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Harmonic Duality

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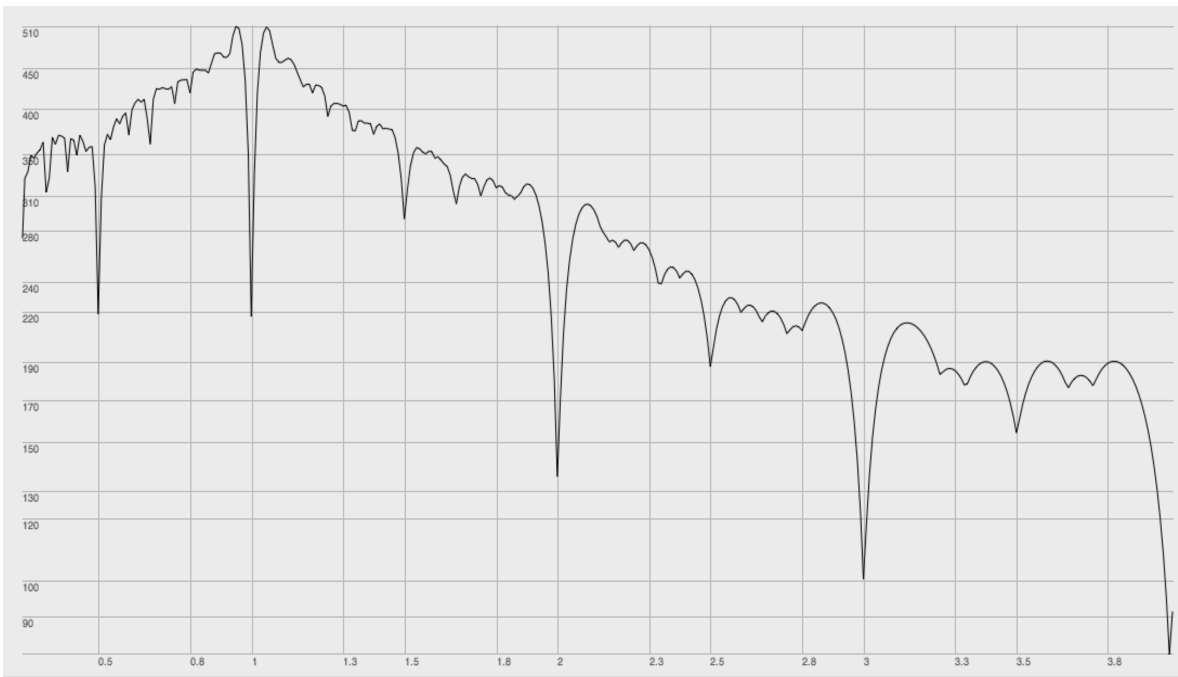
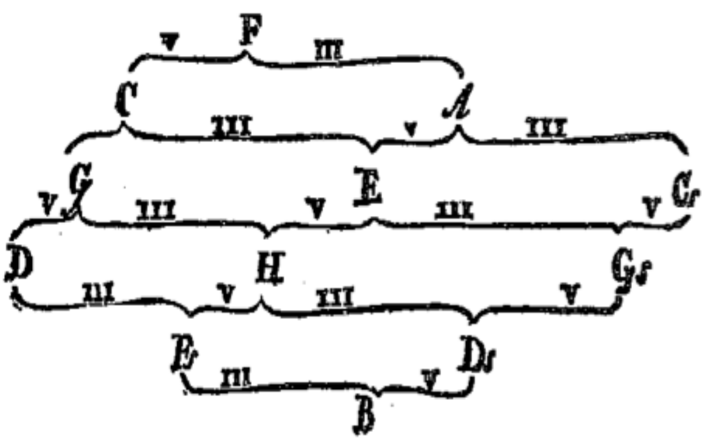
Juan Sebastián Lach Lau





Harmonic Duality

From interval ratios and pitch distance to spectra and sensory dissonance

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Dit proefschrift is geschreven als een gedeeltelijke vervulling van de vereisten voor het doctoraatsprogramma docARTES. De overblijvende vereiste bestaat uit een demonstratie van de onderzoeksresultaten in de vorm van een artistieke presentatie.

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Abstract

Dissonance curves are the starting point for an investigation into a psychoacoustically informed harmony. The main research hypothesis, harmonic duality, proposes an understanding of harmony as consisting of two independent but intertwined aspects operating simultaneously, proportionality and linear pitch distance. The former is related to intervallic characters, the latter to 'high', 'low', 'bright' and 'dark', therefore to timbre. The ideas and outcomes of this study proceed from the development of tools for algorithmic composition which extract pitch materials from sound signals, analyzing them according to their timbral and harmonic properties, and putting them into motion through diverse rhythmic and textural procedures. The tools, the music making and the reflections derived from their use offer fertile ideas for the generation of instrumental scores, electroacoustic soundscapes and interactive live-electronic systems.

This exploration of contemporary harmony begins with a review of scientific accounts of pitch perception read through the lens of harmonic duality. To restore it from the ossified rules of conservatory harmony it also delves into Greek harmonics and Pythagoreanism. Describing the development and musical uses of the algorithmic tools, the research puts forward several compositional strategies that connect the timbral qualities of sounds to compositional pitch materials. Atonality and the modernist turn are also visited in order to attest to the becoming continuous of compositional materials during the twentieth century and to make a call for a renewed attention to proportions and discrete elements. This leads to a detailed practice-based compositional account of harmonic space, a mathematical structure that gives away information about interval ratios. Other approaches to pitch materials are also explored, both stemming from the author's work ('harmonic fields') and from several pioneers in the field, to conclude with some speculations on what harmony can mean in more abstract and general terms beyond music.

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I dedicate this work to my nephews Julián and Emiliano as well as to the memory of Rita Guerrero whose example of living and making music I will always strive to reach.

Introduction

> "If we are not to be faced eventually with the splitting apart of the art of music into an art of pitched sound and a separate art of non-pitched sounds, we must greatly refine our understanding of pitch relationships. Such understanding must be not only theoretical (intellectual) but also practical (audible by ear in actual musical compositions). The relations between component pitches of very complex sounds include a great many with which our traditional pitch system is powerless to deal. We are, therefore, accustomed to hear, more and more, relationships which demand a more comprehensive vocabulary of pitch intervals than we now have." (Ben Johnston, *Proportionality and Expanded Musical Pitch Relationships*[1]).

This study will search for the qualities that lie at the heart of diverse types of sounds in terms of their periodicities and spectral properties in order to extend and build upon them at several temporal levels. The point of departure is a compositional practice influenced and augmented by perceptual theories of sound, with the aim of providing schemes for creating and manipulating musical ingredients, mostly from the perspective of pitch. Much of it has been done with computer assistance, understood as a laboratory for experimentation and development, using it to generate, classify and understand these *materials*, investigating their relations to the *methods* that set them moving in time and into musical *forms*. I am interested in algorithmic composition and harmonic discovery.

Much of the work stems from the implementation and use of 'dissonance curves'. Based on psychoacoustic models of cochlear pitch processing and the phenomenon of roughness, they provide pitch sets that are perceptually attuned to timbral aspects of sound objects, intervals that are concordant − compatible − with spectral qualities of sounds. They started out as a curiosity, as something I wanted to listen to, but have become the stepping stone from which to delve more deeply into harmony, raising questions into what can be understood of it nowadays, beyond psychoacoustics. Apart from supplying ample materials for composing, they are also a departure point for a reflexive consideration of microtonality and harmony in general. The materials they produce are analyzed and understood from the standpoint of the main hypothesis of the research: harmonic duality, or the fact that harmony has two facets embedded within it, one of linear pitch distance and another of proportionality. Linear pitch distance is associated with 'high', 'low', 'dark' and 'bright, that is, with the timbral character of the sounds that carry the pitch; proportionality is associated with the perceptual qualities of intervals, their identities and relations. Even though it is possible to switch between both types of pitch representations (one is the inverse logarithmic function of the other), each pertains to a different musical realm, conveying quite different information. We will go into much more detail regarding the properties and features of the duality and its entwinement as we go along.

The process of this research is speculative or experimental in the sense that the theorization and the musical work are not separate but intertwined. The questions posed spring from the musical involvement at the same time as they are used to produce unforeseen music. The intermediate goals are local in scope: advancing to the next stage in the creative process from the standpoint of the previous step. Theorization is done for the purpose of understanding and hypothesizing new approaches that permit this process of experimentation to continue. The objective is to theorize the knowledge gained through my compositional/musical experience and be able to share it.

Not all of my music has to do with perception, but this interest has inspired a style of composing within my works and provided a toolbox and repository of materials, yielding compositions that

1 Johnston, B. (2006 [1966]). Proportionality and Expanded Musical Pitch Relations. In B. Gilmore (Ed.), *Maximum Clarity and Other Writings on Music*. Chicago: University of Chicago Press, 35.

make use of a wide range of tunings, some of them related to timbres. The search for a 'timbral grammar' is made in the interest of making harmony include any sound whatsoever. It has electronically synthesized chords, timbres and textures from harmonic structures, through what I call 'dissonance chorales' and procedures which might resemble a 'granular harmony' of sorts. It has also provided strategies for manipulating abstract pitch materials relating them to metric rhythms and formal proportions, as well as traversing 'harmonic fields' in which textures derive from the statistics of continuously changing harmonic potentials.

Regarding this written thesis, it is not meant to be a finished theory nor a a closed system. Its main characteristic is to present harmonic and compositional subjects from the viewpoint of harmonic duality. The presentation exhibits the exploration and inquiry itself, probing into some depth into the topics, even if there is a tendency to wander off from time to time from the main narrative, pointing to issues which haven't been explored but which are felt to be relevant from the standpoint of the inquiry. It is a recounting of the investigation rather than a book about harmony, not meant to be a clean and synthetic account of its results.

The investigation process resembles landing in the midst of things and then departing in different directions, sometimes to come back again to the same topics from other angles. Conversely, some preoccupations, such as the form-rhythm-pitch analogies surface frequently in different sections. Another symptom found in this writing is that of jumping around a few perspectives (history, composition theory, a bit of philosophy and mathematics) in a brief amount of space. The perspectives provide complementary accounts that are compositionally fruitful, even if this is not always completely explicit. Being a personal assessment on harmony, it does not pretend to be unique or all-embracing, as there are many others approaches to it. It is also not axiomatic, even if harmony, through its connection to arithmetics, sometimes gives the feeling of being so. Instead it has a number of equally valid entry points and I find it difficult to disconnect the parts between each other or to choose a narrative that can avoid jumping forward, so the structuring is sort of 'vertical', in a way.

Notwithstanding these considerations, the writing is structured similarly as the duality researched: the first chapter delves into it, the second and third ones probe, respectively, into each of the two facets, while the fourth and last one deals with compositional approaches and pieces, also to conclude with speculative considerations as well as an attempt to define harmony out from these discoveries. Each chapter is itself structured in two parts, both having a different register of thought from each other.

Chapter 1 delves quite extensively into pitch and pitch perception models extracting, from the phenomena that science investigates, compositional possibilities and a characterization of harmonic duality. The second part on Greek harmonics is meant to clean the slate for a harmony not so loaded by theories from recent centuries, paying attention on the duality that stems from the different theories as well as exhibiting how it also derives from a horizontal, melodic conception. It also provides an assessment of the duality vis a vis the psychoacoustic approach and concludes by reflecting on the meaning of Pythagoreansim from today's perspective.

Chapter 2 begins by describing the compositional uses of dissonance curves, showing the connection they provide between spectra, consonance and harmony. Afterwards I concentrate on their psychoacoustic basis and the kind of consonance and dissonance conception they imply in order to better understand the timbral facet of harmonic duality. The second part of the chapter reviews some twentieth century approaches to pitch, especially atonality and some of its descendant techniques, showing how it has become pervaded by an increasing timbral relevance. This characterization is done in order to make a case for the incorporation of proportionality into the current situation and this is why the section touches on topics such as modernism, spectralism, timbral microtonality and fluid or continuous forms.

Chapter 3 delves into proportional harmony by exploring in some detail the language of ratios, harmonic space, commas, tuning tolerance and harmonic metrics. The first section ends by giving an illustrative demonstration of how I work with these notions in my computer aided approach. The second part of the chapter presents my development of 'harmonic fields', their workings, some musical results and further consequences regarding harmonic duality and how it also applies to forms.

Chapter 4 takes a more panoramic approach. It begins by reviewing the harmonic strategies of other composers and harmonic developers of the last century, later to describe some of my own as applied in the pieces written for this doctorate. The second part points to further developments and topics that depart a bit from the main research but that open up to future or unfinished pursuits. There is a section on verticality as a static conception of time that springs from harmonic thinking, pointing towards what I will develop after this research regarding harmonic objects and their quadruple constitution. To conclude I have a section that presents a bundle of possible definitions and denotations of harmony after what has been developed during the research.

Preliminary considerations

This project seeks to rehabilitate a concept of harmony compatible with present-day practices, conceived not in normative terms of rules and prohibitions (as it is normally taught in conservatory textbooks), but instead considered in an experimental sense, as an open space for the discovery and treatment of sonorities. At the same time as it constructs a theory of harmony that recounts my musical approaches, it is also meant to provide notions which are as general and neutral as possible – i.e., separate from my own uses.

It is my contention that harmony, preliminarily defined as musical relations that correspond to the perceptual qualities of pitch intervals, cannot be accounted solely by naturalistic approaches that try to explain it in terms of causal relations between sound waves and the human mind. Neither can it be understood (or more precisely, dismissed as a problem) in terms of cultural conventions: there are aspects of sounds which do not depend on conditioning, even though conditioning plays a role further along the way in the development of aesthetic approaches to those features. It is a mixture of both: the two poles of culture and nature (conditioning and sensation) are never separate until theorized or 'purified' as such, otherwise they are always intermingled.

Moreover, the theoretical elements I draw from have a distinct and particular slant and flavor. Owing much to some twentieth century schools of composition, this approach will not disown them. Influenced by parametric thinking, which comes down from serialism, it also has a strong influence from american experimentalism, as well as from electroacoustic and computer music. There is also the influence of specific composers: James Tenney, Clarence Barlow, Ben Johnston, Karlheinz Stockhausen, Iannis Xenakis and John Cage, to name the ones that stand out most and whose ideas and arguments in relation to the subject we will get to know further along the way.

Historically, most thinking on harmony by past traditions took music to be more than just an art form, connected with the cosmos at large, with metaphysical order and in relation to mathematics. This kind of thinking began to diminish around the seventeenth century, when the concerns began to transition from metaphysical towards physical groundings of sound and music. If I want to argue for incorporating elements from these previous traditions it is because I do not see them as incompatible with experimentalism and the search for novelty. The enterprise should not be understood as conservative or nostalgic, but as pertinent to present concerns, as timeless ideas that are appropriated from one world to another, indifferent to the world in which they first appeared, capable of providing inspiration and answers to existing issues. In this sense, experimentation is neither a break with some anachronistic past nor defined in opposition to it. The use of past ideas should not be seen as an exercise in collage of bits and pieces from throughout history, but as the contemporary relevance of mixing heterogeneous epochs and ideas. The organization of these ideas is to be done in the present, as elements of our own time and from the standpoint of current problematics[2].

Since the time of my doctoral studies and throughout the readings in philosophy and musicology that have led to this dissertation, I have felt that there is an atmosphere of uneasiness about harmony in music as it implies, in a condensed way of putting it, some sort of 'bad metaphysics', in the sense of generalized essences of an idealized world, and in the long run some kind of social domination between those who know and construct it and those who don't, ultimately silencing dissent and creativity. Harmony is considered as having 'pseudoscientfic' aspirations, as part of a positivistic, nostalgic, institutionalized music theory whose pretensions are ultimately illusions. Even though these terms may sound a bit far-fetched or caricatured, they are in fact taken from Susan McClary[3], and it is important that I mention this anti-systematic viewpoint as the backdrop to my position before delving into the topic, even if by now it is falling more and more out of the mainstream discourse on music.

Jacques Attali, theorist of the social construction of musical codes and forerunner of some of these positions has likely been read too literally or taken to relativistic extremes. I find his idea of harmony as a social order which is disrupted by the noise of new social codes very compelling (though derivative to the core of this project). In any case, my contention is that it still makes sense to talk in terms which have been devalued by these approaches: rhythms, pitch, meters, harmony, and so on, albeit in the current context of indeterminate, generative, textural music.

The question is not how knowledge produces and exerts its power, authorizing who speaks. What this research pursues is not only how harmony and harmonic objects (intervals, scales, their qualitative characters, spectra, timbres, textures etc) are *known*, but what they *are*, even if it is not possible to define them fully . Music has a certain autonomy over the many domains with which it crosses paths. The different fields of knowledge that revolve around musical objects are all translations, not complete renditions of them. An object is always more than any account, theory, use or any other kind of translation of it, none ever exhausting its reality, though all of them provide diverse points of view as to what it is. A bit like the Indian story of the elephant and the blind men, who each touch a different part of the elephant and give diverse, conflicting accounts of it. Objects cannot be exhausted by the dimension of social construction inherent to them, nor is this construction an alienating feature meant to oppress or deceive. Instead it is just one aspect of objects (a very important one): their relationships. The other aspect is their individuality, not reducible to their relation to other objects (ourselves humans included) or to some deeper hidden layer of reality[4]. This shows the reasons behind the multi perspectival approach, the goal of which is to

2 This viewpoint on 'nature vs culture' comes from Bruno Latour. See Latour, B. (1993). *We Have Never Been Modern*, Harvard: Harvard University Press. The concept of eternal, transworldy truths is Alain Badiou's. A concise expounding of this notions appears in Badiou, A. (2011). *Second Manifesto for Philosophy.* Cambridge: Polity Press, 20-25.

3 McClary, S. (1985). Afterword. In Attali, J., *Noise. The political economy of music.* Minneapolis: Minnesota University Press, 149-158. If I choose her as the example of postmodern attacks on music theory it is because I actually like her writings and agree with many of her ideas. I also find Attali's book influential and inspiring. This posture nonetheless has become institutionalized and, in order to make its point, has gone to the other extreme of classifying everything in music as arising in culture and power struggles. The attribution of an illusory nature to music theory, comes from Kramer, L. (1992). The Musicology of the Future. *Repercussions,* Spring Issue, 9: "From a postmodernist perspective, music as it has been conceived of by musicology simply does not exist."

4 Here I follow Graham Harman whose 'object-oriented' philosophy has been a source of encouragement for my

construct an amazement for harmony not only as a human creation but also having implications beyond us, which is why Pythagoreanism is also invoked.

Ben Johnston warns us in the epigraph that opens this introduction about the dangers of splitting music composition into two irreconcilable camps of 'sound-based' and 'note-based' composition (to put it in todays terms). This text was written 45 years ago, and the splitting that he fears has already happened. Much electroacoustic and acousmatic music, not to say sound art, noise and other such genres that lie outside the concert situation generally define themselves in opposition to conventional, conservative, conservatory note-based music. What I want is to refine our understanding of pitch in order not to reduce it, but to integrate it to the practices of sound-based art without throwing the baby and the bathtub along with it or, what is the same, without confusing the art of composition with the saturation of its own procedures and probably also with its current means of production, but to tackle it straight where it hurts the most: precisely in the age old department of pitch and harmony.

To close this introduction, a lucid quote from Johnston captures and summarizes the spirit and general direction of this endeavor:

> "To establish connection between the known and rational and familiar, on the one hand, and the unknown and irrational and unpredictable on the other, requires subjecting them to the same measure. *Proportionality* is such a common measure, if we bear in mind the modifying principles of variation and approximation. It is not incompatible with other modes of organization, such as serial ordering. *It applies with equal effectiveness to formal, rhythmic, and pitch organization*. It can be realized best by *ear*, in the case of pitch; by *kinesthetic perception*, in the case of rhythm; and by *intuitive timing*, in the case of formal divisions. Yet it is capable of intellectual formulation and manipulation. Most important of all, such a technique reestablishes a connection which has been broken, a connection with ancient and worldwide traditions of aesthetic order." [5]

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position, subtly permeating this thesis even if it is not dealt with directly. In any case we will encounter him again later on. See Harman, G. (2005). *Guerilla Metaphysics*. Chicago: Open Court and Harman, G. (2011). *The Quadruple Object*. Winchester: Zero Books.

5 Johnston, B. (2006 [1963]). Scalar Order as a Compositional Resource. In B. Gilmore (Ed.), *Maximum Clarity and Other Writings on Music*, 29. Emphasis added.

Chapter 1

Harmonic Duality

1.1 Harmony and its duality

> "Music is in fact not without ambiguity – especially since the Renaissance – because it is at once the intellectual love of an order and a measure beyond the senses, and an affective pleasure that derives from bodily vibrations. Furthermore, it is at once the horizontal melody that endlessly develops all of its lines in extension, and the vertical harmony that establishes the inner spiritual unity or the summit, but it is impossible to know where the one ends and the other begins." (Gilles Deleuze, *The Fold: Leibniz and the Baroque*[6]).

> "Music charms us, although its beauty only consists in the harmonies of numbers and in the reckoning of the beats or vibrations of sounding bodies, which meet at certain intervals, reckonings of which we are not conscious and which the soul nevertheless does make." (Godfried W. Leibniz, *Principles of Nature and of Grace*, §17[7])

As the above quotes suggest (the first being a comment on the second) from a metaphysical viewpoint, musical harmony consists of two separate aspects. One is not directly perceived, 'a measure beyond the senses', while the other is affective and sensual. They both make up its ambiguously interwoven horizontal and vertical dimensions and this chapter will be dedicated to exploring this embroilment, which I refer to as harmonic duality.

These two aspects are normally subsumed under the term of harmony: one, its *proportional* facet, involves rational relationships, concerns fundamental pitches and disregards timbre and register; the other one, which we will call its *timbral* aspect, involves sensation and acoustic constitution. Their embroilment and interrelationships are such that none is able to subsist without the other. It is rather the perspective created by putting them musically in context what makes one of them stand out. In this measure, both facets are active in different degrees in various musics and their thresholds and contexts that produce their mixtures or separations can be composed.

The idea of a harmony polarized between two limits will be the guiding thread and principal hypothesis of this inquiry. Although this split will be shown to have been present in the debates between empirical and mathematical harmonists in ancient Greece, it has not been explicitly thematized as such. Either it is a question of one aspect dominating or substituting the other, or they are conflated without distinction. In any case, the distinction happens with some recent composers, such as James Tenney, Ben Johnston or Clarence Barlow, although only in a fleeting and unsystematic way, so that this research will take their ideas as a starting point in order to develop on their properties and consequences. This duality, with each of its poles in turn oscillating between two limits − those of consonance and dissonance in the case of the timbral pole and harmonicity and inharmonicity in the case of proportion − makes for a fourfold structure of antipodes. This will be the structuring figure in the development of further topics later in the study. It is proposed as means by which to think pitch and harmonic relations in the development of compositions and

6 Deleuze, G. (1993). *The Fold: Leibniz and the Baroque.* Minneapolis: University of Minnesota Press, 128-129.
7 Leibniz, G. W. (1908). *Principles of Nature and Grace.* In *The Philosophical Works of Leibniz*. New Haven: The Tuttle, Morehouse and Taylor Company, 306.

compositional materials. Only secondarily can it be applied to pertain or embrace other musical activities, as it is developed not only from a practice but also with an aim of understanding models and spaces that are directly relevant to a specific way of composing – that of making algorithmic environments for composition. Within a necessarily limited scope and context they are also related to other musics and to auditory perception in general, although this is to be understood only as one factor out of many that are operative when describing these more general aspects. It can nevertheless aid or serve as a specific analytical tool for some of these applications because it theorizes the relationships between quantitative and qualitative aspects of harmonic materials as well as proposing some basic computational tools that might render these ideas useful beyond composition.

Apart from compositional experience and intuition, there is evidence for this duality in various approaches for conceiving intervallic qualities. The analysis in this chapter will be based on two sources testifying to this duality: Greek harmonics and psychoacoustics. The debate between Greek harmonists around the distinction between pitch distance and proportion started by Aristoxenus traverses many centuries of music theoretical thinking. Although in a simplified way we might call Aristoxenus a 'timbralist' and Pythagoreans 'proportionalists', this too easy characterization can miss the fact that both sides include some aspects of the other; I would rather say they supplement one another and together allow us to form a wider perspective on this duality. The longstanding theoretical conflict between proportion and spectrum derives from two orientations towards intervallic qualities that emerge as different approaches to the problem of explaining consonance and dissonance or the perception of pitch and timbre. These approaches have parallels to the distinction between the continuous and discrete in mathematics, having on the one hand arithmetics and the Pythagoreans and Aristoxenus and geometry on the side of the continuum. It is through a musical perspective that correspondences are found between sounding qualities and these mathematical domains. Whole number ratios are related to periodic sounds and intervals, while a continuum of pitch and the infinitesimal gradations of timbre can be conceived as happening in a continuous line or space.

The dichotomy between harmonists is analogous to the one in pitch perception models from auditory science, namely, temporal and spatial models. The former relate to periodicity analysis of waveforms and refer to pitch-*chroma*, while the spatial ones involve pattern recognition in spectra obtained from the physiological filtering in the basilar membrane of the inner ear, and refer to pitch-height or distance. These models map subjective attributes to independent mechanisms taking place throughout the auditory pathway, beginning as early as the cochlea, and taking a complex and crisscrossing course through progressively higher-order auditory stages that end up projected into different cortical areas. The survey we are about to take is meant not only to describe what is known about pitch perception, but will serve to extract from the theories their mappings enough details to better characterize the duality. As we will also discuss, it not only pertains to the level of intervals and chords, or to the current sounding qualities, but is also inherited to higher levels or time frames than those of an immediate sounding present. Although the discussion of the different 'harmonic levels' will take place later on, we will already see in this section some ways in which properties of this harmonic duality are inherited not only to musical harmony as it is normally understood, but also, though in a limited way, to rhythm and form.

1.2 Pitch Perception

> "[P]itch is held to be duplex in nature. Two pitch-like qualities are distinguished. They are given various pairs of names by various authors: tone height and tone chroma, ordinary pitch and chroma, [spectral] pitch and quality, pitch and tonality, etc."[8]

The two main types of pitch perception models explain in terms of time-based processes – periodicity analysis of waveforms and the counting of pulses – and spatial processes – recognition of spectra by means of patterns. Both mechanisms are active in our perception and complement each other, pitch-chroma providing a basis for presenting acoustic patterns that do not depend on the particular sound source, and pitch height providing a basis for segregation of sound objects into streams in order to separate sources.

Pitch models are approximations that explain aspects of the world, in this case that of psychological auditory experience. The models are not only rated or judged according to their capacity to explain many sometimes contradicting phenomena, but also as to how useful and compelling they are to the different approaches and disciplines that participate in their construction – be they acoustics, audiology, neurology, musicology, psychology, etc. They are also marked by their statistical and indirect approach. Some account for resulting effects rather than mechanisms, while other approaches begin with physiology and neurology, providing results which are difficult to compare and combine with the other approaches, attesting to the difficulty involved in bringing the two directions together and somehow meeting halfway[9]. There are many gaps in the scientific comprehension of pitch recognition.

On the other hand, these models explain a lot and have been withstanding falsifiability for many decades, being constantly refined and put to work in usable applications as well as becoming stronger in their ability to account for ever more detailed and diverse classes of stimuli. The fact that they mostly translate emergent psychological effects rather than explaining all the complex steps happening along the way, gives them a economical structure that in many cases agrees better with intuition or introspection than with brute force causality.

Induction and deduction are the basis for physics and mathematics, while the study of perception deals with *trans*duction, defined as the conversion of one form of energy to another. In physiology this means the conversion of one form of stimuli to another, wherein a physical stimulus is converted into action potentials in neurons, transmitted along axons towards the central nervous system where it is integrated. A transducer is an object that mediates between two other objects, the input carrying information that molds the energy of the medium with a specific *form*[10]. Transduction resembles modulation in signal processing, where a usually high frequency carrier waveform is modulated by another signal containing the information to be transmitted through it (like in radio transmission, but also in analogy to the modulation of a note with vibrato or tremolo by a musician). The energy in the transducer is the carrier wave, the information it carries is the modulator. From the 'point of view' of the transducer, both the incoming energy (the specific form given to the carrier) as well as the outgoing energy is information, while from the perspective of the transduced information, the transducer becomes transparent, as it could be replaced by any other transducer. The information is independent of the medium in which it travels manifesting the physicalness of

8 Licklider, J. C. R. (1951). A duplex theory of pitch perception. *Experientia,* VII(4), 128.

9 Cheveigné, A. (2005). Pitch Perception Models. In Plack, C., Oxenham, A., Fay, R., and Popper, A., (Eds.). *Pitch. Neural Coding and Perception.* New York, Springer, 169-233.

10 These ideas are taken from Timothy Morton. See Morton, T. (2011, May 26). Transduction [weblog]. Last retrieved April 15, 2012, from: http://ecologywithoutnature.blogspot.com/2011/05/transduction.html

this information, too often taken to be more ideal than physical.

We will get acquainted with the various transductions involved in pitch perception, but what this short consideration suggests is that pitch not only relates to properties of the vibrating objects that produce the sensation, but some aspects of it are themselves created by the very process of translation, carrying with it information about this transduction. Information and medium, transducer and transduced, participate in its formation and characterization, in a double structure similar to what pitch transmits, which is both itself as a distinct perceptual quality and traces of information about the source that produced it.

1.2.1 What is pitch?

'A regular periodic pattern dominates a pitched sound'[11]

Pitch is a perplexing aspect of auditory perception, consisting in a distinct set of *qualia* that stand out clearly from other aspects of auditory experience. These independent but related attributes are grouped under the term 'pitch', but can and should be distinguished in order to characterize harmonic duality and seek strategies for traversing these aspects in composition.

Pitch is defined as the perceptual correlate of frequency[12]. Because some of its features are induced by the perceptual process itself, this does not imply a one to one correspondence. Not all perceptions correspond to the physical stimuli and there is a fault tolerance mechanism which compensates for 'deficient' stimuli by completing and rounding them off. Pitch can also be understood as a psychophysical magnitude correlated closely to the periodicity of waveforms while being simultaneously blended with the timbre of the sounding source. These timbral features can be thought of as peripheral periodicities within that main periodicity, distinguishing between the main repetition period and those 'inner', faster ones constituting the partials.

In the 4th century B.C. Aristoxenus distinguished between pitch in general, *topos*, the space in which high and low notes are located, and *tasis,* the 'pitching' of this space by the melodic voice, indicating that there are two senses in which we can talk about melodic pitch, a continuous one and another in which this continuum is pierced at certain points of stability. This 'pitching' is a 'steady motionlessness of the voice', a resting place where the melodic motion stops within pitch space, therefore a *locus* within the continuum, indicating a calibration, in melody, of this space.[13]

Aristoxenus proposed an empirical account which can be considered phenomenological before its time. This is relevant in order to understand pitch as a perceptual category not reducible to natural processes. Pitch (as is also the case of many musical categories) commences as a perceptual fact that is prior to any scientific conception. Experience provides the 'what', psychoacoustics the 'how'. Still, the musical priority belongs to the 'what'.

Pitch, then, is not one dimensional magnitude but a mixture of several features. A way to tease out these other aspects is to limit the stimuli to the most elemental sounds. Sine waves are considered elemental sounds with respect to pitch, and it is only with these types of sounds that we can say that frequency and pitch actually coincide, so maybe the best and most stringent definition of pitch should limit it to the perception of sine waves and leave other aspects of it to other terms (such as *tone*).

The modern definition of pitch is similar to the one given by Arabic music theorist Safi al-Din (13th

11 Johnston, B. Scalar Order as a Compositional Resource, 12.
12 American Standards Association. (1960). *Acoustical Terminology S1*. New York: American Standards Association.
13 Barker, A. (2007). *The Science of Harmonics in classical Greece.* Cambridge: Cambridge University Press, 146-149.

century): “a sound for which one can measure the excess of gravity or acuity with respect to another sound.” The ANSI (American National Standards Institute) definition is: “that auditory attribute of sound according to which sounds can be ordered on a scale from low to high.” It could be written more specifically as ‘that auditory attribute in terms of which sine tones can be ordered on the low-high dimension’[14], but this is useful for research rather than musical purposes because other attributes associated with pitch are left orphaned and ‘tone’ or ‘complex sound’ become insufficient terms for sounds which don’t have a single or clear pitch but have scalable attributes.

Pitch should then be related to the repetition rate of the waveform rather than to frequency. This repetition rate is equivalent to the periodicity envelope, the recurring amplitude pattern resulting from the overall energy of all the components of a sound. This periodicity envelope, being the rate common to all or the most prominent partials components of a tone, does not necessarily coincide with the frequency of the lowest component. Only when sine and complex tones have the same repetition rate can we say that pitch and frequency are equivalent.

A broader definition of pitch is given by J. F. Schouten (1940): ‘The pitch ascribed to a complex sound is the pitch of that component to which the attention, either by virtue of its loudness or of its contrast with former sounds is strongest drawn. Therefore the pitch of a complex sound may be different depending upon the circumstances under which it is heard.’[15] Pitch is induced or ascribed from composite sensations as well as embedded within other sounds.

This link between pitch and periodicity will be connected further on to harmony and rhythm:

> ‘Nature abounds with periodic phenomena: from the motion of a swing to the oscillations of atoms, from the chirping of a grasshopper to the orbits of the heavenly bodies. And our terrestrial bodies, too, participate in this universal minuet—from the heart beat and circadian rhythms to monthly and even longer cycles.
>
> Of course, nothing in nature is exactly periodic. All motion has a beginning and an end, so that, in the mathematical sense, strict periodicity does not exist in the real world. Nevertheless, periodicity has proved to be a supremely useful concept in elucidating underlying laws and mechanisms in many fields.’[16]

Periodic phenomena being so ubiquitous, pitch has evolved as a mechanism for detecting periodicity in an environment. Because strict periodicity does not exist, being delimited in time and always accompanied by some sort of noise, the mechanism is tolerant of this divergence. Pitch perception consists *both* in the factoring and rounding off of (near) periodicity from an auditory scene as well as in the segregation of sources from this scene. If periodicity is understood as an invariance with respect to translations in time, then we can see that periodicity gives rise to discreteness, puncturing holes in the time continuum by bringing about units through repetition. From this logic, another way of describing this double function of pitch with respect to unities would be: to discretize the world and to detect simultaneities. Both these functions are related to harmonic duality: periodicity detection corresponding to, and likely (partly) responsible for, the harmonic or proportional facet, while the task of stream segregation relates to the enmeshment of pitch and timbre, and many composite harmonic configurations inherit the properties of this more general function.

Pitch perception can therefore be seen as involved in two somewhat inverse functions: the segregation and the unification of sound components, both simultaneous as well as sequentially. Stream separation is a continuous process, while bundling of elements into individually identifiable impressions accounts for discreteness. This integrating aspect might condense inner modulations,

14 Terhardt, E. (2001, May 8). Definition of Pitch. Retrieved from http://www.mmk.ei.tum.de/persons/ter/top/defpitch.html

15 Schouten, J. F. (1940). The residue, a new component in subjective sound analysis. *Proceedings of the Dutch Royal Academy of Sciences*, 43 (361).

16 Schroeder, M. (1990). *Fractals, Chaos, Power Laws.* New York, W H Freeman and Company, 1.

transients or otherwise timewise separate components into unitary perceptions. Harmonic duality maps to this polarity: integration into units and differentiation of streams, the former serving the purpose of identifying proportionality (relations between units, happening in a independent dimension from that of high and low pitch distance), while the latter is of use in the identification of separate sound colors and sources (for which pitch distance is of great help).

In a sense pitch is perhaps the most distinct qualia in auditory perception – and for that matter, also with respect to other sensory modalities – in that its qualitative seemingness, the way it appears as an experience, permits a hugely gradated and finely tuned immediate apprehension that differentiates itself in a measuredly way. Pitch provides a reproducible, memorizable figure for identifying and comparing relative (but sometimes even 'absolute') stimuli, capturing relations in a qualitative, immediate manner. Intervallic hearing, which in a sense is equivalent to 'pitch to the second degree', the quality of two (or more) pitches, provides and even finer spectrum or graded scale for which to single out periodicities, this time not in the sense of high and low but as highly identifiable 'characters' that, with a bit of training, can be immediately apprehended.

1.2.2 Theories of pitch perception

> 'Historically, theories of pitch were often theories of hearing [...] It is conceivable that pitch grew out of a mechanism that evolved for other purposes, for example to segregate sources, or to factor redundancy within an acoustic scene. The 'wetware' used for pitch certainly serves other functions, and thus advances in understanding pitch benefit our knowledge of hearing in general.'[17]

The science of pitch was inaugurated by Greek Pythagoreans. Archytas (4th century B.C.) proposed the first known psychophysical account of sounds and their perception, attributing for each pitch a different speed in the propagation of vibrations, instead of what is now known to be a different *frequency*, the speed of propagation being independent of pitch. This issue was not settled until the seventeenth century, when the quantitative dependance between pitch and frequency was established by Marin Mersenne (1636) and Galileo Galilei (1638). Later on Joseph-Guichard DuVerney (1693) established the first resonance theory which suggested a 'tonotopic' (correspondence of pitch height to spatial distance) projection to the brain, followed by Joseph Sauveur's (1701) observation that a string could vibrate simultaneously at several harmonics (coining the terms 'fundamental' and 'harmonics'). Later on, theories explaining superimposed vibrations on a string were developed by various mathematicians (Taylor, Bernoulli, d'Alembert), but it was Leonhardt Euler who condensed these ideas with the concept of linearity, which implies the principle of superposition (that these partial waves propagate independently and that the total sound at each point corresponds to their sum). The thinking of vibrations as sums of more fundamental ones, with their periods at integer submultiples of the longest period lead to Joseph Fourier's theorem (1822), which proves that the sum can be expressed as a set amplitudes and phases and that this sum is unique.

Nowadays two quantities related to pitch are distinguished: what is called spectral pitch, usually denoted as f_{LOCUS} and periodicity pitch, or F0. August Seebeck, who first utilized acoustic sirens in auditory research, presented observations (1841) suggesting that a pitch sensation was determined not only by the fundamental, but also by other higher partials, even to the point of having acoustic signals that could elicit a pitch without possessing a fundamental partial at that frequency. He concluded that pitch corresponds to the period of the overall periodicity. A battle ensued with Georg Ohm, for whom the ear performs a Fourier analysis of signals into their partial components from

17 Chevaigné, *op. cit.*, 221. Most of this section is structured around the topics dealt with in this article.

which pitch is determined by the frequency of the spectral fundamental. Ohm's dismissal of Seebeck was to cause many future misunderstandings, delaying the development of non spectral theories. This spectral theory was based on his law (1843) which extended the principle of linear superposition to the sensory domain. Hermann von Helmholtz (1863) refined and developed it, explaining how the process happens in the cochlea. If a complex sound is composed of sinusoids, the sensation itself can also be decomposed into simple sensory components. He also stated that: (1) only vibrations with a nonzero fundamental evoke a pitch related to that period; (2) other partials may evoke additional pitches; (3) relative partial amplitudes affect timbre but not pitch; and (4), relative phases of partials affect neither quality nor pitch. He proposed a model according to which the cochlea (and specifically the basilar membrane inside it) behaves like a bank of resonators, to be proved physiologically by Georg von Béckésy's (1928) discovery of the process of transduction happening in the inner ear, later refined by Plomp and Levelt's (1965) critical band model[18].

This history surveys the basics of what is now called place theory, also known as pattern matching. It consists in estimating pitch based on a spectral analysis of the provoking stimulus through patterns formed by the spacing and amplitudes of its partials. It is spatial for it follows the tonotopical metaphor in the way it was proposed by Helmholtz, suggesting that pitch is perceived through 'conscious inference' as each nerve attached to the spatially arranged resonators in the cochlea carries with it 'specific nervous energies', each representing a different quality of pitch.

Nowadays the cochlea has been thoroughly studied and is indeed considered a tonotopic transducer, but there are more aspects to this. The basilar membrane inside it transduces the vibrations transmitted from the timpani by moving inside a liquid spiral chamber and transmitting its vibrations to the hair cells disposed along the spiral and associated with nerve fibers that respond to specific frequencies[19]. There is still a heated debate as to what kind of vibrations the basilar membrane undergoes. Béckésy's model is that of a traveling wave; the problem with it is that, being a serial phenomenon, it does not allocate frequency to place along the cochlear duct. Resonance models, on the other hand, account for tonotopy through parallel vibrations. It may be that what stimulates the basilar membrane and the cells has to do with a combination of traveling waves, resonance, plus an active amplification scheme where the outer hair cells compensate and change the physical properties of the system[20].

The cochlea is an active and non-linear system. Active, because in addition to receiving acoustic energy, it also has a 'regenerative', 'undamping' mechanism which adds energy to the very signal it is trying to detect, effecting a 'sharpening' of the resonance required for the transduction, in order to change its tuning characteristics and resolution. The outer hair cells, through chemical, mechanical and electrical interaction with the basilar and tectorial membranes, change their and the liquid's physical properties, producing a negative damping that injects energy into the system, under control from the central nervous system.

The cochlea is non-linear because it distorts the incoming signal, producing signals which are byproducts of its functioning, such as combination tones and spontaneous subjective pure tones

18 We will discuss in the next chapter Plomp, R & Levelt, W. (1965). Tonal Consonance and the Critical Bandwidth. *Journal of the Acoustical Society of America*, 38(4), 548-560, from which the model for dissonance curves derives.

19 There are inner hair cells which do the actual transduction and outer hair cells which account for compensatory processes within the cochlea. Also, as an interesting fact, it happens that auditory hair cells are the only plant cells in the mammal body, or at least the only ones having the characteristic of being pressurized like plant cells. See for example, Baylor Collegue of Medicine. Outer Hair Cell is Pressurized. Last retrieved May 21, 2011, from http://www.bcm.edu/oto/index.cfm?pmid=15267

20 This heated debate happened between Alain de Chevaigné, Martin Braun, Richard Lyon and others at the Auditory Mailing List http://lists.mcgill.ca/archives/auditory.html started March 2, 2010 and continuing until February 2011.

known as otoacoustic emissions[21]. Another nonlinear characteristic is that it behaves differently according to frequency. Recent modeling of the basilar membrane has been achieved through a cascading filter bank, so-called gammatone filters[22]. The proponents of this model contend that one of the main functions of the cochlea, arguably more important than frequency filtering, is to compress the enormous range of amplitude variations in sound we are capable of hearing into a more manageable dynamic range that can be transmitted through the nervous impulses transduced by the inner hair cells. This compression is frequency dependent and highly non-linear, compressing the input dynamic range by up to six orders of magnitude. Inner hair cells can only transduce around 60 dB, so to explain the 120 to 140 dB of dynamic range perception in humans (able to detect differences in amplitude of up to 14 orders of magnitude!), the outer hair cells have been shown to reduce this range by compressing another 60 dB[23]. Given that the frequency resolution of the cochlea is somewhat broad, most of the auditory filtering occurs at higher neural processing centers in the brain, lending additional support to dynamic compression as the main function of the cochlea. These details regarding the cochlea can help understand the discussion of dissonance curves we will have in the next chapter, as they are offshoots of basilar membrane models.

For Ohm and Helmholtz, the pitch sensation was straightforwardly the lowest partial. As we know, this does not hold for many kinds of stimuli, for which other more refined algorithms have been proposed. They may estimate pitch from the spacing of the partials, from loudness patterns, or through sums of subharmonics of the partials. A same pitch can be evoked by widely varying spectra, including those without a fundamental, as in Ernst Terhardt's model (1974), where specific loudness patterns are the basis for a derived virtual pitch 'gestalt', inducing a pitch from partials other than the fundamental, distinguishing 'spectral' from 'virtual' partials[24]. Virtual pitch is related to what he calls analytic listening, a learned mode of listening distinguished from the innate, synthetic listening where Ohm's law follows. Pattern matching models thus require a learning process and the existence some sort of internally stored templates to which the input is compared. The fact that a string behaves like a pattern-matcher makes for possible mechanisms for which no learning is required, though. Furthermore, a string operates directly on the waveform, not on a spectral pattern, so the Fourier decomposition might not even be needed.

Temporal theories of pitch were brought back to life by Schouten (1938) when he showed many instances where F0 was not the fundamental partial. His said that a 'residue' pitch is responsible for the overall periodicity. This residue arises from the combination of high, unresolved partials – partials too high and close together to be separately distinguished and processed in the basilar membrane – and is present even when a masking noise obstructs the fundamental frequency. Later on it was found that those residues were not limited to unresolved partials but also included the resolved partials, meaning that the residue emerged from the sound as a whole.

Time models assume that the ear 'counts' vibrations instead of guiding itself by the metaphor of calibrated resonators. Contrary to place models, in time based ones it is only possible to suppose that

21 "Otoacoustic emissions are small sounds caused by motion of the eardrum in response to vibrations from deep within the cochlea. The healthy cochlea creates internal vibrations whenever it processes sound. Impaired cochleae usually do not. Some healthy ears even produce sound spontaneously as internal sounds are processed and amplified.", from Kemp, D. Understanding and Using Otoacoustic Emissions. Last retrieved May 31, 2011, from http://www.est-med.com/OAE/understanding-using_OAE von Kemp.pdf. This phenomenon is mostly of interest in audiology and hearing impairment research.

22 Lyon, R., Katsiamis, A. & Drakakis, E. (2010). History and Future of Auditory Filter Models. *IEEE International Symposium on Circuits and Systems (ISCAS),* 3809-3812.

23 Lyon, R., Katsiamis, A. & Drakakis, E. (2007). Practical Gammatone-Like Filters for Auditory Processing. *EURASIP Journal on Audio, Speech, and Music Processing*, Article ID 63685, 2.

24 My implementation of dissonance curves includes a derivation of this model that calculates virtual pitches from a spectrum. It is used to accompany the intervals produced by the dissonance curves as it combines with them very well. More in Chapter 2.

the counting takes place in the brain, not in the cochlea. In time theory, which has its roots in the Pythagoreans and Boethius, the behavior of a string is a guiding metaphor, as it can make many sounds, one sound encompassing others but coming to the ear integrated in the unity of a single pitch. Here the elementary components of sounds are not partials (sine waves) but discrete 'phonons': percussions or pulses. Galileo's account of consonance explains it in terms of *commensurability*, as the blending of two simultaneous sounds due to the proportionality of their pulse trains. This is related to the harmonic metrics we will discuss in Chapter 3.

Joseph Licklider (1951)[25] first put forth the theory of autocorrelation, the main model behind time theories. A measure of self-similarity across time, it reveals the close relationship between periodicity and self-similarity, and as a concept it can also be used to describe and explain musical phenomena at several time scales. Autocorrelation (AC) proceeds after cochlear filtering and hair cell transduction, happening as an analysis of trains of nerve impulses. It is a two dimensional pattern with the axes of CF, characteristic frequency, and lag time, τ. The delayed and direct signals are multiplied and the result is summed up, so that for lag times corresponding to the period of the signal the sum will be maximal. There are also peaks at time intervals related to the sub periodicity of the signal, such as harmonic partials, which also contribute to the main peak. Licklider speculated that neural circuits in the lower centers of the auditory system can perform the three operations necessary for AC: delay, multiplication and temporal integration. The neural arrangement happens in two dimensions: the frequency projection from the cochlea and delayed versions of these projections. At another further network, this matrix of information is summed and integrated for the sought period to emerge. AC has affinities with pulse counting theories of consonance: "when the frequencies of two sounds, either sinusoidal or complex, bear to each other the ratio of two small integers, their autocorrelation functions have common peaks."[26]

AC requires accounting for the low firing rates of neurons which max out at around 300 spikes/second for which the so-called 'volley theory' explains firing rates higher than this limit[27]. Time based processing has a different frequency range of operation than place mechanisms, the former limited with respect to the latter in high frequency resolution, while the latter is limited with respect to the former in the low regions, for which time processes can go below the usual cochlear pitch range at 16-30 Hz, all the way down into the infrapitch regions. Both models overlap within 30-2500 Hz, where most musical activity happens. The breakdown of time mechanisms occurs between 3.5 and 5 kHz, coinciding with the limit of melodic pitch and intervallic recognizability.

AC is a computational model, borrowed from mathematics, of which there have been many variations and refinements, some better equipped that others to predict empirical behaviors. They are usually called Auditory Image Models. The string metaphor can be seen as belonging to the AC model family as it is in essence a delay line that feeds back into itself. AC, the string and pattern matching are closely related, their difference lying in their temporal resolution. At each instant, AC reflects a relatively short interval of its input, the string metaphor reflects the past waveform over much longer time intervals. Other AC models capture regularity over longer periods and are 'subharmonic' counterparts to 'harmonic' pattern matching schemes, showing strong connections between AC and pattern matching, with the concept of the string as an analogy bridging both. Sine waves are elementary in place models, in time models they behave just like any other signal.

Both temporal and place models require a temporal integration mechanism to account for the continuity heard in sounds despite them consisting in trains of pulses or waves following each other. This integration must be limited for the perception of trills and other fast modulating articulations

25 Licklider, J. C. R., o*p. cit.*, 127-134.
26 *Ibid*., 131.
27 See Plack, C. & Oxenham, A. (2005). The Psychophysics of Pitch. In *Pitch. Neural Coding and Perception.* New York, Springer, 12.

not to become smeared. It is also subject to an inherent tradeoff in wave phenomena between frequency and time, where the accuracy of frequency determination depends on a sufficiently large time window for the analysis, and the larger this window, the coarser the time resolution. Dennis Gabor's formula[28] (1945), derived from the uncertainty principle in quantum mechanics, says that:

$$\Delta f \cdot \Delta t \geqq k$$

The product of the uncertainties in frequency and time is always larger than a positive constant. Time and frequency cannot be simultaneously defined in an exact way, there always remains an uncertainty. Increasing the accuracy of one quantity increases the inaccuracy of the other and vice versa. This has consequences for thinking perceptual boundaries, and some topics related to harmony also relate to this inequality, its constant k determining those borderlands which, if crossed, cause sonic forms to experiment a 'perceptual phase change', a change in their properties by which they pass from one uniform perceptual state into another with different emergent characteristics (and this might either happen abruptly or gradually). In the case we are dealing with now, it is pertinent to the transition between pulses and pitch, or to fluctuations belonging to the 'zone of articulation' such as vibrato and tremolo which, when speeded up sufficiently, are conflated into the timbral modulations within a tone. This boundary can be called the rhythm-pitch boundary. Its 'integration zone' has a boundary beginning at around 50 ms (20 Hz) for its lower threshold, with a higher one lying around 10-15 ms (66-100 Hz), although this upper bound is more of a grey zone, much more difficult to determine. Within this region, pitch is not yet fully fused into a unit apart from its sonic components[29]. There are several layers of temporal integration simultaneously embedded on top of each other, an insight relevant also to the fact that harmony involves different time scales, something which will be dealt with during the course of this study.

Several kinds of temporal integration have been incorporated into AC models. They involve two kinds of frequencies: those arriving from the cochlea, which are subject to the uncertainty relation, and those produced by the synchronization of neural circuits to the periodicity of incoming waves from the cochlea ('phase locking'), determined with arbitrary accuracy. They allow transients to reset the integration process, avoiding integrating for longer than necessary. These facts suggest a combination of time and place models.

There has been no way to eliminate either time or place models as they both explain, in their most current versions, many kinds of inputs and predict most kinds of behaviors. To a great extent they are found to be complementary. Licklider proposed a 'duplex' theory which included a learned neural network to integrate them. There is still strong support for a two mechanism hypothesis, its main drawback being the lack of parsimony in conceiving two mechanisms instead of one (plus a possible third mechanism to integrate the two). Place processing adapts well to resolved harmonics, while time processing handles unresolved ones. Despite the evidence for both mechanisms and the fact that they provide a better fit to empirical behaviors, a two mechanism hypothesis also compounds the difficulties of both.

As for this research and my position as composer, evaluating this double mechanism hypothesis from the standpoint of music and harmony shows that the problem should not be limited to pitch perception on its own, but that it may be revealed more clearly in musical and particularly harmonic situations. A concept of a 'single' pitch is already a reduction, as pitch always happens beyond laboratory conditions within an auditory environment, rarely unaccompanied by other pitched and non pitches sounds. Nevertheless, a compositional hypothesis such as harmonic duality, derived by awareness, introspection and experience gathered by exposure and handling of musical materials,

28 Gabor, D. (1946). Theory of Communication. *Journal of the IEEE,* 93(26), 429 – 441.

29 More details in Warren, R. (2008). *Auditory Perception. An Analysis and Synthesis.* Cambridge: Cambridge University Press.

can provide support for the fact that two parallel, supplementary processes are active because they have corresponding affinities with each of the harmonic facets and are likewise entangled with each other. The perspective of composition can't directly intervene in advancing these multidisciplinary models, although it could suggest targets and situations on which the models can be experimentally tested, helping to challenge and refine them through the elucidation of the qualities and properties of harmonic duality. In this way, harmonic duality not only takes some of its features from pitch perception models, but can also give and suggest some ideas back to auditory science.

1.2.3 Neurobiological studies of pitch from the bottom-up

Parallel pathways are followed by the signal transduced by the cochlea in its way from the auditory nerve to the auditory cortex, each matching the properties of time and place based processes. Piecewise neurological descriptions, where each step increases the complexity and time frame of the perceptual process is what cognitive psychologists call a 'bottom-up' description, experiences understood in terms of smaller processes that accumulate into emergent contents of consciousness. They are 'data-driven', proceeding from the sensory towards the cognitive. In contrast, 'top-down' strategies refer to approaches that explain in terms of higher level *qualia* or learned 'schemas', expectations or concepts which are analyzed into subsystems of 'black-box' components (where it is foremost to know their effects rather than their functioning) interacting all the way down towards physiological sense mechanisms.

Bottom-up and top-down processes are simultaneously involved in perception, though their terminating points are hard to conciliate. High level concepts or expectations mediate upon the data acquisition process. Also, most lower level features have been quite well studied, but as the descriptions progress up through the auditory pathway from the brain stem towards the auditory cortex, the gaps in understanding become larger and less detailed. As the processing ascends through the auditory pathway, it involves more subsystems and larger time windows, becoming more contextualized and imbricated with memory and learning. The information in the brain is not processed sequentially but works in a massively parallel way, accounting for the fact that the coordination between relatively slow data processing units can produce relatively fast reactions for complex stimuli.

From the top-down perspective, psychometric studies have shown the mathematical spaces that describe subjective qualities related to pitch, intervals and tonality. These models are a sort of 'average' or overall outcome of the many interactions taking place physio- and psychologically. The evidence for a combined temporal/place operation given by bottom-up descriptions is complemented by the top-down methods with the insight that pitch requires more than one dimension to be accounted for in subjective terms. Both methods provide a wider picture for the duality of pitch perception and its consequent supervention on harmonic situations.

The bottom-up description of auditory processing in the brain goes, in a simplified way, as follows: once the auditory information has been transduced into the auditory nerve, it proceeds to several auditory centers in the brain stem. The first one is the Cochlear Nucleus, where several kinds of neurons specialize in detecting onsets of tones, patterns of peaks or the ending segments of stimuli, information which will be further processed when projected higher up in the auditory pathway. Other neural circuits perform more complex roles, such as decodification of amplitude modulations in the signal. Various types of amplitude modulations are relevant for pitch processing, the periodicity envelope being an amplitude variation resulting from the overall effect of many interacting partial components.

Temporal autocorrelation has been neurologically explained through delayed and undelayed responses of neural networks to the periodicity carrying envelopes of a signal. Two neural circuits, an oscillator and an integrator, are synchronized to the signal envelope, producing an output signal when they coincide. This signal corresponds to integer multiples of the carrier waveform in the amplitude modulated signal. This model (supplemented with 'volley principle' mentioned earlier) can account for the temporal codification of periodicities up to 1 kHz and even higher (up to 3-5 kHz). Many of these circuits running in parallel are necessary to cover the known spectral and temporal ranges of hearing[30].

The next station where axons from the Cochlear Nucleus end up is a crucial auditory structure called the Inferior Colliculus (IC)[31], the main center of tone perception. Its central nucleus has an arrangement of neural nets where the temporal information concerning periodic signals (synchronized spikes in nerve fibers) is transformed into a spatial representation (a neural map). This is the station where the 'high' and 'low' spatial attributes of pitch arise out of temporal periodicity information. It is also the processing center where the constitution of the auditory filters and critical bandwidths is achieved. Its tonotopic structure enhances and alters the cochlear tuning curves through a further filtering performed by neurons tuned to specific frequencies.

This three dimensional matrix consists of around 30-40 laminae of around 30x50 neurons each. One axis (the one with 30 neurons) codifies *tonotopy*, the frequencies of the spectral content of the signal arriving from the basilar membrane. The second axis (of around 50 neurons) corresponds to *periodotopy,* arriving from the temporal analysis performed in the Cochlear Nucleus. Periodotopy lies orthogonal to tonotopy and has a range of variation of around five octaves for each column of constant characteristic frequency CF, in some cases going all the way down to 10 Hz, and going up to a fourth of the CF. Periodotopy is also known as 'best modulation frequency', indicating its derivation from the amplitude demodulation having been performed lower down the pathway. Both characteristic and modulation frequencies can be thought in terms of amplitude modulation, as carrier and modulator. Perception mechanisms concurrently codify both periodic and timbral features of sounds and these modulations cover a span that goes from the rate at which musical articulations happen (such as tremolo and other changes in the amplitude envelope related to onsets and release portions of sounds) all the way up to the 'inner' modulations in the timbre (roughness and timbral effects related to AM such as spectral brightness – as in the electronic music technique of ring modulation).

A 500 Hz tone with an amplitude modulation of 50 Hz will activate not only the neuron tuned to detect 20 ms, but also a neuron tuned to 100 Hz (10 ms) because 20 ms is a multiple of 10 ms, as well as one tuned to 200 Hz (5 ms). A neuron tuned to 6-2/3 ms (150 Hz) will also be activated, but to a lesser extent because dividing by 3 may result in a weaker coincidence as, belonging to a prime number power other than 2, it might involve a longer neural path. These responses behave more like comb filters than band pass filters in that one period provokes not one but a whole series of firings. Of all these firings that project higher up into the auditory cortex, their sum and integral will be maximal at the point of highest coincidences, thereby concluding the process of autocorrelation through neural networks with a robust pitch estimation.

Higher up from the IC, information processing in the cortex is linked to short and long term memory. Schemas[32] arise in connection with long term memory: functional organizations of

30 Langner, G. (1997). Temporal Processing of Pitch in the Auditory System. *Journal of New Music Research*, 20, 118-125.

31 La., *small posterior hill*. Most functions in the auditory system pass through this important and relatively large nucleus, which also includes multi sensory connections related to visual, tactile and olfactory pathways.

32 "Cognitive psychologists attempt to specify, through the interpretation of statistical data obtained from experiments, how the mind works. And they often express that working in terms of "mental structures" and "mental processes." In his book on memory, Sir Frederic Bartlett (1886–1969) had introduced this distinction to explain how the memory of a story is first encoded (a process) into a schema (a structure), and then subsequently decoded (another process) as a recollection that may depart in significant ways from the original experience." Gjerdingen, R. (2002). The

neurons that codify structured information, developing characteristic responses to environmental stimuli. They arise through three types of mechanisms: projection, self-organization and association. Projection, as with tonotopy, is the result of a strict order in the wiring between neural centers, maintaining these structures from the periphery up to the highest cortical areas. Self-organization has been the focus of computer simulations on how higher level response functions arise out of the activation patterns of simple neural networks exposed to musical stimuli. This is how structures resembling torus-like spaces have been found in simulations regarding pitch and tonality. Association is less close to perception and more an attribute of higher cognitive functions, linking, relating and unifying diverse modes of information processing (such as sense modalities) into higher level structures, also involving the imagination.

With respect to music, brain scanning techniques have shown some general functions of the auditory cortex[33]. The primary auditory cortex identifies basic musical featured such as pitch and intensity, the secondary cortex deals with melodic, harmonic and rhythmic patterns, while the tertiary auditory cortex is thought to integrate these patterns into an overall perception of the music. These three functions correspond to the three main time scales in music (sound materials, mid size phrases/textures and large scale forms).

1.2.4 Pitch height and chroma

Pitch height has 'low' and 'high' as its main attributes, carrying also within itself timbral aspects such as spectral envelope (defining source, instrumental family and register). Pitch chroma, also called 'pitch tonality' or 'tonal quality', refers to periodicity pitch as well as to the musical categories of pitch classes, which are independent of register and timbre. It is specified as the quality that makes a pitch different from others inside an octave, such as the fact that all C's, C#'s, D's, etc., in different octaves possess a *qualia* of their own. The fact that melodies can be transposed to any arbitrary pitch shows that chroma happens not in relation to absolute, but to *relative* pitch, hence concerning intervallic qualities ('octave', 'fifth' or 'seventh') rather than absolute pitches. In relative hearing it is meaningless to refer to a 'G-sharpness', but nevertheless chroma is usually understood as the 'colors' through which pitch perception traverses a 'registerless' octave, a conception which is very different from the usual experience of listening to melodic 'characters' as functions within a harmonic or scalar context (such as tonic, mediant, dominant, leading note, etc). These aspects are not distinguished by the concept of chroma, rendering this concept less useful for harmony as one might have expected.

