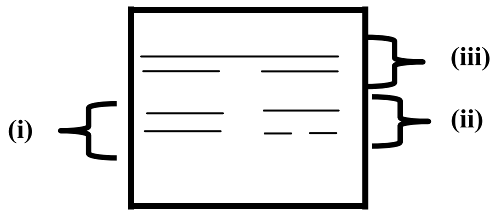
(i) Flagx is Green or_{incl} Flagy is Green, i.e. at least one of Fx or Fy is G
(iii) ET
(ii) *neither [Flagx is not Green] nor [Flag y is not Green], i.e. there isn't any Flagx or Flag y that is not Green: no Fx or Fy is not G*.

In the case of *and*, nothing is stipulated for any flags beyond Flag1 and Flag2. A third, etc. flag existing and being or not-being green is not incompatible with the lexical content expressed. In the case of *all*, however, the rotation of the variables over flags due to x and y has the effect of making meaning component **(ii)** universal ("neither [Flagx is not Green] nor [Flag y is not Green]", i.e. there isn't any Flagx or Flag y that is not Green: no Fx or Fy is not G.), which therefore lexically excludes upward compatibility. In sum, the semantic difference in terms of upward compatibility between *and* and *all* is predicted by the lexical decomposition proposed.

4.7 The difference between (i) in an entailer and (i) in a contradictor

In the descriptions of the lexical decompositions of entailers and contradictors, attention was drawn to an *asymmetry* in that the pivotal meaning **(i)** is in both cases logically prior to the added feature of the other hubs. For contradictors, this was pointed out in § 4.5, box (223), and evident from the fact that in *n-or* and *n-one*, the negative morpheme is a bound morpheme dependent on a free morpheme to attach to (cf. Löbner 1990). For entailers *and* and *all*, the asymmetry consisted in the semantic fact that meaning element **(ii)** in an entailer configuration

(248)



was shown to be semantically dependent: it cannot denote a set of selectees on its own (cf. (193) in § 4.4.5.1), but can only narrow down the extension of a logically prior pivotal meaning **(i)** to yield the extension of the whole entailer. This must mean that the conjunctive meaning element **(iii)** – a case of ET – is asymmetric and that its pivotal component **(i)** is logically prior to **(ii)**.

Notwithstanding the foundational nature of **(i)** for both E-corner and A-corner operators, there is also a difference between the status of **(i)** in a contradictor and an entailer. In a contradictor, **(i)** it is not presuppositional but only a provisional "null hypothesis" which is rejected by means of the application of NON which produces the E-corner operator. In an entailer, on the other hand, it *is* presuppositional and hence prior to **(ii)** if Seuren, Strawson and our version of their analyses are on the right track.

4.8 The two-dimensional model and Löbner's duality squares

The aim of this section is to list the main differences between the /NEC/-decomposition of logical operators and Löbner's (1990) duality approach. Their effect is to make what remains of the Boethian square even more solidly asymmetrical. The first issue that will be touched is the pivot/non-pivot asymmetry. Secondly, a feature of Löbner's Asymmetry Hypothesis, according to which pivots (Type 1) are less marked than entailers (Type 2), contradictors (Type 3) and O-corner operators (Type 4), will be criticised. Specifically, Löbner considers the conceptual complexity of an operator to be an important criterion for its place in the hierarchy, but from our viewpoint that is incorrect. A final difference concerns the meaning structure of the entailer, which is more complex in my proposal on account of the structural presence of the pivotal meaning as subpart **(i)** in the meaning of an entailer.

4.8.1 The pivot/non-pivot asymmetry

By defining the nature of all set-demarcation in Boolean algebraic terms as the repeated application of a single binary operator NEC, the first expressible operator that emerges from subtractive operations on the Universe 1 is *inescapably* the I-corner operator. Its pivotal status is thereby established as a necessary property. Given all the other features of operators which fall into place (their relationship, the fact that the same operator can connect them all, the different epistemological drive they generate, etc.) when the NEC-hypothesis is adopted, the pivotal status of the I-corner has come to be grounded on even more solid foundations than before.

4.8.2 The basic operator and binary negation

The pivot is composed by means of exactly the same operation that is behind all the other derivational steps needed to generate the whole system. This constitutes a considerable reduction of primitives. Indeed, only one of the three types of negation distinguished by Löbner (outer negation, inner negation (= subnegation) and dual negation) is a primitive, namely outer negation, which is really the same selection variety NON of the binary operator NEC. The other two varieties of negation are both complex constructs definable in terms of NEC. Given that the pivot is itself entirely composed by means of NEC, it is much less of a mystery now why it is more basic and why the other operators can be constructed by means of the same NEC operator in the different forms of negative appearance that characterize duality squares.

4.8.3 The symmetry-asymmetry contrast and processing effort

Duality groups are as a rule symmetrical: "Die Dualitätsgruppe ist durchgängig symmetrisch." (Löbner 1990: 71). Given their definition, the three operations of outer, inner and dual negation are indeed idempotent: two consecutive applications of each type of negation always lead back to the original operator:

(249)

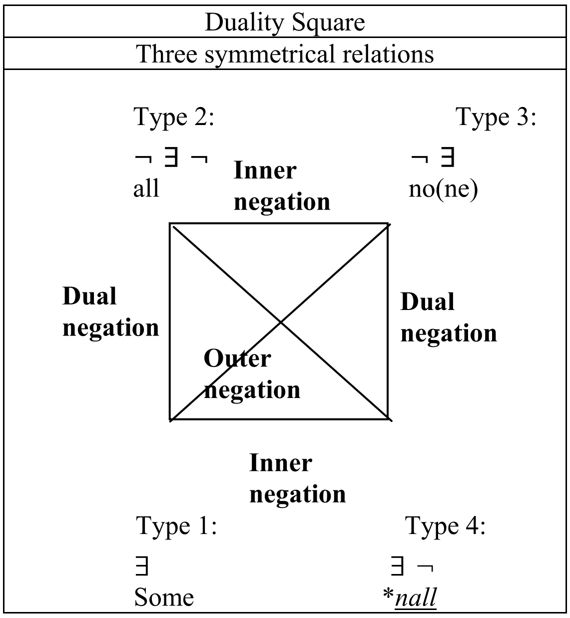
Q = quantifier; S= Subnegation (inner negation); N= Negation (outer negation); D = Dualnegation (dual negation)

S(S(Q)) = Q	daher	$Q \neg\neg$	$\leftrightarrow$	Q
N(N(Q)) = Q	daher	$\neg\neg Q$	$\leftrightarrow$	Q
D(D(Q)) = Q	daher	$\neg\neg Q \neg\neg$	$\leftrightarrow$	Q

(Löbner 1990: 71)

If you apply dual negation D to Type 1 in (250), you get Type 2; next reapply D to type 2 and you end up with Type 1 again, etc.

(250)



With the corners of the square so symmetrically related to one another, the entire burden of asymmetry between the operators comes to rest on their own internal conceptual complexity, given Löbner's view that "Die komplexere Bedeutung is die Ursache für die höhere Verarbeitungskomplexität" and thereby also the cause of higher markedness on the Type hierarchy (Löbner 1990: 105). But are complexity of meaning and processing effort indeed proportional? Consider the entailer (Type 2) and the contradictor (Type 3). Both in traditional terms and in terms of NEC, outer negation – which generates the contradictor – is a less complex operation than dual negation[107]. To me, this can only mean that the internal structure of a contradictor is *less* complex than that of an entailer, so that if Löbner were right it should be less marked on the markedness hierarchy.

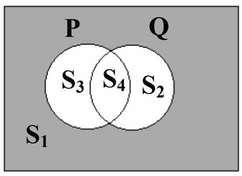
Empirically speaking, Löbner's markedness ordering entailerT2 - contradictorT3 is of course well-motivated. It is easy to see that affirmative corners are invariably lexicalized while negative corner lexicalizations are much rarer. And when the latter do occur, they are morphologically or syntactically more complex than the affirmatives. But given these considerations there is a problem for Löbner's claim that processing effort is proportional to complexity if T2 is configurationally more complex than T3.

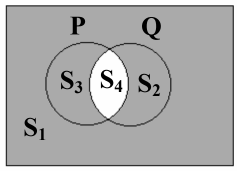
Under the NEC-conception, however, no such problem arises. Internal conceptual complexity is in certain cases even inversely proportional to processing effort. Indeed, recall that though the pivot word *or* has a less complex meaning structure than *and*, acquiring it requires more effort. So the conceptual complexity generated by the number

[107] Max criticizes Löbner for postulating three different negations. For him (and us), outer negation stands for the operation "put a negation in front of the entire expression" (Max 1995: 164)); dual negation: "put a negation both in front of the entire expression and in front of each argument of the operator" (Max 1995: 164)

of applications of NEC does not add much (if anything) to processing complexity, it is the result of comparatively cheap underground activity.
A more important and better measure for processing difficulty and position on the markedness hierarchy, so I postulate, is the impact a derivational step has on the denotation of an operator. Starting from the I-corner pivot (say /OR/) with its three unshaded cells and a single shaded one, the step towards the entailer results in a reversal of the values for cells S2 and S3, which are switched from 1 to 0, i.e. *two* value switches in all. Assuming that the I-corner has *switch level* 1 (ignoring the underground stages), the entailer then has switch level 1+2=*3* and is for that reason more marked.

(251)

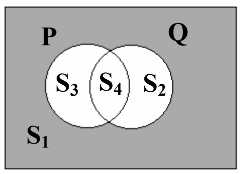


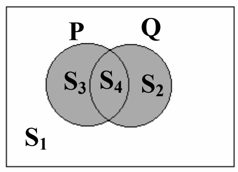


/OR/	/AND/
1	1+2 = 3

The step from the pivot to the contradictor, for its part, involves a reversal of the values of all four cells. It switch level is consequently 1+4=*5*.

(252)





/OR/	/NOR/
1	1+4= 5

It is in this straightforward sense that coming from a pivot, the step towards the contradictor (Type 3) is more marked and takes more processing effort than the step towards the entailer (Type 2)[108]. Counting this way, NAND is at switch level seven from the pivot, hence more marked (Type 4) than all the rest, as required[109]. In sum, the above measure of processing effort yields the right Type asymmetry in a plausible and simple way.

[108] Löbner's explanation for the T2-T3 order is part of his analysis of aspectual adverbs and ultimately rests on an assumption for which as far as I can see no proof is provided, namely that a reversal of a presupposition (e.g. between *already* and *still*) is less marked than a reversal of polarity (e.g. between *already* and *not yet*).
[109] In chapter 6, it will be argued that O-corner lexicalisations are impossible (at least for the calculi considered in this study).

There is interesting further confirmation for this line of thinking. Suppose one followed the normal path from the pivot to the entailer, but then chose to take a step toward the E-corner: since two cells change their value from the A-corner to the E-corner, namely S1 (from 0 to 1) and S4 (from 1 to 0), we add +2 to the switch value of the entailer in (251), namely 3, and consequently end up at switch value 5 for this two step operation. That is exactly the same switch level as the single step from /OR/ to /NOR/. If, as I have been arguing, the number of steps is not a relevant criterion but switch level is, then the prediction of this equal switch level result for two derivational paths is that E-corner lexicalisations should vacillate between lexicalisations based on the pivot and lexicalisations based on the entailer. Now that is precisely correct. That E-corner lexicalisations are often directly based on the pivot has been amply illustrated (*n-or, n-one*, etc.), but A-corner based lexicalisations are not so hard to come by either, witness the following cases:

(253) a. Latin *Ne-que* (or *Nec*), literally “not-and”
b. Dutch *n-immer*, literally “n-always”

Although these two different A-corner based lexicalizations are formally negations of an A-corner operator and might therefore at first sight be viewed as O-corner items, they are not. The negations in question are really inner negations: Lat. *Neque* does not express the O-corner meaning “not and” but rather the E-corner meaning “nor”; and Dutch *nimmer* does not express “not always” but rather the E-corner meaning “always not”, i.e. “never”, with always scoping over negation.
There is an even more surprising example, namely Old English *nalles*, which at first glance looks very much like the missing O-corner item we have been looking for. But again, appearances deceive: its meaning is not “all not”, but again the E-corner meaning “not”, “not at all”. So, as Hoeksema (1999) writes:

> "Even in cases that look very promising (like Old English, which has an item *nalles*, derived from *alles* "all" by adding the negative prefix *ne-* - the same that is used in words like *never, naught, nor, neither*), we end up empty-handed. *Nalles* does not actually mean "not all" or "not everything", but "not at all" (Horn 1989: 261)."
>
> Hoeksema (1999)

The combination of the degree of markedness as determined by the switch level count and the fact that in terms of acquisition entailers may be easier to acquire than pivots (at least for conjunction vs. disjunction this is the case) provides a plausible account why a two-step E-corner operator based on an entailer, namely with internal negation, is quite common across languages.

4.9 Conclusion

The point of departure of this chapter was the two basic relations of the logical 2D Cartesian Coordinate System. Our analysis of them was a standard deductive one, which defined the relationships between the pivot and the other operators in terms of entailment and yielded the static relations CD and ENT between propositions containing the operators in question. Since lexical items are listed in long term memory and sentences/propositions stacked in a discourse domain upon the process of their production, there can be nothing against such a ‘static’ approach postulating relationships among stored products.

Deductively oriented system		
PIVOT	relation	
SOME/ANY/ONE OR	CD	NONE NOR
	Entailment	ALL AND

A look at the properties of logical operators qua lexical items led to an additional perspective. On grounds of morphological and semantic complexity of the non-pivotal lexical items, a pivot-based system with asymmetric *operations* was proposed, identifying the lexical form of contradictors and entailers as constructed from the pivot by means of the operations negation (<u>NON</u>) and conjunction (<u>ET</u>), respectively. The latter are two variants of a *negative* operator <u>NEC</u>.

Inductively oriented system				
Input		operation	Output	
pivot	SOME/ANY/ONE OR	Negation <u>NON</u>	contradictor	NONE NOR
		Conjunction <u>ET</u>	entailer	ALL AND

As the linguistic facts have to dictate the analysis, empirical evidence was sought for this hypothesis that entailers and contradictors are composite lexical items, decomposable into the meaning of the pivotal input and an additional meaning feature added by means of the dynamic operation. For contradictors, morphological evidence made it transparently clear that this is the right approach: *n-or*, for instance, is visibly more complex than *or*. Semantic arguments pointed in the same direction. For entailers, no similar morphological evidence was found, so the idea that they are more complex than pivots had to be motivated on semantic and epistemological grounds. To that end, the relationship between *and* and *or* was tackled. This could be done in empirical detail, as strong evidence (involving *the same, too*, and *so*) was readily available. The claim that *all* has a complex semantic structure which has the same bar-code representation as *and* was defended on the grounds that it provided a solution to the problem of existential import.

The analysis of the propositional operator *and* posed an interesting problem regarding the notion “conjunction” that needed solving: if *AND*, i.e. the meaning of the lexical item *and*, was to be viewed as a “conjunction” of the meaning of *or* and something else, then the conjunctive operation involved had better not be identical to *AND* and as complex as *AND* itself. This led to the postulation of a difference between the lexical item *and* on the one hand, and the less complex, abstract conjunctive operator ET. The latter was independently needed, as it does not enter only into the lexical specification of the meaning of the lexical item *and*, but also into the lexical decomposition of the predicate calculus operator *all*.

It is the hypotheses that emerge from the decomposition analysis which will constitute the guiding intuitions for the next chapter: logical operators are interrelated lexical items, their lexical properties are the source of differences between the operators themselves, amongst others differences in morphological and semantic complexity. At the level of the lexical items that function as operators in the predicate calculus, the most primitive one is the pivot, and the rules of Negation and Conjunction are the sources of the two derived hubs. They are the operations whose effect at the level of the entire propositions is to establish the two basic 'static' relations of CD and entailment of Aristotle's predicate calculus and the Stoics' propositional calculus. While the two relations are different from one one another, the two operations NON and ET that were proven to generate them are variants of a single negative-disjunctive operator NEC, to which the whole system consequently reduces.

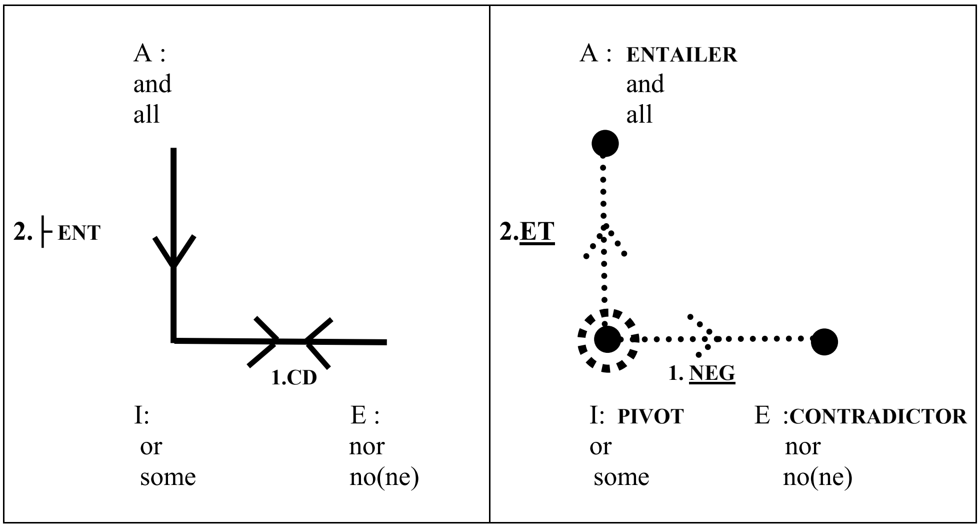
5 THE PIVOT OF THE 2D CARTESIAN COORDINATE SYSTEM

5.1 Why are pivots pivotal?

In our quest for a minimalist natural logic, three steps have been taken. In chapter 2, the Boethian Square of Oppositions was reduced to a two-dimensional system turning on the relations of entailment (ENT) and contradictoriness (CD). Chapter 3 provided an analysis in which these basic relations were postulated to obtain in the first place between lexical items, i.e. proposition types, in the natural language lexicon and only derivatively between the concrete propositions in which the operators are used (cf. (254)a below). In view of this, the attention gradually shifted from the logical relations between sentences containing logical operators to the lexical content of the operators themselves. They were decomposed in such a way that the meanings of the non-pivotal focal points – the *entailer* and the *contradictor* – are based on that of the pivot (cf. the direction of the arrows in (254)b). The rules by means of which the complex non-pivots are composed from the meaning of the less complex pivot were identified as negation (NON) and Conjunction (ET), two variants of the negative-disjunctive operation NEC. These are Language of Thought operations below conscious control in natural language acquisition. The resulting picture adds substance to the view that IN-logic is a two-dimensional Cartesian coordinate system, as it identifies the least complex pivotal I-corner as the Cartesian "origin" of IN-logic. It is the point of intersection where the quality relation (NON) leading to the contradictor and the quantity relation (ET) leading to the entailer meet.

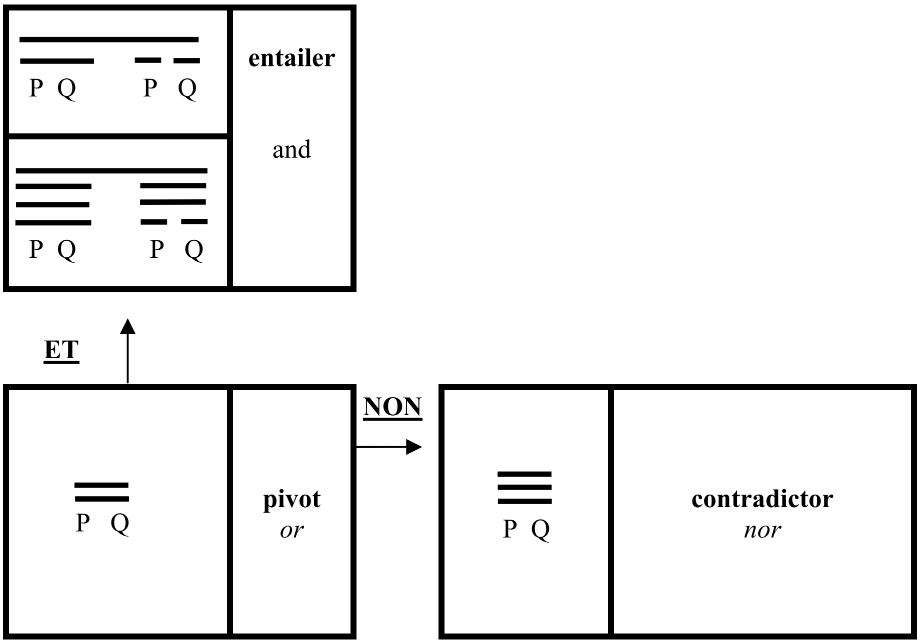
If correct, this makes the pivot the "odd one out" in that its meaning is (a) less complex than those of the two non-pivots and (b) a subpart of both of them.

(254)



Concretely, the following Peirce-style decomposition was argued to represent the lexical content of the propositional calculus triad *and, or, nor*. It is stated in a bar-code which represents a universal code as part of the Language of Thought. The codes show clearly how the meaning of the pivot is indeed less complex than those of the two non-pivots, while at the same time entering into both. This is not a code-specific phenomenon and therefore not an artefact of our notation – other more traditional formalizations would produce the same differences in elaborateness between the pivot and the other two hubs.

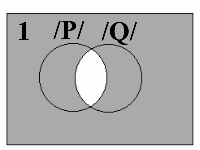
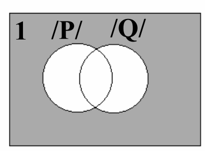
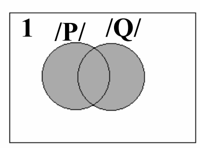
(255)



Auspiciously, not only the bar-code analysis supports the claim that entailers and contradictors are lexically more highly specified than pivots. Lexically more highly specified items are expected to leave fewer possible situations in the extensional universe of discourse which are still compatible with their more elaborate, hence more restrictive description. In our case, the prediction then is that if indeed entailers and contradictors are intensionally more complex than pivots, they should have comparatively more cells excluded in their Venn-diagram representation. This is precisely what obtains (and is standardly recognized). Recall the proposition calculus, set-theoretic and algebraic operations which the three corners of the 2D Cartesian coordinate system represent and the corresponding Venn-diagrams, with the number of excluded cells indicated.

(256)

	logic	**set-theory**	**algebra**
A-corner (and)	conjunction	intersection	multiplication
	$X \wedge Y$	$X \cap Y$	XxY
I-corner (or)	disjunction	union	addition
	$X \vee Y$	$X \cup Y$	X+Y
E-corner (nor)	negation of disjunction	complement of union	universe minus addition
	$\neg (X \vee Y)$	$\overline{X \cup Y}$	1-(X+Y)

A-corner Entailer intersection $/P/ \cap /Q/$	I-corner pivot union $/P/ \cup /Q/$	E-corner contradictor complement of union $\overline{/P/ \cup /Q/}$
		
3 cells excluded Only 1 cell unshaded	1 cell excluded 3 cells unshaded	3 cells excluded only 1 cell unshaded

The logical relations perspective (ch. 1 & 2), the bar-code decomposition (ch. 3) and the set-theoretic exclusionary outlook all converge to the same conclusion: a pivot is systematically different from non-pivots by being lighter in lexical content:
a. it is the 'origin' of the 2D Cartesian Coordinate System, on which both CD and ENT are defined;
b. it is the intensionally least specified item of its operator triad (cf. bar-code);
c. it is the extensionally least exclusive item of its operator triad (cf. three non-excluded cells).

The entailer is even more radically different from the pivot than the contradictor. It has a dyadic internal structure (with *two different* successive twin selections) whereas the pivot and the contradictor have a monadic structure (a single twin selection or at most multiple same-selections which then get conflated and leave a single twin selection representation in the end), as can be seen in (255). This difference, more specifically the semantic contribution of the second twin selection **(ii)** (chapter 4, (248)), is the source of a well-known asymmetry of entailers relative to pivots and contradictors, namely that entailers are not convertible, i.e. *all F are G* does not imply *all G are F*. Pivots and contradictors, for their part, are convertible – *some F are G* implies that *some G are F*, and *no F are G* implies that *no G are F.* This 'convertibility' or 'symmetry' (Barwise and Cooper 1981) was "taken by Aristotle and his followers to belong to the basic logical facts about the square of opposition" (Westerståhl 2005: 3). Among the convertible operators, the affirmative one, the pivot, is the least marked, given that it has fewer bars and is hence derivationally less complex than the contradictor, a final asymmetry.

To find empirical evidence for the pivot-nonpivot, the nonconvertible-convertible and the pivot-contradictor asymmetries, the present chapter will be entirely devoted to the third question that was raised in § 2.3:

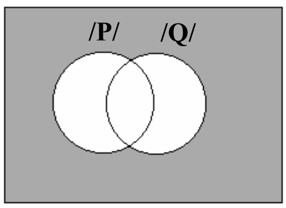
> What are the main semantic features of the element which functions as the *pivot* of a formal calculus of entailments in natural language and how does it differ from the other two hubs?

5.1.1 Pivots versus non-pivots

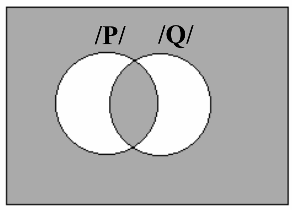
The nature of the difference between pivots and non-pivots will be addressed in § 5.2. Its main observation is that pivots are more variable in meaning than non-pivots and its main assumption is that this can be tied to their lexical "lightweight" status. Specifically, since the entailer and contradictor have their extension sets narrowed down to a single cell in the Venn-diagrams of (256), there is little room for contexts to cause further modification. By contrast, the sparser intrinsic lexical content of pivots means that their lexically specified extension set is larger (three cells unshaded), which leaves more room for further cell exclusion by context[110]. Take the following sentence pair:

(257)

a. Have you met John or Mary?
("John or Mary or both?")
inclusive



b. She met John or Mary.
("John or Mary, but not both")
exclusive



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[110] The general point that is here instantiated in pivots is that less intension (lexical content) leaves more extension and hence more room for context-determined variation. That this generalization is much more generally valid, can easily be illustrated by means of reputedly lightweight functional categories such as the verbs *be*, *go,* etc., the preposition/infinitival particle *to*, the preposition *of,* etc., all of which can get different readings in different contexts (cf. Gruber 1965, 1976; Jackendoff 1983, ch. 10).

(i)
a. The car *is* at the corner (spatial)
b. The meeting *is* at twelve noon (temporal)
c. Elisabeth *is* bright (identificational)
d. The book *is* Elisabeth's (possessional)

(ii)
The car *went* to Bill (= it moved; spatial)
The car *went* to Bill (= change of owner; possessional)

The difference between the lexical meaning (3 cells unshaded) and the extensionally smaller exclusive reading (2 cells unshaded) correlates with the difference between an interrogative and an affirmative context.

Aside from the inclusive-exclusive variation of (257), further chameleonic behaviour of pivots will be shown to come in two categories. First, there are cases of pivots in different languages which have over time acquired a contradictor corner reading, in which case they become semantically indistinguishable from lexical E-corner operators. This will be called *I→E-shift* and can be illustrated here for the French temporal adverb *jamais*. Originally, its meaning was purely that of an existential pivot ("ever") as in (258)a., but it developed an additional contradictor reading (258)b (Grevisse (1980: 1053)).

(258) a. I-corner (jamais = "ever"):
Si JAMAIS vous changez d'avis
"If *ever* you change your mind"
b. E-corner (jamais ="never"):
Vous avez toujours été orateur, JAMAIS philosophe (Fén., Dial., 33 ; quoted from : Grevisse 1980: 1053)
"You have always been an orator, *never* a philosopher."

Secondly, in certain syntactic contexts several pivots can get a reading akin to that of an entailer, an interpretation that will be called *I→A-shift*[111].

(259) a. I-corner: If you see anybody, give me a ring (existential)
b. ≈A-corner: Any man can do that (quasi-universal: ≈ Every man can do that)

Entailers and contradictors, for their part, are quantificationally less malleable. A-corner operators which acquire E-corner interpretations or vice versa, are to my knowledge not attested.

