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Nonzero temperature effects on antibunched photons emitted by a quantum point contact out of equilibrium

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Electrical current fluctuations in a single-channel quantum point contact can produce photons (at frequency ω close to the applied voltage $V\times e/\hbar$) which inherit the sub-Poissonian statistics of the electrons. We extend the existing zero-temperature theory of the photostatistics to nonzero temperature T. The Fano factor $\mathcal{F}$ (the ratio of the variance and the average photocount) is <1 for $T<T_c$ (antibunched photons) and >1 for $T>T_c$ (bunched photons). The crossover temperature $T_c\simeq\Delta\omega\times\hbar/k_B$ is set by the bandwidth $\Delta\omega$ of the detector, even if $\hbar\Delta\omega\ll eV$. This implies that narrow-band detection of photon antibunching is hindered by thermal fluctuations even in the low-temperature regime where thermal electron noise is negligible relative to shot noise.

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I. INTRODUCTION

It is a celebrated result of Glauber that classical fluctuations of the electrical current $I(t)$ produce photons with Poisson statistics.[1] The variance Var n of the number n of photons detected in a time $t_{\rm det}$ is then equal to the mean $\langle n\rangle$. The photons produced by a classical current thus behave as independent classical particles. Quantum fluctuations of the current (with $I(t)$ and $I(t')$ noncommuting operators) change the photostatistics. The bosonic nature of the photons would naturally lead to photon bunching, with Var $n>\langle n\rangle$. Photon antibunching, with Var$<\langle n\rangle$, is also possible, if the photons can somehow inherit the sub-Poissonian statistics of the electrons.[2] One then speaks of nonclassical light.[3]

While nonclassical light emitted by a quantum mechanical current has not yet been observed, the theory is well developed[4–6] and there is an active experimental search.[7,8] Nonclassical light emitted by a quantum conductor such as a quantum point contact[9] would occur in a continuous range of GHz frequencies, in contrast to the discrete frequencies produced by electronic transitions in quantum dots or quantum wells.[10,11] Various methods of measuring the photostatistics have been developed, such as detection by means of photo-assisted tunneling,[12–14] or by means of the Hanbury-Brown-Twiss effect.[8,15]

The theoretical prediction[5] is that photons emitted by a single-channel quantum point contact should have a Fano factor $\mathcal{F}=\text{Var}\, n/\langle n\rangle$ smaller than unity at zero temperature, for frequencies ω close to the applied voltage $V\times e/\hbar$. More specifically,

$$\mathcal{F}=1-\tfrac{2}{3}(\gamma_0\Delta\omega)\tau(1-\tau), \tag{1.1}$$

for photodetection with efficiency γ_0 in the frequency interval $(eV/\hbar-\Delta\omega, eV/\hbar)$. The transmission probability τ through the quantum point contact is assumed to be energy independent on the scale of eV. Equation (1.1) is derived in the limit of weak coupling ($\gamma_0\Delta\omega\ll1$) of electrons to photons, so that the deviations from Poisson statistics remain small. It is also assumed that the photons can be detected individually, see Ref. 6 for an alternative detection scheme.

It is the purpose of the present paper to extend the theory of Ref. 5 to nonzero temperatures, in order to identify the conditions on the temperature needed to observe the photon antibunching. Clearly, photon bunching should take over when the electrical shot noise drops below the thermal noise, which happens when k_BT becomes larger than eV. While $k_BT<eV$ is the condition for photon antibunching in the case of wide-band detection, a more stringent condition $k_BT<\hbar\Delta\omega$ holds for narrow-band detection.

More precisely, we obtain a crossover temperature $T_c\approx\hbar\Delta\omega/4k_B$ at which $\mathcal{F}=1$ for $\Delta\omega\ll eV$. In this low-temperature regime shot noise still dominates over thermal noise, yet the photon antibunching is lost. One qualitative way to understand this is, is to compare the coherence time $t_{\rm coh}\simeq1/\Delta\omega$ of the detected radiation with the coherence time $t_T\simeq\hbar/k_BT$ of thermally excited electron-hole pairs. For $t_{\rm coh}>t_T$ the detected photons result from many uncorrelated electron-hole recombination events, and the one-to-one relationship between electron and photon statistics is lost.

In the next section, we give the nonzero temperature generalization of the theory of Ref. 5, and then in Sec. III, we specialize to the shot-noise regime $k_BT\ll eV$. General results in both the shot noise and thermal noise regimes are presented in Sec. IV. Technical details are summarized in Appendixes A and B.