Both pitch dimensions are represented by a helical image where chromas spiral around a circle which is stretched upward like a spring by the height dimension, each circle completing a cycle at the octaves[34]. Compounding the problem with the definition of chroma is the fact that most researchers assume only 12 chromas per octave, picturing the circle as equally spaced, conflating a proportional with a logarithmic conception. It's as if at each semitone a new chroma would appear, which is not the way musical intervals behave. This mistake is linked to an uncritical inheritance of a much simplified functional music theory. The loci in the pitch distance line (or octave equivalent circle for that matter) where proportions arise corresponds to an irregularly spaced grid related to the stacking of divisions by whole numbers. Moreover, it is known since antiquity that there exist

Psychology of Music. In Christensen, T. (Ed.), *The Cambridge History of Western Music Theory.* Cambridge: Cambridge University Press, 971-972.

33 Abbot, A. (2002). Neurobiology: Music, maestro, please! *Nature*, 416, 12-14.

34 This model comes from Roger Shepard, who in the 1960's developed it from ideas going back to Drobisch (1846) and Révész (1913). See Shepard, R. (1964). Circularity in Judgements of Relative Pitch, *Journal of the Acoustical Society of America*, 136(3), 2346–2353. See also Figure 1.

much more than 12 intervals in an octave, and even in the case of twelve note equal temperament the same interval can have different functions depending on its context, each function related to a different intervallic identity.

Of interest for our discussion are studies whose aim has been to map the subjective dimensions of pitch height and chroma in the auditory cortex[35]. Each feature is independently varied by means of synthetic probe tones. These variations are made in order to perform functional magnetic resonance imaging mappings of the brain responses of test subjects. The analyses of these images, both for the group of subjects as a whole and for individuals, have shown that temporal and spatial extraction processes map to distinct regions of the secondary auditory cortex, reflecting the psychological findings of the two component subjective pitch. There is a region in Heschl's gyrus in the secondary auditory cortex which deals with both height and chroma; sections anterior to the gyrus were activated by changes in chroma-only test tones, while sections posterior to the gyrus corresponded to height-only tones. This latter area is known to be specialized in the segregation of multiple sound objects from an auditory scene (timbral identification of sources), while the areas activated by chroma have been shown to relate to melodic processing as well as to the extraction of prosody in speech. Chroma processing relates to the extraction of "coherent information streams that can be analyzed independently of the specific sound source"[36], alluding to the abstract nature of pure pitch patterns devoid of timbre from which melodies are constituted, generally proceeding from a single source. This is a more useful conception of chroma, one which alludes to the 'pitching' of melodic space more than to colors. The results also suggest a connection between prosody and melody.

By encompassing its material and perceptual principles, the theorization of harmony can benefit from these traits of tensions and entanglements. Here we see that both harmonic dimensions are related to dualities such as multiplicity/unity, source/pattern, value/character, and so on. They play themselves out in different but parallel ways, and to greater or lesser degrees at several levels of organization, ranging from the levels of timbre, pitch, melody, chords, tonal and metric fields, and even higher up to formal levels such as sections, movements and pieces as a whole.

1.2.5 Top-down psychological studies of pitch

Psychometrics and other empirical techniques from music psychology have provided the main source of knowledge for top-down descriptions. The literature on the subject is large and outside the topic of our present research, so we'll focus only on aspects that pertain to the multi dimensional character of pitch perception in order to link them to harmonic duality and harmonic space.

Apart from the aforementioned chroma and height spiral proposed by Shepard, Carol Krumhansl[37] has arrived at so-called probe tone ratings of contextual pitch. Listeners were asked to rate how well a single tone fitted against a fixed tonal background (scales, chord cadences or small pieces), varying the tone over the whole chromatic range for each rating. The statistics agreed quite well with the ratings of consonance having been proposed by some music theories, especially in the case of listeners with a musical background. Intervallic sizes were seen to be less important than their tonal function, implying that context and tonal hierarchies have influence beyond the perception of intervals in isolation, and that invariant structures are abstracted at the level of tonal centers or keys

35 Warren, J. D., Uppenkamp S., Patterson, R. D., Griffiths, T. D. (2003). Separating pitch chroma and pitch height in the human brain. *Proceedings National Academy of Sciences*, 100, 10038–10042.

36 Ibid., 10042.

37 Krumhansl, C., Kessler, E. (1982). Tracing the Dynamic Changes in Perceived Tonal Organization in Spatial Representation of Musical Keys. *Psychological Review*, 89(4), 334-368 and Krumhansl, C. (2004). The Cognition of Tonality – as We Know it Today. *Journal of New Music Research*, 33(3), 253-268.

beyond the immediate setting of single notes and chords.

These probe-tone profiles were measured for major and minor tonalities and then compared by correlating all major-major, major-minor and minor-major key pairs to arrive at an 'interkey' distance matrix. This matrix was subjected to a multidimensional scaling analysis, a statistical technique for interpreting psychological data which produces spatial representations based on distance metrics. What obtained was a four dimensional solution construed as a toroidal map depicted as two circles, one of fifths and the other of thirds, together with corresponding relative and parallel minor and major tonalities, which are represented by the variables φ and θ as the angular rotation along each circle in the torus (see *Figure* 1, **B**). This map, obtained independently from music theory, is similar to the harmonic space that we will discuss in Chapter 3, as well as to Hugo Riemann's tone maps or Arnold Schoenberg's regions chart. As has already been delineated previously, my criticism here is that these kinds of studies want to prove aspects of functional tonality and twelve tone equal temperament from a psychological point of view by way of studies which secretly assume the very theories they want to prove, especially because the simple examples given to the test subjects are derived from these theories. In this case though, the researchers are aware of this limitation and do not make claims outside cadential equal-tempered music and what is interesting for us is that these visualizations show to how a more generalized kind of harmonic space can arise as a mental schema.

Through 'cognitive modeling', Leman & Carreras[38] computer simulated a perceptual learning system whose input is acoustic data. It implements inner ear filtering and dynamic range compression, neural firing patterns and periodicity analysis by autocorrelation, together with a second cognition module based on a self-organizing network which, after learning by self-organization, yields a schema of tone center perception as is thought to be carried by neural networks in the brain. Afterwords, it is tested with musical sequences in a similar way to the probe tone ratings, but as a computer simulation, producing results that fit very well with the psychological studies (see *Figure* 1, **D** & **E**).

Probe tones relate to the Aristoxenian concept of *dynamis*, to be dealt with in the next section. They also relate to the statistical weights of pitch sets that are part of the development of stochastic harmonic fields (elucidated in Chapter 3). Other memory and patterning processes in pitch sets, such as Markov chains, which add orderings to the probabilities of each interval, can be layered on top and understood in relation to probe tone ratings: as potentials or probabilities that structure pitch at a level beyond the immediate intervallic time frame, a level lying between the note, the motive and reaching out towards the phrase.

38 Leman, M. (1997). The Convergence Paradigm, Ecological Modeling, and Context-Dependent Pitch Perception. *Journal of New Music Research*, 26,133-153. Also see Leman, M. and Carreras, F. (1997). Schema and Gestalt: Testing the Hypothesis of Psychoneural Isomorphism by Computer Simulation, in M. Leman (Ed.), *Music, Gestalt and Computing*, Berlin: Springer, 144-168.

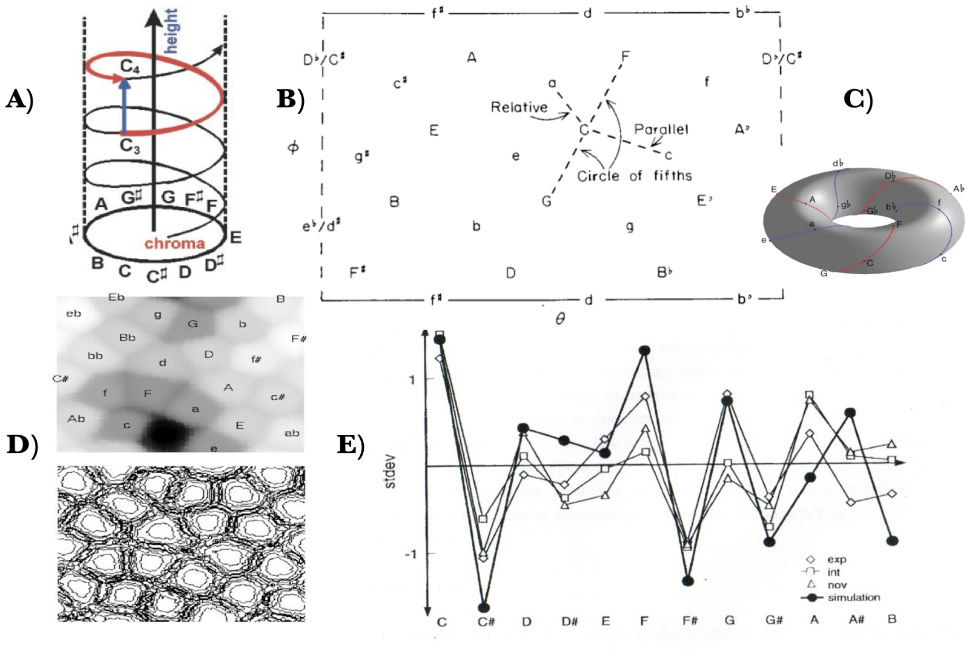

Figure 1. A) Shepard's helical pitch structure. (From Warren et al., 2003.) B) Toroidal map from multidimensional analysis of interkey probe tone profiles. (From Krumhansl, 1982.) The dimensions are closed in on themselves, so that they wrap around at the edges to form a torus, as can be seen in C). D) Self-organizing map derived from Leman's cognitive model. E) Probe tone ratings (for experts, intermediate and novices) for major chords in the key of C, plus computer simulations arrived at from the previous mapping. (From Leman & Carreras, 1997.)

1.2.6 Pitch research in relation to harmony, melody and timbre

In the past, research into pitch was guided mostly by musical considerations but nowadays it encompasses a much wider field of sonic phenomena. After having run through the workings of pitch perception models, now it is of interest to see how they relate back to musical topics. We will do this in relation with harmonic duality and also to sum up some of the distinctions and findings that have been made up to this point.

No pitch model explains nor predicts octave equivalence. It is neither an assumption nor an emergent property of the models. Why, then, is it one of the most striking features of pitch perception? Moreover, it seems to be an exclusively auditory phenomenon having no equivalent in other sense modalities – to consider musical tone color as relating to visual color and frequency leads to the conflicting fact there are no visual octave relations[39]. There is physiological evidence of laminae with octave architecture in the Inferior Colliculus and in the next station in the auditory pathway, the Mediate Geniculate Body in the thalamus, which seem to process pitch classes within each lamina. Both the basilar membrane and the tonotopic/periodotopic maps in the IC behave as approximately logarithmic transducers, especially for higher frequencies. Ernst Weber's law states

39 Visual perception does occur within a span of slightly less than an octave (from 390 to 770 trillion Hz), although the main difference between the two senses has to do with superposition. In audio, superposed frequencies are perceived as increasing in quantity from single to multiple, whereas visually, color frequency superposition results in different hues of single colors. The analogous visual percept of pitch would more likely be the direction of incident light rays, as they strike different regions of the retina in a transduction similar to that of the basilar membrane. See Roederer, J. (2008). *Physics and Psychophysics of Music*, Berlin: Springer, 174 and its footnote.

that most senses transduce physical stimuli logarithmically, meaning in this case that a double amount of physical intensity would correspond to a single step increase in sensation. Putting aside the fact that transducers follow the law only approximately, physically there would be no reason to expect its exponent to be exactly the integer 2. Many sensory transductions have been measured, none of them having an integer constant[40]. Its prominence could arise not because octaves are special to perception, but conversely, that is, because the acoustic structure of octaves produces singular perceptual effects. Being the only interval where the partials of the upper tone match all the frequencies of the lower tone could mean that octaves require less operations for the pitch processor, and hence stand out from other intervals. It seems that both motivations must be operative and that some perceptual mechanisms might have evolved as adaptations to the specific makeup of octaves.

Similar relations of equivalence can also exist within other intervallic spans, the most fundamental being those with prime factors other than 2, such as 3, 5, 7, etc., (quintal, tertial, septimal equivalences), but it's possible to conceive other intervals than those. There are many problems, however, in making compositional materials out of these new equivalences, firstly, because octave equivalence is stronger and will mask and interfere with the 'chromas' within the other fundamental intervals, so appropriate musical contexts have to be devised in order for these different relationships to stand out; secondly, because many of these 'chromas', say, within a twelfth (3) or tenth plus octave (5), happen to be the same than those within an octave, making the differentiation more difficult; thirdly, obtaining these chromas is not a trivial question, they don't arise unavoidably, but have to be generated by some process of division (by proportional – harmonic or arithmetic – or continuous – geometric – means)[41], and this does not in any way guarantee that these 'chromas' will be experienced as belonging in any way to their generator. The case of 7 as a generator seems to be promising, and I will discuss in due course some strategies for making some kind of perceptible septimal equivalence pitch sets.

Another topic of pitch perception of interest to harmony is that of dynamic pitch, as it happens in speech as well as with glissandos. It poses big challenges to pitch models, whose design is based on stationary pitch. It can conceivably arise through different mechanisms from static pitch, and it has been hypothesized that stable pitch could even arise from dynamic pitch mechanisms. It also involves the question of how much frequency modulation the pitch mechanism is capable of tracking and to what extent frequency modulation might be transformed into amplitude modulation in the extraction process. Regarding harmonic duality, dynamic pitch belongs to the timbral dimension as it lacks a fixed periodicity. Whatever the pitch gradient detection mechanism might consist in, it most likely interacts with periodicity processors, so depending on the situation, a constantly moving pitch might make the entwinement of the two aspects give way to the domination of the spectral facet or provide space for a combination of both[42].

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40 E. H. Weber's psychophysical law, 'the size of a just noticeable difference is proportional to the stimulus intensity', 1834, was furthered by G. T. Fechner, '$S = k \log I + C$; S = sensory magnitude, I = stimulus intensity, k and C = constants', 1860, later refined by J. A. F. Plateau, 'equal stimulus ratios produce equal sensation ratios', $S = k\,I^n$; sensory magnitude = a constant times a stimulus intensity to the power of n, 1872. This began to be measurable in practice until the 1930's, giving way to the first subjective scales (Fletcher-Mundson curves for loudness, mel scale of pitch, and many others). Stanley Stevens later generalized the law, measuring many sensory conditions (heat, color, intensity, taste, smell, cold, tactility, etc, as well as auditory pitch and loudness, 1950's). He does not encounter any integer proportionality constants. (Information gathered from Warren, *op. cit*, 108-109 and Weber-Fechner Law. In *Wikipedia.* Last retrieved November 16, 2012, from http://en.wikipedia.org/wiki/Weber-Fechner_law

41 To be discussed in the following section on Greek harmonics. The best known example of a tuning and scale which has a different module than an octave is the Bohlen-Pierce scale, which divides the 'tritave' (twelfth) geometrically into 13 equal divisions.

42 James Tenney's glissando pieces, starting from *For Ann (rising)* (1969), pertain to the ambiguity between chroma and pitch height as embodied in Shepard tones. In *Koan* for String Quartet (1984), discrete periodicities are made to stand out from within the continuum.

A further issue related to pitch, even more associated to musical composition, is the blurring of timbre and pitch, or more precisely, the situations that bring out multiple pitches within timbre, in connection to what is called 'multiple pitch' in auditory research. It is interesting to conceive timbral variations in terms of pitch, contemplating a territory that lies between a steady single pitch (a sine tone) and very fast frequency fluctuations of that pitch producing different kinds of noise, depending on the speed and span of the variations. The realms in this continuum extend from simple to more or less complex overtone structures, transmuting into ever more inharmonic dispositions of partials, reaching out all the way up to saturated and unstable overtones, of which there are a large amount of varieties and densities culminating in the full range fluctuations of white noise[43]. The territory within these poles where the perception of pitch transitions from single to multiple lies more or less where partials become inharmonic and begin to fluctuate, before their density and speed surpasses certain perceptible complexity. Perceptually, this threshold is dependent on loudness, involves context, requires time and usually implies an analytic mode of listening. When it does happen (for instance after a sustained and somewhat loud beating of a tam-tam or a triangle[44]), recognition can easily transmute between a timbre with pitch and multiple pitches within timbre, depending on various factors, some physical (related to the properties of the sound emitting materials as well as the movement of waves within the acoustic space), some psychoacoustic (involving thresholds of fusion/separation of partials) and some mental (dependent on modes of attention). The zone that hovers above single timbre and multiple pitch is interesting for composition inasmuch as its delineates regions of ambiguity and separation between periodicity and spectrum. Compositional mediation can amplify, zoom-in, break/attract, slow down/accelerate, or accentuate/soften these timbral components.

We should finally also mention a distinction made between a source of excitation and a filtering-resonance system. Timbre has several facets in relation to pitch: up-down height (register) and spectral constitution. They are named pitch-height and tone-height, and pertain to the spectral fine structure (the relative amplitudes and distances between the partials) and the spectral envelope of a sound (its global spectral profile), respectively. Both can be independently varied as when a vowel is changed in pitch, changing the fine structure (the excitation pattern) but not the resonance regions (the formants); inversely, one can change the vowel but not the pitch, varying only the tone-height. This source-filter model shows two dimensions operative within timbre, discerning between source recognition (related both to excitation and filter), variations of timbre within a single source (related mostly to filter), differences between families of sources (wind, string, percussion, etc) and internal range of *tessituras* (reliant on the excitation mechanism)[45].

1.2.7 Infrapitch and rhythm

Even though this larger section is mainly concerned with pitch in relation to harmony, an important aim in this study is to show how harmony can be conceived at several time scales and what properties and differences it has in each domain. We will open up the discussion of the rhythmic realm in its relation to harmony by pursuing some of these links.

43 This line of thought comes from Clarence Barlow's ISIS synthesis method. See Barlow, C. (2005). ISIS, an alternative approach to sound waves. Royal Conservatory, The Hague.

44 I can think of some compositions which take as a point of departure the phenomenon of multiple pitch in its relation to timbre, beginning with LaMonte Young's *Composition 1960 #1* (a very long drone of a perfect fifth), through Tenney's *Having never written a note for percussion* (1971, a very long tam-tam crescendo-diminuendo) and Alvin Lucier's *Silver Streetcar for the Orchestra* (1988, for triangle, also producing moving hyperbolic interference patterns in space).

45 See Patterson, R. D., et al. (2010). The Perception of Family and Register in Musical Tones. In M. Riess, R. Fay and A. Popper (Eds.), *Music Perception.* New York: Springer, 13-50.

Some features of pitch spill outside its normal range of operation, mapping aspects of harmonic duality into other perceptual registers. There are studies which have thrown insights into the ranges of operation of the two aspects and mechanisms of pitch perception, charting their registers and showing how periodicity analysis works well below the range of hearing in the cochlea and is connected to the zone of rhythm and articulation[46]. 'Repeating Frozen Noise' (RFN) tones, consisting of iterated repetitions of white noise segments, have been used to probe into the infrapitch regions. They are generalized periodic stimuli: their periodicity can lie below the audible range but still have audible overtones; deriving from noise, their partials have uncorrelated random amplitudes and phases, not restricted in their waveform or spectrum; they also possess a rich timbral quality. At infrapitch periods, the close spacing of these partials means that they are always unresolved by the basilar membrane, mapping their pitch periodotopically while projecting a 'pitchless' timbre tonotopically. Each different noise sample will have a different but rich timbral quality.

These noise segments can be heard as global percepts of iterance over a range of around 15 octaves, from 0.5 Hz to around 8 kHz (see Figure 2). Above that, all harmonics lie beyond the range of hearing, so the tones become indistinguishable from sine waves. Below that they have a timbre which, due to the spacing of partials becomes especially rich below 1000 Hz. Below 100 Hz, it begins to have a hissing component, being a continuous percept from 100 to 70 Hz and a pulsed one from 70 to 20 Hz. According to their features and qualities, the 5 octaves in the infrapitch range (20 to 0.5 Hz) have been divided into high and low ranges. Periodicities from 4 to 20 Hz are reminiscent of machines, lacking discrete component events, their quality described as 'motoboating'; the low range (4 to 0.5 Hz) have been described as 'whooshing'. High range periods correspond in time scale to phonemes, the low range corresponds to syllables and words. Periods higher than 2 s do produce iterance percepts, although they are broken into limited portions of the waveform. Even so, periods can be detected for up to 20 s. They apply well into the musical range of melodic phrases and small scale form.

For most part of the hearing range, place and time mechanisms overlap, precisely where music happens: between 20-40 and 3-5,000 Hz. In this region proportion and timbre are entangled. Place perception spans from the limit of low pitch up to the end of the hearing range, the highest octave belonging exclusively to timbre (labelled 'amelodic'). Time perception comprises periodicity below the auditory range of cochlear perception, encompassing both rhythm and pitch and involving two mechanisms within its operation. One, pertaining to unresolved harmonics, spans from infrapitch up to around 2,5 kHz, the limit of unresolved harmonics, using the mechanism for periodicity detection ('complex pattern') of periodotopic activation of neurons in the inferior colliculus. The second mechanism, based upon resolved harmonics, involves phase locking in the cochlear nucleus lower along the pathway.

Beyond the laboratory, extra cochlear aspects of periodicity perception also incorporate other sense modalities belonging more to the body than to the head and ear, such as proprioception and kinesthesia[47], as it happens in a situation of a collective of musicians synchronizing to a common rhythm (the fine tuning of their articulations to some groove), realized more through the body than the ear, by means of low frequency resonance.

46 Warren, R. (2008). *Auditory Perception. An analysis and synthesis.* Cambridge: Cambridge University Press, 82-91.

47 Proprioception is "the sense that indicates whether the body is moving with the required effort, as well as where the various parts of the body are located in relation to each other." Proprioception. In *Wikipedia.* Last retrieved September 12, 2012, from http://en.wikipedia.org/wiki/Proprioception. It involves muscle memory, equilibrium, balance and position and is also conveyed by internal organs such as bowels. Kinesthesia refers more exclusively to bodily perception of (self) movement. Many internal organs resonate to low frequency sound waves and serve as sensors by transmitting their movements to the cerebellum, where the signals are integrated.

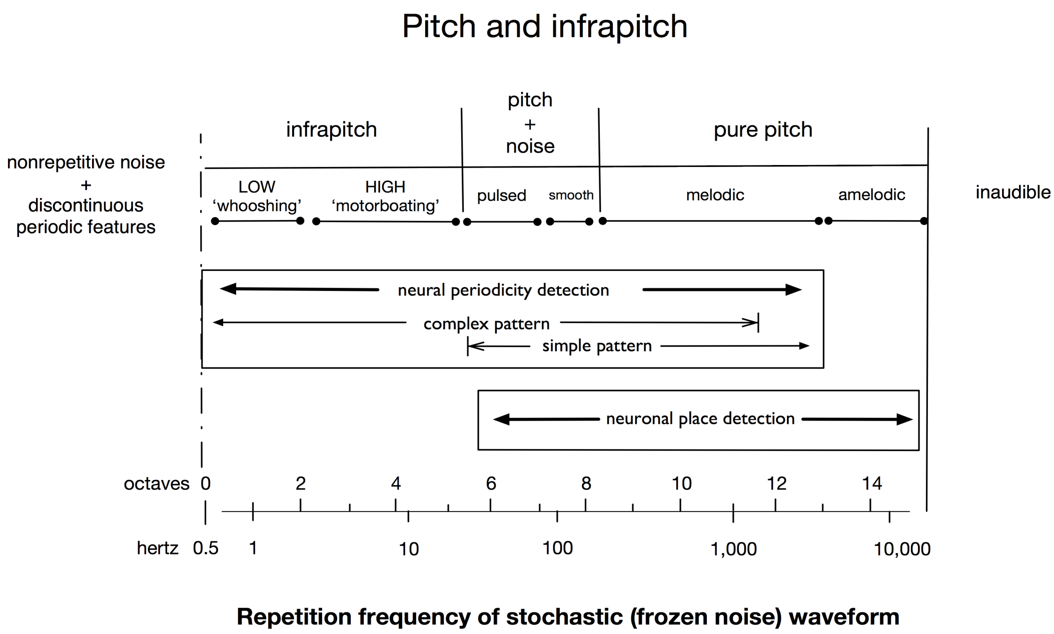

Figure 2. The iterance continuum. 15 octaves over which global percepts can be heard for repeated segments of noise. The upper portion of the figure describes perceptual characteristics; transitions between the given qualities are gradual, category boundaries shown at the approximate positions along with their neurological mechanisms. (From Warren, R. 2004).

Rhythm entails periodicities of acoustic patterns that correspond to slowed down versions of intervallic proportions, meaning that there is a quality of consonance or dissonance to rhythmic patterns. Understood as infrapitch analogues of pitch-range phenomena, harmonic concepts can pertain to the rhythmic domain, though the parallelism is not straightforward since each domain has its own specificities. Rhythmic consonance should be more sensitive to the complexities of periodicities involved, as is the case with quintuplets, which are already quite more difficult to 'digest' (to use a term from Clarence Barlow) than their analogues in the pitch range (corresponding to thirds). Another difference is that phase phenomena are crucial and inherent to rhythm, while in pitch they are mostly inaudible (or blend as part of the timbre but are not relevant proportionally). Phase plays an important role in metric accents and other kinds of rhythmic displacements. Rhythm is made of pulses, concerning onsets more than continuations.

With the aid of mathematical graph theory, Justin London[48] analyses the psychological spaces corresponding to pitch and rhythm, showing them to be topologically different and arguing that pitch and rhythm phenomena do not correspond isomorphically to each other. Perceptual phase transitions ensue different emergent properties for objects in each realm, so more than trying to ensure isomorphic relations, what these spaces reveal are key differences between the two domains. Pitch properties have structures similar to Riemann *Tonnetz* and the psychological spaces we have reviewed, while 'metric spaces' have disconnected graphs that are non-planar (their edges cross) as well as highly dependent on tempo, changing the limits and perceptibility properties of each metric graph as a function of tempo. Tonal spaces, on the other hand, are planar (no crossings between nodes and edges) and independent of the tonic, showing how, for pitch, there is no 'tempo': all pitches behave in an 'absolute' way with respect to their inner rhythmic constituents (see Figure 3). Perceptual phase transitions are categorical borderlands that register changes belonging to different

48 London, J. (2002). Some Non-Isomorphisms between Pitch and Time. *Journal of Music Theory*, 46(1/2), 127-151.

qualitative realms, each with its own properties, despite there being a continuity in the physical processes grounding them. The same types of phenomena are recognized differently at both sides of the borders, making for autonomous attributes in each realm. This continuum generates a perceptual discontinuum that all the same inherits, transforms and disregards some of these properties.

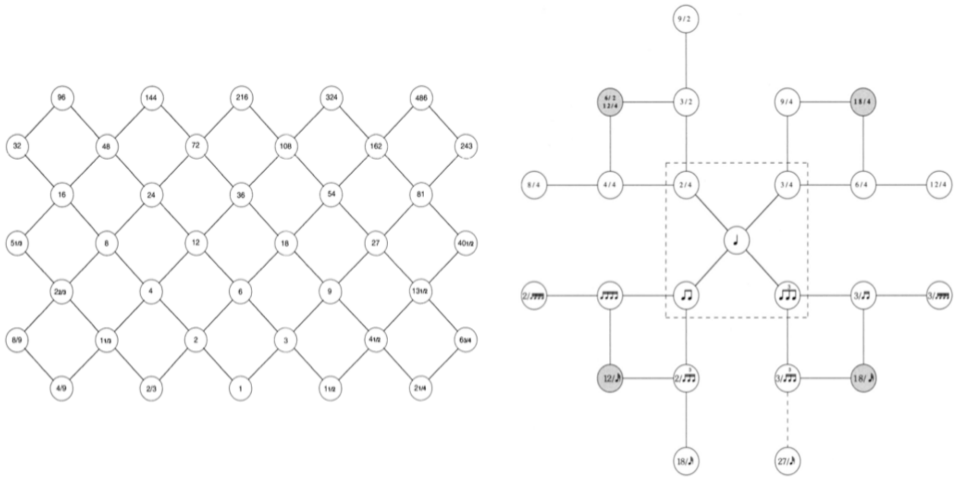

Figure 3. Difference in graph properties between a tone network representation (left) and a metric representation (right). (from London, 2002).

At the pitch/pulse boundary of around 16-20 Hz temporal integration breaks down, so around it lies the zone of disconnection between sound elements. These discrete sound elements become pulses, acoustically different from the integrated sound they were part of. I have the impression that there could be a perceptual analogue to the physical phenomena of acoustic impedance[49]. As is the case in the resonance of tubes in the physics of wind instruments, there happen two types of impedance: bipolar and unipolar. Crossing a boundary results in a change from a bipolar towards a unipolar condition, from alternate current impedance (positive-negative air displacement) in pitch-integrated waves, to direct current impedance (from no displacement to maximum displacement) in the differentiated individual pulses of the rhythmic realm. Each rhythmic element might contain a timbre inside each pulse, but in another sense it is just a DC pulse, an on-off switch. Rhythm and meter are composed of patterns of these on-off elements, beyond the actual sounds that fill them(which are themselves AC: bipolar and integrated). This AC/DC boundary thus marks a distinction within melody between its horizontal (DC) patterns and the vertical (AC) timbres that make up its notes. More research needs to be done with respect to this topic because even if this transition is not happening acoustically but only perceptually, it provides an interesting way in which to conceive a crucial difference between pitch and rhythm.

Notwithstanding their differences, there are some important similarities between pitch and rhythm,

49 "Acoustic impedance Z is the ratio of the acoustic pressure p, measured in Pascals, to the acoustic volume flow, measured in cubic metres per second." *Specific* acoustic impedance z "is an intensive property of a medium. We can specify the z of air or of water. The acoustic impedance Z is the property of a particular area and medium: we can discuss for example the Z of a particular duct. Z usually varies strongly when you change the frequency. The acoustic impedance at a particular frequency indicates how much sound pressure is generated by a given acoustic flow at that frequency." Additionally, "DC (direct current) means constant or slowly varying current. AC (alternating current) means any current in which the movement is alternating backwards and forwards (oscillating) with no overall motion. AC is more interesting because the impedance can vary with the frequency of oscillation of the current.". Taken from Wolfe, J. (2010). What is acoustic impedance and why is it important? Last retrieved June 12, 2011, from http://www.phys.unsw.edu.au/jw/z.html

conducive to an analysis of rhythm in terms of harmony and pitch as well as the other way around. Rhythm shares the same perceptual mechanism as pitch periodicity, but lacks the spectral one. We can nevertheless speak of the timbral and proportional aspects of rhythm, although 'timbral' in this sense would have a different connotation than with pitch. By omitting or binding (slurring) some of its constituent pulses, simple rhythmic relationships can become fluid, blurring their periodicities and appearing more complex and 'floating', incorporating elements of noise within rhythmic intervallic proportions[50]. The 'irrational', slurred rhythms and cloud-like formations archetypal of atonal modernism, are perceived as unit-free, and can be therefore be considered 'timbral' in that they seem more like a flux than a pattern.

Another sense in which there is a timbral or continuous facet to proportional rhythms lies in the way metric patterns are articulated. The proportional facet deals with rhythmic layers in terms of relationships and numeric ratios, even if these ratios are not exact in performance: there is tolerance with respect to rhythmic phenomena, even more sensitive in its effects than in the pitch domain, maybe because of the higher allowance given by the slower speed at which this happens. 'Groove', the systematic displacement of rhythmic positions, can be understood in a harmonic light as a timbral tolerance acting on proportional patterns, as they are still inferred as being exact, even if conveyed through this distorted medium. This special, sensitive and not easily describable musical feature can provide varied 'colorings' or 'enhancements' to the same pattern, a crucial aspect of many musical styles. Once a groove fails to convey the rhythmic pattern it belongs to by transgressing its tolerance limits, it stops being a groove to a pattern, becoming instead another pattern.

Rhythm relates to multiples and divisors of its pulses much like pitch, and analogies such as tempo octaves and other rhythmic partials (subdivision or multiplication by 2, 3, 5 etc.) make a lot of sense, with augmentation/diminution of patterns being an octave equivalence of sorts. This concerns rhythm's vertical, multiplicative dimension, in contrast to its horizontal, additive dimension (its metric sense). There are also pitch range analogues of rhythm, as with early consonance/dissonance theories, which were stated in rhythmic terms. Accents and other phase phenomena in rhythm also relate to spectral features when accelerated up to pitch speed. The concept that best bridges the similarities between the two domains is also the string and *resonance* is the term that best applies to both harmony and rhythm. We will also see further on that some of these aspects also inherit to higher levels of composition, such as larger sequences, sections, and forms, as there are formal analogues to rhythmic and harmonic phenomena, although their appearance is also altered with respect to their original realms, though sharing properties which are still related to harmonic duality.

1.2.8 Concluding remarks

The pitch related *qualias* which we have been surveying, such as chroma, timbre, and their supervenient properties of harmonicity and consonance are established principally on a subcortical level of auditory processing (from the ear to the brain stem). Notwithstanding the role of cognitive top-down processing, as well as the various modes of attention, it is psychoacoustic constraints that for the most part determine the harmonic properties of compositional materials. These constraints provide the starting points for a perceptually informed (and computationally assisted) harmonic research from which the potentials and possible functionalities arising out of these materials are experimented with, in order to build and tryout, in practice, the methods or 'logics' that are the consequences and extensions of these harmonic properties into the wider scale of musical forms. This is done in rapport with a a theorization that picks up from these findings in order to generalize and systematize them, serving as a platform for the formulation and speculation of strategies to be put again to the test, leading again back into the first stage. This cycle of musical experimentation

50 As mentioned in Stockhausen, K. (1957) ...How Time Passes... (C. Cardew, Trans., 1959). *Die Riehe*, 3, 29.

consists of trials and examinations, reflections and modifications, jumping back and forth between the levels of praxis and theory, intermingled with evaluations, analysis, intuitions and serendipities that lead as much to interesting discoveries as to dead ends. Its repeating form resembles more a spiral than a circle, opening up with each new turn in ways that are quite similar to intervallic spirals, never returning to the same interval twice.

As can be seen after laying the ground in this section for establishing the psychoacoustic aspects of pitch, the crux of this research hinges on compositional materials, although the main focus will revolve around methods for incorporating these materials into a musical structure, discovering and inventing the 'logics' that can set these 'onto-logics' into motion. Around the second half of the twentieth century arose what is now a more or less established line of thought that considers three levels of composition, namely material, method and form, as set out by some important experimental composers[51]. We could now say that with respect to our harmonic research, sonic materials function, as it were, ontologically, in the sense that they define basic musical entities. This is not ontology in the strict philosophical sense, as the term is used to denote the basic constituents or objects used for composition as seen from a perceptual perspective; not as the inherent being of any generalized musical material whatsoever, but only in relation to their 'perceptual being', and it is in this sense that this project subscribes to a 'perceptual ontology' of sorts. These materials are also ontological in the sense that they lie 'outside time' (Xenakis), as their properties are considered in isolation from their musical disposition, as is the case of intervals, scales, probabilities, dissonance metrics, rhythmic patterns, etc.

The next compositional level, that of method or logic, focuses on the coherence and consistency of these sonic entities, their relations and motions 'inside time', that is to say, in a musical context. This is where most of the fun happens in compositional research, where theorization is put to use. If this chapter serves to set the stage for the characterization of harmonic materials, then it will be in the other chapters that the development of relations and logics between them shall take place.

Regarding the third level of what Tenney calls 'aesthetic experience', we could mention that it revolves around the musical dramaturgy of a work. It involves relations of relations, or structures and forms, and we will try to see how far some harmonic concepts can be extended to this level of scale. It also involves extra-musical considerations, be they the kind of experience that is intended for a piece, or, in more contemporary terms, the kinds of narratives, connections, causalities (or lack thereof) that string together the musical forms contained in a work, also in sonic situations that lack directionality, as with non-narrative forms. They are also extra-musical in the sense of pertaining to ideas beyond music itself, be they inspired by other artistic disciplines, political or social issues, myths, maths, specific circumstances of a piece or installation, and many other etc's. This is the level in which thought invests matter, being in close rapport with the level of compositional logics, as it defines, selects and alters these logics according to an overriding aesthetic design, concept or dramaturgy. The three levels influence each other in different and complex ways (in a manner reminiscent of 'bottom-up' and 'top-down' processes), and behind these considerations lies the intention to keep the theorization of materials as indifferent as possible to their aesthetic uses in order to take full advantage of their behaviors, leaving open the prospect of materials influencing and suggesting dramaturgies or, conversely, knowing when aesthetic decisions require us to ignore or modify the way the qualities of these materials are put to use.

We can escape the danger of falling prey to the dispute between 'nature' and 'culture', or between 'cultural conventions' and 'biological determinism' by having a clear idea of where and how the

51 These concepts begin with Arnold Schoenberg, but I'm thinking more on the formulations by John Cage, who focused more on the first two categories (and divided form into structure and form), and also others such as Iannis Xenakis and Karlheinz Stockhausen. This approach is quite independent of aesthetic school, though they are all share an interest in systematic composition.

levels of imbrication of these aspects stand and by keeping a healthy dose of skepticism for those approaches that might reduce musical entities to either of the two extremes. It is important to be able to 'modulate' all the given and inherited materials and arbitrarily bend them into whatever direction is required by the compositional decisions. It is also as important to be able to listen and pay attention to the materials, letting them kick back and have their say, not serving only to be meddled with. The neurobiological basis of harmonic structures assists by delineating some of their material characteristics, with the intention that they remain aesthetically neutral. However their characteristics are used, their main requirement is for them to open up paths for compositional speculation and experimentation.

Having said all this, it may sound paradoxical to say that the psychological aspect of pitch and harmonic materials is not even be that important after all. I do not claim that harmonic materials arise exclusively in the human mind. Beyond psychology or biology, some of these properties are inherent to the sound emitting objects themselves, to the acoustic waves or to the numbers lying behind the definition of their patterns. Some proportional aspects of these materials lie beyond psychology, and involving, as we will see in the next section regarding *aisthesis*, not only sensation but also intellection. The cognitive approach to the study of consciousness focuses on efficient causes, when, as we will see, it is the other kinds of Aristotelian causes that are as or even more important for music, precisely the causes that lie outside temporality (formal and material cause). .

Our survey of the psychological and cognitive processes responsible for pitch and lying behind harmony has been done not to reduce harmonic materials to biological processes, but to find in all these loosely connected bunches of facts evidence for and a characterization of harmonic duality in order to map out possibilities to be exploited in algorithmic composition. As we will see in the following section, harmonic duality can be characterized from a very different point of view, that of Greek harmonics, and both readings, the biological and the historical-metaphysical help to delineate and underpin a harmony much more robust and interesting than if it came from a single perspective. What's more, and I see it as an advantage, these accounts are supplementary and do not correspond easily to each other, each involving a quite different register of thought.

1.3 Greek Harmonics

The duality of harmony can be traced back to the Greek harmonists by journeying through some of their ideas in the interest of differentiating and characterizing its attributes. The excursion also serves to introduce and discuss many elements and concepts that a renewed and present day harmony needs to incorporate. A rereading of Greek theory can help unmuddle, restore and reinterpret ideas masked or ignored by conservatory harmony as a result of layers accumulated over the centuries. Being historical, the review is nonetheless focused from a present perspective with a view on opening up new compositional possibilities. A reevaluation of some implications of Pythagoreanism, much underestimated in our times, concludes the section.

The science of harmonics[52] (*harmonkê*) was a companion to the sciences of rhythmics and metrics whose task was together to classify and describe the regular and repeated patterns of form and structure underlying the diversity of melodic, rhythmic or metric sequences in music. Metrics deals with patterns formed by lengths of syllables in verse, rhythmics with patterns within which sequences of long and short syllables are divided and grouped into repeated structures, and harmonics with the structures underlying melody (*melos*). The harmonists set out to identify the varieties of scales and tuning systems which could be reckoned as musical, a task that implied finding quantitive representations for intervals and melodies, classifying scales and their transformations, and seeking underlying fundamental principles behind these structures. Questions such as their rooting in human culture or in something independent of humans, or in mathematics, as well as the status of their applicability beyond the musical sphere were the kind of issues raised and discussed by harmonists. As such, it was a full blown science in the sense of a discipline to discover and demonstrate a body of truths, regardless of whether they could be assimilated to mathematical sciences or to the 'sciences of nature' (*physiologia*). Needless to say, this is very much in sync with our project.

1.3.1 The two schools of harmonics

The two main doctrines within harmonics are the mathematical and empirical. They are fundamentally and irreconcilably opposed in their premises, methods, and aims. The first group is epitomized by Pythagoras, even though there are only second hand accounts of him. Instead there are many Pythagorean scholars, some more mathematical, more philosophical or metaphysical than others, some more practical and linked to musical practice. Their basic premise is that there is a strong connection between pitch intervals and whole numbers, the account ranging from straightforward correlations up to outright cosmological accounts. The earliest mathematical harmonist of whom there are surviving fragments of text is Philolalus, whose ideas fall into the metaphysical kind, identifying the structure of the cosmos with the proportions of the main concords inside the octave and describing the world as a harmony (*harmozein*, 'to fit together') between the unlimited (continua) and the limited (which set limits through shapes and other discrete structures). Beyond its philosophical aims, it already gives us an initial and generalized definition of

52 Most of the information here comes from Barker, A. (2007). *The Science of Harmonics in classical Greece.* Cambridge: Cambridge University Press, and from Crocker, R. (1964). Pythagorean Mathematics and Music. *The Journal of Art Criticism*, 22(2 and 3). There is no intention of providing an overview of Greek music or music theory, but to give most of the relevant information pertaining to harmonic duality. Other sources are Chalmers, J. (1992), *Divisions of the Tetrachord.* Lebanon, NH: Frog Peak, and a bit of Partch, H. (1974). *Genesis of a Music.* New York: Da Capo Press. Other sources shall be mentioned when discussed.

harmony: the fitting together of disparate elements as well the tension between continua and discontinua: a musical scale is the limiting of the continuum of sound by ratios[53]. Plato follows this line of reasoning in the *Timaeus*, portraying the Demiurge and the creation of the World Soul from principles stemming from harmonic science, though they are put to use in a purely metaphysical way, beyond any musical reality. Devoid of sensory considerations, concords and discords were properties of numbers, having no relation to actual sounding tones.

Of the other Pythagoreans, Archytas is far more interesting for us as he takes a route that correlates numbers, interval ratios, sounds as heard, music as practiced, and physical accounts of sounds. He provides what is perhaps the first example of psychophysics, inaugurating musical science. Later harmonists, the most significant living between the fifth and the third centuries B.C., included Aristotle, who concentrated on metaphysics, and later on Euclid, who tried to systematize earlier systems to the greatest extent possible. Ptolemy's approach, in the second century A.D., summed up the most relevant developments of mathematical harmonics along with empirical backing while furthering the music of the spheres in connection with his astronomical interests.

On the side of the empirical harmonists the important figure is Aristoxenus, whose treatise *Elementa Harmonica* is the largest and most complete extant treatise on empirical harmonics (and of any kind of harmonics for that matter) of that era, providing much information regarding his school and the critiques leveled against mathematical (and other empirical) harmonists. His purpose is to make of empirical harmonics a science as rigorous and systematic in method as mathematical harmonics (he was a student of Aristotle and took cue from his account of what good science should be), despite the fact that their aims and procedures were incompatible.

The main characteristic of empirical harmonics is its emphasis on music as heard and the ear as the ultimate judge of musical materials. Instead of measuring intervals with discrete ratios, Aristoxenus measures them in terms of distances in a continuous linear space. Instead of associating the consonance of an interval to the arithmetic properties of ratios, he took their consonances and magnitudes as given facts. Since intervals could be slightly mistuned but still perceived as belonging to the same intervallic category, this was taken to mean that even the principal concords of the scale had a narrow range of variation. Notes exist as points along the continuum enclosed within tolerance ranges. This is the first instance of the crucial concept of harmonic tolerance.

Instead of taking sides with one point of view or the other, what we want is to sort out various themes that traverse harmonic duality because they stand out from the approaches and theories of both schools – even though each doctrine has certain dependencies on the other and does not map cleanly into each side of the duality. What I have referred to as the proportional aspect of harmony is connected with mathematical, while the timbral attribute is connected with empirical harmonics. Furthermore, the opposite poles of the discrete and the continuous are associated, respectively, with ratios and pitch distances, and each connected with a mathematical science, arithmetics and geometry.

Harmonic duality may be seen in the light of these couples. I call *timbral* the continuous aspect because it has to do with sensation, with intervals perceived as actual *sounding qualities* (which is a definition of 'timbre'), continuously variable and in a state of flux. Proportionality, on the other hand, is independent of the timbre with which it is instantiated, lying 'behind' and exceeding sensory qualities. Proportionality is a pattern inferred from the actual sounds, being more like platonic *eidos*, in the sense of pure Forms detached from their sensory presentation, not directly

53 This is probably the earliest definition of musical harmony we find Greeks harmonics, and it is noteworthy to point out its resonances with contemporary thought in order to construct of a concept of harmony fit for our times. Thus, Philolalus could be paraphrased in Deleuzian/Badiouian terms as: incompossible elements coexisting in a 'disjunctive synthesis'. See Badiou, A. (2000). *Deleuze: the clamor of Being.* Minneapolis: Minnesota University Press, 44 and 58.

accessible to hearing. They are not, however, eternal Ideas preexisting and separate from their presentation, but rather, to use an Aristotelian concept, *substantial forms*, that is to say, intelligible rather than sensible. These *eide* are the essential patterns indwelling at the core of sense objects, after their accidental or inessential qualities have been subtracted by reason, but they are not separate from their individual instantiation. A ratio (*logos*) is the *definition pattern* or *form* of an interval, the word *logos* being synonymous with all three of these notions, as well as with 'speech', 'discourse' and 'reason', depending on the context.

We never encounter the harmonic aspect in isolation, devoid of timbre, not even if the interval is made out of sine waves, as they still have a timbral quality. These timbral qualities are the medium through which harmonic pattens are encountered. A harmonic pattern or form always comes sensually attached with timbre. Timbre is the sensual medium for these patterns which are alluded from it.

One fundamental difference between the two schools concerns measurement. The empirical approach seeks to find the minimum interval which can serve as a unit of measurement for others (called a *diesis*). The search for this 'just noticeable difference' of pitch was pursued by Aristoxenus through comparing two closely tuned strings until the difference between them was no longer distinguishable. From this small interval springs the analysis of tunings and scales (composed out of *genos*, and forming *systema*). Aristoxenus achieved his analysis and divisions of the tetrachord (which has the size of a fourth, a *diatessaron*) inside a grid of 30 steps, corresponding to a sixth of a semitone.

Ratios, on the other hand, not being directly accessible to hearing, represent either relations between lengths of strings or pipes, or correspond to aspects of the physical events that cause the perceptions. Being indirect to perception, they are arrived at through reflection and observation by means of measuring devices like the monochord. Aristoxenus' criticism of mathematical harmonics was aimed not at denying the existence of ratios and the phenomena they explain but at their relevance to the study of music and to harmonic science. Contrary to this view, our research finds it important to maintain a flow of information between the accounts given by both domains in a way similar to Aristotle, for whom empirical harmonics provides the facts while mathematical harmonics – which works in a different domain – provides the principles from which these facts are explained and demonstrated. This resonates with our experimental approach to harmony in relation to sciences of perception and mathematics, which in my opinion is compositionally the most productive.

Concerning units of measurement, it is paradoxical that the basic ratio from which other proportions derive is the octave, 2/1, which is not a small interval. Aristotle tried to find these units of measurement to no avail, precisely because it is the smaller ratios which are derived from the octave, but not the other way around; also because most ratios – the ones that matter – cannot be divided into equal parts, so there cannot be a fundamental measure that adds up to them. We can also think of units of harmonic ratios – units of harmony or *harmonemes* – as corresponding to the prime numbers that factorize their terms, or the prime numbers that constitute the axes of harmonic lattices, but that is an extrapolation only present in latent form in Greek harmonics.

Empirical accounts describe facts about music and musical perception. They provide information which cannot be proved from a mathematical perspective. The fact that the octave, fourth and fifth are consonant is primary and cannot be demonstrated from arithmetic alone, not following from any mathematical theorem. Once this consonance is acknowledged as an irreducible fact, it is relatively easy to correlate it with properties of numbers and ratios – Euclid actually tried, unsuccessfully, to prove consonance from mathematics alone. The mathematical approach, on the other hand, once the connection between arithmetics and music has been made, can provide explanations and reveal underlying patterns for some more complex and derivative facts, some theorems of numbers having relevant consequences for music. This shows, however, that the mathematical approach, when its aim is not completely metaphysical, actually requires an empirical departure point, a 'sensory axiom'. The empirical route, by the way, is not a purely qualitative discipline, but one that seeks out quantities from which the notation of melodic sequences and thus the classification of scales and tunings is made possible. These quantities, consisting of half, quarter, third, sixth or full tones will provide the idea on which temperaments will be based much later on, although Greek music had no need for them. Both approaches, even though irreconcilable in spirit, provide complementary information that feeds into each other.

1.3.2 Pythagorean tuning systems

Mathematical harmonics provides a connection between qualitative attributes of sensation and properties of numbers. It explains the 'how' of consonance and dissonance (as we saw, it cannot explain the 'why' of it) by relating properties of numbers and ratios with sounding qualities in formal terms, which is how Pythagorean harmonists classified ratios. They had six categories: equal, multiple, *epimore*, *epimere*, multiple *epimore* and multiple *epimere*. Equal ratios are those whose terms are the same (unison in musical parlance), multiple are those whose terms are multiple of each other (2/1, 3/1, 4/1, etc, forming overtone series); *epimores* are those whose difference between terms is 1 (3/2, 4/3, 5/4, 6/5, etc, also known as 'superparticular' ratios) and *epimeres* those whose difference is not a portion of the smaller term – in *epimores* 1 is a part of both numbers and the mark of their difference – but some more complex part, or rather 'parts'. Relative to today's overtone series, multiple ratios occur between a fundamental and its overtones; *epimores* happen between successive adjacent overtones and *epimeres* between non adjacent ones. This classification, together with the precedence given to ratios with small numbers provides the hierarchies with which to organize the qualities of intervals. This hierarchy has fallen into disuse by now, but I find no intrinsic objection to reject it as a possible harmonic classification among others.

Pythagorean music theory originates before Pythagoras with Near-Eastern civilizations such as Babylonians and Egyptians. A distinction should be made between the well known Pythagorean tuning (based solely on multiples of perfect fifths, thus limited to numbers with prime factors not larger than 3, or '3-limit') and Pythagorean music theory, as the latter is not restricted to the former, which is more a theoretical construct than a derivation from musical practice. The tunings and theory used in Greek music come from the Sumerians, who based their number system on the number 60 and its divisors, corresponding to scale intervals and correlated to their main deities. In this mythological framework, numbers had, in connection to the attributes and powers of the gods, important qualitative properties. Pythagorean arithmetics also classified numbers according to their qualities (square, triangular, cubic, oblong, etc) and not just their size. The prime factors of 60, {2, 3, 5}, form 5-limit intervals, 3-limit Pythagorean tuning not corresponding to ancient musical practice. Whence, the divisor set (the set of all numbers that divide another) of 60 can be considered the seed of most heptatonic scales.

Early Pythagoreans found their scalar unit in the tone 9/8, which appears among the main consonances as the measure of their difference: that between two fifths and an octave, an octave and two fourths, and as the space separating the fourth and fifth within the octave. The Pythagorean tetrachord can be seen as two 9/8 tones and their residue with respect to the fourth which is the *limma* ('remnant'), 256/243 (a step of around 90¢). This parsimonious method of dividing the octave, as five tones plus a *limma*, is one that was adopted in the West up to the Middle Ages:

> "It uses the principles inherent in the beginning of the integer series more economically than any other. For if we reflect on the matter, we see that in some sense the fourth itself is a *limma*, left over

> from the projection of the fifth back into the octave. The fifth, in its own way, is a *limma*, left over from the projection of the octave forward into the twelfth (3/1). Only the octave seems to remain aloof from this process, being generated in some more mysterious way directly from the womb of unity itself."[54]

The Pythagorean *harmoneme* was, for practical purposes, 9/8, though it is clear that the only irreducibly given interval not deducible mathematically is the octave, which in my contention has more reasons to be considered an 'atomic' constituent of proportional harmony, though not the only one.

Of the Pythagoreans, it was Archytas who took on a different and novel route by dividing intervals according to mathematical means, namely the geometric, arithmetic and harmonic, which he introduces for the first time into theory[55]. The geometric mean is the simplest, used to test the equality of ratios. However, it deals mostly with continuous proportions not expressible as whole number ratios (as they are *alogos*, irrational), and cannot be applied to consonances (*epimoric* ratios). It is correctly named as it belongs to the continuous constructions of geometry. The arithmetic mean may have arisen from purely numerical considerations, but it is in harmonic science where it shows its usefulness, as it permits the division of consonances. Its drawback, however, was dividing intervals 'the wrong way around', with the larger one at the bottom, inducing Archytas' search for a 'subcontrary' mean, also called *harmonic* because of the solution it provides to a musical problem. Taken together there is a symmetry between the three: the arithmetic and harmonic means gradually approximate, as their terms grow, the geometric mean, one from above and the other from below. They divide the first multiple proportion, 1:2, the octave, by considering it as the multiple-multiple 6:12. The arithmetic mean yields 9 because 12 - 9 = 9 - 6; the series 6:9:12, equal to 2:3:4, gives a fifth and a fourth. The harmonic mean, a relationship between ratios instead of a difference between their terms, yields 8 because (12 - 8)/(8 - 6) = 12/6; this 6:8:12 series is the same as 3:4:6, a fourth and a fifth. Taking both means together as 6:8:9:12 we can see the 9/8 tone in between the divisions. This is the most important construction in Pythagorean mathematical harmonics, the one that provides Plato with his cosmogony and which Richard Crocker calls *the* harmony.

Archytas then applied this process once more to both the fifth and the fourth, yielding a sequence of *epimores* which sorted become: 1:2, 2:3, 3:4, 4:5, 5:6, 6:7, 7:8 and 8:9. This is not merely a mathematical exercise, but the starting point for a correlation with the way musicians tuned their instruments by, for instance, tuning slightly off from a ditone 81/64 to a 'sweeter' 80/64 = 5/4 by what is called the 'method of concordance', in which the ear is the judge. He adapted mathematics to musical practice as it was impossible to measure these ratios by ear, so they were arrived at through a combination of mathematics and musical skills, adapting these ratios to the given genera of his time, which meant producing, by addition or subtraction, other intervals needed to adjust these deduced ones to the different tetrachords. Some of these derivative intervals are the 28/27 third tone, the 36/35 quarter tone, the semitone 16/15, the 32/27 Pythagorean minor third and a very strange 243/224, close to a neutral second of 141¢. They don't mean much by themselves until we consider the constructions within the pentachord formed with a note a whole tone below the tetrachord, as John Chalmers suggests, revealing important intervals that appear between these degrees, such as the 6/5 and 5/4 thirds, the notable 7/6 (a subminor third, 266¢ in size which was ubiquitous enough to deserve a name of its own, *ekbole*), the 9/7 (a supermajor third, the difference between the fourth and a third tone) as well as the large whole tone 8/7.

54 Crocker, R., *op. cit.*, 197.

55 In today's terms: Geometric mean: $(a-b)/(b-c) = a/b = b/c$; $ac = b^2$. Arithmetic mean: $(a-b)/(b-c) = a/a = b/b = c/c$; $a + c = 2b$. Harmonic or subcontrary mean: $(a-b)/(b-c) = a/c$, $1/a+1/c = 2/b$; $b = 2ac/(a+c)$. From Chalmers, J., *Divisions of the Tetrachord*, 29. Crocker gives a detailed and fascinating account of these means in the second of his aforementioned articles.

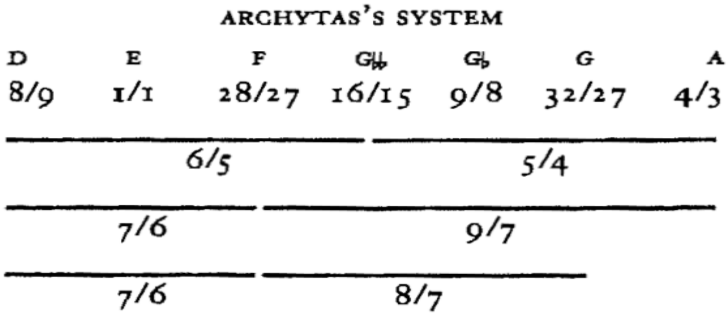

Figure 4. Archytean intervals within an enharmonic pentachord. (From Chalmers, 1992.)

His constructions are subtle, not merely theoretical but derived in conjunction with a keen sense for musical practice and the perception of intervals.

> "Archytas' work opened the way for richer and more detailed explorations of ... abstract patterns of order, most notably by turning the spot-light on the special status of epimoric ratios, by demonstrating techniques for manipulating them in the construction of harmonic divisions, by proving these ratios' resistance to equal division, by his classification and definition of the three 'musical means' and by his deployment of these means in his analysis of attunements. ... [H]is studies in physical acoustics point also to a scientific interest in sound and pitch themselves, and reinforce the impression given by his tetrachordal divisions that he was concerned, much more directly than earlier Pythagoreans, with the domain of the audible for its own sake." [56]

"Archytas is interested in the numbers by which phenomenal things are known and of which they give signs."[57] This again resonates strongly with the aims and procedures of the present project. He is a paradigmatic harmonist[58], combining mathematical and empirical approaches, as well as an interest in physics and the correlations between the domains. Archytean intervals diverge sharply from 3-limit tunings, going as far as 7-limit and including some non-*epimoric* ones as well. These ratios are more complex and rare than most intervals used nowadays, some of them still presenting problems as to their use. Through Boethius' reading of Pythagorean music theory, medieval theories and the practice of plainchant became anchored to 3-limit tunings. Ptolemaic 5-limit tuning, which accounts for the heptatonic modes of Greek music, remained in use in secular and folk musics, to be recovered in the practice of vocal polyphony since the 14th century and given theoretical legitimation, within a Neoplatonic framework, until the 16th century, in Gioseffo Zarlino's *senario*. He proceeded by dividing the fifth with harmonic and arithmetic means, this time interpreted as chords with 5/4 major and 6/5 minor thirds, hence authorizing major and minor chords as the harmonic units of tonal harmonic practices.

1.3.3 Consonance and Dissonance

This is an initial discussion into one of the most intricate topics within harmonic theory. It will now

56 Barker, *op. cit.*, 306.
57 Huffman, C. (2005). *Archytas of Tarentum.* Cambridge: Cambridge University Press, 424.
58 Furthermore, he was, amongst other things, prince, warrior, mathematician (with long lasting contributions to the field of physical mechanics) and a teacher of Plato (for whom he served as a model of the 'philosopher king'). *Ibid.*

revolve around harmonic duality and Greek music theory, in section 2.1.4 it will involve the psychoacoustics and psychology of the late nineteenth and early twentieth century, and in section 3.1.5 it will revolve around harmonic metrics.

We should recognize, as James Tenney makes us aware, that there are at least five ways of conceiving consonance and dissonance in the history of Western music [59]. Each conception can span different styles and genres as it refers more to the underlying assumptions behind the consonance/dissonance distinction than to the aesthetic attitudes toward them, as these can vary considerably within each conception. They can also be coextensive, operating simultaneously in different musics of a single historical period. The one relevant for the Greeks, the horizontal conception, is still in use today in many sorts of heterophonic and tonal melodic music.

For the Greeks, *symphona* and *diaphona* (concordance/discordance) were understood melodically, employed to discern 'degrees of affinity, agreement, similarity or *relatedness between* pitches sounding successively'[60]. *Harmonia* is the attunement of strings to a scale and concordance/discordance refers to relatedness between tones. Both empirical and mathematical harmonists set a sharp cutoff point in demarcating the line between *sym-* and *dia-phona*, allowing only octaves, fourths and fifths as consonances. For mathematical harmonists its was required that a ratio be *epimore* and that its terms lie within the *tetraktys* of integers 1 to 4. This restriction entailed that only 2/1, 3/2, and 4/3 could qualify as consonances, when it was clear that some consonant intervals, particularly 8/3, an octave plus a fourth, would have to be considered, but not perceived as dissonant. Greek theorists swept this aporia under the carpet until Ptolomy solved it with a law stating that octave compounds of consonances are themselves consonant. This further evidences the octavicity of harmonic relations, the octave being a *harmoneme* under which other relationships are confined.

The problem with 8/3 also points to the fact that consonance and dissonance can be conceived either as a continuum of gradations or as being antinomic, in a binary opposition, where an interval belongs either to one category or the other. The former grasp suggests a coloristic approach to intervallic use and choice, abounding in gradations of hues, suitable for transitional purposes, the actual sounding qualities of intervals being the main focus of attention, however broad the possible logics constructed upon these materials might be. This is timbral conception, pertaining (but not limited) to late nineteenth and early twentieth century composition. It lies behind Schoenberg's concept of the emancipation of dissonance, which describes the harmonic situation at his time and place: there is no absolute dissonance but rather a spectrum of 'sonances' to which the ear must accustom itself as they progress further towards unexplored areas. By the combination and compounding of even a few intervals together one can plainly see that a high number of potential sonance levels arise from this conception. These levels are closely connected to matters of voicings, registers, dynamics and sound source. This type of harmony encompasses aspects of Tenney's CDC-2 − vertical, polyphonic − and CDC-5 − timbral, psychoacoustic − conceptions, including various degrees of fusion between proportionality and sound for its own sake.