A-corner Entailer		*I-corner pivot*		*E-corner Contradictor*
every		any		no(ne)
always		ever		never
both		either		neither
And		or		nor

[111] The pivot *some* is an exception to the I→A-shift in English.

5.1.2 The lexical meaning of a pivot

If pivots are variable in meaning due to low lexical content, it is important to be clear about the core lexical meaning that underlies them all. Our starting point in this respect will be the semantic component that was identified for the PROPC operator *or* in the decompositional bar-code and the equivalent Venn and placemat diagrams, namely (AT LEAST) ONE OF P AND Q HAS VALUE 1.

(260)

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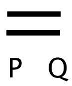
<td>
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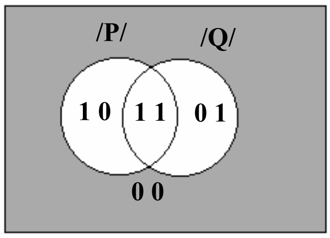
<td>
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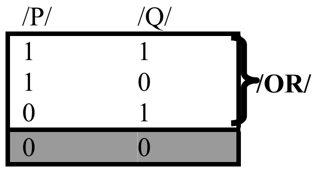
<td>
</td>
</tr>
<tr>
<td>Exclusion (=highest bar) of what is neither P nor Q; rest selected</td>
<td>1 cell excluded, namely that of neither P nor Q; rest selected</td>
<td>1 cell excluded, namely that of neither P nor Q; rest selected</td>
</tr>
</table>

It was observed that there is a certain correspondence between this pattern and a semantic feature of pivots in APC, namely that when *some flags are green*, there exists *at least one* member of the class of flags: there is *at least one* flag. It is the common denominator of these two descriptions, namely a notion (AT LEAST) ONE that is also traceable in other pivots. Since these pivots obviously have different meanings, the element in question is a substrate that enters into them, but does not constitute their whole meaning. The aim of § 5.3 is to illustrate the presence of the substrate for the different pivots on the basis of their common sense lexical meaning descriptions in dictionaries and to tie it to the negative bar code (cf. (219) in § 4.4.6), the placemat diagram (§ 3.3.2) and the valuation space of *or*.

5.2 Shifts away from the I-corner

A good indication that I-corner operators are lexically less specified than the other operators is that their meaning can be affected in a number of ways which even turns them into E-corner operators at times or makes them very similar to A-corner operators.

5.2.1 The I→E-shift

Morphological evidence (*or, n-or; ever, n-ever; either, n-either*) was appealed to in chapter 3 to conclude that E-corner operators are derived from pivots by application of negation. Those facts are sufficient proof of the foundational nature of the pivots relative to contradictors for the purpose of the lexical decomposition analysis. But there is another unidirectional I-to-E phenomenon which lends further support to the same conclusion, namely cases where I-corner operators get an E-corner operator interpretation in particular syntactic contexts.

Take the following examples from Grevisse (1980: 636):

(261) a. Penses-tu qu'*aucun* d'eux veuille subir mes lois? (Boil., Ep., 2.)
b. *Aucun* de nous ne serait téméraire…(Corn., Rod., IV, 1)
c. Lui connaissez-vous des ennemis ? *Aucun.* (Ac.)
(Grevisse 1980 : 636, § 1270)

In conformity with its etymological origins (< popular Lat. *alicunus, from the Latin pivots *aliquis* ("someone") and *unus* ("one")), the meaning of the pronoun *aucun* was pivotal "someone" until the 17th century (Grevisse 1980: 636). While the original meaning no longer survives in modern French, except in phrases such as "*d'Aucuns ont dit que…*" (Grevisse still mentions interrogatives such as (261) a., but they are no longer used in modern French) *aucun* normally has a universal negative interpretation nowadays, primarily in negative contexts like (261) b. Grevisse (1980 : 636) observes : "Le plus souvent *aucun* est accompagné de la négation *ne* : ainsi, par contagion, il a pris la valuer négative de *nul*."[112] While the observation is correct, the precise nature of this "contagion" remains to be made precise. In any case, what is important for our purposes is the directionality of the process: a pivotal I-corner category can acquire an E-corner meaning over time. Moreover, whether the lexical item that has acquired an E-meaning can still have its original I-meaning in the current stage of a language is immaterial to the observation. Thus, instead of *aucun*, for which the I-reading is obsolete, one could consider *jamais* (cf. (258) above) or *personne*, for instance, where the same I-E-shift has occurred, but where the original meaning still survives independently.

A description of what underlies the"contagion" referred to above is given by Jespersen (1924: 334) where he discusses the original English negative morpheme *ne*[113], which he takes "to be (together with the variant *me*) a primitive interjection of disgust consisting mainly in the facial gesture of contracting the muscles of the nose." (…) "When the negative has become a mere proclitic syllable or even a single sound, it is felt to be too weak, and has to be strenghtened by some additional word, and this in turn may come to be felt as the negative proper" (Jespersen 1924: 334). Such reinforcement or strenghtening typically comes from I-corner elements like *ā-wiht, ō-wiht*, meaning "something, anything" (*ā* (ever)-*wiht* (thing)) or "a small thing (*not a bit, not a jot, not a scrap,* etc., Fr. *ne…mie, goute, point, pas*) or by an adverb meaning *ever* (OE. *na* from *ne* + *a* = Goth. *ni aiws,* G. *nie)…*" (Jespersen 1924: 336). Jespersen's emphasis on the nature of the strengthener as an I-corner element, adopted here, is more supportive of our analysis than the position of van der Auwera & Neuckermans (2004: 460-461), who consider a contradictor (their quantifier negator) as a more natural strengthener[114]:

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[112] Note however that universal negative aucun also occurs in elliptical sentences (261) c.
[113] To be precise, several languages have two negators, e.g. Vedic Sanskrit has *ná* in clause-initial and preverbal position, and prohibitive *mā* in imperative contexts. (Horn 1989: 447)
[114] For the terms predicate negation and quantifier negation, cf. van der Auwera & Neuckermans (2004: 475), who mention that predicate negation is also often referred to as sentence negation; quantifier negation as constituent negation and negative concord.

> "First of all, the element that strengthens the original negator and that will grow into the new negator may be a quantifier negator. There is no necessity, though. French *pas* 'step' or Latin *oenum* 'one' are not quantifier negators. Dutch *niet*, on the other hand, or English *not* are. Their original meaning was 'nothing'."
>
> (van der Auwera & Neuckermans 2004: 460-461)

Semantically, however, it is pivots like *pas* and *oenum* which which get inserted to play this strengthening role because they have the effect of turning the negation ("not") into a more emphatic but extensionally equivalent universal negative ("not a single one/anything"). The same type of strengthening is observed in German and Dutch by means of I-corner *icht/ocht,* again meaning 'something'. This was followed in many (but not all) languages by negation copying on the particular affirmative quantifier and dropping of the weak predicate negation, yielding the following version of Jespersen's cycle:

(262) a. 'ik *ne* ga' (predicate negation)
b. 'ik *ne* ga icht. (reinforcement of predicate negation by an <u>I-corner element</u> meaning "something/anything/at least one thing")
c. 'ik *ne* ga (*ne*)-icht' (optional quantifier negation on top of predicate negation as a "double"/"multiple", but without negative force since the interpretation is not that of a double negative, but a case of negative concord)
d. 'ik *ne* ga *ne*-icht' (bipolar negative concord, i.e. quantifier negation and predicate negation)
e. 'ik (*ne*) ga *ne*-icht' (quantifier negation gradually takes over from predicate negation)
f. 'ik ga *ne*-icht/*n*-iet' (dropping of *ne*)

In Scandinavian, no quantifier negation was added after the original *ne* was strengthened by additions and its morphological expression finally ousted by these, hence the current negative *ikke*, which at first had no negative meaning but now does: a clear I→E-shift. (Jespersen 1924: 335).

Do E/I-shifts exist as well? Looking at the bipolar negative concord stage in (262) d. in isolation, one could still argue that the item *ne*-icht might represent a case of E/I-shift rather than vice versa, on the assumption that *ne*-icht is a contradictor, but acquires pivotal meaning for the combination with the sentence negative to result in a universal negative. Both the diachrony of Jespersen's cycle as the following facts with a morphologically explicit I-corner item *pas* (a minimal amount) prove, however, that no E/I-shift can be involved.

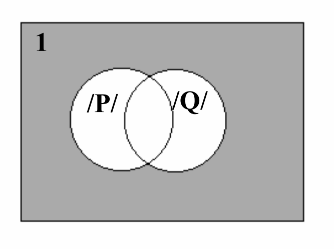
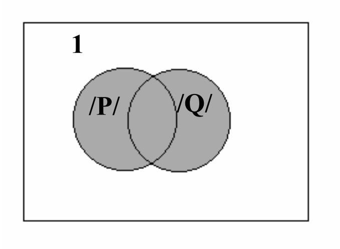
(10) 1 *jeo ne dis*
2 *je ne dis* (*pas*) the negator is strengthened
3 *je ne dis pas* the strengthener bleaches and becomes part of the negator
4 *je* (*ne*) *dis pas* the original negator loses ground
5 *je dis pas*

(van der Auwera & Neuckermans: 459)

In the case of bipolar negative concord, I-corner *icht* is historically prior to the form with the negative copy *ne*-icht, hence the shift was I→E, not the other way around. In the *pas* example, though the strenghtener "bleaches" into part of the negator, it was originally, as its etymology betrays, an undeniable I-corner element: *passus* = minimal amount "(at least) one step".

The description just given leads to the conclusion that, diachronically, items like *aucun* and *ikke* used to have only the type of meaning that we identified for current pivots in the previous section. The later acquired universal negative meaning was then attributable to the effect of the negative operator NEG in its syntactic context, (which may but need not be morphologically realized (*ne*, *n'*, *en*)).

(263)

Former lexical meaning of *aucun:* Pivot meaning	Effect of negation (*ne*): Contradictor meaning
a. Penses-tu qu'*aucun* d'eux veuille subir mes lois? (Boil., Ep., 2.) union $/P/ \cup /Q/$ 	b. Aucun de nous ne serait téméraire…(Corn., Rod., IV, 1) complement of union $\overline{/P/ \cup /Q/}$ 

The diachronic I→E-shift analysed in this section reinforces the conclusion drawn from the morphological evidence for the pivotal role of I-corner elements relative to E-corner operators, this time on the basis of the effect of negation in syntax on a lexical pivot, causing the kind of "contagion" alluded to above. Its effect is to turn an I-corner pivot into an E-corner contradictor.

5.2.2 The I→A-shift

Turning from the CD leg of the 2D Cartesian Coordinate System to its entailment leg, we note that pivotal I-operators in certain contexts acquire a reading akin to that of an entailer in the A-corner. Thus, *any* functions as an existential (=I-corner) when used as a negative polarity item (NPI)[115] – also called polarity sensitive *any* (PS).

[115] Van der Wouden (1994: 5) defines them as "expressions which can only appear felicitously in negative contexts". The contexts involved do not just include negatives, but also "nonassertive speech acts (questions, imperatives, exclamatives), the protasis of conditionals, the scope of strong intensional verbs like *want* and *hope*, and the restriction of certain universal quantifiers"

(264) a. I do not have any solutions. (NPI-*any;* nonaffirmative negative)
b. Has he said anything interesting? (NPI-*any;* nonaffirmative interrogative)

But it can also acquire a free choice (FC) reading (Kadmon and Landman 1993: 353), which has been argued by many to be a universal quantifier (Horn 1972; Carlson 1981), hence an A-corner item.

(265) a. Any beaver builds dams. (FC-*any*)
b. Every beaver builds dams. (universal quantifier)

In this section, this *I→A-shift* will be illustrated for a range of pivots.

A first case involves indefinites, which get a particular reading in the same contexts in which *any* is existential and shift to a generic reading in those contexts where *any* is free choice.

(266) a. A beaver builds dams (generic)
b. John saw a beaver (particular)

This gives us reason to believe that indefinites are pivots, a point to which I return in (280) and § 6.3.1.

The domain of quantification of the quantificational pivot *either* in (267) is a two-entity universe. While it has a pivotal I-meaning ("a or b or both a and b") in negatives and interrogatives, it has an interpretation akin to an A-reading ("both a and b") in the affirmative sentences of (267) b.

(267) a. Have you seen [either] of them? (I: "a or b or both a and b")
I haven't met [either] of them. (I: "a or b or both a and b")
b. There are trees on [either] side (A: "both sides") of the river.
[either of them] can fill you in. (A: "both of them")

The fact that the I→A-shift occurs in (267), in the realm of quantification in the two-element universe of APC_2, suggests that it might well show up in the propositional calculus too, since we have seen before that the properties of the two-element predicate calculus and the proposition calculus are very similar. And indeed, Horn (1972) and Kamp (1973), Zimmermann, (2001), Chierchia (2001, 2004) have all observed that in the same contexts where FC *any* is allowed, *or* gets a free choice interpretation (referred to as FC disjunction or conjunctive disjunction), and becomes semantically akin to a conjunction:

(268) a. Have you talked to [Henry or Bill]? (I: Henry, Bill, or both?)

(Giannakidou 1998: 8). Notwithstanding these insights in the nature of the contexts, the precise nature of the licensing principle (syntactic, semantic/pragmatic and/or both) and the question whether it is an independent principle is still under debate (cf. Horn and Kato 2000: 9).

b. [Henry or Bill] could tell you that (A: "conjunctive disjunction": Henry and Bill can tell you that, so the choice is yours[116])

Chierchia (2001, 2004) has extended the observation to other operators. In (269), it occurs with the locational adverb *anywhere*. Once more, there is an I-reading in interrogatives and negatives, but an A-reading in some other environments. In (270), the I→A-shift involves pivotal adverb *ever*

(269) a. Have you seen him [anywhere] ? (I: place x, place y, or more places)
I haven't seen him [anywhere]. (I: place x, place y, or more places)
b. [anywhere] the wind blows. (A: "place x and place y")

(270) a. I haven't [ever] met her. (I: at time x, at time y, or at time x and y)
Won't they [ever] learn? (I: at time x, at time y, or at time x and y)
b. The most remarkable president [ever] (A: of *all* times x and y)

With respect to the I→E-shift, it was observed that there is no inverse shift. The question whether the I→A-shift has an inverse A/I-variant is a harder one to resolve, since the situation for the propositional operator *and* is mixed. Thus, in a sentence

(271) I have not seen Johan and Guido today

the meaning expressed is that of a joint falsehood: I have seen neither of them. But that would seem to entail that in this context *and* has to be disjunctive. Only then can the combination with negation result in a universal negation (*neither…) nor*. This would mean that an A-corner element *and* can shift to an I-corner disjunctive interpretation. I believe the question can be resolved by claiming that the conjunction remains what it is, but scopes over rather than below negation, which gets distributed over the conjuncts, as in the Dutch sentence below. In that case, the higher operator has to be a regular conjunction to yield a joint falsehood:

(272) Ik heb geen van beiden gezien – [[Johan niet] en [Guido niet]]
I-saw-neither-(of both) [[Johan-not]-and-[Guido-not]]

For predicate calculus operators, however, the situation is unambiguous: there is no A/I-shift. While pivotal existential operators can acquire universal readings, the opposite

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[116] Carlson (1981: 17) gives further examples of conjunctive *or* in FC *any* environments.

(i) John likes cats or dogs = John likes dogs and John likes cats (on one reading).
(ii) For Bob or Bill to do that is a nuisance = For Bob to do that is a nuisance and for Bill to do that is a nuisance.

He notes that the set of environments in which *or* is interpreted as equivalent to *and* "goes beyond the FC *any* environments to include a goodly portion of the affective contexts as well (LeGrand (1975), Ladusaw (1980))". I do not agree, as will is e clear from (268)a.: in affective contexts like interrogatives (negatives, the protasis of conditionals, etc.) the interpretation is that of an inclusive disjunction, not a conjunction. The fact that the disjunction in such contexts *includes* the inclusive part (*or both*) is why it is felt to be close to a conjunction.

does not occur: universal operators cannot get a contextually determined existential reading (in interrogative contexts, for instance):

(273)

universal quantification ∀	*existential quantification *∃
All doctors can tell you that. Everything will be well. Always ready to shoot trouble Everywhere the wind blows Both of them can fill you in.	*Have you got all complaints? *Has he said everything interesting? *When will they always learn? *Have you seen him everywhere? #Have you seen both of them?

If the basic assumption that we are defending is correct, namely that A-corner operators lexically include the meaning of I-corner operators, but have an additional meaning feature that narrows down the extension set from a union to an intersection (cf. (256)), the facts of (273) are not surprising. To turn lexically specified universal quantifiers into existentials would require canceling part of the lexical meaning of the former to widen the extension set, in other words, canceling information originally drawn in from the lexicon, which goes against the informativity constraint: the role of new intensional information (except for a disjunction) can only exclude, narrow down the extension set, not widen it. What would be the point of first pulling in information from the lexicon if one then has to annul it by syntactic means afterwards? It would then clearly have been much more economical to simply select the lexically less specified item, the pivot, so that no superfluous information is entered into the derivation in the first place which then has to be removed again.

Critical readers might object at this point that the variability of universal operators is being downplayed, since there certainly is a lot of semantic variation among them (Vendler 1967, Gil 1991, (for Dutch) Dik 1975). For instance, *all* can get both a collective and a distributive interpretation, while *each* and *every* are exclusively distributive:

(274) a. All men kissed a girl/gathered
b. Every man kissed a girl/*gathered
c. Each man kissed a girl/*gathered

The crucial point for our discussion, however, is that this variation does not affect the *domain* of quantification, which remains universal throughout, but only its set-theoretical *organisation*. Whatever the reading, the domain of quantification is invariably the set of all men, so the operators never shift away from the A-corner. In this sense, universal operators are less malleable than pivots, whose I→A-shifts represent genuine shifts affecting the *domain* of quantification.

A final comparative point that needs to be made is that the I→A-shift illustrated here is different in character from the diachronic I→E-shift described in § 5.2.1. In the latter case, the I-corner meaning has disappeared entirely and the shift is a case of change of lexical meaning over time. The I→A-shift, however, is rather a case of *enrichment* of lexical meaning in a given context, whereby the lexical meaning does not disappear. That is why many have emphasized that the result of I→A-shift is a meaning 'akin to' but not identical to that of an entailer.[117] The following example pair illustrates the correctness of the claim that I→A-shift is not to be identified with a pure A-corner universal.

(275) a. I→A: John will say anything
b. A: John will say everything

In (275)a., divulging one or other thing (= I) that is adequately *representative* of everything (= A) that could be said, is enough to make the sentence true. In other words, the I-corner meaning does not disappear. In (275)b., however, John has to divulge the whole range of things relevant to a certain domain of discourse (= A).

The same conclusion is valid for I→A-*or* versus A-corner operator *and*, as is illustrated by the following contrast:

(276) a. A: I don't think [employers and union leaders] can meet on Friday.
b. I→A: I don't think [employers or union leaders] can meet on Friday.

While the real conjunction in (276)a. can result in a collective reading whereby employers and union leaders meet one another in a single meeting rather than each group having a separate gathering, only the latter reading is available in (276)b., which shows that the meaning of *or*, though "conjunctive", i.e. "akin to a conjunction", is not identical to that an A-corner conjunction. Its lexical I-corner meaning, which is never collective, is not undone by the shift. But what then is that core meaning of pivots?

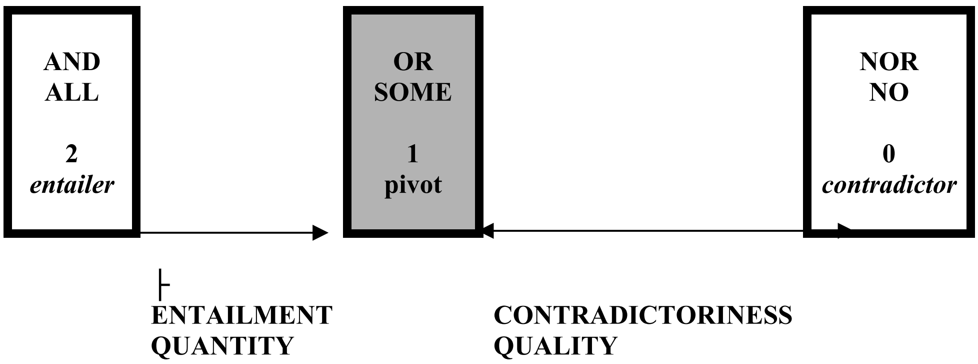
5.3 Pivots and (AT LEAST) ONE

In this section, it will be shown that although pivots are semantically variable, there is nonetheless a common core. Notably, the semantic notion (AT LEAST) ONE keeps reappearing. This joint feature of pivots will be described for each case and represented by means of similar placemats and corresponding valuation space diagrams as were set up for *or*.

For a first case, observe that there is a striking resemblance between numerals and the operators analysed in the second chapter. If *two flags are green*, this entails that *one flag is green*, a conclusion which is analogous to the conclusion that (i) if *John and Peter are in the garden* it follows that *John or Peter is in the garden*; (ii) if *all my friends are here,* it is also true that *there are some of my friends here*. The relation between these propositions with *2* and *1* is thus a well-behaved quantity relation (entailment). And affirming that *zero flags are green* is analogous to saying that (i) *neither one nor any other flag is green* or (ii) *no flags are green*. In other words, the relation between

[117] . Cf. Carlson (1981: 8; my underline, DJ) with respect to the pivot *any*: "English *any* is ambiguous, (…) instantiating an existential quantifier as well as something akin to a universal quantifier, the latter henceforth referred to as "free-choice" *any*." Cf. also Löbner's (1990: 27-29) "Homogene Quantor".

propositions with *1* and *0* represents the same quality difference identified earlier for the other triads : contradictoriness. The numeral triad {0, 1, 2} can therefore easily be added to the triads of propositional {and, or, nor} and predicate calculus operators {all, some, no(ne)} in the scheme below. The relationships between the elements are the same ones, and while *1* is a pivot, *2* is an entailer and *0* the contradictor of *1*.



This perspective on numerals is inspired by their inclusion by Horn (1989: 232) as scalar values belonging to sets of semantic concepts ordered on a pragmatic relative informativeness scale. Horn gives the examples in (277), which do not only include numerals, but also the other quantifiers we are studying in this work (and more):

(277)

<all, most, many, some>; <always, usually, often, sometimes>; <and, or>; <6, 5, 4, 3, 2, 1>; <must, should, may>; <necessary, (logically) possible>; <certain, {probably/likely}, possible>; <obligatory, permitted>; <boiling, hot, warm>; <freezing, cold, cool, (lukewarm)>; <beautiful, pretty, attractive>; <hideous, ugly, unattractive, plain>; <adore, love, like>; <loathe, hate, dislike>; <excellent, good, ok>; <{terrible/awful}, bad, mediocre>; <no(ne), few, not all>.

The ordering in such scales is from strong to weak, and entailment relations among the elements of logical operator scales are the same as in the 2D Cartesian Coordinate System and standard logic: the stronger scalar value entails the weaker one, e.g. *nobody has come* entails *not all have come*. In pragmatics, scalar orderings are crucial for drawing invited inferences, i.e. implicatures: when a speaker chooses to utter a sentence with a weaker value, the hearer will conclude that the speaker wishes to express that s/he either has no evidence that the stronger value obtains or is certain that it does not. On the assumption that the speaker is both well informed and cooperative, the hearer of *not all my brothers have come to Brussels* will infer as an implicature that the stronger value *none of my brothers have come to Brussels* is not the right one and hence that at least someone has come. *Not all* will consequently be interpreted as “not all, but not nobody either”. Not to let the whole onus of explanation rest on pragmatic principles, however, the relevant relations will be tied to the 2D Cartesian Coordinate System. That is a first reason why the number triad is treated on a par with the other operators.

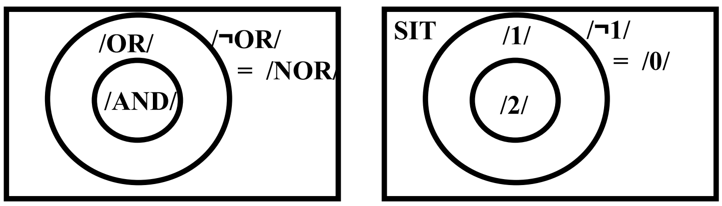
Another source for the inclusion of numerals among the triads studied so far is the hypothesis in Montague grammar and formal semantics more generally that the basic meaning of *one* is 'at least one'. This is none other than the numerical realm counterpart to claiming that the basic meaning of *or* is inclusive, e.g. *Have you seen A or B?* ('at least A or B, i.e. A, B or A and B'). And the fact that in *John has one book on linguistics* and sentences with *one* more generally the 'at least' reading is blocked by a more restrictive one ('one but not more', i.e. 'precisely one') is similar to the fact that the inclusive reading of *or* is more often than not blocked and narrowed down to the exclusive interpretation ('A or B, but not A and B')[118].

Using the valuation space approach and abbreviations of earlier chapters, the extension of propositions with the three basic numeral predicates can therefore be modelled after those of propositions with the propositional operators (and predicate calculus quantifiers).[119]

(278)

$/P\wedge Q/$ is written as **/AND/**
$/P\vee Q/$ is written as **/OR/**
$/\neg(P\vee Q)/$ is written as **/NOR/**

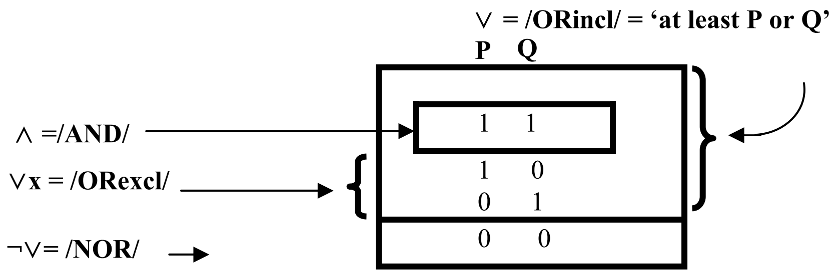
/2 F are G/ is written as **/2/**
/1 F is G/ is written as **/1/**
/0 F is G/ is written as **/0/**

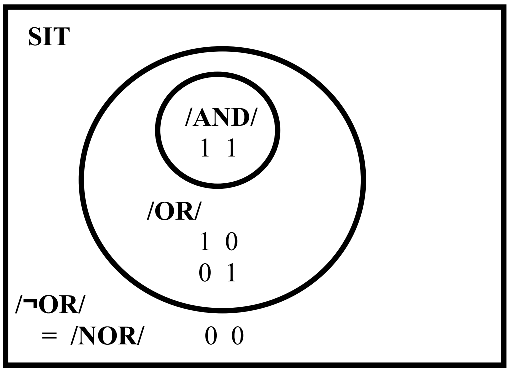


These diagrams indicate for the numeral-propositions that the set of possible situations in which *two flags are green* is true is a subset of the set of possible situations in which *at least one flag is green* is true, and the relation of the latter and *zero flags are green* is one of contradictoriness.

For the propositional operators above the truth tables were related to valuation space diagram by means of a placemat-construction (§ 3.3.2). Given the two-argument nature of the operators (P and Q, P or Q, P nor Q) and the two values true and false, the number of possible value combinations was $2^n = 2^2 = 4$. Thus conceived, the diagram with two arguments, i.e. binarity or *twin selection* (i.e. P and Q) turned out to be both necessary and sufficient to set up the triad.

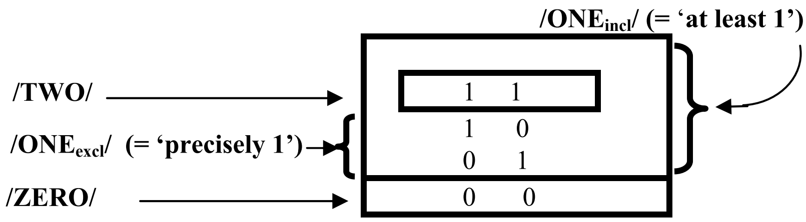
118 The relationship is more complicated, but that issue will be ignored for the moment.
119 As before, the extension of P is the set of possible situations in which P is true and is called the *valuation space* of P, represented as /P/ (Van Fraassen 1971, Seuren 1998: 331, Seuren et al. 2001).