II. GENERATING FUNCTION AT NONZERO TEMPERATURE

We seek the non-zero-temperature generalization of the formula[5]

$$F(\xi)=\left\langle\prod_{m=1}^{N}\text{Det}(1+T_m[e^{Z}e^{Z^\dagger}-1])\right\rangle, \tag{2.1}$$

for the factorial-moment generating function $F(\xi)$ of the photocount. We first introduce the notation and then present the required generalization.

The photons are produced by time-dependent current fluctuations in a quantum point contact, characterized by transmission eigenvalues $T_1, T_2, \ldots T_N$, with N the number of propagating electronic modes (counting both orbital and spin degrees of freedom). The current flows between two reservoirs, with Fermi functions

$$f_L(\varepsilon) = (1 + \exp[(\varepsilon - eV - E_F)/k_B T])^{-1}, \tag{2.2}$$

$$f_R(\varepsilon) = (1 + \exp[(\varepsilon - E_F)/k_B T])^{-1}. \tag{2.3}$$

The current fluctuations can be due to thermal noise (at temperature T) or due to shot noise (at a voltage V applied over the point contact). We take the transmission eigenvalues T_m as energy independent in the range $\max(eV, k_B T)$ near the Fermi energy E_F.

The photons are detected during a time t_{det} in a narrow frequency interval $\Delta\omega$ around frequency Ω, as determined by the detection efficiency $\gamma(\omega)$. Antibunching of the photons requires that Ω is tuned to the applied voltage, $\Omega \simeq eV/\hbar$. (In the following we set $\hbar$ and e equal to unity.)

The average $\langle \cdots \rangle$ in Eq. (2.1) indicates a Gaussian integration over the complex numbers z_p,

$$\langle \cdots \rangle = \prod_p \frac{\gamma_p}{\pi} \int d^2 z_p e^{-\gamma_p |z_p|^2} \ldots . \tag{2.4}$$

The matrix Z has elements $Z_{pp'} = \xi^{1/2} z_{p-p'} \gamma_{p-p'}$, depending only on the difference of the indices p and p'. This difference represents the discretized frequency $\omega_{p-p'} = (p-p') \times 2\pi/t_{\text{det}}$ of a photon emitted by an electronic transition from energy ε_p to $\varepsilon_{p'}$ and detected with efficiency $\gamma_{p-p'} = (2\pi/t_{\text{det}})\gamma(\omega_{p-p'})$. Since $\gamma(\omega) \equiv 0$ for $\omega \leq 0$, the matrix Z is a lower-triangular matrix. The discretization of frequency and energy is eliminated at the end of the calculation, by taking the limit $t_{\text{det}} \to \infty$.

The expansion

$$F(\xi) = \sum_{k=0}^{\infty} \frac{\xi^k}{k!} \langle n^k \rangle_f \tag{2.5}$$

of $F(\xi)$ in powers of ξ gives the factorial moments $\langle n^k \rangle_f = \langle n(n-1)(n-2)\cdots(n-k+1) \rangle$ of the number of detected photons. Antibunching means that the variance of the photocount $\text{Var}\, n = \langle n^2 \rangle - \langle n \rangle^2$ is smaller than the average, or equivalently that the Fano factor $\mathcal{F} = \text{Var}\, n / \langle n \rangle < 1$.

As outlined in Appendix A, at nonzero temperature we have instead of Eq. (2.1) the generating function

$$F(\xi) = \left\langle \prod_{m=1}^{N} \text{Det} \begin{pmatrix} 1 + T_m f_L (e^{Z} e^{Z^\dagger} - 1) & \sqrt{T_m(1-T_m)} f_L (e^{-Z^\dagger} - e^{Z}) \\ \sqrt{T_m(1-T_m)} f_R (e^{-Z} - e^{Z^\dagger}) & 1 + T_m f_R (e^{-Z} e^{-Z^\dagger} - 1) \end{pmatrix} \right\rangle. \tag{2.6}$$

The Fermi function $f_L(\varepsilon)$ in the left electronic reservoir is contained in the diagonal matrix f_L, with elements $(f_L)_{pp'} = \delta_{pp'} f_L(\varepsilon_p)$, $\varepsilon_p = p \times 2\pi/t_{\text{det}}$. Similarly, the Fermi function $f_R(\varepsilon)$ in the right reservoir is contained in the diagonal matrix f_R.