The second conception, on the other hand, is governed by contrast. Consonance/dissonance can be the cause of both melodic and harmonic movement by means of tensions and resolutions, as in CDC-4, functional tonality, or can be an effect resulting from movement, as in counterpoint, CDC-3. A decision has to be made as to where to draw the line between the two poles, and this alters the resulting logics that set the materials into motion. It is less attached to sensory considerations than to functional and contextual ones, as notes can be treated as dissonant without actually sounding dissonant. This is the case when a note in functional tonality does not belong to either the prime,

59 Tenney, J. (1988). *A History of Consonance and Dissonance.* New York: Excelsior Music Publishing. He proposes five Consonance/Dissonance Concepts, labelled neutrally as CDC 1-5. These notions can be succinctly described as melodic, polyphonic (or vertical), contrapuntal, functional and timbral (or psychoacoustic).

60 Ibid., 4.

third or fifth of a triad but nevertheless forms a consonance with the fundamental root (sixth chords for example, or a six-four major chord which is considered dissonant without being aurally so). This kind of proportional harmony is less dependent on the sounds which populate its intervals, leading to the consideration of widely different chords as belonging to the same harmonic category, as is the case with triadic inversions, or when a fundamental is implied but not sounded.

In terms of harmonic duality, the terms 'consonance' and 'dissonance' refer to the timbral dimension. This aspect depends on spectral constitution, perceptual salience of the components (their degree of fusion, called *tonalness*), and the interaction between partials which produces psychoacoustic roughness (or sensory dissonance). On the other hand, the terms 'harmonicity' and 'inharmonicity' refer to the proportional feature which is independent of spectrum, only pertaining, as it were, to the fundamentals that carry the interval and thus, to the commensurability or simplicity of the numbers defining their ratio, as is the case with time based pitch perception theories[61]. Historically there has not been much of a distinction between these two senses and some aspects of both have been conflated together in different ways. In the case of the Greeks, they were mostly referring to harmonicity, but they also discussed consonance in relation to two simultaneous sounds and the occurrence of fusion between them, involving the timbral aspect.

The following is list of factors involved in the harmonicity/consonance of an interval. Some of these factors involve both attributes, some only one, but for now they mainly concern harmonicity more than consonance. They are:

- The *form* of an interval's ratio. The dividing line is drawn between *epimore* and non-*epimores*;
- The magnitude of the pitch distance (a timbral consideration always present within proportionality). For pure sounds (sine waves), intervals larger than a critical band are more consonant, but with complex timbres it depends on the interaction between partials;
- The independence of an interval's harmonicity with respect to register, which is the same as saying that octave compounds of intervals retain their harmonicity (Ptolemy's law);
- The 'simplicity' of the numbers involved in its ratio:
  - Considerations based on the ordinal index of a number, i.e. its size, such as their inclusion in the *tetraktys* for the early Pythagoreans, or within {1..5} for Ptolemy;
  - Consideration of the prime factors of the terms in a ratio, a concept which we do not find in the Greeks although it is clearly a corollary of Greek harmonics and math;
- The melodic/harmonic function of an interval in context: the way intervals follow and precede each other, along with their function within a scale (their *dynamis*);
- The metric weight or rhythmic accentuation of an interval;
- The 'aesthetic attitude' or compositional/cultural conventions towards an interval's sonority. Perceptual qualities can be ignored or pursued in different and arbitrary directions. This consideration stems from the point of view of twentieth century composition, but has been present in all kinds of musics and epochs.

There is no doubt to the fact that *epimore* ratios posses a special quality. They are easier to tune by ear, as it happens with acoustic instruments, where non-*epimores* lack a special resonance that *epimores* do possess. Archytas' theorem proves that *epimores* have no integer geometric mean, that is, that they cannot be divided into halves. This is in direct conflict with the Aristoxenian practice that divides

61 What 'simple' and 'commensurable' mean is not so obvious and will be developed in section 3.1.5.

any pitch distance by half (or any other part) without difficulty by considering them as linear distances. We can now make sense of this apparent contradiction by distinguishing each procedure as referring to a different and independent dimension of harmony. Proportionally it makes no sense to talk about the middle point of an interval (half an octave, say) as they are unitary characters. It is also absurd to talk of an interval not expressible as a ratio, as would be the strict case with any interval of equal temperament, excluding octaves. On the other hand, notes can be placed wherever we want in the pitch-distance continuum, but they only acquire harmonic meaning by being understood as tolerably close to a proportion. The size of this tolerance depends on harmonicity as well as the function of the intervals in context. Thus, there are intervals whose function is timbral, when they do not refer to a harmonic context (as in the case of ornamentation, transposition, register changing, doublings, and so on) and intervals that function proportionally, independently of register and timbre and which do not have to be exactly tuned to function as such.

There is also the issue of complementarity or inversion. An interval can be complemented modulo another interval, the most common case being inversion modulo octaves. A ratio is reflected in its opposite direction and then octave transposed back to its original register. For instance, the 5/4 major third reflects into a 4/5, which when moved up an octave gives 8/5, a minor sixth. The proportional harmonicity of complements is maintained but its timbral aspect of height changes, so the resulting consonance perception will diminish or increase depending on the simplicity of the new interval with respect to the first as well as its pitch distance. Intervals other than octaves can also serve as moduli, but as with the case of non-octave related equivalences, their use has to be clearly constructed with an appropriate context.

The symmetry stemming from interval complementarity is a fundamental harmonic feature. These symmetric properties can be applied to whole intervallic systems, as an arbitrary pitch set is only 'complete' when all its intervals have complements, making the set a mathematical group under the operation of intervallic addition, where every interval has an inverse and there is a neutral element, 1/1. Much like overtone and undertone series, o-tonalities and u-tonalities in Harry Partch, the quadrants in harmonic space and the relation between arithmetic and harmonic means (which tend towards the geometric mean), inversion is a harmonic feature that forms symmetric clusters around intervallic classes. It is likewise expressed inside an interval by the property of standing either 'up' or 'down', that is to say, of the two notes that compose it, which of them has the most weight and feels like the root. If it is the lower note, then the interval is standing up, otherwise it is 'upside down'. There is an intriguing interrelation between the topics of complementarity, symmetry and the tendency from above and below towards logarithmicity, the linear pitch distance embodied by the geometric mean and approximated asymptotically by the integer means. Logarithmicity is a way to fit or compress more information into a limited space. It is the final limit tendency of periodicity, a tendency achieved from above and below. Together, periodicity and logarithmicity form a dialectic which lies at the heart of the thinking about harmonic duality.

1.3.4 *Dynamis*

Dynamis is a technical term found in Aristoxenus' *Elementa Harmonica* referring to the ability of intervals to follow each other and the ways in which this happens. It is a relational potential inherent in notes, independent of their intervallic size. Melodic function (*dynamis*) is a higher perceptual concept in harmonic science that is encountered through hearing (*akoe*) and thought (*dianoia*) over the course of a melody. This incorporation of thought and perception is referred to as *aisthesis*: perception sensitive to musical meanings of notes and intervals in various contexts.

The Greek word *dynamis* has several meanings, all of them useful for elucidating harmonic properties: it is most literally potential, or capacities *in potentia*, powers obtained through relations; it is also movement, dynamics as the capacity to produce (melodic) movement or change; finally, it is translated from Aristoxenus, who refers to it as one of the non-quantitative discriminations found in harmonic science, as function. This function is independent of interval sizes since melodic characters can remain stable while the magnitude of the intervals that instantiate them can vary (within limits). The concept is closely connected to the Greek *genera*, and it is valuable that we pay them a short visit.

We have seen that the octave is the fundamental interval from which harmonic and arithmetic means are used to divide it and produce other concords, namely the fourth and the fifth. Proceeding a step further, each of these intervals can in turn be divided. The fifth produces harmonic intervals (4:5 and 5:6, as mentioned above) while the fourth produces melodic ones (7:8 and 6:7, which are not used harmonically but melodically). This suggests that the fourth is a primary unit from which to generate melodic divisions. Dividing it with an 'infix', a semitone or a tone, it produces 3 tones, giving rise to pentatonic scales when two of these structures are conjoined within an octave (for instance, dividing the fourth A-d to form A-c-d and then adding the complementary fourth e-g-a to complete the scale A-c-d-e-g-a). Adding another infix to fill-in the wide interval produces a 4 note tetrachord which yields heptatonic scales. As with the Pythagorean harmonic derivation of intervals, we can make sense of melodic intervals as spanning from divisions of larger, stable ones.

Tetrachords are the building blocks of scales, consisting of two tetrachords, either conjunct (a-d-g) or disjunct (a-d and e-a, separated by a tone and forming an octave). The notes bounding the tetrachords are fixed, forming the scalar structural framework. Changes in the positions of the two movable notes inside tetrachords determine their *genera*, which are of three types: diatonic, chromatic and enharmonic. Each *genos* has different variations or 'shades' in tuning (*chroai,* such as 'soft', 'tense', 'tonic' and 'even'). Diatonic has no interval smaller than a semitone or larger than a tone, but chromatic and enharmonic have crowded intervals at the bottom and larger ones at the top. Intervals in all *genera* are unequal in magnitude, and the fact that Aristoxenus divided the fourth equally into 30 steps does not imply the common reading according to which this idea anticipates equal temperament: there is no place for equal intervallic spacing in Greek music, equal temperament arising out of different musical problems, namely modulation in just intonation for keyboard and fretted instruments. Aristoxenus' divisions are theoretical means of measuring and classifying the unequally spaced intervals of his time, and never did he refer to the actual stacking of more than two or three equal intervals together[62].

Enharmonic was considered the principal genera by Aristoxenus (its meaning was to be in tune, a *harmonia*, although already by his time it was falling in disuse and was alluded to as 'old style'). Chromatic is a deviation from enharmonic (*chroma* meaning coloring) and diatonic was initially only used in certain regions, but in time this changed and ended up dominating the music in Roman times, later inherited to Europe. Using the letters *q*, *s*, *t*, *x*, and *d* to stand for quartertone, semitone, tone, three semitones (trihemitone) and ditone, their basic structures are the following:

enharmonic: *q*, *q*, *d*

62 An example of this misunderstanding can be seen in Xenakis, I. (1985). Music composition treks. In C. Roads (Ed.), *Composers and the Computer.* Los Altos, CA: W. Kaufmann, and Xenakis, I. (1992). *Formalized Music: Thought and Mathematics in Composition.* Stuyvesant, NY: Pendragon Music Press, 182. He claims that Aristoxenus 'invents, in theory, a complete, equally tempered chromatic scale with the twelfth of a tone as the modulus (step).' Moreover, his terminology designates Pythagorean theory as geometric and Aristoxenian as additive. This because adding proportions is a multiplicative operation, while distances become additive through their logarithmicization. His terminology is founded upon an operational, algebraic, viewpoint, not on the correlation between music theory and the two main fields of mathematics, as in our case (the former related to arithmetic, the latter to geometry). More than an inconsistency it is a difference in terminology.

chromatic:	*s*, *s*, *x*	(Aristoxenus called this shade 'intense')
diatonic:	*s*, *t*, *t*	(also 'intense')

There are more variations on each *genos* depending on how some of the intervals are defined for each shade, a change in one of them evidently modifying the others. The intervals, once defined in one of the possible shades cannot be further divided: they are *incomposite*, no matter how large. A tetrachord is characterized by its largest interval: enharmonic is distinguished by the incomposite ditone (construed by Archytas as a 5/4 but without tertial implications, used as a melodic *step,* not as a harmonic *jump*; other rationalizations including 81/64 – the Pythagorean ditone – and the unusual 19/15); chromatic is characterized by some variety of trihemitone (regarded as 32/27, 6/5 or 7/6); and the diatonic by a tone, which could be different from the disjunction tone (interpreted as 10/8, 9/8, 8/7, and even 7/6).

Melodic functions appear when each note in a tetrachord is identified as carrying a specific behavior. Because several interval sizes are shared by different genera, this recognition depends on function, not size. Notes are not just pitches, in the sense of only occupying a position within a system, but functions acquired through their relational roles within that structure. A 'route' (*hodos*, the root word for 'method') is a progression, a specific melodic formula reminiscent of melodic patterns that form part of the structure of hindu *raags* and arabic *maquam*. It refers to the possible movements melodic successions can take when traveling between stable pitches, defining the features of intervallic progressions, where each note has the potential to influence future alternatives that depend on a wider pattern of relationships beyond the previous, current and following note. It is not just a fixed point in the structure, but something with its own 'power' or dynamic properties which impel the melody to move forward.

1.3.5 Corollary. *Dynamis*: the horizontal core of harmony

Harmonic and melodic tensions (the propensities to move) are not limited to the intrinsic properties of chords and notes, but result from context. Differences between genera were comprehended through *aisthesis*: through hearing and thought, which also means through training, and this is where cultural schemata play a role in the organization of these contexts. Harmonists were interested in explaining the nature (*physis*) of *melos*, not the conventions built on these structures. Greek *systema* were not given phenomena but constructions created by theoreticians, so the question centered on why these systems and not others reproduced the melodic roles taken by the notes. The rules of melodic progression are not just arbitrary but stem from the features of pitch that produce melodies and determine the scales. This is the reason why *dynamis* is a non-quantitative aspect of *melos*: an interval is not to be defined by a magnitude but by a *character* present to the ear. This is one of Aristoxenus' greatest contributions to harmonic science and was to be taken on by harmonists of every persuasion: it belongs both to the proportional and to the timbral facets of harmony. Proportional (although harmonic seems to be the better term here) in the sense that it defines characters and roles, potentials spanning beyond immediate sonic qualities, defining structures at larger time frames, one level of organization above intervals and notes in isolation. Some roles can belong to the timbral sphere (ephemeral, ornamental and coloristic roles, tensions aiming towards goals – the 'leading note effect', – etc), while some are harmonic (stabilities, goals, the use of intervals larger than fourths, pivot notes that can change function in modulation and so on).

Potential for movement is a significant feature of harmony and *melos* is its operating principle, whether it is movement that produces tension (as in contrapuntal conceptions of consonance and dissonance, Tenney's CDC-3) or tension that produces movement (as in functional tonality, CDC-4). In this way, context, progression and potential are properly harmonic roles, expressed through melody whether it is explicit or merely implied, which is another important lesson to be taken from *dynamis*. A scale is not a neutral structure, it involves not just a collection of notes or intervals but also specific and conditional comportments. If scales are associated with colors or moods, this functional conception plays a salient role in defining those higher level features by involving characters, routes and functions. These roles determine the tension and movement from which a melody is said to imply a harmony and from which a verticality induces a melody.

With ratios we are dealing not so much with magnitude as with qualitative attributes, so magnitude belongs to the timbral aspect and character to proportion. There is no such irresolvable incompatibility between the mathematical and the empirical approach: they describe different aspects of music, each having advantages and disadvantages over the other in different circumstances, but without overlap or contradiction. It is similar to the way in which harmonic duality plays itself out, as an intertwining or entanglement, always in tension, no aspect being independent of the other. Their paradoxical relationship can be turned into a concept.

From this perspective, we must also distinguish in intervals the difference in function between commas, alterations, steps, and leaps[63]. A comma changes the tuning of an interval without changing its scale degree, as it happens when an interval is tuned in accordance with one set of fundamental intervals rather than another (as when a third is tuned according to fifths as opposed to pure thirds, for example). An alteration does not alter the degree but changes its mode or quality into that of another genera, as in the case of a change from minor to neutral to major within a degree. A step is an adjacent change of scale degree, independent of its size (in enharmonic it could range from a quarter tone to a ditone). A leap is a non-adjacent change of degree. Moreover, inversion can be considered an operation upon these roles, altering their properties in interesting ways: intervals larger than a fourth can be inverted to fall within the fourth, so that their roles are related to the smaller versions with additional characteristics brought by size and direction.

Functions within the tetrachord could also be related to posterior functional harmony such as tonic-dominant, supertonic-submediant, mediant-leading-tone and subdominant-tonic, but the difference is that these are harmonic properties of chords constructed on these scale degrees while the functions we're discussing are eminently horizontal. Modulatory harmony takes *dynamis* a step further to higher levels of organization by applying it to chords progressions and, even further, to tonalities.

1.3.6 Numbers and perception: Pythagoreanism

> 'There is no difference between composing music and thinking about the stars' (Karlheinz Stockhausen[64])

The mathematics issuing from arithmetic and harmonic means are by no means trivial, even if their initial musical application might seem a bit innocent. The harmonic mean, discovered as a result of musical problems, shows that musical phenomena testify to patterns lying beyond the auditory realm, providing departure points for mathematics, and not the other way around, as the relation between music and mathematics is usually understood[65]. The discoveries of Pythagoreans have since

63 For more on this topic, refer to the discussion in Lekkas, D. (1999). The Rationale for Ratios and the Greek Experience. In C. Barlow (Ed.), *The Ratio Book.* Cologne: Feedback Papers 23, 24-43.

64 Taken from the documentary film *Tuning In*, by Robin Maconie, 1983. Last retrieved August 13, 2012, from http://www.youtube.com/watch?v=qGnkZnm9MPw

65 '[Around 500 B.C.,] music gives a marvelous thrust to number theory and geometry [...] Music theory highlights the discovery of the isomorphism between the logarithms (musical intervals) and exponentials (string lengths) more than 15 centuries before their discovery in mathematics; also a premonition of group theory is suggested by Aristoxenos.', Xenakis, I., Music Composition Treks, 171-192.

taken a life of their own, integer harmonic means providing further inroads into puzzling and deep number-theoretic insights. Numbers with integer harmonic means are quite rare and from them derive harmonic divisor numbers, positive integers whose *divisors* have an integer harmonic mean – they are non trivial, the first few being 1, 6, 28, 140, 270, 496, 672, 1638, 2970, 6200, 8128, 8190, possessing interesting number-theoretic properties[66]. Euler proved that there are infinitely many primes – a result having been proved through *reductio ad absurdum* by Euclid – with the aid of reciprocals of harmonic series. This resulted in Riemann's zeta function which gives clues to the distribution of the primes over the integers, providing insights into the relation between the continuous and the discrete as well as their relation to periodicity, which, as we saw, lies at the heart of pitch[67]. The related prime number theorem states that $x/ln(x)$ (a real number divided by its natural logarithm) approximates $\pi(x)$ (the number of primes smaller than x) as x approaches infinity, connecting primes (integers) and logarithms (reals) in a single expression.

Simple questions related to integers and arithmetic (counting, addition and multiplication) lead quite quickly to the frontiers of knowledge. Perception arrives very early to the shores, but knowledge also gets there quite fast, either to extremely complex mathematics or simply to questions that nobody, not even the best mathematicians know how to answer.

It would be quite far fetched to try to read these results back into music, but it nevertheless manifests that the notion of harmonic duality, giving evidence of the polarity between the continuous and the discrete intrinsic to musical phenomena, an hypothesis not resulting from cultural conventions, is closely involved with a long tradition of thinking about the opposition between integers and the reals, the mathematical dialectic between arithmetic and geometry. These mathematical results attest to a deep link between the two aspects on which auditory harmonic perception rests, even if these connections lie far beyond the direct field of apprehension.

Music (perception, theory and practice) offers awareness of these mathematical structures. Better still, it can convey some mathematical ideas *as* phenomena. The connection happens through *aisthesis*: not only as immediate sensation but requiring thought and reflection for its recognition. *Aisthesis* captures only the lowest confines of these numerical traits, but it is exposed to their whole 'frequency range', not isolated from their full ramifications, which could also re-fold back into the *aisthetical* spectrum. Discussion of Pythagorean topics at least refreshes (or resuscitates) the problem of the relation between mathematics and music, indicating that the issues at stake are intricate, complex and quite relevant today. Musical phenomena are caught between continuity and discreteness, sharing forms and properties with numbers and ratios, communicating to and fro between abstract ideas of order, pattern, relationality and the empirical regions of 'tone color', intervallic character, degree of dissonance, potential, movement, rhythm and other musical *qualias* known through sensory experience. Musical questions open up to issues bearing on the nature of numbers and their relation to the empirical universe (or, which is the same, of mathematics to physics), as well as the relations of the latter back into music. This is all closely related to Pythagorean science, which incorporates mathematics, philosophy, natural science *and music*. Music understood not simply as an art form among others, but also as a gateway, a sensory entry into and between these disciplines.

Though arithmetic has very little relevance in today's mathematics, it still has contemporary

66 They are a recent discovery. See the online lecture notes by Goto, T. On Ore's harmonic numbers [PDF document]. Last retrieved May 18, 2011, from http://www.ma.noda.tus.ac.jp/u/tg/files/uts.pdf

67 The zeta function can be expressed as series of periodic functions: "A physicist will think of a sum of periodic functions as a superposition of waves, a vibration or sound. This is what the physicist Sir Michael Berry meant by 'we can give a one-line nontechnical statement of the Riemann hypothesis: The primes have music in them.'" Webpage of Jeffrey Stopple: Stopple, J. (n.d.). Riemann's Explicit Formula. Last retrieved May 12, 2011, from http://www.math.ucsb.edu/~stopple/explicit.html

relevance is in musical harmony. The relation between qualitative and quantitative aspects of numbers in and through music is a good reason to glance back at Pythagoreanism. Pythagoreans noticed that operations on numbers applied to natural phenomena, conceptualizing this as 'reality is structured by number', a kind of mathematical empiricism. This came to an end with Hypassus, who discovered quantities exceeding numbers as they were understood then: the diagonal of a square of size 1, the area of a circle, the golden ratio, all being *alogos*. These irrationals found their intervallic equivalence in Aristoxenian *dieses* (the diagonal of the unit square, $\sqrt{2}$, corresponds to the tempered tritone). Contemporary mathematicians had the tools (extraction of square and cube roots) to compute the string lengths for tempered scales. Aristoxenian dieses are equivalent to taking 3 geometric means (square roots) and 2 cubic roots to the octave[68]. The Greek musical system did not require temperament nor logarithms. Unless we acknowledge music as emerging solely within the mind, rational intervals have more musical primacy than the otherwise more practical logarithmic intervallic divisions, which are closer to sensation, approximating but not replacing their musical meanings.

This Pythagorean catastrophe leads to Aristotelian instrumentalism, where sublunary phenomena are subject to degradation, as opposed to the celestial realm, where exact mathematical relationships still hold: beauty was not terrestrial but cosmic. Kepler reunited both realms under a single mathematical physics, renewing the Pythagorean dream. Music, particularly harmonics, was crucial to this dream, figuring prominently as a model for the universe in the thought of many of the involved thinkers up to the Renaissance. The quadrivium of sciences inaugurated *avant la lettre* by Archytas set music side by side with mathematics – divided into the discrete/continuous twofold of arithmetics and geometry – and astronomy – with which it shared its cosmological perfection. The music of the spheres was a realm for contemplation and amazement: a correspondence between the structure of the heavens, that of music (particularly the proportions determining consonance and dissonance) and the human soul. Microcosm and macrocosm were in accord, mediated through music through sympathetic resonance.

Newton's decomposition of planetary orbits into terrestrial linear movements ruined the separation between cosmos and earth, displacing it into an intra-terrestrial one between nature and culture. Sound is now understood within the domain of the laws of motion and not as a separate phenomenal field, and the theory of sound propagation is founded on Newton's model of the harmonic oscillator (the pendulum) which was inspired by harmonic science, later loosing its musical derivation and taking on an independent history as a mathematical abstraction which would serve physics a great deal. Since the Enlightenment, harmonics lost its importance as an intellectual model in a transition that began with Galileo and Mersenne and went all the way up to Helmholtz, whose aim was explaining the consonance of simple ratios, the Pythagorean problem par excellence, from a sensory instead of a formal standpoint – effecting a change from a proportional to a timbral explanation. Music theory's foundations underwent an 'empirical turn', speculation being replaced by physical and physio-psychological explanations. Furthermore, after Kant's suspension of metaphysics as dogmatic and naïve, speculation lost whatever impetus it had left. When all that is permitted or possible to talk about is the conditions of access for subjective experiences, and not things in themselves, which are confused as grounded objectively instead of subjectively, reasoning in music turns toward the perceptual. As the human subject becomes the new center of the universe, the aim of theorizing about music is transformed from that of gaining knowledge about music in general to that analyzing individual compositions.

The qualitative aspects of numbers have gradually been lost since those days, regarded as they are in purely quantitative terms and leading to a 'quantocentrism' which cedes the monopoly over

68 Dividing an octave into 72 equal steps: 72 is $2^3 \cdot 3^2$, equivalent to taking 3 square and 2 cubic roots to 2.

numeric qualities to new age numerology[69]. Mathew Watkins[70] proposes that to recover these qualitative aspects without loosing some kind of verifiable support implies seriously studying number theory, 'because as far as I'm concerned, that is numerology – you're looking at the properties of integers and if you study it to a certain depth it takes you into the realms of what you could only call the mystical or uncanny, where cracks seem to open in your normal understanding of reality.'[71] Music, though rarely mentioned in relation to this topic, provides one of the most direct instances of the qualitativeness of numbers, and in this sense harmony can be considered an audible interface between these realms.

> 'Although number is widely considered as a mental construct, at the same time it manifests directly in the world of matter: when you consider a quartz crystal or a five-petalled wildflower, it's hard to deny there's an essential "sixness" or "fiveness" there. So, number itself is a bridge of sorts between psyche and matter.'[72]

Philosophy was in closer contact with music in its Greek origins. These numeric qualities are audible proportionally, both as intervals and durational distributions: 'twoness' is one of its basic attributes, relating to octave equivalence and interval classes, as well as basic rhythmic binary divisions; 'threeness' has to do with fifths and fourths, hemiolas and sesquialtera; 'fiveness' with thirds, sixths; and so on, each being a distinctive and easily recognizable audible *qualia*. Moreover, from purely harmonic considerations, we should discern in 'sixness' a combination of 'threeness' and 'twoness': the fact that composite numbers adopt and combine the qualities of their prime factors, which become of *prime*-ary importance not only to arithmetic, as in the Greek Fundamental Theorem of Arithmetic[73], but also to harmony. They are the atomic constituents of proportional intervals, their formal causes or 'definition patterns'. If according to Aristotle the formal cause of the octave is 2/1, then 2 is the main content or the interval's quality. In something more complex like 15/8 (a major seventh), its formal causes would be 2, 3, and 5 (the 2 compounded 3 times)[74]. Another issue is how high in the prime series can human auditory *aisthesis* encompass. Some say up to 7, some up to 13 and higher: this contemporary debate will have to be waged with musical rather than purely theoretical hypotheses.

'Pythagoreanism' can refer to many schools and eras beginning with early Pythagoreans, split between the *acusmatici* – a mystical, non-scientific tradition based on the aurally revealed word of Pythagoras, emphasizing ethical precepts for living ascetically – and the *mathêmatici* – referring to rational Pythagorean science: mathematicians and natural philosophers, culminating with Archytas.

69 I am correcting this paragraph on 11/11/11 and cannot believe all the vacuous frenzy around such a fortuitous date. While all this complacency takes place, yesterday the Western Black Rhinoceros was declared extinct. Many of today's superstitions pass unexamined (at times with awful consequences, as the news shows) while the depth and subtlety of reasoning behind some of the most influential models of the universe in history tends to be lost and considered unsophisticated belief compared to our current 'advanced' knowledge.

70 Watkins, M. (2006). Prime evolution (interview). *Collapse*, 1, 93-189.

71 Ibid. 166-7.

72 Ibid, 183.

73 Proven by Euclid, the Fundamental Theorem of Arithmetic 'states that any integer greater than 1 can be written as a unique product (up to ordering of the factors) of prime numbers.' That these factorizations exist is evident, that they are unique is not so trivial. Extended to harmony it implies that every interval is determined by a unique combination of fundamental intervals. Fundamental Theorem of Arithmetic (2010). In *Wikipedia*. Last retrieved November 16, 2010, from http://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic

74 '"Cause" means (1) that from which, as immanent material, a thing comes into being, e.g. the bronze is the cause of the statue and the silver of the saucer, and so are the classes which include these. (2) *the form or pattern, i.e. the definition of the essence, and the classes which include these (e.g. the ratio 2:1 and number in general are the causes of the octave), and the parts included in the definition.* (3) That from which the change or the resting from change first begins; e.g. the adviser is a cause of the action, and the father a cause of the child, and in general the maker a cause of the thing made and the change-producing of the changing. (4) The end, i.e. that for the sake of which a thing is ...', Aristotle. (1996). Metaphysics. In J. Barnes (Ed.), *The Complete Works of Aristotle.* Princeton, NJ: Princeton University Press, Book V. §2 (1013a). Emphasis added.

Pythagoreanism is also attributed to other thinkers and schools who assimilate and transform some of their chief ideas such as the dyad of the limited and unlimited and the correlation between the ideal and empirical realms. Plato and Aristotle are examples, as well as Aristoxenus, who provides the earliest direct account of their theories, having lived under Archytas' rule. The unification and creation of a canonic tradition was pursued by the Neopythagoreans of later centuries (Porphyry, Iamblichus) and carried further by medieval neoplatonists (Boethuis, Nichomacus) all the way up to the Renaissance (Copernicus, Zarlino, Kepler, Galileo) and the Baroque (Euler, Leibniz). Even today, 'many modern scientists accept the basic tenet that knowledge of the natural world is to be expressed in mathematical formulae, which is rightly regarded as a central Pythagorean thesis, since it was first rigorously formulated by the Pythagoreans Philolaus and Archytas and may, in a rudimentary form, go back to Pythagoras himself.'[75]

Recovering this scientific, philosophical and mathematical strand is central for us, not dismissing it as numerological mysticism (or superstition) nor as a highly innovative but easily surpassable idea. This would miss the subtlety, importance and permanence of some of these discoveries, which stretch beyond the possible interpretations and uses given to them. It refers more to the *structure* of these ideas rather than their content. In fact, we can state in the same vein that harmony is both discovered and created (and composition an act of observation more than just authorship). The model of the string is still in use and provides one of the most tangible links between the discrete, the continuous, the mathematical and the physical, as well as supplying the link between place and time theories in pitch perception models. Moreover, probably the principal characteristic of this Pythagoreanism is the gesture 'number is a bridge between psyche and matter' more than the regularly acknowledged 'being is number'. It is with Archytas then, that the inspiration for a renewed Pythagoreanism must be found, where perception and empirical observation are coupled with deduction and logical rigor. The 'pitching' of melodic space, as Aristoxenus named the puncturing of the pitch continuum with stable pitches, is discovered by induction from observation. Through this abstraction, an 'astonishing orderliness'[76] is uncovered, attesting to a *physis*, a nature or essence of *melos*. An orderliness revealing patterns of rational numbers.

1.3.7 Concluding remarks

The Romantic nature-culture pair that ensued from the Enlightenment is replaced by a modern symbol of history as a non-human, non-natural force driving an impersonal process[77]. Modernity shifts the focus from the human strife with nature towards indifferent forces, and in music this involves turning our ears from self-expression towards receptivity, abstraction, aloof forms, as well as all kinds of things 'out there'[78]. This very contemporary concept of attuning to all sorts of entities is implied by Pythagoreanism and consists in paying attention to the structure of what surrounds us. It

75 Pythagoreanism. (2010). In *Stanford Enciclopedia of Philosophy.* Last retrieved November 13, 2011, from http://plato.stanford.edu/entries/pythagoreanism

76 Aristoxenus as quoted in a discussion of this topic in Barker, *op. cit*. 150.

77 These 'symbols' of history (cosmos-earth, nature-culture, history) are acquired from a translated preview of Quentin Meillassoux's yet unpublished *Divine Inexistence*, in Harman, G. (2011). *Quentin Meillassoux. Philosophy in the Making.* Edinburg: Edinburgh University Press.

78 In this vein, the avant-garde serialists of the 1950's (Eimert, Stockhausen, Pousseur, etc) are also weird heirs to Pythagoreanism, mostly due to their absolute and anti-language approach to music. Nevertheless, if there is any paradigmatic Pythagorean composer inspiring this research it is Iannis Xenakis (even if his approach to pitch is highly Aristoxenian). If we make sense of Pythagoreanism as in the Renaissance, as a conjunction of music, philosophy, mathematics, poetry as well as the common origin of arts, then is not he a most Renaissance figure who conjoins heart, mind, science and philosophy, adapting it to the needs of his own time? He is perhaps the greatest emancipator of the continuum in music which is a point of departure for our advocating a reassessment of discreteness and proportionality.

should not be understood either in a mimetic nor positivistic sense of mechanical knowledge or as an optimistic-progressive metaphor, but instead as something to probe or dig into, to be reckoned in its full uncanniness. According to Graham Harman's metaphysics of objects[79], essences, although having been considered either elsewhere (outside this world) or non-existent, are in fact in objects, yet withdrawn: they are not simply present. They are somehow like substantial forms in relation to proportions: definition patterns that establish objects as autonomous realities that emerge over and above their constituent parts as well as being independent of their outward relations. An object's relation to other objects siphons some of their withdrawn qualities creating a sensual, phenomenal realm in the object that makes contact with another, though the object itself is never exhausted by these relations, always holding some novelty in reserve (its withdrawn, noumenal realm). These qualities are highly singular, not 'bare particulars', each imbued in the unique style of its object, not being pale specters of ideal ones (a 3/2 is the 3/2 of the timbre that instantiated it, not a chimerical interval whose timbre is a deficient approximation of an ideal one). For that matter, these essences are not indestructible nor preexist the object. There is more inside the object than outside it. As we will see with more detail further on in defining proportional and timbral aspects of harmony in relation to rhythm and form, in this static model space and time are emergent features of objects and we are fundamentally embedded in an infinite regress of objects, not in a meaningful, blissful way, but in an uncanny, expressionist, Lovecraftian, disturbed manner. It is not matter which is at the service of subjectivity, but a matter of subjectivity caught inside the strangeness of other entities.

This suggests an aesthetic stance of attuning to musical and sonic phenomena, instead of trying heroically to dominate them (which would fall into the Romantic attitude). This position leads to sincerely (instead of ironically) approach all kinds of objects for translation into music and sonic forms, seeking some of their hidden harmonies. The task calls for the sonification of the formal as well as the empirical worlds, rendering entities into sound, as well as proceeding from sounds towards abstract entities.

One aim of this research has been to give back to harmony its speculative and arithmetic dimension without disregarding but even emphasizing the developments made after the study of music and harmony changed from having a metaphysical towards a physical and subjective grounding. Many concepts of Greek harmonics can be incorporated and used in a contemporary harmony, chiefly by admitting *melos* as one of its pivotal aspects, a structuring and motion producing device established through the relationality of tones. Other notions such as division by means, atomic constituents, tetrachordal (and other moduli) divisions, functional conceptions, etc., can be further extended, inviting us to recuperate long forgotten intervals, with ideas about possible generative and relational strategies to deal with them. The demarcation between the proportional and timbral aspects of Greek harmonics as well as the detailed delineation of melodic consonance/dissonance also provides a refreshing perspective on these issues, which have been clouded by layers of sedimented theories covering them up throughout the centuries. Finally, its metaphysical perspective can be transposed today by rethinking the question of numbers in relation to perception, of mathematics in relation to music and of music in relation to structural aspects of reality.

Attuning, contemplating, ceding control, receptivity: to reinterpret ideas from the past as part of the construction of the future and in order to have the largest possible present.

If the first section on pitch perception ended by delineating a perceptual ontology of sorts, this section has tried to identify a link between perception and arithmetic, but this time perception is linked with a phenomenological first person perspective and the arithmetical with the *aisthetical* deduction of the formal causes behind these perceptions. The arithmetic standpoint is a kind of 'formal ontology' for harmony, while the phenomenological attitude belongs to its precondition, its

79 The main books where I draw these ideas from are *Guerilla Metaphysics* and *The Quadruple Object*.

‘axiomatic’ starting point. What must now be brought forward is the connection of these in the direction of a properly musical ontology, not dependent on neither arithmetic nor perception, lying at a certain distance from them, but in constant tension and correspondence.

Chapter 2

Timbral Harmony

2.1 Dissonance Curves

2.1.1 Dissonance curves from a compositional perspective

> '[T]o attune noises does not mean to detract from all their irregular movements and vibrations in time and intensity, but rather to give gradation and tone to the most strongly predominant of these vibrations.' (Luigi Russolo, *The Art of Noise.*[80])

Dissonance curves have been the driving impetus of this research, being a practical and fertile means to produce microtonal intervals out of spectra with the aid of the computer. These spectra can proceed either from empirical sounds as well as from abstract mathematics. Overall, their basis corresponds to timbral principles of harmony, namely the phenomenon of psychoacoustic roughness, though by the fact that they also produce coincidences with proportional intervals, working with them leads to thinking the relation between spectrum and proportionality. The use of the generated intervals, of which there are a great variety, has induced conceiving ways of sorting, classifying, filtering, partitioning and deploying these intervallic sets. It has also implied working out their relationships with the sounds that generate them as well as extracting differentiated harmonic areas inherent in each of them.

This set of algorithmic composition tools has been developed as en extension library, *DissonanceLib*, for the composition and sound synthesis programming language *SuperCollider*[81]. We'll review below in some detail the psychoacoustics behind them, stemming from Helmholtz up to Plomp and Levelt and a bit beyond, but its more important first to understand them in compositional rather than scientific terms. The following considerations characterize and summarize these aspects.

Dissonance curves indicate how certain timbral characteristics of sound behave when transposed. They provide a profile conveying the transpositions at which a sound is most sensory-consonant with itself. They display the behavior of a spectrum, within a determined intervallic span, according to roughness.

Their implementation takes as an input a set of partials (frequencies and amplitudes), and an intervallic range in which to do the analysis. Their output is a *pitch set*[82] of frequency ratios corresponding to the intervals at which the dissonance profile reaches a local minimum. These minima correspond to intervals at which the original partials are less rough with respect to the transposed partials.

80 Russolo, L. (1913). *The Art of Noise.* Unpaginated. Last retrieved February 2, 2012, from http://120years.net/machines/futurist/art_of_noise.html

81 They are available as an extension (a 'quark') for *SuperCollider*, a programming language for audio synthesis and algorithmic composition. McCartney, J. (2002-2012). *SuperCollider* (version 3.4-3.6) [software]. Available from http://supercollider.sourceforge.net/ Its documentation details the many functions and procedures for composing developed during this research some of which we don't have space to cover here. The reader is therefore directed to the help files contained in *DissonanceLib*.

82 A pitch-set is a raw collection of intervals, not yet a scale, lacking a melodic or functional structure, being '*dynamis*-less'.

Within the intervallic scope of their analysis, dissonance curves relate compound tones to frequency ratios. The peaks and valleys of their silhouettes occur at ratios that frequently lie within tolerance from well known proportions. These intervals coincide with proportionality from the point of view of timbre. A good example of this happens when inputting harmonic series and obtaining just (or extended-just) intonation intervals (see *Figure 1*).

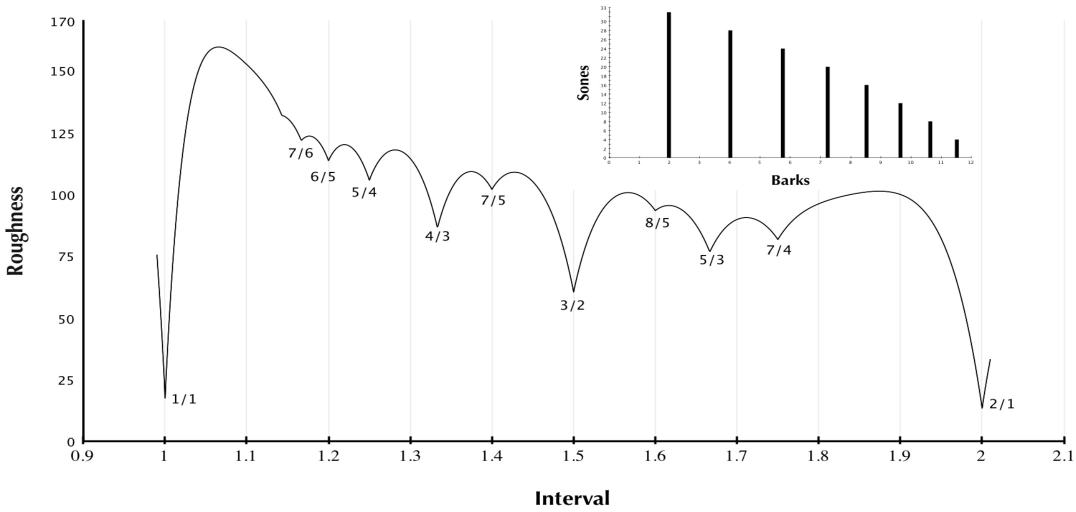

Figure 1. Dissonance curve derived from a mathematical spectrum, that of a sawtooth wave, over the range of a little more than an octave. The proportions obtained through rationalization are shown beneath each local minima; they correspond to intervals from just- or extended just-intonation. The spectrum is shown above the curve, frequency and amplitude have been converted into the subjective psychoacoustic scales of *barks* and *sones*. The figure was made from information generated with *DissonanceLib.*

The intervallic sets produced by the outputs of dissonance curves have the attribute of 'cooperating' with their source spectrum. They can be described as 'coherent', 'compatible', 'concordant', 'consonant', 'minimally rough', and other similar qualities, for that particular spectrum. No single term is able to describe the type of auditory sensations they produce, though all of them give good indications of their features, which vary also according to their settings in a musical context (see *Figure 2*).

Some of the intervals produced coincide with the partials of the spectrum while others are different, some arising from combinations of partials (as when, for example, partials 6 and 7 in an overtone series produce the interval 7/6, not corresponding to a partial), and others from intervallic inversion (as when a 4/3, an interval not contained in overtone series, is produced as an inversion of the third overtone). There are intervals of other kinds as well, not easily typified according to they way they arise. They depend both on the sweeping interval over which the curve is made and the relative amplitudes of the partials. The range over which dissonance curves are calculated is usually quite different from the ambit of the spectrum.

Dissonance curve analysis can be done between the spectra of two different sounds. However, most of the present research has been done by analyzing spectra from single sounds, mainly because this approach is very fertile and with two spectra the interpretation of the results is not so straightforward. The pitch sets resulting from the analysis of two spectra correspond to intervals for which the timbral compatibility between the two sounds is maximal, producing inter-timbral pitch sets and opening the way for future endeavors.

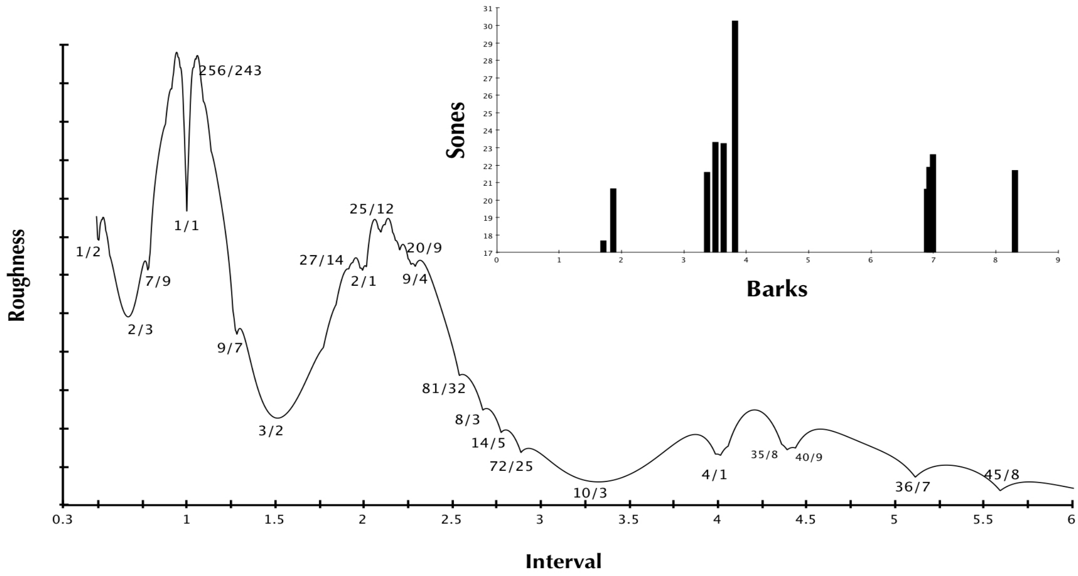

Figure 2. Dissonance curve obtained from an empirical spectrum, that of the vowel 'ee' (see its spectrum on the inner box). The ambit of the curve ranges from the sub-octave to two octaves and a fifth above the unison. Known intervals from just intonation can be seen, as well as others that are quite rare, whose function is more timbral than harmonic.

Interpreted as scales, the generated pitch sets have irregular microtonal structures, not repeating patterns within octave or other equivalents. Their behavior varies with register and at high transpositions increases the sensory consonance of the intervals as their spacings become widened (as in rightmost half of *Figure* 2). Ranges below 1.0 yield intervallic inversions resembling subharmonics with a similar timbral behavior.

In my implementation each interval in the set is represented as a distance in *cents*, a frequency ratio (a decimal number), an integer ratio (rationalized proportions, approximated to significant harmonic intervals), and as vectors within harmonic space. Additionally, each interval stores its roughness and calculates its harmonic measure[83]. For 7-limit configurations, a harmonic function is derived[84].

The intervallic sets can be deployed in timbral and proportional ways, which is why they are furthermore partitioned into timbral and harmonic subsets. Not having yet discussed harmonic space, it is still worthy of mentioning that the latter sets are usually confined to small regions near the origin, while the former lie farther out from the center. Different roles can be assigned to the separated interval sets, based on their characteristics:

- Timbral intervals, holding a close spectral relationship with the source sound are prone to be deployed in fluid and ephemeral roles, associated with time scales ranging from the micro temporal to the psychological present. They can be used as granular particles or as colorings and enhancements enveloping concrete sounds, as well as for electronic sound

83 We are now only panoramically reviewing these topics, which will be discussed in detail in the first section of Chapter 3. Harmonic measures quantify the harmonicity of proportions. There a various measures such as *harmonicity* (Barlow), *harmonic distance* (Tenney) and *gradus suavitatis* (Euler). Each produces a distinctive sonority.

84 The functions are sub- and dominant, sub- and mediant, sub- and septimal in an extrapolation of Hugo Riemann's ideas. Implemented from Wohl G. (2005). Algebra of Tonal Functions. Last retrieved December 2011, from http://sonantometry.blogspot.mx/2007_05_01_archive.html

synthesis or emulations with acoustic instruments.

- Harmonic intervals fall within certain compact zones in harmonic space, I call these zones 'islands' because they contain autonomous harmonic worlds. Each island is coupled to a specific temperament in equal divisions of the octave that approximates it and, by treating the proportions as degrees, allows for transformations, modulations and combinatory operations to be performed on them. Harmonic intervals are also compatible with the source spectrum but in less immediate, more abstract or formal ways, suggesting longer time frames than timbral intervals. They can function as fundamentals, pedals, drones, notes, chords and larger textures/progressions.
- Other partitioning schemes are available, such as separating the pitch sets according highest prime number, yielding subsets arranged according to combinations of primary intervals. Its is also possible to filter intervals lying within a certain harmonicity span or according to their absolute roughness.

Another way do deal with pitch sets is to extract their intrinsic harmonic areas, for which a 'stochastic harmonic field' is constructed. A harmonic metric is interpreted as the probabilities for choosing each interval, and this can be varied by scaling the probabilities according to the field's 'strength'. This permits generating textures with fine-grained transitions between different harmonic zones (between tonal, atonal and 'anti-tonal'). The details of this important aspect of my research will be put forward in section 3.2.

Further psychoacoustic models have been used in conjunction with dissonance curves. By providing conversions between different subjective scales (*bark*, *ERB*, *mel*, for pitch; *phones* and *sones* for loudness[85]), *DissonanceLib* permits to fine tune the generation of the curves. Additionally a 'pitch salience' model allows compensation by masking and virtual pitch. The latter is a subharmonic (greatest common divisor) of the main partials and provides a pitch lying in the lowest register of hearing, usually different from the spectral fundamental, which combines very well with the pitch sets derived from the curves[86].

2.1.2 Dissonance curves in relation to my musical research

> 'The thread of time has knots all along it ... Reality does not stop flickering around our abstract reference points. Time, with its small quanta twinkles and sparks.' (Gaston Bachelard, *The Dialectic of Duration*[87])

This is a first of two sections on my musical research, here providing a chronological overview of the compositional work involving dissonance curves by focusing more on the programs, general approaches and paradigm involved, rather than on individual pieces, which will be the concern and development of section 4.1.

The outcomes of this research have consisted in sound experiments, sketches, tryouts and pieces. The compositions are for instruments with and without real time or fixed electroacoustics and have sprung from the algorithmic composition tools developed to experiment with materials generated by dissonance curves. The first version of dissonance curves, their minimal implementation, obtained intervals out from their local minima. Further versions added more sophisticated intervallic analyses

85 Scaling amplitudes of the partials according to equal loudness contours.

86 Masking, virtual pitch and salience derived from Parncutt, R. (1994). Applying Psychoacoustics in Composition: 'Harmonic' Progressions of 'Nonharmonic' Sonorities. *Perspectives of New Music*, 32(2).

87 Bachelard, G. (2000). *The Dialectic of Duration*, Manchester: Clinamen, 81.

such as rationalization, representation in harmonic space, timbral/harmonic partitions, visualizations and the establishment of their harmonic fields. Major developments have gone hand in hand with compositions, in turn accompanied by sketches and preliminary experiments. This thesis is the theoretical upshot of the questions raised in practice, but it was also used as a source of speculations and experimentations with which to further the practice. Experimentation in this research is meant not only in the sense of John Cage, i.e. music, the outcome of which is not foreseen, but also in the sense of experimental science, where one experiment leads to new questions, hypothesis, tests, surprises, evaluations and thus to further experimental cycles, never quite reaching a conclusive ending but opening up to new musical experiences.

The first piece after the research began was a piece for solo harpsichord based on recursive pitch structures, which 'modulate' further away from the initial configurations in accordance with the level of recursion[88]. This was just before dissonance curves, however. The first composition to spring from them involved a single pitch set derived from a mathematical spectrum (the sawtooth wave of *Figure 1*). The piece transitions between different combinations of these intervals, filtering them according to harmonic measures[89].

Later on, the work focused on following and enveloping the source sounds as they change in time with what I call 'dissonance chorales': chordal and other textural accompaniments adhering to the surface of concrete sounds, usually happening at a fast pace. Each pitch set can be treated either as a chord or as a texture. It was the upshot of developing the program *Dissophonos*, built atop the basic tools. It permits spotting regions of a sound files to extract and listen to dissonance curves at those points. This enables making 'dissonance chorales' out these selections: pitch sets corresponding to the spectra at those moments. The maximum number of selectable points in the sound is limited by the resolution of the spectral analysis, varying from around 8 to 20 per second, which is quite dense in terms of the requirements of the synthesis engine to render the textures, implying non-real time work. The chorales can be saved to disk to be retrieved and used later, allowing different kinds of electronic 'orchestrations' and accompaniments to the source sounds. They are saved as collections of dissonance curves, sometimes containing many thousands of them.

The experiments produced by this program have also led to a classificatory typology of the outcomes of dissonance curves according to the source spectra (whether it is mathematical, empirical, instrumental, phonetic, varieties of randomness and noise – such as the already mentioned 'frozen noise', – and so on). The groupings tend to highlight the kinds of intervals characterizing these sounds. The classification has not been pursued in an extensive nor controlled manner as it falls outside the aims of my compositional approach. What it has done, though, is provide a connection between acousmatics and harmony, showing that one way of using dissonance curves is through a '*harmonie concrète*' of sorts, providing a timbral (*concrète* or sonic) logic to harmony while conversely complementing 'sound-object *solfège*' with proportionality and other harmonic resources: timbral harmony, on the one hand, as well as the harmony within timbre ('harmonic timbres') on the other.

The next important step was the real-time implementation of dissonance curves, entailing that the curves were to be triggered manually at certain moments, instead of being continuously generated. This is because of the amount of calculations needed as well as (mainly) because the work that can be realized by a single pitch set requires enough time to be musically interesting. These pitch sets extracted from the sounding audio input are deployed as different types of electronic textures. Each texture can run for a while on the same pitch set, to be replaced by a new pitch set when triggered anew, transitioning either smoothly or abruptly between the two sets. Another possibility is to

88 *discrete infinity* (2006) for harpsichord. The piece was almost abandoned due to circumstances, but later finished in 2008. It has not yet been performed.

89 *rolita pa Modelo* (2007), for ensemble (Fl, B. Cl, Trp, Hrp, Guit, Vln, Vlc, DB). Written for ensemble *Modelo62*.

change the type of texture (both in terms of layers, rhythmic patterns, timbres, tempos) with every change in dissonance pitch set. These alternatives imply dealing aesthetically with the pace at which to deploy these harmonic textures and the kinds of interactions and feedback between a performer (or the audience in an installation) and the textures. One of the main aims of my interactive piece for guitarist and computer is to have the performer imitate the computer in an acousmatic manner, playing his instrument by reacting not only to pitch but to the whole timbral environment, giving rise to gestures and sonic aggregates which are used by the computer to further imitate her/him producing further textures built on dissonance pitch sets from his input, engaging in a timbral-imitative feedback loop[90].

Instead of treating pitch sets as chords or groups of pitches with which to lay a shroud over other sounds, relating and constantly varying the intervals according to the sonic context, the following phase of the research concentrated on wresting different qualities from single pitch sets, uncovering their 'internal', rather than 'external', consistency (as was the case with dissonance chorales), in accordance with their harmonic properties. From this idea the harmonic fields generator program, *Harmonic Fields Forever*, was developed. It creates gradual, almost imperceptible transitions through the space of configurations brought forward by these interval sets. It uses a lesser amount, though more complex and larger, pitch sets, usually just one, distributed over longer periods of time, providing ways to delimit and explore their regions and modes. The principal parameter, the field's strength, variable between zero (all intervals equally probable) and one (harmonic intervals more probable), produces a continuum of differing pitch configurations ranging from atonal to tonal. When the strength is reversed to reach minus one, priority is given to the least harmonic pitches, yielding a zone which I call *antitonal,* for being relatively harmonic between the chosen intervals but highly inharmonic with respect to the overall fundamental. The program can work in two modes: 'tonic', which relativizes the probabilities with respect to every pitch in the set, providing a distinct modes, and 'atonic', which uses the probabilities of all the modes, making each new chosen pitch the tonic with which to choose the next one. There is a striking difference in sound between these two types of strategies[91].

These approaches can be summarized as follows:

- Composing with a wide range of tunings related to timbres
    - Using higher than 5-limit intervals with aid of a timbral logic. This implies paying attention to the connection between intervals and the sounds from which they are obtained.
    - *Acousmatic* harmony: extending and complementing 'sound-object *solfège*' by providing harmonic analyses to spectral materials. Harmony consisting in levels of 'sonance' instead of poles of consonance/dissonance.

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90 *'strings'* (2007) for guitarist, speakers and computer, an open, improvisatory piece composed around the principle of computer-performer feedback. It was made in collaboration with guitarist Tom Pauwels and varies quite a lot between performers (it has also been played with Matthias Koone in 2008 and Carlos Iturralde in 2010). It was part of the project *A Search for reNoise* with composers Paul Craenen and Cathy van Eck in the *Transit Festival* in Leuven, 2007. More recently a derivation of this program has been used to create the sound installation *Ahí estése* (2011) for computer, microphone and multichannel setup, as part of electronic arts festival *Transitio MX* in Mexico City, 2011. More details in Chapter 4.

91 This will be explored in detail in section 3.2, here it is mentioned in relation to dissonance curves, but it also includes the ability to work with pitch sets derived by means other than the curves. This program has been used to generate the *Logos Sessions* (the first batch in 2008, the second in 2009), algorithmic improvisations with harmonic fields performed with the musical automata of *Logos Institute* in Ghent. It is also the basis for *Circular Limit* (2008) for bass recorder and electronics, written for recorder player Tomma Wessel, as well as for electroacoustic textures used in several other projects.

  - To provide the tuning and harmonic characteristics of soundscapes: their virtual, spectral, dissonance pitches and their representation and separation into intervals sets in harmonic space from which diverse deployment strategies can be built.

- Synthesis of dissonance chords, timbres and textures
  - ‘Dissonance chorales’: the harmonization of recorded sounds.
  - ‘Granular harmony’, *harmonie concrète*: when the pace of the chorales is fast enough to become granular and follows the transients, formants, and other fast fluctuations with its fast textures.
- Real-time analysis-synthesis
  - The pace at which to change the dissonance textures: how much work can be accomplished by each one and the speed at which timbral changes in the source surpass the congruence of its harmonic background in an interactive situation.
  - Harmonic feedback between the player and the computer, both responding to each other.
  - The harmonization of a sonic environment (and its social interaction).
- Harmonic fields
  - Choosing the notes of a pitch set according to probabilities correlated with the harmonic measure of the intervals.
  - Extraction of sonority regions within a single pitch set.

Theorization and research intersect with the compositional work. They come after the music has led to new questions and findings (or lack thereof!), but also have a retrospective effect of opening up new speculative possibilities to try out and incorporate into the cycle. For instance, the hypothesis of this study, namely harmonic duality, is a consequence of working with dissonance curves. At the same time it has informed their development to the point of becoming a concept that almost outweighs their original purpose. A review of Greek harmonics also infuses the musical work with new hypotheses and tools (arithmetic functions from Pythagorean harmonists such as *katapyknosis*, musical means, and others have been implemented and used), understanding and acknowledging their subtle and (almost forgotten) ideas that seem to shine brightly in light of today’s harmonic situation. This has led the compositions less towards materials derived from empirical spectrums and more towards abstract harmonic structures, also deployed at the scales of rhythm and form. These approaches are to be mentioned in section 4.1 None of this could have been suspected when I started implementing the curves at the end of 2006.

Another influential development related to dissonance curves was the development in 2009 of ‘polyrhythmia’, a collaboration with sonologist and physicist Alberto Novello. An algorithm that connects at various time scales elements from rhythm, pitch and form, it is basically a rhythmic acceleration → steady-state → deceleration process in several layers that interprets rhythm spectrally as the stratifying of simultaneous periodicities. This is equivalent to a spectrum: each partial is regarded as periodically repeating at a certain phase shift; any kind of metric rhythm can be reproduced this was if enough partials (rhythmic elements) are present[92]. From out of a single

92 A similar spectral approach to rhythm, though not involving accelerated/decelerated transitions between steady states has been developed by Sorensen, A. (2010). Oscillating Rhythms [webpage]. Last retrieved November 3, 2012, from http://www.acid.net.au/indexb565.html?option=com_content&task=view&id=99&Itemid=164

spectrum (say, from a 4-5-6-7 polyrhythm harmonically equivalent to a natural seventh chord) it transitions from vertical chords into that rhythm by accelerating/decelerating each element at a precise rate so that it falls into place in the 'steady' section, later to be decelerated so that they all fall together at the end. The process can be applied at several speeds and densities and together with pitches spawned by dissonance curves[93].

The effect of this process when used together with pitches is that of polyrhythmic cannons. Their time scale can be varied drastically, so the process can take from around several minutes to fractions of a second, also transitioning between the levels as an acceleration into a steady state can be further accelerated into another steady state at a succeeding time scale, and so on (and conversely for decelerations). This is the starting point for the electroacoustic multichannel piece done in collaboration, putting these ideas in motion. It stems from the dissonance analysis of a sound recording. The pitches are used to reconstruct the sound as a chords of bandpass regions of the sound, which begin their canonic process of separating pitch and rhythm-wise into a long process in which the pitches transition into sine waves, then up a time scale into ring-modulation, then up another rung into FM, then (already very fast) into complex spectra and finally into very condensed accelerations with impulses. This middle section is a sort of free plateau where anything can happen, after which the process is repeated in reverse manner, decelerating towards the reconstruction of the sound at the end (*Figure 3*).