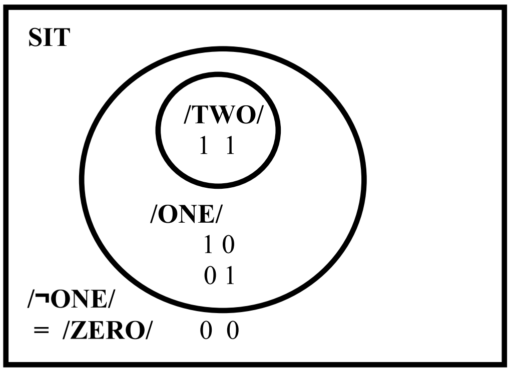




On the basis of the analogies between logical operators and numerals, a version of the placemat analysis developed for the operators can therefore be transferred to the numeral triad. Again, twin selection of *two arguments* and hence $2^n = 2^2 = 4$ combinations are precisely what is necessary and sufficient to generate the triad:

(279)





On further inspection, the semantic element (AT LEAST) ONE is a surprisingly recurring feature of the content of pivots and can even serve to extend the set brought up so far. Thus, the entry for the word *one* in Webster's Revised Unabridged Dictionary (1913) contains a reference to the morphologically more complex pivot *any*, but also (and here is the extension) to a new pivot, the indefinite article *a*.

> **<u>One</u>** \One\, a. [OE. one, on, an, AS. ["a]n; akin to D. een, OS. ["e]n, OFries. ["e]n, ["a]n, G. ein, Dan. een, Sw. en, Icel. einn, Goth. ains, W. un, Ir. & Gael. aon, L. unus, earlier oinos, oenos, Gr. ? the ace on dice; cf. Skr. ["e]ka. **<u>The same word as the indefinite article a</u>**, an. [root] 299. Cf. 2d A, 1st {An}, {Alone}, {Anon}, {**<u>Any</u>**}, {None}, {Nonce}, {Only}, {Onion}, {Unit}.]
>
> Webster's Revised Unabridged Dictionary (1913)
> (emphasis mine, DJ)
> http://smac.ucsd.edu/cgi-bin/http_webster?one

The connection with *any* confirms earlier attribution of pivotal I-corner-status to the latter, while the link with the indefinite article *a* correctly predicts that the latter can get the (AT LEAST) ONE reading too, in fact both the inclusive version with AT LEAST (280)a. and the exclusive variant (280)b without[120].

(280) a. AT LEAST ONE: Have you talked to [a student] yet? ('<u>at least</u> student x or student y, i.e. x, y or x and y, with x and y variables').
b. ONE: John bought [a book on linguistics]. ('one <u>but not</u> more', i.e. one or other book, but not more: 'precisely one')

From a cross-linguistic viewpoint too, the inclusion of the indefinite article *a* among the (AT LEAST) ONE pivots is warranted, given that the words for the numeral 1 form one of the few sources from which indefinites are derived cross-linguistically (the other two

[120] The specific presuppositional reading with QR of the bracketed DP is left out of consideration here. Since it involves QR, the original lexical meaning is further affected by the movement, but in a way that would lead me away from the general line of thought in the text.

sources are interrogative pronouns and generic ontological-category nouns such as *person, thing, manner, place, time*, etc. (Haspelmath 1997: 26)).

And for a historical example: the same element (AT LEAST) ONE characterizes the pivotal operator *ullus* which Aquinas used to formulate the linguistic puzzle that started our quest. *Ullus* is a combination of *unus* ("one")+*-ulus (*diminutive*)*, i.e. *unulus* ("little one") > *ullus* (Ernout-Meillet 1960). Once again, it is the numerical unit word *unus* (in its diminutive cloak) which occupies the I-corner. This fact adds some historical backing to the idea that its cognates *a*, *any* and *one* have a common ingredient (AT LEAST) ONE, not just formally, but also in their standard dictionary definitions.

In view of the facts above, extension of the usage of the terms inclusive (=AT LEAST ONE OF x AND y HAS VALUE 1) and exclusive (= (precisely) ONE OF x AND y HAS VALUE 1) to other pivots than *or* is not metaphorical language, but motivated by the hypothesis that the same Boolean placemat is a common feature of all of them. That is what the data indicate.

5.4 Conclusion

This chapter was devoted to proving that I-corner operators are the pivots of their paradigms. This was done by showing that they are the operators of the triads which easily acquire A-corner(like) and/or E-corner interpretations in different contexts. These semantic properties were explained by postulating that their basic meaning is set-theoretically that of the union of two sets. By being placed in certain restrictive syntactic contexts or by genericity, however, that meaning can be affected, resulting in a denotation which has (quasi-)universal force, set-theoretically either intersective (linguistically "conjunctive") or the complement of the union of two sets. In other words, in certain syntactic contexts existential quantifiers acquire "akin-to"-universal affirmative quantifier readings and behave very much like lexical A-corner operators, in other contexts they acquire a universal negative reading and thus become semantically indistinguishable from lexical E-corner operators. These facts support the hypothesis that rather than having three operators on an equal footing in logical triads, there is relief: the pivot is the most versatile and hence by hypothesis the lexically least specified item and the meanings of the two other corners are functions of its meaning: the entailer can thus be viewed as a combination of the meaning of the pivot and an additional specification – say *pivot-plus* -; the contradictor is the result of applying the complement function to the pivot – say *non-pivot.*

(281)

A-corner
entailer =
pivot^{+}

E-corner
contradictor =
$\overline{\textbf{pivot}}$

I-corner
pivot

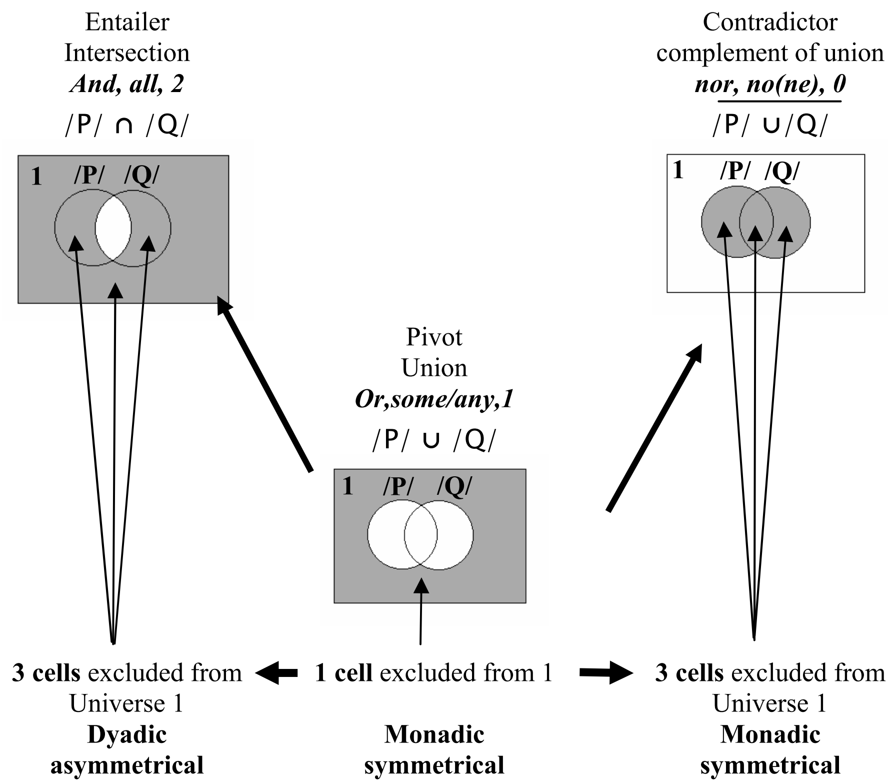
Besides the pivotal role of I-corner operators in the system, the present section has also established how tightly the operators cohere. For one thing, as table (282) expresses, the steps away from the pivot to the other operators are always cases of adding intensional information to the pivot, thereby affecting the extension set of the pivot by (a) *exclusion* of more cells in a second step **(ii)**, a negative canceling operation (cf. (282) below), which yields the entailer; (b) a set-complement switch which yields the contradictor. The fact that the entailer is created by means of two specifications, makes it dyadic and non-symmetrical. The switch to the contradictor happens by means of same-selection and conflation and consequently results in a monadic lexical structure. Because of this, the meaning of the contradictor is symmetrical, like that of the pivot. In this respect, the entailer is the odd one out.

The present conception also permits a more general statement of informativeness than so far. Up to this point, the informativeness requirement was viewed as a requirement that each step in the derivation has to produce a nonnull subset of the denotation at the preceding stage. It was observed that this notion of informativeness is only valid for the entailer-leg of the logical system. Note however, that a nonnull subset requirement on the quantitative entailer-leg always amounts to a reduction of the set of cells of the universe. If we take the latter perspective as what is behind informativeness, it is easily seen that the contradictor leaves fewer cells open than the pivot, so that in that respect it is also more informative than the pivot. All in all a simple but efficient generalization of informativeness to the qualitative E-corner leg of the logical system.

Though less exclusionary and hence less informative than the rest, pivotal elements themselves also have an undeniable exclusionary effect (1 cell excluded) on the Universe SIT (= 1) of possible situations: *John is in the garden or Bill is in the garden* excludes the possibility that neither of them are. That was the real reason for our adoption of the *exclusion* device /NEC/, not only as the key operation required to relate the universals to the pivots, but in effect also as the generator of the pivot itself from the universe SIT (= 1) by two-step subtraction (cf. chapter 4) and hence of the whole triadic system.

On the whole, the pivot-level is more basic than that of entailers and contradictors as indicated by the two types of shifts. Yet, it is not rock bottom (see the two shaded underground stages in (219)). /NEC/, the operator operative at the prelexical level is the most basic category, given that decomposition of a pivot in terms of it and derivation of entailers and contradictors by repeated application of the same /NEC/-operation was carried out successfully in chapter 4.

(282)


Entailer
Intersection
And, all, 2
/P/ ∩ /Q/
1 /P/ /Q/
Contradictor
complement of union
nor, no(ne), 0
$\overline{/P/ \cup /Q/}$
1 /P/ /Q/
Pivot
Union
Or,some/any,1
/P/ ∪ /Q/
1 /P/ /Q/
3 cells excluded from Universe 1
Dyadic asymmetrical
1 cell excluded from 1
Monadic symmetrical
3 cells excluded from Universe 1
Monadic symmetrical

6 THE VARIABLE MEANING OF PIVOTS

The present chapter will shift the perspective from the common semantic core of all pivots to factors which are the source of variation among pivots. To begin with, a lexical distinction will be drawn between more flexible pivots such as *or* and more rigid pivots such as its counterpart *either* in APC_2. The latter are restricted to negative polarity-contexts (cf. § 5.2.2), whereas the former are not subject to that constraint:

(283) a. Henry is talking to [Mary or Bill] at the moment
b.*Henry is talking to [either of them] at the moment

Aside from the flexible-rigid distinction, stress too plays an important role in the variability of the meaning of pivots, an issue which will be discussed in § 6.2.
Finally, a concrete illustration of the effects of the flexible-rigid difference and the role of stress will be given in § 6.3 by means of a close comparison of the indefinite article *a* and the existential quantifier *any*. On the theoretical front, that section will remove all possible doubt that IN-logic really is a 2D Cartesian Coordinate System.

The overall goal of this chapter is to provide sufficient empirical evidence that because pivots are the lexically least specified items, they are therefore also the semantically most variable operators of natural logic triads.

6.1 Flexible and Rigid Pivots

The description of pivots in chapter 5 suggested that they can all get both an inclusive AT LEAST ONE- reading and an exclusive ONE-reading. This is however not quite accurate. There is a bifurcation between those operators which can get both, the flexible pivots, and operators which can get only one of these interpretations, the rigid pivots. Specifically, the propositional calculus pivot *or*, the indefinite article *a* and the predicate calculus operator *some* can get both, whereas *either, ever* and *any* are more restricted. The relevance of this difference is double. First of all, it testifies to the degree of variation that is found among pivots. Secondly, the fact that *a* and *or* are more flexible than items such as *either* and *ever* correlates with the fact that the former are functional categories while the latter are lexical categories, which are generally thought to be richer in intrinsic lexical content. This correlation confirms the less-lexical-content/more-variation generalization introduced in § 5.1.1. Let us consider the flexible and rigid types in turn in example sentences.

Or and *a* get an inclusive interpretation in (i.a.) the protasis of a conditional, as in (55)a. and b. They can often (though they need not always[121]) get an exclusive reading in affirmative sentences, as in (55)c. and d.

[121] The crucial distinction is not affirmative – nonaffirmative. Thus “Every man loves a woman” is affirmative, but does not impose an exclusive reading on a woman, given the plausible continuation “but no man is monogamous”. A better characterization is in terms of a distinction “veridical-nonveridical” (Giannakidou 1994, 1995, 1997; Zwarts 1995). Applied to the example above, the timeless/habitual nature of “loves” does not pin the predicate down as referring to a

(284) a. *If [P: Mary is in the garden or Q: Bill is in the garden]], John will be there too* : P, Q, or both (inclusive)
b. *If [you meet [a girl] at the party], ask her to dance* : girl x or girl y, i.e. x, y or x and y, with x and y variables': inclusive.
a. *[Mary talked to John or Mary talked to Bill]*: to John or to Bill, not to both (exclusive)
b. *Henry talked to [a doctor]*: doctor x or doctor y, but not more: exclusive

The crucial contrast between these two sentence types concerns the constituents in square brackets. In (55)a. and b. those constituents – i.e. both the event in the protasis and the characters involved in it – have an *irrealis* construal (Palmer 2001 , Jackendoff 2002: 403; for the notion *irrealis context*, see also Haspelmath (1997: 40, 108-109) and Croft (1983)). The bracketed constituents in affirmative sentences (55)c. and d., on the other hand, have a realis construal.

For the kernel difference between a realis and an irrealis construal and the referential dependence between events and their characters, consider the following pair:

(285) a. [Did [a woman]$_1$ buy [a car]$_2$]$_3$?
b. [[A woman]$_1$ bought [a car]$_2$]$_3$.

Jackendoff (2002: 399)

In question (285)a., the irrealis context, there is no claim that a buying "event took place; rather the speaker wishes the hearer to assert whether or not it did" (Jackendoff 2002: 399). In the absence of the claim that an event$_3$ actually occurred, *a woman*$_1$ and *a car*$_2$ have no established existential force either: "we need not be able to identify either of them." (Jackendoff 2002: 399)[122]. In our terms, they are inclusive on the placemat. The semantic equivalent of that is a lack of existential commitment with respect to the individuals N in the NPs. Since there is no individuation from AT LEAST ONE to ONE, there can be no existential commitment. In (285)b., the realis context, the existence of a past event is asserted, which has an important consequence for *a woman* and *a car*. "If the event is conceptualized as having taken place, then its characters are conceptualized as existing." (Jackendoff 2002: 400) In other words, the characters of an event in a realis construal cannot have an irrealis construal themselves.

Giannakidou (1994, 1995, 1997) and Zwarts (1995) have provided a formal characterization of irrealis, non-referential environments and corresponding operators, which they define as non-veridical[123] (Zwarts 1995, Giannakidou 1997):

(286)
"Let Op be a prepositional operator. The following statements hold:

---

single, actual event, but rather to a multitude of virtual, non-veridical or irrealis events. It is because of the non-episodic nature of the predicate that "a woman" can easily get an inclusive, non-veridical reading. The concept of (non)veridicality will be detailed in (286) below.
[122] Jackendoff attributes this notion of referential dependence between an event and its characters to Csuri (1996) and Erteschik-Shir (1998).
[123] The notion *veridical* is Montague's (1969, 1974).

> (i) Op is *veridical* just in case $Op\ p \rightarrow p$ is logically valid. Otherwise, *Op* is nonveridical.
> (ii) A nonveridical operator Op is *antiveridical* just in case $Op\ p \rightarrow \neg\ p$ is logically valid."
>
> (Giannakidou 1998: 8)

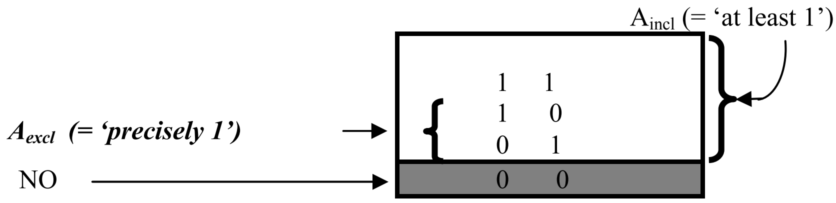
> "An operator is nonveridical if we don't know whether the embedded proposition is true or false. Adverbs like possibly and modal verbs are typical nonveridical operators:
>
> a. Paul has possibly seen a snake-/→ Paul saw a snake
> b. Paul may hit Frank -/→ Paul hit Frank
>
> Disjunction is also nonveridical (see Zwarts 1995, Giannakidou 1997 for more details). Other nonveridical environments include negation, nonassertive speech acts (questions, imperatives, exclamatives), the protasis of conditionals, the scope of strong intensional verbs like *want* and *hope*, and the restriction of certain universal quantifiers. PIs are generally grammatical in these environments.[124]"
>
> (Giannakidou 1998: 8, with her example (30))[125]

Now compare *a* and *any* in such non-veridical and veridical contexts context respectively:

(287) a. *If [you make [a/any mistake]], you'll be punished* : mistake x or mistake y, i.e. x, y or x and y, with x and y variables': inclusive.
b. *[Henry talked to [a/*any doctor]]*

Both get a grammatical inclusive reading in nonveridical contexts. In (287)b., however, characters in the *talked*-event lose their freedom of referential commitment: they have to be conceptualized as referential, as actually existing. This is reflected in the exclusive interpretation of *a*, more traditionally referred to as a particular, existential reading. *A*, though inclusive lexically, is no longer interpreted as an inclusive phrase in this context.

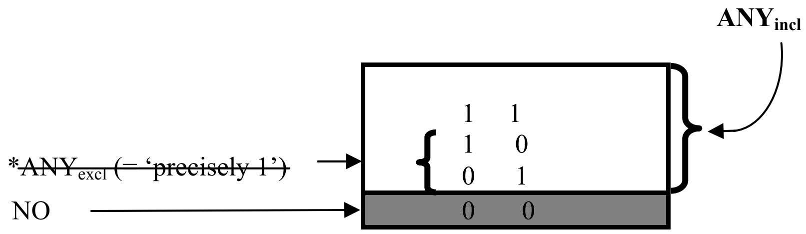
(288)



124 PI= Polarity items. Underscore mine [DJ].
125 The formal definition of nonveridicality as given in Zwarts (1995) runs as follows.
(i) *(Non)veridicality* (Zwarts 1995:287)
Let *O* be a monadic sentential operator. *O* is said to be *veridical* just in case $Op \Rightarrow p$ is logically valid. If *O* is not veridical, then *O* is *nonveridical*. A nonveridical operator *O* is called *averidical* iff $Op \Rightarrow \sim p$ is logically valid.

For *any* the consequences are more serious: it cannot be salvaged in such a context at all, which must mean that as opposed to regular indefinites, it is a *rigid* pivot, i.e. a *necessarily* inclusive one (= inherently modal, non-referential or irrealis[126]) which cannot be made exclusive by context. In other words, it completely resists a veridical construal with referential commitment.

(289)



The strictly nonveridical contexts in which it occurs are either non-affirmative sentences – where *any* is usually NPI-*any* – or affirmative irrealis contexts, where *any* has its free choice interpretation (FC-*any*). As observed in § 5.2.2, the interpretation of *any* in these contexts is never uniquely universal, but only "akin" to it (Carlson 1981: 8). We now know why if *any* is indeed a rigid pivot: the inclusive nature cannot be undone and remains the indelible lexical contribution to its final interpretation. *Any* is inherently I+A (= a union of I and A-meaning). The polarity of the syntactic context may single out the lower-end I-part of *any*'s meaning (NPI) or its upper-end A-part (FC), but the inclusive lexical meaning is never deleted. In other words, NPI-*any* is also 'akin to' to an exclusive I-reading, but certainly not to be identified with it, since its semantic scope includes the whole scale. This is where the present proposal connects with the concept of scalarity as developed by Fauconnier (1975a,1975b, 1979)[127]. Applied to the case above, on a scale of alternatives (in casu the inclusive scale I + A) the more informative endpoint on the non-reversed scale is the A-corner since A entails I. Therefore, singling out the A-corner suffices to entail the whole inclusive scale, as required by *any*'s rigid lexical specifications. On the reversed scale the more informative endpoint is not-I, which entails not-A (cf. the law of contraposition in logic), hence keeps the whole inclusive scale as part of the final interpretation.

Either and *ever* are like *any*: rigidly "inclusive" (cf. Higginbotham (1991) for *either*; Israel (1997) for *ever*[128]). Due to this, they too are ungrammatical when an exclusive ONE reading is enforced by a realis context, as in sentences (290) and (291).[129]

---

[126] Israel (1997: 37) uses Fauconnier's term *phantom indefinite* "to designate the peculiar way these forms seem to designate an instance which, in some sense, isn't really there [= irrealis, DJ]. Phantom indefinites do not refer directly – they cannot, for example, introduce a new discourse referent to a mental space. Rather they pick out an arbitrary and schematic "ghost" of an instance and thereby trigger inferences to the set of all other possible instances in a category (cf. Israel 1995: 164)." (Israel 1997: 37)

[127] Cf. also Haspelmath (1997: 117) and a formal proposal for scalar *any* in 6.3.1.4 below.

[128] Cf. also http://www-linguistics.stanford.edu/semgroup/archive/abstracts-97-98/israel.txt , where an abstract of Israel's paper states "*Ever* functions in the temporal domain much like *any* in the nominal domain. In the paradigm of temporal adverbs, *ever* occupies the same position as does *any* in the paradigm of nominal quantifiers: as *always* is to *all, usually* to *most, often* to *many* and

(290) *either*:
a. *Affirmative episodic sentences*: *(Yesterday,) John played with [either of them][130]
b. *Existential constructions*: *There was [either of these books] on the desk
c. *Progressives*: *He is reading [either book] at the moment.
d. *Epistemic attitude intensional verbs*: *She believes that we bought [either car] for her birthday

(291) *ever*:
a. Affirmative episodic sentences: *John [ever] played with them
b. *Existential constructions*: *There [ever]were three books on the desk
c. *Progressives*: *He is reading a book [ever].
d. *Epistemic attitude intensional verbs*: *She believes that we [ever] bought car for her birthday

Where no reference to a single event is involved, on the other hand, as in the negative sentences of (57), they are grammatical as predicted:

(292) a. Jules does not speak either language (neither language x, nor language y, nor languages x and y (in a two-unit universe))
b. Marita has not ever been to South Africa (not at moment x, nor at moment y, nor at moments x and y, with x and y variables)

In many usage contexts, *any* and *some* are complementary in interpretation: while *any* is invariably rigidly inclusive as stipulated by its lexical properties, cf. (289), *some* is mostly "exclusive" (in the sense of non-all-inclusive) in interpretation (293),

(293) Some flags are green (flags x or flags y, but not all of x and y)

(294) I haven't got any problem: not a problem x, nor a problem y, nor problems x and y, with x and y variables

As a consequence of its lexical meaning, *any* is excluded in the same sentence types that do not tolerate *either* and *ever*, e.g. (5).

(295) *any*:
a. *Affirmative episodic sentences*: *Yesterday, John played [any game].
b. *Existential constructions*: *There was [any article] on the desk
c. *Progressives*: *He is reading [any book] at the moment.

---

rarely to *few*, so is *ever* to *any*." It is in terms of the valuation space diagrams in placemat format that I try to bring out the parallelism.

[129] These types of actual, single-event contexts were identified by Giannakidou (2001: 677).

[130] Higginbotham (1991: 145) observes that this type of sentence, e.g. *any/either of them worked* is "strictly speaking […] not ungrammatical, but has uniquely an interpretation as a generic in the past tense, as in "In those days, any/either of them worked."

d. *Epistemic attitude intensional verbs*: * She believes that we bought [any car] for her birthday

Though *some* is mostly exclusive, it is generally viewed as a flexible rather than a rigid pivot. The main reason for this stance is to do with data involving weak *some,* e.g. in existential sentences, where the meaning of *some* is not the proportional/ exclusive reading "some but not all".

(296) There are some rabbits in my garden.

Clearly, the description of their lexical meanings is only a fraction of the full story for *some* and *any*. Whether they are stressed or not is an important factor. That topic will be taken up in the next section § 6.2, which will thereby pave the way for the comparison of a flexible pivot (the indefinite article) and a rigid one (NPI-*any* and FC-*any*) in § 6.3.

On the whole, the data analysed in this section warrant the conclusion that there is a basic division within the set of pivots between flexible pivots and rigid pivots. They also showed that in context and due to stress further meaning can supervene on lexical meaning and hence affect interpretation. That is the topic we now turn to.

6.2 The role of stress

Consider the following set of examples, which illustrates that *some* and the negative polarity item *any* are parallel in that they can bear stress or not and have their lexical semantics affected by it.

(297)

a. NPI-*any*	i'.Did he DRINK anything? (*indefinite, non-emphatic NPI*) ii'.Did he drink ANYthing (at all)? (*emphatic NPI*)	c. *some*	i. There are sm MEN in the garden (*weak*) ii. SOME flags are green (*strong*)
b. ¬NPI-*any*	i'.Didn't he DRINK anything? (*indefinite, non-emphatic NPI*) ii'.Didn't he drink ANYthing (at all)? (*emphatic NPI*)	d. ¬*some*= *no(ne)*	i.There are no MEN / aren't any MEN in the garden (*weak*) ii. NO man / Not ANY man is in the garden (*strong*)

When stressed, NPI-*any* is called *emphatic* – a term borrowed from Haspelmath (1997: 125) -, when not it is called *indefinite* or *non-emphatic*. Unstressed *some* is called *weak*, stressed *some strong* (Milsark 1977). In these paragraphs, it will be shown that such stress differences have implications for the semantics of pivotal *some* and *any*. In particular, stress adds meaning to the lexical meaning of the pivots in question. In § 6.2.1. the focus is on *any* and the emphatic-non-emphatic contrast, in § 6.2.2. on *some* and the difference between weak and strong readings.

6.2.1 Emphatic and non-emphatic *any*

In the previous chapter, it was observed that the sparser intrinsic lexical content of pivots means that their lexically specified extension set leaves more room for modification by context factors. In this section, the factor involved is stress and its effect on the pivot *any*. The phenomena discussed contribute both to the conclusion that there is a lot of variation among pivots and to the idea that logic and language (i.c. stress) are inextricably intertwined.

6.2.1.1 The presence/absence of a comparison base

The difference between the emphatic/stressed NPI and the non-emphatic/unstressed NPI in (297) a.-b. involves the presence versus absence of a comparison base, in this case a comparison base of *scalar* alternatives. Emphatic NPIs express the low endpoint on a scale (*Did he drink ANYthing = the least bit*), whereas non-emphatic NPIs do not[131]. Actually, they cannot: "constituents have to bear sentence accent in order to be interpreted as scalar endpoints." (Haspelmath 1997: 123). Note the parallel with the weak vs. strong reading of *some,* where the accent of the strong reading also comes with the comparison base of the proportional reading. This link between stress and the activation of a set of alternatives not only brings strong readings and emphatic NPIs together, it is also strongly reminiscent of how a set of alternatives is made available by focus accent (Rooth 1985). Actually, this link is established in Lee & Horn (1994), who analyse the lexical meaning of NPI-*any* as an indefinite combined with an incorporated scalar focus particle *even* (cf. paraphrase (299) for (298)), thus explicitly connecting NPI *any* and focus.