Following the steps in Appendix A, the expression (2.6) can be reduced to the more compact form

$$F(\xi) = \left\langle \prod_{m=1}^{N} \text{Det}(1 + T_m [\bar{f}_R e^{Z^\dagger} f_L - f_R e^{-Z} \bar{f}_L] \mathcal{M}) \right\rangle, \tag{2.7}$$

with the definitions $\bar{f}_L = 1 - f_L$, $\bar{f}_R = 1 - f_R$, $\mathcal{M} = e^{Z} - e^{-Z^\dagger}$. The zero-temperature limit [Eq. (2.1)] follows from Eq. (2.7) by setting $f_L = 1$, $f_R = 0$ in the energy interval $E_F < \varepsilon < E_F + V$. (There are no current fluctuations outside of this energy interval for $T = 0$.)

III. SHOT NOISE REGIME

The result [Eq. (2.7)] holds for any temperature, provided that the energy dependence of the transmission eigenvalues may be neglected. In particular, it describes both thermal noise and shot noise. A simpler formula is obtained in the shot noise regime $k_B T \ll V$. Thermal noise can then be neglected and only the finite temperature effects on the shot noise are retained. We assume $\Delta\omega \ll \Omega \simeq V$, so even if $k_B T \ll V$, the relative magnitude of $\Delta\omega$ and $k_B T$ is still arbitrary.

A. Generating function

The first simplification in this regime is that we may set $f_R e^{-Z} \bar{f}_L \to 0$, since $f_R(\varepsilon) \bar{f}_L(\varepsilon') \to 0$ for $\varepsilon' \leq \varepsilon$. Equation (2.7) reduces to

$$F(\xi) = \left\langle \prod_{m=1}^{N} \text{Det}(e^{-Z^\dagger} + T_m f_L \mathcal{M} \bar{f}_R) \right\rangle, \tag{3.1}$$

where we have multiplied by $\text{Det}\, e^{-Z^\dagger} = 1$.

The second simplification is that we can ignore energies separated by pV with $p \geq 2$, because V is the largest energy scale in the problem. Since Z^p and $Z^{\dagger p}$ connect energies separated by $p\Omega \simeq pV$, we may set $Z^p, Z^{\dagger p} \to 0$ for $p \geq 2$. From Eq. (3.1) we arrive at

$$F(\xi) = \left\langle \prod_{m=1}^{N} \text{Det}(1 - Z^\dagger + T_m f_L (Z + Z^\dagger) \bar{f}_R) \right\rangle. \tag{3.2}$$

Following the steps in Appendix B, the determinant may be rewritten in the more convenient form (bilinear in $Z, Z^\dagger$),

$$F(\xi) = \left\langle \prod_{m=1}^{N} \mathrm{Det}(1 + T_m(1 - T_m) f_L Z \bar{f}_R Z^\dagger) \right\rangle. \tag{3.3}$$

B. Moment expansion

The generating function (3.3) is of the form $F(\xi) = \Pi_m \mathrm{Det}(1+X_m)$ with X_m of order ξ. An expansion in powers of ξ can be obtained by starting from the identity

$$\prod_m \mathrm{Det}(1 + X_m) = \exp\left[\sum_m \mathrm{Tr}\ \ln(1 + X_m)\right], \tag{3.4}$$

and expanding in turn, the logarithm and the exponential. Up to second order in ξ we have the expansion

$$F(\xi) = 1 + \left\langle \sum_m \mathrm{Tr}\ X_m \right\rangle - \frac{1}{2}\left\langle \sum_m \mathrm{Tr}\ X_m^2 \right\rangle + \frac{1}{2}\left\langle \left(\sum_m \mathrm{Tr}\ X_m\right)^2 \right\rangle + \mathcal{O}(\xi^3), \tag{3.5}$$

from which we can extract the first two factorial moments,

$$F(\xi) = 1 + \xi\langle n\rangle + \frac{1}{2}\xi^2(\langle n^2\rangle - \langle n\rangle) + \mathcal{O}(\xi^3). \tag{3.6}$$

