The Generosity of Artificial Languages

A revolution in language heralded the birth of modern science. Latin was replaced by formal languages, such as algebra, born of artificial notations and practical devices like new numerals. Frits Staal argues that some of the roots of that revolution lie in Asia.

Frits Staal

Still ignored by the majority of Asian scholars who should know better, the Euro-American idea that 'science' is 'Western' has long been discarded. Long before the modern period, Asian contributions to ancient and medieval science were expressed through classical languages such as Old-Babylonian, Chinese, Sanskrit, Greek, Arabic, and Latin. In their scientific uses, some of these languages were formalised to some extent, but they were not designed to express abstract relationships in a systematic manner. They were intimately linked to different civilisations and lacked universality. What happened next and culminated during the 17th and 18th centuries was a revolution in language. The construction of formal languages grew out of natural language, artificial notations and special devices such as numerals. The replacement of Latin by such universal languages, in particular the languages of algebra, was a greater revolution than the so-called European scientific revolution.

The birth of artificial or formal languages

Some of Newton's laws provide simple examples. They were not, at first, written in an artificial form. Newton formulated his law of motion in cumbrous, ambiguous and obscure Latin. Less than a century later, it was disambiguated, clarified and formalised by Euler by making use of an artificial language. It is now taught to children as $f = ma$.

A more dramatic example provides a demonstration of the thesis that some of the roots of modern science lie in Asia. Madhava of Kerala, Southwest India, who lived around 1400 CE, invented infinite power series that are expansions of **pi** and the trigonometric functions **sine**, etc. by using methods that led to the infinitesimal calculus. Similar developments led to similar findings by Leibniz and other European mathematicians three centuries later. In her forthcoming book on the history of Indian mathematics, Kim Plofker refers to this discovery as the Madhava-Leibniz series of *pi*.

The accompanying illustration depicts, on top, the infinite power series that expresses the circumference of a circle with diameter D (i.e., two times the radius R) in Sanskrit. It is followed by a translation into English by Kim Plofker. At the bottom is the series in its modern form which is basically the same as what was written by Leibniz.

Rarity of artificial notations and absence of an artificial language go far towards explaining why modern science did not originate in India or China. Old-Babylonian, Indic, Chinese and other early forms of Asian mathematics inspired the algebra of the Arabs, but to what extent was that an artificial language? India developed a formal or artificial language for linguistics. It is now a science worldwide, but how could it have originated earlier by more than two millennia?

Jeffrey Oaks answered the first question in "Medieval Arabic Algebra as an Artificial Language." It provides our account with an important missing piece: a historical survey of algebra applicable to Arabic and European languages. Starting in the 9th century with systematic verbal solutions of equations, it reached a symbolic form in the 12th century in the western part of the Islamic world.

Brendan Gillon's "Panini's Ashtadhyayi and Linguistic Theory" gave a brief overview of Panini's grammar, showing that it could address all of the central concerns of a formal grammar, including what pertains to not only the syntax of Sanskrit but also its semantics. He then showed that three concerns that are central to current linguistic theory - compositionality, implicit arguments and anaphoric dependence - figure centrally in Panini's grammar.

vyāse vāridhinihate rūpahṛte vyāsasāgarābhihate

triśarādiviṣamasaṃkhyābhaktamṛṇaṃ svaṃ pṛthak kramāt kuryāt

labdhaḥ paridhiḥ sūkṣmo bahukṛtvā haraṇato'tisūkṣmaḥ syāt /

Add or subtract alternately the diameter multiplied by four and divided in order by the odd numbers like three, five, etc., to or from the diameter multiplied by four and divided by one.

The result is an accurate circumference. If division is repeated many times, it will become very accurate.

$$4D - \frac{4D}{3} + \frac{4D}{5} - \frac{4D}{7} + \ldots\ldots$$

Infinite Series Expansion of the Circumference of a Circle

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Frits Staal explained how the surprisingly early development of an artificial meta-language for linguistics in India is explained by early Vedic ideas about a hierarchy of languages of which the lowest is our common spoken language. He wondered "to what extent the innate faculties of language and number may be dissociated from each other and from other features of civilisation?" During the preceding workshop (of which the Proceedings are now published in The Journal of Indian Philosophy, Vol.34: 2006), Karine Chemla and Charles Burnett demonstrated that the Chinese and Latin written traditions led to greater separation between natural and artificial expressions but not to greater clarity. Does an oral tradition like the Vedic maintain a closer connection between the two innate faculties of language and number? Do artificial languages result from a fusion of the two faculties?

Generosity

The French mathematician d'Alembert wrote: "algebra is generous: she often gives more than is asked of her." It means that notations and equations achieve far more than that for which they were originally designed. A simple example is the expression (a + b) = (b + a). It applies to integers, but also to rational, real and complex numbers, then to vectors, various geometric and other figures, etc. It also applies to natural language, though there are exceptions as philosopher Gilbert Ryle pointed out: "She took arsenic and died."