(298) There isn't *any* replacement drive available right now.
(299) There isn't *even a single/even one* replacement drive available right now.

The presupposition introduced by even ranks the proposition that even is added to along a scale, in this case a quantitative scale of alternative propositions: *there aren't two replacement drives available right now, there aren't three replacement drives available right now, etc.* On this scale, the proposition *there isn't one replacement drive available right now* makes the strongest, most excluding statement ("even sets consisting of a single replacement drive are not available, let alone sets of two, three, etc. replacement drives") and hence outranks the set of alternatives.

6.2.1.2 Formal and semantic differences between emphatic and non-emphatic any

Formally speaking, many languages use different lexicalisations for unstressed and stressed NPIs corresponding to English *any*, as illustrated by the following sample of non-emphatic (a.) vs. emphatic (b.) forms from Haspelmath (1997: 125-126).

---

[131] There is a long tradition of analyses of *any* as involving the endpoint of a scale, including Schmerling (1971), Fauconnier (1975a,b), Krifka (1994), Lahiri (1995).

(300) French

a. Si **quelqu'un** vient, réveille-moi.
If someone comes, wake-me
'If anyone COMES, wake me up.'

b. Si qui que ce soit vient, réveille-moi.
If anyone comes, wake-me.
'If ANYONE comes, wake me up.'

(301) Polish

a. *Jeżeli* ***co-ś*** *zobaczysz, odrazu mnie obudź.*
If what-INDEF see:2SG immediately me wake:IMPV
'If you SEE anything, wake me up immediately'

b. *Jeżeli* ***co-kolwiek*** *zobaczysz, odrazu mnie obudź.*
If what-INDEF see: 2SG immediately me wake:IMPV
'If you see ANYTHING, wake me up immediately.'

(302) Hindi/Urdu

a. Agar **koii** fon kare, mujhe bataanaa.
if someone phone calls I:DAT tell:IMPV
'If anybody CALLS, tell me.'

b. Agar **koii bhii** fon kare, mujhe bataanaa.
If someone INDEF phone calls I:DAT tell:IMPV
'If ANYBODY AT ALL calls, tell me.'

(303) Chinese

a. *Wǒ bù xiangxin* ***shénme*** *rén lái le.*
I not think what man come PFV[132]
'I do not think that anyone CAME.'

b. *Wǒ bù xiangxin* ***rènhé*** *rén lái le.*
I not think any man come PFV
'I do not think that ANYONE came.'

Haspelmath (1997: appendix A) provides further pairs from Russian, Hungarian, Bulgarian and Italian.

On the semantic side, Haspelmath considers the difference between emphatic and non-emphatic NPIs like *any* "very subtle" and "not easy to identify across languages" and believes that they are "emphatic and non-emphatic indefinites without a difference in the truth conditions" (Haspelmath 1997: 125). While the semantic difference is indeed subtle, it has nonetheless been argued that there is a difference in truth conditions (Linebarger 1980, Heim 1984, Kratzer 1989, Rullmann 1996). The difference can be brought out by embedding *any* in sentences expressing accidental and non-accidental (or causal) generalizations (Heim 1984: 104-106; Kratzer 1989; examples from Rullmann 1996: 346)

(304) a. Everyone who ate anything got sick.

[132] PFV: perfective

b. Everyone who ate anything was actually wearing blue jeans.

Sentences like (304) a. can be used to express a non-accidental generalization: there can be a causal connection between eating something and getting sick. A causal link between eating something and wearing blue jeans, however, is unlikely, and (304) b. therefore normally expresses an accidental generalization. Heim (1984: 104), basing herself on a contrast noted in Linebarger (1980), notes that certain NPI's – all scalar – are not licensed in statements normally expressing accidental generalizations while they are ok in non-accidental generalizations:

(305) a. ?? Everyone who ate *a single bite* was actually wearing bluejeans.
b. Everyone who ate *a single bite* got sick

The oddness of (305) a. (with a minimizer[133] NPI (Bolinger 1972: 121) *a single bite*) is due to the nature of the NPI, which invites the reading that the link between eating something and wearing blue jeans is causal[134]. Now crucially, emphatic NPI's behave exactly like *a single bite*:

(306) a. ?? Everyone who ate ANYthing (at all) was actually wearing bluejeans.
b. Everyone who ate ANYthing (at all) got sick.

No causality-restriction affects non-scalar, non-emphatic unstressed *any*, witness (304), repeated here.

(307) a. Everyone who ate anything got sick.
b. Everyone who ate anything was actually wearing blue jeans.

Rullmann concludes that this non-emphatic item is equivalent to the unstressed indefinite pronoun "*iets"* ("something/anything") of Dutch and defines non-emphatic *any* as "an indefinite determiner (in English, *a(n)* or ∅), the only difference being that in its distribution *any* is restricted to downward entailing environments." (Rullmann 1996: 348). Note, however, that the latter qualification does indicate that something is still amiss in Rullmann's analysis, as pointed out by Israel (1997: 41):

> "The plain implication of this [Rullmann's, DJ] claim is that sentence pairs like those (23-(…)) must be semantically and pragmatically synonymous.
>
> a. Glinda doesn't eat seafood.
> b. Glinda doesn't eat any seafood.
>
> I must confess that this conclusion runs counter to my own intuitions. (…) Pace Rullmann, even without stress, these forms do make a contribution to

[133] A minimizer is an NPI referring to a minimal quantity, situated at the lowest point of a scale.
[134] The claim that emphasis comes with the postulate of a causal link does not reveal anything about actual causality in extralinguistic reality. The causal link is established in the speaker's mind and there is no telling what causal links an individual can think up. Note that it is easy enough to conjure up a story context in which people who eat a single bite suddenly turn into bluejeans-wearers.

> sentence meaning and their contributions are, if not identical, at least very analogous."
>
> (Israel 1997: 37-38)

Since I have claimed that *any* is a rigid pivot, I am in agreement with Israel.

On the whole, the correspondence between Rullmann's analysis and Heim (1984) is that while emphatic NPIs can be described in Lee and Horn's terms as NPIs with an inherent scalar focus particle *even* (*even*-NPIs), non-emphatic NPI's have no similar emphasis on scalar alternatives. It is because of this that their meaning stays very close to that of an indefinite. Stress affects interpretation in that it comes with a causal interpretation, scalar alternatives and focus. It emphasizes the full lexical meaning content of pivot *any* as specified in (289). The way in which stress thus affects lexical meaning will now be explained in the section on weak and strong readings.

6.2.2 Weak and strong readings

The difference between weak and strong readings of /(¬)SOME/ in (297) c. and d. figures prominently in Milsark (1977). Postal (1966) and Milsark provide the following sentence to illustrate it:

(308) Some unicorns appeared on the horizon

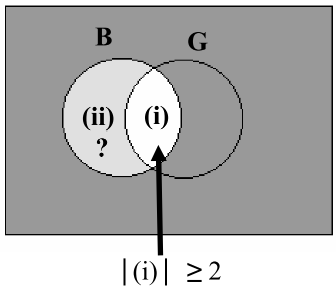
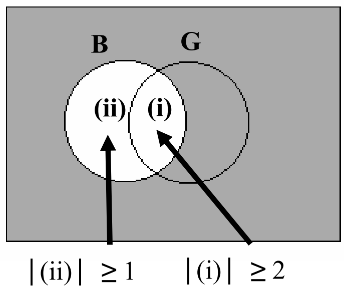
The strong, stressed reading (also called proportional or partitive) can be paraphrased as "at least two[135] but not all": some unicorns appeared, others did not. The reading is quantificational, i.e. involves reference to a set of relevant alternatives fitting the predicate *unicorns* that functions as a comparison base. The weak, unstressed reading, on the other hand, is purely cardinal: only the cardinality of unicorns that appeared matters. Whether there are other unicorns who did not appear on the horizon does not enter into the equation and, hence, neither can the proportional relationship between the unicorns that appeared and the unicorns, if any, that did not. In the previous chapter, the difference was described as the difference between an intersective (=cardinal, symmetric) and a dyadic (=quantificational, proportional) reading. The label "dyadic" signifies that the meaning involves both cells of the restrictor set B, i.e. (i) and (ii))[136].

(309)

		Reading	*(i)* $B \cap G$		*(ii)* $B - G$
Some_{pl} B G	is true iff	cardinal	$\lvert(i)\rvert \geq 2$		
		quantificational	$\lvert(i)\rvert \geq 2$	<u>ET</u>	$\lvert(ii)\rvert \geq 1$

[135] "Two", because *unicorns* is plural, not singular.
[136] For the corresponding /NEC/-bar codes, see § 4.6.3.1.

(310)

Cardinal/intersective/monadic	Quantificational/proportional/dyadic
(i) = B ∩ G; (ii) = B - G	*(i) = B ∩ G; (ii) = B - G*
	

Since the dyadic reading is the one with more specifications, the natural hypothesis in our framework is to assume that the intersective reading is the lexical meaning and that the additional specification is and added contribution, either by context or by stress, which has an exclusionary effect and narrows the extension down to a smaller, hence more informative set.

The more basic nature of the intersective meaning is naturally describable in terms of an epistemological procedure, more particularly the way in which induction generalizes from particular knowledge (*some boys are in the garden* or more briefly *some B are G*) to universal or general knowledge (*all boys are in the garden: all B are G* or *no boys are in the garden: no B are G*)[137]. Let us have a look at the nature of this (idealized) procedure by which one arrives at epistemological conclusions with *some* to illustrate how the meaning of strong *some* differs from weak *some*.

Let G(x) be the matrix predicate function and the range of the substitutional variable x be restricted to the restrictor set B = {a, b, c, d, e}, i.e. an (in casu finite) set of boys. G(x) is then a function from the set of boys B to propositions with a truth value **1** or **0**, since each insertion of a boy name in the position of the substitutional variable will produce one of those two values. Before any checking starts, the initial situation of ignorance is embodied in a neutral and/or the emphatic yes/no question:

(311) Are (=1) there any boys in the garden? (neutral)
‘ARE (=1) there any boys in the garden (or are there ‘NOT (=0))? (emphatic)

Given the stress, the emphatic yes/no question drags in the complement set of situations as a comparison basis, i.c. “are there ‘NOT (=0)”? Both the fact that questions signal (partial) ignorance and that the set of alternatives is introduced by means of the I-corner

[137] Thanks to Pieter Seuren for crucial help with this procedure. The link with Peirce, the en route notion and its elaboration in terms of the intersective vs. dual lexical structure of operators are my own. The procedure bears resemblance to Van Benthem’s (1987) analysis of first order quantifiers as regular “semantic automata” and Löbner’s “Berechnung des Quantifikationsergebnisses” (1987: 190-197).

disjunction *or* indicate that questions (as opposed to affirmatives and negatives) are yet another incarnation of an I-corner item relative to affirmatives and negatives. The triadic structure is too obvious to be ignored and matches the triadic nature I believe to characterize epistemology.

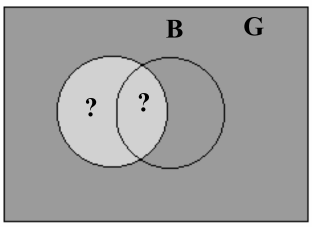
(312)

A-corner	I-corner	E-corner
	- Disjunction (P or not-P?) - “knowledge is incomplete” (De Morgan) - Epistemological doubt (Peirce) -“what we know we don’t know” (Bromberger 1992)	
affirmative	interrogative	negative
+	?	-

Clearly, in this pattern too there is an epistemological drive away from the I-corner. Recall Peirce (1877): “doubt is an uneasy and dissatisfied state from which we struggle to free ourselves”. This means that in the triad above the disjunctive unresolved state of the interrogative is the engine that sets the desire and quest for reduced ignorance (of the A- c.q. E-corner type) in motion. The reason is as always the conviction that reality is determinate (+ or -), not unresolved (?).

The checking procedure that can settle the issue for (311), now, is about BEING IN THE GARDEN (and in the emphatic question also the comparison base NOT-BEING IN THE GARDEN), and the set of elements which are in the running is restricted to the set B (= property of conservativity; cf. § 4.6.3.1.). At this initial point all the boys are still *selectable* for garden-presence or garden-absence, which is as usual represented by means of light shading of the selectable-but-not-yet-selected restrictor set.

(313)



Take a statement of the type Ga to represent the particular empirical knowledge after insertion of a in the position of the substitutional variable that “boy a is in the garden” and Gb the particular empirical knowledge that “boy b is in the garden”. When one has such particular empirical knowledge, a provisional answer can be given to the initial question, this time an emphatic affirmative:

(314) There ARE sm boys in the garden (= $|(i)| \geq 2$).

The meaning expressed is both intersective and has an at least-interpretation. This is crucially to do with the fact that (314) is an answer to the question. Compare:

(315) a. There are two boys in the garden
b. ARE there two boys in the garden? – Yes, there ARE (two boys in the garden) – there are even four.

While the a. sentence, if volunteered as a piece of information without preceding question context, normally gets the interpretation "precisely two", the at least interpretation is perfectly legitimate in the emphatic yes/no-question context (cf. Gazdar 1979: 138, Scharten 1997: 71-76; Seuren 1998: 409)[138]. In our theory, this is to be explained semantically (not pragmatically) in terms of the veridical-nonveridical contrast and the inclusive lexical meaning of pivots such as *two, some, a*. The question raised is whether there is *a* set of two boys in the garden. Given its occurrence in a question environment, the NP *two boys* has a nonveridical, inclusive interpretation: is there one or other or who knows more than one set of two boys (cf. (287) above). This means that the intended reading is "at least two boys". The answer *yes* to the question is a *confirmation* that there indeed exists at least one set of two boys in the garden. In the more highly specified affirmative realis context of (314) a., on the other hand, the direct *affirmation* is the "exclusive" one that there is one or other set (hence not more) of two boys in the garden (cf. (288)), yielding the precisely two reading[139]. While the "at least" reading brings out the lexical meaning, the "precisely two" reading is a narrower interpretation due to the realis nature of an NP in an affirmative context (as opposed to its irrealis nature in a yes/no-question and a confirmation).

The criterion reading ("at least") of (314) has the epistemological status of an interim report on the basis of partial evidence: so far it has been established that there are two boys in the garden, whether there are more is not determined yet (and maybe not even relevant). This is the reading when the variable x of G(x) has not been rotated over the whole set of boys, and the idealized checking procedure is still *en route*: there might be discovered to be 2, 3, 4 or 5 boys in the garden upon further checking. The "at least" reading of this type of weak *some,* is indicated by the three dots in the description of the incomplete extension of the propositional function:

(316) $\|G(x)\| = \{\langle a,\mathbf{1}\rangle, \langle b,\mathbf{1}\rangle, \ldots\}$

[138] This reading is also known as the criterion-reading, as in the situation where anybody with three children meets "welfare benefit eligibility criteria" (Gazdar 1979: 138). In such a context, the at least reading is true in the truth-conditional sense:
(i) Who has three children? – I do, in fact I have four
(ii) Does John have three children? Yes, he has four.

[139] The theoretical hypothesis/assumption has been throughout that what is in the A-corner (i.c. an affirmative) is more highly specified and hence extensionally more restrictive/informative than what is in the I-corner (i.c. an interrogative). This is confirmed by the interpretation of the NPs: an (inclusive) irrealis denotation is less restrictive than an (exclusive) realis interpretation. As will be clear from this remark, the systematicity of the I-corner vs. non-I-corner contrast is remarkably uniform across calculi.

Since so far there is no boy who has been identified as not being in the garden, no information is available about cell (ii). It is still possible that at the end of the checking procedure either *all* the boys or just *some, but not all* of the boys turn out to be in the garden. At this en route stage, however, knowledge is still incomplete. As usual, the I-corner feeling of incomplete knowledge (here incarnated as weak intersective *some*) will cause irritation if the desire is for the entire truth (Peirce 1877) and it gives the urge to go on checking and try to settle belief by arriving at more complete information. Observe that the present description of this intermediate en route stage of the epistemological procedure and its association with an I-corner quantifier clearly confirms our general hypothesis that the I-corner is prior to the A-corner: *en route some* is prior to and more basic than posterior A-corner *all* in the epistemological checking procedure[140].

Now imagine that the third boy checked for garden-presence yields a negative value:

(317) $\| G(x) \| = \{\langle a,\mathbf{1}\rangle, \langle b,\mathbf{1}\rangle, \langle c,\mathbf{0}\rangle,\ldots\}$

This is a new situation: we now know that there is at least one boy in cell (ii) ($= |(ii)| \geq 1$), so that it is no longer possible to arrive at the conclusion that all the boys are in the garden. Whatever the values for boy d and e will be during the rest of the checking procedure, the conclusion will remain the same: "some, but not all boys are in the garden". This is the proportional meaning of strong *some*, as in *'SOME boys are in the garden*. As the informal characterisation "some, but not all" betrays, its semantic structure is dyadic: $|(i)| \geq 2$ <u>ET</u> $|(ii)| \geq 1$.

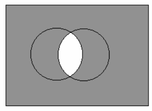
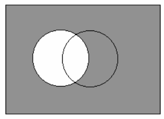
Alternatively, imagine that the third boy checked for garden-presence still yields a positive value:

$\| G(x) \| = \{\langle a,\mathbf{1}\rangle, \langle b,\mathbf{1}\rangle, \langle c,\mathbf{1}\rangle,\ldots\}$

Since the set B is finite, the checking can go on till the final stage when the last element is checked. This can result in two possible conclusions: (strong) *some but not all B are G* (namely when either d or e or both yield a zero value), or the universal *all B are G,* namely when d and e have value **1** like the other elements. Note that in each of these two possible scenarios, the quantifier bears stress, is quantificational, and is dyadic in that it involves both (i) and (ii). A final diagnosis at the end of checking cannot but yield a dyadic interpretation, the whole restrictor set B having been run through:

[140] The distinction en route (incomplete) vs. final diagnosis (complete) is the quantificational counterpart in the realm of predicates of the I-corner vs. A-corner distinction among quantifiers: *some* episodes of the checking procedure defined by the predicate have been run through vs. *all* of them. In other words, the Boolean placemat used earlier for quantification over individuals in quantifier NPs also enters into the meaning of predicates. This is in accordance with Bach's (1986) and many others' defense of a Boolean algebraic approach to quantification over events.

(318)

Boethian corner	*Sentences D (B,G)*		*Size of Domain*	*(i)* $B \cap G$		*(ii)* $B - G$
A-corner Entailer	*all B G*	is true iff		$\lvert(i)\rvert \geq 1$	and	$\lvert(ii)\rvert = 0$
I-corner Pivot	*Some$_{pl}$B G*	is true iff		$\lvert(i)\rvert \geq 2$	ET	$\lvert(ii)\rvert \geq 1$

Returning to sentences with the numeral *two*, note that the difference between the weak "precisely"-reading of (315) a. and a proportional reading ("two of the boys") is that in the former case it is irrelevant whether at least one boy is absent from the garden or not: attention remains restricted to the garden and the precisely reading is the result of the realis nature of the sentence. The empirical fact that attention remains restricted to the subset of boys *in* the garden in such cases is to do with the nature of a *there*-insertion sentence, which generally resists proportional, strong quantifiers in the indefinite NP position:

(319) a. *There are all (of the) boys in the garden.
b. *There are most (of the) boys in the garden

I believe this has to be accounted for in terms of the property of conservativity, which was defined as restricting the part of the universe that matters for *some/all/... F are G* to the restrictor set F. When applying conservativity to a *there*-insertion sentence

(320) There are two boys in the garden

it should be borne in mind that there are actually two restrictor subjects: (a) the postposed indefinite subject *two boys*, which restricts the part that matters for checking to the set B and (b) the dummy subject *there*, which arguably marks the scope of matrix predicate G (≈ *the garden has two boys in it*) and thereby restricts the part of the universe that matters to G by conservativity. The combination of (a) and (b), now, has the effect of narrowing the part of the universe that can possibly matter for checking down to the *intersection* of B and G and hence bars any non-intersective, dyadic quantifiers from the indefinite subject position. This means that non-intersective operators such as *all*, *most*, which are crucially dependent on information about cell (ii) are excluded from *there*-sentences. This is also why the strong, dyadic proportional reading "two of the boys are in the garden" is barred from *there*-insertion sentences, as it requires that the checking procedure reveal that at least one boy be outside the garden.

The most important overall observation so far, is that the pivotal quantifier *some* has just one lexical meaning, the intersective "at least" meaning, and that the two other possible readings, namely the "precisely" cardinal reading and the proportional quantificational reading are cases of enrichment: they are a function of context (veridical) and stress (comparison basis) respectively. This testifies to the chameleonic nature of pivotal operators.

As opposed to *some*, *all* is *lexically dyadic* according to the lexical decomposition in chapter 4: its use in *all B are G* embodies the claim that not only (i) there is at least one boy in the garden, but also (ii) that there is none that is not.

(321)

(i) $B \cap G$		(ii) B - G
$\lvert(i)\rvert \geq 1$	and	$\lvert(ii)\rvert = 0$

That this meaning is lexical can be proved by means of cases of so-called inductive leaps. Image a checking procedure in which set B is infinite or too large to be checked in its entirety and all checked cases have so far proved to be G. An inductive leap will occur at a certain point, which is really the addition of the cell (ii) conclusion $\lvert(ii)\rvert = 0$ (i.e. “there are no boys who are not in the garden”) to the (necessarily) partial cell (i) knowledge $\lvert(i)\rvert \geq 1$ (“there are one, two, three, … boys in the garden”) gathered up to that point. This time, the second part (ii) of the dyadic structure of *all* does not yield certain knowledge at the end of exhaustive checking of the elements of B, but rather constitutes an inductive “leap of faith” that occurs at a certain stage of *en route* checking, calling the procedure off since it is endless or too long and everything points in the same direction anyway. The fact that the word *all* is indeed used both in utterances about a finite set of individuals and for infinite-cum-induction procedures supports the claim it has the dyadic lexical content specified.

no(ne) (= ¬*some*) is the final operator that has to be given its place in the epistemological system and the weak-strong pattern. Recall the basic assumption of our whole analysis of logic, namely that the I-corner is pivotal and the two other corners derived. For epistemological procedures this implies that they always start in the I-corner with an initial weak hypothesis: (weak) *some (= at least one) B are G*. If the first step of the checking procedure immediately gives a negative result, the hypothesis has to put on hold and the new weak knowledge is that *no* boys are in the garden *so far (= en route)*. As long as the epistemological procedure has not run its course and the results that come in keep being negative the semantic status of *no(ne)* remains that of an *en route* weak reading. The hypothesis is at this point still reversible into strong *some (but not all) B are G* if suddenly a boy should be detected in the garden. If that does not happen and the set of entities in question, in casu boys, is finite, the epistemological procedure can run its course without changing the negative hypothesis. If the set to be checked is infinite, there is once again an inductive leap, with the usual risk involved.

This description of the epistemological checking procedure for *no(ne)* is a perfect match for the intersective meaning description given earlier: the weak pivotal hypothesis $\lvert(i)\rvert \geq 1$ is consistently rejected, either *en route* or as a final hypothesis, so that $\lvert(i)\rvert = 0$:

(322)

E-corner Contradictor	*f. No B G = Not one/any B is G*	is true iff		$\lvert(i)\rvert = 0$, i.e, it is not the case that $\lvert(i)\rvert \geq 1$		

For the strong reading "none of the boys are in the garden", there is the added knowledge that cell (ii) is not empty, but that is once again independent of the lexical meaning of *no*.[141]

[141] The present footnote is added to complete the picture of the epistemological procedure and its relation to Peirces ideas about the irritation of doubt and partial knowledge. It does not affect the text proposal for the meaning of the operators themselves, which is why it has been relegated to this note.

While an *all*-sentence attains general knowledge of the type that is useful in causal deductive 'theories', a strong *some-but-not-all*-sentence remains pivotal, though different from a weak *at-least-some*-pivot. Its pivotal status, i.e. the nature of its indeterminacy, is of a different nature than before. Peircean irritation no longer concerns an incomplete checking procedure as with weak *some*, but rather the imperfection of the irreversible knowledge attained: it turns out that some boys are in the garden and some not, but why is the end-result so mixed? How can better general, causal knowledge about the set B be reached? The only option at this point is to try one or more new predicates instead of either B or G, since due to the mixed results ("some but not all B are G"), general knowledge in the form of a universal proposition is no longer attainable without a change of predicate. Alternative hypotheses which could be checked could for instance be about darkhaired boys being in the garden, or boys being at the party. Such changes of the restrictor or/and matrix predicate could in principle yield better results:

(i) all dark-haired boys (=DB) are in the garden
(ii) all the boys are at the party (=P)

This is clearly the point where the principle of *abduction* has its place in the epistemological model: it is dependent on how creative someone is in selecting new, different predicates whether general knowledge becomes attainable in a new checking-procedure. The new predicate can be either a conjunction of a prior one with a new one (DB instead of B), or an entirely new predicate (P instead of G). The conjunctive new predicate DB has a smaller extension set than B and may therefore yield a better description and a new possibility that general knowledge as expressed by (i) might be attainable. An entirely new predicate P may shed different light and can thus possibly lead to entirely new general knowledge. Good choices of enriched or different predicates will make good theories, with general knowledge, as desired in causal deductive 'theories'. Human beings seem to be driven to acquire knowledge of this universal, causal type instead of partial pivotal knowledge. Moreover, they seem to have an instinct for making relevant choices, which is why Charles Sanders Peirce (1957: 253) ventured the hypothesis that humans are all innately endowed with a sense of abduction, which "puts a limit on admissible hypotheses" and provides man with "a natural adaptation to imagining correct theories of some kind" (Peirce 1957: 238; quoted from Chomsky 2000: 80).

Together, this *sense of abduction* and the earlier-mentioned *sense of induction* constitute the two points of uncertainty and hence creative decision in an epistemological system. The former is the source of bold but creative choices of new predicates. It may be successful and lead to general knowledge or it may misfire. The latter, induction, is risky and creative too. A good and responsible inductive leap when an infinite set is involved always involves choice and may misfire.

6.2.3 The parallelism summarized

On the whole, there is a parallel between weak and strong readings and non-emphatic and emphatic NPIs. Weak forms and non-emphatic NPIs are purely cardinal and their interpretation is independent of a comparison base. Strong readings and emphatic NPIs are dyadic in the sense that they invoke a scalar comparison base, which results in a proportional or partitive interpretation in the case of strong readings and in a scalar focus reading in the case of NPI-*any.* A scalar model to describe the latter will be worked out as part of the case study comparison in § 6.3 of the rigid pivot *any* and its flexible counterpart, the indefinite article. Overall, this section has provided evidence that pivot meanings show great variation, but that the variation is not lexical. The lexical meaning of pivots is the one with the widest extension and the fewest intensional specifications. Variation on the basic lexical pivotal meaning is generated by context (affirmative vs. interrogative) or stress.

6.3 Flexible *a* and rigid *any*

This section will take the fact that flexible *a* and rigid *any* are both pivots and hence related as its starting point. Its focus, however, is on the main difference between the two. Consideration of that difference will lead to a formal model for for *any* whose purpose is to account for its emphatic FC- and NPI- readings as well as its non-emphatic interpretation. My proposal is inspired by the analysis provided by Kay (1990) for the focus particle *even*, which was proven by Lee & Horn (1994) to be an inherent component of the lexical meaning of NPI-*any*[142]. I will show that Kay's analysis of *even* can be used for *any* provided a modification is made. The resulting scalar model is a version of the 2D Cartesian Coordinate System that is the running thread of this study. Thus this section will render additional support to the theoretical core of this work and at the same time flesh out further the nature of the semantic variability of pivotal meaning.

While abduction involves the choice of predicate and is a qualitative leap, induction involves the infinite range of the argument variable and represents a quantitative leap into the uncertain. Yet without such types of uncertainty, no creative and daring hypotheses and hence really new knowledge would be possible.