We perform the Gaussian averages and obtain the average photocount $\langle n\rangle$ and the variance $\mathrm{Var}\ n = \langle n^2\rangle - \langle n\rangle^2$ in the shot noise regime,

$$\langle n\rangle = \frac{t_{\mathrm{det}}}{2\pi} S_1 \int d\omega \gamma(\omega) \int d\varepsilon f_L(\varepsilon + \omega) \bar{f}_R(\varepsilon), \tag{3.7}$$

$$\begin{aligned}\mathrm{Var}\ n = \langle n\rangle &+ \frac{t_{\mathrm{det}}}{2\pi} S_1^2 \int d\omega \left[\gamma(\omega) \int d\varepsilon f_L(\varepsilon + \omega)\bar{f}_R(\varepsilon)\right]^2 \\ &- \frac{t_{\mathrm{det}}}{2\pi} S_2 \int d\varepsilon \left[f_L(\varepsilon) \int d\omega \gamma(\omega) \bar{f}_R(\varepsilon - \omega)\right]^2 \\ &- \frac{t_{\mathrm{det}}}{2\pi} S_2 \int d\varepsilon \left[\bar{f}_R(\varepsilon) \int d\omega \gamma(\omega) f_L(\varepsilon + \omega)\right]^2. \end{aligned} \tag{3.8}$$

We have defined

$$S_p = \sum_m [T_m(1 - T_m)]^p. \tag{3.9}$$

Since the two reservoirs are at the same temperature, we can write $f_L(\varepsilon) = f(\varepsilon - V - E_F)$ and $\bar{f}_R = f(E_F - \varepsilon)$ in terms of a single Fermi function

$$f(\varepsilon) = (1 + e^{\varepsilon/k_B T})^{-1}. \tag{3.10}$$

We abbreviate $\Gamma(\varepsilon,\omega) = \gamma(\omega) f(\varepsilon) f(\omega - \varepsilon - V)$ and can then write Eqs. (3.7) and (3.8) in the compact form

$$\langle n\rangle = \frac{t_{\mathrm{det}}}{2\pi} S_1 \int d\omega \int d\varepsilon \Gamma(\varepsilon,\omega), \tag{3.11}$$

$$\mathrm{Var}\ n = \langle n\rangle + \frac{t_{\mathrm{det}}}{2\pi} \int d\omega \int d\varepsilon \Gamma(\varepsilon,\omega) \times \left[S_1^2 \int d\varepsilon' \Gamma(\varepsilon',\omega) - 2S_2 \int d\omega' \Gamma(\varepsilon,\omega')\right]. \tag{3.12}$$

The difference $\mathrm{Var}\ n - \langle n\rangle$ contains a positive term $\propto S_1^2$ and a negative term $\propto S_2$. The sign of this difference determines whether there is bunching or antibunching of the detected photons.

C. Crossover from antibunching to bunching

To investigate the crossover from antibunching to bunching with increasing temperature, we take a block-shaped response function

$$\gamma(\omega) = \begin{cases} \gamma_0 & \text{if} \quad V - \Delta\omega < \omega < V \\ 0 & \text{otherwise.} \end{cases}, \tag{3.13}$$

In the low-temperature regime $k_B T \ll \Delta\omega$ the function $\Gamma(\varepsilon,\omega)$ then has a block shape as well and we recover the results

$$\langle n\rangle = \frac{t_{\mathrm{det}}\Delta\omega}{2\pi} \gamma_0 \Delta\omega \frac{1}{2} S_1, \tag{3.14}$$

$$\mathrm{Var}\ n - \langle n\rangle = \frac{t_{\mathrm{det}}\Delta\omega}{2\pi} (\gamma_0\Delta\omega)^2 \frac{1}{3}(S_1^2 - 2S_2) \tag{3.15}$$

of Ref. 5. These correspond to a Fano factor

$$\mathcal{F} = 1 + \tfrac{2}{3}\gamma_0\Delta\omega(S_1 - 2S_2/S_1). \tag{3.16}$$

For a single-channel conductor $S_2 = S_1^2$, so there is antibunching ($\mathcal{F} < 1$) at low temperatures.

At high temperatures $k_B T \gg \Delta\omega$, but still in the shot-noise regime $k_B T \ll V$, we may substitute $\Gamma(\varepsilon,\omega) \to -\gamma(\omega) k_B T df(\varepsilon)/d\varepsilon$ into Eqs. (3.11) and (3.12), which gives

$$\langle n\rangle = \frac{t_{\mathrm{det}}\Delta\omega}{2\pi} \gamma_0 k_B T S_1, \tag{3.17}$$

$$\mathrm{Var}\ n - \langle n\rangle = \frac{t_{\mathrm{det}}\Delta\omega}{2\pi} (\gamma_0 k_B T)^2 S_1^2. \tag{3.18}$$

The Fano factor

$$\mathcal{F} = 1 + \gamma_0 k_B T S_1 \tag{3.19}$$

is now >1—hence, there is photon bunching.