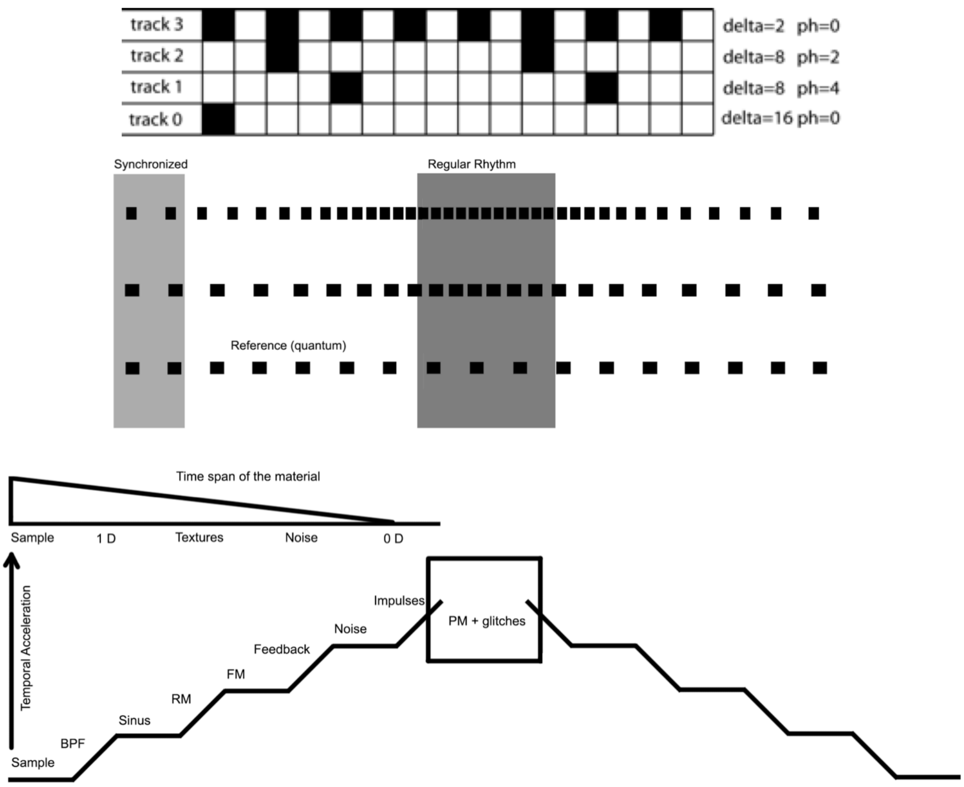

Figure 3. Above: a graphic representation of how a rhythm is conceived spectrally showing the periodicity and phase of each component. Middle: a simplified representation of the polyrhythmic process in three layers, from synchronized chords to steady-state rhythm ('Regular Rhythm') and back. Below: schema of the piece *Clinamen*, going from sound recording, through bandpass filtering in the first acceleration, then

93 For more details on this algorithm, see Lach J. S. & Novello, A. (2010). Musical Scene Analysis: Applying the Laws of Stream Segregation to Music. *Ideas Sónicas/Sonic Ideas*, 2(2), 20-27. The algorithm was initailly used in *Blank Space*, but the piece stemming from this collaboration where it is developed further is *Clinamen* (2011), 4 channel electroacoustic soundtrack, composed jointly. More in Chapter 4.

sine waves, ring-modulation (in the second acceleration), FM, FM-feedback, noise and impulses. The middle section uses PM – phase modulation – and free elements and then the process is reversed. It transitions from sample to 1D (one dimensional sines), texture, noise and impulse ('zero dimensional'). Figures courtesy of Alberto Novello.

From 2009 onwards, there have not been many new developments to the basic tools of *DissonanceLib* – though debugging and documentation has continued – nor to the larger programs developed on top of them. The concentration has been more on composing – which in my case involves quite a lot of programming, – using more or less the same tools although applying them differently and in different combinations and methods. It has also involved reading theory, be it on Greek harmonics or more philosophical materials. The theory has given me ways to ground and give coherence to the whole undertaking, finding in some philosophical readings ideas that authorized me to take a formalistic and Pythagorean approach that goes a bit against the generally anti-harmonic and anti-essentialist ideas of the time, but with an understanding of the dangers and traps involved. Even though the thesis does not tackle these topics directly, they inform it and even some sections (the critical and speculative ones) have been written on the basis of, for example, Alain Badiou or Graham Harman.

2.1.3 The psychoacoustics behind dissonance curves

Dissonance curves go back to the psychoacoustics of Hermann von Helmholtz, in his book *Die Lehre von den Tonempfindungen* of 1862, which in its expanded translation by Alexander Ellis, *On the Sensations of Tone as a Psychological basis for Music* of 1885, is one the few scientific books from the nineteenth century which is still being published and read in the twenty-first[94]. As has been shown, they are based on roughness, a dynamic fluctuation ('intermittence' as described by Helmholtz) caused by the interference between the amplitudes of two periodic sounds. At slow speeds they are known as beatings, at intermediate speeds as tremolos. When their rate is faster than sixteen times a second they produce a continuous and irregular vibration accompanied by a low tone; this is referred to as roughness. Helmholtz believed he had found in roughness the physical, as opposed to metaphysical or number-theoretic, solution to the millenary problem of how consonance and dissonance emerge and can be measured. Even though beats and tremolos are acoustic phenomena, roughness is a mainly psychoacoustic one, influenced by sensorial distortions rather than existing solely in the intermittence of the acoustic waves. We now realize that he discovered the main aspect of 'sensory dissonance', which is also influenced by the nearness of the partials in a sound to a harmonic series (something known as *tonalness*). Sensory dissonance is one of the main components of what we have been referring to as the timbral aspect of harmony.

Helmholtz's theory of hearing models the ear as a bank of resonators. As we already saw, this is the basis for spatial hearing theories, which are physiological, in contrast to temporal theories, which are psychological, happening higher up along the auditory pathway in the mid brain and cortex. Most recent spatial theories are refinements upon Helmholtz. His model pictured the transduction in the cochlea as resonating tubes (or strings, but the stress was given to the tubes) inside the organ of Corti. In was in the 1930's that Georg von Bekèsy discovered the basilar membrane which actually performs it. The other main discovery pertaining to roughness and dissonance curves is the 'critical bandwidth' (Fletcher, 1940's). In the 1960's Greenwood related the bandwidth to roughness[95],

94 Helmholtz, H. (1960). *On the Sensations of Tone as a Psychological basis for the Theory of Music* (A. Ellis, Trans.). New York: Dover. (Original work published in 1862).

95 Greenwood, D. (1961). Auditory masking and the critical band. *Journal of the Acoustical Society of America,* 33, 484–501.

afterward Zwicker and Stevens provided a psychophysical unit calibrated to it, the *bark*, and Plomp and Levelt provided a model to calculate the total roughness for compound tones[96].

The critical bandwidth is the area of the membrane within which partials mask and interfere with each other, producing roughness. The speed of fluctuations between two components reaches a maximum beyond which the sensation of roughness declines. Helmholtz measured this maximum speed to be of around 33 Hz for 100 Hz tones, acknowledging that this speed varies with register: 'as we ascend the rapidity will increase but the character of the sensation remain unaltered'[97]. Plomp and Levelt linked this behavior to the critical band:

> 'Helmholtz's theory, stating that the degree of dissonance is determined by the roughness of rapid beats, may be maintained. However, a modification has to be made in the sense that minimal and maximal roughness of intervals are not independent of the mean frequency of the interval [its register]. A better hypothesis seems to be that they are related to critical bandwidth, with the rule of thumb that maximal tonal dissonance is produced by intervals subtending 25% of the critical bandwidth, and the maximal tonal [sensory] consonance is reached for interval widths of 100% of the critical bandwidth. In all experiments in which the critical bands have been investigated, the width of this band represents the frequency-difference limit over which simple tones cooperate. So it is not surprising that roughness appears only for tones at a frequency distance not exceeding the critical bandwidth.'[98]

They obtain a weighting function over the critical bandwidth, a best-fitting curve approximating the results of psychometric studies of subjective judgements scores for the 'pleasantness' of intervals (*Figure 4*, left). With this weighting curve it becomes possible to measure the dissonance of compound tones, since it allows to account for the influence of higher component partials and not only their fundamentals. It is assumed that dissonance behaves linearly: the total dissonance is the sum of dissonances of each pair of adjacent partials. 'Though these presuppositions are rather speculative, they are not unreasonable as a first approximation, and may be justified for illustrating how, for complex-tone intervals, consonance depends on frequency and frequency ratio'[99] In this conclusion lies a link to proportionality, justifying linearity in the sake of arriving at ratios – which explain consonance – and thus allowing the study of their relationship with spectra. Furthermore, by being based on empirical cognitive subjective data, the model not only captures a physiological function but also carries with it the effects of psychological mechanisms involved higher up in perception.

96 Plomp, R., Levelt, W. (1965). Tonal Consonance and the Critical Bandwidth.", *Journal of the Acoustical Society of America*, Vol. 38(4), 548-560.

97 Helmholtz, Ibid., 171.

98 Ibid., 554-555. The terminology we have been using is presented in brackets to make the relevance of the passage to our discussion clearer. The terms are equivalent.

99 Ibid., 555.

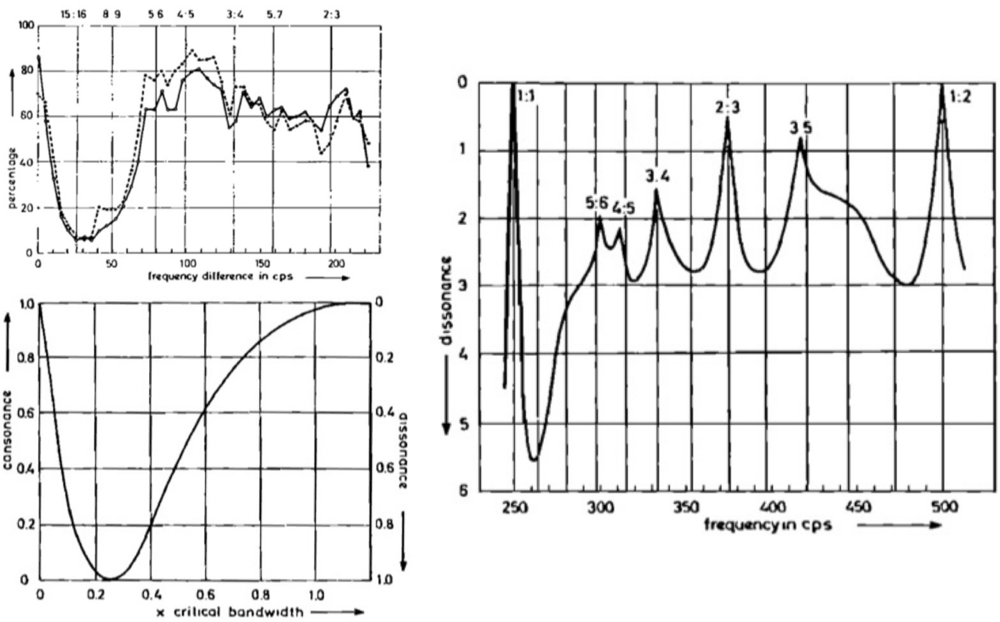

Figure 4. Left, top: a plot of the results from psychometric consonance judgement tests. Left, bottom: the weighting curve after fitting, averaging and calibrating the empirical data to a critical bandwidth. Notice how consonance reaches a minimum at around 0.25. Right: a dissonance curve made from a spectrum of 6 partials, the first harmonic spectrum fixed at 250 Hz, the second one varied between a bit less than 250 Hz and a bit more than 500 Hz. The vertical lines are equal tempered semitones. Note that the vertical axis is inverted with respect to my implementation, but that is only a result of the visualization. (The three graphics are taken from Plomp & Levelt, 1966).

There are two psychoacoustic (also called 'subjective') units calibrated to the basilar membrane, the *bark* and the ERB. The former are useful for pitch related features, while the latter are better suited for loudness models (for calculating the effect of masking, the other auditory function that critical band models explain)[100]. A *bark* is equivalent to a critical band in pitch, $1/4^{th}$ of a bark corresponding to 25% of the curve. Musically, this interval corresponds, for most of the hearing range, to a minor third. This fact shows one of the reasons why this interval is the limit between melodic and harmonic intervals, between 'steps' and 'jumps': below the critical band, partials interact, so intervals smaller than a minor third are rough when sounded together; above this threshold partials produce less roughness and are therefore better suited for vertical arrangements.

A dissonance curve is calculated by measuring the contribution of roughness between all pairs of partials for a compound tone. This makes for a single point in the curve. Measuring a spectrum against a transposition of itself (optionally against the transposition of another spectrum) gives the roughness for that particular transposition. Sweeping the transposition interval in the manner of a glissando (by using small steps in practice), and calculating the total roughness at each transposition level, we obtain a dissonance curve (*Figure 4*, right, shows Plomp and Levelt's dissonance curve).

Going back to Helmholtz one last time, it is remarkable that he and Ellis were able to calculate and draw a dissonance curve for a violin tone before any of the developments related to the basilar membrane or critical bands had been made. The equations they use, based on sympathetic

100 They arise from different methods of measuring the critical bandwidth. ERB stands for Equivalent Rectangular Bandwidth. *DissonanceLib* implements both scales, using ERB for masking compensation and barks for dissonance calculations.

vibrations of resonators in the organ of Corti, are quite convoluted because their assumptions lacked this evidence. It implied some judicious simplifications together with speculation regarding the shape of the weighting curve. The curves themselves were drawn separately for different pairs of partials and later superimposed in the drawing. I cannot fail to be impressed by these drawings. It is first rate science, a reason why this book continues to be influential 150 years after its first edition[101].

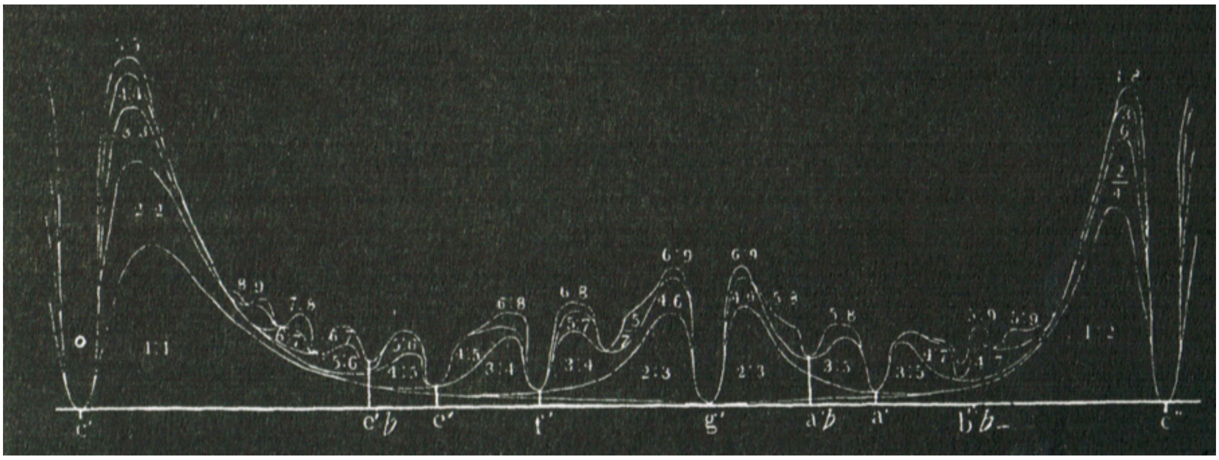

Figure 5. A dissonance curve made by Helmholtz a century before Plomp and Levelt (from Helmholtz, 1960, [1862]).

Dissonance curves have been quite studied and used ever since the ones calculated manually by Plomp and Levelt. Later developments related to them involve the work of Kameoka and Kuriyagawa (1968)[102], a quantitative model of 'dissonance intensity' based on the same premises as Plomp and Levelt but from different empirical data, and Richard Parncutt (1976), who approximated the weighting curve mathematically with an exponential function.

Plomp and Levelt's dissonance curve was calculated without taking into account the interactions between all partials, only between adjacent ones, as well as not considering their amplitudes. Clarence Barlow's approach, probably the earliest compositional use of dissonance curves incorporated both interactions between partials and amplitudes. It is part of the research behind the piece *Çoğluotobüsişletmesi* (1978), using them to calculate the roughness for all the notes of a piano. Instead of obtaining intervals, as is the case with my approach, these measurements were employed to calibrate the priority formulas used to generate the notes of the piece. The parameters influenced were 'melodic smoothness', harmonic (or 'tonal') priority and harmonic 'cohesion'. This research would later be incorporated into the algorithmic composition program *Autobusk.* Another use is made in his program *Dissonometer*, used to calculate the total roughness of chords for a given timbre. This is the inverse of the approach taken by this research: my aim is to obtain intervals from timbres, while his is to obtain roughness from intervals in conjunction with timbres[103].

Other compositional uses include the implementations of Wendy Carlos and William Sethares (1980s). Sethares provides the most comprehensive study on them to date, delving into their mathematical properties and their relation to the source spectra[104].

101 Helmholtz, Ibid., Figs. 60 A and 60 B, 193, as well as the technical explanation by Ellis in Appendix XV, 415-421.

102 Kameoka, A. & Kuriyagawa, M. (1969). Consonance theory, part II: Consonance of complex tones and its computation method. *Journal of the Acoustical Society of America*, 45(6), 1460-1469.

103 Barlow, C. (1981). *Bus Journey to Parametron.* Cologne: Feedback Papers 21-23, 55-70. Also see Barlow, C. (2012). *On Musiquantics.* University of Mainz: Musikwissenschaftliches Institut Der Johannes Gutenberg Universität.

104 Sethares, W. (1999). *Tuning, Timbre, Spectrum, Scale*. Berlin: Springer. Also, Carlos, W. (1987). Tuning at the crossroads. *Computer Music Journal,* 11(1), 29- 43. A very clear, thorough and updated account is given in Benson, D. (2008).

Sensory dissonance and timbre in relation to music correspond to what Tenney designates as CDC-5 in his review of consonance and dissonance conceptions (1988)[105]. It is a a distinct mode harmony, stemming from Helmholtz, relating to orchestration, to timbral combinations of instruments in their relation to harmony, prevalent in the music of the nineteenth century but actually pertaining and embracing much music of the twentieth. It is especially relevant in electroacoustic music and, as we will see in the following section, is also connected to atonality and many modernist approaches to pitch.

My implementation was initially based on Sethares. However, his uses a formula in terms of frequency and amplitude. On recommendation by Barlow, I adapted the code to use Parncutt's approximation, adjusted to psychoacoustic units. It has the advantage of giving finer grained results and is a bit faster to calculate[106]. Had I stopped there, it would not have been too different from Sethares' research, which is restricted to finding the intervals for the local minima of the curves and using them as scales for playing back the timbres that generated them (in 1990s sampler-sequencer style). As mentioned, my implementation furthers this by rationalizing the intervals to find the closest and most harmonic whole number ratios. Their harmonic metrics are calculated and made into pitch sets representing the intervals in harmonic space and partitioning them into harmonic and timbral subsets. The calculations for constructing inter-harmonicity matrixes for harmonic fields are also a unique part of my implementation.

Other contemporary implementations of dissonance curves that I know of (the list does not pretend to be exhaustive) are those by Alexander Porres and Charles Céleste Hutchins. The former implements them more with a focus on sound synthesis/re-synthesis and spectral modeling than algorithmic composition, as well as incorporating other psychoacoustic theories[107]. The latter implements dissonance curves with an emphasis on FM synthesis spectra, and is also available as an extension for *SuperCollider* called *TuningLib*. Also for *SuperCollider*, Nick Collins has developed a unit generator, *SensoryDissonance*, which calculates the instantaneous total roughness for an input sound. It does not, however, transpose or obtain intervals from the data, so it is not a complete dissonance curve analysis[108].

Notable is the similarity between dissonance curves and other methods for measuring consonance, such as Harry Patch's qualitative 'one-footed bride' (1940's) as well as Paul Erlich's harmonic entropy[109]. This concept is combined with dissonance curves in the work of Georg Hadju's and his program *Djster,* which incorporates these ideas into to a version for Max/MSP of Barlow's *Autobusk* program[110].

To end on a speculative note, dissonance curves may be considered as a sort of autocorrelation of spectra in the frequency domain. Recall that in autocorrelation a signal is delayed many times and summed up, the result exhibiting its periodicities. Frequency-wise we substitute partials for signal

Music, a Mathematical Offering. Chapter 4, 139-144. Last retrived March 30, 2012, from http://www.abdn.ac.uk/~mth192/html/maths-music.html

105 Tenney, J. (1988). *A History of Consonance and Dissonance.* New York: Excelsior Music Publishing.

106 Dissonance measure, D, for a pair of partials is: $D = \sqrt{s_1 * s_2} * P(bk_1 - bk_2)$
P is the Parncutt approximation of the weighting curve: $P(x) = 4|x|e^{1-4|x|}$
s_1, s_2 and bk_1, bk_2 are the intensities and frequencies of the partials in sones and barks respectively. See Appendix I for a more detailed explanation of the implementation.

107 Porres, A. (2011). Dissonance Model Toolbox in Pure Data. *Review of the International Meeting of Music, Sound and Art (EIMAS, 2011).* Last retrieved February 19, 2012, from http://www.ufjf.br/anais_eimas/files/2012/02/Dissonance-Model-Toolbox-in-Pure-Data-Alexandre-Torres-Porres.pdf

108 See http://doc.sccode.org/Classes/SensoryDissonance.html Last retrieved November 3, 2012.

109 Erlich, P., Monzo, J. (2004). On harmonic entropy. In J. Monzo (Ed.), *Enciclopedia of Tuning.* Last retrieved March 30, 2012, from http://www.sonic-arts.org/td/erlich/entropy.htm

110 Hadju, G. (2011). *DJSter* [software]. Last retrieved August 3, 2012, from http://djster.georghajdu.de/extras/tonality

and transposition for delay, while the summing is quite similar (the weightings being just a particular kind of integration). The minima obtained exhibit spectral periodicities, resonances within a space modeled after the physical/psychological properties of a membrane. What is interesting is that the 'space' or medium defined by the model can be detached from its empirical background to become an abstract mathematical space in which the peaks corresponding to minimal roughness are seen as tendencies toward 'basins of attraction' associated with periodic or quasi-periodic behavior, as in mathematical theories of dynamical systems (even if this case is static). In this sense the peaks, which connect proportions and spectra, may be thought of as singularities. According to Manuel DeLanda, singularities are mechanism-independent, defining the objective structure of a space of possibilities (minima, maxima, inflection points) which does not depend on the material substratum. It will have to be seen up to what point this is the case for dissonance curves and if their emergent properties are independent of the properties of the weighting curve. It is interesting to conceive that simple frequency ratios could arise independently from these weighting measures and their underlying physical layers, and that therefore their 'coincidences' with just intervals are not dependent on physiological properties but could arise in transduction systems quite different from humans[111].

2.1.4 Consonance and dissonance theories

This section could have been excluded from the chapter as it repeats some topic that have been seen before. However, in the spirit of laying out my findings with respect to consonance and dissonance and to further the discussion began in the section on Greek harmonics I will provide a comparative listing of current accounts of consonance and dissonance, from the standpoint of harmonic duality. It will take advantage of what we have seen until now to also briefly discuss and account for the cultural contexts of harmony. The section is also meant to bridge the discussion into the second part of this chapter, involving a historical and aesthetic account of twentieth century musical modernism with respect to timbral harmony. Although the listing is not exhaustive, it is more or less the way musical science stood at the beginning of the century, after a wave of research had taken place, probably as a response to Helmholtz. The listing will provide the main topics, proponents and features of each conception[112].

i. Proportions. A line of thinking that spans from the Pythagoreans up to Galileo, Leibniz, Euler, and Theodore Lipps[113] at the beginning of the twentieth century. Proportional consonance corresponds to *harmonicity,* distinguishing it from the timbral sort.
   - Intervals are understood as frequency ratios, relationships between fundamentals or pulse counts.
   - Corresponds to discrete mathematics and to time-based pitch perception: to particles rather than waves.
   - As discussed in the previous section on consonance, harmonicity in ratios depends on the properties of the numbers involved, leading to harmonic measures. Greeks required small numbers within the *tetraktys*. Euler provided the connection with prime

111 DeLanda, M. (2011). Emergence, Causality and Realism. In Bryant L., Srnicek, N., Harman, G. (Eds.), *The Speculative Turn*. Melbourne: re.press, 381-392.

112 The structure and some of the content for this list stems from Sethares,*Tuning, Timbre, Spectrum, Scale.*

113 Lipps, T. (1995). *Consonance and Dissonance in Music.* (W. Thomson, Trans.). San Marino, CA: Everett Books. (Original work published in 1905). He offers detailed critiques of all the previous theories con consonance: Helmholtz, Stumpf, Krüger, Wundt and Meyer, proposing his own time-based 'Tone Rhythm' theory, close to Galileo's commensurability extended to include simultaneous sounds as well as melodic successions.

numbers, implying that divisibility is more fundamental than magnitude. From his *gradus suavitatis* function springs Clarence Barlow's harmonic measure.
- With proportionality the concept of tolerance is needed to avoid shameful aporias: a slightly mistuned consonance would correspond to ratios with enormous numbers, implying a huge inharmonicity, which is clearly not the case. Considering a proportion as *referential* to a small zone around it in pitch distance space avoids the problem. We'll delve into the factors involved in intervallic 'rationalization' in the next chapter.

ii. Relationship between harmonics. The theories of Jean Philippe Rameau and William Wundt.
- A naturalist account: tonal harmony as deriving from Nature.
- An extrapolation of the harmonic series to consonance: coinciding harmonics are the basis for melodic intervals.
- *Corps sonore*: an idealized overtone series. The tenets of spectral music extend this further.
- Its main problem as a theory is its failure to account minor chords, the subdominant function or any other structures that require intervallic inversion. It relies too closely to just intonation (i.e. it does not take tolerance into account) and hence is usually limited to a single fundamental, leaving modulation unaccounted for. It also has to impose an arbitrary limit on the series in its analysis and ignores the role of prime numbers. It is a timbral conception.

iii. Beats between partials. Helmholtz.
- A physiological conception.
- Leads to sensory dissonance, composed of roughness and *tonalness.*
- Tenney identifies it as timbral consonance/dissonance, CDC-5. The main component for timbral harmony.
- It has 3 main consequences:
  - individual tones have intrinsic dissonances
  - consonance and dissonance depend not only on relations between fundamentals but also on their spectral structure
  - consonance and dissonance stand in a continuum of gradations (*sonance* levels) instead of being polarities (in contrast to CDC-4, functional tonality)
- Of all consonance theories this one deals both with harmonic and inharmonic sounds.

iv. Difference tones. Felix Krueger.
- Difference tones were made famous in the XVII century by Guiseppe Tartini and studied by Helmholtz. They are 'ghost' tones arising from non-linear processes in auditory perception. The most relevant ones are difference and summation tones, corresponding to the sum and difference of the frequencies of two tones lying near each other. They need to be quite loud to be noticeable.
- A psychoacoustic notion.
- The order and complexity of difference tones serves to determine consonance hierarchies.
- Dissonance is proportional to the number of distinct difference tones in an interval.
- A strong argument against this conception is that because they are weaker than other

psychoacoustic byproducts such as virtual pitch or roughness, their effect is quite limited to account for consonance.

- On the other hand they are compositionally very useful for creating interesting harmonies (as in Tenney's *Koan* for string quartet, for example, where a glissando is harmonized with difference and summation tones) or as a source of interesting psychoacoustics illusions (such as Barlow's *Until* for piccolo).

v. Fusion. Carl Stumpf.

- Happens when a plurality of tones form a unity or whole for consciousness.
- A phenomenological conception which is kind of Aristoxenian and also close to medieval notions of consonance as 'sounding as a single tone'.
- Only given qualities can be the basis for consonance, what Tenney identifies as CDC-2.
- Takes into account deviations from exact tunings and stresses the independence of consonance from timbre, loudness and register: roughness cannot account for the consonance and dissonance of sine waves, since it would imply that any intervals surpassing the critical bandwidth would be equally consonant. In this sense this conception embraces aspects of proportionality (intervallic qualities of fundamentals) and timbrality (sensation as a whole).
- Its main drawback is that there is no method for defining fusion in an unambiguous or quantifiable way. There are no simple physical correlates for it.
- There is a connection that links Helmholtz with theorists who belong to the school of empirical psychology of Franz Brentano, the teacher of Stumpf, Husserl and Freud, himself a student of Wundt. Some of the most important theories of experience and consciousness of the twentieth century (psychoanalysis, phenomenology, gestalt), stem from this school.

vi. Virtual Pitch. Terhardt, Parncutt.

- This conception is much more recent. It is also timbral, derived and related to Helmholtz and place theories.
- *Tonalness* as a measure of how a sound's partials deviate from a harmonic template, *harmonic entropy* (Paul Erlich) as a measure of this deviation, quantifying the uncertainty involved in interpreting intervals in terms of simple integer ratios.

vii. Cultural criteria. Norman Cazden (1940's)[114].

- To consider that natural laws lie behind consonance implies that the foundations of music and art are static, universal and deterministic, from which no possible changes in music can be conceivable.
- It leads to Pythagorean number magic and the mystery of the 'harmony of the spheres'.
- Musical metaphysics consists in a series of domains that go from the acoustic waveform, physiology of hearing and psychology of perception. All of these theories assume an essentialist error: proportions are made eternal & mystic 'noumena', of the 'psychological consonance of tonal isolates', alluding to the laboratory conditions of psychoacoustics and empirical psychology. All of these theories diverge in their predictions and hierarchical orderings of consonance (something we regard positively

114 Cazden, N. (1945). Musical Consonance and Dissonance: a Cultural Criterion. *The Journal of Aesthetics and Art Criticism*, 4(1), 3-11.

instead of a sign of contradiction). The problem of accounting for minor harmonies immerses theorists into gymnastics reminiscent of Ptolomy's epicycles in astronomy to justify what is just a question of cultural 'taste'.

- Against this reductionism, he proposes much more context and learning. Context: expectancy, movement in an harmonic structure over a certain time frame. Culture: common response acquired by individuals and determined by a cultural area. The science of consonance and dissonance is social, not natural: history, conventions, language, comparative musical systems (with other cultures), etc. Our position in this study is that they are neither natural nor cultural objects but hybrids of both, at different levels of scale, each level possessing its own mixtures and properties irreducible to other levels (a standpoint informed by Bruno Latour).
- For Cazden, harmony has nothing to do with intervallic qualities but with movement and resolution in relation with other intervals and chords (that is, to Tenney's CDC-4, functional harmony), so although he does acknowledge that intervallic qualities of "fifthness" and "major-thirdness" exist, he does not consider harmony as happening at this level. There is nothing 'natural' about resolution, it is based *exclusively* on cultural conventional rules.
- Atonality is a proof that culturally made arbitrary rules different from traditional tonality are possible, showing how intervals can be liberated from being mediators into ends in themselves, therefore leading into a timbral CDC-5 conception of sorts (as the next section will detail). Nevertheless, it is not easy to invent new tonal systems without falling back into subsets of functional or modal (poly)tonality. One cannot just posit an arbitrary way of resolving dissonances (such as interchanging fifths with tritones) while speculating that culture will eventually and in ideal circumstances catch up. This has not happened in atonality, and I don't think it will ever happen because its not even the point of atonal music. Cultural conventions are not that arbitrary after all as they are bounded and conditioned by perception.

This study holds that all these constituents are not exclusive but in fact complementary or adjacent. The preceding discussion takes into consideration more recent ideas such as Tenney's consonance-dissonance conceptions, levels of scale, tolerance and harmonic duality. The problem with Cazden is not only that he assumes functional harmony as the only possible harmony, as happens with a lot of later empirical musicology, but that he does not provide much insight into these cultural codes themselves. Cultural aspects operate in different ways at different levels, determining less at the smaller scales of harmony, rhythm and timbre, where perceptual factors predominate, but increasingly influencing larger levels of scale, from phrases to pieces, from methods to performance situations, then to *ouvres,* styles, artistic movements and even higher epochs. It is here where Jaques Attali's type of analyses provide better tools to deal with the emergence of styles and the social codes embedded within music. He interprets harmony as the power relations that maintain the consensus of a society, every person fitting into an allotted role. Noise, on the contrary, harbors subversion and dissent. Music mirrors social reality by coding its relations, controlling but also predicting future social organizations because, being immaterial, it explores the range of possibilities of social codes faster than material reality can. Harmony and dissonance: order and disorder. The harmony of an epoch embodies a syntax, opposed by another syntax which then becomes noise to the previous one: 'What is noise to the old order is harmony to the new.'[115].

115 Attali, J. (1985). *Noise. The political economy of music.* Minnesota: University of Minnesota Press, 35.

2.2 Timbral Atonality

2.2.1 Timbral relevance

> "[T]he rejection of the idea that noise, 'randomness' and ultracomplex pitch are the primary frontiers of avant-garde exploration. [… C]omplexity arrived at as perceptible order rather than as seeming disorder. […] We must learn to differentiate sharply between complexity due to large numbers and complexity which delineates subtlety of relationship." (Ben Johnston, *Rational Structure in Music*[116])

There has been a growing emphasis on the timbral aspect of harmony during the course of twentieth century composition. Methods for the organization of pitch relationships as well as the involvement of materials, rhythms and forms all play a role in creating this overall effect. Today there is a situation where pitch, surpassed by its allegiance to timbre, has lost much structural force, its power to convey restricted to distances, spectrum and profiles. This section will briefly trace this development in order to make a case for the (re)incorporation of proportionality into the current harmonic situation. With this increasing relevance of timbre in harmony, pitch has become absorbed by timbre, while timbres themselves tend towards sonic complexity instead of periodicity, escaping the isolation of traditional notes. This is in fact one of the greatest achievements in the music of the last decades, not itself problematic. However, in most instrumental and electroacoustic music it has reached a point of saturation, an overt complexity-disorder unable to transgress its own limits, which in the extreme case exhausts the configurational options and effects a sense of immobility, deadlock, global predictability ('it sounds the same'). Having become commonplace and prevalent in different ways in many independent aesthetics, approaches and discourses, its capacity to create new musical experiences is worn-out. A possible way out of this condition has to do with recuperating discreteness in general as well as the use of more subtle pitch relations in particular, which is what the opening quote by Johnston refers to.

Many techniques in atonal and electroacoustic music deal with pitch relationships solely in terms of pitch-distance. Atonal series, being the first methods for organizing pitch to be based on equal temperament, are solely distance-based, 'pitch classes' not taking into consideration intervallic qualities such as consonance and dissonance, nor the combination of primary intervals that constitute them, both aspects involving relations other than a mere number of steps. Pitch-class theory deals with the combinatoric and permutational possibilities of interval sets, opening up new alternatives for organizing sounds according to criteria external to their perceptual attributes[117]. In a sense it is much too abstract on its own for dealing with harmony. On the other hand, maybe it is not abstract enough, in the sense that harmonic *aisthesis* and the whole numbers that compose proportions are arrived at indirectly from perception, alluded through the intellect, therefore lying at a higher level of abstraction than sets of distances, while still being immanent to the materials[118]. Proportions are further removed from sensory concreteness than pitch classes even though still connected to their qualities. Despite the fact that permutations are discrete operations over individual notes, in terms of pitch relations they underline continuous timbral characteristics.

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116 Johnston, B. (2006 [1976]). Rational Structure in Music. In B. Gilmore (Ed.), *Maximum Clarity and other writings on Music*, Chicago: University of Chicago Press, 62.

117 As noticed by Milton Babbitt, tonality is related to combinations while atonality to permutations, because in the latter the whole set of intervals is used, no repetition is allowed and order is important. When these two constraints are dispensed with as in later atonality and pitch-class theory, it all boils down to combinatorics.

118 'Categorial intuition' is the term used by Husserl to denote these indirect allusions and is a sort of twentieth century conception of Greek *aisthesis*. Proportionality is related to *eidetic* qualities, which are not directly present to perception, in contrast to the direct sensation of timbre.

Ben Johnston reckons that '[s]erial technique is only an interim solution'[119] by discussing the differences between the four psychophysical scales of audible order of Stanley Stevens[120]: nominal (distinguishing only sameness and difference, as with ABA forms), ordinal (relative ordering scales such as *pp, mf, ff*), interval (ordering upon a basic degree of increment, the semitone in equal temperament or pulses in simple metric rhythms) and ratio scales (as in proportionality, both rhythmic and harmonic). Each scale of order includes the previous ones, adding one more degree of refinement. Atonality and pitch-distance correspond to scales of ordering which lack the subtletly of complexity of ratio scales.

Equal division of the octave is not the only path to timbrality, the other road is through the harmonic series. Since the XVII century when Mersenne introduced the concept, it has become a metaphor to explain the way many just and tonal structures arise and relate. However, the harmonic *series* (embracing all numbers and giving them equal importance) is the limit case of any overtone structure whatsoever, the partials lying in space of continuum variations. Proportions are relations that happen exclusively between fundamentals, not regarding any higher partials and being therefore 'registerless'. As mentioned, overtone series also fail to explain intervallic inversion and do not involve modulation. Characteristic of timbral settings is the way the notes are laid out according to register and in the case of overtone series, the way the spacing between them becomes smaller and smaller. For this reason the music of French spectralists is even more timbral than the serial music from which it originates. Many of these works are an extension of the serial style through the use of spectral overtones. Exciting and original as some of these pieces were (for example, Grisey's early works), they are nevertheless a step further into timbral use of pitch: the consequences of the materials are developed timbrally, not harmonically. Furthermore, the fact that they use quarter tones to approximate the overtones (where only the 11th partial can be said to be close to a quarter tone) evidences a lack of regard for proportionality.

We can extrapolate this even further to see how noise in music is an amplification of a tendency to overflow the spectral space. 'Noise in a purely physical sense is a form of dissonance pushed to the extreme'[121]. The consonance/dissonance axis is replaced by one of sound/noise. A symptom of timbrality is apparent when sonic actions, gesture and movement acquire a meaning that overrules the specificity of the pitches used to convey them. It also means a concern for sonority for its own sake, detached from the relational potential inherent in the sonic aggregates, their reference to a system or syntax at a larger scale which lies over and above their individual properties. Timbral relevance also pertains to time frames other than pitch, such as 'floating' ametric rhythms, dynamic processes of form, complexity and saturation, or any developments that produce impressions of continuity and flux in different sonic attributes at different levels of scale.

There is a pervasive fixation on fluidity in theoretical discourse on music. To give an example, I recently read the manifesto for the launch of a new highly theory-laden publication on experimental music and sound art, *Tacet*[122]. This text purports to avoid seeking meaning in music, instead wanting to delve in modalities and forms of creativity by distinguishing four types of 'discursive movements' that resonate and associate among each other: flux, afflux, influx and reflux. I am disappointed to not find room for any kind of stasis, solidity or individuality in these categories: fluctuation and change, smoothness and flow, uninterrupted and seamless time and discourse being all that can be accounted for by their cutting edge analyses[123].

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119 Johnston, B. (2006 [1963]). Scalar Order as a Compositional Resource. *Maximum Clarity and Other Writings on Music,* 29.

120 Stevens, S. S. (1946). On the Theory of Scales of Measurement. *Science,* 103(2684), 677-680.

121 Saariaho, K. (1987). Timbre and harmony: interpolations of timbral structures. *Contemporary Music Review,* 1, 94.

122 *Tacet. Experimental Music Review.* Last retrieved August 19, 2012, from http://www.tacet.eu

123 I infer from the title of their publication an allusion to the score of Cage's *4'33",* their undertaking being made in alliance with post-Cagean consequences. A study on its own would be necessary in order to deal with the

Electroacoustic methods and theories don't fare much better either: the harmonic properties of materials being the collateral effect of treating pitch as part of the 'sonic object'. Spectromorphology treats pitch relations and their qualitative aspects solely as profiles and movements within pitch-distance. In fact, even much properly harmonic music of the late XX century (say, post-tonality, and minimalism) does not go much beyond the harmonic procedures of the early modernist period such as extended and poly-tonality and other extensions of chromatic equal tempered harmony. They explore new facets of already established harmonic logics but do not delve into new harmonic territory.

None of the musical techniques mentioned above can evade proportionality completely. The enmeshment of both aspects of harmony does not permit the full elimination of one of them. What happens is that timbral harmonies traverse proportionality in an inadvertent, coincidental manner. Compositional harmonic (proportional) control becomes involuntary, as intervallic qualities are limited in their logic to a 'timbral causality' that eludes inter-relationships between components. Proportionality is present even in ultra-complex pitch configurations, but becomes fortuitous if not acknowledged as such. Avoiding proportionality is not the same as ignoring it. A sound aggregate, from a chord to more complex configurations of pitches and rhythms/textures, can be considered in timbral or proportional terms, or as a combination of both, the effects being different in each case. They can be regarded as independent sonic entities, in their relationality (between their internal components or towards other external aggregates), or taking into account both their horizontal, dynamic aspects, as well as their vertical, static ones.

2.2.2 Modernism

'Modernity is a qualitative, not a chronological category'[124]

We may interpret Modernism in music as dealing, both technically and aesthetically, with the 'unfinished business' of Romanticism. The process of increasing timbral relevance in contemporary music can be understood as resulting from the reorganization taking place during Romanticism to the stable thematic and harmonic structures of Classicism towards dynamic and modulating forms. As distinct classical themes become more and more transitional from Beethoven onwards, development enters every aspect of form. Discontinuous Classicism becomes continuous in Romanticism, continuing this tendency in Modernism. The scope of development, the scale at which changes take place in the materials, as when, for example, modulation is incorporated into themes themselves or the boundary between theme and transition is blurred, leads, if taken to the limit (the limit when the scope of development collapses into a single note), to atonality and *Klangfarbenmelodie*. Atonality (especially the early 'free' period of the Second Viennese School) amounts to chromatic modulation at (almost) every note, modulation also embracing instrumentation, timbre incorporated into harmony. 12-tone harmony is the point where timbre and harmony touch: *each note is its own harmonic center and each timbre a note*.

In later dodecaphonic atonality, especially with the elimination of thematicism in Webern, the process reaches its culmination with the decontextualization of dissonances which were once held together as tonal aggregates but are now isolated or 'frozen'. Motifs and themes are crystalized into musical objects, their temporality focused on the present moment. What was once ornament (dissonant excerpts in tonality lying within a frame of preparation and resolution) is now detached as a sonority for its own sake, passing from background to surface, from exception to primary

dichotomy continuity/discontinuity in Cage. In any case, I think *4'33"* has been interpreted way too easily towards the first pole disregarding its potential for allowing to think discrete individuals in 'any sound whatsoever'.

124 Adorno, T. (1974). *Minima Moralia.* London: Verso, 218.

content.

Webern counteracts the increasing drive towards the fluxing of musical materials running throughout the century. His case inaugurates a parallel though far less widespread practice of freezing the material, of rendering static ‘instants of eternity’. His is an discontinuous approach to atonality, where the ephemerality of Schoenberg’s fluid atonality is turned into a tension between a veiled *cantus firmus* stemming from the *Grundgestalt* of the row and the isolation of intervallic configurations into absolute sonorities. This immobility is made possible through surrounding silences and timbral kaleidoscopes, both of which help to fracture the auditory streaming. On the other hand, the tension with the guiding horizontal thread of the *melos* imparts a drive to the static aggregates, providing an overall formal directionality. Webern has not so much to do with dissonance as with inharmonicity, while Schoenberg was looking for maximum dissonance and achieved atonality along the way.

In this second period of atonality, the case of Edgar Varèse is also quite paradigmatic, and I think he and Webern stand at the antipodes of the two main approaches to atonality: those of flux and stasis. In Varèse there is a flooding of partials and sonic fluctuations, a maximization and saturation of sound components. Masses become the main focus of the music; texture, contrast and opposition guiding the flow. Sweeping torrents of sound materials clash into one another, alchemically transmuting into other meteorological sonic forces. As with Webern, each layer consists of sonorities ‘for themselves’ in the sense that they don’t acquire meaning through their outward relationships, even though in this case they are dynamic sonic layers in constant change instead of carefully isolated aggregates. Each layer has its own drive or tendency, a different kind of *melos* than that of Webern, even more timbral and immanent. Varèse’s universe is also a culmination of nineteenth century processes, more related to orchestration and the expansion, in range and kind, of all varieties of sonic features. It consummates the assimilation of pitch within timbre, incorporates continuous pitch (sirens, ondes martenot) and deals with a timbral consonance and dissonance comprising roughness rather than proportion.

Atonality cannot be a general sonic category. Without deflecting the discussion into details, we must distinguish between different types of atonality: ‘proportional’, ‘intervallic’ atonality (Webern), fluxified atonality (Schoenberg), quasi-tonal (Berg), sonorous, massive (Varèse), aleatoric (the Cage of chance music), the very distinctive one of late Nono, almost proportional, the beautiful atonality of mid to late Feldman (close to Webern’s in my opinion, but with very different results), the extreme complexity in pitch of Ferneyhough, the concrete instrumental atonality of Lachenmann, spectral atonality, and so on.

2.2.3 Diagonalization

Alain Badiou has analyzed the irruption of creativity of the *fin de siècle* in a way that can aid to elucidate this becoming-continuous[125]. He considers atonality the ‘truth’ of tonality, its blind spot or ‘evental site’. There is an intensification of elements which, once having been relegated to decorative functions, having possessed a minimum (or small) degree of existence, develop to possess a maximum intensity. What was previously hidden and not considered an aesthetic form on its own, turns into what makes new forms possible.

The notion of ‘diagonalization’, interpreted from the mathematics of Georg Cantor and set theory,

125 Badiou, A. (2006). *Being and Event.* London: Continuum. This is a condensed account of what can be useful for us of Badiou in terms of timbral relevance. It is mostly drawn from meditation 31, (pp. 327-343) of *Being and Event*, though it is imbued with a more general understanding of his philosophy and complemented by the analysis he makes of the music of this era found in Badiou, A. (2009). Scholium: a Musical Variant of the Metaphysics of the Subject. In *Logics of Worlds.* London: Continuum, 79-89.

gives insights into this process. Atonality traverses or diagonalizes functional tonality, both by making visible its historically contingent facet as a system of conventions, as well as by subtracting itself from it. The situation in which this happens is the expressive crisis of Western Europe's music during the late XIX century. There is a language (a *knowledge*), namely functional, chromatic, triadic, equal tempered modulatory harmony. The crisis is an inadequacy of this language to serve as a vehicle for artistic expression, having become formulaic and not able to convey a reality that had changed beyond the culture that gave birth to it, and this includes new subjective experiences as well as many social and technological changes which we need not delve into. The saturation of an artistic system and epoch leads to academicism, textbook rules, to prescriptive and proscriptive conditions of what can and cannot be considered form.

This crisis of language suffused all the arts, but it was music, considered the purest and most abstract of them all, the one which held the promise of finding a new language adequate for its times. What came out, instead of a single global language, was a multiplicity of 'local', private languages that are part of the very creation of art: the composability of music's own infrastructure, what before was conventionally given. Composers, through aesthetic 'operators' (experiments, intuitions, deductions, connections, syntheses, crossbreedings, etc), enquire over the 'determinants' (components) of the situation, encountering 'holes' in its knowledge: materials, methods and aesthetic effects which are unnameable in terms of the worn-out language. Some of these enquiries intersect with elements from the old world of harmonic tonality, but others avoid them. These latter ones are contradictory elements in this old world, as is the case with arbitrary pitch relations, melodic structures that breach thematicism, rhythmic structures breaking out from metric periodicity and the avoidance of arc-like developmental or narrative-rhetorical forms.

For Badiou, a truth (we need not delve into the intricacies of this concept) is a process, a procedure of inquiries gathering elements of the situation into a (generic[126]) subset that both contains and evades its knowledge. For every determinant of the situation, this truth contains at least an enquiry which avoids it: (1) it is unnamable from the resources of the old language, (2) it is subtracted from its rules and (3) is indiscernible in the sense that it just *is* in the situation, referring not to *re*presentation (knowledge) but to presentation, to belonging as such, therefore rejoining the entire situation. Truths are radiographies of sorts of their world. They are 'true', though not in the sense of being logically necessary but historical and contingent, even though they are subject to being recommenced in other epochs, thus being in this sense eternal. It is in this sense that atonality is the truth of tonality, its other face. This truth is constituted by the works that express the counter-effect to tonality, the ones that pass through some of its determinants while also avoiding others, weaving through them a diagonal, traversing its knowledge while simultaneously escaping it. Atonality is a solution to the antagonism already contained within harmonic tonality, a harmonization of dissonance.

Chromatic functional tonality really 'happened' or was fulfilled only in atonality (like when it is said that the 60s only really happened in the 70s). It is not an accident that Schoenberg wrote his *Harmonielehre* at the same time when he found his way out of tonality, signaling that the theorization of the previous period was only possible when the beginning of a new configuration was taking place. Other systematic theorizations of functional tonality come from the same period (Heinrich Schenker, Hugo Riemann), just when the tonal world had been transversalized by atonality. The Romantics didn't understand their technique in the same terms as we do after the fact. It was only

126 This is a complex and central aspect of Badiou's system (taken from the mathematics of Paul Cohen): though we will not enter into details here, enough is to say that not just any subset of the situation constitutes a truth, only generic subsets: infinite sets that traverse the whole situation, both its knowledge and what subtracts itself from it. Because truths are infinite, they are only completed 'at the limit', something not possible in practice. See meditation 31 of *Being and Event*.

possible to theorize them from the point of view of their epoch having reached a limit. According to Badiou,

> "there is no contemporary understanding of the classical style and its becoming-romantic, no eternal and therefore current truth of the musical subject initiated by the Haydn-event, which does not pass through an incorporation into the serial sequence, and therefore into the subject commonly named 'contemporary music'."[127]

This ultimately refers to an *event*, the other of Badiou's main concepts: what can retrospectively be seen as an irruption of novelty. He calls it the 'Schoenberg-event', but that is only a name for an effort that involved many other composers who found their own particular solutions to the same crisis, such as Reger, Strauss, Mahler, Berg, Webern (in Vienna, the main site where this took place) or Satie, Debussy, Stravinsky, Scriabin, Bartok, Ives, Busoni, Carrillo, Cowell, etc, elsewhere, to name but a few. There are revolutionaries, reactionaries and conservatives involved: both the spirit of renovation, destruction and nostalgia for the old world are present. Their contributions should not be effaced by the name 'Schoenberg' as it stands for a musical situation which had multiple lines of attack and correspondingly varied solutions, some of which contributed to other aspects of modernism than Schoenberg's mostly pitch-oriented approach.

There is a *trace* of this event, essentially a proposition which serves as an imperative: 'an organization of sound may exist which is capable of defining a musical universe on a basis which is entirely subtracted from classical tonality'. This is accompanied by a *body* of pieces that compose this new universe, say, pieces since the *Trois Sarabandes* of Satie (1886), towards Schoenberg's first to second period works (his second String Quartet being a renowned example of the transition from tonality to atonality occurring within a single work), *Le Sacre*, Mahler, Debussy, Futurism, etc. This body constitutes a *subject,* not a person, but an artistic movement: 'contemporary music'.

This truth was unknown for those who began it, and, because it didn't seem clear at the time that what they were doing was in fact the creation of a new configuration, it shows why many of them defected to more reactive or conservative styles after the great rush of creativity was over in the years following the Great War. The new configuration established itself but also became full of contradictions. From here on, only a few courageous musicians took on the full logical consequences of the discoveries (Webern, Varèse, Cowell, Ives, Bartok, Revueltas, Antheil, Messiaen, early to mid Cage, and more, each with different degrees of faithfulness, systematicity and success). In any case, some of the deepest assumptions of Romanticism are preserved at the core of Modernism, such as the conception of instrumental music as autonomous and absolute, the need to innovate at all costs, as well as the singularity of each work.

What is to be done today? We are still under the 'Schoenberg-event', even if it has been complemented by the most likely 'Cage-event' ('composing with any sound whatsoever', 'silence as a metaphor', the beauty in contingency, etc), as well as contaminated by other kinds and attitudes towards music making (especially importing influences from popular musics)[128]. The situation of atonality has become similar to late XIX century: academic and prohibitive instead of faithful to the evental trace of 'ways of organizing materials other than harmonic tonality', which is open to more approaches than the purely equal tempered, combinatoric one. True, some of the best routes have been taken in the direction of noise and timbres which cannot be easily inscribed into existing theory. There are many parallel roads open for exploring the yet unexhausted potential of the statement that inaugurated contemporary music even if the designation 'contemporary music' is even more problematic nowadays that it was then. Minimalism is a also continuation of the

127 Badiou, *Logics of Worlds*, 85.

128 Composer Michael Pisaro pursues Badiou's conception of truth, event and its consequences for art, expanding it in terms of new music by exploring the 'Cage-event'. See Pisaro, M. (2006). Eleven Theses on the State of New Music. Last retrieved March 3, 2012, from http://www.timescraper.de/pisaro/mp11theses.pdf

Schoenberg event, even if at a level of counter-effect to the serial sequence, a movement which provided a fresh air to atonality's academization, addressing the audibility of process without sacrificing pitch and rhythmic complexity. It now suffers, though, from saturation through too much exposure the same problem of institutionalization.

We strive for an organization that brings arbitrariness of order and perceptual intuition together, both quantitative and qualitatively. To go past atonality as a generic description or as the negation of tonality into the awareness of various and singular atonalities, a profusion of regions of differentiated *sonances* (distinguishing cons/dis-sonance from in-/harmonicity) within a larger harmonic field that embraces both. The recognition and means to arrive at more nuanced and subtle constitutions of timbre/pitch combinations at the scales of timbre-harmony-rhythm-texture and form, confronting the objects at each level, obeying their characteristics as irreducible to other higher or lower levels, without loosing concern for their articulations and interrelations. There is a pervasive emphasis today on a generic atonality a sort of 'greyish-goo' that is the pervasive default for many configurations in today's composition. There is too much emphasis on concreteness and 'embodiment', all of them related to timbral pitch. Atonality is a region of harmony and not its contradiction.

We seek to restore the 'rights of the abstract' to its proper place: attention to harmonic proportionality and timbral perception welded together in reciprocal supplementation, an aggregate that combines the advantages of both into one hypothesis. Attention to synthesis and experimentation in addition to analysis and axiomatization. From atonality to *a*-tonality (using the resources of both matheme and poeme)[129], leading towards tonal multiplicity: the multiple composition of tonalities, tunings and the assemblage of qualitative, heterogeneous, continuities/discontinuities.

Badiou makes reference to 'atonic worlds', those in which there are no 'points' (a technical term) which compels decisions to be made that can change the situation. Although he is referring mainly to parliamentary democracies, where everything is organized and guaranteed, but there is no possibility of making decisions that can transform the structural inequalities, it does have an analogy with music: atonality is the absence of points, a situation where the concern for the distinctiveness of intervals is gradually lost. It is like a state of equilibrium where the forces cancel each other's effects, a democracy of intervals excluding notes that can act as quilting points. What we seek is to define atonality as a region within a larger field where tonality and antitonality are also operative and there is a continuum of gradations between the three states, as we will se in the next chapter.

This review of the Badiou's conception of change and truth in art is not only meant as historical and philosophical, but is meant to link many topics related to timbral harmony and the becoming continuous of musical materials. The set theory from where this interpretation is derived is the one that formalized the continuum in mathematics. Cantor's diagonal argument (1891)[130] is a method for finding irrational numbers out of the densely packed rationals. Dense because for every pair of ratios, no matter how close together, there is always a ratio sitting between them, and therefore an

129 "I have always conceived truth as a random course or as a kind of escapade, posterior to the event and free of any external law, such that the resources of narration are required simultaneously with those of mathematization for its comprehension. There is a constant circulation from fiction to argument, from image to formula, from poem to matheme." Badiou, A. (2000). *Deleuze: the clamor of Being*, 57. Elsewhere, and against too much concreteness (what I associate with timbrality), Badiou argues about supporting the "real rights of the abstract [...] what there is of conceptual stability in the order of theory, of formal equilibrium in the order of art, amorous consistency in the existential order, and organization in the political", *Ibid.,* 99.

130 There are many references for this topic, but I especially recommend an accessible, ludic and insightful account in Gardner, M. (1989). Aleph-null and Aleph-one. Chapter 3 of *Mathematic Carnival.* Washington: The Mathematical Association of America, 27-40. Also see Cantor's Diagonal Proof [webpage]. Last retrieved March 3, 2012, from http://www.mathpages.com/home/kmath371.htm

infinity of others in even the tiniest interval. In diagonalization, an infinite set of rational numbers within an interval (say, between 0 and 1) is listed as digits in their decimal (or binary) representation. For each number, a digit in the diagonal (the first digit of the first number, the second of the second, and so to infinity, hence the name), is changed. The number resulting from this thought experiment will differ from each of the numbers on the list in at least one digit. It lies inside the interval but does not belong to the list: it escapes the rationals (it is 'uncountable'). There are many more numbers like this one than the infinity of rationals from where they detach, marking the discovery a new kind of infinity, the 'power' (or 'cardinality') of the continuum, larger that the classical infinite of the natural, countable numbers. This is the first formal account of the mathematical continuum, the gesture that completed the real numbers.

Badiou reads this into explaining paradigm shifts in the arts, sciences, politics and personal experiences (love). What we have surveyed not only shows a detailed process of how paradigm shifts happen in art but, more importantly, exhibits the infrastructure behind the becoming continuous of musical materials, what we have associated as the fluid movement of kaleidoscopic contours in the materials. The formalization of mathematical continuity 'fills the gaps' lying between the proportional grids by weaving through them, 'touching' while also eluding them. To understand the continuum as a saturation of discreteness leads to a parallel perspective on musical atonality: filling the space left open by harmonic tonality, subtracting itself from proportional grids (both rhythm- and pitch-wise) to occupy an intermediate smooth space which is only arrived at after having traversed these proportions.

2.2.4 Timbral microtonality

> "[T]here's something absolutely fascinating about a straight line. Rays of sunlight can be seen when looked at through clouds. The rays of sun which converge near the ground are, in reality, parallels. A laser's beam line is something absolute, the line of a mason's edge is also absolute. The straight line, therefore, exists in nature. But as an intellectual entity, it's most fascinating from the point of view of speed, direction, and also continuity. From the point of view of continuity, it's impossible to imagine anything simpler than a straight line." (Xenakis, Arts/Sciences: Alloys[131])

There is a lineage of work concerning schemes of other than 12 equal divisions of the octave. In the early decades of the 20th century we have the pioneering cases of Julián Carrillo proposing his 'thirteenth sound'[132], dividing the octave 96 times (into sixteenths of a tone), as well as Alois Hába who proposed quarter tone (and later sixth and fifth tone) tunings. These approaches are timbral, limited to degree and pitch-distance, without conscious control over harmonic meaning, and, as a result, instead of increasing the transparency of the intervals, produces an effect of out-of-tuneness[133]. This is witnessed by some of Carrillo's compositions which have a character of improvisatory exploration over new materials. Hába's music is the first example of dodecaphonism extended to smaller intervallic divisions. These approaches are contrasted to the proportional microtonalities of Augusto Novaro (1930's), Joseph Yasser (1920-30's) and Harry Partch (1940's) who started from the premise of ratios, acknowledging their proportions as more fundamental than distances.

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131 Xenakis, I. (1985). *Arts/Sciences: Alloys.* New York: Pendragon Press, 75.

132 The thirteen in the name usually refers metaphorically to 'more than twelve tones'. Like most twentieth century pioneers in microtonality, he experimented and tried out many different tunings. Nonetheless it is said that what set in motion his endeavor was the experience of listening to13th harmonic around 1895. See Nieto, V. (2008). Escuela del continuo en México. *Perspectiva Interdisciplinaria de la Música,* 2, 66.

133 '[T]his creates a sort of soup in which the peas loose their potential individuality, causing everything to sound more or less out of tune', Barlow, C., *Bus Journey to Parametron*, 2.

Novaro's approach is interesting in that he discovered how pitch-distance temperaments relate and draw their harmonic meaning from proportional sources[134]. Without thematizing harmonic tolerance explicitly, he found how certain equal divisions approximate harmonic proportions better than others, showing how they help to solve the practical problem of tuning instruments to unequally spaced just ratios. The approached intervals nevertheless derive their properties proportionally (it has no proportional or intervallic meaning to talk about an interval of 400 cents, but to speak of 5/4, 9/7, 32/25, 14/11 or 81/64, all of which lie tolerably close to 400 but have distinct harmonic meanings). This made him commend the importance of the 12 note equal temperament as a great solution instead of just condemning it outright, as Partch did, for whom no compromise with temperament was possible. Harmonic duality clarifies the apparent contradiction of the two approaches because it separates continuous pitch from the proportional intervals and points to their interrelations. Novaro discovered the main equal divisions of the octave that are usable in terms of proportionality: 12, 19, 22, 31, 41 and 53. There is also Aristoxenus' 72 divisions per octave, even though its original purpose was to measure and not to approximate harmonic systems[135]. Dividing the octave differently has no special proportional meaning. Some of those intervals do lie close to proportions, but as whole systems they do not provide any harmonic use beyond the use of arbitrary 'timbral grids'. As we saw earlier, Aristoxenus sowed the seeds for harmonic tolerance, but the concept did not develop fully until Barlow in the late 1970s and Tenney and in the 1980s (with some important contributions from Fokker in the 1960s). Novaro found out this divisions by trial and error, now there have been many ways to graph the harmonic properties of different equal divisions to see peaks that stand out at those particular divisions.

As we mentioned regarding the French spectralists, atonalities involving quarter tones (after Hába I can think of evident cases of Boulez and Ferneyhough) are even more timbral than twelve tone ones. Quarter tones provide a different sounding atonality to the twelve tone serialism which was becoming saturated in the second half of the century, an atonality further removed from any kind of organized proportionality. Needless to say, the aleatoric use of pitch takes this tendency even further. There is also the rejection of providing clearly identifiable intervals in the cases of those styles whose ideal is to arrive at ultracomplex pitch relations, which ultimately turn out to be even more out-of-focus (tune) than ordinary tempered intervals. Ben Johnston's approach in his first two string quartets is the opposite: he rationalizes just intervals from out of his series, regaining the proportionality of atonal aggregates, paying attention to the context surrounding the intervals and hence going beyond the limitation of 12 intervals of tempered atonality, opening toward a just-intoned atonality, with a very acute sense of intervallic colors, a verily proportional atonality.