[142] Sir William Hamilton's 19th century characterisation of the lexical meaning of *any* (quoted from Horn 2000a: 159) already contains an explicit reference to the focus particle *even*:

> "Our English *"any"* (aenig, anig, Ang.-Sax.) is of a similar origin and signification with the Latin *"ullus"* (unulus), and means, primarily and literally *(even) one, even the least or fewest.*"
> (Hamilton 1858: 615)

Maybe surprisingly, Hamilton's descriptions *(even) one* and *even the least or fewest* have survived to this day as the lexical meaning of the NPI part of *any*. They have been taken up and given more theoretical and formal substance in Lee and Horn's (1994) analysis of NPI *any* as "equivalent to *one*", more specifically, as denoting a minimal element on a quantity scale, meaning *"even a single/the least bit"*. The basic features were apparently recognized early on: the link with pivotal *one*, the claim that *any* incorporates *even*, the role of superlatives (*the least or fewest*) and hence implicitly also of scales.

6.3.1 *Any* as a rigid and widened indefinite

Describing the difference between the readings of *any* as context-determined modification of a single basic lexical meaning (see § 6.2.1) brings *any* in the same theoretical fold as *indefinites*, whose various readings are now also generally regarded as highly dependent on their context of appearance (cf. Lewis 1975, Kamp 1981, Heim 1982; Diesing's (1992) Existential Closure[143]). Recall that their meaning was claimed to be pivotal in § 5.3. It is the semantic variability of both *any* and the indefinite article *a* and the distributional analogy between their different readings that led Jespersen (1924), Perlmutter (1970), Carlson (1981), Kadmon and Landman (1993), and others to postulate the existence of a close relationship.

Thus, in a discussion of the generic singular, Jespersen (1924: 203) writes that in "*a cat* is not as vigilant as *a dog*; the article may be considered as a weaker *any*", and expresses the idea that in generic readings "one ("a") dog is taken as a representative of the whole class". Jespersen's remarks, however brief, contain the basic elements which are crucial for a good understanding of *any* and *a*: (i) there is a *difference* between them: the indefinite is *weaker* than *any*; but (ii) there is also a close *relationship* between them: *a* and *any* are both characterized by an existential(I)- universal(A) connection in parallel contexts. More specifically, in those contexts they are existentials which have universal import: "<u>one ("a") dog</u> (∃) is taken as a representative of the <u>whole class</u> (∀)"[144]. This is an instantiation of the more general phenomenon which I have called I→A-shift in § 5.2.2, where the emphasis is on the A-side interpretation. In other contexts, the emphasis is more on the particular I-side

(323) A:
a. [An owl] hunts mice
b. [Any owl] hunts mice
(324) I:
a. I don't have [potatoes]
b. I don't have [any potatoes]

Kadmon and Landman's (1993) outlook on *any* as an emphatic indefinite takes the same distributional analogy as its empirical starting point: *any* gets a free choice (FC) interpretation, i.e. a reading "akin to" a universal, in precisely those contexts where an ordinary indefinite gets a generic interpretation. It gets an existential NPI reading, however, in contexts where ordinary indefinites are non-generic indefinites (cf. Perlmutter 1970, Carlson 1981).

(325) a. generic: $[\text{An owl}]_{\text{GEN}}$ hunts mice
b. free choice: $[\text{Any owl}]_{\text{FC-}\forall}$ hunts mice
(326) a. non-generic: I don't have $[\text{potatoes}]_{\text{non-GEN}}$

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[143] For Dutch, the case for this conception was made by Postma (1994) (and later also by Bennis (1995)) who developed a contextual dependency description of the semantically diverse nature of the Dutch interrogative / indefinite pronoun *wat*.
[144] Underscore and added symbols mine [DJ]. For this hybrid existential/universal-meaning, see also Carlson (1981) and Löbner's (1990: 27-29) "Homogene Quantor".

b. NPI: I don't have [any potatoes]$_{NPI\text{-}\exists}$

But as said, there is also an important difference between indefinite *a* and *any*, namely that the indefinite article "is a weaker *any*", as Otto Jespersen (1924) put it. In a sense, Kadmon and Landman (1993: 357) turn this observation on its head and affirm that *any* is an enriched or stronger *a*, witness their definition:

(327) ***any* CN** = the corresponding indefinite NP *a CN*
with additional semantic/pragmatic characteristics
(widening, strengthening) contributed by *any*

One of the main objectives of their article is to spell out in detail the precise nature of the "stronger" meaning of *any* and hence of these "additional semantic/pragmatic characteristics". A core notion in their analysis is the notion of widening. I will give evidence that their conception of the notion needs to be modified. First their definition will be given and illustrated, after which the problem with it will be brought to light and solved. That will pave the way for a formal model for the semantics of *any.*

6.3.1.1 Kadmon and Landman's notion of widening

(328) **Widening**
In an NP of the form *any* CN, *any* widens
the interpretation of the common noun
phrase along a contextual dimension. [145]

Kadmon and Landman (1993: 359) illustrate this notion of widening by comparing examples with *any* and examples with the generic use of indefinite article *a* and indefinite bare plurals, as in (329)-(331).

(329) a. I don't have potatoes
b. I don't have any potatoes
(330) a. Every man who has matches is happy
b. Every man who has any matches is happy
(331) a. An owl hunts mice
b. Any owl hunts mice
Kadmon and Landman (1993: 359)

In all of these pairs, it is felt that the b. examples express reduced tolerance of exceptions relative to the a. examples. Kadmon and Landman begin their explanation by pointing out that a context of utterance sets up a "local" domain of quantification, from which all sorts of things are excluded.

> "For example, in a given context, rotten potatoes or sick owls may be excluded as irrelevant. For that reason, you can accept (26) a [= (329) a., DJ]

[145] The first detailed analysis of *any* stressing its systematic "no-matter-which"-widening effect, was Vendler's (1967) description in terms of the notion *blank warranty.*

> as true even if you know that I do in fact have a few rotten potatoes in the back yard, and you can accept (28)a. [= (331) a., DJ] as true even if you don't think that sick owls hunt mice. The effect of *any* in the (b) sentences, expecially when it carries main or emphatic stress, is to indicate that even things that could previously be regarded as irrelevant (in a given context) are no exception to the claim being made. Thus *I don't have ANY potatoes* may imply: not even the rotten ones; *ANY owl hunts mice* may imply: even a sick one – the use of *any* indicates that even rotten potatoes or sick owls (which might otherwise have been disregarded) are no exception"
>
> (Kadmon and Landman 1993: 359).

On the whole, this means that the effect of *any* "is to widen the previously given domain of quantification", as the normal interpretation of the following conversation from Kadmon and Landman (1993: 360) illustrates.

(332) YOU: Will there be French fries tonight?
ME: No, I don't have potatoes.
YOU: Maybe you have just a couple of potatoes that I could take and fry in my room?
ME: Sorry, I don't have ANY potatoes…

The use of *any* here widens *potatoes* from larger to small quantities as well (*a couple of*). Kadmon and Landman emphasize that such widening always involves a restricted <u>contextual dimension</u>, here large vs. small quantities, to account for the fact that the conversation could naturally proceed as follows:

(333) ME: …unless you want that rotten potato over there.

Though use of *any* widens *potatoes* from larger to smaller quantities in (332) and thus reduces tolerance of exceptions, all sorts of potatoes – like rotten potatoes – may well continue to be exceptions to the statement. They fall outside the contextual dimension determining the domain of widening (i.c. quantity) in (332). In sum, tolerance of exceptions is reduced by *any* but in most cases does not reach zero tolerance.

Though contextualization to a particular dimension correctly makes widening a relative ("reduced tolerance") rather than an absolute notion ("zero tolerance"), even this more restricted notion is too strong.

6.3.1.2 The problem with Kadmon and Landman's widening

A first objection to the Kadmon and Landman's version of widening was raised by Lee and Horn (1994), who observed that Kadmon and Landman's account cannot explain why the following two sentences are equally odd:

(334) a. #I don't have *a* pen to write with, or at least not enough.
b. #I don't have *any* pen to write with, or at least not enough.

If the domain of quantification of *any pen* is wider than that of *a pen* along one or other dimension, as Kadmon and Landman would contend, it is predicted that (334) a. should be at least somewhat better than (334) b., quod non. The lesson Lee and Horn draw from this factual observation is that widening, though possibly well-suited for FC *any*, which involves a *kind* or *quality* scale (*even the $\underline{A_{superlative}}$ $\underline{X}$*)) cannot be an essential feature of NPI *any,* which denotes a minimal element on a *quantity* scale (*even a single $\underline{X}$*).

> "Our analysis predicts that there is no difference in the domain of quantification between *a CN* and *any CN* if *a* denotes the minimum quantity and *any CN* is associated with a quantity scale. As discussed in §3.2, in the case of a quantity scale, the least likely quantity of CN for which the proposition schema holds happens to be the minimum quantity, which is exactly the semantics of *a:* It is the lexical property of *a* which makes *any CN* the low point of the quantity scale, and whatever implicature *any CN* induces by being the low end of the scale, its counterpart *a CN* (in which the quantity feature of *a* is salient) induces as well."
>
> (Lee and Horn 1994: §3.4)

Krifka (1995) signals yet another context where Kadmon and Landman's notion of reduced tolerance of exceptions runs afoul.

> For example, we can say, referring to a particular sequence of numbers: *This sequence doesn't contain any prime numbers*. It seems implausible that *any prime numbers* induces a widening of the precise concept 'prime number' here, or even a contextual widening from 'small prime number' to 'small or large prime number'.
>
> Krifka (1995: 5)

On Lee and Horn's (1994) account one could conclude: since *any* denotes a minimal element on a *quantity* scale here, it has the same semantics as *a*, so there is no widening[146].

And yet, in light of (335), Lee and Horn's solution cannot be the full answer.

(335) a. I don't have a car.
b. ?I don't have any car.

When uttered without a special context, b. is clearly felt to be slightly awkward, the reason being that replacement of *a* by *any* introduces the counterintuitive interpretation that people *normally or stereotypically* have more than one car and that I violate expectations by being an exception to that rule. In other words, the car-quantity I have widens or extends the extension of the expectation generated by the stereotype to the smallest quantity possible: a single car. This can only mean that Lee and Horn's idea that NPI-*any* never involves widening only holds true at the level of the *objective* domain of

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[146] Cp. my analysis for the uncertainty about the A-part of *any*'s rigidly non-referential lexical meaning in non-emphatic weak reading contexts in § 6.2.2. That is the context in which *any* is very similar to *a.*

quantification, not at the level of the *subjective domain of quantification governed by expectations*.

Note that exactly the same expectation-based widening effect as in (335) is present in the *even* paraphrase[147]:

(336) ?I don't even have a single car.

The oddness of both (335) b. and (336) is that they run counter *normal expectations* about car possession by suggesting that the normal situation is for people to have more than one car. Now compare car possession to car selling, where standard expectations are different and where we get the following perfectly natural pair:

(337) a. CAR SALESMAN: I haven't sold any car today.
b. CAR SALESMAN: I haven't even sold a single car today.

The stereotypical perception of car salesmen being that they sell as many cars as possible and preferably more than one a day, it is in this context quite normal to find one of them disappointed at having sold not even a single car on a particular day.

Though this analysis confirms Lee and Horn's (1994) analysis of *any* as incorporating *even*, it adds a crucial feature, which is to "subjectivize" widening and define it relative to an *expectation*. The oversight of Kadmon and Landman (1993) is that they approach widening as operating exclusively on an *objective* domain of quantification rather than on a subjective "expected" one[148].

Observe that the expectation-based analysis also captures Krifka's objection to widening, which is really an objection to widening of the *objective* domain of quantification only. Suppose someone is given a huge list of numbers, which is presented to him as a randomly selected sequence. Under those circumstances and with common sense intuitions about chances and maths, the receiver of the list will normally expect the list to contain prime numbers. Against the background of that expectation, s/he will probably and rightfully be surprised when the actual domain of quantification turns out to be different from the anticipated one, namely extended from a larger quantity (more than one) to the smallest quantity (*even a single*):

(338) a. This sequence doesn't contain *any* prime number.
b. This sequence doesn't *even* contain *a single* prime number.

Krifka's objection that the precise concept *prime number* cannot be widened is thus correct for the objective domain of quantification, but it dissipates when the domain of quantification is taken to be the subjective or expectation domain.

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147 This parallellism is striking confirmation of Lee and Horn's overall perspective on *any* as incorporating the semantics of *even*.

148 In Lee and Horn's (1994) case, it is just a "local" oversight: in his own analysis of *even*, Horn (1969, 1971) does invoke *expectation* violation. Cf. also Fillmore (1965), Karttunen and Peters (1975, 1979) for an expectation-based perspective on *even*.

6.3.1.3 Kay's scalar model for *even* minimalised

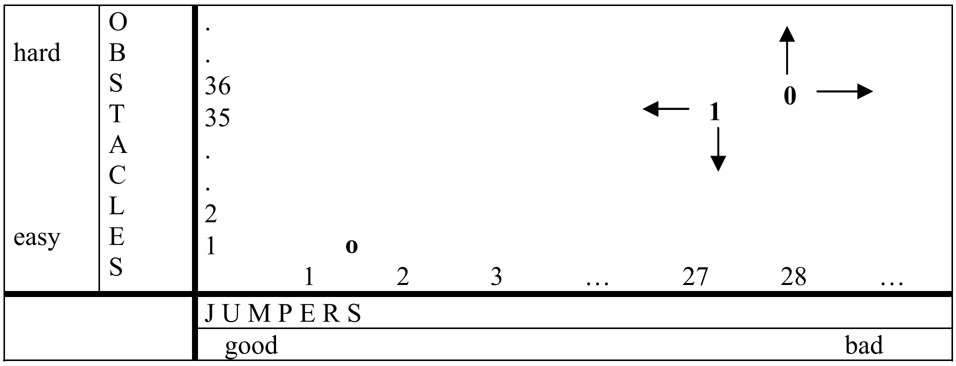
In view of Hamilton's (1858: 615) and Lee and Horn's (1994) insight that *any* is "indefinite plus *even*", this section turns to an analysis of *even* to help solve the problem of how to formalize subjective widening for emphatic *any*. The theoretical tool used is a variant of the type of scalar model developed by Fillmore, Kay and O'Connor (1987) and elaborated by Kay (1990) for *even*. A modified version of his model will be proposed to capture the essence of the subjective widening contributed to the meaning of a sentence by *even*. In § 6.3.1.4, that model will be transferred to *any*-sentences. Crucially, the logical structure of this model is that of a two-dimensional Cartesian coordinate system, so that a link with our triadic logic can be established.

For Kay, a scalar model (SM) "is taken empirically to contain a set of propositions which are part of the shared background of speaker and hearer at the time of utterance." (Kay 1990: 63)[149] Such background conditions have to hold for a sentence with *even* to be used felicitously. For instance, to utter *even John did it* felicitously, the speaker needs to have scalar background information about John and others which s/he does not need to use the proposition *John did it* felicitously. To illustrate its structure, Kay sets up a two dimensional example of a scalar model, with *multiple values* on each axis. Thus the model

> "might include a set of jumpers ordered with respect to jumping ability and a set of obstacles ordered with respect to difficulty. In a given state of affairs we may not know which, if any, jumpers can jump which, if any, obstacles, but we do know that if any jumper can jump any obstacle then the best jumper can jump the easiest obstacle. Similarly, if there is any jumper who can't jump some obstacle, then the worst jumper can't jump the hardest obstacle."
>
> (Kay 1990: 63)

The scalar model is represented in Kay's Figure 1, where the so-called "origin" of the whole argument space for the template *X can jump Y* is the point of the two scales closest to the intersection of the abscissa and ordinate of the two-dimensional Cartesian coordinate system, i.e. the best-jumper/easiest-obstacle *n*-tuple. It is the bottom-left point **o** in the argument space Dx "such that for any state of affairs if the proposition corresponding to any point in Dx is true then the proposition corresponding to **o** is true." (Kay 1990: 64)

[149] For formal definitions, see Kay (1990: 63-70)



In such a scalar model, there is a system of entailment relations:

> “if we know for a given state of affairs that a particular cell has a ‘1’, say that jumper 27 can jump obstacle number 35, then we know that in that state of affairs for any jumper X who is better than jumper 27 and for any obstacle Y that is easier than obstacle 35, X can jump Y. Similarly for the negatives: if jumper 28 can’t jump obstacle 36, then no jumper worse than 28 can jump any obstacle harder than 36. This pattern of entailment in a scalar model is indicated in the diagram by the arrows pointing leftward and downward from the cell <27, 35> and rightward and upward from the cell <28, 36>.”
> (Kay 1990: 65)

From this it follows that the ‘1’ entries “form an unbroken cluster around the origin of the space” (Kay 1990: 65) and also that informing the hearer that jumper 27 can jump obstacle number 35 is more informative than telling him that, say, jumper 20 can jump obstacle number 15, given that the former statement entails more other propositions than the latter. This pattern of entailment relations leads Kay to define “the Gricean notion of *informativeness*, that is, what the Maxim of Quantity tells us to optimize.” In particular, “given a scalar model SM containing two distinct propositions *p* and *q*, *p* is more informative than *q* iff *p* entails *q*.”
These tools suffice to state Kay’s basic analysis of *even*: “*even* indicates that the sentence (or clause (…)) in which it occurs expresses, in context, a proposition which is more informative (equivalently ‘stronger’) than some particular distinct proposition taken to be already present in the context.” (Kay 1990: 66) The *even* sentence he calls the “text sentence or text proposition (tp) and the taken for granted, less informative proposition the context sentence or context proposition (cp)”. *Even* is felicitously used when the tp entails the cp in the scalar model. So, if the sentence is *Even John went to Spain*, this means that for the template *X went to Spain* there is, over and above the most informative proposition tp *John went to Spain,* an outranked alternative true proposition cp expressing a more likely situation (say *someone other than John went to Spain*) entailed by the tp.

Kay stresses the pragmatic nature of his scalar model and illustrates its workings by means of two multi-valued scalar dimensions (*jumpers* and *obstacles*). I will try to prove that the logical structure of the system is not pragmatic but much more deeply ingrained in the cognitive system. Moreover, the multi-valued nature of Kay's scalar dimensions hides the fact that it is really a binary system which is at the root of his model and does the work. Thus, Kay's system turns on <u>*two*</u> dimensions, and on the distinction between the <u>*two*</u> propositions cp and tp, binary contrasts. Though that basic system is clearly often embedded in larger scales with multiple values on each dimension, its core does not itself exceed binarity and is arguably not pragmatic in its logical structure[150].

Regarding Kay's dimension binarity, observe that in *even an ignoramus can pass the exam*, there is a clash between prior *expectation* and actual *reality*. The clash concerns the set of people for whom the template *X can pass the exam* is <u>expected</u> to yield the value true, and the set for which it does in actual <u>reality</u>. The speaker's expectation, embodied in the cp which he takes "for granted", is that geniuses (and a number of more moderately gifted minds too) can normally pass exams. Ignoramuses, however, who are less than moderately gifted, are not normally expected to belong to the set for which *X can pass the exam* is true. This set of expectations generates an expectation entailment. If an ignoramus manages to pass the exam (without cheating), this is entails the expectation that all geniuses have managed to pass too. What the speaker has come to discover in *even an ignoramus can pass the exam*, is that in this case the set denoted by *X can pass the exam* in *actual reality* is larger than the set which constituted the background *(normal or stereotypical) expectation*. In breach of expectation, ignoramuses are actually included in the set of those who passed.

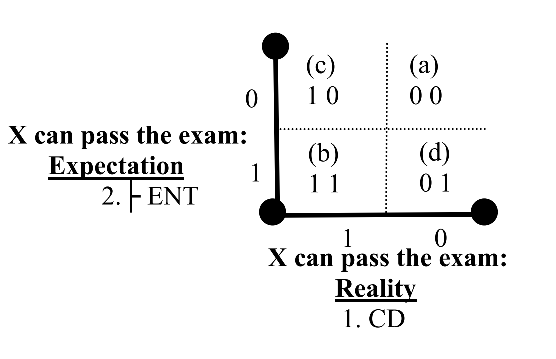
This reality vs. expectation two-dimensionality fits a version of our 2D Cartesian Coordinate System. Its x-axis will be used to express reality, i.e. the binary contrast between the set of possible situations for which the propositional function *X can pass the exam* yields the value actually true (1) and the set for which it yields actually false (0). The y-axis will be used for (necessarily virtual) presuppositional expectations, i.e. the binary contrast between the set of possible situations for which *X can pass the exam* was *expected* to be true (1) and the set for which it was *expected* to be false (0). Since the reality set and the expectation set do not coincide in *even*-sentences, there is a system with four Cartesian quadrants:

(a) the set of situations in which *X can pass the exam* is actually false and was also expected to be false: 0 0;
(b) the set of situations in which *X can pass the exam* is actually true and was also expected to be true: 1 1;
(c) the set of situations in which *X can pass the exam* is actually true and was expected to be false: 1 0;
(d) the set of situations in which *X can pass the exam* is actually false and was expected to be true: 0 1;

[150] Such "bare minimum systems" sometimes surface in the functional categories of a language. Thus, alongside the upwardly recursively infinite scale of the natural number series, for instance, there is APC2 with its basic binary oppositions between *neither(0)* and *either(1)* on the x-axis and between *either(1)* and *both(2)* on the y-axis. APC2 illustrates the core of the cardinality system without any recursion, the natural number sequence shows the effect of recursion on the y-axis.

(339)

2D Cartesian Coordinate System for *even*
x-axis: reality
y-axis: expectation



Above it was said that expectations come with an *entailment* relation: if an ignoramus manages to pass the exam (without cheating), the entailment in one's expectations is that geniuses should have managed to pass too. This makes the y-axis not only the presuppositional subjective leg but also the entailment-(ENT)-leg of this system. The contrast between 1 and 0 on the x-axis, for its part, is the *polarity* contrast between the complementary notions actually true and actually false: this makes the x-axis both the objective and the CD-leg of this system. The parallel with the 2D Cartesian Coordinate System developed for logical operators is striking.

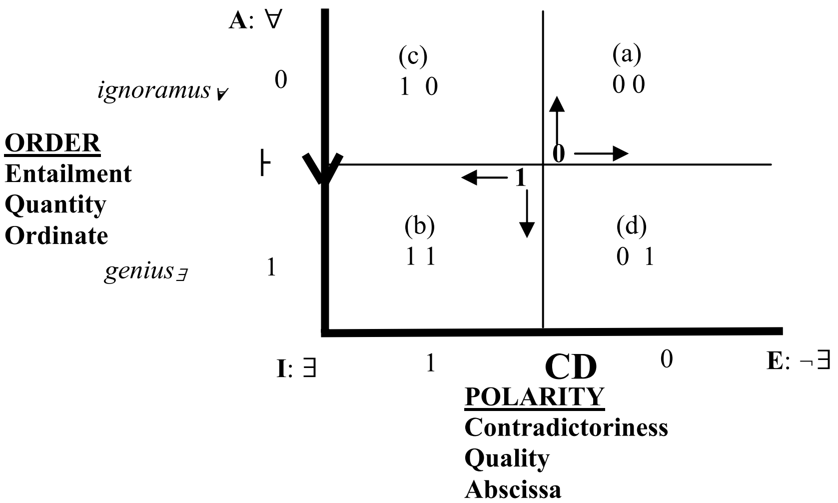
The parallel with the logical triadic system can be made even stronger, as the following *any*- and *even*-data from Horn (2000b) illustrate. They bring in an ambiguity involving the logical corners A (universal) and I (existential).

(340) a. I don't think that anyone$_{\forall/\exists}$ can pass the exam.
b. I don't think that even {an ignoramus$_{\forall}$/a genius$_{\exists}$} can pass the exam.

(340) a. is ambiguous between the quasi-universal reading "I don't think that just anyone$_{\forall}$ (≈ everyone/even an ignoramus) can pass the exam" and the existential reading "I don't think that there is anyone$_{\exists}$, a single student/even a genius who can pass the exam".
To capture this phenomenon, the 2D Cartesian Coordinate System of (339) is enriched with labels for its three corners: the origin of the system, where the CD abscissa and the entailment ordinate of the structure intersect and on which the two relations CD and ENT are predicated, is the pivotal I-corner of the Boethian logical structure as usual. On the abscissa we find the second focal point: the E-corner item $\neg\exists$. The abscissa is therefore the *quality* or <u>polarity</u> axis, i.e. the axis where the CD relation between the set denoted by *x can pass the exam* (1) and the set denoted by *x cannot pass the exam* (0) is represented. The universal affirmative A-corner, for its part, is on the y-axis where subjective expectations and entailments between them are represented. For instance, the set of possible situations in which the A-corner sentence *All the students (including*

ignoramuses) passed the exam is true is expected to be a subset of the set denoted by the I-corner sentence that *some students passed the exam*[151]. The former proposition entails the latter and is therefore more informative and higher on the y-scale. Clearly, the y-axis ordinate is the *quantity* axis of the system, where inferential set-subset relations among scalar expectations are determined. I take it to be the order axis where our general expectations about student performance and entailments between them are represented in a minimal linear scalar ordering of set-subset relations[152].

(341)



The quadrants (a) and (b) represent those situations in which actual ability and expected ability are not in conflict: situations where ignoramuses cannot pass and geniuses can pass the test. The propositions (b) and (a) expressing these situations will function as the expected background cp-s for expectation-violating (d) and (c), respectively.

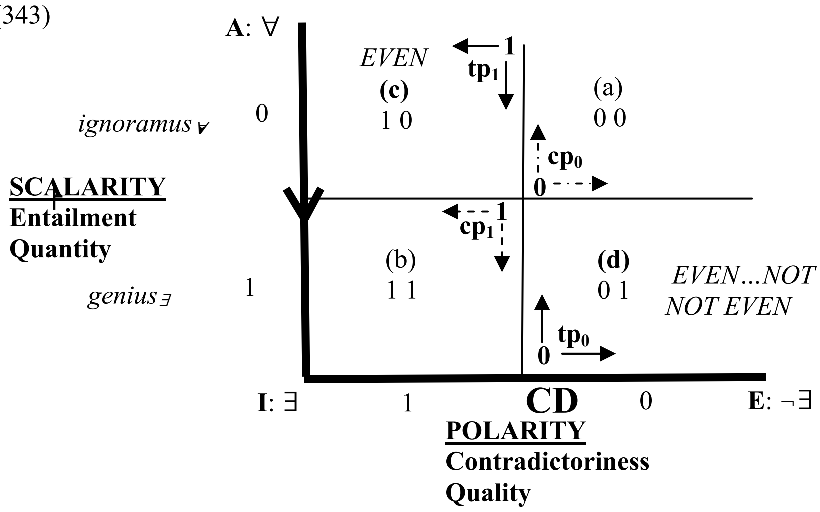
(342) (a): an ignoramus$_{\forall}$ cannot pass the exam: 0 0 (actually unable; not expected to be able)
(b): a genius$_{\exists}$ can pass the exam: 1 1 (actually able; expected to be able)
(c): even an ignoramus$_{\forall}$ can pass the exam: 1 0 (actually able; not expected to be able)
(d): even a genius$_{\exists}$ cannot pass the exam (or: not even a genius$_{\exists}$ can pass the exam): 0 1 (actually unable; expected to be able).

This captures the fact that (c) and (d) are comparatively less expected, more informative situations: situations with ignoramuses who can pass the test and geniuses who cannot. It is for these latter situations and the propositions expressing them that speakers resort to *even* to express the distinction between the less informative cp alternative and the

151 Crucially, talk is about the number of situations, not the number of participants in situations.
152 As will be shown in § 6.4, the y-axis is also involved in partial ordering, which is why the label *order* is preferred to the more specific labels *scalarity* or *linear ordering.*

more informative tp. In each case, the tp is more informative, 'stronger' than the cp, given that (and here is the connection with our modification of Kadmon and Landman (1993)) it functions as a subjective widener extending the expected domain of quantification (boldface arrowed **1** and **0**) to the less expected. Thus (c) extends the subjective domain of quantification from the normally expected (b) situations to (c), which entails (b). In negative contexts, we get the usual reversal of entailment relations, with (a) the less informative alternative (the cp) and (d) the more informative text proposition (tp) which entails its alternative. Subjective widening and resulting entailment from (c) to alternative (b) and (d) to alternative proposition (a) is expressed by *even* and has the following Kay-type representation.