An example of generosity from modern logic started in 1942 with J.C.C. McKinsey coming to Berkeley to study intuitionistic logic with Alfred Tarski. Tarski had already seen that the work would best be reformulated in algebraic terms, and so the two of them tied three topics together in "The Algebra of Topology." In the 1970's, the computer scientist Edgar F. Codd developed a method for dealing with relational databases. Later it was shown that that was another notational variant. Such unexpected generosities explain that Dirac declared of his own equation: "it is smarter than I am."

Over-generosity

Jens Hoyrup examined several examples of over-generosity. One is the extension by a 14^{th} century Italian mathematician of rules like:

$$\frac{a^4}{a^2} = a^2$$

to rules like:

$$\frac{a^2}{a^4} = \sqrt{a}$$

Such generosity is unwanted. The same holds for Cantor's unrestricted acceptance of sets as members of other sets.

These over-generosities correspond to over-generalisations in natural language. If we know the English plural *trees* we can make the plural *plants*. Children pick it up soon but may go too far as in *mans* or *sheeps*. Philosophers, European as well as Indian, have always done it – claiming, for example, that the world may be explained in terms of substances and qualities because sentences consist of subjects and predicates.

Panini's grammar is very generous. The techniques he uses to refer to groups of sounds, called "condensation" (pratyahara), are also used to refer to groups of nominal and verbal endings.

John Kadvany's "Positional Notation and Linguistic Recursion" compared ancient relationships between linguistics and mathematics to modern ones. He used Sanskrit positional number words and the formal techniques of Panini's grammar to explain how modern mathematical computation is constructed from linguistic skills and language structure.

The distinction between natural and artifical

Joachim Kurtz supplemented Jeffrey Oaks' contribution with an account of the surprising adventures of European Syllogistics - medieval reformulations of Aristotelian logic – in Late Imperial China. Since it involved the introduction of some 800 unintelligible new terms, it relied on Kanji characters found in logic textbooks imported from Japan.

Martin Stokhof's "Hand or Hammer?" discussed 'grammatical form' and 'logical form' in early 20^{th} century Euro-American analytical philosophy. Adding linguistics and the philosophy of language, he wondered whether the distinction between natural and formal languages can be maintained.

In "Can the world be captured in an equation?" Robbert Dijkgraaf discussed a variety of examples, some of them suggesting that physics benefits from the generosity of mathematics, others (especially in the quantum theory of strings) that they develop simultaneously, others again that reductionism plays a role or that a sense of playfulness or beauty is decisive.

The Indic contribution

The Indic approach to the exact sciences has generally preferred computation to theory, and so assigns a role to language, natural or artificial, different from that in European science. Roddam Narasimha showed how the best example of this approach is the Bakshali Manuscript of around 800 CE. Here computational tasks are displayed in an artificial language that is written with the help of symbols for arithmetical operations that foretell the algebraic equations of modern science. These displays did not lead to equations like the Newton/Euler $f = ma$, but their spirit survives in the famous diagrams that the self-confessed Babylonian Richard Feynman invented for doing calculations in quantum physics.

Most of the works of the Kerala school of mathematics are in Sanskrit, but one is composed in a Dravidian language. In "The First Textbook of Calculus: Yuktibhasa," P.P.Divakaran examined a Malayalam work of the mid-16th century which describes the development of infinitesimal calculus for the geometry of the circle and the sphere, together with all proofs. These proofs are written almost entirely in natural Malayalam, without the help of a formal notation or even diagrams. Divakaran presented translations of two passages to illustrate the point that the lack of an artificial language did not hinder the communication of the subtle reasoning involved in this new mathematics. He then argued that, nevertheless, an efficient artificial language is a prerequisite for abstraction and greater generality and that its absence may have played a role in preventing the Kerala work from realizing its potential.

The story of generosity has not come to an end. One afternoon in Bangalore, at the time of writing this report, the author had a long conversation with Roddam Narasimha and P.P. Divakaran, both primarily physicists, and Vidyanand Nanjundiah, who started out as a physicist but is now responsible for Molecular Reproduction and Development Genetics. He declared and illustrated that "Every structure is generous." It's a good place to stop and think again.

Robbert Dijkgraaf referred to "the great little meeting in Amsterdam" and added: "it was a gem." The event owed much of its success to the lively rulings of the chairs who included Henk Barendregt, Kamaleswar Bhattacharya, Dirk van Dalen, Fenrong Liu, Kim Plofker and Bram de Swaan. Like the *Proceedings* of the first, the papers will again be published in the *Journal of Indian Philosophy*. The present report owes much to conversations with Roddam Narasimha and P.P. Divakaran, strengthened by emails from Kim Plofker. The author thanks them all and expresses his sincere gratitude to Shri K.S. Rama Krishna of the *National Institute of Advanced Studies* at Bangalore for his generous computer and general IT assistance. <

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