The crossover temperature T_c, at which $\mathcal{F} = 1$, can be calculated numerically from Eqs. (3.11) and (3.12). In the single-channel case, when $S_2 = S_1^2$, we find

$$k_B T_c \approx 0.25\Delta\omega. \tag{3.20}$$

The crossover is shown graphically in Fig. 1.

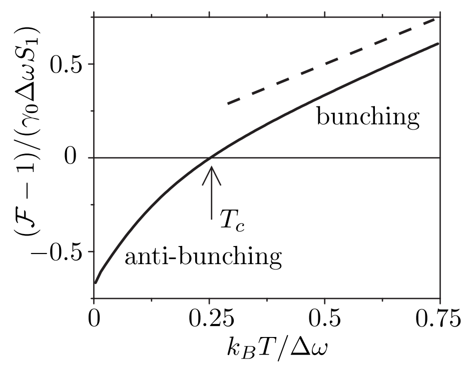

FIG. 1. Crossover with increasing temperature from antibunching (Fano factor $\mathcal{F}<1$) to bunching ($\mathcal{F}>1$) of the photons produced by a single-channel quantum point contact in the shot noise regime ($k_BT \ll V$). The solid curve is calculated from Eqs. (3.11)–(3.13). The dashed line is the asymptote [Eq. (3.19)]. The crossover temperature T_c from Eq. (3.20) is indicated.

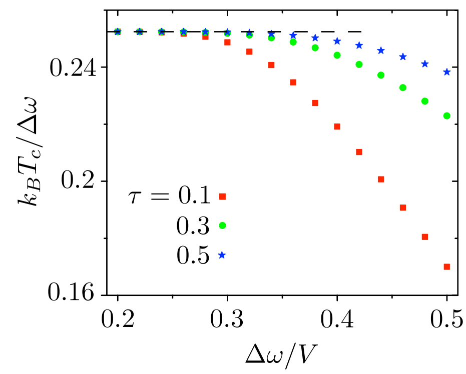

FIG. 3. (Color online) Dependence of the crossover temperature T_c (at which $\mathcal{F}=1$) on the bandwidth $\Delta\omega$. The points are calculated from Eq. (2.7) for three values of the single-channel transmission probability τ. For $\Delta\omega \ll V$ all points converge to the shot noise limit [Eq. (3.20)] (dashed line).

IV. BEYOND THE SHOT-NOISE REGIME

In the previous section we assumed $k_BT \ll V$ (shot noise regime). For arbitrary relative magnitude of k_BT and V, the general formula (2.7) can be used. With the help of Eq. (3.4), this general expression of the form $\mathrm{Det}(1+X)$ was expanded to second order in powers of ξ. In this case however, since $X=\mathcal{O}(\sqrt{\xi})$, terms up to order X^4 had to be retained. The first two moments of n are obtained as integrals over energy and frequency, similar to Eqs. (3.11) and (3.12) but containing many more terms in the integrands. The results shown in Figs. 2 and 3 are for the case $N=1$, $T_1=\tau$ of a single channel, and for the box-shaped response function (3.13).

As expected, all curves converge to the shot-noise results when $k_BT \ll V$ (shown dashed). At higher temperatures, the Fano factor lies above the shot noise limit due to the appearance of thermal noise. The temperature T_c at which antibunching crosses over into bunching, so when $\mathcal{F}=1$, follows the shot-noise limit [Eq. (3.20)] for narrow-band detection ($\Delta\omega \ll V$). With increasing bandwidth, T_c drops below the shot noise limit, in particular for small transmission probability τ. For $\tau=0.5$ the shot-noise limit remains accurate even for bandwidths $\Delta\omega$ as large as $V/2$.