Xenakis' approach to pitch began as a furthering of Varèse's. Beyond his trademark glissandi and the stochastic use of pitch in the early works, his later microtonality based on sieves is the first revitalization and direct tackling of Aristoxenian ideas in the century. They are inspired by Messiaen's modes of limited transpositions and Byzantine music structures, in amalgamation with set theory. Congruence classes act as sieves (filters, masks) which, applied to grids of equal pitch distance, provide formal mechanisms allowing the creation of a proliferating variety of asymmetric scalar configurations. These procedures still have lots of untapped potential, offering a path to what he referred to as a 'general harmony'[136]. Sieves are more modular and comprehensive than other approaches to equal octave divisions, and, being based on arithmetic and combinatoric properties of linear spaces, are 'faithful to the medium' of pitch-height. In conjunction with intervallic rationalization that gives proportional meaning to these pitch sets, they are a rich source of microtonal materials.

134 Novaro, A. (1951). *Sistema Natural de la Música.* Mexico City: Author's Edition (The work was written in the1930s). An english translation is available at http://www.anaphoria.com/novaro.html Last retrieved March 29, 2012.

135 They will be discussed in the following chapter.

136 Xenakis, I. (1992). *Formalized Music: Thought and Mathematics in Composition,* 182.

Xenakis assesses atonality, by placing an emphasis on what he refers to as the *in-time* ordering of elements in detriment to a 'degradation' of *outside-time* structures.[137] This interpretation concurs with the viewpoint expressed in this section, in which we have identified much of this music as stressing movement over structure. We identify structure in terms of discrete, proportional terms, what is ultimately a reference to harmony, not in the sense of tonal harmony, but to outside-time abstraction: order, symmetry, reversibility, all which cannot strictly happen in time. They do happen, though, by way of implication, through memory and *aisthesis*, which is what we have been stressing regarding the mode in which many of these architectures are conveyed. Much of twentieth century music's stress on continuous variation, including much of Xenakis' own – and this should not be understood as an attack, but as acknowledging its importance and influence – overrides other categories of musical thought, leading to the aspiration or imperative to attain a sort of 'state of flux', the embodiment, of a complex sensorial intricateness. We will talk more on topics relating to outside of time in chapter 4, when we deal with 'the time of qualities' or the inversion of the primacy of time to the primacy of qualitative features of sounds, each with its own time, the time it needs to unfold its characteristics, instead of considering time as just a blank blackboard over which sounds are inscribed.

It is a bit puzzling that all these discrete methods (combinatorics) over disjunct materials (degrees of scales, partials in spectra) can be on the side of the continuum, but our analysis has been conducted from the perspective of pitch relations, where things do not quite map into discontinuity but into linearity, even if proportionality is casually traversed in certain cases. Nonetheless, this also happens at scales other than those of pitch relations, where continuity can be the resultant of discrete elements[138]. This review does not advocate against noise, timbral complexity and pitch-distance, but seeks to draw some kind of structure and abstraction from them, to move the fluid rhetoric toward relational units as well as attention to the 'prominent vibrations' that Luigi Russolo – a proponent of noise if anyone – mentions. As Xenakis puts it:

> '[T]he listener doesn't stay at the lower level of the specimen's microscopically individual event, and he perceives noise as a macroscopically individual whole; in other words as something possessing regularity, an order!'[139]

2.2.5 Continuous forms

Having seen how continuity is achieved via a filling-in of the gaps of a harmonic grid, we can turn to its statistical aspect, discerning it in terms of the distribution of the elements involved. The timbral use of pitch can be seen as the dissolution of hierarchies that give equal prominence to the newfound materials. There is an 'atonalization' of all aspects and scales of musical composition, from Schoenberg's tone rows towards the egalitarian use of the 'phase space' (space of configurations) defined by the combinatory potentials of all composable aspects of sound. The usual designation for these composable aspects is *parameters* and they constitute a space of configurations wherein all possible musical states are virtually contained.

The tendency of forms to become continuous begins with the equally distributive use of discrete

137 "[I]t is necessary to distinguish structures, architectures and sound organisms from their temporal manifestations. It is therefore necessary to take 'snapshots', to make series of tomographies over time, to compare them and bring to light their relations and architectures and vice versa. [...] [F]undamental outside-time architecture [...] has been thwarted by the temporal architectures of modern (post-medieval) polyphonic music. These systems, *including those of serial music*, are still a somewhat confused magma of temporal and outside-time structures, for no one has yet thought of unravelling them.", Ibid., 192-3. Emphasis added.

138 "One may produce continuity with either continuous or discontinuous elements", *Ibid.,* 9.

139 Xenakis, I. *Arts/Sciences: Alloys*, 78.

elements (pitch in dodecaphony, timbre as instrumentation in Varèse, early Cage's *gamuts* of aggregates, non-pulsed rhythm, and so on). This tendency intensifies and expands, especially during the second rush of creativity after the Second World War, to include a smearing of the intervals lying between these elements, broadening them into fuzzy areas instead of simple units, producing an impression of continuous degrees of intensity. All aspects of composition begin to resemble the dynamic aspect of loudness. This is epitomized by Stockhausen's agenda of 'using all components of any given number of elements, ... with equal importance and try[ing] to find an equidistant scale so that certain steps are no larger than others. It's a spiritual and democratic attitude towards the world.' A serial, quantitative, idea of the 1950s continued and extended in the 60s to embrace qualitative features of musical materials.

This attitude has expanded the resources of composition enormously, embracing ever more nuanced and diverse aspects of sound amenable to being composed, which is the same as saying that these sonic aspects are susceptible of being invested with all kinds of forms, many coming from extramusical provenances, mainly concerned with the counterpointing of unbroken lines in parallel aspects of sound. This happens at several levels of scale. The material level (the *medium* of music, the site of so much innovation during the century), is carried over into the larger formal areas of sequences, textures, sections and whole pieces. This multidimensional counterpoint of continuous parameters seeks to attain some kind of what Julio Estrada has named 'macro-timbre'[140] as a resultant of these interactions. The search is for the emergence of something which is beyond the sum of the parts, the mixture, interaction and con-fusion stemming from the interface of all these features, comprising several levels of materials, from small vibratos to performative actions.

At formal levels, the rendering continuous of materials leads to what Tenney has called *ergodic* forms (similar but more general than Stockhausen's *momente* form): forms which result from the many possible egalitarian ways of traversing and covering the phase space of its parameters, not giving any areas more preference than others (statistically speaking), producing the illusion of an 'heterogeneous yet continuous' music.

From a spectral perspective this process of filling-in the multidimensional phase space can be graphically portrayed. A spectrum graph represents discrete elements as lines, the height of which are proportional to their probability of occurrence. In dodecaphony this 'amplitude' has the same height for all the elements involved. What makes for the specific sonority of each piece is their ordering. Later on, serialization embraces rhythm (indirectly through duration), dynamics, instrumentation, register, spatial movement, etc, opening the way for the full parametrization of composition in the 50s, the parameters of which can be defined by the composer as any sonic, musical or structural feature prone to be manipulated and composed. This 'parametrization' not only applies to the serialists but also to aleatoric and stochastic composition: to Cage, Xenakis, *et al.*

Spectrally, this interrelation of parameters can be seen a broadening of the discrete *lines* to become spectral *bands*, whose probabilities now are not only defined by their heights, but also by their width, turning from lengths more and more into areas broadening out from the elements as originally defined, filling the space that separates those elements. This tendency can be radicalized even further, as in many noise musics, going from the reinventions of instrumental techniques and unusual sonorities in Lachenmann all the way up to industrial noise musics in which the sonic spectrum is saturated by giving the impression of covering all possible sonic states. These approaches flatten the spectrum of the constituent musical ingredients, widening the noise bands into flat horizontal lines that comprise the whole range of conceivable combinations of sonic materials. This is analogous to white noise, whose energy is uniformly distributed over the whole frequency spectrum. It is described by the height and the slope of this line (in case of a slope it

140 Nieto, V., "Escuela del continuo en México", 66.

represents some variety of colored noise), which at the time scale we are dealing with corresponds to the formal, procedural and material possibilities of a compositional phase space being traversed in a uniform, egalitarian manner.

This tendency progressively transitions from points to lines to areas, thereby involving what in chaos theory are called 'attractors': multi-dimensional complex forms determined by the space of compositional parameters. This use of compositional space produces a 'flooding', in the sense used by Stockhausen of flooding a space with sound, but referring to a compositional space of combinatorial possibilities, abolishing discreteness even while involving discontinuous elements. The overall effect is an impression of heterogeneous continua, projecting the space of composition into a plane where any position is potentially equal in importance as any other. This is the logical conclusion of atonality taken to all aspects and scales of composition.

Chapter 3

Proportional Harmony

3.1 Harmonic Space

> 'Cage has always emphasized the multidimensional character of sound-space, with pitch as just one of its dimensions. This is perfectly consistent with current acoustical definitions of pitch, in which — like its physical correlate, frequency — it is conceived as a one-dimensional continuum running from low to high. But our perception of relations between pitches is more complicated than this. The phenomenon of "octave-equivalence," for example, cannot be represented on such a one-dimensional continuum, and octave-equivalence is just one of several specifically harmonic relations between pitches — i.e. relations other than merely "higher" or "lower." This suggests that the single acoustical variable, frequency, must give rise to more than one dimension in sound-space — that the "space" of pitch perception is itself multidimensional. This multidimensional space of pitch-perception will be called harmonic space.' (James Tenney, *John Cage and the Theory of Harmony*[141]).

This chapter will deal with the proportional facet of harmony, delving into some of its technical particulars. The aim is to comprehend the way in which harmonic space and ratios behave in order to expand on the possibilities afforded by these ideas and show how music can and has been made taking proportionality into account. As a way to exemplify, this will be done with a focus on my implementations of these concepts in *DissonanceLib*.

Harry Partch called for a retrieval of the harmonic aspect of intervals to compensate for the limited variety engendered by twelve tone equal temperament. He called 'the language of ratios' the concern for the properties of intervals and the whole numbers constituting the ratios. It is important to combine information from both ratio and size representations: distance, as Aristoxenus debated, takes place in a linear space, its elementary units are calculated additively and it corresponds sensibly to musical intuition; in contrast, the multiplicative nature of ratios makes them harder to calculate, they need to be reduced to their most compact form and do not convey any inkling as to their intervallic size. Ratios can also be a source of confusion as a result of there being so many of them close to any single point in pitch-distance space, entailing interpretation: there are several candidate ratios for any single pitch height and the choice depends on several factors and context. What ratios convey are the types and complexities of pitch relations, and this can be understood as taking place in a multidimensional integer lattice involving 'fundamental intervals' which are linked to prime numbers.

The first representation of tonal relations relating chords and keys in more than one dimension was first proposed by Leonhard Euler. It was later called *Tonnetz* − tone-network − by Arthur von Oettingen in the second half of the nineteenth century, a grid of parallel horizontal and vertical lines that intersect at nodal points representing tonal relations between third and fifth related chords and keys, a feature called forth by the contemporary use of mediant relations in composition, taken later on by Hugo Riemann in his *Tonnetze* of 1913 and since used extensively by his followers. A similar approach that refers mainly to tonalities and large scale form can be seen in Schoenberg's

141 Tenney, J. (1984). John Cage and the Theory of Harmony. In Garland, P. (Ed.), *Soundings 13: The Music of James Tenney.* Santa Fe, NM: Soundings Press, 55-83. Also available at www.plainsound.org/pdfs/JC&ToH.pdf Last retrieved May 15th, 2012.

'regions chart' in his *Structural Functions of Harmony* (1930s). In a different vein, Alexander Ellis introduced the harmonic *duodene* and *duodenarium* in his appendices to Helmholtz's *On the Sensations of Tone*[142]. This concept is closer to intervals than to tonalities, to the time frame of immediate perception rather than contextual functions, not being based on tempered cycles but on just rational relationships and combined with pitch distance information.

Other composers and scientists have also made use of this lattice. In composition, I think it is best developed in the work of the usual suspects we have come to meet: Johnston, Barlow and Tenney. The latter develops the topic from a compositionally insightful way of opening up experimental possibilities[143], basing it on John Cage, who despite having disliked harmony in the sense of *a priori* (that is, logical and arbitrary) thinking upon sounds, opens the way for the rediscovery of a harmony apt for 'composition with any possible sound', simple and complex, capable of dissolving the opposition between sound and note based composition. A harmony which is in accord to the nature of sound and perception, related to an ultimately parametric view (parallel with serialism, Xenakis) of what Cage referred to as the 'total sound space'[144].

Harmonic space is but one aspect of this total sound space, helpful for classifying and grasping the proliferation of new and unusual intervals brought by dissonance curves. Beyond their use as merely novel or strange materials it links some of their qualitative, harmonic, aspects in quantitative ways: not all the features of an interval are quantified, only the fundamental harmonic 'hues' and their harmonicity is linked to harmonic space, while their timbral sensoriality is disregarded. Analyses in harmonic space are an aid in the development of strategies for creating, traversing and deploying interval collections. It does not impose a determined order nor set a limit to the possibilities, but provides assessments as to what results under given sets of conditions, showing approaches to general problems rather than a constraining and narrowing down to proscribing rules.

We will be learning the language of ratios along the way as we pay a visit to harmonic space.

3.1.1 The harmonic lattice

> "Starting from an original pitch level, by adding and subtracting fundamental pitch intervals, an infinite variety of musical notes will be produced. For fundamental intervals I shall take the octave, representing the harmonic frequency ratio 2/1, the perfect fifth with the ratio 3/2, the perfect major third, ratio 5/4, and the concordant perfect seventh, ratio 7/4.
>
> By common general agreement all notes differing by an arbitrary number of octaves only, are considered as unison, and as one and the same note. Thus we are left with only a threefold variety of notes. A note will be defined by the numbers, positive or negative, of the respective intervals constituting the note. It is quite natural to visualize these three numbers by coordinates in a lattice, thus making a harmonic note lattice. The nearest part around the origin (0,0,0) will comprise 3^3 = 27 notes." (Adriaan Fokker, *Unison vectors and periodicity blocks in the three-dimensional (3-5-*

142 *Duodenes* are built upon on chords and scales, extending them to encompass intervallic regions which aid in the creation of new chords and modulations for playing a just harmonium called 'the Harmonical'. The *duodenarium*, on the other hand, is closer to the harmonic space we are discussing in that it generalizes *duodenes.* It is limited to two-dimensional representations, with extra primary notes inhabiting its corners. See Helmholtz, *On the Sensations of Tone,* Appendix XX, Section E, Articles 11- 26, 461-466.

143 Tenney, J. John Cage and the Theory of Harmony.

144 Deserving another study, it would be very interesting to appraise the notion of 'total sound space' in light of the non-All and inaccessible closure of worlds in Alain Badiou's set-theoretical ontology according to which there is no set of all sets, so the dissemination and totalization of the parts of this sound space is inexhaustible and sound holds a limitless reservoir of novelty. See Badiou, A., *Logics of Worlds*, Book IV, Section 1 and 3: 306-310 and 331-335.

7-)harmonic lattice of notes[145])

Fundamental intervals, comprising prime numbers, are to intervallic listening what primary colors are to visual sensation, that is, the elementary building blocks of harmony, *harmonemes*, chiefly at the time scale of intervals and chords rather than that of texture and form. Primes have been mentioned much in the literature, especially for the purpose of filtering and classifying intervals in a system (such as 3-limit, 7-limit[146]), or to define the axes of the multidimensional space. Fokker, from a purely mathematical viewpoint, isolates and defines primary intervals, but does not allude to their qualities. This linkage has mainly happened from a compositional perspective with Partch and particularly Johnston. Most empirical studies focus more on tonal contexts than on the psychological or intersubjective link between prime numbers and their perceptual seemingness in auditory perception[147]. From a compositional perspective, however, the connection is phenomenologically quite evident and here we have a small but decisive point where composition can suggest targets for music cognition.

The positions and relative distances of ratios represented in a multidimensional space of the harmonic lattice reveal their harmonic information: mixtures of fundamental intervals, harmonicity and relations to other intervals – their derivation or 'function'. The bases that form the axes comprise the first *n* primes. An interval is expressed as an *n*-tuple of exponents of the prime bases, corresponding to the combination of fundamental intervals in its constitution. Positive exponents denote the amount of accumulation of that fundamental interval in the upward direction, while negative ones refer to inversion, i.e., towards the low end. These factors engender the numerator and denominator of the ratio.

The following list illustrates this in a 5-limit space, with a base {2,3,5}:

<0, 0, 0>	=	$2^0 \cdot 3^0 \cdot 5^0$	= 1	=	1/1	unison;
<-1, 1, 0>	=	$2^{-1} \cdot 3^1 \cdot 5^0$	= 3/2	=	3/2	fifth, an octave lower than the third harmonic;
<1, 1, -1>	=	$2^1 \cdot 3^1 \cdot 5^{-1}$	= (2x3)/5	=	6/5	minor third: a third lower than a fifth;
<-5, 2, 1>	=	$2^{-5} \cdot 3^2 \cdot 5^1$	= $(3^2x5)/2^5$	=	45/32	tritone: a third up from a ninth (two fifths);
<6, -2, -1>	=	$2^6 \cdot 3^{-2} \cdot 5^{-1}$	= $2^6/(3^2x5)$	=	64/45	tritone: a third below a seventh (two fourths);
<-4, 4, -1>	=	$2^{-4} \cdot 3^4 \cdot 5^{-1}$	= $3^4/(2^4x5)$	=	81/80	*syntonic* comma: difference between 4 fifths and a third.

It is useful to ignore octaves to understand how composite intervals derive from combinations of primes higher than 2. By abstracting register other fundamental intervals are isolated. As the example of the *syntonic* comma shows, it is conceptually easier to grasp this interval as the difference between two ways of arriving at the third diatonic degree: one through four fifths (degrees V → II → VI → III) and the other directly through a single just third. The complete calculation is (81/64) / 5/4 = 324/320 = 81/80, but by having skipped the octaves and followed the degree routes, we arrived qualitatively without the need of calculations. In another example, 25/24 is easily identified as two major thirds ($5^2 = 25$) upward and a fifth downward ($24 = 3 \times 2^3$, throwing out the 2s we see

145 Fokker, A. (1969). Unison vectors and periodicity blocks in the three-dimensional (3-5-7-)harmonic lattice of notes., *Koninklijke Nederlandse Akademie van Wetenschappen,* Proceedings 72(3), 1. Last retrieved November 14, 2010, from http://www.huygens-fokker.org/docs/fokkerpb.html

146 These nomenclature comes from Harry Partch, but in his and other studies who use the term there is no clear distinction between primes and composite odd numbers, such as 7-limit and 9-limit.

147 A psychological study that links quanta and qualia in pitch perception, although devoted mostly to tonal contexts rather than individual intervals, is Krumhansl, C. (1990). *Cognitive Foundations of Musical Pitch.* New York: Oxford University Press, Chapter 5, 111-137. Also, Sethares, *op. cit.*, Chapter 5, lists many sources of empirical studies related to this topic, although it is focused more on roughness and dissonance than proportionality and harmonicity. However, what I have not seen is an empirical research that explicitly considers prime numbers as the basis for harmonic 'chromas; or, as I call them, fundamental intervals.

the 3 representing the fifth), a route which in terms of diatonic degrees is III → VI → II.

I agree with Fokker that fundamental intervals should be considered as lying within the octave: 3 is a 3/2 fifth and not a twelfth (as overtone 3), the same holding for 5 and 5/4, 7 and 7/4, and so on. The fundamental *numbers* behind fundamental *intervals* are nonetheless the primes themselves, {2,3,5,7,11...}. However practical octave reduction is, it is a 'common agreement', and we should not take it for granted in all situations. The fact that it is the strongest of all harmonic relations masking other less strong equivalences does not make it absolute. As shown in the previous chapter with dissonance curves, many of the pitch sets I use span many octaves (mostly in unsymmetrical patterns) and no assumptions are made regarding intervals related by octaves, so that their differences in register, polarity and harmonicity (which are close but not equal) can be accounted for.

Ratios and harmonic coordinates do not give an indication of their pitch-distance, which needs a projection from the lattice into logarithmic pitch-distance space. Still, we can get a rough sense of the interval's type and size by its degree derivation. In the above 5-limit list, it is clear that the first tritone, a third above a major second, is an augmented fourth. The other tritone is a third below a minor seventh, a diminished fifth. It does not tell us (except for the unison), their exact sizes, however, which in cents are: 590 and 610.

The `HarmonicVector` class in *DissonanceLib* represents a ratio in harmonic space and calculates all its other features. For example, the code: `HarmonicVector.from([45,32]);` results in →

```
HarmonicVector( [ -5, 2, 1 ], 45/32, [ 2, 1 ], 45/32, 590.22¢, 2DM)
```

Displaying[148] coordinates, ratio, octave-reduced coordinates and ratio (the two being the same here), distance, and function (the code designates D/d = dominant/subdominant, M/m = mediant/submediant, and S/s = septimal/subseptimal, in this case showing two dominants plus a mediant).

For the sake of clarity and to give a more complex (septimal) example:

```
[[20,7],[7,5], [21,32], [28,15]].asHvector;

→     HarmonicVector( [ 1, 0, 1, -1 ], 20/7, [ 0, 1, -1 ], 10/7, 617.49¢, Ms)
      HarmonicVector( [ 0, 0, -1, 1 ], 7/5, [ 0, -1, 1 ], 7/5, 582.51¢, mS)
      HarmonicVector( [ -4, 1, 0, 1 ], 21/32, [ 1, 0, 1 ], 21/16, 470.78¢, DS)
      HarmonicVector( [ 2, -1, -1, 1 ], 28/15, [ -1, -1, 1 ], 28/15, 1080.56¢, dmS)
```

Non-octave reduced ratios (20/7, 21/32) are also represented in reduced form. It is from this reduced form that their size and function are derived, so all the information is available. Additionally, there is a function that returns the names of the intervals from a database obtained from the Huygens-Fokker foundation[149], yielding: (1) *Euler's tritone*, (2) *septimal or Huygens' tritone/Bohlen-Pierce fourth,* (3) *narrow fourth* (which, as its function – DS – implies, can be understood more easily as the harmonic seventh of the dominant), and (4) *grave major seventh* (which as degree is more precisely a *diminished octave*: dmS indicating that we go down a fifth to IV, then down a third to II, and then up a seventh to VIII, which, by its size of 1081¢ turns out to be quite diminished)[150].

148 They are displayed here in the *SuperCollider* language format. Numbers within brackets represent collections of numbers, in this case representing interval coordinates in harmonic space. The following example shows collections of collections, representing interval ratios ([7,5] represents 7/5). This is for reasons of consistency within the language and because they are usually used not for display but for calculation purposes.

149 Compiled by Manuel Op de Coul, it is available at http://www.huygens-fokker.org/docs/intervals.html Last accessed April 22, 2012.

150 With the exception of a some historiacal names, the nomenclature of the Huygens/Fokker intervals is derived through a quite consistent method explained in Keenan, D. (1999). A note on the naming of musical intervals. Last retrieved April 12, 2012, from http://www.dkeenan.com/Music/IntervalNaming.htm. It deals very well with the distinction between *subminor, minor, neutral, major,* and *supermajor* intervals (which differ to a certain extent between intervals that admit major and minor varieties – thirds, seconds and their inversions – and those that admit perfect or just varieties – unison, fourths and inversions). The problem is that it does not take function into account so it

Harmonic vectors can be added, subtracted, and exponentiated, their attributes calculated automatically, hence facilitating harmonic arithmetic. Adding and subtracting intervals involves moving through the coordinates in the lattice and is equivalent of to multiplying or dividing the ratios. Exponentiation means compounding an interval as many times as the exponent. Together, these operations provide a harmonic calculator useful for combining, extending, finding remainders, sizes and functions for ratios. Their help file in *DissonanceLib* gives several examples, some of them taken form the article by Fokker. For instance, they were of crucial aid in finding the microtonal accidentals for the table in Appendix II. We will see more in-depth examples where they are put to use in section 3.1.6.

3.1.2 Harmonic qualias, hues

In the spirit of the Archytean Pythagoreanism that sustains this study, we will now pursue the link provided by musical harmony between the qualitative and the quantitative aspects of sound, digging a bit into the qualitative aspect of the prime numbers behind harmonic intervals. This phenomenology of auditory numbers is inserted in the midst of the discussion of quantitative topics of harmonic space, intended to not lose a compositional perspective, before we delve further into the more arithmetically dense topics of commas, tolerance and harmonic metrics.

Ben Johnston gives a qualitative assessment of the axes in the harmonic lattice[151]. He designates the first dimension as 'cyclic' since it produces the repetition of pitch classes. Assuming the possibility of intervallic equivalences other than octaves, this axis refers to smallest prime involved. The next axis will be 'tonal', in analogy to dominant/subdominant relations. Further axes will be 'modal' in reference to major/minor modes provided by thirds. Sevenths and elevenths offer further modal hues.

Aside from their function as bases of the lattice, Johnston had previously proposed the following 'psychoacoustic meanings' of fundamental intervals[152]:

2 → recurrence, repetition, cycle;
3 → polarity, gravity, root-five or tonic-dominant;
5 → major/minor coloration: mode;
7 → contributes to a sense of 'centralized stability' to a 4:5:6:7 chord, suspending the dominant-tonic polarity as well as providing consonant tritones, sevenths and seconds (7/5-10/7, 7/4, 8/7);
11 → ambiguity, neutrality (lying a quarter-tone between a fourth and a tritone and producing neutral intervals)

There is a lot of compositional discernment to these descriptions, characterizing chief harmonic features through the mediation of subjective metaphors. Harry Partch, once his teacher, had already given an account of qualities of intervals in his 11-limit, 43-note system. For him intervals that comprise a given degree share the same quality, so that emotion is shared, for example, by thirds all the way from the 435¢ supermajor 9/7, down through 14/11 and 5/4 major, the neutral 11/9,

does not distinguish diatonic degrees, as we have just shown. 'Intervals can have more than one name', something which is true for pitch distances and involves tolerance, but is not true when dealing with the nomenclature of definite ratios. According to this scheme, a 7/5 can be either a fifth or a fourth, which is not correct. I have not yet developed an algorithm to do the naming automatically, though it should not be too difficult to implement in a future version of *DissonanceLib*. See also Appendix III where a list of septimal intervals is investigated and their names compared between my functional nomenclature and the one in the Huygens/Fokker list.

151 Johnston, B. (2006 [1971]). Tonality Regained, *Maximum Clarity*, 46-47.

152 Johnston, B. Scalar order as a Compositional Resource, Ibid., 27-28.

minor 6/5, 32/27, to the 266¢ subminor 7/6 *ekbole*. The confusion is compounded because he does not account for divisibility but only for the size of the numbers involved. He puts forward the quality of 'power' to unisons, octaves, fifths and fourths (all the perfect intervals); 'suspense' to tritones (close to ambiguity); 'emotion' to 3rds and 6ths; and 'approach' to seconds. I think the latter pertains to a more horizontal rather than vertical qualia since it is not connected to the numbers that determine some of these degrees[153].

I had taken some notes on the auditory qualias of primes before reading Johnston's seminal articles, which were a relatively late discovery (2011, my assessments were done in 2010). The first discrepancy is the importance I placed on interval inversion and its lack of quality-wise symmetry. Not only is a fifth upward different than a fifth downward (and in my experience this is exacerbated with thirds and even more with sevenths), but their octave reductions which place them in the same direction but different size (such as 3/2 with 4/3 or 2/3 and 3/4) and polarity, make this even more clear: a fourth and a fifth, although sharing some similar characteristics, are quite different qualitatively and functionally. Furthermore, the accumulation of a fundamental interval changes its quality quite drastically, except in the case of octaves, which is why I give 2 a quality of neutrality, not in the sense of a 'neutral third', but in the sense of 'transparency', producing register and closing harmony on itself (which is why it is close to Johnston's notion of cycle). The distinction between 3 and 3^2 or 3^{-2} shows that ninths, seconds and sevenths are quite different in quality from fifths. Seconds are not directly produced by any of the first primes (except if we take into account the inverted seventh: 8/7, 231¢, but it does not correspond to any commonly used versions of the major second). It is not the same to consider the 9/8 second as the fifth of the fifth than to regard it directly. Maybe this is why steps are usually deemed melodic rather than harmonic (even while considering harmonic relations as also occurring horizontally). Their harmonic use stems from octave reductions of ninths and sevenths (9/8, 16/9), their function being mostly (sub)dominant of other degrees (9/8 is the dominant of 3/2, 16/9 the subdominant of 4/3) and not an independent function in itself. Johnston's account and general approach stays within close chains of relationships, considering dominant-tonic chains, but not direct relations between the tonic and double or triple subdominants or dominants. Direct seconds are disconnected in the lattice by more than a single step in any direction from their tonics, making them more inharmonic and difficult to establish as tonally independent sonorities, in contrast to mediant and dominant functions which are adjacent.

In my 'psychoacoustic' assessment I distinguish between a prime number in its upward and downward direction and in its combination with other fundamental intervals:

3 → stability; up: openness; down: attraction;

5 → a very 'colored' interval; up: stands out and is connected with motion (maybe associated with Partch's e-motion?); down: even more mobile (as minor 6th) but with quite a different qualia; together with 3 it gives the minor 3 and major 6 which share the poignancy but are darker (up) and brighter (down); combined with 7 it yields stable tritones;

7 → gloomy, uncanny; up: stable seventh, very distinct color; down: unstable, large second; stable tritones and an unstable third (7/6). There is a connection with 'blue notes' in blues and jazz, connoted as sad or painful: seventh, tritone and minor third (which is used as augmented ninth against a major third), corresponding to the combinations of seven with 2, 3 and 5, respectively: 7/4, 7/6, 10/7 & 7/5. They function as stabilities, not requiring resolution;

153 Instead of primes, Partch bases his system on intervallic 'identities' – octave reduced odd numbers. Although I think his music was not a full consequent experimentation upon his discoveries and some of his concepts are not quite correct, his influence as *the* microtonal pioneer of the twentieth century is considerable. One aspect of his work that my research assumes as central is the emphasis on intervallic inversion as a fundamental harmonic feature. He calls it by the terms of symmetry, dualities, and over- and under-tonalities, arriving at at 'tonality diamonds', which comprise upper and lower components of fundamentals.

11 → 'tritoneness', ambiguity, due to the schizoid nature of tritones, which in octave equivalent systems are the pivot intervals under which intervallic inversion occurs, owing to the fact that they can be arrived at in many symmetric ways.

The higher we go up the prime ladder, the more fragile, singular and uncommon the harmonic hues become. Considering, as we do, prime intervals as lying within an octave, their series does not follow any predictable order in terms of distance: the series of octave, fifth and third make it seem like the intervals are getting smaller, but then seventh and fourth-plus-a-quarter-tone (11^{th}) break the pattern. The thirteenth is a very strange interval, close to a neutral sixth (841¢), with a very distinguishable somber tone, even though its harmonicity is quite low to allow easy manipulation. Primes 17 and 19 have the characteristic of lying very close to the tempered minor ninth (or minor second) and the minor third, respectively. Even if their perceptibility as such is very low, they don't really contribute to new hues because they lie too close to intervals derived from combinations of lower primes.

To add a few more overtones to the discussion, we must mention that Johnston, with reference to LaMonte Young's work, also describes intervals in terms of number symbolism [154]:

2 → 'repetition of the same thing on another scale of magnitude'; cycle;
3 → stability and strength;
5 → (human) emotions;
7 → 'could be said to symbolize sexuality' and has relation to blues music;
11 → androgyny as a symbol;
13 → death or at least the 'emotions commonly associated with the possibility of death' (he makes very effective use of prime 13 in his 5^{th} String Quartet by isolating and pursuing its connotation with death).

Beyond single intervals, the most common harmonic situations involve combinations of them. A major triad can then be considered in the lattice as a structural point of stability for mixtures of {2, 3, 5}, which in its most compact (closed) form gives a major triad in root position. This is symmetrical with respect to intervallic inversion, thus also extending to minor chords. These chordal structures can be seen as invariant shapes within the lattice that can be translated, rotated and scaled in size to yield other related sonorities, so their geometric structure can serve to generate related but distinct harmonic sonorities. Johnston's starting point for his approach is rooted in traditional triadic and chordal harmony but generalized to chords with *n* notes (*n*-ads), where each note in the chord comprises a step (in any direction) in each of the axes of the lattice, beginning with the cyclic, then the tonal and continuing with the modal ones. If the direction in the modal axes is negative the resulting chords contain 'minor' ingredients.

3.1.3 Commas

As has been seen in passing, a comma is a small interval measuring the difference between two notes close enough to be considered unisonous. These very similar intervals (in our previous example 5/4 and 81/64) are reached through different paths in the lattice, which is the same as saying that they are composed of independent mixtures of fundamental intervals. Commas result from the fact that powers of different primes are never equal to each other, so two separate constitutions cannot coincide. The most ancient and well known example arises with 2 and 3 and the fact that $2^7 \simeq (3/2)^{12}$ ($128 \simeq 129.7463$), their ratio being 1.01364 = 23.46¢, the Pythagorean comma, a difference

154 Johnston, B. (2006 [1995]). Regarding LaMonte Young. *Maximum Clarity*, 251-258.

resulting from arriving at almost the same note by following either 12 fifths or seven octaves. More generally, the aim is to find two exponents such that $(2/1)^n \simeq (3/2)^m$ and it turns out that the first n,m pairs following {7, 12} correspond to well known divisions of the octave: {41, 24} and {53, 31}. They make for 'ultra-chromatic' tunings (41 and 53) which contain 'ultra-diatonic' scales (24 and 31).

Many kinds of commas exist, some famous, some not quite so, some really obscure, all of them involving independent routes to near-enough unisons. The way I refer to them is by the primes involved in each separate route, so the Pythagorean comma we've just seen is an example of a 2-3 comma, the other pairs being also 2-3 commas of higher order. Commas pertain to any combination of tuples of primes.

To elucidate with examples, in the 3-limit lattice the Pythagorean comma is the difference between 12 fifths <0,12> and the origin <0,0>, equal to $3^{12}/2^{19}$ = 531441/524288. The *syntonic* comma which we met earlier lies between four fifths <4,0> and a single third <0,1> in 5-limit, octave reduced space. Subtracting the latter from the former gives <4,-1> = 81/80 = 21.51¢. In a 7-limit space, a triple seventh <0,0,3> (343/256, 506¢) is practically in unison with a single fifth downwards <-1,0,0> (4/3, 498¢), their difference being <1,0,3> (1029/1024, 8.4¢). An example involving three primes (a 3-5-7 comma): <2,2,0> (225/128, 977¢) and <0,0,1> (7/4, 969¢) differ by <2,2,-1> (225/224, 7.7¢); two fifths and two thirds (functionally 2D2M, V → II → IV+ → VI+, an augmented sixth) lie very close to a minor seventh (VII-).

Johnston devised microtonal tuning notations based on commas. Marc Sabat and Wolfgang von Schweinitz have developed the idea further with their 'Helmholtz-Ellis' notation[155]. Special symbols (derived as much as possible from historical practice) specify some of the commas that turn intervals from 3-limit Pythagorean tuning to other prime interval combinations. No symbol (or natural accidentals) implies this base tuning, and there are symbols for *syntonic* (3 to 5), septimal (64/63, the difference between 16/9 and 7/4, a (2, 3, 7)-based comma), 11, 13, 17, 19, 23 up to prime 61 (most of them being too high for being effectively perceived in my opinion). All symbols have their corresponding inverted sign and there are special signs for equal temperament and cent deviations from it, useful for inharmonic tunings.

The coordinates of the two near-unison intervals usually lie quite far away in harmonic space, showing how they are different routes to similar intervals. Fokker connected these coordinates with vectors, realizing that placing these 'unison vectors' one after the other (summing them) results in further near-unisons. Unison vectors delimit regions in the lattice in which equivalent intervals repeat themselves, producing a periodic tiling of the lattice in the vector's direction. By using three appropriately chosen unison vectors, three-dimensional blocks of periodicities (parallelepipeds) are formed.

> 'The whole lattice will be divided in countless periodicity blocks, each of them representing in a way the whole of the lattice. Obviously there is a considerable economy in handling the finite number of notes within a periodicity block instead of the numberless notes of the complete lattice.'[156]

According to the choice of these vectors, blocks of varying sizes will be produced. Choosing them is not trivial, but Fokker found several sets of vectors both in two dimensional (5-limit) and three dimensional (7-limit) spaces[157], forming blocks that contain 12, 19, 22, 31, 41 and 53 notes. These blocks correspond to the most well known tempered approximations of proportional intervals: 31, 41 and 53 arise once again, the discrepancy between 22 and the 24 we found previously probably

155 http://adagio.calarts.edu/~msabat/ms/pdfs/notation.pdf Last Retrieved May 27, 2012.

156 Fokker, Ibid., 3, in the pdf version. The diagrams in this article illustrate this concept quite clearly.

157 Let us not forget that he assumes octave equivalence, so that 7-limit is three- instead of four-dimensional. The vectors lie orthogonally to each other and together involve commas relating to the three main primes. Intervals inside different periodicity blocks differ by one (or several) of these commas.

having to do with rounding and the centering in the lattice of these parallelepipeds. As we saw last chapter, these are the principal equal divisions of the octave that best correspond to approximations of proportional systems. What we have now seen is how these numbers emerge by recourse to the lattice. Though the task is not trivial, there should exist unison vectors for the only famous approximation that is missing, the Aristoxenian 72 divisions. There should also be vectors for producing these divisions in four-dimensional 11-limit space (Fokker provides a single set of vectors for this one). We have also seen that the two best approximations for 7-limit intervals, 53 and 31, stand in a relation analogous to chromatic and diatonic (12 and 7), explaining why 53 is probably the most accurate approximation of all (although this will definitely depend on the particular system and aims chosen), the comma that defines it spanning only 3.61¢ and containing within it the next to best approximation (31) as its ultra-diatonic subset[158].

Within each block is contained a complete harmonic world; I have called them 'islands' since they enclose these domains. They have been implemented to deal with pitch sets arising out of dissonance curves. Dissonance curves produce irregular geometries within harmonic space (refer to *Figures* 2 and 3 of section 3.1.6 for an illustration). Periodicity blocks can be used to partition these intervals into harmonic and timbral subsets according to whether they lie or not within the central block. The intervals obtained from the curves lying inside a block can specify scalar structures within the tuning system implied by the block, providing specific patterns within 12, 19, 22, 31, 41 or 53 divisions of the octave. The intervals that lie outside the block can be esteemed alien to the main harmonic realm and given different (timbral) behaviors. How these partitions are to be interpreted is a compositional decision: I have mostly used this distinction to 'orchestrate' them. Chords made out of dissonance curves can be rendered with different timbres in order to distinguish the closer periodicities of the harmonic subsets to the more distant ones of timbral subsets, for example, or by giving each subset a different rate of movement (timbral intervals usually moving faster than harmonic ones, but there is no reason why these behaviors have to be preestablished).

The separation of pitch sets coming out from dissonance curves into timbral and harmonic islands does not alter or round off the intervals themselves, it just serves to classify them. Alternatively they can be mapped into the tempered approximations implied by the periodicity blocks. In this case the intervals lying in parallel blocks, differing from intervals in the central island by a comma, may be considered equivalent and transposed back into the central block. This option opens the way for treating them as degrees of these systems instead of only as ratios, allowing to transpose, invert and modulate the intervallic geometries to other degrees. An advantage with these approximations (which can be used only for working out materials but applied to the actual non-approximated intervals) is that scales and modes are more readily visible in terms of pitch distance degrees, also opening the way for operating with them combinatorially, something which is not so straightforward when dealing with ratio representations.

The `PitchSet` class in *DissonanceLib* is a collection of `HarmonicVector`s that are partitioned into timbral and harmonic subsets. To give a simple example, consider the following four-note septimal chord:

```
PitchSet.with([[1,1],[7,6],[3,2],[7,4]])
→	Harmonic Set[ 1/1, 7/4, 3/2 ]
	Timbral Set[ 7/6 ]
```

This example uses the default unison vector set (12 divisions of the octave in 7-limit space), and we can see that intervals with negative exponents of 3 (7/6 contains a fifth below 1/1) are situated

158 The comma derives from the fact that $(3/2)^{53} \simeq (2/1)^{31}$. $3^{53}/2^{81}$ = 19383245667680019896796723/19342813113834066795298816 = 3.16¢ (also known as Mercator's comma). Additionally, the size of 53rd division of the octave is 22.64¢, lying between the *syntonic* (21.5¢) and the Pythagorean (23.46¢) commas, probably explaining (or at least illustrating) why many of its intervallic combinations approximate some harmonic intervals so well.

outside the block. Changing to a larger block does not necessarily solve this:

```
PitchSet.with([[1,1],[7,6],[3,2],[7,4]], PitchSet.unisons.dim3.et31[1])
→     Harmonic Set[ 1/1, 7/4, 3/2 ]
      Timbral Set[ 7/6 ]
```

The unison vector is set for 31 divisions of the octave, but the block is still centered to the 'right' of 1/1 in the axis of fifths. Several options are available for each equal division [159], and it turns out in this case that the third one gives us a block centered to cover the whole chord:

```
PitchSet.with([[1,1],[7,6],[3,2],[7,4]], PitchSet.unisons.dim3.et31[3])
→     Harmonic Set[ 7/4, 3/2, 1/1, 7/6 ]
      Timbral Set[  ]
```

All intervals have been now been included inside the island. This example is not yet musically meaningful, as we still need to know what to do with the intervals, or if we want to use these intervals against others in a larger system. Usually much larger pitch sets are used and the choice of an appropriate block is related to the musical task, being quite different for electronic than for instrumental performance. I have used them mostly in electronic 'dissonance' chorales that accompany sound field recordings as well as in real-time computer interaction with instruments or as an installation (more will be said about the strategies and pieces later on).

To approximate into temperaments, the method `asETdegrees` returns the equivalent degrees in equal temperament (the default is 53 in 7-limit space[160]), thus posting the following for the first example:

```
TET: 53
Harmonic Ratios: 1/1, 3/2, 7/4
Harmonic Degrees: [ 0, 31, 42 ]
Timbral Ratios: 7/6
Timbral Degrees: [ 11 ]
```

This shows how harmonic ratios map to degrees 0, 31 and 42 in 53ET, while the other interval maps to degree 11. We will follow a more thorough and contextual example that encompasses these topics in section 3.1.6.

3.1.4 Tuning tolerance

> '[G]enerally, by some numbers that the tones are expressed by, if the proportions are too complicated, the ear will substitute a close approximation that is simpler. Thus the heard proportions are different than the true, *and it is from them that we must judge the true harmony and not from the actual numbers*.'
>
> 'Our hearing is accustomed to taking all proportions that differ very little from uncomplicated ratios as such. The simpler the proportion, the more sensitive our hearing is to noticing small aberrations. This is the reason why we can hardly stand any deviation in octaves; we intend that all octaves be exact and that they do not differ at all from doubling.' (Leonhard Euler, *Conjecture on the Reason for some Dissonances Generally Heard in Music*[161])

---

159 These alternatives are defined by the sets of unison vectors, spanning several approximations in 2 and 3 dimensions, with a single alternative for 4 (11-limit) dimensions. In this example there are 3 possibilities associated with 31ET in 7-limit space. They are accessed as a dictionary, `PitchSet.unisons`, from which further keys give access to the matrices or sets of vectors that define the block. The first codifies the dimension (`dim3` in this example), then the number of divisions per octave (`et31`), and finally one of the possible alternatives. This is all documented in the software.

160 These approximations are commonly referred to as *n*-(T)ET – (tone) equal temperament – or *n*-EDO (equal divisions of the octave).

161 Euler, L. (1766). *Conjecture on the Reason for some Dissonances Generally Heard in Music.* (J. A. Scaramazza, Trans. From

Aristoxenos understands the fault-tolerance mechanism in pitch perception as narrow nodes in pitch-distance space inside of which intervals preserve their identity. This mechanism is responsible for many musical phenomena, including out-of-tune renditions of songs when singing in the shower, making temperaments feasible and allowing nuanced tuning around notes. Disregarding tolerance in proportionality would imply that slightly mistuned consonances would have to be represented by very big ratios, entailing very high inharmonicities, which is clearly not the case.

Tolerance permits this distortion, perceptually 'rounding off' to the nearest and strongest harmonic ratio, the amount resulting from the difference in tuning between the nominal and the actual ratio becoming its timbral coloration or residual *clangtint*. This coloration involves the presence of beatings between spectral components, but is also present within the fundamentals themselves, as if this phenomenon could pertain only to sine waves abstracted out from the actual spectral surface. This variance separating the correct (true) ratio and the actual (heard) deviation imposes the timbral facet of harmony upon proportionality, twisting and blurring it.

The mechanism for tolerance, its intervening factors and ranges are not well understood yet. Its range is inversely proportional to the harmonic complexity of a ratio – a harmonic interval has a smaller tolerance range than an inharmonic one, an octave is very sensitive to deviations, whereas a tritone much less so, – but there is still no way of knowing how to quantifiably measure this, maybe another target of study for music cognition suggested by composition.

Due to tolerance the same note may have more than one function or identity. In equal temperament there happen cases where the harmonic meaning of a note changes depending on the preceding and following ones, implicating a horizontal aspect on the time axis, as well as metric stress. Temperament benefits from these ambiguities by approximating proportional scales of unequally spaced intervals into equal or near equal steps where some sharps and flats are fused into single enharmonic notes. In minor tonalities, for example, the interval between the leading note below the tonic (15/16) and the minor third (6/5) is a diminished fourth (32/25, 427¢)[162]. In standard temperament both this interval and the major third (5/4, 386¢) are conflated into a single 400¢ third that can acquire either role while sounding the same. The difference between both intervals is large, 41¢ (larger than the 16¢ deviation of minor thirds in 12-ET), showing that the span of tolerance is not fixed. One could assume that within the major mode this diminished fourth does not arise, so the tolerance range is stricter. We see this happening in the various meantone and well-temperaments, each of which is better adapted to certain tasks and styles than others.

Tolerance depends also on attention, experience, stylistic attributes and musical education. Yet another factor that intervenes is the verticality of the intervals in question: an unaccompanied melody has wider tolerance than when it is held together to other simultaneous notes. Arriving at a quantitative relation for tolerance would require isolating and abstracting these contextual considerations, something which complicates the task, maybe rending the formula difficult to apply in real world situations. All the same, it would be interesting to pursue this as it might reveal unknown details

In his study on the harmonic lattice Tenney adds a few points that are relevant here. He talks about the 'activation' of nodes in the lattice, which require saliency and stability of pitch as well as sufficient time. The activated points persist after the sounds have ended as a result of some kind of psychological resonance. This is interesting compositionally, suggesting that inharmonic intervals may require more time to activate than harmonic ones, inverting the usual priority of durations

Conjecture sur la raison de quelques dissonances generalement recues dans la musique). Mathematics Department, Rowan University, §9 and 12. Available at http://www.math.dartmouth.edu/~euler/docs/translations/E314.pdf. Last retrieved November 15, 2010.

162 A well known example occurs in Fugue #4 in C sharp minor of J. S. Bach's Well Tempered Clavier BWV 849: C#, B#, E, D#. The diminished fourth lies between B# and E.

given to harmonic notes, inversely connecting duration with proportionality. Their persistency manifests a limit to the speed at which harmonies can be processed (the more harmonic the longer they take to deactivate). Another topic he mentions is what he calls 'harmonic containment cones', a way to visualize the lattice as a timbral spectrum, as if the activation required enough points in the 'cone' (corresponding to its harmonic partials seen as projected into harmonic space). The more points it has, the clearer the sense of fundamental and the less time required for its activation. It integrates timbre into tone and harmony, actual timbre stemming from the relative amplitude of the individual partials; when the points are contiguous either in pitch or harmonic space, they are perceived as unitary and not as separate components[163].

In harmonic tolerance the two facets of harmony touch each other and for composition this suggests clues for composing the duality itself: transitions, mixtures and separations between timbral and harmonic materials and logics. In tolerance also lies the key for simplifying and making instrumental writing practical. Contrary to the aim of many proponents of just intonation, who I think are too attached to exact tunings and neglect tolerance, it is possible to write microtonal music without having to exactly tune every interval. To extract a proportional logic to intervals means we can manipulate them with that logic without their tuning being just. This is why standard equal temperament can be used with other kind of harmonic priorities and stabilities than the usual ones, provided the tolerance analysis and the right context are accounted for, as when the harmonic logic involved is quite different from usual tonal-atonal sonorities (timbral logics like the ones derived from dissonance chorales are a good example, they can be used with standard temperament without loosing much of their logic and consistency, despite missing the sonority of some of the strangest intervals). Dissonance pitch sets can also be understood proportionally without their tuning being changed. The harmonic discretization of the pitch continuum that produces nodes within which the identity of a string interval is maintained, masks the strange, eerie sounds of higher primes lying inside them. One way to make these fragile and unusual sonorities stand out in stable or functional ways is to avoid some of these strong intervals, or to remove a prime axis (as in LaMonte Young's Well Tuned Piano where intervals based on 5 are excised in order for 7-based intervals to emerge), making room in pitch space and avoiding ambiguity in harmonic space. Mixing these intervals with lower primes in order to form harmonic 'regions' of relative harmonicity can also anchor them functionally, reducing their ambivalence and their conflation into the *clangtint* of simpler ratios. Another factor involves duration (and therefore rhythm, articulation, meter, etc): the more inharmonic an interval, the more time it needs to be activated, showing that formal levels above the immediate sonorities of these 'regions' are also involved. These differentiations in harmonic space contribute to reduce the tolerance range, increasing the focus that allows these intervals to stand out on their own.[164]

3.1.5 Harmonic metrics, rationalization

Euler's work on music culminates a strand in the lineage of Pythagorean harmonists, spanning such figures as Galileo – *commensurability* and the origin of 'pulse counting' or 'coincidence' theories –, Kepler – *harmonia mundi*, – Descartes – *mathesis universalis,* the communication of proportions from sense to intellection, – Mersenne – the theory of simultaneous vibrations in strings and the ensuing harmonic series, – and Leibniz, whose famous phrase 'music is a hidden arithmetical exercise of the

163 Tenney, John Cage and the Theory of Harmony, 29-35.

164 The subject of tuning tolerance is also developed, particularly with regard to simultaneous tuning systems, in Mogini, F. (2000). Alternative Tuning Systems (Master's thesis, Lansdown Centre for Electronic Arts, Middlesex University, 2000).

spirit unconscious of enumeration'[165] had much influence on Euler, as acknowledged in his *Tentamen novae theoriae musicae*[166] (written at age 24 in 1731), where he introduced the *Gradus Suavitatis* function and offered the first multidimensional representation of intervals.

As we have seen, the main premise behind this Greek lineage is that there exists a relation, involving numbers, between intervals and their translation by perception, between their primary and secondary qualities, as it were. Primary are qualities of an object in itself (in this case a sound complex or an interval), independent from its relation to perceivers, being quantitative and mathematizable: extension, length, weight, frequency, etc; secondary qualities are effects of these features upon the senses, not located in the object: colors, smells, tastes, pitch, harmonic qualias, and so on. The Pythagorean hypothesis attempts to provide an explicit relation between these two realms, to link *quanta* and *qualia* through arithmetics, the senses and intellection: qualities of sound carry a proportional structure. This structure is elucidated through *aisthesis* and is the source (the 'formal cause' in Aristotelian terms) of its specific hue: proportions describe a physical feature of the object which stands in correlation to its subjective properties, to 'harmonicity in the first person', as it were. It is the *formal* aspect of this structure that constitutes its definition pattern: a non-accidental quality of the interval arrived at by abstracting its *eidetic* core out from the enclosing timbral surface. As the 'remainder' of this reduction, the contingent timbral features that envelop the *eidos* of the interval account for its accidental qualities. In harmonic terms, timbre and proportion complement each other, each pertaining to different kinds of qualities.

Euler offered an explicit account of this process, the arithmetical operations underlying perceptual translation, clarifying what is understood by the complexity of a ratio from the metaphysical perspective of order and perfection with regard to music: the formal causes behind 'musical pleasure'. Helmholtz later contrasted his position by stating that according to Euler,

> 'the human mind perceives commensurable ratios *as such*, according to our method, it perceives *only the physical effects* of these ratios, namely the continuous or intermittent sensation of the auditory nerves,'[167]

shifting the explanation to efficient causes (which happen sequentially in time) and ignoring the indirect allusion of *aisthesis* taking place after the fact in intellection. It is not that numbers come to mind when listening to proportions, but that proportional relations have a 'sound' of their own and the audible realm leads to the discovery of these relations. We should therefore distinguish between auditory and calculating numbers, the former being alluded, formal, and *eidetic*: they are 'perceivable' only indirectly. Eidetic or essential qualities are what is left over after accidental qualities are stripped off from an interval in categorial intuition, also known in phenomenology as *eidetic variation*; in the case of musical harmony, *aisthesis* is the name we have given to this process[168].

Instead of musical harmony giving a glimpse of metaphysical harmony, it is metaphysics which comes to the service of music perception. After Euler, arithmetic-based speculative harmony would have to wait until Augusto Novaro and Ervin Wilson in the 20th century for inventions/discoveries that would pursue the Pythagorean hypothesis further, opening up new possibilities. It is interesting that these two figures are neither mathematicians, philosophers nor exactly composers, their unorthodox approach isolated from most of the main musical currents of such an Aristoxenian

165 '*Musica est exercitium arithmeticae occultum nescientis se numerare animi*.' Appears in a letter (1712) to Christian Goldbach in Leibniz, G.W. (1734). *Epistolae ad diversos* (Chr. Kortholt, Ed.). Leipzig, p. 240ff.

166 Euler, L. (1739). *Tentamen novae theoriae musicae.* St. Petersburg: Ex typographia Academiae scientiarvm. See also Bailhace, P. (1997). Music Translated into Mathematics: Leonhard Euler. Last retrieved November 15, 2010, from http://sonic-arts.org/monzo/euler/euler-en.htm#T2

167 Helmholtz, *On the Sensations of Tone*, 231.

168 For a more thorough discussion of *eidetic* qualities and allusion, refer to Harman, G. (2011). *The Quadruple Object.* Winchester: Zero Books, Chapter 2, 20-34.

century.

Euler related consonance with order and this with arithmetic divisibility rather than sheer numeric size. Divisibility implies that there is more to coincidence theories than just the period it takes for pulses to fall back in sync with one another, outweighing the sizes of the numbers involved. Partch was intrigued by the fact that 9/8 is more consonant than 8/7[169]. Coincidence theories would imply that the latter should have a shorter period of coincidence than the former (56 against 72), but when it comes to measuring complexity, the coincidences of periods must refer to the factors of the numbers: 7 is more inharmonic than 9 because 9 = 3^2. The compounding of threes does not suffice to overcome the incommensurability of a single period of seven. But by how much? How can this be quantified? Euler came up with a formula involving the factors of the numbers in the interval. For any natural number *a,* expressed as a product of prime powers,

$$a = \prod_{k=1}^{n} p_k^{e_k}$$

its *Gradus Suavitatis* (degree of 'softness' or 'sweetness') is defined as:

$$\Gamma(a) = 1 + \sum_{k=1}^{n} e_k(p_k - 1)$$

The measure of a prime is itself. For composites, the measure is the sum of each of its primes compounded as many times as its exponent. The influence of each prime in the sum is one less than the prime itself. One is added at the end to the sum so that unity does not yield zero and primes engender their own value. The *gradus* function of a ratio is the *gradus* of the combined factors of numerator and denominator. Symbolically, for a ratio *a/b,* its *gradus* is:

$$\Gamma(a \cdot b)$$

The weights of the first primes are {1,2,4,6,10,12,16}, showing an increasing difficulty of comprehension for each new fundamental interval. The function, though, does not discriminate, for instance, between a 3 compounded twice and a 2 compounded thrice, so different intervals can yield the same measure.

Gradus suavitatis can be applied to chords, scales and tunings, something very useful compositionally. It is calculated by expressing all the intervals as belonging to a harmonic series and taking the *gradus* function of their least common multiple:

$$\Gamma(\mathrm{lcm}(a_1, a_2, ..., a_n))$$

A dominant seventh chord {1/1, 5/4, 3/2, 16/9} is equivalent to harmonics {36,45,54,64}[170], their least common multiple being 8640 and its *gradus* 17. By exchanging a 9/5 for the 16/9 seventh, the harmonics become {20,25,30,36}, their lcm 900, *gradus* 15. By changing 64 to 63 in the first series, Euler reduced the chord to {4,5,6,7}, lcm 420, *gradus* 15, making a case for the natural seventh. However, it yields the same measure as the chord with the 9/5, not telling them apart despite their difference being as large as 35¢, so a more nuanced measure is required[171].

Other harmonic measures have been devised by composers in the last decades. One is Barlow's *harmonicity* (*H*), the other Tenney's *harmonic distance* (*Hd*). The first is close to Euler's but refined so that no two intervals yield the same measure. It is rooted in the psychological 'indigestibility' of whole

169 Partch, *Genesis of a Music*, 150. The heading is telling: 'Enigma of the Multiple-Number Ratio'.

170 There are functions in *DissonanceLib* that convert from ratios to harmonics and vice versa. They are documented in the 'number extras' help file.

171 It is also clear that they are different kinds sevenths. 9/5 has a function of 2Dm, 16/9, 2d, and 7/4, S. The two former ones are related by being double dominants or sub-dominants, and are usually considered tensions requiring resolution, while the latter should be deemed as a separate function, a subminor instead o a minor seventh. See the comparative table of sevenths provided in Appendix I.

numbers. Having been deduced from a musical perspective, it was later found to be consistent with studies that classify the psychological effects of numbers according to their complexity, magnitude and whether they are prime/composite and even/odd.[172] For a number expressible as a product of prime powers[173],

$$\mathcal{N} = \prod_{r=1}^{\infty} p_r^{n_r}$$
$$\mathcal{N}, n, p \in \mathbb{N}$$
$$p \in prime$$

its indigestibility is:

$$\xi(\mathcal{N}) = 2\sum_{r=1}^{\infty}\left(\frac{n_r(p_r-1)^2}{p_r}\right)$$

Notice the similarly to the *gradus* function in that there is a sum of primes subtracted by one and compounded by their exponent, though in this case the weight is squared and the compounding is divided by the prime itself, thus preventing equal measures for different numbers[174]. This makes for more nuanced degrees, the difficulty in perceiving each prime increasing by

$$\frac{(p-1)^2}{p}$$

instead of $p - 1$. The weights for primes up to 13 are {0.5, 1.333, 3.2, 5.143, 9.091, 11.077}. They have a higher slope than the *gradus* function, both behaving in approximately linear ways, but with sharper discrimination in this case. The 2 at the outset of the formula is a normalization factor so that unity gives zero and octaves (powers of 2) yield a natural number series.

The *Harmonicity* of a ratio is the reciprocal of the sum (instead of product as in *gradus*) of the indigestibilities of the numerator and denominator. Additionally, the sign of the difference between the indigestibility of denominator and numerator indicates the polarity of the interval. For an interval P/Q it is:

$$\mathcal{H}(\mathcal{P}, \mathcal{Q}) = \frac{sgn(\xi(\mathcal{Q})-\xi(\mathcal{P}))}{\xi(\mathcal{Q})+\xi(\mathcal{P})}$$

The *harmonicity* of the unison is infinite[175], an octave yields 1 and continues to decrease with intervals of higher complexity. For the purposes of measuring the relative strength of intervals polarity can be ignored, Barlow calls this absolute value *harmonic intensity*. He also provides a measure for intervallic systems, named *specific harmonicity*, more complex than Euler's, pertaining to combinatorics and indigestibility, about which I will say a bit more below.

The second harmonic measure, Tenney's *harmonic distance*, is based on the harmonic lattice, on the number of 'blocks' or steps to be traversed through the shortest possible path between intervals. These steps are first projected from harmonic into pitch-distance space or into pitch-class space, which is a lower dimensional collapsing of some of the axes, as happens in the case of octave reduced lattices. These projections are logarithmic, so *harmonic distance* is proportional to the logarithms. The measure for a reduced ratio *a/b* is:

---

172 Barlow, C. (1980). *Bus Journey to Parametron.* Cologne: Feedback Papers, 21-23, 21-23. This formula was obtained 'experimentally', meaning intuitively and through trial and error out of the premises of Euler's *totient* function, as well as *gradus*, and later compared to the psychological complexity tests.

173 I have decided to preserve the notation of each author so even though the nomenclature changes slightly, it means the same thing as in Euler's formulation: the decomposition of a number into a sum of prime powers.

174 In practice I've found that there is a case of coincidence: 27 and 256 have the same indigestibility. This causes the rare interval 256/27 to yield a harmonicity of zero, crashing the software or giving wrong results on rare occasions.