(343)

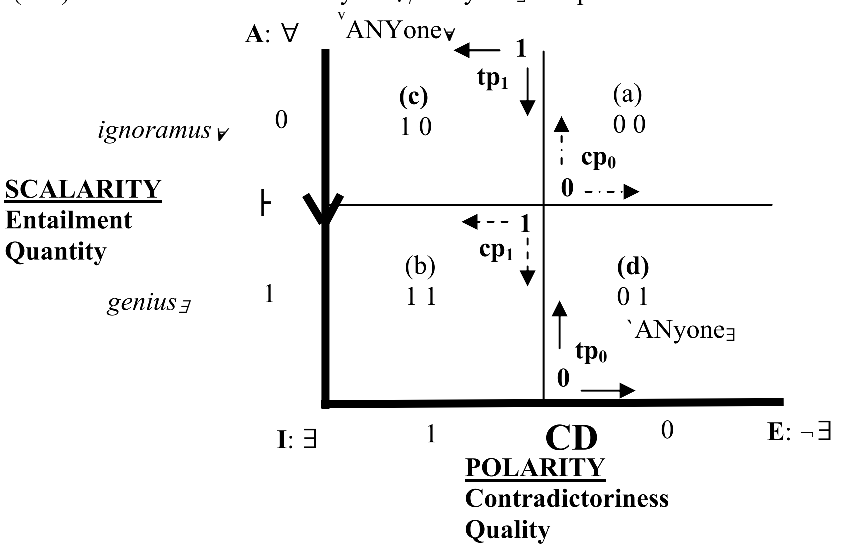


Note that both affirmative and negative widening with *even* has the effect that the subjective domain of quantification is extended from an expected quarter (cp) to a less expected, hence more informative half (tp) of the argument space.

6.3.1.4 *Any* and subjective widening

Adopting Lee and Horn's (1994) insight that the semantics of *any* incorporates that of *even*, the subjective widening by expectation violation that *any* expresses comes out as follows:

(344) I don't think that ^{v}ANyone$_\forall$/`ANyone$_\exists$ can pass the exam.



Not only in the case of *even* but also in the case of *any* the same lexical morpheme is employed to effect both the positive A-widening (cp_1 to tp_1) and the negative I-widening (cp_0 to tp_0). But not only on the formal side but also on the semantic side we find an unambiguous core: the intrinsic semantic content of the emphatic widening operation itself, namely extension of the subjective domain of quantification from cp to tp, is always the same throughout, only the polarity-based directionality can change.

In sum, what emphasis and attendant subjective widening add to the rigid meaning of *any* is a focus-type presupposition-reality opposition, where the presuppositional cp is subjectively or emphatically widened to the more informative expectation-violation. Depending on polarity, the widening will give rigid *any* an FC-profile or an NPI-profile[153].

[153] Note the interesting correlation between having a cp-tp meaning and being a polarity item. This is not only true for the negative polarity item *any*, but also for positive polarity items such as the Dutch aspectual particles *al, nog*. Indeed, the two-dimensional Boolean system outlined here can be extended to many other well-studied particles which involve a distinction between an expectation and an assertion axis, including the aspectual adverbs *already, not yet, no longer, still* in English and in several other languages (for Dutch *al, nog niet, (al) niet meer, nog* see Vandeweghe (1992); for German cp. Löbner (1990), who opposes two-dimensional treatments however).
With a few changes, the set-up of (344) is what is needed for (affirmative) *still* and (negative) *no longer*. The y-axis is now a temporal axis and embodies the presupposition of a phase shift (cf. Löbner 1990: 117) from 1 to O, i.e. from a time interval during which, say, *John is in the garden* to a phase when *John is not in the garden* and an expectation about a point in time at which the shift will take place. Extension of the temporal domain of quantification from an expected quarter (cp1) to a less expected, hence more informative half (tp1) of the space will on the affirmative side capture the meaning of *still:* for a longer stretch of time on the y-axis than up to the point at which the presupposed shift from 1 to 0 was expected, the value for *John is in the garden* actually

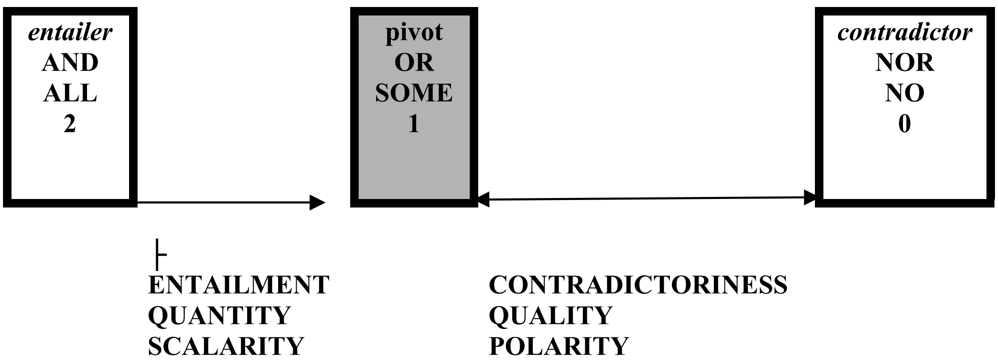
6.3.2 The meanings of *any*

The lexical meaning of the predicate calculus quantifier *any* was identified as that of a rigid pivot. When used in emphatic contexts, *any* acquires a scalar profile which puts emphasis on either the lower end or the upper end of its scale. The former situation obtains in non-affirmative irrealis contexts and is called NPI-*any*. The latter occurs primarily in non-episodic affirmative modal contexts and is called FC-*any*. When non-emphatic, *any* gets a weak reading. This variability testifies to the general property of pivots that has been highlighted in this chapter, namely that their lexical content is comparatively sparse, which enables contextual enrichment by stress and influence of the nature of the predicate on its ultimate interpretation.

(345) a. NPI: I haven't seen ANYbody today.
b. FC: ANYbody can tell you that
c. Non-emphatic weak: There aren't any unicorns in the garden. (neither some unicorns x, nor some unicorns y, nor more)

6.4 Singulars are pivots

In chapters 4 and 5 it has been established that the pivot is special in that it is the origin of the 2D Cartesian Coordinate System. The latter is a very minimal system which has no more than three focal points: the pivot, the E-corner on the quality (polarity; CD) leg and the A-corner on the quantity (scalar; ENT) leg. This much suffices to account for the operator-sets studied so far, all triads.



remains 1 (= 1 0 in (344) (c)); for *no longer* the presupposition and expected point of phase shift on the y-axis are the same, but the extension is downward from cp0 to tp0, expressing that the asserted shift from 1 to 0 occurred before the expected transition point and is consequently ordered before the latter on the temporal y-axis.
Just reverse the presupposition on the y-axis to a shift from 0 to 1 (i.e. from *John is not in the garden* to a phase when *John is in the garden* (Löbner 1990: 117)) and a downward extension on the affirmative side will yield *already*, an upward extension on the negative side *not yet*. The present system of representation illustrates clearly Löbner's point that these four items – though a single duality group – cannot be captured in a (single) Aristotelian Square on account of the pairwise different presuppositions.

In this section I will provide a final argument for the claim that pivots stand out and are more basic than the other focal points. The argument involves the fact that the 2D Cartesian Coordinate System can be expanded on the entailment leg to yield multiple entailers. By contrast, there can only be just one pivot in a calculus[154]. This point will first be made for the number system. Then, it will be shown that there is a relationship between the unique pivot/multiple entailer contrast in the number system and a corresponding contrast in the domain of grammatical number (singular versus plural). In view of this parallelism, the singular will make its appearance as our final newcomer pivot to the same type of Boolean 2D Cartesian Coordinate System that underlies logical operators and numbers.

6.4.1 Generating Linear Scales

Consider the numerals scale, which is a linear ordering. In the minimal model, the pivot is the lower endpoint of a Horn scale <2,1> and there is one other value, the A-corner entailer value 2. But if we look again at the Horn scales (1989: 232) in (277), partly repeated here, with pivots in italics and underlined:

(346) <all, most, many, *some*>; <always, usually, often, *sometimes*>; <and, *or*>; <6, 5, 4, 3, 2, *1*>; <must, should, *may*>; <necessary, (logically) *possible*>.

we find that the numerals scale included there is not just <2,*1*>, but the larger linearly ordered scale <6, 5, 4, 3, 2, *1*>. As before, the pivot is the lower endpoint of the scale.

[154] A similar point can be made about the contradictoriness leg of the system in terms of the opposition between a unit and fractions, but that would lead me too far afield (and the special status of 0 would have to be discussed). Moreover, when the contradictoriness leg functions as a polarity leg where 1 stands for truth and 0 for falsehood (cf. the scalar model developed for *any* in § 6.3), the claim I am driven to for my analysis of the special status of the pivot to be correct, namely that there is only one truth buth that there can be more than one falsehood has already been proved by Seuren (2001b: 221). On the basis of the following sentences, he distinguishes between 'radical' and 'minimal' falsehood:

(i) The present king of France is bald.
(ii) Rosemary's favourite lover is bald.

Sentence (i) is "radically false when said now, or in Russell's day, since there is no really existing individual in the world answering to the description 'present king of France' (nor was there one in 1905)" (Seuren 2001b: 221). Sentence (ii), "said of a person with a hirsute scalp, is minimally false, and its minimal negation (…) is true and entails both the existence of the man spoken about and the fact that he has hair on his head" (Seuren 2001b: 221). "Radically false sentences are made true by the radical negation, which is represented by an emphatically accented *not* in canonical negation position (for English: with the finite verb form), and preferably accompanied by a specification of the presupposition which is violated." (Seuren 2001b: 221) Seuren gives the following example (his (17) b.):

(iii) The present king of France is NOT bald: there is no king of France!

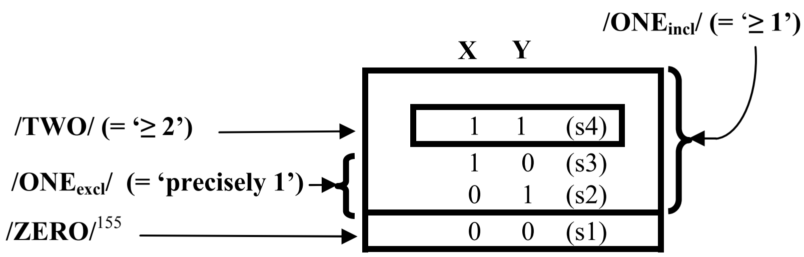
Since Seuren's analysis proves there is more than one type of falsehood, and since there is only one truth, his analysis supports the claim that pivots stand out.

This time, however, it is special in another respect. First, it is the only element that is entailed by all other elements on the scale. Secondly, it is the only value on the scale that does not itself entail a number, given that the relation between 1 and 0 in the 2D Cartesian Coordinate System is not on the scalarity leg of the system (hence no entailment), but a contradictoriness relation on the polarity leg.

This special status of the pivot on the entailment leg can be expressed by specifying how the minimal model developed so far can be extended to accommodate extra values beyond 2. What has to be found is a way to make infinite recursion possible, since the linear number scale obviously does not stop at 6.

A solution to this problem is to view the Boolean placemat introduced earlier as a recursive rule set (s1)-(s4) consisting of instructions to form the union of X and Y, and recursive in that the output set can be used to reapply the same rule:

(347)



Consider a three-entity universe[156] with elements a, b, and c as possible members of the sets to be demarcated. (s1) is an instruction to form the union of an X and an Y, where neither contains an element from the universe. This yields the null set, whose cardinality is 0 (zero). However often the output is used for reapplication of this rule, the result does not change. The extension of *zero* is the set of all sets with no members, which is just the null set.

(348)

		| output set |
/ZERO/	Ø	0

(s2) and (s3) yield unions where one or other entity from the universe is a member, but not more than one. This yields singletons: cardinality of membership is 1. Here again, recursion is to no avail in the sense that it cannot increase the membership.

(349)

					| output set |
/ONE$_{excl}$/ (= 'precisely 1')	{a}	{b}	{c}	](s2-s3)	1

[155] *Zero*, but also synonyms restricted to specific contexts: *nil, naught, love, o, …*
[156] As before, all cognitively realistic set demarcation occurs within a universe of discourse.

A first application of (s4), finally, forms the union of two sets, each with a single (different) element from the universe, say {a} and {b},yielding two-member sets, i.c. {a,b}. This time, using the output in a recursive reapplication of the rule does get one further: the union of {a,b} and {c}, for instance, yields {a, b, c}, a three-member set

(350)

				$|$output set$|$
	{abc}		(s4)	3
{ab}	{ac}	{bc}	(s4)	2
{a}	{b}	{c}	(s2-s3)	1
	Ø		(s1)	0

It is clear how the system can go on when operating on a universe with more entities. In principle it could go on indefinitely; in reality life is too short of course and only an inductive leap can bring in ∞ (infinity).

At this point, a remark has to be made from a cognitive viewpoint. First of all, recall that the Boolean placemat used here is actually generated by the negative-disjunctive Peircean bar-system introduced in § 4.4, so to get a fully cognitively realistic (hence negative) description, what I have stated here in terms of a positive recursive rule set, has to be translated into that negative bar-code and employ conjunction for recursion as follows, to get the entailers beyond 2.

(351)

Logic	**Bar-code**	**cardinality**
$\neg(P \vee Q)$	$\overline{/P/\ /Q/}$	0
$P \vee Q$	$\overline{\overline{/P/\ /Q/}}$	≥ 1
$P \wedge Q$	$\overline{\overline{/P/\ /Q/}\ \overline{/P/}\ \overline{/Q/}}$	≥ 2
$P \wedge Q \wedge R$	$\overline{\overline{\overline{\overline{/P/\ /Q/}\ \overline{/P/}\ \overline{/Q/}}\ /R/}\ \ \overline{\overline{\overline{\overline{/P/\ /Q/}\ \overline{/P/}\ \overline{/Q/}}}}\ \overline{/R/}}$	≥ 3

This *via negativa* is more accurate too. Thus, in the positive rule set, the “exclusive”, singleton sets (‘precisely’ one) are generated separately from the higher cardinalities. What actually happens in the cognitively realistic negative bar-code system, is that after the null set, one first gets by <u>NON</u> to all the sets with cardinality $|\geq 1|$(“inclusive”), after which that set is narrowed down to its subset with cardinality $|\geq 2|$ by further fractioning exclusion, and so on. So, in the cognitively realistic bar-code version, the extension of *one* is the set of all sets with *at least one* member, the basic meaning I have systematically attributed to pivots throughout this study. The extension of the lexical predicate *two* is a smaller set: the set of sets with *at least two* members, the extension of *three* the set of sets with *at least three* members, and so further and so on, a standard interpretation in formal semantics. [157]

The nature of the cognitively realistic process illustrates a general point about the negative-subtractive theory defended in this study. When an entailment scale is enriched from its binary opposition I-A to a larger set of values, those values are always added "in between", i.e. by *fractionizing* the I-A scale into intermediate values, not by literally extending the scale. In a negative subtractive system, you cannot add anything since you start from the universe, but you break up what you have into ever finer bits. This conception matches the factual observation that enrichment of the quantifier scale yields intermediate values between *some/any* and *all* (such as *many, most*, etc.) and nothing beyond *all.*

The same conclusion obtains for the polarity leg of the 2D Cartesian Coordinate System, where the perspective outlined here can give Boole's (1854 [1958]: 2) traditional distinction between a Logic of Classes and a Logic of Probabilities a natural place. The system as sketched so far (e.g. the Kay-type scalar model) worked with no more than the basic binary opposition true (1)-false (0) on the polarity axis and thereby represents the binary Logic of Classes. But the binary polarity opposition can be made quantificational and fuzzified by means of *fractions* representing transitional values between 0 and 1. Such an introduction of quantity relations and a quantificational approach to the contradictoriness axis turns the binary Logic of Classes into the Logic of Probabilities and (though differently[158]) fuzzy sets: the predicate is no longer unequivocally true or false, but borderline fractional cases are introduced, which makes transitional values (1/2 true, 1/3 true, etc.) inevitable. The fact that a Logic of Probabilities is generally felt to be a complication of the binary Logic of Classes thus finds a natural and intuitively correct characterization. While the polarity axis is the axis of set-complement set relations (CD) – simple (i.e. binary) or complex (fuzzy/intermediate values) –, the scalarity axis is the axis of entailment (ENT), simple (i.e. binary A-I) or complex i.e. with multiple entailers and recursive inferential set-subset relations.

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[157] The reason why I have used the positive approach too, is double. First of all, it enables me to illustrate clearly that the negative-subtractive theory has greater cognitive reality. Secondly, the positive picture will simplify discussion of the singular-plural distinction in the next section.

[158] The logic of probabilities and fuzzy logic (Zadeh 1965) are of course not the same. Take the following two examples:

(i) (He is a pensioner) He's 30% likely to be bald.
(ii) He is 30% bald.

While the first statement is the result of a calculation of probabilities in a population, the second is about partial membership of a set: it fuzzifies the predicate bald itself and hence the boundary nonbald-bald. The similarity between probabilities and fuzzy logic which is relevant to the text proposal is that they both become identical to binary Boolean logic if all the partial memberships are either 0% or 100%.

6.4.2 The singular-plural distinction and partial order

In this section, it will be illustrated that singulars are another (and final) category of pivots with AT LEAST ONE meaning. They stand out relative to plurals in the same way in which all other pivots relate to entailers.

Chierchia (2003) showed that an algebraic perspective is very fruitful for the description of the meaning of bare semantic predicates, e.g. the denotation of the noun *table* and its plural form *tables*[159]. In Chierchia's (2003) words: "The role of lexical nouns (without the determiner) is that of singling out a class of objects (which will have common traits: they may share a form, a function or what have you). Plural morphology (-s, in English) turns nouns that apply to singularities into nouns that apply to groups." Take a universe of discourse IND_p (in formal semantics often referred to as the domain E of entities/individuals in the model of discourse) with five individuals a, b, c, d, e. Assuming that only a, b and c are tables, the lexical noun *table* singles out this class of atomic objects between the lowest angles in the following representation[160]:

(352)

{abc}

{ab} {ac} {bc}

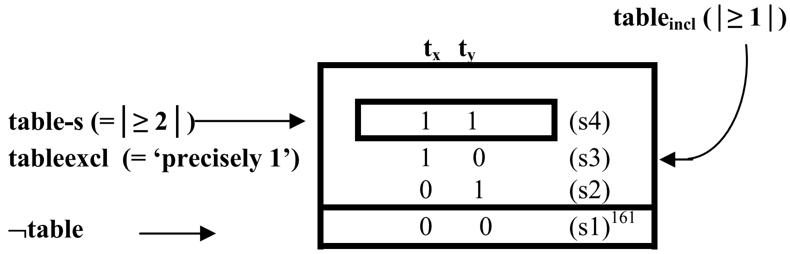
a b c

Since a, b and c are tables, it "follows inexorably that a, b and c taken together are also tables; and also for instance *b* and *c* taken together are tables." (Chierchia 2003) According to Chierchia, this is something we know a priori: "Once we individuate through a singular noun a set of atoms, all of the groups (of various size) made up of those atoms will be something the corresponding plural is true of." But the next question is then what the precise nature and structure of this a priori knowledge is. What generates the pattern? This is where the negative twin selection placemat comes in and adds an interesting touch to proposals like Chierchia's, namely by specifying how and by which means the partitioning of the denotation of a count noun into singularities and pluralities of ascending cardinality comes about. Concretely, the pattern resulting from twin selection ($table_x$, $table_y$,) and its $2^n = 2^2 = 4$ possible value combinations looks as follows.

[159] In the past decade, several people have worked out the hypothesis that an algebraic approach should be adopted to the meaning of NPs, e.g. Scha (1983), Link (1983), Krifka (1986), Landman (1989), Schwarzschild (1996), Doetjes (1997), Winter (1998), Chierchia (2003)).

[160] The curly brackets round the non-atomic combinations are borrowed from set-theory, but Chierchia does not take sides in the debate whether plural entities *are* really sets. In view of the analysis developed in this study, I am forced to view singularities and pluralities all as sets. Singularities or atoms too are viewed as (singleton) sets (pace Chierchia, who leaves out the curly brackets). The difference between singular and plural is then a *sort* distinction between singletons and non-singletons and not a *type* difference (pace Bennet (1974), Winter (1999:6)). Opinions are divided, but if the same set-forming Boolean algorithm used for other set demarcation is active here, I believe I am driven to the set-position.

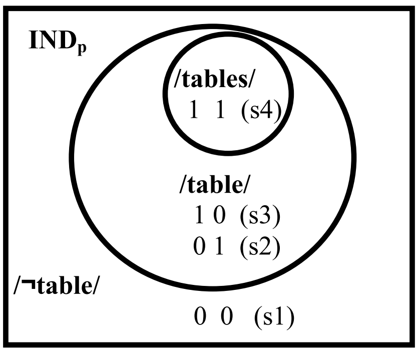
(353)



Interpreting (s1)-(s4) as a recursive rule set, we get:

a. (s1): twin selection yields a set with neither one, nor any other table: 0 0. Hence the predicate does not apply (but *non-table* does).
b. (s2-s3) John has *one or other table*, i.e. twin selection yields one (0 1) or other (1 0) singularity, but exclusively: not more than one table at once.
c. (s4) John has *table-s.* Twin selection yields one *and* another table (1 1), e.g. two tables {ab}. On the earlier assumption that s4 can be thought of as an instruction which applies recursively to its output, we can now take {ab} as a first argument for a new application of the same (1 1) rule and combine it by set union with a second argument {c}, which means that recursive reapplication of (1 1) now results in {abc}.

(354)



Merging the denotation of *table* vs. *tables* as conceived by Chierchia with our placemat analysis, we get the following partial ordering[162]:

(355)

[161] S1 = all sets of type 1, i.e. all sets of non-tables (neither one nor any other table) in the domain of discourse (physical objects) over which the predicate *table* ranges. How set demarcation by means of /NEC/ turns the Boolean algebra including this set into the final description of the predicate as a join semi-lattice (with S1 cut off) was explained in chapter 4, §4.3.
[162] The order is partial because while there is vertical ordering (set-subset), the horizontally juxtaposed sets of each cardinality are not ordered relative to one another.

					/table/ $(\lvert \geq 1 \rvert)$
		{abc}			
/table-s/ $(= \lvert \geq 2 \rvert)$	{ab}	{ac}	{bc}	(s4)	
/table$_{excl}$/ (= 'precisely 1')	{a}	{b}	{c}	(s2-s3)	

First of all, the binary placemat version of the underlying algebraic structure used imposes a partial ordering separating out the denotation of singular and plural forms of the noun with finite means, exactly as required by Chierchia's analysis. While the rule system that generates this partial ordering is a very small finite algorithm, there is no limit in principle on the cardinality of the pluralities. It does not really matter how large the universe of discourse IND_p (= the domain IND of individuals in the model of discourse) that s1-s4 operates on is, though a realistic conception of it will keep it finite. At the same time our system maintains a link between singular and plural in the sense that the pluralities (s4) are still conceived of as part of the denotation of the basic, underlying "inclusive" $(\lvert \geq 1 \rvert)$ predicate /table/. This conception of the nature of the a priori knowledge is empirically more justified than one which restricts /table/ to the singular, as a singular form does not always apply to atoms only, witness the following non-referential negative and interrogative contexts:

(356) a. I haven't got a table. (neither one, nor more than one)
b. Have you got a kitchen table? Yes, I have two.

This is different from Chierchia's position when he says that the "role of plural morphology is that of turning something that applies *only* [italics mine, DJ] to atoms into something that applies to the corresponding groups". The present modification seems empirically more correct (cf. (169)), and it also makes it possible to incorporate Chierchia's results in the negative-disjunctive Peircean algorithm developed in this study. If I am on the right track, the meaning of a singular such as *table* incorporates the notion AT LEAST ONE (cf. § 5.3) and is in that sense a regular pivot. Plurals, for their part, are semantically more complex and derived from the singular by a morphological procedure that excludes part of the meaning of the singular, analogous to the way in which entailers more generally relate to pivots. And a singular, like other flexible pivots, can be inclusive or exclusive in meaning. And as with other A-corner I-corner relations, plurals entail singulars, i.e. the set of all sets with more than one x is a subset of the set of all sets with at least one x. Singulars, however, cannot entail in this sense.

6.5 Conclusion

The basic contrast that pervaded this chapter is the distinction between flexible and rigid pivots, the latter of which are confined to nonveridical contexts. For the flexible pivots, the distinction between veridical and nonveridical contexts yielded different readings. In addition to the lexical bifurcation and the semantic variability of flexible pivots depending on the (non)veridical nature of their environment, futher variation is induced by the difference between stressed and unstressed pivots. In a final section, it was illustrated how partial orderings are generated by Boolean means and how the set of pivots can be extended to include singular Ns.

7 CONCLUSIONS

7.1 IN-Logic and the *nand-puzzle

The main objective of this study has been to find evidence for the hypothesis that the foundations of logic are to be sought in the mind. The general programme was described in the first sentence of Boole's *The Calculus of Logic* (Boole 1848: 183) as "the application of a new and peculiar form of Mathematics to the expression of the operations of the mind in reasoning". Evidence was collected for the hypothesis that logic in general and logical operators in particular betray their true nature in natural language, i.e. in the material expression of thought.
The path traversed in our quest will be summarized briefly. That will be done, however, with special emphasis on the puzzle triggering it (*why neither *nand nor *nall*?) and proposals that have been advanced for the O-corner problem. It will be shown how the original question became a different one as our model developed. More importantly, a solution to the **nand*-riddle is now within reach.

7.1.1 The O-corner problem and the 2D Cartesian Coordinate System

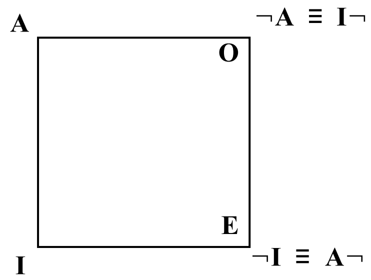
On the first pages of the present study, a small logical calculus devised by Aristotle (384-322 BC) and referred to as APC (Aristotelian Predicate Calculus) was presented in a didactic format introduced by the late Roman statesman and philosopher Anicius Manlius Severinus Boethius (480-524 BC). As said there, Boethius labelled the four sentence types of APC A, I for the AffIrmative propositions and E and O for the negative ("nEgO") statements (Seuren 1998: 306):

(357)

type A: All F is G	type E: All F is not G / No F is G
type I: Some F is G	type O: Some F is not G / Not all F is G

For the benefit of his beginning students, Boethius cast the four types into a geometrical figure: the Square of Oppositions (Seuren 1998: 306-307). A surprising linguistic feature of this Square observed by Thomas Aquinas and many others since was that while the A, and I corners have single word lexicalisations in language after language, there is variation with respect to E-corner lexicalization, and the O-corner systematically resists lexicalisation. There is no **nomnis, *nall,* etc. This has led to a spate of proposals, some of which have sought a "robust" exclusion in terms of innate linguistic principles (Huybregts 1979), while others do not rule out O-corner lexicalization a priori, but claim that "it would have to be a rare phenomenon" (Hoeksema 1999: 6). In this respect, my own proposal is on Huybregts' side, though different in its content. By and large, the proposals made come in two categories: there are "blocking" (Huybregts 1979, Barwise & Cooper 1981, Horn 1989) solutions and "geometrical" solutions. To illustrate each type, a very simple representation of the Square will be used, namely with no more than the labels A and I of the affirmative lefthand axis, and negation (external negation before the letter, internal negation behind it):

(358)



Huybregts' proposal is the following universal lexicalisation principle:

(359) Lexicalization Principle:
Lexicalisation of ¬Q is possible just in case lexicalization of Q¬ is impossible

This rules out lexicalization of the O-corner (¬A) if A¬ is lexicalized, which is the case in English, Dutch and many other languages. As pointed out by Hoeksema (1999), the generalization fails for many Asian languages (Japanese, Hindi, etc.), where there are no single word lexicalisations for negative quantifiers at all. In those languages, the fact that lexicalisation of E-corner operator A¬ is impossible should make lexicalization of the O-corner available, contrary to fact[163].