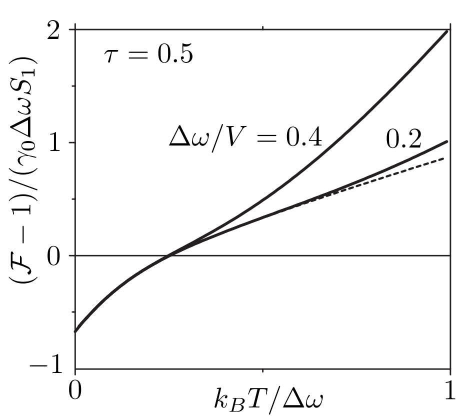

FIG. 2. Same as Fig. 1, but now without making the restriction to the shot noise regime (so without assuming $k_BT \ll V$). The two solid curves are calculated from Eq. (2.7) for two values of $\Delta\omega/k_BT$ (both for the single-channel case with transmission probability $\tau=0.5$). Both curves converge to the shot noise result at low temperatures (shown dashed).

V. CONCLUSION

In conclusion, we have investigated the effects of a nonzero temperature on the degree of antibunching of photons produced by current fluctuations in a quantum point contact. Antibunching crosses over into bunching as a result of thermal noise in the point contact, but this is not the dominant effect in the case of narrow-band detection. In that case, the finite coherence time of electron-hole pairs governs the transition from photon antibunching to photon bunching, which occurs at a temperature $k_BT_c \simeq \Delta\omega$ even if $k_BT_c \ll V$ (so even if thermal noise is negligible relative to shot noise).

The optimal conditions for the observation of antibunched photons are reached for a bandwidth $\Delta\omega \approx V/2$ and a transmission probability $\tau \approx 1/2$ through a (spin-resolved) single-channel quantum point contact. In that case $k_BT_c \approx V/8$ has the largest value at any given applied voltage. The currently available[8] detection range ($4\ \mathrm{GHz} < \omega < 8\ \mathrm{GHz} \Rightarrow \Delta\omega = 4\ \mathrm{GHz} = V/2$), should make it possible to detect antibunching at temperatures below $1\ \mathrm{GHz} \cong 50\ \mathrm{mK}$.

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APPENDIX A: DERIVATION OF THE GENERATING FUNCTION AT NONZERO TEMPERATURE

We briefly describe how the analysis of Ref. 5 can be generalized to nonzero temperatures, in order to arrive at Eq. (2.6). Referring to the equations in that paper, the first equation which changes is Eq. (5) of Ref. 5, which now reads

$$F(\xi) = \langle e^{-a^\dagger DZa} e^{b^\dagger DZb} e^{b^\dagger DZ^\dagger b} e^{-a^\dagger DZ^\dagger a} \rangle. \tag{A1}$$

The four factors correspond to the four current operators that need to be taken into account: $I_{\text{in}}^\dagger$, $I_{\text{out}}^\dagger$, I_{out}, and I_{in}.

The operator $a^\dagger$ creates an incoming electron, while $b^\dagger$ creates an outgoing electron. The matrix D projects on the right lead, where the current is evaluated. (Since D commutes with Z, we can write DZ instead of DZD.) One can relate $b = Sa$, with S the unitary scattering matrix, so one can write the entire generating function in terms of the operators a. The expectation value $\langle \cdots \rangle$ is both an expectation value over the fermion operators a, as well as the average over the Gaussian variables $Z, Z^\dagger$.

Following the steps of Ref. 5, we calculate the expectation value of the fermion operators by means of the identity

$$\left\langle \prod_n e^{a^\dagger A_n a} \right\rangle = \text{Det}(1 + AB), \tag{A2}$$

$$A = \left(\prod_n e^{A_n} \right) - 1, \quad B_{ij} = \langle a_j^\dagger a_i \rangle. \tag{A3}$$

We have $B_{ij} = \delta_{ij} f_i$, with f_i the Fermi occupation number in channel i. The matrix A is given by $A = e^X e^Y e^{Y^\dagger} e^{X^\dagger} - 1$, with $X = -DZ$ and $Y = S^\dagger DZS$. Notice that $X^p = D(-Z)^p$ and $Y^p = S^\dagger DZ^p S$.

We now make the assumption of an energy independent scattering matrix, so $S, S^\dagger$ commute with $Z, Z^\dagger$. The determinant is invariant under a change of basis, and by working in the eigenchannel basis we can reduce S to a 2×2 matrix S_m for each eigenchannel,

$$S_m = \begin{pmatrix} \sqrt{1 - T_m} & \sqrt{T_m} \\ \sqrt{T_m} & -\sqrt{1 - T_m} \end{pmatrix}, \tag{A4}$$

with T_m, $m = 1, 2, \ldots N$ the transmission eigenvalue. The matrix structure of f, D, and Z in this basis is

$$f = \begin{pmatrix} f_L & 0 \\ 0 & f_R \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad Z = \begin{pmatrix} Z & 0 \\ 0 & Z \end{pmatrix}. \tag{A5}$$

Substitution of Eqs. (A2)–(A5) into Eq. (A1) leads after some algebraic manipulations to the result [Eq. (2.6)].