175 In the software this is handled by converting infinite (or 'not-a-number') to 2, which is higher than the harmonicity of any other interval.

$$Hd(a,b) \propto \log(a) + \log(b) = k \log_x(ab)$$

The proportionality constant *k* and the logarithm base *x* are used to scale the formula. When the constant is 1 and the base of the logarithm is 2, the metric can be said to be in octaves, as they yield integer series. The size of the rungs for each prime axis are proportional to the logarithm of the prime, accounting for the decreasing influence of higher primes, but in this case following a convex curve, meaning that the differentiation is sharp at first but becomes less acute as the primes increase. It differentiates differently than Barlow's *harmonicity*. For instance, 5/4 and 9/8 are inversely regarded by both metrics; for *Hd*, 5/4 is more harmonic than 9/8, lying one rather than two steps away from 1/1, while for *harmonicity* two threes do not overcome the complexity of a single 5. Moreover, *Hd* regards 7/4 as more harmonic than 9/8, suggesting that its weighting curve is probably a bit too gentle with respect to perception. As far as I can tell, it should not be easy at all for music cognitivists to empirically measure how the difficulty in perceiving the prime intervals increases. The indispensability measure benefits from having been correlated to studies of visual and intuitive assessments of numbers, though more detailed auditory evaluations should be pursued. The following tabulation compares the way the three metrics hierarchize a just-chromatic scale as well as a scale made with septimal intervals:

H	1/1	2/1	3/2	4/3	9/8	5/4	5/3	8/5	6/5	9/5	15/8	16/15	45/32
Hd	1/1	2/1	3/2	4/3	5/3	5/4	6/5	8/5	9/5	9/8	15/8	16/15	45/32
gS	1/1	2/1	3/2	4/3	5/4	5/3	9/8	6/5	8/5	9/5	15/8	16/15	45/32
H	1/1	2/1	7/4	8/7	7/6	12/7	9/7	14/9	7/5	21/16	10/7	27/14	49/48
Hd	1/1	2/1	7/4	7/5	7/6	8/7	9/7	10/7	12/7	14/9	21/16	27/14	49/48
gS	1/1	2/1	7/4	8/7	7/6	9/7	7/5	12/7	10/7	14/9	21/16	27/14	49/48

Table 1. Comparison between the orderings provided by *harmonicity* (*H*), *harmonic distance* (*Hd*) and *gradus suavitatis* (*gS*) for a just-chromatic (above) and a septimal scale (below). The colored intervals in *gS* yield the same measure and are placed in order of size. At the bottom of each ordering is shown where all metrics agree (red) and differ (gray). In the bottom example (in light gray) a series of inverted and forward intervals of 7 follow a numeric series from 4 to 12 (the 11 is missing by design), revealing the stepwise character of *Hd*.

The three measures are built into *DissonanceLib* and beyond their theoretical adequacy all produce different sonorities that are musically fruitful. Other possible measures can be added, such as the criteria used in ancient Greece, Erv Wilson's complexity function, Paul Erlich's harmonic entropy (proportional to Tenney's *Hd*), or any arbitrary mathematical function operating on ratios[176].

In terms of scales and chords, *harmonic distance* does not distinguish between permutations of sets of intervals while *harmonicity* fails to distinguish intervallic retrogrades (*a*,*b*,*c* from *c*,*b*,*a*).

We discussed last chapter that ratios are harmonically more fundamental than distances in the sense

176 Chapter 5 of John Chalmers' *Division of the Tetrachord* gives a detailed account of many kinds of metrics and classification schemes used to analyze and characterize tetrachords.

that they provide distances with an intervallic characterization, function and harmonicity measure, and that failure to consider this fact leads to confusions in harmonic theory. We also referred to distances as providing fertile means of generating materials (i.e., Xenakis' sieves). To cross from ratios to distances is straightforward using logarithms, but to traverse the opposite direction, from distances to ratios, is not so trivial. Points in pitch space are harmonically neutral until they they are given meaning, with many ratios lying close to any point, their choice depending on criteria which vary according to the task.

Rationalization is the name given by Barlow to this process. It involves tolerance, metrics, context, an implied harmonic system or aim and, in the last instance, educated choices. He draws a list of candidate ratios lying close to the sought pitch distance. This list is generated by a combination of the prime powers in the harmonic space coordinates, excluding those that lie outside the tolerance range and those whose *harmonicity* value is below a predefined minimum. He came up with a formula, implemented in *DissonanceLib*, which calculates the highest prime powers below which the generated intervals are guaranteed to have a minimum harmonic intensity. Once this list is obtained, the intervals inside the tolerance range are damped with a Gaussian curve in order to give prominence to intervals lying near the middle of the range and those with high *harmonicity*. The alternatives are then chosen according to the task and context at hand. If the rationalization is done to a set of pitches then *specific harmonicity* is used to aid in the selection of the strongest candidate sets out of the many combinatorial constellations that arise[177].

DissonanceLib takes a simplified approach to rationalization by reading from tables of intervals generated using Barlow's combinatorial method as implemented in his JST program[178]. The choice does not involve a Gaussian damping curve, but is instead made by picking the interval with highest harmonic intensity within the given tolerance range. The implementation could be refined in the future to include the damping, but this is not a pressing need as it produces adequate musical results.

3.1.6 Visualizations, navigation

"Geometry is the art of reasoning well from badly drawn figures." (Henri Poincaré[179])

We can proceed at this point with a practical discussion into the topics seen so far. Even though it is used for building more complex systems, *DissonanceLib* can also be used as a harmonic calculator of sorts, in the style of interactive sessions in interpreted computer languages such as *SuperCollider.* I have written a few pieces based on the information provided by these sessions, basing the composition process on paper and pencil elaborations of intervallic tables, analyses and graphic visualizations calculated with the library. The point of this section is to further, converge and elucidate the musical usefulness of the ideas presented so far.

The `HarmonicVector` and `PitchSet` classes have just been presented, while dissonance curves were reviewed in the previous chapter. The central class in *DissonanceLib* is the `Dissonance` class that

177 Barlow, C. (1987). Two Essays on Theory. *Computer Music Journal*, 11(1), 47-53. This process is also explained in detail in Barlow, C. (2012). *On Musiquantics.* University of Mainz: Musikwissenschaftliches Institut Der Johannes Gutenberg Universität. Also see Gräf, A. (2002). Musical Scale Rationalization – a graph-theoretic approach. *Bereich Musikinformatik & Medientechnik*, 45.

178 Barlow C. (1986-2002). *Autobusk* [software]. Last retrieved May 23, 2012, from http://www.musikwissenschaft.uni-mainz.de/Autobusk . The program calculates all the combinations of primes that produce intervals more harmonic than a specified minimum. Five tables are included in *DissonanceLib,* according to their minimum *harmonicity.* They are made from minimums of 0.03, 0.035, 0.04, 0.045 and 0.05, each spanning 9 octaves above and below unison. Other values are also possible, but these ones are practical for most applications.

179 Quoted without attribution by Stillwell, J, translator, in his introduction to Poincaré, H. (2009). *Papers on Topology.* (J. Stillwell, Trans.). Last retrieved June 10, 2012, from www.maths.ed.ac.uk/~aar/papers/poincare2009.pdf.

calculates the curves to produce a `PitchSet` made up of `HarmonicVector`s. Once a dissonance curve has been calculated from a spectrum, a scale is obtained from the frequency ratios of its local minima (in decimal, floating-point numbers). These frequency ratios are then converted into cents and given a first preliminary crude approximation into ratios[180]. Afterwards a proper rationalization is performed (the one just discussed, involving tolerance, a harmonic metric and a filtering of the candidate intervals – eliminating ones above a certain numeric size or limiting the prime numbers involved). The harmonic metric of the resulting ratios is calculated and a `PitchSet` generated, partitioning the intervals into timbral and harmonic subsets, each interval in the subset represented as a `HarmonicVector`. Lets follow this with an example. The code,

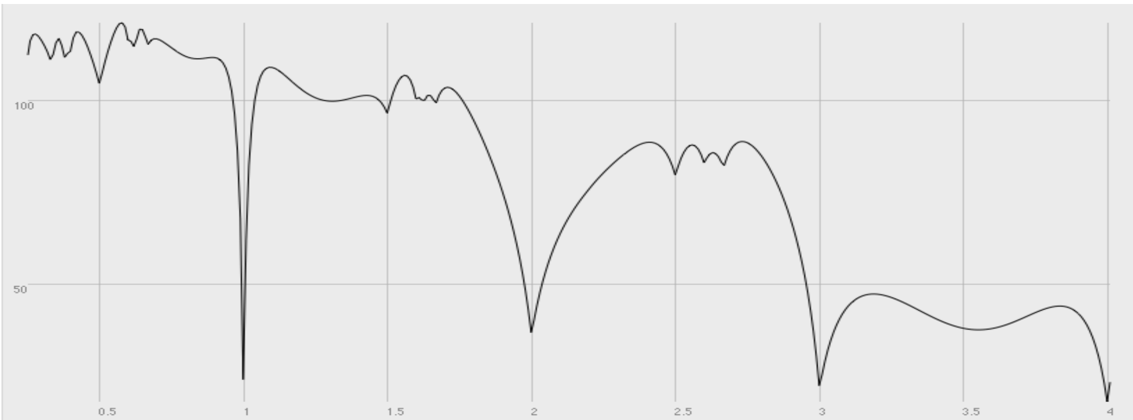
```
a = Dissonance.make([100, 200, 300, 500, 800, 1300], [32, 20, 15, 13, 10, 7.5], 0.249, 4.01);
```

makes a `Dissonance` object (stored in variable 'a') from an invented mathematical spectrum consisting of an array of frequencies (I chose a Fibonacci sequence of frequencies that yields both common and strange intervals) and an array of loudnesses in *sones*. The last two arguments correspond to the range of the analysis, namely (a bit more than) two octaves below and two above unison. The output is:

```
Dissonance( 25/76, 36/95, 1/2, 13/21, 2/3, 73/87, 1/1, 72/55, 3/2, 8/5, 101/62, 5/3, 2/1, 5/2, 13/5, 8/3,
3/1, 181/51, 4/1 ),
```

showing the first crude approximation of ratios for the curve. In terms of harmony, we see that some intervals are a bit strange, that is, they are not common, and their routes and combinations of primary intervals do not make much harmonic sense, especially 36/95, 73/87, 72/55, 101/62 and 181/54 (73, 101 and 181 being extremely high primes, beyond human harmonic perception). A plot of the dissonance curve shows how these intervals were obtained:

```
a.plot →
```



We can appreciate the many minima and the clear low roughness peaks corresponding to octaves and fifths at frequency ratios 1, 2, 3 and 4. For a full rationalization we type:

```
a.harmonicAnalysis(12, \harmonicity, \size, 64, PitchSet.unisons.dim3.et12[2]);
→ Dissonance( 21/64, 8/21, 1/2, 28/45, 2/3, 5/6, 1/1, 21/16, 3/2, 8/5, 18/11, 5/3, 2/1, 5/2, 13/5, 8/3,
3/1, 32/9, 4/1 )
```

The first argument to the function is the tolerance range, in cents (above and below each pitch, i.e., ± 12¢); the second is the metric (could also be `\harmonicDistance` or `\gradusSuavitatis`), the third is the type of filtering (either `\size` or `\prime`) of the candidate intervals, followed by the maximum size or

180 This is done through the Farey fractions algorithm for converting floating point numbers to ratios. It is integrated into *SuperCollider* (after a proposal I made to its mailing list around 2005), with some added 'hacks' (i.e., changes made to the code to use it beyond its original purpose) made in order to handle rounding errors that are inherent to the floating point mathematics of computers for the correct harmonic interpretation of periodic decimals (so that, for instance, 0.333 is interpreted as 1/3 and not as 333/1000).

maximum prime allowed to pass; finally the unison set used for partitioning. We can check out the pitch set that is created as an attribute of the `Dissonance` object, displaying its timbral/harmonic partitions:

```
a.pitchSet
→ Harmonic Set[ 21/64, 2/1, 1/2, 1/1, 3/1, 8/5, 4/1, 21/16, 5/2, 3/2 ]
Timbral Set[ 8/3, 32/9, 2/3, 13/5, 28/45, 18/11, 8/21, 5/6, 5/3 ]
```

Before continuing with the partitions, let us take a look at the rationalization, observing that the ratios now make more harmonic sense: 73/87 is replaced by a common 5/6, for instance; 101/62 by 18/11, 72/55 by 21/16, and so on. Say we want to avoid using high primes such as 13 and 11, limiting ourselves to prime 7:

```
a.harmonicAnalysis(12, \harmonicity, \prime, 7, PitchSet.unisons.dim3.et12[2]);
→ Dissonance( 21/64, 8/21, 1/2, 28/45, 2/3, 5/6, 1/1, 21/16, 3/2, 8/5, 81/50, 5/3, 2/1, 5/2, 70/27, 8/3, 3/1, 32/9, 4/1 )
```

It is a good idea to broaden the tolerance range in order to allow intervals further apart from the original distances to become candidates and be able to pass through:

```
a.harmonicAnalysis(19, \harmonicity, \prime, 7, PitchSet.unisons.dim3.et12[2]);
→ Dissonance( 21/64, 3/8, 1/2, 5/8, 2/3, 5/6, 1/1, 21/16, 3/2, 8/5, 81/50, 5/3, 2/1, 5/2, 21/8, 8/3, 3/1, 32/9, 4/1 )
```

It is after a tolerance range of ±19¢ that we find replacements such as 8/21 → 3/8, 28/45 → 5/8, 70/27 → 21/8 that make more harmonic sense (smaller numbers, lower primes, closer harmonic functions). There is no general formula for finding the optimal compromise between tolerance and numeric limits to obtain automatic results. This example shows how the process involves musical aims, trial, error and educated choice, as there is no single method to obtain optimal results, depending as they do on musical purpose.

To illustrate a concrete musical purpose, let us see how we can adapt these pitch materials to be performed on a piano, involving further approximations determined by the tuning constraints of the instrument. First of all, the sevens must be filtered out since the 12ET of the piano corresponds well with intervals up to 5-limit. Tolerance should then be increased to allow simpler 5-limit intervals to be included, even if they lie a bit further away from the pitches we are approximating. By trial and error, it is found that simpler ratios appear around ±23¢, a tolerance of close to a quarter of a semitone:

```
a.harmonicAnalysis(23, \harmonicity, \prime, 5, PitchSet.unisons.dim2.et12[2]);

→ Dissonance( 1/3, 3/8, 1/2, 5/8, 2/3, 5/6, 1/1, 320/243, 3/2, 8/5, 81/50, 5/3, 2/1, 5/2, 125/48, 8/3, 3/1, 32/9, 4/1 )
```

The three natural sevenths of the dominant have been replaced: 21/64 → 1/3, a fourth; 21/8 → 125/48, an augmented third (plus an octave); 21/16 → 320/243, a grave fourth (function: 5dM, 5 fifths downward to a Pythagorean minor second 256/243 and then up a major third, 477¢). This happens because these intervals are not really related by octaves but come from approximations to different distances yielded by the dissonance analysis, in this case 466¢. The other strange intervals remaining are 81/50, stemming from an original distance of 844¢, almost a neutral sixth or a 13th partial, having been rationalized previously as a 18/11. These intervals will not be able to be correctly approached by the piano, suggesting other instruments or electroacoustics for the job (this would of course depend on the setting of the music, if the piece were for solo piano, these intervals would not be able to be included).

Notice that the unison set is 2-dimensional (5-limit) and fit for a 12-note periodicity block. The corresponding pitch set is partitioned as follows:

```
Harmonic Set[ 2/1, 3/2, 5/8, 3/1, 3/8, 1/1, 8/5, 4/1, 5/2, 1/2 ]
Timbral Set[ 320/243, 81/50, 8/3, 2/3, 32/9, 125/48, 1/3, 5/3, 5/6 ]
```

We can also seek the corresponding 12ET degrees closest to the intervals in the pitch set:

```
a.pitchSet.asETdegrees(12,5)
→ 12-TET
Harmonic Ratios: 3/8, 1/2, 5/8, 1/1, 3/2, 8/5, 2/1, 5/2, 3/1, 4/1
Harmonic Degrees: [ 7, 0, 4, 0, 7, 8, 0, 4, 7, 0 ]
Timbral Ratios: 1/3, 2/3, 5/6, 320/243, 81/50, 5/3, 125/48, 8/3, 32/9
Timbral Degrees: [ 5, 5, 9, 5, 8, 9, 5, 5, 10 ]
```

This function takes a *n*-TET argument and a prime limit, returning the corresponding degrees for each partition. The harmonic ratios are the common intervals approximated by temperament: unison, major third, fifth and minor sixth. Most of the timbral ones are also common: 4/3, 5/3, 16/9 (as octave reduced identities). The three strange ones, however are conflated into the perfect fourth (320/243 and 125/48) and the minor sixth (81/50), being the ones that cannot be adequately supported by the piano. Using 31ET would solve these ambiguities as no same degrees are shared between different intervals:

```
31-TET
Harmonic Ratios: 3/8, 1/2, 5/8, 1/1, 3/2, 8/5, 2/1, 5/2, 3/1, 4/1
Harmonic Degrees: [ 18, 0, 10, 0, 18, 21, 0, 10, 18, 0 ]
Timbral Ratios: 1/3, 2/3, 5/6, 320/243, 81/50, 5/3, 125/48, 8/3, 32/9
Timbral Degrees: [ 13, 13, 23, 12, 22, 23, 12, 13, 26 ]
```

Returning to the 12ET example, the combination of timbral and harmonic sets yields a total {0,4,5,7,8,9,10} degree set, with a suggested 'division of labor' between {0,4,7,8} and {5,8,9,10}. *Figure 1* displays this in musical notation, in octave reduced versions and according to the registers yielded from the dissonance curve. It also separates the prime mixtures to show some possible treatments and notations.

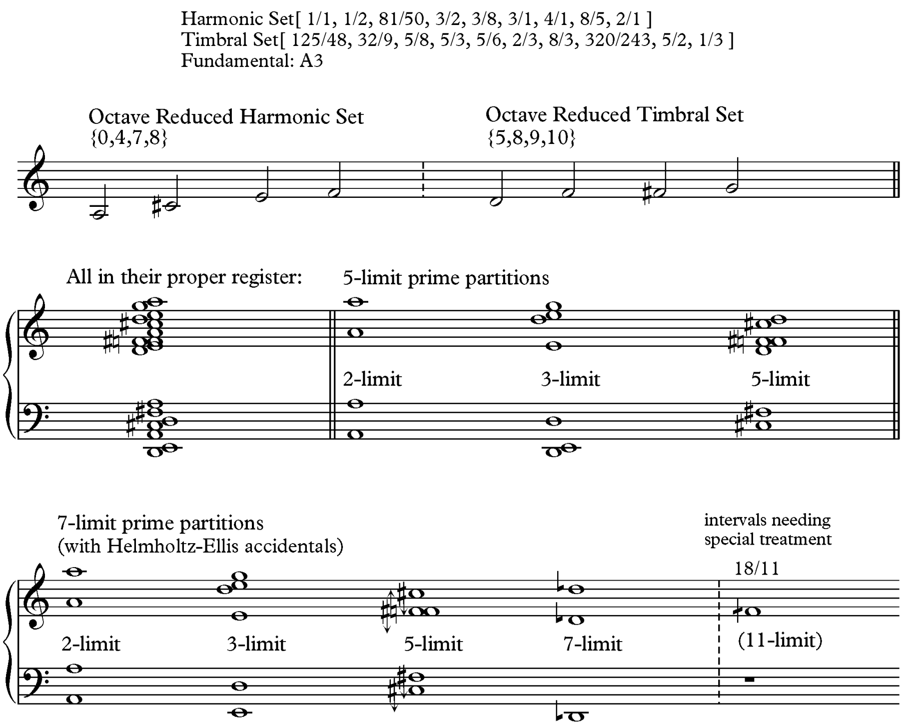


Figure 1. Transcription of the pitch set example into the piano together with prime interval separations in 5- and 7-limit approximations and their corresponding notations.

To display the intervals in a pitch set in various useful representations, for example to aid in the transcription to musical notation in the above figure, the code: `a.pitchSet.asScale`, produces:

Complete (absolute): 1/3, 3/8, 1/2, 5/8, 2/3, 5/6, 1/1, 320/243, 3/2, 8/5, 81/50, 5/3, 2/1, 5/2, 125/48, 8/3, 3/1, 32/9, 4/1 Size: 19

Complete (adjacency): 1/3, 9/8, 4/3, 5/4, 16/15, 5/4, 6/5, 320/243, 729/640, 16/15, 81/80, 250/243, 6/5, 5/4, 25/24, 128/125, 9/8, 32/27, 9/8

Complete (cents): [-1902, -1698, -1200, -814, -702, -316, 0, 477, 702, 814, 835, 884, 1200, 1586, 1657, 1698, 1902, 2196, 2400]

Reduced: 1/1, 5/4, 125/96, 320/243, 4/3, 3/2, 8/5, 81/50, 5/3, 16/9 Size: 10

Reduced (adjacency): 1/1, 5/4, 25/24, 2048/2025, 81/80, 9/8, 16/15, 81/80, 250/243, 16/15

Notes: [[D3, -2], [E3, 2], [A3, 0], [Db4, -14], [D4, -2], [Gb4, -16], [A4, 0], [D5, -23], [E5, 2], [F5, 14], [F5, 35], [Gb5, -16], [A5, 0], [Db6, -14], [D6, -43], [D6, -2], [E6, 2], [G6, -4], [A6, 0]]

Harmonic Set[1/1, 3/2, 1/2, 81/50, 3/8, 3/1, 4/1, 8/5, 2/1]

Timbral Set[125/48, 32/9, 5/8, 5/3, 5/6, 8/3, 2/3, 320/243, 5/2, 1/3]

We see adjacency (the intervals lying between each interval) and absolute (intervals with respect to the unison) versions of the pitch set ('complete' meaning the conjunction of the timbral and harmonic subsets), both in ratios and as distances. We also see octave reduced representations, helpful for isolating the different distinct hues outside their register. Finally, the pitch set in terms of note names displayed with their cent deviations from 12 tone equal temperament.

To visualize the pitch set in harmonic space: `a.pitchSet.plotHarmonicSpace`, yields the graph shown in *Figure 2*.

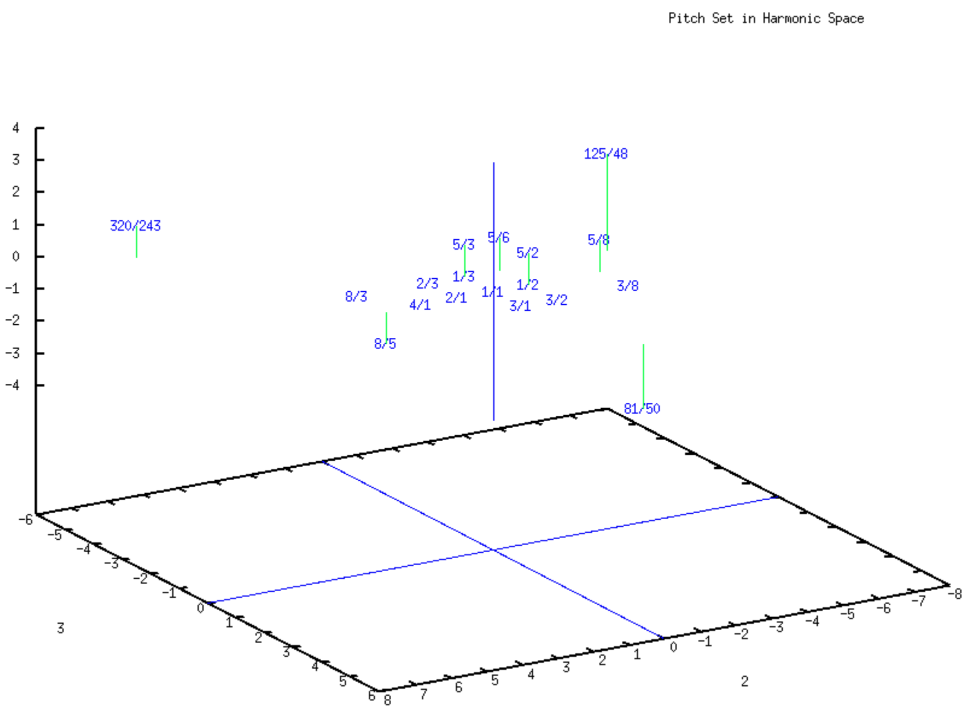


Figure 2. Plot of the pitch set in 2-3-5 space. Each axis is labelled with its generating number, octaves go from left to right, fifths from front to back and thirds from top to bottom[181].

Visualization makes it is easy to spot harmonic features as well as routes and combinations: we can spot the mainland and some of its islands. There are the solitary and far away intervals (the three aforementioned timbral ratios) as well as two close and less lonely islands, one with 3/8 and 5/8, the

181 The visualization was made assisted by the *gnuplot* program, integrated into *SuperCollider/DissonanceLib*. http://www.gnuplot.info/ Last retrieved May 26th, 2012.

other, symmetrical to this one, with 8/3 and 8/5. It is also apparent that the mainland pitch set forms an irregular geometry within its periodicity block. By this I mean that it does not use up all the lattice points contained inside the periodicity block, therefore defining a scalar structure within this tuning. All of these features can be used compositionally, adapted and dependent to the principle behind the music to be made. It is clear, though, that the mainland should be of central importance harmonically and that the different islands can gravitate around, against, or towards it. I will give some examples further below regarding how some of my pieces have tackled some of these structures.

A problem with the previous visualization is that the three dimensions are difficult to grasp and pitch sets with more intervallic dimensions cannot be displayed. To circumvent these problems, a visualization technique known as 'multidimensional scaling analysis', first applied to harmony by Barlow in 2001, is used[182]. I have tried implementing it directly into *DissonanceLib* without having full success, but it is easy to export the data into a format appropriate for one of the many open source programs available for this purpose. The following code writes a text file to be used as input to a visualization program:[183] `a.pitchSet.makeMDSfile("/Users/jsl/plotPitchSet.txt");` →

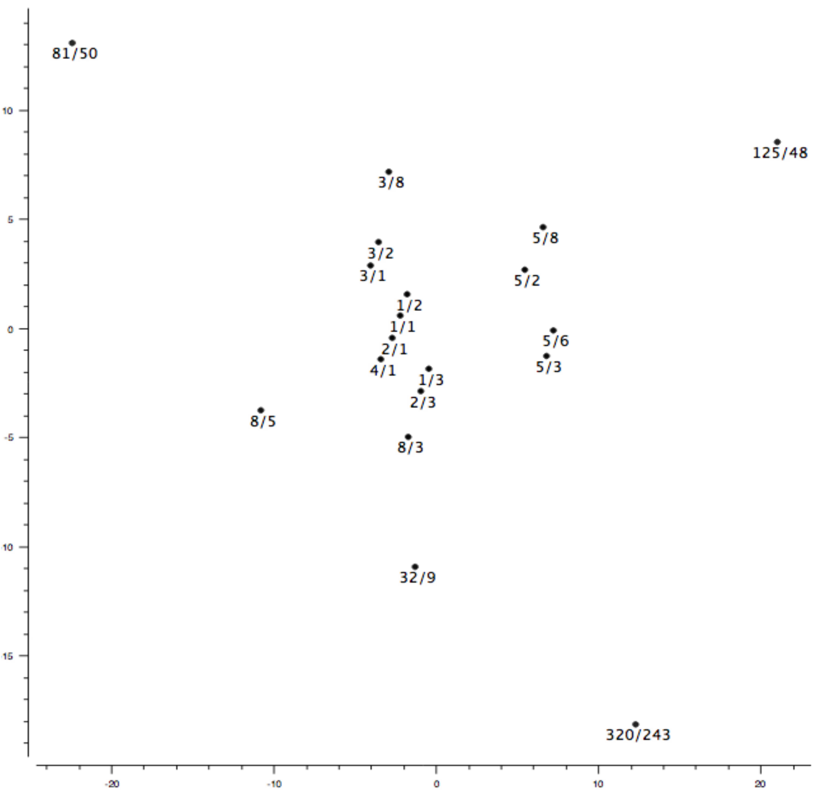


Figure 3. A way to visualize pitch sets, especially when their harmonic spaces have more than 3 dimensions, is through *multidimensional scaling*, a statistical technique that collapses multiple dimensions into lower dimensional projections (for our purposes the projections are performed into a bidimensional plane), preserving the relative distances between the elements. The axes in the graphic are in arbitrary units. Relative distances, aggregates

182 This is a very well known procedure in statistics, with a large body of literature describing different approaches and applications. While working on my homemade implementation (abandoned for being too complex for reinventing the wheel, but left at a point close to working), I read a good short introduction: Groenen, P., van de Velden, M. (2004). Multidimensional Scaling. *Econometric Institute Report*, 15. http://publishing.eur.nl/ir/repub/asset/1274/ei200415.pdf Last retrieved March 9, 2010.

183 The program is called *Orange*, based on the programming language *Python*. http://orange.biolab.si/ Last retrieved April 21st 2012.

and intervallic zones can be observed, as well as symmetries by inversion (between 5/8 and 8/5, for example).

The dimensions (which in this case do not exceed 3) are collapsed into the plane, the relative distance between intervals preserved, allowing a clear view of their positions and possibilities of deployment. Apparent are the relations between octaves (much more clear than in the 3D plot), as in 1/2, 1/1, 2/1 and 4/1 or between 1/3, 2/3 and 8/3, etc. We can also see a direction of 3s (fifths) vertically and a direction of 5s (thirds) horizontally. Depending on the criteria determined by compositional intention, different islands could be drawn around interval clusters, taking into account their symmetric properties, relative distances, and so on. Chords could be made only of members of single islands or, in contrast, be made by combining intervals from each of the different islands (which would likely produce more spicy, inharmonic sonorities). The transitions between the chords can follow a geometric logic drawn from the plot or, by contrast, follow a combinatorial route (all the possible combinations of inter-island-intervals, for instance). We could also have the piano play notes from the large islands and another sound source surround and interfere with it using the timbral notes. There is no doubt that these outer intervals could likely acquire a special role, one which could even be central to the principle behind the music.

Besides these considerations, the pitch set can also be separated according to its mixtures of fundamental intervals: `a.pitchSet.separateIntoPrimes →`

```
List[ [ [ 1, 2 ], [ 2, 1 ], [ 4, 1 ] ], [ [ 1, 3 ], [ 3, 8 ], [ 2, 3 ], [ 3, 2 ], [ 8, 3 ], [ 3, 1 ], [ 32, 9 ] ], [ [ 5, 8 ], [ 5, 6 ],
[ 320, 243 ], [ 8, 5 ], [ 81, 50 ], [ 5, 3 ], [ 5, 2 ], [ 125, 48 ] ] ]
```

This output indicates that the set has been partitioned into the subsets {1/2, 2/1, 4/1}, which only involve a 2; {1/3, 3/8, 2/3, 3/2, 8/3, 3/1, 32/9}, involving 2 and 3; and {5/8, 5/6, 320/243, 8/5, 81/50, 5/3, 5/2, 125/48}, which are combinations of 2, 3 and 5. This suggests other ways of combining and deploying the intervals. Cases of pitch sets whose prime limit is higher that 5 yield separations with more interesting possibilities.

The example we have been following is a relatively simple one, used to expound the applications and theory behind this research. In Appendix III a detailed intervallic analysis of a septimal pitch set is offered that includes coordinates, functions, 53ET approximations and accidentals, classifications, and visualization of its regions with both *mds* and harmonic lattice diagrams. It is built from the logic of the coordinates in harmonic space and is meant as a research into the practical use of septimal harmonies and ways of realizing them with instruments.

3.2 Harmonic Fields

3.2.1 Stochastic uses of pitch sets

After having reviewed the theory and some practice behind harmonic space, metrics, dissonance pitch sets and rationalization, this section will take us to the other principal approach and application of dissonance curves, one much more linked to algorithmic composition, taking us even further away from conventional uses of pitches into a terrain shared with stochastics and textural composition. As in our discussion of the second half of Chapter 2, where we advanced the idea of combining textural and timbral composition with a renewed focus on discreteness and proportionality, this approach combines some of the most forward developments of twentieth century composition together with an emphasis on harmonic properties of intervals. It is here, I think, that many new findings pertaining to my research are situated.

As much as harmonic metrics[184] are useful for analytical purposes such as measuring and classifying intervals, they can also be made the starting point for the synthesis of harmonic materials. Metrics can be interpreted as the probability of occurrence of an interval in a weighted random choice: the more harmonic (or inharmonic) an interval, the more likely it is to be elected. A harmonic field is created when the probabilities themselves are varied in intensity to delimit shades of distinct sonority. These choices can be made both in sequence or vertically to form chordal aggregates and this can also be made to vary, creating a shifting distribution of densities of a granular, textural sort. This statistical approach is consubstantial with the medium of computer aided composition and suitable for large pitch sets such as those generated by dissonance analysis, extracting their various intrinsic harmonic zones and modulating between them. Changing among different sets is also possible, analogous to 'extrinsic' rather than 'intrinsic' modulation.

Harmonic fields are constructed by calculating an inter-interval matrix containing the ratios formed between all pairs of notes in a pitch set. From out of this matrix another one with harmonicities is computed (which can correspond to any metric, not just Barlow's *harmonicity*). This matrix is scaled to vary the discriminating ability of the probabilities, so that at its maximum (or minimum) very few different intervals are chosen. The scaling factor of the probabilities, the field's 'strength', is a continuous parameter ranging from -1.0 to 1.0, demarcating three principal zones: at zero all notes have the same likelihood of being chosen (its 'atonal' region, even if the pitches consist of tonal collections), between zero and one, more and more harmonic notes are chosen ('tonal'), and between zero and minus one an increasing amount of inharmonic intervals are selected (I call this zone 'antitonal': the intervals tend to be harmonic between themselves but inharmonic with respect to the global unison[185]). The field created by these variations of strength (which can be as slow and imperceptible as wanted) can be understood both in the sense of a force field of attraction and that of a plane whose territories can be traversed.

The matrix's data can be interpreted in two manners: by using one column of the matrix at a time, the 'tonic' mode relativizes the probabilities with respect to a single pitch of the set, providing

184 *Harmonicity* and *harmonic distance* are not only distance functions but also metrics in the mathematical sense of the term: they are always positive, zero only if the points they measure are the same, symmetrical: $d(x, y) = d(y, x)$, and satisfy the triangle inequality: $d(x, z) \leq d(x, y) + d(y, z)$. The consequences of this are not relevant for this topic, so this clarification boils down to hair splitting.

185 The probabilities for antitonality are read from an inverse weight matrix which is calculated from the rankings of the harmonic metric. The formula to convert ranks to weights is: $(rank + add)^{pwr}$ the ranking having been calculated from the metric (the 'priority of election' for an interval) is added an offset and then taken to the power of *pwr* (by default *add* is 1 and *pwr* 15). The higher the power, there higher the difference in probability between harmonic and inharmonic intervals.

different 'modal' sonorities (as many as there are pitches in the set); the 'atonic' mode, in contrast, uses the whole matrix at a time: each chosen interval becomes the new tonic from which the next interval is to be chosen. Each strategy produces distinct sonorities. Tonic mode allows modulations between different tonics within which the whole range of variations in strength is available. Atonic mode makes for a wider availability of pitches at each strength position, fluctuating a little even at the poles, where the tonic mode tends towards steadiness[186]. It also differs with the tonic mode in that it has no center of gravity, no tonic, which is why 'back to back' changes between the two modes offer forceful contrasts in sonority (as exemplified in many of my *Logos* improvisations). See the following *Figure 5* for a visualization of the probability matrices.

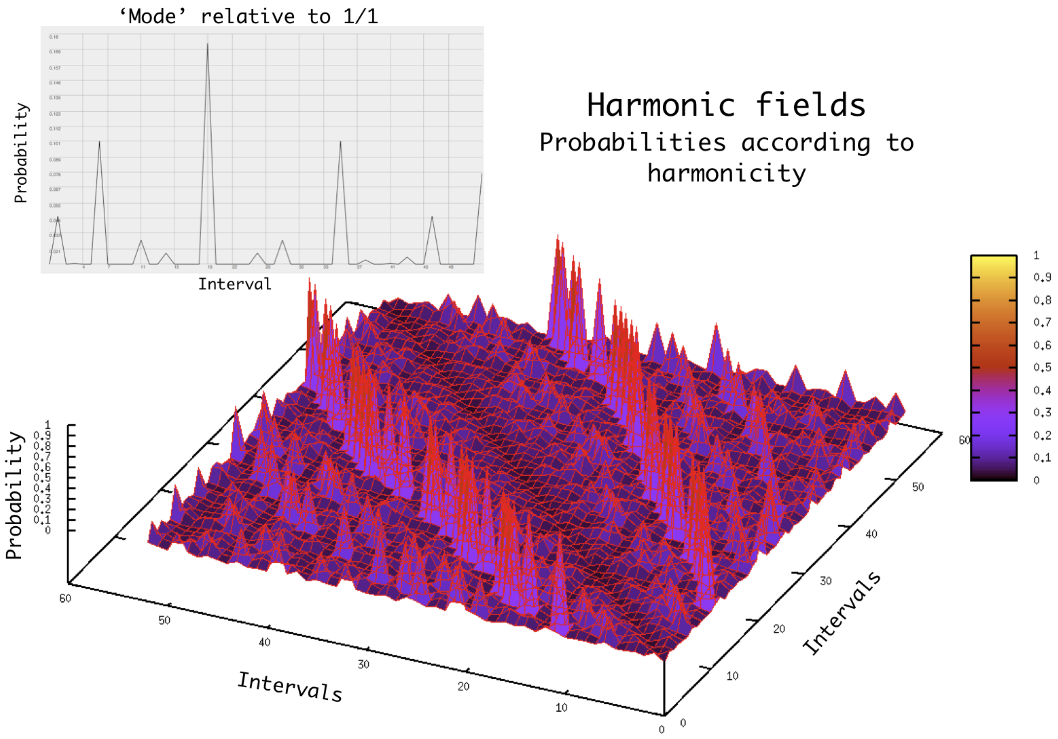


Figure 5. In the lower area of the figure a visualization of the harmonicity matrix for an intervallic set of around 60 pitches can be seen. The axes on the plane represent indices for these pitches, the elevation of a point in the plane showing the harmonicity between two intervals, regarded as a probability. The central diagonal is rendered as zero so that the whole matrix can be seen, because otherwise it would block the view as the intersection of intervals with themselves yield 1/1 and thus the highest harmonicity. This rendering of zero for the diagonal is for visualization purposes only, not happening in the actual matrix used for choosing notes.

The 'tonic' mode uses a transversal cut of the matrix (a column) corresponding to a 'tonic', while the 'atonic' mode uses the whole matrix. In the upper left area is shown one of such transversal cuts, corresponding to the modal zone of 1/1. The figure is based on graphics rendered by *DissonanceLib* (the lower one with *gnuplot*, the upper one

186 Technically, the atonic mode is made possible by applying a Markov analysis of order 1 to the matrix and navigating it by scaling the Markov probabilities. These probabilities pair the likeliness of notes to follow each other, making them behave as though they had a bit of memory. The choices made have some history and the pairings depend on the specific harmonic relations in a pitch set. I have not yet been able to devise how to make Markov analyses of orders higher than one, something which would be suitable for linking the choices to a broader history and hence to a longer range of sonorities.

with the graphic system of *SuperCollider*).

Calculating the matrices for the pitch set takes quite a lot of computation (at least 5 matrices have to be calculated, for a large pitch set this can become a bottleneck in real-time situations), which is why it is not performed automatically after a pitch set is derived from a dissonance curve. The following code generates the matrix of the previous figure, made out of a large pitch set in a dissonance object saved to disk:

```
d = Dissonance.load("even_harmonics_[16,4_oct].dis"); //load a large set
d.pitchSet.makeProbMatrix(15,1);
```

The arguments follow the formula shown in footnote 186. The first one is the power factor that contrasts the probabilities in proportion to the harmonic metric, so a high value will make very few intervals likely, while lower values will permit more pitches at the poles (tonal and antitonal) of the field. The other number is an offset adjustment, usually left in its default value of 1. To plot the field (as in the lower portion of the previous figure): `d.pitchSet.plotHarmonicField;`

Notes are thought of in terms of grains and this is where the other parameters intervene in the harmonic field generator, coupling themselves in various reconfigurable ways. Control of the textures is done through vertical density (number of 'voices' or monophonic streams), tempo, articulation, accents and timbre controls. Strength, mode and tonic supply the pitches while durations provide the rate at which they are chosen. If no specific duration pattern is provided, a single value is repeated. Together with the arpeggiation parameters, providing a minimum and maximum range of random dispersion, it spreads the notes to range from vertical chords to clouds. With more complex durational patterns, the variety of textural possibilities become quite enlarged.

All these parameters can be changed in real time, making the generator an interactive music system. The program can generate either electronic sounds or MIDI notes to be played by other sound generating devices or used for transcription. In the case of electronic synthesis, the timbral constitution of the particles, as well as their duration and the density with which they are unfolded have an effect on the types of aggregates produced. When the density is high and the notes are short, they tend toward fusion, coalescing into timbral streams. I call these 'harmonic timbres', because of their continuously changing harmonic logic. At low densities and with larger particle sizes, the aggregates tend toward fission, being heard as textures with separate components. They are stochastic harmonic textures in their sonority and logic, but could also be called 'timbral harmonies' if they possess a relation to the empirical sound from which they derive, as in dissonance chorales mentioned in Chapter 2 and earlier in this chapter. From the premises of harmonic duality, the harmonic field can be visualized as the intersection of a tonal-antitonal axis, one of consonance-dissonance and a third, textural one, of fission-fusion. (See *Figure 6*)

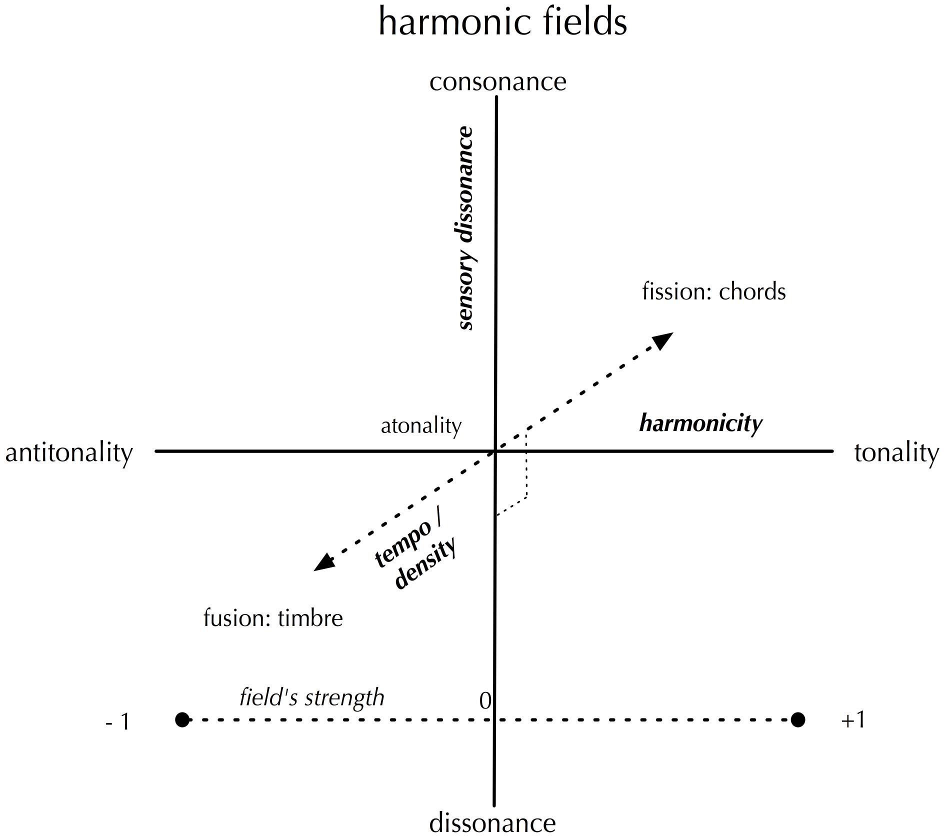


Figure 6. Schematic diagram of the zones in harmonic fields. The horizontal axis concerns harmonicity (proportional harmony), the vertical sensory dissonance (timbral harmony) and the suggested third dimension the axis of 'aggregateness' of fusion/fission of the sonic particles. This field is navigated indirectly on the basis of the parameters of the harmonic fields generator.

This figure not only pertains to the harmonic fields generator, but could be also understood as a diagram that encompasses the kind of harmony we have been theorizing about all along this study. We see its dual aspects as the two main vertical and horizontal axes, each with its own antipodes (consonance/dissonance, harmonicity/inharmonicity). Furthermore, there is the 'aggregateness' axis of texture relating to the distribution in time of the sounds, within its poles of fission and fusion. Depending on its harmonic, timbral and textural properties, one could picture many kinds of harmonies and music as lying within different areas of the diagram[187]. To the left of the atonal center lie the antitonal quadrants (one consisting of consonant, the other of dissonant timbres), the music of which has yet to be produced. Some of my pieces have ventured into this territory, but it is mostly still unknown. Moreover, I think most interesting is to combine musical materials along the four quadrants as well as conceiving both continuous (barely perceptible) as well as abrupt, discontinuous, transitions between the different areas.

187 For example, one could say that most tonal music played with timbres which are close to the harmonic series (that is, most 'classical' Western music played with orchestral instruments) lies along the upper right quadrant, the more dissonant and rough in its orchestrations, the more it will move towards the lower quadrant and to the left (say, something like Varèse or Schoenberg). Dissonant, inharmonic music such as industrial noise would lie in areas along the lower right quadrant. I will refrain from pursuing these musical comparisons further, as they have been made mainly in order to illustrate the ideas behind this diagram.

3.2.2 Navigating the field

The program's principal parameter, its strength, is used to traverse the antitonal-tonal axis – with the added facet of tonic and atonic modes – but the other axes are navigated indirectly. The sensory dissonance axis depends on timbral constitution and cannot be traversed straightforwardly because sensory dissonance and timbre depend on many sonic dimensions. Even if we abstract out attack portions of sounds, the 'stable' portions of the spectra cannot produce a one-to-one correlation with roughness, influenced as they are by non-linear interactions between partials, as we saw when studying dissonance curves. However, materials derived from the curves contain information regarding the roughness measure of each interval, so even if their deployment in the harmonic field has an oblique effect on their perception, they can be ordered according to this information. Also, timbral partitions in pitch sets are to be considered closer to the dissonant pole than harmonic ones. It is easy to make and classify sounds that belong to certain relative regions in this axis (harmonic spectra tend towards consonance and their dissonance is increased by shifting some of the spectral components towards to 25% of a *bark*). Even so, what is most difficult is to modulate continuous timbral transitions between the two poles.

The aggregation dimension involves tempo, particle size and articulation. The fission or fusion of the particles depends on many factors, one of them being their harmonicity. Inharmonic/dissonant notes are less prone to fusion than harmonic/consonant ones. In any case, from a certain speed onwards, the particles tend to aggregate into textures that continuously change according to their position in the field. The program is able reach tempos higher than 1000 beats per minute, putting into motion grain particles of very small size and yielding granular harmonies.

Rhythm is the most independent variable in the harmonic fields generator. Various kinds of rhythmic logics or 'patterns' can be supplied to the program, as it is well-suited for 'plugging' code into the part of the program that generates the durations, also having access to the information contained in the pitch set as well as to the notes currently being played. In this way the rhythmic logic can be coupled to the harmonic behavior. One approach has been to make duration congruent with the pitch ratios *à la* Henry Cowell and Stockhausen. Durations can also be based on the fundamental primes of the ratios, as well as being filtered, transposed and inverted (simpler ratios have more complex durations and vice versa). Another strategy is to couple duration to harmonicity (longer/shorter according to the individual notes' harmonic measure) or to the strength parameter. They can also be independent from the harmonic state but linked to timbre or dynamics. The arpeggiation ('strum') controls give an added flexibility to these textures, compacting or rarefying them. Any arbitrary rhythmic pattern can be supplied and there is a control for accenting/de-accenting notes according to their harmonicity, something which gives the textures a metric quality. Various simultaneous streams can be generated, each with its own rhythmic patterns, timbres, densities, etc, but all coordinated by the strength of the harmonic field.

The harmonic field generator, *Harmonic Fields Forever,* is built atop the `Dissonance` and `PitchSet` objects as an application that generates patterns in stochastic harmonic fields, coupling and controlling the interaction between its parameters. The arbitrary coupling between controls is also done by 'plugging' code where their behavior is defined. The first piece composed with this program was *Circular Limit* (2008), for amplified bass recorder and live electronics, based on a single dissonance pitch set obtained from a recording of a low G tone of the instrument. The first half of the piece presents the material with a 'tonic' logic, generating melodic sequences that traverse the tonal-antitonal axis of each of the possible modes constituted by the pitch set. The computer part accompanies the recorder by generating in real time the same kind of harmonic journeys but providing a wider *ambitus* of notes, polyphony and timbres, deployed in various vertical/horizontal combinations. A harmonic journey was improvised and transcribed for every modal degree, making for eight sections which were are interspersed with 'timbral interludes' making use of extended recorder techniques and intensified by computer processing of the instrument (through vocoders tuned to the dissonance pitch set). In contrast, the second part of the piece is a navigation through the field in 'atonic' mode, transitioning slowly from the tonal to the antitonal poles. The computer and the recorder are allied by having the former generate in real time bass lines and chordal accompaniments for the latter.

Following this piece, the next music done with the program consisted in 'algorithmic improvisations' with the musical automats of the *Logos Foundation* (in two weeks of sessions taking part in 2008 and 2009). Many possibilities and extensions to the program were pursued in an improvisatory manner, taking advantage of the possibilities offered by plugging code to reconfigure the systems behavior and made according to the musical requirements of the moment, as part of a day's development. Due to the nature of these live coding sessions, where many portions of code are replaced and transformed on the spot making impractical to reconstruct the strategies after the fact, many developments did not leave a compositional trace apart from the music produced by them at the moment.

Simultaneous independent layers were pursued (sometimes a different one for each automat), new pitch sets developed (adapted to the instruments, especially the quarter tone organ and xylophone) and partitioned – timbral/proportional, according to prime mixtures, by register or by harmonicity regions – allotting them to different instruments, each with its own independent rhythmic behavior. Each automat can be coupled to a separate pitch set and harmonic field, or it can be part of a global shared field. The interactions that occurred produced interesting and unforeseen effects, as when opposing fields against each other, one being the antitonality of the other – I called these contrasting pitch sets 'nemeses' – or proceeding from different pitch sets.

Many of the textural and instrumental variations in density were controlled by way of dynamic thresholds depending on the disparity between accented and non-accented notes (themselves dependent on strength and harmonicity, with antitonal inversions too) when crossed by the offset parameter, permitting only certain notes to pass through and diversifying the textures and instrumentations. The generator also has a variable tempo which goes from 10 to around 720 beats per minute. Nowadays, with computers running faster, it should be possible to go even faster, although the nature of the MIDI control of the automats, this is already close to the mechanical limits of the instruments.

These algorithmic improvisations were made both through discovery and by controlling variables, many things emerging interactively, the music not traceable back to what was done. I gathered many hours of material, including at least an hour of improvisations that function as complete pieces, as well as many fragments to be used for other purposes. The generator has since also been useful for generating sections of either electronic soundscapes or instrumental score by transcribing the output of the field generator into notation programs. More details regarding these and other harmonic strategies pursued in my pieces will be given in the next chapter.

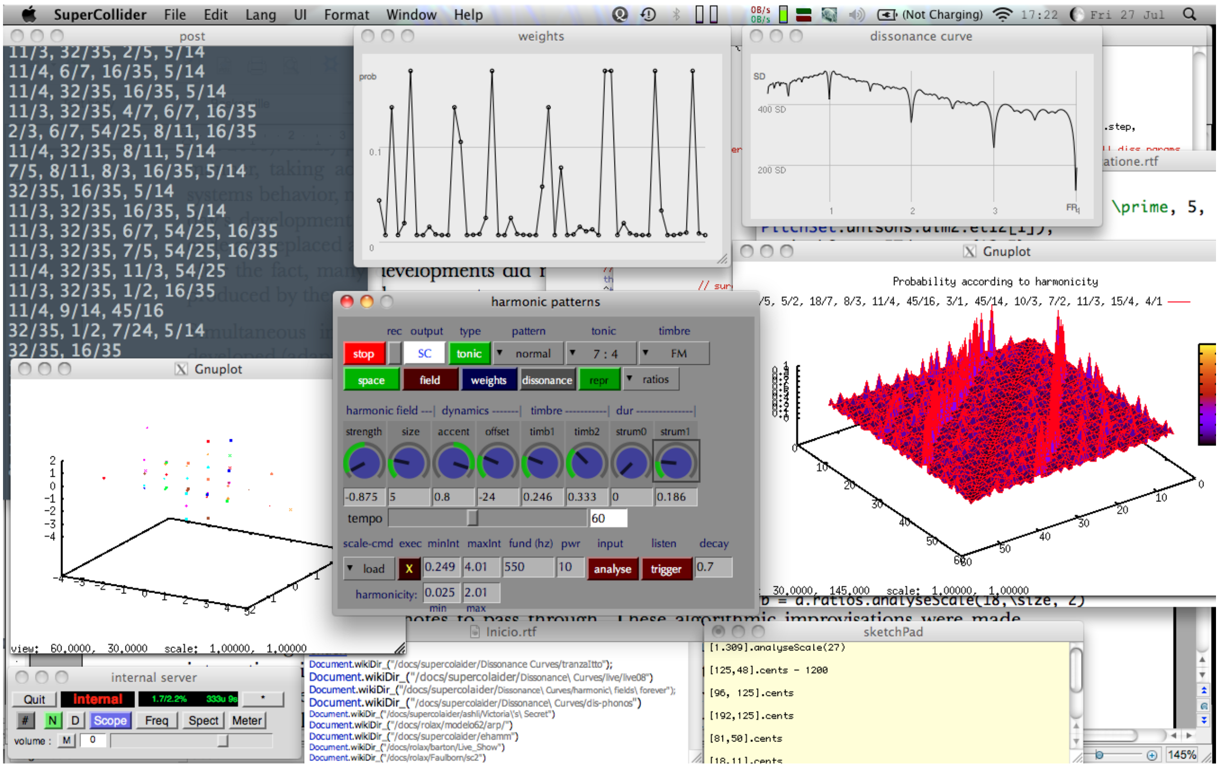


Figure 7. A display of the harmonic field generator showing its main controls and some visualizations. The generator has 8 controls (seen as knobs) that control the harmonic field parameters (the strength of the field, the number of simultaneous streams), dynamic couplings (accent, offset), generic timbre parameters (affecting the sounds synthesized, but not relevant when the output is MIDI) and arpeggiation limits (strum0 and strum1, lower and upper random bounds for the dispersion of the particles). The upper buttons serve to play/stop the generator, change the output from MIDI/synthesis, the type of patterns, modal degree and kind of synthetic timbre. Below are the controls for visualization and in the lowest section the tempo and the parameters to generate dissonance curves from the sound input. Above the generator we can see the probabilities relating to the current modal tonic (which is 7/4). Next to it is the dissonance curve from which the pitch set is derived. The dark window with white text at the left displays the intervals currently being generated. There are also two visualizations, one for harmonic space (below the text window) and another showing the probability matrix (to the right of the screen). Both visualizations are of the same kind as the ones shown in previous figures.

3.2.3 The field in terms of form: structure and morphology

> "Durational proportionality exists on a note-to-note scale, on a beat-to-beat scale (since beats have proportions other than equality), an a measure-to-measure scale, on a phrase-to-phrase scale, on a section-to-section scale and even on a movement-to-movement scale" (Ben Johnston, *Scalar Order as a compositional resource*[188])

The examination of harmonic fields leaves us at the shores of the topic of form in relation to harmony. Tenney, incorporating ideas from Gestalt and phenomenology into music, discussed form

188 Johnston, B. Scalar Order as a compositional resource, *Maximum Clarity*, 15.

as comprising the facets of structure and morphology[189]. Morphology is the variation in time of an attribute of sound expressed as *contours* or profiles ('forms'), while structure corresponds to both vertical and horizontal *relations* between sound configurations. Shapes and structures bring forth relationships between parts and wholes, implying a series of forms (in both senses of the term) embedded at many levels of scale. These scales comprise three principal musical 'zones' (each possibly containing several levels): that of 'materials' (the basic ingredients and textural elements), 'methods' (or what I would term 'logics', larger scale relations between those primary organizations) and 'aesthetic experience', what is properly regarded as form (the processes, concepts or narratives that happen at the level of large scale sections or complete pieces).

Each zone could be seen as a perceptual phase transition, encompassing a regime of musical perception with its own characteristics. The larger the forms, the more memory and differentiation, rather than perception and integration, are involved. There can also be sub-element nano-forms and even longer times scales pertaining to the collective, historical, and cultural levels of situations, concerts, styles, genres and epochs, but that is not relevant here.

One of the aims of this study is to expand this conception of form and blend it with notions derived from harmonic duality. The morphological aspect involves continuous phenomena while relationships between discrete parts and wholes are involved in structure. As in harmonic duality, these two aspects are intertwined. This analogy in not merely coincidental but shows how this tension is carried out at all levels of embedding. Harmony as usually understood happens at the time scale of 'materials', timbre being understood as micro-temporal continuous changes, proportion corresponding to discontinuity between pitch relations formed out of the timbral flux.

We have seen in Chapter 1 how some tensions in harmony are 'inherited' to the level of rhythm, how some harmonic principles operate within it, correlating with its continuous and proportional aspects (even if they are qualitatively different, not perceived nor behaving in the same way as pitch). These perceptual discontinuities are founded upon continuous physical processes. I propose to interpret Dennis Gabor's uncertainty principle, originally created for understanding communication of information, as applying to perception: the constant k in his formula $\Delta f \cdot \Delta t \geq k$ delimiting phase transitions at specific thresholds of pitch and time. The dualities we are discussing (harmonic duality and Tenney's morphologhical/structural duet) map to this formula: f corresponds to morphology and t to structure, meaning that within a perceptual regime, morphological and structural intervals are integrated into units larger than a minimum size, a size determined by the regime's constant k. A consistency of behavior is contained within the bounds of the minimum size and those that define the minimum size of the following phase transition (perhaps with a grey area between them).

It is productive to think some aspects of rhythm in terms of dissonance/consonance while pitch can be thought of in terms of pulsations, recurrence, accents and syncopation. We are trying to extend the notion of harmony to encompass the various time frames of sequences, sections and large scale forms, harmony understood as a concern for relationships in abstract terms of continuity/discontinuity, flux/stasis, multiplicity/unity, source/pattern, and so on, each with its own poles corresponding to order/disorder, coherence/incoherence, stability/instability, fission/fusion, homogeneity/heterogeneity, similarity/dissimilarity, and so on.

Tenney's approach is also interesting in that it accounts for form and content as being constituted out of the same perceptual phenomena but at different scales: forms at one level are the content for the next one, not differing in their fundamental makeup. Forms are not ontologically different from

189 Tenney, J. (1969). Form in 20th century music. Last retrieved May 29. 2012, from http://www.plainsound.org/pdfs/Form.pdf. He also deals with this topic in his longer study Tenney, J. (1986 [1964/1975]). *META/HODOS and META Meta/Hodos.* Lebanon NH: Frog Peak.

contents (materials), but forms at one level emerge as contents at the next, from which new forms (contours or profiles) are manifested, to be integrated into contents at the following level. Timbre at one level is the contours of partials and (micro) sound fluctuations, at the next is the content for the forms and relationships between pitches. These embeddings continue through higher levels pertaining to phrases, motifs, textures, sections, and so on.

From this standpoint we can imagine a two-axis graph for form similar to the one of harmonic fields. Vertically it represents morphological conditions, horizontally, structural relations. The poles of the axes would then have an analogy to consonance/dissonance, as mentioned above: a degree of similarity/dissimilarity between shapes, or the heterogeneity/homogeneity (or any other measure of variety) between structural parts. Two Tenney associates have pursued further his research into form by offering insightful work on each of its two aspects. Larry Polansky, with his morphological metrics[190], provides many methods to measure the similarity/dissimilarity between profiles at any level of scale. Michael Winter applies this to structure, furnishing techniques to estimate the degree of randomness in structural relations, offering ideas (stemming from the mathematics of computation) of what is to be considered a minimal description of a structure as applied to musical logics[191].

A fully developed investigation into this type of formal harmony is outside the scope of this work. What we wish is to draw out connections for further study, to point at how the notion of harmony can be extended further into form while also passing back into (pitch- or rhythm- based) harmony some aspects of form. Each level behaves according to its own specific laws, patterns not translating identically between levels, emerging as different kinds of forms and contents when transposed. Stockhausen's idea of transposing one level directly (isomorphically) into another is not enough: the change in properties of each regime must be taken into account. The point is not to make a strict parallel, but to show some invariant abstract connections, not pertaining to content (to 'musical objects'), but to formal relations as such (to correspondences between these objects). Each level brings with it its own content, itself emerging from forms on a lower level of scale, and there is no level that acquires more importance because of being smaller and more 'fundamental' or for being larger and more conceptual or cultural. Each level is as genuine as any other and possesses its own logic and characteristics[192].

One way to understand harmony is not as pre-established, *a-priori* relations, but as the concern for *types of relations* as such, as in one of its original Greek meanings, where harmony is meant as 'interlocking'.