Horn's (1989) blocking proposal is that lexicalization of O-type quantifiers is blocked by lexicalized I-type quantifiers due to pragmatic equivalence. Though I and O quantifiers are logically different, they implicate each other:

(360) a. I: some boys are in the garden; implicature: not all of them are;
b. O: not all boys are in the garden; implicature: some boy is.

Thus, though they are not logically equivalent, they are pragmatically equivalent. On the additional assumption that affirmation is unmarked and negation marked, the blocking of O is explained. A major problem for this analysis, however, is that pragmatic equivalence does not always obtain, as remarked by Hoeksema:

> "In contexts where the speaker has only partial knowledge, there is not even pragmatic equivalence. If I say that some of my students are gay, one should not infer immediately that not all my students are gay. Perhaps I am unaware of the sexual preferences of the remainder. But if I and O are often not even pragmatically equivalent, because the conditions for the Gricean implicatures are not met, then why should O be superfluous?"
>
> (Hoeksema 1999: 4)

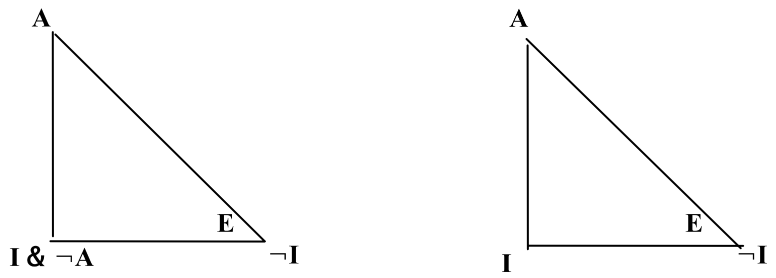
[163] For discussion of Barwise & Cooper's blocking principle, namely their "monotonicity correspondence universal", cf. Hoeksema (1999:2)

For this reason, but also because more tangible, formal aspects of language such as the distinctions flexible-rigid, veridical-nonveridical, presuppositional-nonpresuppositional and stressed-unstressed can help account for the facts in question, an account in semantic terms, wherever possible, is to be preferred over a purely pragmatic one.

This is the only reason why H. P. Grice (1913-1988) has not been mentioned more often in this study. In many ways, Grice was the first to systematically analyse "divergences in meaning" between "formal devices" in logic and "their analogs or counterparts in natural language" (Grice 1975, 1978, 1981, 1989) and a version of his concept of informativeness and other notions (e.g. implicature) play an important role in the present account. But certain concepts of his theory are problematic, namely his "Cooperative Principle". Implicatures - which refer to something implied by the speaker in the conversation but not stated - are taken to be effects of people obeying (or deliberately flouting) an ethical code of cooperative verbal conduct. I believe, however, that at least in the case of the logical operators studied here a Gricean appeal to such social-ethical communicative principles should be replaced by the individual-epistemological principle hinted at on several occasions and embodied in Peirce's "irritation of doubt": our individual search for causal knowledge and patterns in reality drives us away from particular I-corner statements to universal statements of the affirmative A-corner or the negative E-corner variety. It is this drive which is not only behind the phenomenon of (sometimes unwarranted) leaps of induction to A or E, but also behind the orientation of the I→A and I→E shifts detailed earlier and behind the implicature that when we make a particular I-corner statement such as *some flags are green*, it can safely be assumed that more informative knowledge is not available. If we had had more satisfying information, we would no doubt have supplied it given our epistemological preferences for A and E-expressions. It is only because we don't have better information that we are forced to make do with the epistemologically disappointing conclusion that *some* (implicature: "but (irritatingly enough) not all") *flags are green*. The advantage of such a Peircean individual-epistemological principle is that the distinction that it revolves around is already incorporated in the logical system, namely the categorial I-corner versus universal corner distinction. It seems much harder to me to tie Grice's cooperative principle and maxims to a concrete feature of the logical apparatus. A mentalist IN-principle (individual, internalist, intensional) like Peirce's irritation of doubt and attendant "flight from the pivot" whenever possible seems to me to be more promising and more accurate than a social-ethical set of principles and maxims of verbal conduct.

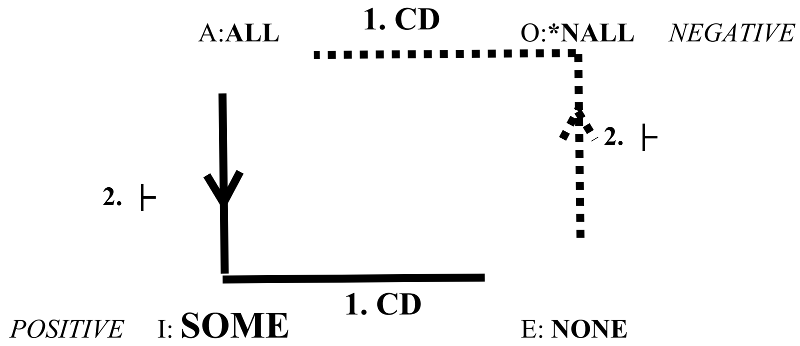
On the geometrical front, alternative quadrangular schemes (Löbner 1985, 1987, 1990) to that of Boethius and all kinds of representations with more corners (five, six, etc.) have been proposed (see van der Auwera 1996). A number of scholars (including De Morgan 1858: 121; Jespersen 1924: 324-325; Horn 1989: 253; Seuren (2002, 20-21)) dropped Boethius' Square for a triangular representation, either (a) collapsing the O-corner with the I-corner or (b) dropping the O-corner altogether, leaving a basic <u>Triangle</u> of Oppositions.

(361) a. b.



The collapsed construct (Jespersen 1924) was shown in § 2.2.5.1 to fail both theoretically and empirically. The O-deletion triangle, for its part, just erases the fourth corner, and is therefore, crudely put, an attempt to solve the question by "dissolving" its appearance in the geometrical figure. While the representation has changed, the original question survives unscathed. In slightly reformulated terms it now becomes: why *could not* there be a fourth corner? Surely, the meaning it expresses is not logically inaccessible: by combining the meanings of *not* and *all* into the phrase *not all* the required meaning is easily obtained.

The first model to propose a two-legged system rather than a triangle was Löbner's (1990) account in terms of the two relations of (outer) negation and dual negation (cf. § 2.2.5.2 and § 4.8). Our own attempt to find an economical system stayed within the confines of Aristotle's calculus in chapter 2, but nonetheless succeeded in producing a system which is more minimalist than a triangle. It *derived* all relations of Boethius' square from two arguably foundational relations, namely *contradictoriness* between I-corner propositions and their negation in the E-corner on the one hand and *entailment* between A-corner propositions and the I-corner propositions they entail. As this turned out to be possible, the O-corner was thereby reduced to the status of a non-basic corner of the system and the triangle lost one of its sides, namely the (derived) contrary relationship between A and E, leaving a two-dimensional Cartesian coordinate system. But the lexicalization issue was not conclusively settled yet: why cannot a *non-basic corner* be lexicalised as a single word? Or more fundamentally: even if one admits that the logical system is a two-dimensional Cartesian coordinate system with CD and ENT as its axes, why then cannot the system be built on the O-corner just as well as on the I-corner? Why this positive-negative asymmetry?



The issue has two sides: (α) proving that the 2D Cartesian Coordinate System is undeniably based on the I-corner, not the O-corner, (β) finding a reason why on top of not being a hub of the system, the O-corner resists lexicalization as well. While the latter task has been saved up till this conclusion, chapters 2-4 were devoted to (α).

7.1.2 The I-corner as the positive pivot of the 2D Cartesian Coordinate System

In the third chapter the two relations CD and ENT were approached in set-theoretic and algebraic terms and the claim was developed that they are relations between items in the lexicon rather than directly stated at the level of complete propositions. Both relations were argued to be of an internalist nature and a cognitively realistic theory of set demarcation was worked out. Furthermore, an account for unnatural entailments was provided in terms of a principle of informativeness. Though this chapter provided no answer to the lexicalization question, it did pave the way for progress by bringing in set-theory and algebra and by providing a lexical anchoring for the discussion.

The fourth chapter, then, consisted of a semantic decomposition of the lexical meaning of the focal points of logical calculi. These decompositions were carried out in terms of a basic negative operator. In the logical literature, two candidate negative binary operators had already been established to which all others can be reduced (so in that sense we did not have to do the really tough base work). One possible candidate was Sheffer's stroke, whose meaning is identical to that of the non-existing O-corner element **nand*, a negative conjunctive operator. The second candidate was Peirce's dagger, expressing E-corner "joint falsehood" (*nor*) and hence a negative disjunctive operator. For several reasons, Peirce's negative-disjunctive approach turned out to be the correct choice and superior to Sheffer's (also historically later) alternative:
(i) our excursion into set-demarcation had taught us that the latter is effected via twin selection, which involves set-theoretic union, hence logical disjunction[164], not intersection/conjunction;

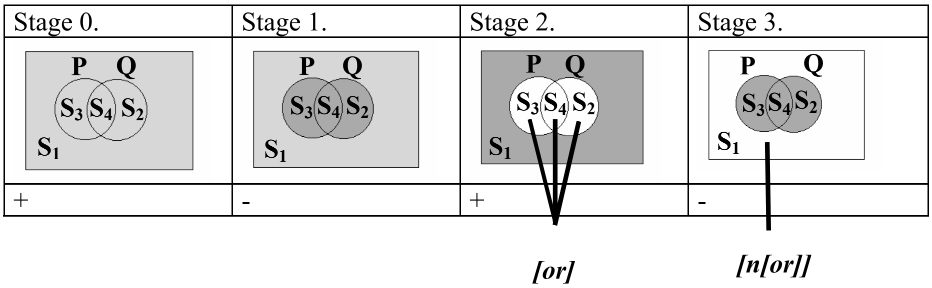
[164] As mentioned a number of times, from the viewpoint of Boolean algebra there are models of the set-theoretic, arithmetical and logical type, which can be mapped onto each other. Set theoretic union, for its part, maps into addition and disjunction respectively ($\cup \approx + \approx \vee$)

(ii) union and its algebraic counterpart addition are procedurally less complex[165] – hence by assumption also more primitive - operations than conjunction and its algebraic counterpart multiplication;
(iii) in Peirce's system the most basic lexicalised operator is the particular affirmative one (*or, some*) and the two which are derived in a single step from the pivot are the universal operators (affirmative and negative respectively). From the viewpoint of informativeness, that makes sense: the two universal, hence more informative notions are constructed from the less informative particular one. No such symmetry and sense in Sheffer's system. Since it is based on a nand-operator, the pivot of a Sheffer-based system – i.e. the operator reached at stage 2. after two applications of Sheffer's stroke - is the universal affirmative operator in the A-corner. Consequently, in a birelational system the two operators that would then have to be derived in a next step are the particular affirmative on the one hand, the universal negative on the other. Not only are these a mixed bag (they differ both in quantity and in quality), the resulting orientation of the system does not make sense from the viewpoint of informativeness. Its pivot (the universal affirmative A-corner item) is more informative than one of the derived operators (the particular affirmative I-corner item) and equally as informative as the other derived operator (the universal negative E-corner item).

So Peirce's operator as generator of all the focal points had to be adopted and the operator was rebaptised /NEC/ to emphasize that from our mentalist perspective it is not just a logico-mathematical object in a Platonic realm like Peirce's dagger, but arguably an operator with cognitive reality, an innate a priori feature of the mind. Operating on a selectable universe of possible situations, which is a *positive* concept (represented by means of 1), the exclusionary activity of /NEC/ gives a purely *negative* and nonlexicalizable result at stage 1. A second application of /NEC/ yields a first lexicalizable result, which is both *positive* and of the *I-corner variety*, thereby establishing the pivotal role of I in a logical calculus *as a matter of inescapable theoretical principle*[166], as well as the marked nature of operators derived from them, including the negative operator (stage 3.) in (362). These points are illustrated in the following sequence for the propositional operators *or* and *nor*.

[165] The adverb procedurally is crucial: there is of course no difference in denotational complexity, the difference is only in constructional complexity.
[166] That is: if /NEC/ is the only primitive operator, then the I-corner operator is necessarily the pivot of the logical system. This is where the present proposal manages to "anchor" the pivot theoretically.

(362)



From a theoretical angle, that is enough to decide issue (α) above, namely proving that the 2D Cartesian Coordinate System is based on the positive I-corner (i.c. *or*), on which the non-pivotal hubs (here the E-corner operator *nor*; for *and*, see § 4.4.5.1) are based. But another theoretical argument emerged: there is a difference in epistemological status between the operators. Operators of type A and E are *general* (*universal* in APC) while only I-corner pivots are used in propositions that provide *particular empirical knowledge*.

Empirical evidence compiled in chapters 4 and 5 confirmed the theoretical analysis, including the relief pattern in logical triads by which the I-corner stands out. The search for confirmation for the meaning of pivots as developed in the decomposition analysis led to linguistic evidence that indefinites and singulars are in the class of pivots too. On the basis of a comparison of the features of the indefinite article and the pivotal predicate operator *any*, it was found that the pivotal I-corner is not only more basic than the rest, but also more malleable: I-corner operators can get different contextually enriched interpretations. Being more basic was consequently identified as being intensionally less specified as a lexical item. This rendered conclusive support to our cognitive restyling of Peirce's decomposition. It proved that the two universals incorporate the meaning of the less highly specified pivot, which is the positive origin of the calculus.

7.1.3 Non-lexicalization of the O-corner: why neither **nall* nor **nand*?

In this section, the final question will be addressed: why is it impossible to have lexical items such as **nand* and **nall* in natural language? So far, all we have is a plausibility argument: if the 2D Cartesian Coordinate System has the I-corner as origin and there is no fourth hub, it is not unexpected that lexicalization of Boethius' O-corner is problematic. However indicative, this does not of course decide the issue.

7.1.3.1 The problem

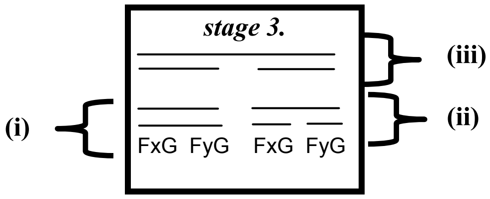
Our account will take the analysis for *all* as its starting point and determine why application of negation to *all* cannot result in morphologically complex single word lexicalization **nall*.

(363) **ALL =**

“**(i)** Flagx is Green or$_{incl}$ Flagy is Green, i.e. at least one of Fx or Fy is G

(iii) ET

(ii) neither [Flagx is not Green] nor [Flag y is not Green], i.e. there isn’t any Flagx or Flag y that is not Green: no Fx or Fy is not G.



In view of this decomposition structure of *all* and the nature of the operation NON as developed so far, there is at first sight no problem whatsoever to change the lexical item *all* (at stage 3) into stage 4. *nall* by application of NON, witness NEC-flowchart (364). Application of NON (= 1-…) to stage 3. constitutes a switch from *all* to its complement set. But if this switch is so easily obtained, why then is the corresponding label **nall* not attested?

(364) Stages*:* ***1.*** (1-(F_xG+F_yG))
2. (1-(1-(F_xG+F_yG)) **[**+ (1-(F_xG+F_yG)))**]**
3. (i) (1-(1-(1-(F_xG+F_yG))) **[**+ (1-(1-(1-(F_xG+F_yG))))**]** (ii) (1-(1- F_xG) + (1- F_yG))
4. 1- stage3

		cell structure of the universe	***0.*** SIT = 1	***1.***	***2.***	***3.***	***4.*** ***1- stage3***
F_xG	F_yG		*value*	*value*	*value*	*value*	*value*
1	1	S4	*1?*	***0***	***1***	***1***	***0***
1	0	S3	*1?*	***0***	***1***	***0***	***1***
0	1	S2	*1?*	***0***	***1***	***0***	***1***
0	0	S1	*1?*	*1?*	***0***	***0***	***1***

/SOME/ /ALL/ **[all]** **[*n[all]]**

Mere technical application of the 1-…-device apparently cannot prevent the generation of **nall*. But there is an element of the analysis provided in chapter 4 whose effects have not yet been fully acknowledged in the presentation so far, namely the (F-class) non-emptiness presupposition that is carried by the universal quantifier (Strawson 1952: 174-176; Seuren 2002: 31). It is this property which will be shown to cause the impossibility of **nall.* To reach that end-result, the following steps will be taken. First, the nature of a presupposition will be briefly described (§ 7.1.3.2). The next sections are devoted to the effect of negation on a presupposition (§ 7.1.3.3) and the way in which presuppositions can be represented by means of valuation spaces (§ 7.1.3.4). It is at that point that the question *Why no nall?* can be answered and the conclusion can be transferred to **nand* (§ 7.1.3.5). A few final arguments in favour of a presupposition approach (§ 7.1.3.6 and 6 7.1.3.7) will wrap up the discussion.

7.1.3.2 Presuppositions

(365) a. All flags are green
PRESUPPOSES the set of flags is non-empty, i.c. there exist flags
b. All gnomes are fictitious
PRESUPPOSES the set of gnomes is non-empty

The effect of a presupposition is to make its carrier sentence (i.c. *all flags are green* and *all gnomes are fictitious*) ill-suited for use in any context in which the presupposition does not already hold. Since *green* is normally an extensional G-predicate – it has existing entities in its extension set –, the non-emptiness presupposition amounts to a presupposition of existence and makes *all flags are green* ill-suited for use in a context where the set of flags is empty (cf. Seuren 2002: 34). *Fictitious*, for its part, is an intensional G-predicate, i.e. a predicate with no existing entities in its extension set. Consequently, no strengthening from a bare set-theoretical nonemptiness presupposition to an ontological claim is warranted. But the purely set-theoretical non-emptiness presupposition itself stands nonetheless: the thought-up set of nonentities described by *gnomes* is presupposed to be non-empty. The assertive content which is added is that not a single element of the non-empty set described by *gnomes* is outside the set described by the adjective *fictitious*, again a set that contains no existing entities, but only virtual, intensional elements. In sum, no existing entities are involved at all in the two sets that are related in this sentence, yet set theory and the set-theoretical non-emptiness presupposition operate as usual.

7.1.3.3 The Effect of Negation

Since our interest is in the impossibility to derive **nall* from *all* by negation, we need to be in the know about the effect of negation on presuppositions.

In some cases, sentential negation with emphasis on *not* can cancel the presupposition:

(366) All flags are NOT green, there ARE no flags.

In most cases, however, no such cancelling can occur (Seuren 1985: 260-266), a point of immediate relevance for our concerns. Let us look at sentences with an intensional F-predicate *gnomes* and an extensional G-predicate *be married*. The latter imposes a presupposition of *existence* on the F-class[167]. But since the former is purely intensional or virtual, the conflicting requirements cause an intensional-extensional clash. For this reason, sentences like those in (367) are the clearest cases to show where presupposition-cancelling can and cannot occur.

[167] Cf. Seuren's (2002: 34) remark that it is not quantifiers which have existential import, but that the latter is derived "from the extensional character of extensional predicates".

(367) a. All gnomes are NOT married, there are no gnomes! (True)
PRESUPPOSITION CANCELLED
b. All gnomes are unmarried (Spurious/radically false)
PRESUPPOSES the set of gnomes is non-empty
i.c. there exist gnomes

Only sentential negation with emphatic tone (367) a. is presupposition-cancelling. When the negative is a bound morpheme, as in (367)b, the F-class non-emptiness presupposition survives. In the present case, it is a presupposition of existence due to the extensional nature of the G-predicate. There being no gnomes in this world the presupposition does not correspond with reality and hence, the sentence in question is spurious (or radically false).
The case that is crucial for **nall*, is (367)b.: *n-* is a bound morpheme (as is *un-*), and those cannot cancel the non-emptiness presupposition. This means that in **nall*, the presupposition of non-emptiness associated with *all* remains active. It is this fact which will turn out to be the source of the impossibility of **nall*. But how precisely? To answer that question, we first set up a valuation space representation for presupposition.

7.1.3.4 Presuppositions and valuation spaces

Consider the following joke:

(368)
"A hippie walks down the street; friendly person remarks:
- Young man, you've lost a shoe.
- Oh no, I've found one." (Gullberg 1997: 219)

This joke illustrates that the friendly man's use of *lose* came with a hidden presupposition: namely that the man had the shoe before.

(369) Q: The hippie lost a shoe PRESUPPOSES P: The hippie had that shoe

In other words, proper use of the carrier sentence Q is confined to a context which is already restricted by the information embodied in the presupposition P which is part of the lexical meaning of the predicate *lose*. In valuation space terms, what the above means is that the domain for demarcation of the set /Q/ - i.e. the set of possible situations in which Q is true – is not the complete universe of possible situations SIT, but its proper subset /P/, even though P is never explicitly asserted, but rather contained as a presupposition in the lexical item *lose*. Such domain restriction to a narrower section of the universe SIT is what the presupposition brings about, a point which has been made in discourse terms by several people (Gazdar 1979; Heim 1982; Seuren 1985, 2000).

7.1.3.5 Neither **nand* nor **nall*

Transferring this hypothesis to sentences with *all*, we are driven to the following picture: the nature of the non-emptiness presupposition is such that it confines the interpretation of *all* to a subset of the full universe SIT. Concretely, the presuppositional part **(i)** of

(363) has the effect or restricting the universe SIT to the domain S2-S4, so that the further demarcation of the assertion part **(ii)** of the meaning of *all* occurs within that domain, rather than within the whole universe SIT (S1-S4). In other words, any NEC-operations beyond stage 2. occur within the subset S2-S4 of the universe SIT and cannot involve S1 anymore.

(370) Stages***:*** ***1.*** (1-(F_xG+F_yG))
2. (1-(1-(F_xG+F_yG)) **[**+ (1-(F_xG+F_yG)))**]**
3. (i) (1-(1-(1-(F_xG+F_yG))) **[**+ (1-(1-(F_xG+F_yG))))**]** (ii) (1-(1- F_xG) + (1- F_yG))
4. 1- stage3

F_xG	F_yG	cell structure of the universe	***0.*** SIT = 1 *value*	***1.*** *value*	***2.*** *value*	***3.*** *value*	***4.*** ***1- stage3*** *value*
1	1	S4	*1?*	***0***	***1***	***1***	***0***
1	0	S3	*1?*	***0***	***1***	***0***	***1***
0	1	S2	*1?*	***0***	***1***	***0***	***1***
0	0	S1	*1?*	*1?*	***0***		*

/SOME/ /ALL/ [**all**] **[*n[all]]**

Now, if the domain for stages 3. and 4. is indeed restricted to S2-S4 because cell S1 is excluded by presupposition and hence no longer available, then further application of negation to stage 3. can never result in selection of S1-S3 anymore, i.e. the intended meaning content of **nall.* In sum, due to the domain restriction contributed by the presuppositional part **(i)** to the meaning of *all*, the S1-S3 meaning of **nall* does not represent an accessible meaning in natural language.

Recall that in Russell's Modern Predicate Calculus, *all* is taken not to have existential import and hence does not incorporate a Strawsonian non-emptiness presupposition. If that were correct, not only would the sentences of (367)b.-d. all have to be judged true, counter to natural language intuitions, but it would also be a mystery anew why **nall* (with the meaning S1-S3) is impossible. This indicates that Russell's hypothesis is foreign to IN-logic and natural language.

But the account is not fully satisfying yet. Why cannot **nall* be used in a presupposition-preserving way to express the meaning S2-S3? Here the answer is lexical blocking: the relevant meaning is identical to that of (exclusive) *some*, which – being a pivot (in an appropriate exclusive context) – is a stage 2. item and hence less complex than stage 4. **nall* in our bar-notational system. Consequently, the latter is blocked.

Note that a phrasal version of the meaning S2-S3 is available in the form *not all*. To explain this, there are two possible avenues. One might attribute the grammaticality of *not all* to it being phrasal, so that lexical blocking cannot exclude it. One could then claim that *not all* is a functionally useful addition to *some* when large quantities are involved: it covers the large quantities approaching *all*, leaving the smaller quantities to *some.* But there are some data from Horn (1989) which indicate that another explanation is more accurate.

(371) a. All that glisters is not gold.
b. Not all that glisters is gold
c. Tout ce qui reluit n'est pas or.
d. *Pas tout ce qui reluit est or

Not only is English (382) b synonymous with (382)a, the French equivalent of the term negation is ungrammatical (Horn 1989). This indicates to me that the *not* in (382)b. is a case of sentential negation which shows itself via negative copying on the term, rather than actual term negation. If so, the French facts are more revealing and direct negation of the universal term is impossible throughout. English *not all* merely looks like term negation. Support for this conception comes from the corresponding phrase in the proposition calculus: **not and*, which is ungrammatical. Not accidentally, I think, the sentential negation escape route is not available for this phrase (*and* is not a term in a proposition). In sum, the claim that direct negation of an entailer lexical item is impossible throughout is probably correct.

Let us now turn from *all* to *and.* Since the first part **(i)** of the meaning of *all* was identified as the presuppositional non-emptiness condition in the predicate calculus, the corresponding part **(i)** of the meaning of *and* is bound to represent the corresponding non-emptiness presupposition contributed to AND(P,Q) by OR(P,Q) in the realm of the propositional calculus.

In the present case, this means that the demarcation of the meaning of part **(ii)** of AND(P,Q) occurs within the restricted domain S2-S4 that is contributed by the presuppositional part **(i)** (= OR(P,Q)) and hence not within the whole universe SIT (S1-S4).

(372) natural language *and*; no natural language **nand*

Stages***:*** ***1.*** (1-(P+Q))
2. (1-(1-(P+Q)) **[**+ (1-(P+Q)))**]**
3. (i) (1-(1-(P+Q))) **[**+ (1-(1-(P+Q))))**]** (ii) (1-(1-P) + (1-Q))
4. 1- stage3

P	Q	cell structure of the universe	*0.* SIT = 1 *Value*	*1.* *value*	*2.* *value*	*3.*	*4.*
1	1	S4	*1?*	***0***	***1***	***1***	***0***
1	0	S3	*1?*	***0***	***1***	***0***	***1***
0	1	S2	*1?*	***0***	***1***	***0***	***1***
0	0	S1	*1?*	*1?*	***0***		*

/OR (P,Q)/ /AND (P,Q)/ **[and]** **[*nand]**

Once again, the domain for stages 3. and 4. is restricted to S2-S4 and hence cell S1 is no longer available. Consequently, further application of negation to stage 3. can never result in selection of S1-S3 anymore, i.e. of the meaning of **nand.* In sum, due to the domain restriction contributed by OR(P,Q) to the meaning of AND(P,Q), the intended meaning S1-S3 of **nand* does not represent an accessible meaning in natural language.

Note that if Russell's analysis of Modern Predicate Calculus entailer *all* is transferred to *and*, and hence strips it of its presuppositional part **(i)**, then *nand* is perfectly accessible (like *nall* was). It then has precisely the S1-S3 meaning so useful in a number of scientific contexts (e.g., nand-gates). This is yet another indication that the Russellian solution is an EX-logical one, with great value for science and engineering, but – if our analysis is correct – inaccurate as an analysis of entailers in natural language. Natural language words and scientific terms are different categories.

(373) science terminology: *and* and *nand*

Stages***:*** ***1.*** (1-(P+Q))
2. (1-(1-(P+Q)) **[**+ (1-(P+Q)))**]**
3. (i) (1-(1-(P+Q))) **[**+ (1-(1-(P+Q))))**]** (ii) (1-(1-P) + (1-Q))
4. 1- stage3

P	Q	cell structure of the universe	*0.* SIT = 1 *Value*	*1.* *value*	*2.* *value*	*3.*	*4.*
1	1	S4	*1?*	***0***	***1***	***1***	***0***
1	0	S3	*1?*	***0***	***1***	***0***	***1***
0	1	S2	*1?*	***0***	***1***	***0***	***1***
0	0	S1	*1?*	*1?*	***0***	***0***	***1***

/OR (P,Q)/ **[and]** **[nand]**

On the whole, the Strawsonian approach in terms of a non-emptiness presupposition for *all* – and by extension for *and* – is not only empirically more satisfying for natural language, but it also predicts that S1-S3 meanings can not be naturally acquired as lexical items, but only crafted artificially for scientific purposes. They are EX-logical constructs rather than part of natural IN-logic.