The determinant in Eq. (2.6) can be reduced by means of the folding identity

$$\text{Det}\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \text{Det}\, M_{11}\, \text{Det}(M_{22} - M_{21} M_{11}^{-1} M_{12}), \tag{A6}$$

leading to

$$F(\xi) = \left\langle \prod_{m=1}^N \text{Det}[1 + T_m f_L(e^Z e^{Z^\dagger} - 1)] \text{Det}(1 + T_m f_R(e^{-Z} e^{-Z^\dagger} - 1) - T_m(1 - T_m) f_R(e^{-Z} - e^{Z^\dagger})[1 + T_m f_L(e^Z e^{Z^\dagger} - 1)]^{-1} f_L(e^{-Z^\dagger} - e^Z)) \right\rangle. \tag{A7}$$

We continue the reduction of the determinant, using first the identity

$$[1 + T_m f_L(e^Z e^{Z^\dagger} - 1)]^{-1} f_L(e^{-Z^\dagger} - e^Z) = -f_L(e^Z e^{Z^\dagger} - 1)[1 + T_m f_L(e^Z e^{Z^\dagger} - 1)]^{-1} e^{-Z^\dagger}, \tag{A8}$$

then multiplying the determinant by $\text{Det}\, e^{Z^\dagger} = 1$, and finally combining the product of three determinants into a single determinant. In this way we eliminate the matrix inversion, arriving at

$$\begin{aligned} F(\xi) &= \left\langle \prod_{m=1}^N \text{Det}([1 + T_m f_R(e^{-Z} e^{-Z^\dagger} - 1)] e^{Z^\dagger} [1 + T_m f_L(e^Z e^{Z^\dagger} - 1)] + T_m(1 - T_m) f_R(e^{-Z} - e^{Z^\dagger}) f_L(e^Z e^{Z^\dagger} - 1)) \right\rangle \\ &= \left\langle \prod_{m=1}^N \text{Det}(1 + T_m[(1 - f_R) e^{Z^\dagger} f_L - f_R e^{-Z}(1 - f_L)](e^Z - e^{-Z^\dagger})) \right\rangle. \end{aligned} \tag{A9}$$

This is Eq. (2.7) in the main text.

APPENDIX B: DERIVATION OF THE GENERATING FUNCTION IN THE SHOT NOISE REGIME

Starting from the expression (3.2) for the generating function in the shot noise regime $k_B T \ll V$, we give the steps required to arrive at the bilinear form (3.3). We group terms with Z and with $Z^\dagger$ in the matrices $A_m = T_m f_L Z \bar{f}_R$ and $B_m = T_m f_L Z^\dagger \bar{f}_R - Z^\dagger$, so that Eq. (3.2) can be written as

$$F(\xi) = \left\langle \prod_{m=1}^N \text{Det}(1 + A_m + B_m) \right\rangle. \tag{B1}$$

Because energies separated by V^p with $p \geq 2$ can be discarded, we may set $A_m^2 \to 0$, $B_m^2 \to 0$. For any pair of matrices A, B which square to zero, one has the identity

$$\mathrm{Det}(1+A+B)=\mathrm{Det}(1-AB). \tag{B2}$$

This leads to

$$F(\xi)=\left\langle \prod_{m=1}^{N} \mathrm{Det}(1+T_m Z\bar{f}_R Z^\dagger f_L - T_m^2 Z\bar{f}_R f_L Z^\dagger \bar{f}_R f_L) \right\rangle. \tag{B3}$$

Eq. (3.3) follows by noting that $Z\bar{f}_R f_L \to Z\bar{f}_R$ for $k_B T \ll \Omega \simeq V$, since the Fermi function f_L in this term is evaluated at energies near E_F, where it can be replaced by unity. Similarly $Z^\dagger \bar{f}_R f_L \to Z^\dagger f_L$, since $\bar{f}_R$ is evaluated at energies near E_F+V where it can be replaced by unity.

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