Anyway, now that we have made the connection, it must be completed. In addition to morphology and structure, Tenney talks about an additional factor that determines form: that of statistics or 'state', the average behavior in time of the sonic configurations, their density (vertical, horizontal), size, range, which corresponds to the aggregation dimension we have already discussed. This brings about the possibility of accounting for layers of texture and simultaneity (heterophony, holophony – in the sense of various kinds of simultaneous textural streams and their relations). Texture and its relation to harmony and form is a very interesting subject, left for future undertakings. Here we only a hint at the territories it opens up.

This work is mainly in the service of composition, so these abstract, relational spaces are meant to

190 Polansky, L. (1996). Morphological Metrics. *Journal of New Music Research*, 25(4), 289-368.

191 Winter, M. (2010). *Structural Metrics: An Epistemology* (Doctoral dissertation, University of California, Santa Barbara, 2010).

192 Still, *clangs* have a perceptual priority as the main aural gestalts as they pertain to the perceptual present. They are called 'strong *gestalts*'. My point, though, is that no level is more fundamental in the sense that it produces or explains the others. See Miraglia, R. (1995). Influences of Phenomenology: James Tenney's Theory. *Axiomates,* 2, 273-308.

be useful not so much as analysis tools, but for synthetic purposes, for compositional approaches to relationships at several levels of scale. *Figure 8* schematizes these perceptual regimes in harmonic, formal and perceptual terms, mapping a span of around 22 octaves within which these forms and contents reside[193].

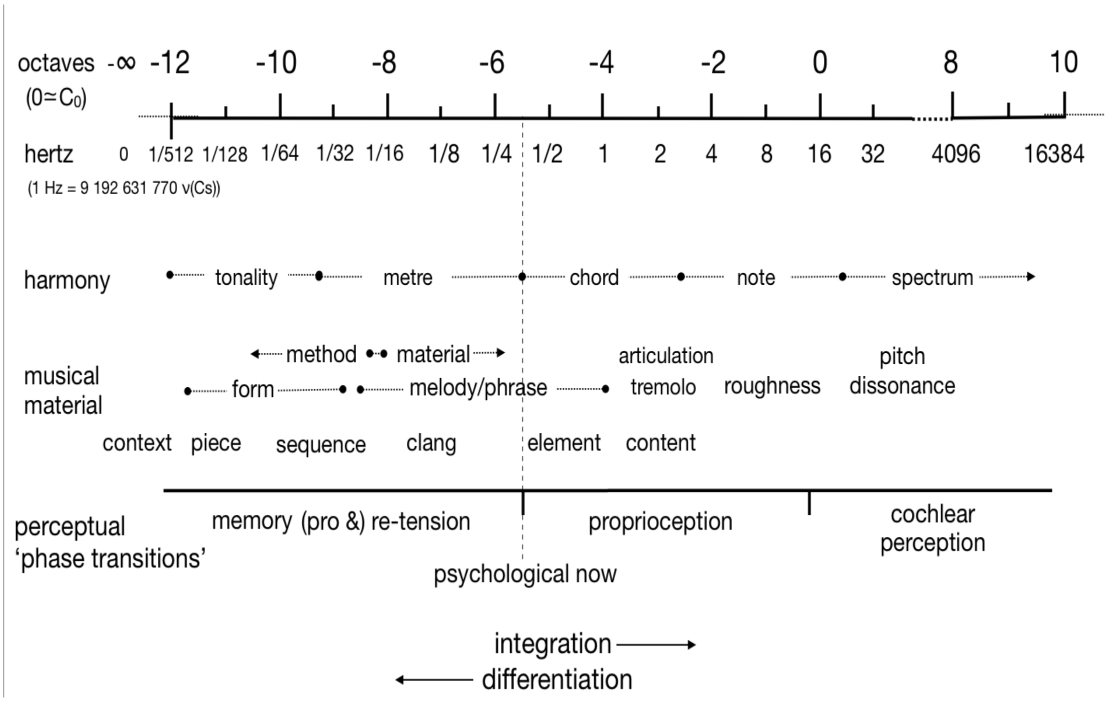


Figure 8. Formal and rhythmic analogues of harmony. The first horizontal line is the time scale expressed as frequency of vibration in Hertz and octaves. The second line displays harmonic analogues: from spectra (timbre) at right towards the separation into notes, chords, meters and tonalities. Below they are shown in terms of musical materials, from compositional technique toward pitch, from form to dissonance, also presented in terms of Tenney's formal regions: element, clang, sequence and piece. Below that, regimes of perception are shown, from cochlear hearing to proprioception and memory. Toward the right correspond processes of integration, to the left, of differentiation.

To conclude, I designate the main levels and their formal/musical regimes with respect to harmony as follows. This incorporates Tenney's levels together with harmonic duality to arrive at terms which appoint the levels according to their continuous/timbral and proportional/discontinuous aspects:

	timbre	proportion
material	*sonos*	*eidos*
method	*morphe*	*logos*
form	*drama*	*nomos*

In terms of proportion (discreteness) and timbre (continuity), at the *micro* level of materials I propose the terms *sonos* and *eidos*, the latter having been discussed as the formal causes of

193 Curtis Roads develops the topic of the times scales of music in depth. See Roads, C. (2001). *Microsound.* Cambridge Massachusetts: The MIT Press, 1-42.

proportions in sound, the former belonging to the sounding, sensory aspect. At the *meso* level of 'method' (rhythm, textures, clangs) they become *morphe* (shape, profile) and *logos* ('pattern' or logic), the former being the continuous aspect of form and the latter the way the smaller forms are structured at the *meso* level. At the largest level of form is the continuous *drama* (or lack thereof, which is also a kind of neutral drama or anti-narrative) together with *nomos* (law) the (apparent) principle governing the aesthetic experience of the piece. This might seem as an arbitrary game conjoining Greek words with continuity/discontinuity and the levels. I agree that these points may not further the science of musicology very much, but for the aim of synthesizing and composing, I think it can provide useful points of reference that hold a harmonic perspective at different hierarchies of perception. Future work will expand them to incorporate the Aristotelian causes together with object-oriented philosophy so that they are for now only part of a further-reaching research into abstract properties of dual harmony at different levels of scale. For further details, consult the terms in the glossary.

Chapter 4

Practical and speculative harmony

4.1 Some harmonic strategies

4.1.1 Harmonic logics of Tenney, Barlow, Johnston, Novaro and Wilson

Having presented most of the research done so far by visiting harmonic space and the language of proportionality seen under the hypothesis of harmonic duality, we can take a more panoramic view that pushes into compositional territory by reviewing and pursuing some tuning, system and navigation strategies.

When it comes to microtonality, Tenney, like most composers belonging to the tradition of the American Mavericks (Ives, Partch, Harrison, Johnston, Young would be also included here, other experimentalists are not so relevant for harmonic microtonality) accepts, as it were by decree, higher primes as harmonically assimilable. This is partly because of an experimental acceptance of sonic speculations, each departure point suggesting further experimentation and pieces. On the other hand, by pursuing different harmonic approaches throughout his *oeuvre*, we see various tendencies in setting materials into motion, with several extents and perspectives that depend on aesthetic effect. He uses pitch systems based on subsets of harmonic series which could be called 'spectral' in that they engage in chains of fundamentals and tend towards timbral outcomes (*Clang*, *Quintext*, *Spectral Canon for Conlon Nancarrow*, the series of *Harmonia*). Combined with his minimalist, anti-narrative, perceptual aesthetic that slowly reveals austere ideas and patterns, it makes a music which quite different from the French spectral movement of those same years. He also works with difference tones as generators of harmony in *Koan for String Quartet*, a harmonization of a long, slow glissando, that weaves proportional (micro) harmonies together with a pitch-distance logic, falling in and out of harmonicity as the different nodes in pitch space are traversed: similar to the rhythmic in and out of phasing of early Steve Reich's *Piano Phase* but rendered in the harmonic domain. Other logics include the use of harmonic means in *Critical Band*, a long, sustained and slow transition from unison, beatings, roughness and timbral towards proportional harmony made possible through harmonic divisions: as the interval of division grows larger the timbral/proportional borderline is crossed at the critical bandwidth, when the timbral mixtures clarify into proportional stabilities. The last approach is that of different routes for crossing through harmonic spaces (*Bridge, Changes: 64 Studies for 6 Harps*, the series of *Spectra*, *Arbor Vitae*), all of these quite complex algorithmic works that bring together his interests in gestalt theory, ergodic form and harmony[194]. It is in these last category of works that he takes tolerance more into account.

His harmonic aesthetic can be divided into two strands: that which tends towards consonance, limiting the proliferation of primes (the *Harmonia,* for instance) and that which is highly experimental, privileging inharmonicity and high primes (*Spectra*, *Bridge*, *Changes*, *Arbor Vitae*, or

194 This is not an exhaustive list of works nor an attempt to characterize of all of Tenney's music. I am leaving much his output aside (such as his percussion, processual or equal tempered pieces), mentioning only the pieces which I think are important for delineating harmonic materials and logics. For more details on his microtonal music I recommend Wannamaker, R. (2008). The Spectral Music of James Tenney. *Contemporary Music Review*, 27(1), 91-130. For details on all his music previous to the early 80's, see Polansky, L. (1983). *The Early Works of James Tenney.* In *Soundings,* 13, 119 – 297. Last retrieved June 7, 2012, from http://eamusic.dartmouth.edu/~larry/published_articles/tenney_monograph_soundings/index.html

Critical Band which transitions between the two aesthetics). *Changes* is particularly interesting in that he explores the harmonic space of 72-ET parametrically, in a stochastic spirit, following an ergodic logic[195] in harmonic space, a logic which is also taken over to the time scale of texture and sequence, pursuing sound configurations based in terms of temporal, dynamic and vertical densities, all in a state of continuous transition made possible by algorithmic interpolation between discrete states. The constraints and textural guidelines are supplied by an hexagram corresponding to each study, the aim being to achieve a maximum of variety of parametric states within a multi-level 'holarchy' and a multi layer approach (some studies are 'monophonic' some 'polyphonic'). The harmonic logic also has an additional modal comportment that includes various tonics and limits the pitches at a time to scalar subsets of 72-ET[196].

Elsewhere he proposes algorithms to build compact configurations in harmonic space through a growth path incorporating pitches that minimally increase the total harmonic distances of the set. This follows a crystallographic metaphor, some sort of chemical logic of least increase. The approach of *Arbor Vitae* is an organic metaphor of descent (usually he rises) from the high branches of a large tree (as high as harmonic 1300, octave reduced) all the way down to the root. The branchings consist of the harmonic series of each new prime, and they connect between themselves through the coinciding primes which are both roots of their branch and harmonics of some lower branch forming a mesh which, when the angles between primes are accommodated to be less than ninety degrees, looks very much like an arborescent structure. Locally, the branches and their leaves are harmonic amongst themselves though the higher they are in the tree, the more inharmonic they are to the main trunk and lower branches, something very similar to what happens with regards to antitonality in my applications.

The case of Ben Johnston is quite different as he evolves from a just-intoned atonality passing through different phases of harmonic and stylistic experimentation. The main pieces I refer to are the string quartets, perhaps his main playground for microtonal development. His approaches almost always involve some use of variation technique in which the very tuning or fundaments of the harmonic materials are varied. The fourth quartet is a series of variations on a folk tune, each variation involving different tunings together with the rhythmic and metric analogies of these ratios. The fifth is also a variation on tunings and a folk melody but this time with a focus on the 13th harmonic, sometimes juxtaposing different tunings against each other and effectively conveying the sonorities of the thirteenth harmonic as stabilities, even if very alien ones. His middle quartets, 6-8, explore further into unknown territory, being the most daring, especially the 7th which has been called the 'Mount Everest' of string quartets[197] (the first recording of it will come out around 2013, so I cannot comment on it yet). The 6th is a sort of expressionist quasi-atonal world in what seems like 11 or 13-limit intonation in a style reminiscent of early modernism, Silvestre Revueltas and Béla Bartok come to my mind. Quartets 9 and 10 on the other hand explore a speculative style that imagines what Classical music might have been had it not taken the 12-ET route, instead exploring extended chromatic harmonies. These spring from his theoretical research into ultra-chromatic spaces in the lattice, in which the small patches that define common tonality are expanded so that modulation can encompass more chords and tonalities than usual. Ultra-chromaticism can be extended in several dimensions, starting with 5-limit, in which classical Western harmony is expanded without introducing sonorities related to alien primes but by building chords on the

195 Ergodic behavior is, according to Tenney, a statistically homogeneous distribution of a sonic parameter over some structural time frame. In this case it means a constrained random movement through harmonic space. See his article on Form: Tenney, J. "Form In 20th century music", http://www.plainsound.org/pdfs/Form.pdf Last retrieved May 29th 2012.

196 Tenney, J. (1987). About 'Changes: Sixty-four Studies for Six Harps'. *Perspectives of New Music,* 25(1/2), 64-87.

197 See Gann, K. (2010). The Mount Everest of String Quartets. [weblog]. Last retrieved June 6, 2012, from http://www.artsjournal.com/postclassic/2010/03/the_mount_everest_of_string_qu.html

intermediate degrees and exploring tonalities not reachable in 12 ET. The other, more experimental approach includes both this expansion as well as the introduction of new primes. Johnston's strategies are more proportional than timbral, rarely using relations which involve a separation of steps in the lattice, instead following close-knit chains of relationships and without avoiding traditional connotations but pursuing them into unknown territory. Theoretically he makes a distinction between melodic and harmonic uses of harmony in order to produce scalar frameworks that are made simultaneously in harmonic- and distance-space by filling in each diatonic degree with prime-limited mixtures of intervals and doing this symmetrically within intervallic equivalences. He does not distinguish between consonance and harmonicity, his duality being more a horizontal/vertical rather than a timbral/proportional one[198].

Clarence Barlow's approaches are quite varied although not pursued for more than a single piece or groups of pieces normally. His timbral logics have reached into phonetics, with *Im Januar am Nil* (1984) being the one where he follows this path to the fullest extent, making musical patterns out of phonetic spectra. This has been taken further towards what he calls 'synthstrumentation' and 'spectastics'. The former is the use of spectral information used for instrumentation, some kind of electronic synthesis with acoustic instruments – as in the case of *Im Januar am Nil*, but there are quite a few other pieces involved, such as *Septima de Facto* (2007) and sections of his widely embracing orchestra piece *Orchidea Ordinaria* (1986). The latter method is the use of spectra as statistical probabilities for composing granular textural behaviors (used in conjunction with the other method). Other timbral approaches can be appreciated, as in *Approximating* π (2007), an electronic piece made out of a single overtone series in square waves, where the amplitude of each of its ten partials changes according to the 'Newton approximation algorithm' for obtaining the digits of π, creating a static harmony with an extremely fast internal movement that slows down into a stable timbre (taking 76 minutes to come to a standstill). His proportional developments include long, imperceptible transitions within an harmonic field, such as the different versions of *Until* (1975-78), each consisting of a melody against a drone which slowly changes from being consonant to dissonant, with an almost unnoticeable effect, the version for piccolo having an additional behavior of creating psychoacoustic difference tones with the sine wave drone. His most intricate developments include modal stochastic fields (*Cogluotobusisletmesi*), from which his algorithmic generator program *Autobusk* stems and where metric and tonal fields are coupled together, to be varied and traversed in continuous and imperceptible amounts. He also delves into what we could call 'hybrid' harmonies that consist of manipulations of harmonic effects of various cultures and styles (such as the mixture of Clementi, Schumann and Ravel in *1981* (1981), the Western and Indian logics of *Ludus Ragalis* (1974-2006), or a study of septimal blues harmony in *otodeblu* (1997).

Augusto Novaro is one of the pioneers of microtonality, following Julián Carrillo's footsteps but eventually taking a proportional route that precedes and differs quite a lot from that of Harry Partch. As recounted in the introduction to his self-published book *Sistema Natural de la Música*[199] which wraps up into a systematic treatise many years of research, he discovered/invented, after the deception of having unsuccessfully experimented with equal intervallic divisions, a musically fruitful arithmetic method of intervallic division. He names the smaller of the numbers of a co-prime ratio[200] the *fundamental*, the larger *co-fundamental* numbers, building from them arithmetic series that divide any interval into any number of ratios, all within whole number arithmetic. He then finds reciprocal series, corresponding to inversion and to harmonic (as opposed to arithmetic) means. Together with transposition of these scalar/chordal structures (called gradual series) a wealth of

198 Johnston, B. (2006 [1976]). Rational Structure in Music, *Maximum Clarity,* 62-76.

199 Novaro, A, *Sistema Natural de la Música*, Mexico City: Author's Edition, 1951.

200 'Co-prime' is just the mathematical jargon meaning that the numerator and denominator of a ratio have no factors in common, they are prime amongst themselves. It is what we have called a reduced ratio, meaning its terms cannot be further factorized.

combinations that produce scales and chords is made available (complex series, which combine the four kinds of series).

The first part of Novaro's book investigates his arithmetic series, turning afterward to geometric series (logarithms) that lead him to investigate tolerance and approximations to just and unequally spaced intervals. His visualizations revolve around logarithmic spirals, from which he developed the resonance chambers for his *novaro clave* microtonal piano. The second part of the book concerns practical music or ways to perform and approximate these discoveries as well as the instruments invented for this purpose, such as the 'acoustic boxes' made to experiment with 15, 19, 22, 31 and 34 divisions per octave. 53 ET has a dedicated section that explores all of its intervals (the fifty third root of two is very close to a 81/80, a bit less so to the 64/63 seventh comma, and a bit more than the Pythagorean comma, showing perhaps why this temperament approximates so well many 3, 5, 7 and even some 11 and 13-limit ratios). This temperament is studied in a diatonic manner, having up to 8 levels of sharps and flats between each degree. Since Archytas' harmonic means there have been no developments in harmonic theory concerning arithmetic ways for generating of intervals. Novaro does not take Archytas as his departure point though. As far as it goes, and as it is common in harmony, he came up through independent *a priori* deductions after years of tribulations which almost led to him abandoning music. The series are implemented in *Dissonance Lib,* providing departure points for some current compositions I am pursuing[201].

In terms of innovation in the proportional realm and the harmonic lattice, Ervin Wilson holds a prominent place, and I think his ideas will take some years to be assimilated and pursued, as he is not completely a composer, instead balancing between a theorist and an inventor of tuning systems. His logics are arithmetical, exploring patterns and numeric structures lying close to the confines of number theory and diophantine equations ('pairs' and 'triplets'). These arithmetic discoveries include inventions such as 'moments of symmetry', 'combination product sets' and 'co-prime grids'. The latter are interesting because they embrace several kinds of arithmetic series: the 'lambdoma' which is a form of Farey series, as well as the Pierce, Fibonacci and the Novaro series. These grids are made through the combinations of co-prime numbers found through triangular and rhomboid graphical structures out of which musical ratios are derived. Wilson acknowledges his indebtedness to Novaro and Joseph Yasser, concluding his investigation into the grids with a beautifully weird Pythagorean metaphor: 'An Hyperdimensional co-prime pattern fills the paradisal infinitude'[202].

His arithmetic structures are expressed in geometrical ways, forming subsets of the harmonic lattice that represent primes and connections between consonant intervals with different angles, forming projections of hyper cubes, hexagons and other figures, some of which have been used as templates for microtonal keyboards. He names them hexanies, eikosanies, stellate harmonies, diamonds, mandalas, spirals, scale trees, zig-zag patterns and even more[203]. He combines both arithmetic and geometric logics and offers an epistemology of pitch in which he talks about three levels of musical

201 His series rapidly grown into high dimensions in harmonic space, not being prime limited, so in terms of the lattice they are a bit complex to follow. Beyond the scope of this thesis I am laying the ground for an article researching how his series relate to the classical Greek means and *katapyknosis* as well as how they comport in harmonic space. As is the case with arithmetics, apparently trivial constructions give rise to complex structures which can be musically fertile.

202 Wilson, E. (2000). Pecan Tree Patterns, in a Nut-Shell. [PDF photocopies of author's research]. Last retrieved June 6, 2012, from http://www.anaphoria.com/peach.pdf I will leave to the reader any interpretation or link regarding this phrase that I could not help including even if it does not follow from the argument.

203 Most of his research consists of pencil and paper sketches of his discoveries, as well as letters to prominent microtonalists. His disciples are taught in the way of an oral tradition so he has not published any systematic books. Most of his letters papers and letters reside in http://www.anaphoria.com/wilson.html Last retrieved June 8, 2012, but more information (including explanations for lay people) can be obtained at http://www.thesonicsky.com Last retrieved April 5, 2012, and the integration of his theories into wider microtonal research is found in the Xenharmonic wiki: http://xenharmonic.wikispaces.com Last retrieved June 8, 2012.

abstraction. The first corresponds to musical reality and feeling (most music), the second to the theories, rules and methods behind musical reality, while the third is the 'master set of ingredients' in which developments of the infrastructure of music, the amplification of its gene pool, takes place: tuning systems, microtonality. He also mentions a mystical fourth level beyond the whole number grids he develops, a 'skylight' level of the creative act, inseparable from perception, where musical systems become dynamic.

4.1.2 Some of my approaches to harmonic space

Furthering the descriptions of approaches to dissonance curves delineated in section 2.1.2, I would like to focus now on harmonic strategies and the pieces composed during this research. Here the topics intermix as the ordering is done with regard to individual pieces and not to the chronology of the research topics. Some span from the theory, some provided a platform for theorizing, some both, but it is difficult to tell because of the intricate nature of the research. The review will not be a full description of the pieces, but will be centered mainly on their harmonic workings. I have already delved into some detail on *Clinamen* in section 2.1.2 and *Circular Limit* and the *Logos sessions* in 3.2.2. In this section I will talk about four more pieces plus a bit of what lies ahead at the moment of writing.

4.1.2.1 *rolita pa modelo* (2007)

Generally I work by inventing a tuning or harmonic modus operandi, then devising a logic or navigation scheme in order to either discover what may happen through that logic, or mix it with and be loyal to a drama/affect/concept overriding the work. My first probings into dissonance curves were of the first kind, in the piece *rolita pa modelo* (2007) for chamber ensemble. It is based on a rather static single pitch set derived from a mathematical spectrum (a sawtooth wave)[204]. This set was traversed through random weighted choices, producing different subsets of the set through the harmonicity windows through which it was filtered. These windows work like 'tendency masks' (a term coined by composer Gottfried Michael Koenig) that let through only intervals within a minimum and maximum harmonicity threshold. These thresholds continuously change during the piece. The work adapts to the 8 piece ensemble by approximating to 12ET for most instruments with a few important notes of the set intoned outside ET (relatives of partial 11 played in quarter tones in the flute and 7^{th} harmonic related sonorities obtained by retuning a few of the harp's strings). Most of the algorithmic generation was used to create the soloist harp part, which was later enhanced, embellished or counterpointed with the other instruments, with various functions assigned to different sections of the ensemble. The guitar, for example, plays the role of commentator to the harp, the other instruments generally support it by extending with resonances and furnishing it with decorations. Throughout the piece there are some interludes that interrupt the process, where the instruments emancipate from their assigned roles and the harmonies become transposed and more complex, resting in crucial chords that are required to be played in exact microtonal tuning. Here instruments other than the flute and harp also play microtones.

The general consistency I was looking for in the piece was to arrive at a polyphony that is a by-product of texture (of notes chosen randomly within the harmonicity limits) and not the usual way,

204 The pitch set comprises 3 octaves upon a fundamental G_3: 1/2, 8/15, 5/9, 3/5, 5/8, 2/3, 31/45, 7/10, 11/15, 3/4, 7/9, 4/5, 5/6, 13/15, 7/8, 8/9, 9/10 (lowest octave), 1/1, 11/10, 10/9, 9/8, 7/6, 6/5, 5/4, 13/10, 4/3, 11/8, 7/5, 13/9, 3/2, 14/9, 8/5, 13/8, 5/3, 7/4, 16/9, 9/5, 11/6, 15/8 (central octave), 2/1, 13/6, 11/5, 9/4, 7/3, 12/5, 5/2, 13/5, 8/3, 11/4, 14/5, 3/1, 16/5, 13/4, 10/3, 7/2, 11/3, 15/4, 4/1 (upper octave). They are more or less common just scales with extended clusterings of intervals around the thirds, tritones, sixths and sevenths.

where texture is a result stemming from fixed polyphonic assignments. In this way the density of voices and their vertical/horizontal relationships change constantly between independent, chordal and timbral (supporting) roles. I like to listen to it as if it were an immersion into a single resonance (with some added intermezzos) from different sonic perspectives.

4.1.2.2 *'strings'* (2007) and *Ahí estése* (2011)

The next project done that same year involved the real time triggering of dissonance curve materials for the piece *'strings'*, for guitarist, speakers and live electronics. Most of the technical work involved 'fine tuning' the interactive music system in order to obtain a perceptually clear link between what was played by the guitar (or any other kind of input) and the resulting chords and textures generated by the computer. This meant finding optimal windows for the spectral analysis to be further processed by extracting the highest partials and deciding which of them was to be considered the fundamental. There is also a separate pitch detection running in parallel in order to compare with the spectral fundamental and because having both options is useful as they produce different results. It is from the most prominent partials, usually between 8 and 10 of them, as well as from the overall amplitude and the fundamental, that the curves are calculated. After that, they are rationalized and their harmonicities calculated. The timing aspect is crucial as the curves take a few seconds to calculate and make their effects heard (this was in 2007, now with faster computers this time has gone down by at least a factor of 10).

Once the dissonance curve pitch sets are obtained, several kinds of textures are built upon them, some being simple chords, some with moving layers, others involving scalar canons, random textures as well as several combinations of these. They are made with synthetic timbres, mostly based on derivations of sine waves and filtered noise. A single texture combines several timbres in various layers. The musical interaction is based on a feedback process of '*acousmatic* imitation' between performer and computer, the former deciding the moments of mimesis by triggering the curves and textures with a foot pedal. The reaction to the computer is through a global mode of listening, embracing not only pitch but pertaining also to timbre and texture. The resulting actions consist not only of notes, but also sounds aggregates ('clangs') called for by the sonorous context. The score has instructions which refer the musician to follow or go against the computer, so that the resulting sounds blend and 'concord' with the electronic textures or contrast and stand against them. It is an interaction with the computer through *écoute réduite*, as it could be said that the machine listening implied by dissonance curves is of this kind. The interaction process is improvisatory but not an improvisation: the performer is not supposed to 'jam' but to stay closely connected with the sounding environment and be open to wherever this process might take him/her and the computer. The causal relationships have to follow the timbral setting in connected chains of sonorities where spectra and pitch are the main centers of action. The piece uses timbral harmony almost exclusively.

On top of this feedback process, the structuring of the piece is carried out by means of a graphic score that states the general guidelines for each of the six panels that comprise a performance: articulation, texture, pitch, dynamics and 'style' – such as '*cantus firmus*', '*bailey-esque*' (Derek Bailey type gestures), '*Mississippi delta blues*', and others. The guitar has a microphone in the neck, with which computer controlled feedback is produced when it approaches any of several speakers placed over the performance space. This begins to happen in the last third of the piece. Its overall principle is a change in focus from 'micro' pitch-timbral responses of the beginning, gradually growing towards 'meso' gestural textures, eventually making the guitar player play with his whole torso, moving the guitar neck around the speakers and provoking feedback to be interacted with through movement. In the last two panels the process continues its movement outwards to the 'macro', when the performer walking around the space, producing feedback with the extra speakers in the room. The piece deals with a gradual change of attention from micro-time and spatial aspects of timbre towards the macro aspects of performance and space, passing through the middle phases of texture and instrumental gesture. It ends with the guitar player walking out of the room or falling still and quietly back in the center of the speaker arrangements. The tuning of the folk guitar (acoustic guitar with steel strings) is based on a dissonance curve of a low D2 string: 1/2, 1/1, 14/5, 13/4, 7/4 and 5/2. Many extended techniques are called for (though suggested by the performers, one of the reasons why the piece is quite different at every performance and with different guitar players), including the use of e-bow and bottleneck.

The interaction includes a role and a score for a computer performer (me), controlling the scope of the behavior of the computer for each panel. It permits some room to modify the textures and the feedback section, so there is an extra layer of direction over what is happening. The piece has been played with several speaker configurations and with three different guitar players: Tom Pauwels, Matthias Koone and Carlos Iturralde.

Last year, I had the opportunity to present a modified and updated version of this system as an installation. Many improvements were made to the machine listening strategies (beginning with the fact that my laptop now is around 8 times faster) permitting more analysis and less time between triggering and the sounding of the textures. The main change in the system, however, was that it is now triggered automatically by amplitude thresholds. The installation is called *Ahí estése* which is a reference to *aisthesis* with a very Mexican way of saying ‘stay put’. The allusion to *aisthesis* is a metaphor for the ‘sensory ether’ where perception occurs, as the analysis extracts and puts into motion the harmonic qualities inherent in sounds in the same way as intellection (*noesis*) infers abstract forms (*eide*) from sensory data.

The intention is to harmonize an environment with which the audience can interact. The setting is with quadraphonic speakers in the corners of a space and a microphone in the middle. There are several behaviors programmed into the system, akin to the textures in *‘strings’* but covering more possibilities and layers. The dissonance curve analysis goes further than before, including harmonic space partitions and virtual pitch, so every texture has 3 strata of ‘orchestrations’, each possibly comprising more than one layer: a timbral one, a harmonic one, and another with a virtual fundamental (usually very low and derived from a virtual pitch algorithm). The thresholds act both in time and amplitude, so that only long enough sounds cause triggers, making the computer screen flash like a camera to notify the users that a ‘sound photograph’ has been taken. The thresholds automatically change their values slightly, so occasionally relatively quiet sounds can trigger the analysis, sometimes only louder and longer ones. It is also designed to be triggered by itself every so often: the sounding textures will provoke a trigger, keeping the sonic results in variable constant change even when no interaction from the audience is happening.

The behaviors/textures also include a wider range of variability than in *‘strings’*: tempos, timbres, rhythmic arrangements, ‘orchestrations’, along with spatialization. The behaviors are quite contrasted between each other. For example, one of the behaviors produces canons of harmonic intervals in ways inspired from (but not sounding like) Tenney’s *Spectral Canon for Conlon Nancarrow*, the canons making circles around the quadraphonic field and accompanied by fast swirling sparks of high timbral intervals and a slow drone in the virtual pitch stratum. Another behavior is very rhythmical and staccato, providing several layers of tuplets derived from the intervallic proportions. Each layer has its own comportment, usually at a different time scale from the others. In some behaviors the tempos change automatically, in others they are fixed.

The installation contemplates interventions by musicians and sound artists to incite and make tangible the sonic potential of the system. They have been barely tapped upon as the installation was presented during a limited amount of time. These interventions are planned as controlled improvisation sessions guided by open schemes and using some microtonal and timbral strategies to drive the system. There is also a plan to present it in a larger space which has 21 speakers around and above the room. The idea is to position the intervals in the room according to their coordinates in harmonic space, rendering the proportions spatially.

4.1.2.3 *Blank Space* (2009)

Many topics discussed in this research concur in this piece. It includes dissonance pitch sets chorales, partitions and granular harmony together with the *polyrhythmia* algorithm discussed in Section 2.1.2 regarding *Clinamen*, here used for the first time (still in an old and buggy version, as it was one of its first compositional tryouts). It is for clarinet, piano and soundscape.

Material is derived from a sound recording in order to structure many aspects of the piece, such as the harmonies, rhythms and form. I decided to use sounds relating to war, at first wanting them to relate to Mexico's president's insane and brutal 'war on drugs' – by now, after some 80,000 estimated deaths, he does not call it 'war' but 'struggle', but in late 2008 he still did, – but after not finding sonic examples that could be specific to it, I delved into the by then stereotypical Iraq war – for which I had already done a piece in 2004, – which is why the piece begins with an Iraqui Assyrian funeral chant. By the beginning of 2009 however, when the piece was underway, the bombing of civilians in Gaza overtook everything else and it is from this conflict that the rest of the sound material is taken.

The score was generated algorithmically. The first part is based on the moving canons produced by the accelerations/decelerations in several voices of the *polyrhythmia* algorithm. The beginning section proceeds like regular contemporary music: abstract, 'interesting' gestures and rhythms over a wide *ambitus* in both instruments, accompanied by rhythmic layers of synthetic electronic sounds. The pitches derived from the Assyrian chant which introduces the section follow naturally from it. The *polyrhythmia* steady state rhythm is a 4 to 3 polyrhythm in 4 voices; it begins with vertical chords that begin to diagonalize, producing shifts and melodic fragments that gradually fall into the steady rhythm; afterward the process is inverted, taking about as much time to fall back again into vertical sync (each component process lasts about a 45 seconds). The whole process is repeated twice but because of the changing pitches and the way the different coincidences where transcribed, it turns into a quite different variation. The textures are produced when coupling the rhythmic process with the pitches, which are selected from variable harmonicity windows (tendency masks), as in *rolita pa modelo.* The texture was transcribed freely for the two instruments, writing manually in an improvisatory manner over the generated textures, like 'connecting the dots' between the isolated abstract notes generated by the algorithm or as with a star map, looking for melodic and harmonic coincidences or interesting interactions between the instruments, varying the articulations and dynamics at will in the interest of creating a layer of musicality upon the 'dry' algorithmic process.

At a certain stage, after having been a good example of well behaved contemporary music, the music steps out of itself at the same time as the world and reality enter into it. As this reality is related to events that happened at the time of composition, they where not completely chosen. First there is an electronic transition from the last chords of the *polyrhythmia* process that begin with the Assyrian voice but turn into a mixture of chords and voices, finally to emerge as the voice of Israeli Foreign Minister, Tzivi Livni saying 'we are going to change reality'. It is at this point that the piece goes astray and several episodes ensue where the piano and soundtrack harmonize sound recordings of voices and bombs by way of solos separated by electronic chords. The voices are taken from statements by Livni relating to 'reality' and the minimization of civilian casualties. The dissonance textures are transcribed for the piano and electronics in several different ways and rates simultaneously, going up to granular speeds by the fourth interlude, the harmonization of an explosion turned into an angry piano solo (whose performance in the recording done in 2012 by pianist Gabi Sultana is particularly stunning). The harmonizations include all features of dissonance harmony: partitions, virtual pitch, textures at several times scales and granular harmony. Together, piano and electronics make for multilayered dissonance chorales which follow the contours and inner harmonies of the voice recordings, sometimes even harmonizing the very noise of the recording medium (highly compressed audio downloaded from the internet). In the case of the bomb, the electronics make a chorale out of the piano solo, in a way making for a dissonance chorale of the second order: one made of the piano which itself is a texture derived from the kaboom[205].

Afterwards the clarinet makes a solo accompanied by a recording I found of the bombing of a school that took place in those days. Here I used the *polyrhythmia* procedure in a different way, one which still holds promising possibilities. The core rhythm/spectrum which is fed to the algorithm is a transcription of the Assyrian chant fragment, here pitch and rhythm are specified together. It is then deployed melodically, instead of polyphonically: the several layers that comprise the canonic acceleration/deceleration are conflated into a single voice, producing a beautiful ornamented melody which feels like variations on the archetype and hands a middle eastern kind of flavor to it[206]. It begins with the phrase/cell, wonders around for a while, arrives in the middle at the melody again to vary differently till the end of the process, where it falls again into vertical chords where the piano resumes. During this clarinet solo, the horrible soundscape begins to include a chorale made out of harmonies derived from the bomb of the piano solo, which lead to the ending chorale. It is a somewhat tonal chorale, the analysis having extracted the core periodicities from the noise of the bomb, most of them used by the instruments in equal temperament. The electronics have two layers made out of timbral and harmonic subsets, providing both types of microtonal pitches, making for a harmony that is both tonal and close to the center of harmonic space but also has outer orbiting constellations at various speeds with timbral components. Even the harmonic pitches sound at the same time congruent with the chords and are a bit outside normal tuning, making for a mixture which I find compelling.

After some performances I decided to include a voice into the ending chorale, feeling that the political nature of the piece was a bit drowned in the musical process. It consists of a computer generated voice speaking quotes from architect and theorist Eyal Weizman, relating to the 'blanking out' of civilian populations and the policies of war[207]. The title of the piece is itself taken from a mention by Joseph Conrad of 'the blankest of all blank spaces' when referring to the disastrous colonization of Congo by the Belgian King Leopold II in the late XIX century. The title is also related to philosopher and political theorist Bolívar Echevarría's concept of 'whiteness' ('*blanquitud*'): 'Whiteness is a concept than can serve to explain the reasons for selective genocide in the contemporary world: why do we deliver certain populations to sacrifice, why do we condemn them to die.'[208]

205 Dissonance analyses of several orders are an interesting future avenue for research: generating a texture from dissonance curves and then making an analysis of that texture to produce yet another one, and so on. It would produce always changing but related textures and harmonies within a clearly perceivable process.

206 The uses of *polyrhythmia* where 'quantized' to sixteenth notes for this piece. In *Clinamen* (2011), as discussed in section 2.1.2, the process is deployed 'as is', with a much more refined formula for the tempo changes and in many more voices (more than twenty as opposed to four at the beginning of this piece and about 7-8 in the melodic rendering we are discussing now).

207 Weizman, E. (2010). Political Plastic (Interview). *Collapse*, VI, Falmouth: Urbanomic, 257-303. The quotes I took are 'lesser evil', 'tolerated sin', 'violence itself legislates', 'how much will still be tolerable', 'not by the victims, of course, but by others who are watching', 'moderation of harm', 'the magic number was 30'.

208 Echevarría, B. (2010). Occidente, Modernidad y Capitalismo (interview by Carlos Oliva Mendoza, in Spanish). *La*

It is a piece I like a lot, quite unlike anything else I have written and, at the same time, a piece I feel quite uncomfortable with, very exposed and touching delicate matters. It has an aura of prayer or some kind of supplication, something I never noticed during its making, probably due of the solemn subject matter and also because chorales, be them of any speed and kind, tend towards this kind of music.

4.1.2.4 *Chamba de um acorde* (2011)

This piece pursues my interest in the relation between pitch and duration. This time a pitch set is constructed that does not derive from dissonance analysis but from mathematical curiosity. Working in this piece is where I came up with the method to separate a pitch set into its mixtures of fundamental intervals, entailing collecting mixtures for each prime and its combinations with all the lower primes. Hence, octaves will form a set, mixtures of 2s and 3s another one, 5 combined with 3s and 2s, 7s with 5,3,2, and so on. The pitches were generated through arithmetic series from primes up to 11, limiting their size to 36:

```
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36
5, 10, 15, 20, 25, 30, 35
7, 14, 21, 28, 35
11, 22, 33
```

With duplicates removed and sorted, it looks like this:

```
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36
```

Converted to ratios:

```
1/1, 3/2, 2/1, 5/2, 3/1, 7/2, 4/1, 9/2, 5/1, 11/2, 6/1, 7/1, 15/2, 8/1, 9/1, 10/1, 21/2, 11/1, 12/1, 25/2, 13/1, 27/2,
14/1, 15/1, 16/1, 33/2, 17/1, 35/2, 18/1
```

Octave reduced (useful for showing distinct harmonic hues, with rounded cent values below):

```
1/1, 33/32, 17/16, 35/32, 9/8, 5/4, 21/16, 11/8, 3/2, 25/16, 13/8, 27/16, 7/4, 15/8
0, 53, 105, 155, 204, 386, 471, 551, 702, 773, 841, 906, 969, 1088
```

The unreduced pitch set is then separated into prime mixtures and affixed to a fundamental of A1, 55 Hz, displayed here ordered by their highest prime in the mix (left), note names with cent deviations from equal temperament, and as ratios (below each note):

2:	A2, 0	A3, 0	A4, 0	A5, 0				
	2/1	4/1	8/1	16/1				
3:	E2, 2	E3, 2	B3, 4	E4, 2	B4, 4	E5, 2	F♯5, 6	B5, 4
	3/2	3/1	9/2	6/1	9/1	12/1	27/2	18/1
5:	C♯3,−14	C♯4,−14	G♯4,−12	C♯5,−14	F5,−27	G♯5−12		
	5/2	5/1	15/2	10/1	25/2	15/1		
7:	G3,−31	G4,−31	D5,−29	G5,−31	B5,−45			
	7/2	7/1	21/2	14/1	35/2			
11:	E♭4,−49	E♭5,−49	B♭5−47					
	11/2	11/1	33/2					
13:	F5, 41							
	13/1							
17:	A♯5, 5							
	17/1							

Up to here the material was calculated with the computer in interactive sessions such as the one described in section 3.1.6. Once the premises were decided on, most of the piece was written

Jornada Semanal, 805. Last retrieved August 5, 2012, from http://www.jornada.unam.mx/2010/08/08/sem-carlos.html. My translation.

spontaneously, in the way of an improvisation over its materials, in about a week (in contrast, the piano solo and the transition to the end took almost a month). This idea is to distribute the pitches among the instruments, setting them into proportional rhythmic grids, making the piece a set of variations upon these subsets and their instrumental and rhythmic allotments. The duration of each variation was determined in part with the falling in sync of the rhythmic phases, but also having to do with trying to extend each panel as much as I felt was plausible before the setting exhausted itself.

The distribution of pitches allocates 2s and 3s together, the 5s by themselves and the higher ones being packed together in various manners. During most of the piece, the intervals are approximated to equal temperament which means that the more complex ones get conflated into pitches shared by other ratios. There are breaks in the rhythmic process, though, when the instruments play vertical arrangements and the flute and clarinet are asked to play exact pitches. This means the higher, difficult to perceive primes (11, 13, 17) rear out their heads from time to time. I was seeking for a harmony at the same time static (in the sense of non modulating) but also colorful, neither tonal nor atonal. I found that the premises for the writing opened up to fruitful discoveries instead of wearing themselves out quickly.

The rhythmic assumption was to 'transpose' the rhythmic analogies so that 2s and 3s are played in rhythmic multiples of twos, 5s are in triplets and higher intervals in quintuplets. This because I think (and argued in section 1.2.7) that rhythm is more sensitive to periodicities than pitch, so it is reasonable to play the intervals in simpler rhythmic relationships than their literal counterparts, one step below, as it were: 3 goes to 2, 5 to 3, and 7 and higher to 5, retaining the rhythmic complexity from getting too complex. The piece is fixed most of the time on 3:4:5 rhythms (together with 'octave' transpositions of them) in diverse settings, which meant incorporating metric modulations to diversify the speeds and relationships. The pitch contours result from patterns of permutations together with accents and grouping that produce conflicts between the additive meters and the multiplicative rhythmic grids, as when 7/8 meters collide with 5 against 3 multiplicative relations. Most of these processes tend toward staccato textures, sprinkled with long contrasting tones.

Once it was felt that several possibilities had been used, I wrote a piano cadenza consisting of three variations that incorporate many of the previous configurations within a single instrument. It increases in intensity, leading to the entrance of the trio in a section of 10:3:2 rhythms in the maximum density and difficulty whose culmination leads to the last large section of the piece where repetitions of periodic cells in 3:4:5, deployed against a 4/4 bar, are gradually infected with silence, this leading to the ending section where chordal islands are formed out of the remains of the process. Chords and bits of the rhythms that survive the eruption of silence form isolated and progressively sparser textures. This section was aided by algorithmic calculations for the introduction of silences into a the process, letting the holes suggest the formation of these islands as if by overlays or masks intersecting the inexorable rhythms.

Chamba is a polyphony of cycles and combinations of periodicities, producing an effect analogous to various bicycle wheels spinning at different speeds and interfering with each other, a sort of auditory equivalent of a visual *moiré* pattern, a bit like the way Galileo explains commensurability as the conjunction of different but related cycles. The writing of this piece broke a spell of almost a year in which I lost many of my reasons to write music. The title alludes to Antonio Carlos Jobim's *Samba de uma nota*, but in Mexican Spanish 'chamba' means 'work' or 'labor' and its about a chord instead of a note.

4.1.2.5 Future directions

I am sure this research will bear most of its compositional fruit after the period in which it was realized. Now that I am about to conclude this thesis I would like to mention some of the ideas I have for a piece that will sum up and pursue further some of these findings, intended to be performed as part of the graduation.

The planned piece is for ensemble *Modelo 62* and is intended as a piece in multiple parts, each exploring a different but related idea or a similar idea in a different form. I have been pondering on how to set up a different relation between the score and the performance, to include more open settings that can produce multiple results within a single specified material, logic and form instead of fixed notations. I want to specify pitch sets and temporalities that are subject to decisions in performance that can unfold the harmonic aggregates, with a view on providing multiple ways of interaction within the ensemble. The tunings must be quite different and subject to idiomatic adaptations, pursuing, for example, *scordaturas* and extended techniques but also involving equal temperament and mixtures of approaches. The various kinds of rules will determine the character of the music, which can change according to different options for the rules, so that they can together make series of pieces or sections. For example, the relation between duration and harmonicity, as in Tenney's notion of 'activation of nodes' in harmonic space, can be established so that far away intervals have long durations and low loudnesses while harmonic intervals become short and loud; inverting this relation (producing relations which are more 'conventional') can produce interesting variations/contrasts within a piece or between pieces. There are other possible variations on this single idea.

During a recent course I was involved in which Michael Pisaro gave a lecture on his music, I was stimulated by an idea present in some of his pieces in which sound field recordings are 'framed' with sine waves of random frequencies. The effect is that they always tend to coincide with some components in the sound field, or that there is at least a tendency to make the relation in the listener. This made me think that the connection between random, fortuitous pitches and specific harmonies can be established in an effective way, opening prospects for approaching harmony simultaneously in timbral and proportional ways. This can be coupled with a tendency towards sparse textures, where there is not so much activity but a space where there is enough information to bring the listener in, contrasting the usual tendency of composers (like me!) to anxiously fill every moment in the music with 'exciting' events. The inclination is to have few but requisite sounds in order to pursue a different approach to the deployment of pitches, seeking the subtle complexity that Ben Johnston talks about not only in the intervals themselves but also between them and in relation to their background, a kind of harmony that summons the listener instead of emitting a great deal of sound waves and information. Complexity as seduction or allurement instead of pouncing and oversupply. The ending section of *Chamba de um acorde* already moved toward this situation, which I want to pursue further: rarefied textures, rhythmic structures framed through overlays that result in textural islands surrounded by silence or other types of framings. I want to take this further both in terms of diaphanous configurations as well as divergent processes either towards densification/sparseness or without going anywhere, simply retaining aloof states of affairs (ergodicity, in Tenney's terms).

The rules and procedures that govern these processes can be inspired by some methods of Cage's school for setting materials into motion, but in a way that does not seem forced, but that arises, one may say, 'organically' from out of my musical approach and style of composing. For this, I think Christian Wolff and Earl Brown's notations and strategies can be of more aid than Cage himself. Only time will tell what will come of this, and this is an example where the thesis will leave off into new musical territory, one which will be still included in the doctoral research but which, in all probability, will bear fruits further along the way, jointly with newer approaches to pitch, harmony, texture and form.

The harmonic strategies that are glimpsed are:

- Vectorial (i.e, based on the coordinates of the harmonic lattice), functional, approaches for generating and moving within harmonic space, by using, for example, chains and combinations of functions: chords with tonic, dominant, mediant, seventh, elevenths, each of which can move its coordinate forward or backward in harmonic space, thus producing varieties of chords, all of which are close to each other, closely connected, and at the same time being complex and with a consistent sonority. The ensemble setting can be done by giving musician or sections a single harmonic function in which to concentrate their performance, each moving in a single axis of harmonic space instead of embracing many complex intervals at a time.
- A Novarian approach using his series of divisions to densify harmonic intervals, from large to small ones (or the other way around), abstractly generated and probably in combination with computer interaction, for example, by putting musicians 'inside' a varied version of my sound installation *Ahí estése*, with different behaviors customized for the purpose, where the intervals played by the musicians serve as *provocateurs* that direct the process. Any interval can be divided into any number of parts, with and without reciprocal intervals, producing varied chords and pitch sets from which to choose relationships. The crux of the matter is to devise a logic to the way the divisions are realized: which intervals are divided, into how many parts, and in which order, something which has to be determined by experimenting with them.
- 53ET investigations such as the one shown in Appendix III, from which large partitioned pitch sets will be the starting point for setting up processes from which the ensemble will perform over rule based open structures. This temperament has many seven based intervals, together with a few interesting eleven based ones plus many of the usual known ones in five limit. They will be organized proportionally around fundamental intervals as well as in pitch distance space. There is the idea of specifying the open pitch gamuts to be played depending on specific rules that take into account the microtonal possibilities of each instrument. I want to also add rhythmic gamuts to the pitch partitions, giving each pitch region a corresponding temporal grid and letting the instrumentalists handle the connections. A conductor could give a common pulse but that is not completely necessary because the clouds and textures that may ensue need not be synced to a common pulse, and this may function even more flexibly and musically than in a completely specified written out form. A bit like the way the coincidences between sine waves and sound fields happen but in the domain of note aggregates.
- Partitions: musicians will play gamuts taken from many kinds of subsets obtained by partitioning the previously mentioned pitch generation schemes. I can also think of 'meteor clouds' of timbral, distant intervals, generated by the computer and set against the ensemble's textures.
- Twofold harmony: the two aspects of harmonic duality being composed against each other. A melody with a pitch distance space logic (say thirds or quarter tones in equal temperament) against proportional configurations, opposing two instrumental groups and producing a hybrid result which is thought as complementary in the sense that many unforeseen connections will happen between both harmonic worlds, leaving that aspect partially open so that interesting connection can take place.

Furthermore, I will compose a dissonance soundscape to be played in a concert in a few months from now, in which sound field recordings will be accompanied by electronic materials derived from dissonance analysis. I will try to make the resulting textures quite different in timbre and texture from the dissonance chorales I have been pursuing until now.

4.2 Loose ends, speculative harmony

To conclude, this section will define and discuss a contemporary meaning of harmony. It is after having traversed the more practical and compositional research that it is pertinent to speculate in a panoramic manner, both to conclude and produce insights, opening up to other perspectives and future involvements while connecting some of the ideas about harmony that have been discussed in the previous chapters.

4.2.1 What is harmony? Metaphysics, Noise

> 'The world is not respectable; it is mortal, tormented, confused, deluded forever, but is shot through with beauty, with love, with glints of courage and laughter: and in these, the spirit blooms timidly and struggles to the light among the thorns.' (George Santayana[209])

Harmony is not an originally musical and technical term, stemming instead from philosophy. It is interesting to contrast and confront the practical, auditory and musical ideas regarding harmony with those pertaining to conceptual issues that reach beyond music. The above quote could refer to harmony as the extraction of beauty from chaos, as the attention paid to exceptional things in a world filled with difficulties and sufferings. Having an ethical character, it could be translated metaphorically into sonic terms and related to the quote by Luigi Russolo which opens Chapter 2, which I interpret as referring to the harmony associated with dissonance curves that 'attunes' to the most prominent vibrations in noise[210]. This attuning is not a detraction from 'the irregular movements and vibrations in time and intensity', but a process of revealing its 'gradation(s) and tone(s)'. It is not turning away from the mortal, tormented world, but accepting and including it through selection. Attention not only to its more prominent 'glints of beauty', but also a concern for the relation between confusion and laughter, delusion and courage. Beauty as the contrast between elements of the world and not as a necessary or preexisting model. The 'light among the thorns' is not predefined and could be defined very differently depending on the perspective. The way harmony has been examined throughout this work, with dissonance curves, spectra and rationalization, implies that noise is not its opposite. Their difference is not of kind but of degree: of ranges of gradations and inclusions. In fact, noise is a too general and problematic term that I prefer to replace by (sonic) 'complexity', connecting more easily to a harmony that comprises simplicity and complexity. This is already from the outset one of the main premises behind this study.

209 This quote was taken from a posting by a close friend of mine. I have not found the original book in which it was published, but it can be found here: http://thinkexist.com/quotation/the-world-is-not-respectable-it-is-mortal/347861.html Last retrieved August 17th 2012.

210 Russolo, *The Art of Noise*. Here is it again, for reference: '[T]o attune noises does not mean to detract from all their irregular movements and vibrations in time and intensity, but rather to give gradation and tone to the most strongly predominant of these vibrations.'

Harmony not as an retraction from chaos but as the concern for seeking forms within and through complexity, the separation and dissection of its qualities, as in the case of the timbral *sonances* we have encountered. Furthermore, harmony embraces the opposites of dissonance and consonance, it is not the subordination of order to disorder or of dissonance to consonance. Moreover, taking harmonic duality into account, with each of its two aspects possessing its own polarities (consonance/dissonance; harmonicity/inharmonicity), reveals a field that encompasses atonality, high dissonance and antitonality as harmonic *regions* not exterior or contrary to consonant and harmonic zones.

If harmony touches on aspects of beauty, order and chaos (*logos*-*alogos*), continuity and discreteness, relationality and the connection between the micro to the macro, it is because since its Pythagorean inception it has always had a metaphysical dimension. *Harmos* means joining, harmonious denotes interlocking, concerning the relationality of the elements involved more than the elements themselves. Another Pythagorean sort of figure is Leibniz, defining universal harmony both as 'diversity compensated by identity' and as 'identity compensated by diversity'[211], emphasizing that both variety and unity must be operative: 'there is greater harmony when there is greater diversity, which nonetheless is reduced to identity. (For there cannot be grades in identity, but in variety)'[212]. This is the metaphysical question of whole/part, or unity/multiplicity, very close to the way harmonic duality has been elucidated, unity corresponding to proportional pitch (*logos*) and multiplicity (variety) to linear continuous pitch-timbre (*alogos*). On the musical/auditory side of our research this leads to the question raised by Tenney regarding Cage: 'Under what conditions will a multiplicity of elementary acoustic signals be perceived as a "single sound"?'[213]

Late in his life Cage became sympathetic to the idea of harmony, albeit one of an anarchic type, embracing both 'legal' and 'non-legal' harmony. He states that 'the simple *togetherness* of art – I mean of sounds – produces harmony. That harmony *means* that there are several sounds ... being noticed at the same time, hmm? It's quite impossible not to have harmony, hmm?'[214], also to add a few paragraphs later (in relation to Giacinto Scelsi, but still referring to harmony) that it is 'sameness and difference as being together'. This is close to Leibniz's definition above, although I believe that the two aspects Cage mentions (coexistence of diversity of elements and the relation between sameness and difference) should be managed separately as two different characteristics. Harmony includes but is not exhausted by coexistence. The relation between sameness and difference (identity and diversity) can imply relations that are not necessarily vertical nor adjacent in time. Some of the consequences that have been extracted from this study, especially those concerning proportionality, suggest that vertical coexistence is not enough, that there also exists an overall (even if local) measure or reference, an interlocking, that puts the elements into relation, into 'being together': a unity that makes a whole more than the sum of its parts.

Cage's idea of an anarchic harmony arising as the coexistence, through chance encounters, between diverse elements, focuses on the workings and relations of collectives (of sounds, but also of persons), resonating with Cage's approach of optimistically affirming aesthetic forms as models for collective life. That in this kind of anarchic harmony it would be 'quite impossible not to have harmony' should be understood, in my opinion, not as an 'anything goes' stance (quite the opposite to the Cagean approach), but as the idea of an inclusive harmony we have just considered, one embracing the entire field of sound and which cannot be opposed to a 'non harmony'. The problem with accepting any possible combination of sounds as well as means of producing and giving them

211 Stated in his *Elements of Natural Law.* I have taken the quote from Mercer, C. (2004). *Leibniz's Metaphysics: Its Origins and Developments.* Cambridge: Cambridge University Press, 214.

212 Ibid.

213 Tenney, John Cage and the Theory of Harmony, 15.

214 Cage, J., Retallack, J. (1996). *Musicage: Cage Muses on Words, Art, Music.* Hanover: Wesleyan University Press, 108-109.

continuity resides not the possibilities it opens up but the fact that it obliterates any distinctions, erasing the harmonic regions. It could insinuate, for example, that any collective undertaking could be equally valid, and I don't feel Cage meant this by saying that it is impossible not to have harmony. For Cage there was a right spirit for doing things, with careful disciplined actions and against 'improvisation', 'intention', 'mind', 'ego', etc, giving preference instead to very specific types of sounds and their combinations throughout his work. These constraints delineate a field and an aesthetic posture, a style with which it is so easy to identify Cage's music despite its diversity and indeterminacy. What is significant to my position is to consider a harmony which, within this all-embracing situation (which, of course, includes silence), does not ignore the properties of sounds and therefore makes possible the distinction and inquiry of its various zones, placing them in relief against each other.

This directly leads to Tenney's definition of harmony, one motivated by Cage but developed perceptually, a definition which has been a basic premise throughout this study:

> 'We can now define *harmony* as *that aspect of musical perception which depends on harmonic relations between pitches — i.e. relations other than "higher" or "lower"*. Thus defined, "harmony" will still include all of those things it now includes — the "vertical aspect of music," chord-structure, etc. — but it is no longer limited to these, and it is certainly not limited to the "materials and procedures of the diatonic/triadic tonal system . . ." It would, for example, also include pitch-relations manifested in a purely melodic or monophonic situation, and — by this definition — nearly all music will be found to involve harmony in some way (not just Western "part-music"). In addition, the model of harmonic space outlined here suggests an important "first principle" for a new theory of harmony — that *there is some (set of) specifically harmonic relation(s) between any two salient and relatively stable pitches*.'[215]

These 'specifically harmonic relations between pitches' give rise to the examination of harmonic space and also extend harmony from vertical into horizontal correspondences not requiring simultaneity, an aspect that Cage's conception does not explicitly tackle. A melody can be regarded both outside of time, as structure, as well as inside time, as a flow and distribution of this structure. Simultaneity and horizontality have an intimate relationship, the vertical induces the horizontal and vice versa. Harmony has the capacity to go beyond the single sonority and is able to produce systems of relations, comprising multiple sonorities beyond the actual sounding present. These connections correspond to the traditional term of *tonality* (antitonalities being not their opposite or negation but the far side of harmonic tonalities). In terms of harmonic duality, we could name the horizontal aspect, as in Chapter 1, the DC (direct current) feature of harmony in that it consists of pulses, on-off switches ('unity') inside of which vertical AC components (bipolar waves, chords, timbres) happen ('diversity').

For Barlow, 'harmony is the study of that which is intervallically intended or at least understood'[216], a definition which accommodates anarchic harmony as long as it is listened to as a relationship of intervallic characters, thus pertaining more to harmonicity than to consonance. Both of these previous definitions pertain to compositional and musical characteristics of harmony, and it is from these premises that the formal, structural and metaphysical aspects have been examined. I have arrived at a Pythagorean account of harmony, namely that there is a connection, in hearing, between whole numbers and intervallic characters, from musical questions which eventually lead to abstract ideas that go beyond music, the perspective of musician/composer being always the site from which other external notions are connected. Because it does not evade the lines of reasoning behind the models of the universe that have been put forward by this long standing tradition, this route avoids many of the caricatured and superstitious accounts which surround us to this day and which have given such a bad reputation to the serious efforts of so many proportionalists throughout

215 Tenney, John Cage and the Theory of Harmony, 34-35. Emphasis in the original.
216 Barlow, *Musiquantics*, 20.

history.

There is a metaphor for harmony we could call 'chemical' or 'synthetic': putting together of elements into something which is qualitatively different from their mere gathering. Chemically this corresponds to a solution rather than a physical suspension. The latter pertains more to textures and aggregates which do not coalesce into a new substance, while the former is a mixture different from the aggregation of its parts, as is the case with timbre, which is more than the breakup into its partial components. This synthetic, as opposed to analytic, feature relates to harmonic duality by showing how both of its facets share the unifying and splitting forces that make possible the various kinds of musical objects at various levels of scale.

From the above considerations, I venture a definition of harmony of my own:

> *Harmony is an sonic assemblage mixing concrete and abstract elements that produces a result which is greater than the sum of its parts and which produces (and requires) time while also giving the illusion of space.*

Harmony must be *assembled*, it is not pre-given: 'harmony is a result, not a guiding principle'[217]. The arithmetic correlations of harmony do not exhaust it and cannot make of it sets of *a priori* rules that legalize or prohibit certain configurations over others. Harmony must be discovered. As we saw, it cannot be purely deduced, but needs to have an empirical axiom to get started. Secondly, this produces something which is qualitatively different from the mere aggregation of its components and this is what produces a specific temporality and a spatiality. It is not necessary to have multichannel arrays of speakers and sophisticated spatialization (all so unassumingly prevalent in electroacoustic circles) to give the illusion of movement and spatiality: this can already be done with pitch alone, a property that is usually not considered in thinking the spatiality of sound. Temporality is also intimately linked to harmony: not only does pitch and pitch relations require time but they also produce an experiential sense of it at various levels, including sometimes an 'out of time' sensation that directs the listener outside the situation where the music is happening.

Harmony is not a happy-ending complacency that resolves conflicts by pointing us to a higher sense of agreement or peace (as in the German name for harmony which is *Eintracht*). It is the joining (*harmos*) as well the splitting and tearing apart, keeping in touch with the emancipation of noise brought forward by atonality and modernism. It implies beauty, but not necessarily past beauty, instead inviting us to extend the notion of beauty. Neither nostalgia (origin) nor ultimate purpose (eschatology). By not positing any entity as a explanation or ground for all else closure is avoided, so origin and ultimate fate are not relevant. The harmony I seek is a coexistence between the real and the ideal. Both cannot be purely *a priori* realms but imperatives to discover the novelty in sonic (and other kinds of) objects. There is no higher goal or meaning: harmony is not global, but local, not a harmony of the whole disseminating into its parts, but the relation of parts with wholes in an indefinite interlocking of embeddedness with no end in sight (therefore it is not holistic), including in the system all its 'symptoms', antagonisms, and inconsistencies, as integral parts of it.