7.1.3.6 A Final Argument

There is an additional argument that a presupposition approach to the internal semantics of entailers is on the right track. When the notion ET was introduced in chapter 4, it was argued that it cannot be directly applied to the universe SIT to yield *and* (4.4.5.1). It is however crucially needed at stage 3. as part (ii) of the formula for *and*. Our account why ET cannot be applied at the initial stage was based on the assumption that exclusion (*1?*-…) can only apply successfully to an initial cell *if there is something that can be excluded*. (This is in line with the general observation that negatives needs something positive to apply to). I adopted this hypothesis and applied NEC and ET respectively to the initial values for P and Q. The NEC sequence (194) came out fine, since S2-S4 all have at least one positively specified (boldface) item P or Q, so that *1?* – **1** yielded 0 as required.

(374)

universe		cell structure of the universe	***0.*** SIT = 1	***1.*** (1-(P+Q))
P	Q		*value*	*Value*
1	**1**	S4	*1?*	*(1? –* **1** *=) 0*
1	0	S3	*1?*	*(1? –* **1** *=)* ***0***
0	**1**	S2	*1?*	*(1? –* **1** *=)* ***0***
0	0	S1	*1?*	*(1? –* 0 *=) 1?*

But sequence (193) failed, because there is no positively valued P or Q in S1 for subtraction to yield *1?*-**1**=0, which is however needed for successful application of ET. Indeed, the latter requires that S1 end up with a zero value, given the formula (1-((1-P)+(1-Q)). In sum, under this conception application of ET to the initial stage is impossible because S1 fails to be excluded.

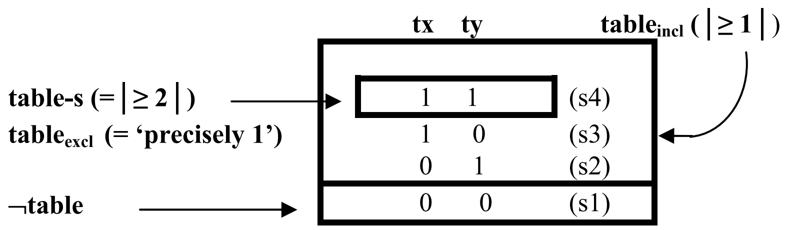
(375)

universe		cell structure of the universe	***0.*** SIT = 1	***1.*** *(1-((1-P)+(1-Q))
P	Q		*Value*	*Value*
1	1	S4	*1?*	***1?***
1	0	S3	*1?*	*(1? –* **1** *=)* ***0***
0	**1**	S2	*1?*	*(1? –* **1** *=)* ***0***
0	0	S1	*1?*	**(1? – 0 =) 1?*

Now note that if the Strawsonian hypothesis about the restriction of the universe at stage 3 to the presuppositional part S2-S4 is correct, ET can suddenly apply unproblematically at that stage, since the recalcitrant cell S1of sequence (193) is no longer part of the computation at stage 3. Hence all cells with a zero value for P or Q, namely S2 and S3, can now be excluded (as required by the formula (1-((1-P)+(1-Q))) because the other value of S2 and S3 is always 1 so that subtraction *(1? –* **1** *=)* ***0*** takes place.

In sum, the arguments in favour of universe restriction by presupposition are solid, as is their desirable effect that they exclude **nall* and **nand*. This conclusion is most welcome, since the pattern behind the **nall* and **nand* problem is very general. Recall the predicate *table* from § 6.4.2. It is easy to think of a predicate such as the S2-S4 predicate *table* and of an S1-predicate for its complement: *non-table*. Yet, it is hard and unnatural work to conceive of a S1-S3-predicate, say **ntable*, that unites every singular table and anything that fits the predicate non-table in a single set as opposed to all pluralities of tables. The latter would be the nominal counterpart of **nand* and **nall*, and it is clearly felt to be as unnatural as the latter. In other words, the solution to the nand-puzzle is more widely useful than for logical operators alone.

(376)



7.2 The negative logic of natural language

The shift of perspective in this study from a purely mathematico-logical to a cognitive logical approach has had some consequences for our implementation of Peirce's derivation of all propositional operators from joint falsehood. For expository purposes and transparency, a more manageable bar-notation representation was introduced. While easier to work with, the resulting bar-codes still looked far less "user-friendly" than the simple separate words that are the existing manifestations of the lexical meanings at the linguistic surface. But that was no great cause for concern: from a minimalist perspective, mental underground computation works with an algorithm with as few primitives as possible. This keeps the algorithm optimally small and economical, which is the kind of simplicity that is important in a computational system (cp. the underlying binarity of computation in a computer). But we do not of course converse in straight universal mentalese. One of the roles of the lexicon in natural language was argued to make the repetitive underlying elaborateness (i.c. of the bar-codes) of the lexical semantic structures expressible at the linguistic surface. Thus, lexicalisation in the sense of matching a complex underlying semantic configuration with a single designated surface label (PF-features) was claimed to make the underlying meanings expressible in a more language-particular format. Luckily, such encapsulation of information occasionally leaves traces. Compact lexicalisation does not always completely erase underlying semantic compositionality – the case of bimorphemic *n-or* was mentioned several times.

At the underlying computational level, the postulated interrelatedness of the universal bar-codes for the different operators achieved an important result: it accounted for the intuition that operators are semantically cognate elements united in a single paradigm. It also predicted that conjunction, disjunction, joint falsehood have a robust core meaning across languages, whatever variation may be latched on to the basic meanings in particular languages.

The solid patterns which were found were argued to be located in a realm of grammar below the lexical level. This level is below awareness in normal language use and hence closed to conscious interference in such contexts. It was argued that logical rules have their ultimate home in that realm and that logical primitives and operations are hard and fast and inviolable at that level. This provided an answer to a paradox that has haunted attempts to make logic natural and internalist since Kant raised it: if logic is an aspect of human psychology, so the problem went, one would not expect the rules to be violable at all. Our answer was that they actually are not at the underground level of natural concepts. In the realms of morphology, syntax and discourse rules, however, they *are* violable because their application involves conscious choice. Still, such violations are experienced as transgressions of a rule, which confirms the existence of the stable logical substrate in the Language of Thought.

Throughout the study, emphasis was put on the nature of cognitively realistic set demarcation. This line of thought confirmed the core idea that the procedure employed had to be of a negative nature, as is expressed in the subtractive exclusion of cells from a Venn-diagram. Part of the set-theoretic excursion was the insight that in order to be able to generate sets from a *single* operator, the latter has to be binary, i.e. to involve *twin selection,* disjunctive and negative. These features of set-demarcation and logical operations more generally accounted for the binary nature of most operators and had no problem explaining the unary nature of negation at the linguistic surface either. In light of all the above, the conclusion to which I am led is that standard logical operators are regular lexical items and that the natural logic of language is negative.

hie sî der reden ein ende
"Let this be the end of the story"
Hendrik van Veldeke, *Eneide*,12th century, Spalbeek.

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Index

Samenvatting in het Nederlands

Dit proefschrift beoogt een beschrijving te bieden van de lexicaal-semantische eigenschappen van de Engelse woorden *and, or, nor* en hun Nederlandse equivalenten *en, of, noch*, alsook van *all, some/any, none* en *alle, enkele, geen*. In de geschiedenis van de logica en vervolgens ook in de formele semantiek werd aan deze woorden, de standaardoperatoren van de propositie- en de predikatencalculus, vaak een speciale status toegekend. Daardoor verschilden ze wezenlijk van andere woorden en werden op een aparte manier (syncategorematisch) geïntroduceerd. Het theoretische uitgangspunt van onze studie is dat het lexicale elementen betreft die eigenlijk net zo benaderd moeten worden als de rest van het talige lexicon.

Het empirische startpunt van het onderzoek is de vaststelling in hoofdstuk een dat er een gat zit in de lexicaliseringsmogelijkheden van dit type logische operatoren in natuurlijke taal. In wellicht alle talen ter wereld bestaan er woorden voor de universeel-affirmatieve en de particulier-affirmatieve kwantoren, in het Nederlands resp. *alle* en *enkele*. Woorden voor universeel-negatieve kwantoren, zoals *geen* in het Nederlands, zijn zeldzamer dan de affirmatieve vormen, maar komen niettemin in een minderheid van talen voor. Er is echter nog geen enkele taal gesignaleerd waarin een particulier-negatieve kwantor met één enkel woord wordt gelexicaliseerd. Een vorm als **nalle* met de betekenis "niet alle", bijvoorbeeld, kan weliswaar kunstmatig worden aangemaakt als een soort gelegenheidswoord, maar in natuurlijk verworven woordenschat wordt het met de bedoelde betekenis nooit aangetroffen.

Dat hier een systematisch gat in het geding is, kan voor het Engels geïllustreerd worden door de bovenvermelde reeks *all, some/any, none* uit de aristotelische predikatencalculus (APC) te vergelijken met de overeenstemmende hoeveelheidwoorden in een domein met slechts twee elementen (APC_2) en vervolgens met corresponderende operatoren uit de propositiecalculus. Als *<u>alle</u> vlaggen zijn groen* (*all flags are green*) waar is, maar het domein vlaggen F blijft expliciet beperkt tot twee vlaggen, dan zeggen we: *<u>beide</u> vlaggen zijn groen* (*both flags are green*). En als *beide vlaggen (*$vlag_1$ en $vlag_2$*) zijn groen* waar is, kunnen we ook zeggen: *$vlag_1$ is groen <u>en</u> $vlag_2$ is groen* (*$flag_1$ is green and $flag_2$ is green*). Dezelfde redeneertrant kan worden toegepast op de particulier-affirmatieve kwantoren (zij het in een negatieve context[168]): als *I do not think that any flag is green* waar is en het domein F bestaat uit twee elementen, dan zeggen we *I do not think that either flag is green* en daarmee stemt in de propositiecalculus *I do not think that $flag_1$ is green or $flag_2$ is green* overeen. Gelijkaardige beschouwingen voor de universeel-negatieve en particulier-negatieve kwantoren leveren het volgende patroon van correspondenties op.

(377)

	logische types	APC	APC_2	PROPC
a.	universeel affirmatief	all	both	and

[168] De negatieve context is nodig omdat *either*, net als *any*, beperkt is in zijn gebruiksmogelijkheden en niet zomaar in alle contexten kan worden ingevoegd.

b.	particulier affirmatief	some/any	either	or
c.	universeel negatief	no	neither	nor
d.	*particulier negatief*	*nall	*nboth	*nand

De vier logische types a.-d. berusten op kruising van twee basisopposities: een kwantitatieve oppositie (universeel–particulier) en een kwalitatieve oppositie (affirmatief-negatief). De vormen in (377) d. zijn echter onmiskenbaar en systematisch artificieel: ofwel bestaan ze helemaal niet, ofwel alleen in een heel gespecialiseerde wetenschappelijke context. Wat dit laatste betreft: de term *nand* komt inderdaad voor in de elektrotechniek voor de beschrijving van digitale logische schakelingen. Het bestaan van die wetenschappelijke term zet het beginraadsel over het lexicale gat in *natuurlijke* taal evenwel niet op de helling. Wetenschappelijke termen worden immers niet op dezelfde natuurlijke wijze verworven als alledaagse taal, maar “artificieel” – je moet ervoor naar school of op zijn minst een doelgericht, bewust leerproces doorlopen. Ze behoren dan ook niet tot wat Chomsky “natural language” of I-taal noemt, een relatief stabiele toestand – de zogenaamde “steady state” – van de moedertaalkennis in de individuele menselijke geest (= *intern, intensioneel, individueel*), maar veeleer tot wat hij E-taal noemt, die als extern, extensioneel en sociaal wordt gedefinieerd[169]. Vermits het doel van dit taalkundige proefschrift de begrenzing is van lexicalisering in natuurlijke IN-taal, betekent de doorbreking van die grenzen in artificiële EX-taal helemaal niet dat het oorspronkelijke raadsel vervalt. Integendeel, de internalistische aard ervan is nu nog preciezer omschreven. En de precisering onderstreept tevens de hier voorgestane individueel-psychologische, mentalistische benadering van de logische intuïtie die in natuurlijke taal wordt uitgedrukt (cf. Macnamara 1986, Seuren 1998, Ludlow (in prep.)). Die “IN-logische” benadering is niet nieuw: ze stemt grotendeels overeen met de opvattingen van George Boole in *The Laws of Thought* (1854). Hij toont zich in dat boek met zijn expliciet mentalistische titel een cognitiewetenschapper *avant la lettre* (cf. Grattan-Guinness 1982 en Devlin 1997), zonder echter te vervallen in het door Frege en Husserl zo gelaakte psychologisme (Richards 1980: 30).

Het vastgestelde lexicale gat springt nog het meest in het oog wanneer de aristotelische logische relaties tussen zinstypes geometrisch worden weergegeven in het bekende Oppositievierkant van Boëthius (480-524). De hoekpunten aan de affirmatieve kant kregen bij Boëthius de labels A (universeel) en I (particulier) mee van **A**ff**I**rmo, de negatieve hoeken de labels E (universeel) en O (particulier) van n**E**g**O**. De vier zijden en de twee diagonalen leveren samen zes relaties op, maar slechts vier soorten oppositie, resp. contradictoir (= CD: P en Q kunnen noch gelijktijdig waar, noch gelijktijdig onwaar zijn), subalternantie/entailment[170] (= $\vdash$: als P waar is, is Q noodzakelijk ook

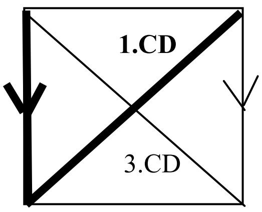
[169] In wat volgt worden de afkortingen IN-taal en EX-taal gebruikt voor dit Chomskiaanse contrast omdat de labels I en E al voor een ander, historisch veel ouder contrast vereist zijn.

[170] Er is sprake van entailment (= logische gevolgtrekking) wanneer men de verhouding bekijkt vanuit de universele zin P en stelt dat de particuliere zin Q een logisch gevolg is van P. De term subaltern wordt gebruikt als men de relatie bekijkt vanuit Q en stelt dat die particuliere zin de “ondergeschikte andere” (“sub-altern”) is t.o.v. de universele zin P. De relatie tussen de twee zinnen is evenwel in beide gevallen dezelfde.

waar), contrair (= CR: P en Q kunnen niet gelijk waar zijn, maar wel gelijk onwaar) en subcontrair (= SCR: P en Q kunnen niet gelijk onwaar zijn, wel gelijk waar).

(378) A:ALL[171] 5.CR E:NO(NE)

2. onmiddellijk gevolg ├
(subaltern affirmatief)



4. ├ onmiddellijk gevolg
(subaltern negatief)

I:SOME 6. SCR O: NOT ALL (**nall*)

In hoofdstuk twee wordt bewezen dat als de in (378) vetgedrukte CD-relatie tussen I en E en de entailment van A naar I als enige basisrelaties worden beschouwd de rest van het vierkant helemaal kan worden afgeleid en op aristoteliaanse wijze kan worden gedefinieerd in termen van onmiddellijke gevolgtrekking (├; = entailment (ENT)). Dit betekent dat de O-hoek, de hoek van het lexicaliseringsgat, alleszins geïsoleerd *kan* worden en dat het meest economische IN-logische systeem niet het uitzicht heeft van een vierkant, maar van een tweedimensionaal cartesiaans assenstelsel, hier met de I-hoek SOME als “oorsprong” en logische “spil” (*pivot*). De dubbele taak voor de overige hoofdstukken van het proefschrift bestaat erin de aard van de assen te beschrijven (hoofdstuk drie) en de uitzonderingspositie van de I-hoek zowel theoretisch te verankeren (hoofdstuk vier) als empirisch verder te onderbouwen (hoofdstukken vijf en zes). De theoretische verankering van I als spil is broodnodig, want een tweedimensionaal systeem dat op CD en ENT draait, zou in principe immers ook opgezet kunnen worden met de O-hoek als oorsprong en de relaties 3.CD en 4.├ in (378) als assen.

In hoofdstuk drie worden de twee gepostuleerde basisrelaties CD en ENT diepgaand bestudeerd, niet alleen vanuit een logisch perspectief, maar ook vanuit een verzamelingtheoretisch en een booleaans algebraïsch oogpunt. Dit gebeurt in de eerste plaats om na te gaan of een variant van deze relaties gedefinieerd kan worden op het lexicale vlak om aldus de relaties tussen lexicale items als *all, some* en *none* of *and, or* en *nor* rechtstreeks in het lexicon te kunnen beschrijven en niet via de omweg van volledige proposities. Die strategie biedt tevens een eerste inkijk in de semantische binnenbouw van die lexicale items. Het verzamelingtheoretisch perspectief levert namelijk het basisinzicht op dat het afbakenen van een verzameling altijd vertrekt vanuit een domein of universum van mogelijke leden en dus noodzakelijk een *negatief* exclusiegebeuren is. Daarbij wordt het universum (in algebraïsche termen: 1) afgeslankt tot het uiteindelijke aantal effectieve leden door uitschakeling van een aantal oorspronkelijke kandidaat-leden. Dit is de kernreden waarom een *negatieve* aanpak de meest aangewezen werkwijze is om de lexicale betekenis van standaardoperatoren en hun onderlinge relaties te ontleden.

[171] All F is G wordt verkort weergegeven als ALL; Some F is G als SOME; No F is G als NO; Not all F is G als NOT ALL.

De verzamelingtheoretische en algebraïsche kijk worden daarnaast ook gehanteerd om na te gaan hoe geschikt de twee gepostuleerde basisrelaties zijn als fundamenten van de natuurlijke IN-logica. Vooral voor de relatie entailment leidt dit tot een verfijning in de vorm van een bijkomende informativiteitsconditie. Die heeft als gevolg dat onnatuurlijke entailments (*ex necessarie falso sequitur quodlibet, verum sequitur ad quodlibet* en entailment tussen twee identieke proposities) passend als EX-logisch kunnen worden gebrandmerkt en aldus buiten de stringentere logica van de natuurlijke taal belanden.

In hoofdstuk vier wordt de lexicale decompositie uitgewerkt vanuit een binaire visie op de afbakening van verzamelingen en de Peirceaanse "joint falsehood" operator NOR (= $\downarrow$). Die laatste is een binaire operator waarvan de betekenis de versmelting (*merger*) is van externe negatie en disjunctie $\neg\vee$. Met behulp van dit instrument en de wetten van de tautologie ($P \equiv P\vee P$)[172], dubbele negatie ($P \equiv \neg\neg P$) en De Morgan ($P\wedge Q \equiv \neg(\neg P\vee\neg Q)$)) bewees Peirce dat één primitieve operator volstaat voor de klassieke propositielogica, een inzicht dat we willen benutten.

(379)

	Gebruikte middelen	Stappen in de conversie
a. NOR		$\neg(P\vee Q)$
	Merger	$P\downarrow Q$
b. NOT		$\neg P$
	Tautologie	$\neg(P\vee P)$
	Merger	$P\downarrow P$
c. OR		$P\vee Q$
	Dubbele Negatie	$\neg\neg(P\vee Q)$
	Merger	$\neg(P\downarrow Q)$
	Tautologie	$\neg((P\downarrow Q)\vee(P\downarrow Q))$
	Merger	$(P\downarrow Q)\downarrow(P\downarrow Q)$
d. AND		$P\wedge Q$
	De Morgan's Law	$\neg(\neg P\vee\neg Q))$
	Merger	$\neg P\downarrow\neg Q$
	Tautologie (tweemaal)	$\neg(P\vee P)\downarrow\neg(Q\vee Q)$
	Merger	$(P\downarrow P)\downarrow(Q\downarrow Q)$
e. NAND		$\neg(P\wedge Q)$
	De Morgan	$\neg(\neg(\neg P\vee\neg Q)))$
	Merger	$\neg(\neg P\downarrow\neg Q)$

[172] $\equiv$ = equivalentie

	Tautologie	$\neg(\neg P \downarrow \neg Q) \vee (\neg P \downarrow \neg Q)$
	Merger	$(\neg P \downarrow \neg Q) \downarrow (\neg P \downarrow \neg Q)$
	Tautologie (viermaal)	$(\neg(P \vee P) \downarrow \neg(Q \vee Q)) \downarrow (\neg(P \vee P) \downarrow \neg(Q \vee Q))$
	Merger (viermaal)	$((P \downarrow P) \downarrow (Q \downarrow Q)) \downarrow ((P \downarrow P) \downarrow (Q \downarrow Q))$

In een aantal opzichten moet de Peirceaanse analyse aangepast worden om dienst te kunnen doen voor de logica van natuurlijke taal. Een eerste belangrijke inhoudelijke aanpassing bestaat erin dat de basisoperator niet opgevat wordt als een reeds gelexicaliseerde woordvorm, maar als een prelexicale cognitieve operator waarvan de activiteit zich situeert op het niveau van de conceptuele structuur. Het is om de abstracte, universele aard van deze onderliggende operator weer te geven dat de onderstreepte Latijnse naam NEC wordt gebruikt in het proefschrift. Met deze aanpassing en een vereenvoudigende streepjescode zien de structuur van NEC en het ermee opgebouwde OR eruit als volgt:

(380)

	Peirce's notatiesysteem	Streepjes-code
a. NEC	$P \downarrow Q$	$\overline{P\ \ Q}$
b. OR	$(P \downarrow Q) \downarrow (P \downarrow Q)$	$\overline{\overline{P\ \ Q}\ \ \overline{P\ \ Q}}$

Er worden argumenten aangevoerd voor de stelling dat een hiërarchie van minstens twee toepassingen van NEC vereist is om een eerste uitdrukbare *lexicale* operator te verkrijgen, waardoor de I-hoekoperator *or*, met de semantische structuur OR van (380)b. er als meest elementaire lexicale operator uitkomt. De formule voor de lexicale operator *n-or* moet wegens de niet-lexicaliseerbaarheid van NEC complexer zijn dan de Peirceaanse in (379)a. en ook complexer als die van *or*, hetgeen trouwens blijkt uit de zichtbare morfologische structuur van het woord. Ook voor de lexicale operator *and* worden argumenten verschaft die leiden tot de conclusie dat zijn conceptuele structuur complexer is dan die van *or*. Meer bepaald wordt betoogd dat de betekenis OR als presuppositie aanwezig is in die van *and*. Vervolgens wordt die asymmetrieanalyse verruimd van de lexicale operatoren van de propositionele calculus naar de predikatenlogica, hetgeen noopt tot een behandeling van het probleem van de existentiële import. Op deze wijze wordt de meer elementaire status van operatoren in de I-hoek ten opzichte van die in de A- en de E-hoek theoretisch verankerd.

In hoofdstuk vijf wordt de spilstatus van operatoren in de I-hoek geïllustreerd aan de hand van twee asymmetrische processen van betekenisverschuiving, die respectievelijk *I→A-shift* en *I→E-shift* worden genoemd. Zo heeft de I-hoekoperator *any* een existentiële I-lezing in (381)a., maar een quasi-universele A-lezing in (381)b. De temporele I-hoekoperator *jamais* ("ooit") in (381)c., van zijn kant, heeft in de loop van de tijd een E-hoekvariant met de betekenis "nooit" gekregen.

(381) a. I: If you see anybody, give me a ring (existentieel)
b. ≈A: Any man can do that (quasi-universeel: ≈ Every man can do that)
c. I : Si JAMAIS vous changez d'avis (existentieel)
"als je *ooit* van idee verandert"
d. E: Vous avez toujours été orateur, JAMAIS philosophe (universeel negatief)
(Fén., Dial., 33 ; quoted from : Grevisse 1980: 1053)
"Je bent altijd een orator geweest, *nooit* een filosoof."

Het feit dat *any* nooit helemaal identiek wordt aan een universele operator in uitdrukkingen als (381)b. en de etymologische samenhang ervan met de particuliere betekenis "one", wijzen erop dat de I-hoeklezing de basis blijft en de quasi-universele lezing daarop geënt is. Voor de *I→E-shift* geldt een soortgelijke asymmetrie: historisch gezien zijn de operatoren die dit type shift hebben ondergaan steeds oorspronkelijke I-hoekoperatoren, nooit omgekeerd. Deze twee shifts worden geïllustreerd voor een hele reeks betekenisverschuivende I-hoekoperatoren.

In hoofdstuk zes wordt de spilstatus van I-hoekoperatoren op nog een andere wijze benaderd: als woorden minder semantische specificaties bevatten, vertonen ze namelijk doorgaans meer contextuele betekenisvariatie. Dit geldt in niet geringe mate voor I-hoekoperatoren. Op lexicaal vlak bestaat er een systematisch verschil tussen I-hoekoperatoren zoals *or* en het onbepaalde lidwoord *a* enerzijds en hun lexicaal meer gespecificeerde tegenhangers *either* en *any*. Terwijl de eerste categorie zich flexibel aanpast aan zowel veridicale als non-veridicale contexten (cf. Zwarts 1995, Giannakidou 1997; zie voorbeelden (382)a.-d.), zijn *either* en *any* rigider: ze gedijen alleen in non-veridicale contexten (382)a. en (382)c.

(382) a. If Henry is talking to [Mary or Bill/either of them] at the moment, he will be punished.
b. Henry is talking to [Mary or Bill/*either of them] at the moment.
c. If [you make [a/any mistake]], you'll be punished.
d. [Henry talked to [a/*any doctor]]

Na een illustratie van de invloed van klemtoon op de interpretatie van I-hoekoperatoren wordt een formeel model ontworpen om het betekenisonderscheid tussen het flexibele lidwoord *a* en de rigide operator *any* adequaat te vatten. Dat model blijkt identiek te zijn aan het tweedimensionale cartesiaanse assenstelsel van de voorgaande hoofdstukken.

In het zevende en laatste hoofdstuk wordt de afgelegde weg kort samengevat, waarna het O-hoekraadsel wordt opgelost. De oplossing rust op twee hypothesen. Ten eerste de in hoofdstuk 4 beargumenteerde stelling dat A-hoekoperatoren de betekenis van de corresponderende I-hoekoperator als presuppositie bevatten; ten tweede de stelling dat negatie in de vorm van een gebonden morfeem een presuppositie niet ongedaan kan maken (Seuren 2002). Als die twee stellingen kloppen, kunnen operatoren met Ohoekbetekenissen niet op natuurlijke wijze ontstaan, vermits die betekenis strijdig is met de presuppositionele inhoud van de A-operator. Ze kunnen hooguit worden aangemaakt als artificiële constructen, met bewuste doorbreking van de natuurlijke grenzen van de IN-logica.

Curriculum Vitae

Dany Jaspers werd geboren op 4 februari 1958 in Spalbeek, België. Van 1976 tot 1980 studeerde hij aan de Katholieke Universiteit van Leuven, waar hij met grote onderscheiding afstudeerde als Licentiaat in de Germaanse Filologie. In 1980-1981 werkte hij als redacteur voor het *Groot woordenboek Engels-Nederlands* van *Van Dale*. In 1981 werd hij assistent Nederlandse en Algemene Taalwetenschap aan de Universitaire Faculteiten Sint-Aloysius (UFSAL) te Brussel, de huidige K.U.Brussel. In 1983-1984 verbleef hij als visiting scholar aan het Massachusetts Institute of Technology, waarna hij tot 1987 opnieuw verbonden was aan de UFSAL/KUBrussel. Sinds 1987 is hij werkzaam bij het departement Toegepaste Taalkunde van de Hogeschool voor Wetenschap en Kunst, campus VLEKHO-Brussel, voor de vakken Engelse Taalkunde en Variatielinguïstiek. In 2003-2004 kon hij zich voltijds wijden aan dit proefschrift dankzij een opdracht als wetenschappelijk medewerker aan de K.U.Brussel.