We should then maybe talk about a 'post-established harmony'[218], the pre-established harmony of Leibniz being an axiomatic (or divine) agreement between sense and thought, world and experience, real and ideal, the condition that permits this world to be 'the best of all possible worlds'. Instead of taking the lead from a postulated agreement, the sought harmony must be invented *and* discovered in a concern for patterns and relations at all levels. Speculative harmony: in a mirror (*speculum*). To find harmony in randomness as well as building ergodic harmonies thorough an exploration of the space of possibilities of its materials: harmony characterized by complexity.

217 Harman, G. (2009). *Prince of Networks: Bruno Latour and Metaphysics.* Melbourne: re.press, 21.
218 Ibid., 21.

To recognize that there is a specific musical relation between relatively stable pitches and whole numbers ('harmonic relations', or the Pythagorean focal point of our research), implies that there are specific features to musical perception; that music possesses its own autonomous rules (*auto-nomos*: having its own law). This goes against postmodern musicology which states that music is made up entirely of (social/cultural) conventions, as I discussed briefly in the introduction, but it does not imply that music is 'absolute' (that it is the only art that transcends this world) in the sense of Schopenhauer/Wagner. Music is always in communication with other arts as well as the sciences, feeding from both of them. However, it should also acknowledge, greet and respond to its own properties. Music reaches into art and science, but should also remain aloof from them, its functions and context far exceeding those of art and science (for example ritual, dance, trance, resonance – different from the mimesis of art –, healing, forgetting, accompanying, etc). Its distance from language is also one of its main sources of its individuality. In some occasions it has became too close to art, in others, to science. It should not forget its own traits and back and forth communication and transmission of information with the other disciplines. Harmony is a specifically musical phenomenon. Even though it has a connection to science (mainly arithmetics, as we have seen) and art (to broader aesthetic principles), its roots lie in musical and sonic principles. This should be acknowledged and maybe shared with other sonic practices which lack a concern for pitch relations, more frequently out of unawareness and because of the traditional connotations of harmony, than for a lack of interest in the possibilities that can be opened up.

To conclude by going back to the beginning of this section, a few words about harmony and contemporary chaos and numbers. According to the philosophy of Quentin Meillassoux, which is complex and in which I cannot (and do not need to) delve into here, randomness is subject to the same (meta) law as everything else. It is not noise and randomness (as with many theorists of noise who think noise music takes us to the uncanny realms) which convey a glimpse of the contingent, but rather proportionality. Randomness is not the opposite of order. It is not different in kind, but in degree. According to his posture on randomness and probability, what is important is not what is likely but what is *interesting*. Harmonic analogies (proportions) become a Symbol amidst chaos of what is worthy of grabbing on to, the 'light among the thorns' that Santayana refers to in the quote above. He argues that Being is contingent, that everything could change at any moment for no reason whatsoever, so that the laws of nature fall into the same contingency as everything else, randomness being as much a law as any deterministic law[219]. It is in the way he questions the way that mathematics can capture something absolute about being, independent of human thought (a Pythagorean question), that he arrives at these considerations. What I'm relating to harmony is the way it implies the precariousness of whole numbers and proportions, which become small islands in the midst of an extended field of chaos.

The philosophical implications of harmony considered in relation to the psychoacoustically and

219 Meillassoux, Q. (2008). *After Finitude. An Essay on the Necessity of Contingency.* R. Brassier, Trans. London: Continuum. The topic is developed closer to art and music in the dialogue Meillassoux, Q., Hecker, F., Mackay, R. (2010). Chez Meillassoux, Paris, 22.7.2010. Falmouth: Urbanomic Documents. Last retrieved August 18, 2012, from http://www.urbanomic.com/archives/Documents-1.pdf. He says: "Randomness means laws. There are laws of randomness, calculations of randomness. It's a way of calculating, that's all. And so it's just a particular mode of the existence of physical laws. It's a way of anticipating, it's absolutely regular, in fact. So, the problem is, if you break laws which are structurally random, you can't find yourself again in randomness, it is not the same phenomenon. But it's very difficult to show this. The problem is that maybe, by examining the way that artists try to show randomness, to make it felt, what did they do exactly? We have something that is 'random', how can we break this? How can I break into this lawful randomness in a way that is other than random? The difficulty is there." Elsewhere, he adds: "[D]etermination and randomness, they are the same. So, at the beginning, for example, you could show it as an opposition, but progressively you see that it is just 'quoted' inside something else. The challenge would be to surprise a musician or an artist of randomness: he thought he was exploring the world of the random, but now he sees that random is just a quotation".

arithmetically informed approach that has been studied can serve to expand and give it a wider compositional assessment. The various perspectives make sense of what has been previously explored, pointing towards new lines of thought that reach beyond the purely technical, sonic and musical, pointing abstractly and aesthetically to ideas to be pursued later both compositionally and theoretically.

Conclusions

I hope this research is valuable to others. It provides many perspectives, some of them left open for the readers themselves to link.

I think that this thesis contributes with 5 main points:

- An argument for microtonal harmony useful beyond microtonal specialists
- To put Tenney's and Barlow's ideas on the map a bit more
- Acknowledge the tools that psychoacoustic and cognitive research can bring to music as well as a critique of some of their approaches: psychologism – everything happens in the brain, something I argued is not completely defendable for intervallic ratios – and the fact that they use traditional theory – equal temperament, functional harmony – in order to prove it in a circular way. I think composition can suggest research which is less based on existing theories and can actually open up to new insights.
- To show my artistic research process. It embraces experimental music understood as:
  - Cage: music whose outcome is unforeseen
  - Tenney: one experiment leads to the next one, as in the sciences
  - Me: experimental in the sense that it might be able to make contributions to the field for others to expand, beyond my specific applications of the ideas and through music
- To be able to use this theorization of harmony and recollection of information regarding its uses, history, metaphysics, etc., in order to make more music. Both in my case and hopefully for others too. Concerning me, and as already mentioned, there are many avenues left open to pursue: Augusto Novaro, Erv Wilson, object-oriented metaphysics, further developments to *DissonanceLib*, to name the ones that stand out most at the moment.

On the other hands, I should mention my specific contributions in order to differentiate them from the ideas of Barlow and Tenney which are very close to my approach. Regarding harmonic duality, my point of departure has been Barlow, although hints of this duality are also present in the writings of Johnston and Tenney. What I have done is to delve into the details of this two features in order to extract and detail the many nuances behind them, characterizing each side and showing how they intertwine. The different functions that derive from this are not immediately obvious nor do they map neatly into each side of the duality. The functions were described both through the evidence provided by the psychoacoustic foundations of pitch and its neurological perceptual path, as well as through the historical route that connects present day concerns with Greek harmonics, both in their mathematical and empirical, almost phenomenological, approach. I have found lots of detailed substantiation for renewing a harmony from a broader perspective than is usually the case, even if this has not led to a full-blown and systematic theory. Another outcome to this extension of the senses in which harmony can be compositionally dealt with has been relating it at various levels of scale as well as in abstract terms such as discreteness/continuity, flux/stasis, and so on. I think this work opens up further investigation and compositional uses of these ideas, some of which are still not completely formulated and developed, especially those that required the more abstract uses of object-orientation and multilevel duality (*eidos-sonos*, *logos-morphe*, *nomos-drama*) and which will be pursued more fully in the future.

Perhaps where my approach diverges mostly from the aforementioned composers is in that much of the theoretical development has been done through my implementation and use of dissonance curves. These curves are analyses of timbral and continuous features of pitch, and the fact that they lead to and intersect with proportionality gives a very specific slant to my examination of harmonic topics. Harmonic duality has been developed from this departure point, intersecting with the compositional process. Harmonic space has been approached from the practical and algorithmic requirements of analyzing the wide variety of intervals produced by dissonance curves in conjunction with instrumental and computer generated music. Applications of dissonance curves such as dissonance chorales and the derived pieces have also benefitted from the multilevel considerations of harmony and have proven to be quite particular to my research, unrelated to the usual spectral routes of relating features of sound to electronic resynthesis methods.

Regarding harmonic space, most of the ideas were already there, but my specific way of implementing them brings independent approaches such as functional representations, degrees, tempered approximations, nomenclature and correct use of accidentals, together, also incorporating visualizations and pursuing strategies for navigating the pitch sets. I also put quite more emphasis than any of the treated authors in the importance of prime numbers in harmony as well as using this notion to produce separations and mixtures of pitch sets in composition. For all that, I think more could have been written regarding adaptations and transcriptions for instrumental settings, although this has been accomplished through some of the compositions and some traces of it are attested in section 3.1.6 and Appendix III.

Another independent line of inquiry was the development of harmonic fields, even though it shares some premises with Barlow's methods in his software *Autobusk*, that is, to regard harmonicity as a basis for the statistical selection of pitches. Nevertheless, my implementation is more textural than his metric approach, and, as a consequence of initially having been based on dissonance pitch sets, has led to new areas discovered through the musical and algorithmic development, notably the region of antitonality, a genuine contribution of this study (more a discovery than an invention) as well as the 'atonic' mode of selection, which has a sound very much of its own. The fact that it is based on a real time programming environment has also guided it towards live interactive behaviors and electronic sound synthesis, installations as well as algorithmic improvisations with live coding.

The section on Pythagoreanism was meant to give emphasis to a line of thinking that is enormously influential throughout Western history and has fallen into relative obscurity in recent times. I consider that the use and understanding of pitch ratios in music is very important and this approach cannot afford to ignore the ideas stemming from this tradition. This is where some ideas from Archytas, Galileo, Kepler, Leibniz and Euler, among others, have resonated with my investigations. Finally, the other theoretical section that deals with harmony in different registers of thought than the purely technical and compositional mode is the review of atonality and modernism as a liberation of continuous forms at many levels, especially concentrating of how the musical materials could be understood as filling the gaps of discrete structures. This makes an analogy with the mathematical continuum in order to show how most contemporary music today is saturated with continuous forms at all levels, suggesting that proportionality and discreteness should be reassessed and re-approached from within this situation.

Much more is still to be pursued, both on a technical as well as a conceptual/aesthetic level, and I hope this work has been an important stepping stone in my development as a composer, more a beginning than a conclusion, the herald of much more music, theory and software to come.

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Glossary

‘algorithmic improvisation’

As with ‘algorithmic composition’, I use this phrase to refer to improvisations conducted with the aid of algorithmic tools, such as the *Logos sessions* I did with the harmonic fields generator.

accidental/essential qualities

Aristotelian terms that denote the difference in an object between the properties it must have to remain what it is (essential) and those contingent ones that arise from its relations to other objects in its environment (accidental). Used in object-oriented thought and applicable to harmony as the difference between the harmonic and timbral qualities of an interval: the former are essential, while the latter are accidental.

aggregation/aggregateness

In my conception of harmonic fields, it refers to the axis of agglomeration, the textural distribution in time of the elements, lying between the poles of ‘fusion’ and ‘fission’, that is, when the elements coalesce or segregate depending on their size, speed, density and harmonic properties (consonance and harmonicity). See *Figure 6* in section 3.2.1.

aisthesis ‘*Aesthesis*’ in Greek means perception. Aristoxenos, in his discussion of *dynamis*, refers to *aisthesis* in music: hearing (*akoe*) complemented with thought (*dianoia*). The difference between simple hearing and this way of ‘perceiving’ is that it requires training and involves discrimination: a sensitivity to the musical meaning of notes and intervals in context. This term is taken by this study to clarify that the proportional meaning of intervals requires both perception and intellection, that ratios do not appear directly in consciousness but nevertheless exist as ‘formal causes’. See also **Aristotelian causes**.

alogos An irrational number, a quantity for which no ratio exists. It could also pertain in the realm of musical form to the absence of perceivable logic at the mid-level of method, suggesting an ergodic/aleatoric kind of behavior. See ***ergodic***.

antitonality In my conception of harmonic fields it is the region where inharmonic intervals are more probable than harmonic ones (the left half of the harmonic fields diagram of *Figure* 6 in section 3.2.1). Antitonal configurations may often contain local harmonic relations, as the intervals are inharmonic with respect to 1/1 but may be more closely related between themselves.

Aristotelian causes

It seems like harmony and music would benefit a lot from understanding some of its objects in terms of the four Aristotelian causes. For a start, they provide the best approach to explaining how intervallic ratios 'arise' as formal instead of efficient causes (as if science would be able to explain the moment in the neuronal pathway where a whole number ratio emerges, the 'ideal' arising somewhere or at some point in time in the brain). Formal causes are inferred from perception but do belong to the sensation itself. Here is where object-oriented philosophy could benefit a speculative harmony, as presented in a embryonic outline in the last Appendix. The two temporal causes (efficient and final) govern upon perceptual qualities while the spatial, mereological, causes (formal and material) are involved in the objects themselves, outside their relations to perceivers. This is only the initial work on what I feel to be interesting prospects for the future. See also footnote 74, page 46.

autocorrelation

The principal model behind time-based pitch perception theories, proposed by Joseph Licklider in the 1950s. A mathematical tool for finding periodic patterns in signals by measuring their self-similarity across time. It is the cross-correlation of a signal with itself as a function of time and it is believed to happen neurologically in the auditory centers of the mid brain and projected upwards into the cortex.

bark, sone Psychoacoustic units calibrated to the psychological responses of of pitch and loudness: critical bandwidth for *barks*, and equal loudness contours for *sones*.

bottom-up, top-down

Two approaches to description used in cognitive science. The former builds up from individual components to ever more complex and larger scale systems. The latter begins from high level 'cognitive' experiences and explains them as the conjunction of smaller 'black box' components. Though it would be desirable for them to 'meet in the middle', that does not happen and there are many gaps between the two descriptions. In music composition one could say it has mainly been approached from the bottom up: from small cells and motives that build up to larger works. Iannis Xenakis, with his use of graph paper in the early 1950s, is probably the first composer to conceive a piece simultaneously from the macro to the micro and from the micro to the macro.

bpm Acronym for beats per minute. A term used for metronome markings.

CDC, *Consonance/Dissonance Conception*

James Tenney's *Consonance/Dissonance Conceptions,* as expounded in *A History of 'Consonance' and 'Dissonance'.* They refer not to theories of consonance and dissonance but to the underlying conceptions behind the terms, each being able to accommodate various theories styles and epochs within it. They are:

CDC-1 Melodic. The relatedness or ability to directly tune melodic pitches.

CDC-2 Dyadic or 'fusion', as in early polyphony, based on the sonorous character of simultaneous dyads, the way pitches tend to fuse into a single percept.

CDC-3 Contrapuntal. An operational conception related to the rules of counterpoint and the relation between the bass and

	other voices. Melodic and textural clarity is central.
CDC-4	Functional. Based on the idea individual tones being consonant or dissonant with respect to the root of a triad, not necessarily connected to their sonorous quality of the interval.
CDC-5	Timbral. Pertains to any sound or aggregates and depends on roughness and the perceptual qualities of the spectrum.

clang In James Tenney's theory of *Gestalt* forms it refers to the primary aural *Gestalt*, a configuration (can be a note, motive, chord or aggregate) that is perceived as a primary musical unit. It is a 'strong' *Gestalt*, in contrast to *elements* (smaller subordinate parts of a *clang*) and *sequences* (larger configurations made up of *clangs*).

clangtint A term I use to refer to the residual timbre that results from the slight mistuning (within tolerance) of an interval from its exact proportional tuning. It is the tonal shading of a proportion, its timbral-harmonic attribute.

***comma: syntonic*, Pythagorean**

Commas are intervals that result from tuning an interval in two different ways, arrived at from separate combinations of fundamental intervals. The most famous ones are the *syntonic* (also called the chromatic *diesis* or comma of *Didymus*) which is the difference between a just 5/4 third and Pythagorean major third (or ditone) 81/64. This difference is 81/80, 21.5¢. The Pythagorean comma is the difference between 12 pure fifths and 7 octaves, the difference being 531441/524288 which is 23.46¢. There a many other commas related to different combinations of primes. See section 3.1.3.

commas, alterations, steps, and leaps

A comma changes the tuning of an interval without changing its scale degree, (the interval is tuned in accordance with one set of fundamental intervals rather than another). An alteration does not alter the degree but changes its mode or quality (major, minor, just, augmented, etc). A step is an adjacent change of scale degree, independent of its size. A leap is a non-adjacent change of degree.

commensurability

The proportional, time-based, theory of consonance according to which the different numbers that compose a ratio are commensurable to each other. Thinking intervals as pulse trains, it means that the sooner they fall back in sync the more commensurability they will be. Galileo Galilei proposed the first full theory of commensurability as the conjunction of different but related cycles. Leonhard Euler was to further refine it in stating that what is of most importance is the divisibility (factorization, therefore prime numbers) rather than the sheer size of the numbers involved.

critical bandwidth

The frequency interval within which sine waves interact inside the basilar membrane in the cochlea. It is of different sizes according to the register of hearing, being more than an octave in size in the lowest range and less than a second in the highest. For most of the hearing range it lies between a subminor

7/6 and a minor 6/5 third. It determines the limits of roughness (whose maximum is reached at 1/4th of the bandwidth) and masking. It is also the limit between melodic (steps) and harmonic (leaps) intervals.

diagonalization Georg Cantor's method for finding irrational numbers out from series of densely packed rational numbers by traversing their them diagonally and changing a digit of each number's decimal or binary representation to arrive at a number not representable as a rational but contained within the interval. It formalizes the continuum in mathematics and discovers a new infinity that lies beyond the denumerable one of the whole numbers (and rationals). See section 2.2.3.

diatessaron A perfect fourth 4/3, the principal interval that engenders melodic harmony. The main interval of division in Greek music theory.

diesis A small interval that alters other intervals into their chromatic counterpart. *Dieses* can be minor, major, septimal, undecimal, etc. It has had many meanings in tuning theory but in pitch distance space it refers to the Aristoxenian *diesis,* the thirtieth part of a fourth, corresponding to a sixth of a tempered semitone or 16 2/3 ¢.

difference tones Discovered by Giuseppe Tartini in the seventeenth century and studied by Hermann von Helmholtz, they are tones arising from two closely spaced pitches, usually in a high register which are produced by non linearities in the perception mechanism. They are of several kinds and orders, the most common being the summation and difference between the frequencies of the tones. They have been used to explain consonance and dissonance and can be used in composition as as harmonic generators, as in the case of James Tenney's *Koan for String Quartet*, where they are used to harmonize a continuous glissando.

diminished, just, augmented

The possible varieties that 'perfect' intervals such as unison, fourth, fifth and octave can have according to their size. See **subminor, minor, neutral, major, supermajor.**

dissonance chorales, dissonance chords and textures

The names I give to the generation of harmonic textures derived from dissonance curves. Usually refers to recorded soundscapes that are harmonized at various rates through this kind of analysis.

DissonanceLib The collection of tools I have developed during the research and available as a *Quark* for the ***SuperCollider*** programming language for sound synthesis and algorithmic composition. Its main classes are Dissonance, PitchSet, HarmonicVector and some others, plus extensions for using and converting psychoacoustic units as well as the implementation of some historical harmonic functions (means, katapyknosis, Novaro series, and other utilities). It is available from within *SuperCollider* and comes with documentation.

Dissophonos The program built in *SuperCollider,* atop *DissonanceLib,* for analyzing sound files with dissonance curves. See **dissonance chorales**.

drama Of the terms I use to describe harmonic duality at different time scales, *drama* corresponds to the timbral/continuous aspect of large scale form. It is named thus because its Greek meaning ('action') is in agreement with the way a piece progresses as a narrative, not necessarily a directional narrative, as it could be constituted by an ergodic distribution of lower level forms, without any expected progression. In this extended sense, it refers to the causality of musical forms at the time scale where memory and extended attention are operative. This level is made up of lower level objects: continuous profiles (see ***morphe***). Its proportional counterpart is ***nomos***, the 'law' or 'logic' behind structural aspect of large scale form.

dynamis/melos The horizontal aspect of harmony. The melodic character of an interval, depending more on context and scalar function than on its absolute size. An interval's *potential* for movement.

eidos/eidetic A essential quality of an object, in contrast to its accidental qualities. In object oriented metaphysics it is a 'submerged' quality: it can only be alluded through categorial intuition (*aisthesis*), but not directly perceived. In the case of a harmonic object it is its ratio, its formal cause. It is the relation between the real sound and the ideal ratio, the process whereby they are related and formalized.

Eidos also is used in this research to refer to the proportional aspect of harmonic duality at the time scale of small scale materials. The term corresponds to the manner in which the proportionality of pitch intervals is encountered in perception, that is, submerged within the sensory aspect (***sonos***) of the interval, alluded indirectly by means of 'intellection' (Aristoxenos) or categorial intuition (phenomenology, also know as eidetic variation), as described in the previous paragraph.

ekbole The name given in Greek harmonics to the subminor third of 7/6, 266¢.

epimore/epimere

Terms referring to whether an interval is super proportional or not. The main criteria for consonance in Greek theory.

ergodic A term used by James Tenney in his article 'Form in 20^{th} century music' to describe forms where the compositional space of possibilities is explored in a statistically homogeneous manner. It can pertain to various or only to a single sonic variable, both in wide ranges and in limited spans, to sections as well as whole pieces. One way to detect ergodic forms is when 'everything is changing but everything stays the same' (with respect to some sonic parameter). The temporality of ergodic forms tends to be static, without direction.

fundamental interval, fundamental number

Fundamental intervals derive from the sonic qualities of prime numbers. They are like the primary hues of harmony, from which other combined intervals derive. I call it fundamental *interval* when it is octave reduced, but refer to fundamental *numbers* in their non octave reduced form (i.e., fundamental *number* 3 is taken as fundamental *interval* 3/2).

Fundamental Theorem of Arithmetic

Every whole number is decomposable into a unique sum of prime powers (Euclid). This translates into harmony as the decomposition of an interval into fundamental ones defined by prime numbers.

genos The different kinds of tetrachordal structures upon which Greek scales are built. There are three main kinds, diatonic, chromatic and enharmonic, each with several variations or shades named *chroai.*

granular harmony

The name I use to refer to harmony which goes very fast, at granular speeds. **Dissonance chorales** can be of this type and it also related to the **aggregateness** axis in harmonic fields.

harmoneme The term used in this study to refer to the building blocks or units of proportional harmony. The timbral aspect of harmony has small distances as its basic constituents, such as Aristoxenian *dieses* or 'just noticeable differences'. Instead, harmonic units are not small intervals but larger ones than get divided. In this study these units are held to be the prime numbers that factorize harmonic ratios and constitute the axes of harmonic space. The octave is one of these *harmonemes*, but also fifths (twelfths) and thirds (two octaves plus a third) are fundamental intervals that provide primary harmonic hues.

harmonic arithmetic

Operations concerning musical ratios. The most basic and common are addition and subtraction (corresponding to multiplication/division of fractions), but can also include exponentiation, octave reduction, and factorization. Harmonic means, ***katapyknosis*** and Augusto Novaro's series could also be said to be more complex kinds of operations in harmonic arithmetic. The class HarmonicVector in *DissonanceLib* allows to perform these operations on harmonic proportions. They can be thought of as arithmetic operations in the field of rational numbers and also as movements within the harmonic lattice where they are represented.

Harmonic duality

The main hypothesis of this research, though which all other topics related to harmony are considered. In a nutshell it deems musical harmony as having two aspects, a proportional, properly harmonic one, related to intervallic qualities and stabilities, and a linear, distance-based one related to timbre and continuous pitch. Both are intertwined and always present to various degrees in different musical contexts. Their separations can be composed and a concern for this duality can help explain and untangle many harmonic topics, both historical and current.

harmonic function: dominant/mediant/septimal

Any ratio can be given a harmonic function that derives from the fundamental intervals that compose it, usually ignoring octaves. These functions have inverses (sub-dominant, sub-mediant, etc) that correspond to accumulation of that interval in the negative direction in the harmonic lattice. For example, 3*D2mS* denotes a triple dominant, double submediant and single septimal. This interval (with no direct musical use, only given to illustrate the point) corresponds to $3^3*7/5^2 = 189/25$; octave reduced it is 189/100 (1102.06¢).

harmonic hues

The perceptual harmonic qualities of fundamental intervals and their combinations. See section 3.1.2.

harmonic metric

A distance function for harmonic proportions that gives a measure of their harmonicity. There are various possible harmonic metrics, the most studied and interesting for this research are Leonhard Euler's *Gradus Suavitatis*, Clarence Barlow's *harmonicity* and James Tenney's *Harmonic Distance.* They are metrics in the mathematical sense: they are non-negative, $d(x, y) \geq 0$; if zero, it implies that the two intervals are the same: $d(x, y) = 0 \Leftrightarrow x = y$; they are symmetric: $d(x, y) = d(y, x)$; they satisfy the triangle inequality: $d(x, z) \leq d(x, y) + d(y, z)$. See section 3.1.5.

HarmonicFieldsForever

The program built atop *DissonanceLib* in *SuperCollider* for generating and exploring **stochastic harmonic fields**. See section 3.2.

harmonicity/inharmonicity – consonance/dissonance

In harmonic duality, harmonicity/inharmonicity refer to proportional agreement or dissimilarity, while the terms consonance/dissonance refer to the timbral aspect of harmony, where they denote sensory consonance/dissonance (**roughness** and the nearness of the partials to harmonic series or *tonalness*).

harmonics (*harmoniké*)

The science of harmonic in classical Greece was companion to the sciences of rhythmics and metrics whose task was together to classify and describe the regular and repeated patterns of form and structure underlying the diversity of melodic, rhythmic or metric sequences in music. In particular, harmonics dealt with the structures underlying melody (*melos*). It set out to identify the varieties of scales and tuning systems which could be reckoned as musical, seeking quantitive representations for intervals and melodies, classifying scales and their transformations, and searching for the underlying principles behind these structures. Questions such as their rooting in human culture or in something independent of humans, or in mathematics, as well as the status of their applicability beyond the musical sphere were the kind of issues raised and discussed by harmonists. As such, it was a full blown science in the sense of a discipline to discover and demonstrate a body of truths, regardless of whether they could be assimilated to mathematical sciences or to the 'sciences of nature' (*physiologia*). There are two main schools of harmonics, the mathematical and the empirical, the former associated with Pythagoras and the latter with Aristoxenus.

HarmonicVector

A class in *DissonanceLib* that represents intervals as coordinates in the harmonic lattice, both as they are and in octave reduction, together with their ratio and functional representations. It also allows for ***harmonic arithmetic***.

holophony A term that describes a kind of musical texture in which various kinds of simultaneous textural streams coexist. It is a generalization of monophony, polyphony, homophony, heterophony as it encompasses combinations of them. Can be said to pertain to many musics (especially electroacoustic) of the 20th century.

hypate, parhypate, lichanos and mese

The names of the identities of notes in the central tetrachord in Greek music.

infrapitch Refers to perception of *iterance* below cochlear pitch. Only ***periodotopic*** pitch detection is involved, rather than a combination of periodicity and ***tonotopy***. It ranges from below 2 Hz to between 16 and 20 Hz which corresponds to the periods of **clangs**, melodic phrases and low to mid scale forms.

inside- /outside-time structures

Terms used by Iannis Xenakis to distinguish between sonic objects in their abstract form (outside time: scales, durations) and their musical deployment in time (rhythms, chords, timbres, sonic transformations).

inter-interval matrix

A matrix which contains all the intervals that happen between each of the intervals in a pitch set (it is actually a half-matrix). Harmonic fields are built on top of this matrix by taking its harmonic metric, then ranking the values and converting them into probabilities. The intervals are chosen from out of this last matrix. See section 3.2.

katapyknosis A procedure for the densification of intervals by dividing them into successively smaller *dieses*, either in pitch-distance space (Aristoxenos) or by producing series of smaller and more complex ratios (Pythagoreans). *DissonanceLib* has functions for performing the Pythagorean version of *katapyknosis* on ratios.

klangfarbenmelodie

Tone color melody. When the scope of development in late Romantic music collapses into a single note at its modernist limit in the *Second Viennese School*, collapsing of harmonic modulation to the note level, embracing instrumentation and timbre. The heart of timbral modernist harmony. See section 2.2.2.

limma An interval which is a remnant of two others. A type of *comma*. It arose with Pythagoreans as the difference between two 9/8 whole tones (81/64), and a fourth 4/3. Their difference is the Pythagorean *limma*, 256/243 = 90.22¢.

logarithmicity/proportionality

Logarithmicity is the linear pitch distance embodied by the geometric mean and approximated asymptotically by the integer means (arithmetic and harmonic). Logarithmicity is a way to fit or compress more information into a limited space. It is the final limit tendency of periodicity, a tendency achieved from above and below. Together, periodicity and logarithmicity form a dialectic which lies at the heart of harmonic duality. See also **musical means.**

logos Definition pattern: form of an interval. Also translates from Greek as 'reason', 'speech', 'discourse'.

The term is also used to define the proportional aspect of harmonic duality at the mid-level time scale of 'method', the way basic constituents (elements and clangs) are structured. Referring to the 'pattern' in which these smaller scale forms are set in time, it denotes 'structure' as well as 'logic'. Its timbral counterpart is ***morphe*** and it is constituted by smaller scale ***eidos***:

proportional materials (intervals, rhythmic elements, smaller scale structures).

Markov analysis A statistical technique introduced for the first time by Iannis Xenakis into composition. A musical (or other kind of) structure is analyzed into states whose probability of transition represent patterns behind this structure. It can then be 'navigated' by means of random choices weighted by the transition probabilities. They are used in harmonic fields to produce the 'atonic' mode, where each new interval becomes the new tonic from which to choose the next interval. See section 3.2.

mathesis universalis

A model of the universe in which reality is amenable to being mathematized. It can be said to be Pythagorean in spirit, although it arose in Europe in the middle ages. It is also name given by René Descartes to the communication of proportions from sense to intellection.

melos See ***dynamis***.

mereology '*Mereology* is the theory of parthood relations: of the relations of part to whole and the relations of part to part within a whole.', (*Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/mereology).

MIDI Acronym for Musical Instrument Digital Interface. A technology from the 1980s for transmitting information between electronic instruments. It is now used only as a legacy protocol. In this research it was used in the algorithmic improvisations with the harmonic fields generator to control the musical automatons of the *Logos Institute* in Ghent. It is also used sometimes to transcribe the algorithmic output of *SuperCollider* into musical notation.

morphe A term referring to the timbral aspect of form at the time scale of mid-level features (phrases, sequences, small sections). Is called like this because it corresponds to the morphological aspect of continuous profiles, not only in pitch but in other sonic parameters. It is what Tenney refers to as the morphological aspect of form and it is at this mid-level that this emerges most clearly. Its proportional counterpart is ***logos***, a term which among other things translates as 'pattern' thereby denoting 'structure' as well as 'logic'. Morphe is constituted by smaller scale *sonos*, or sounding 'elements' and 'clangs'.

multi-dimensional scaling analysis (mds)

A statistical technique for visualizing data in many dimensions into two or three which are easier to analyze. It takes any distance metric (such as harmonic metrics) preserving as much as possible in its dimensional reduction the relative distances between the elements in the data. It is useful for envisaging harmonic spaces of more than two dimensions.

musical means: geometric, arithmetic & harmonic

The three main ways of dividing intervals used in antiquity. For two intervals a and c, their mean b can be geometric, when $a/b = b/c$; arithmetic, when $a - b = b - c$; or harmonic, when $(a - b)/a = (b - c)/c$. The geometric mean gives irrational means for most intervals, while the arithmetic and harmonic are integer means which 'surround' the geometric, one from above and the other from below. The geometric mean of 1 and 2 (an octave) gives a tritone, the square root of 2; the arithmetic mean is a fifth 3/2 and the harmonic mean is a fourth 4/3. It is believed that the geometric and arithmetic means where discovered before the Pythagoreans, probably stemming from Babylonia, while Archytas, during the fourth century BC. is assumed to have developed the harmonic (or subcontrary) mean. See also ***logarithmicity/proportionality.***

***n*-ads** A generalization of a dyad or triad to include chords of *n* notes.

***n*-EDO, *n*-TET, *n*-ET**

Acronyms for *n*-Equal Division of the Octave, *n*-Tone Equal Temperament or *n*-Equal Temperament, the division of the octave into *n* steps of equal size.

***n*-limit tuning**

A tuning whose ratios contain no prime factors higher than *n*.

nomos The term denoting the proportional aspect of harmonic duality at the time scale of large scale form. Its counterpart is ***drama***. Its Greek meaning is 'law' and it is used in here to designate the orders which belong to a piece as a whole, not so much to its narrative and causal chains of musical forms (which is mainly what *drama* refers to), but, as it belongs to the structural (proportional) aspect, it characterizes the underlying principle of a piece. Its is made up of smaller scale *logoi*, that is, of 'patterns' and smaller scale structures.

***n*-tuple** A term used in mathematics to refer to a sequence or group of *n* elements.

octave equivalence

The strongest of harmonic relations, the fact that notes separated an by octaves tend to sound as 'the same'. It is evident and simple yet also startling on subsequent consideration. Many harmonic relations are enclosed within octave equivalence.

octave reduction The operation of disregarding octaves in harmonic analysis. By convention and simplicity it entails displacing a ratio into the octave that lies between 1 and 2.

parametric composition/ parametrization / parameter / variable

A kind of composition that flourishes from the 1950s onwards in which different aspects of sound ('parameters' or 'variables') become amenable to composition. It usually refers to serialism, although it also involves composers such as Iannis Xenakis in his formalization of music (and his introduction into music of density and other statistical parameters) as well as John Cage, whose **'total sound space'** conceives composition as happening in a space that encompasses the whole of possible musical variables. Today it is native to algorithmic and computer aided composition.

periodicity block/harmonic islands

A concept introduced by Adriaan Fokker to delimit in harmonic space blocks or parallelepipeds inside of which a complete harmonic world (or 'island' as I call it) is contained. There are many kinds of periodicity blocks in various dimensions, obtained by choosing the commas or equivalences between unisons ('unison-vectors') which delimit their borders. They are used in *DissonanceLib* to find best approximations of ratios into different kinds of temperaments as well as to separate intervals according to their harmonic and timbral functions depending on whether they lie inside or outside a specific block.

periodotopy The spatial arrangement of periodicity happening in the *Inferior Colliculus* (IC) in the mid brain and stemming from the temporal periodicity analysis of the signal performed in the *Cochlear Nucleus*. Both ***tonotopy*** and *periodotopy* are integrated into a three dimensional network structure in the IC.

pitch The perceptual property that allows ordering sounds on a frequency related scale. A definition which this study adheres to is the one by J. F. Schouten (1930s): 'The pitch ascribed to a complex sound is the pitch of that component to which the attention, either by virtue of its loudness or of its contrast with former sounds is strongest drawn. Therefore the pitch of a complex sound may be different depending upon the circumstances under which it is heard.'

pitch chroma, pitch height

Terms used in psychology to distinguish the two main attributes of intervals: chroma refers to their 'color' or character, height to their continuous pitch distance. Used by Roger Shepard for his helical pitch model. See section 1.2.4 for a description and commentary on these advantages and limitations of these concepts in their relation to harmonic duality.

PitchSet A class in *DissonanceLib* that is a collection of HarmonicVectors that are partitioned into timbral and harmonic subsets. It is also the basis on which to calculate a harmonic field. See section 3.1.6.

polarity (of an interval)

Which of the notes of a dyadic interval seems to be its root. If it is the lower note then the interval has a positive polarity, otherwise a negative one. In the case of a fifth, the lower note seems to have the most weight, which is why it has positive polarity. Its inverse, the fourth has negative polarity as the upper note feels as its root. Clarence Barlow has researched into this feature, incorporating it into his harmonic measure, *harmonicity*.

Polyrhythmia The name of an algorithm developed together with my colleague Alberto Novello for the segregation and amalgamation of rhythms. A base rhythm is input in spectral terms, understood as the stratification of simultaneous periodicities (each with its own phase and period). The algorithm consists of a rhythmic acceleration → steady-state → deceleration process in as many layers as the specified rhythm, transitioning between 3 fixed states: simultaneity of the rhythmic elements, the rhythm itself and back to simultaneity. The transitions between these states are made by means of accelerations and decelerations happening at a specific rate and in parallel for each 'rhythmic partial', so that each one falls smoothly into place in each of the terminating states. This algorithm is used in various of my *LogosSessions* improvisations, in *Blank Space* and in the piece composed jointly with Alberto, *Clinamen*. See section 2.1.2 and 4.1.2.3.

primary/secondary qualities

A term used in classical philosophy (coined by John Locke) to distinguish between those qualities which belong to the object (extension, weight, solidity, motion, number, figure) and those which pertain to the object as perceived (color, taste, smell and sound – including pitch and other attributes).

Pythagoreanism, *acusmatici, mathêmatici*

There are two strands of Pythagoreanism, the mystical and the scientific. The former lead an ascetic religious life while the latter are scientists concerned with the relation between mathematics, physics and music. Archytas is the prominent figure of the latter group, an sort of unsung hero of harmonic science that this study wishes to canonize.

qualia A term used in philosophy to denote the experience of a quality. An individual, subjective, conscious experience. 'The way things seem'.

rationalization The conversion into ratios of pitch sets given in terms of pitch-distance. It involves **tuning tolerance**, **harmonic metrics**, context, an implied harmonic system or aim and, in the last instance, educated choices. See section 3.1.5.

sensory dissonance/roughness

Roughness is an auditory sensation produced by amplitude fluctuations that occur due to constructive and destructive interference of sound waves. At low speeds the phenomenon manifests itself as beatings, at higher speeds as tremolo and when it is faster than around 16 cycles per second, as roughness. Helmholtz first researched the principles behind dissonance perception as amplitude fluctuations generated by spectral components of sound. It is mostly a psychoacoustic phenomenon happening in the basilar membrane, dependent on the alignments of partials with respect to the critical bandwidth. It is now referred to as *sensory dissonance* to distinguish it from other kinds of conceptions and, beside roughness, also involves the closeness of the spectrum to harmonic overtones, an aspect which is called *tonalness.* See section 2.1.3.

sonance When consonance/dissonance (or harmonicity/inharmonicity) are conceived as a continuum of gradations instead of being in antinomic relation. It involves a spectrum of 'sonances', or degrees of relative sonance (instead of absolute con/dis-sonance). This approach is characteristic of late 19^{th} and early 20^{th} century music's coloristic approach to harmony. It is a mainly timbral approach, based in gradations of hues with a focus on the sounding qualities of intervals. This type of harmony encompasses aspects of Tenney's CDC-2 – vertical, polyphonic – and CDC-5 – timbral, psychoacoustic – conceptions, including various degrees of fusion between proportionality and sound for its own sake. See section 1.3.3 and 2.2.

sonos A term used to describe the timbral aspect of harmonic duality at the small scale level of material. Counterpart to the proporional ***eidos***, which represents the way intervallic proportions emerge (indirectly), *sonos* refers to the sensory and directly timbral aspect of intervals: their spectra, and direct sounding qualities. Even though there is no direct Greek word *sonos*, it is used here to refer to the *sounding* facet of intervals.

spectral/periodicity/virtual pitch

Spectral pitch derives from the actual components of a sound in perception, usually referring to the lowest component F0. Periodicity pitch is ascribed from the global envelope created by the components in a sound and may be different from the spectral pitch. Virtual pitch is ascribed or induced by perception from the spectral pitches in some spatial models. This virtual pitch is a common harmonic of the partials in the spectrum.

stochastic harmonic field

The name I give to the method for generating intervals from a pitch set using the harmonic metrics of its intervals as probabilities with which to choose the intervals. It creates gradual, almost imperceptible transitions through the space of configurations contained in the pitch set. Its main parameter, the field's strength, is variable between zero (all intervals equally probable) and one (harmonic intervals more probable), producing a continuum of differing pitch configurations ranging from atonal to tonal. When the strength is reversed to reach minus one, priority is given to the least harmonic pitches, yielding a zone which I call ***antitonal***, for being relatively harmonic between the chosen intervals but highly inharmonic with respect to the overall fundamental. The generator can work in two modes: 'tonic', which relativizes the probabilities with respect to a single pitch in the set, providing a distinct modes, and 'atonic', which uses the the probabilities of all the modes, making each new chosen pitch the tonic with which to choose the next one. See section 3.2.

stochastic, stochastic music

A name given to non-deterministic processes. Introduced into composition by Iannis Xenakis who used different kinds of mathematical models, such as continuous probabilities, laws of thermodynamics, the Poisson law, Markov chains and others to compose music he called 'stochastic'. James Tenney also defines some of his music as stochastic, but in a sense of a constrained or directed random process. The term comes the Greek *stochos*, meaning 'aim' or 'target': 'a good image for the kind of textures that can arise is the pattern of hits on a target. They're clustered around a certain region, and within that region they are random, but they're not all over the place.'*

subminor, minor, neutral, major, supermajor

The possible varieties that intervals such as seconds, thirds and their inverses (sevenths and sixths) can have, depending on their size. See **diminished, just, augmented**.

SuperCollider A programming language for electronic sound synthesis and algorithmic composition from the late 1990s and early 2000s. Programmed originally by James McCartney, it is now a widely used open source project used for composing, sound art, installations and science research. *DissonanceLib* is available as library of functions that extends *SuperCollider*. It can be installed into it as part of its built in 'quarks' system for managing extensions. http supercollider.sourceforge.net/

symphona/diaphona

Consonance/dissonance in the sense of Greek harmonics (a melodic conception, corresponding to Tenney's CDC-1).

systema A continuous sequence of tetrachords makes a system, usually a two octave scale inhabiting no particular pitch range.

* Tenney, J. (1984). James Tenney in conversation with Udo Kasemets, Tina Pearson and Gordon Monahan. *MusicWorks,* 27(Spring), 10.

'the language of ratios'

A phrase given by Harry Partch that refers to the recovering of the harmonic properties of intervals in terms of proportions rather than in terms of their pitch distance to compensate for the limited variety engendered by twelve tone equal temperament.

***tonic* and *atonic* modes**

In the **harmonic fields** generator there are two ways of choosing intervals. *Tonic* reads the probabilities as relative to a pitch of the set, which becomes a sort of modal 'tonic'. 'Atonic' mode, by contrast makes every chosen interval the new tonic. Both modes sound very distinctive. See section 3.2.

'total sound-space'

The name given by John Cage to the perceptual 'space' where music exists. It encompasses all possible variations on 'characteristics' (or 'determinants') of sound. 'The situation made available by these [tape-recording] means is essentially a total sound-space, the limits of which are ear- determined only, the position of a particular sound in this space being the result of five determinants: frequency or pitch, amplitude or loudness, overtone structure or timbre, duration, and morphology (how the sound begins, goes on, and dies away). By the alteration of any one of these determinants, the position of the sound in sound-space changes. Any sound at any point in this total sound-space can move to become a sound at any other point . . . musical action or existence can occur at any point or along any line or curve . . . in total sound-space'[**] These determinants, 'variables' or 'parameters' constitute the 'dimensions' (Tenney) of this space. See also **parametric composition**.

tonotopy The arrangement of frequencies according to spatial distance, as happens in the basilar membrane and its neural projection upwards towards the auditory centers in the mid brain and higher. See also ***periodotopy***.

transduction In psychology it means the transmission or transformation of something from one form, place or concept to another. In physiology it is the conversion of one stimulus from one form or medium to another.

tuning tolerance

A phenomenon of fundamental harmonic importance. A fault-tolerance mechanism in pitch perception that permits the identity of an interval to withstand deviations from its exact tuning. It produces narrow nodes in pitch-distance space inside of which intervals preserve their character. Disregarding tolerance in proportionality would imply that slightly mistuned consonances would have to be represented by very big ratios, entailing very high inharmonicities, which is clearly not what happens. Tolerance permits this distortion, perceptually 'rounding off' to the nearest and strongest harmonic ratio, the amount resulting from the difference in tuning between the nominal and the actual ratio becoming its timbral coloration or residual ***clangtint***.

ultra-chromatic/ ultra-diatonic

Names given by Ben Johnston to the extension of harmonic materials in the

** Cage, J. (1961). Experimental Music. In *Silence.* Middletown, CT: Wesleyan University Press, 9.

harmonic lattice, either by encompassing larger areas within the same dimensions, such as extending classical Western 5-limit tuning to new far away regions not reachable with temperament, or by extending the dimensions of the lattice itself, that is, by introducing new fundamental intervals. The distinction between ultra-chromatic and ultra-diatonic can also be understood as in the cases of 53ET being the chromatic set of 31ET, in the same way as diatonic scales are subsets of 12ET.

vocoder An analysis-synthesis system used to codify speech, originally developed for telephone communications but with many uses in electronic music synthesis. It analyses a the amplitudes of a signal according to a filter bank and uses these amplitudes to pass another signal through it. This widely used technique was part of my piece *Circular Limit* for bass recorder and electronics. See section 3.2.2.

Summary

This research delves into contemporary approaches to musical harmony on the basis of algorithmic composition tools derived from psychoacoustics and microtonality. As an artistic, practice based research, it entails a cycle that involves programming, experimentation, composition and theorization. The experimentation itself comprises findings, reflections, tests, modifications, speculations, intuitions, surprises, and so on, successively. Which paths will produce the most fruitful findings cannot be determined in advance, leading as much to interesting discoveries as to dead ends. What is sought is to discern and comprehend some aspects of the pitch materials produced by the tools through notions such as *harmonic duality*, *harmonic space* and *harmonic fields*, ideas that provide resources for discovering new harmonic possibilities.

The written thesis is one of the outcomes of this research. It presents not only finished results but also bears witness to the way the research was conducted, showing the findings as they are encountered and reflecting the way they mold and influence the main research subject and questions. It begins by postulating the hypothesis of 'harmonic duality', according to which harmonic materials in music have an intertwined, double aspect: one relating to the character of pitch intervals, their proportionality, and the other pertaining to the high, low, dark and bright character of the pitches and sound qualities that comprise these intervals, their timbral facet. The first chapter sets the stage for understanding what this hypothesis means and how its two sides are embroiled. First it describes the hypothesis and the way it surfaces from a compositional practice, subsequently setting out to explore and read the topics of pitch perception and Greek harmonics in light of this duality, detailing and expounding its features and the way its facets are entangled while providing it with evidence and support. The chapter ends by revisiting Pythagoreanism in an attempt to recover some of its landmark ideas in relation to musical microtonal harmony, especially the ones involving the relation between number and perception and between the micro and the macro.

Most of the work has concentrated on compositional uses of dissonance curves, algorithmic devices that relate timbres to harmonic intervals, extracting microtonal pitch materials from sound spectra and analyzing them according to their timbral and harmonic properties. Putting them into motion through different rhythmic and textural procedures, these materials have been used to compose varied kinds of electronic and instrumental music, another principal outcome of this research. The analysis of the generated intervals has led to the study of their arithmetic features, interpreting them in 'harmonic space', a mathematical structure that helps characterize their properties and measure their harmonicity, ranking and separating pitch sets into distinct regions and contributing to the development of strategies for their deployment. The use of harmonic metrics also leads to the notion of 'harmonic fields', the algorithmic generation of pitches through probabilities. This leads to the third main outcome of this research, encapsulating many of these compositional experiments in computer programs and a code library of extensions for the composition and sound synthesis programming language *SuperCollider.*

Chapter 2 studies the timbral aspect of harmony. First it delves into the compositional description and uses of dissonance curves, also exploring the science behind them and some of their musical uses. A small section serves to wrap up most historical notions of consonance and dissonance in order to show their compositional possibilities. The second half of the chapter is an appraisal of twentieth century music in relation to pitch and the increasing emphasis that has been given to the timbral aspect of harmony, in close connection to the way that compositional materials have become ever more continuous at various time scales. It reads these uses from various styles and schools of compositions in an attempt to describe the current compositional situation with respect to

harmony and to suggest the incorporation of discrete elements and pitch proportions back into harmonic practices.

Chapter 3 inquires into the details of proportional harmony, employing ideas from several composers and theorists (Harry Partch, Ben Johnston, James Tenney, Clarence Barlow as well as Leonhard Euler and Adriaan Fokker) in order to treat some technical topics that involve pitch ratios: harmonic space, intervallic hues and fundamental intervals, commas, intervallic tolerance, harmonic metrics, visualization and some practical uses of the algorithmic tools developed by the author. The second section details the development of harmonic fields, the probabilistic generation of continuously varying harmonic textures, wrapping up many of the topics having been traversed during the study. It ends with some considerations on how harmony can be conceived in relation to form at various time scales.

Finally, Chapter 4 gives a recount of harmonic strategies used by the principal composers that have influenced this investigation (again, Tenney, Barlow and Johnston, additionally, Augusto Novaro and Ervin Wilson). Its second part explores the harmonic strategies behind some of the author's own compositions, showing how they fit into the research at large, sometimes emerging from it, while at other times altering the research as a consequence of the findings and experiences gathered in the music making process. The second half of the chapter closes the whole endeavor by discussing some of the speculative meanings of harmony and how they might produce insights that reach beyond what has been examined in the research. There is a consideration of noise in relation to harmony and a brief dissection of John Cage's notion of it. Some philosophical reflections of harmony follow, considering it as a coexistence of diverse elements and also as the relation between sameness and difference. After proposing an abstract definition of my own, the dissertation ends by shortly touching on randomness and the extraction of simple, small whole numbers, from within the complexity of the world.

The many terms stemming from disparate sources and disciplines (perceptual, mathematical, musical, philosophical) that have been either gathered or proposed during the study are given a separate treatment in the glossary, converging in a distilled section that isolates them as different notions that relate to harmony. Three appendixes follow, the first describing the technical details for the implementation of dissonance curves, the second showing a comparative table of sevenths, displaying the nuances involved in composing with seventh based intervals, and a third one showing the results of an investigation into an extended pitch set. This inquiry has been used as a stepping stone to compose one of the last pieces for this research, showing many timbral and proportional properties of these intervals: their nomenclature, accidentals, tempered approximations, degrees and functions, coupled with visualizations in harmonic space and a few examples of how they are used in instrumental writing.

Curriculum Vitae

Juan Sebastián Lach Lau was born October 3rd, 1970 in Mexico City. Since 1982 he begins playing the piano and is later involved with the instrumental jazz band *Psicotrópicos* (1986-1991) and rock band *Santa Sabina* (1991-2001), with whom he recorded 4 studio and 2 live cds and toured Mexico and other countries. He studied mathematics at the National Autonomous University of Mexico (UNAM, 1989-1992) and later studied with Alejandro Velasco at the Centro de Investigación y Estudios Musicales (CIEM, 1992-1996) earning a Bachelor's degree in composition. Studied piano with Julieta More Boone (1991-1998) and obtained the Advanced Degree in piano from the Associated Board of the Royal Schools of Music, London, in association with CIEM (1996). Also took composition lessons with Juan Trigos (1994-1996), Víctor Rasgado (1996-1998) and Ignacio Baca Lobera (1999-2001). In 2001 he travelled to Holland to study composition at the Royal Conservatory in The Hague with Gilius van Bergeijk and Clarence Barlow, also following the sonology course. He obtained a Bachelor's (2003) and Master's degree (2005) from this conservatory after which he began to follow the doctoral artistic research program docARTES in 2005, also taking a course in systematic musicology at the Institute of Psychoacoustics and Electronic Music (IPEM) in Ghent, Belgium. He returns to Mexico in 2008 where he since teaches composition and music analysis at Las Rosas conservatory in Morelia, Mexico.

He started composing for theatre, film, radio and cinema since the late eighties, also associating with theatre directors David Hevia and Juan José Gurrola. Since 1998 his instrumental compositions have been played in various countries and music festivals, performed, among others, by Cuarteto Latinoamericano, Camerata de las Américas, Ensamble 3 and Liminar, in Mexico, as well as Soil, Modelo 62, Klang, The Electronic Hammer and The Barton Workshop elsewhere. He has received support from the National Fund for Culture and Arts in Mexico to study in Holland (2001 & 2003) as well as as grant for recording a cd with his instrumental and electroacoustic music, to be released by Navona records in early 2013. His music has also been featured in other compilation cds. He has written and published an article with Alberto Novello in *Ideas Sónicas/Sonic Ideas* (2010) and two of his recent writings are soon to be published in *Pauta* and *Perspectiva Interdisciplinaria de la Música*. This year he has also seen him been busy performing, both as improviser in collaboration with other musicians and with in duo with Francisco Colasanto in *Dr. Zoppa*.

Samenvatting

Het onderhavige onderzoek gaat diep in op de aktuele benadering van de rol van harmonie in de muziek op basis van algoritmische componeermethoden en -materialen ontleend aan de psychoakoestiek en de microtonaliteit. Als onderzoek in de kunsten toont het een cyclus van programmeren, experimenteren, componeren en theorievorming. Het experimentele gedeelte zelf omvat achtereenvolgens bevindingen, reflecties, proefcomposities, aanpassingen, beschouwingen, ingevingen, verrassingen, enz. Welke strategieën de meest vruchtbare bevindingen zouden opleveren kon onmogelijk van tevoren worden bepaald, hetgeen tot zowel interessante ontdekkingen als doodlopende wegen heeft geleid. Het doel was enkele aspecten te kunnen onderscheiden en begrijpen van het compositorisch materiaal door middel van begrippen als harmonisch dualisme, harmonische ruimte en harmonische velden, begrippen die als hulpmiddel kunnen dienen bij het ontdekken van nieuwe harmonische mogelijkheden.

Dit proefschrift vormt een van de resultaten van dit onderzoek. Het presenteert niet alleen eindresultaten, maar getuigt tevens van de wijze waarop het onderzoek is uitgevoerd door de bevindingen te presenteren in het stadium waarin zij zich voordeden en door te laten zien hoe zij vorm gaven aan en invloed uitoefenden op het voornaamste onderzoeksobject en de belangrijkste onderzoeksvragen. Het proefschrift begint met het stellen van de hypothese van het 'harmonisch dualisme', inhoudende dat er binnen het harmonisch materiaal in muziek twee met elkaar verstrengelde aspecten aanwezig zijn: het ene betreft de aard van de intervallen tussen toonhoogten en hun onderlinge verhoudingen, het andere aspect heeft betrekking op de hoge, lage, donkere dan wel heldere aard van tonen en geluidseigenschappen waaruit die intervallen zijn opgebouwd, d.w.z. het aspect van hun klankkleur. Het eerste hoofdstuk bereidt de weg om te komen tot inzicht in de betekenis van deze hypothese en in hoezeer de twee aspecten met elkaar zijn verweven. Het opent met een beschrijving van de hypothese en hoe deze aan de praktijk van het componeren ontspringt, en onderzoekt vervolgens het dualisme van de thema's toonhoogteperceptie en de harmonieleer van het oude Griekenland met een gedetailleerde uiteenzetting van de kenmerken daarvan en de onlosmakelijke verknoping van al hun facetten, gestaafd met ondersteunend bewijsmateriaal. Aan het slot van het hoofdstuk wordt teruggekeerd naar de leer van Pythagoras in een poging enkele van diens baanbrekende ideeën inzake microtonale harmonie in ere te herstellen, met name die met betrekking tot de relatie tussen getal en perceptie en tussen micro en macro.

Het onderzoek was grotendeels gericht op het componeren met algoritmische hulpmiddelen die klankkleuren koppelen aan intervallen in de harmonische context, die microtonaal toonmateriaal uit geluidsspectra halen en deze analyseren aan de hand van harmonische en klankkleureigenschappen. Dit materiaal, in beweging gezet via verschillende procedures ten aanzien van ritmiek en tekstuur, werd gebruikt voor het componeren van uiteenlopende soorten elektronische en instrumentale muziek, hetgeen een ander belangrijk resultaat vormt van het onderzoek. Analyse van de aldus gegenereerde intervallen heeft geleid tot bestudering van de rekenkundige karakteristieken daarvan, door middel van interpretatie in de 'harmonische ruimte', een wiskundige model dat ondersteuning biedt bij het karakteriseren van de kenmerken van deze intervallen en het meten van hun harmoniciteit, waarbij toonreeksen worden gerangschikt en onderscheiden in afzonderlijke gebieden, en een bijdrage wordt geleverd aan de ontwikkeling van strategieën voor het gebruik daarvan. Het gebruik van harmonische metriek leidt daarnaast tot het begrip 'harmonische velden', het algoritmisch genereren van tonen op basis van waarschijnlijkheidsberekening. Dit heeft geleid tot een derde hoofdresultaat van het onderzoek: het opslaan van een groot aantal van deze compositie-experimenten in computerprogramma's en het creëren van een code library met extensies voor SuperCollider, de programmeertaal voor componeren en geluidssynthese.

In hoofdstuk 2 wordt het klankkleuraspect van harmonie bestudeerd. Eerst wordt ingegaan op de beschrijving en toepassingen van 'dissonance curves' voor het componeren, waarbij tevens de achterliggende technologie en enkele muzikale toepassingen worden onderzocht. In een korte paragraaf wordt een samenvatting gegeven van de meeste historische ideeën over consonantie en dissonantie teneinde de mogelijkheden daarvan bij het componeren te belichten. De tweede helft van het hoofdstuk bevat een beoordeling van de twintigste-eeuwse muziek in relatie tot toonhoogte in combinatie met de toenemende nadruk die daarbij werd gelegd op het klankkleuraspect van harmonie, in nauwe samenhang met de wijze waarop het basismateriaal van de componist in de loop van de tijd steeds meer continuïteit liet zien. Het behandelt het gebruik ervan in verschillende compositiestijlen en -scholen in een poging de huidige situatie in het componeren te beschrijven met name ten aanzien van het gebruik van harmonie, met de suggestie eventuele onsamenhangende elementen en toonhoogteverhoudingen weer te integreren in een harmonische praktijk.

In hoofdstuk 3 wordt gedetailleerd ingegaan op proportionele harmonie aan de hand van de ideeën van een aantal componisten en theoretici (Harry Partch, Ben Johnston, James Tenney, Clarence Barlow alsmede Leonhard Euler en Adriaan Fokker) met het oog op de behandeling van een aantal technische thema's op terrein van toonhoogteverhoudingen: harmonische ruimte, intervalkleuringen en basisintervallen, komma's, intervaltolerantie, harmonische metriek, visualisering en enkele praktische toepassingen van de door de auteur ontwikkelde algoritmische gereedschappen. De tweede paragraaf bevat een gedetailleerde behandeling van de ontwikkeling van harmonische velden, de probabilistische generatie van zich voortdurend wijzigende harmonische texturen, gevolgd door een samenvatting van de thema's die in de loop van het onderzoek aan de orde zijn gekomen. Het hoofdstuk besluit met enkele beschouwingen over hoe harmonie op diverse momenten kan worden bezien in relatie tot vorm.

Tot besluit wordt in hoofdstuk 4 een opsomming gegeven van de harmonische strategieën die zijn gehanteerd door de componisten die in belangrijke mate van invloed zijn geweest op dit onderzoek (wederom Tenney, Barlow en Johnston, en daarnaast ook Augusto Novaro en Ervin Wilson). Het tweede deel van dit hoofdstuk onderzoekt de harmonische strategieën die ten grondslag lagen aan enkele composities van de auteur, waarbij wordt aangegeven welke plaats deze innemen in het onderzoek. Soms zijn zij voortgekomen uit het onderzoek, terwijl zij in andere gevallen leidden tot wijziging in de richting van het onderzoek als gevolg van de bevindingen en ervaringen die werden opgedaan tijdens het scheppingsproces. De tweede helft van het hoofdstuk besluit met een bespreking van enkele van de speculatieve betekenissen van harmonie en hoe die inzichten zouden kunnen opleveren die verder reiken dan hetgeen in dit onderzoek is bestudeerd. Er is een beschouwing over geluid ('noise') in relatie tot harmonie en een korte analyse van de opvatting van John Cage over dat begrip. Dan volgen enkele filosofische bespiegelingen over harmonie, waarbij deze wordt gezien als een samen-zijn van ongelijksoortige elementen en ook als de relatie tussen overeenkomst en verschil. Na een voorstel voor een eigen abstracte definitie ervan besluit ik het proefschrift met een korte bespreking van de toepassing van toevalstechnieken en het ontlenen van eenvoudige, kleine gehele getallen aan de complexiteit van de wereld.

De vele termen − afkomstig uit diverse bronnen en disciplines (perceptueel, wiskundig, muzikaal, filosofisch) − die in de loop van het onderzoek zijn vergaard dan wel naar boven zijn gekomen worden apart behandeld in een termenlijst, die een soort bijlage vormt waarin de verschillende manieren waarop men harmonie kan bezien separaat zijn beschreven. Dan volgen drie bijlagen. De eerste bevat de technische gegevens voor de toepassing van 'dissonance curves'; de tweede toont een vergelijkende tabel van septiemen, waarin de kleine verschillen zijn te zien die spelen bij het componeren met op septiemen gebaseerde intervallen; en de derde toont de resultaten van het onderzoek naar een uitgebreide 'pitch set'. Dit onderzoek is gebruikt als springplank voor het componeren van een van mijn laatste stukken, en toont de vele klankkleur- en verhoudingskarakteristieken van de desbetreffende intervallen: hun nomenclatuur, verhogingen en verlagingen, subtiele benaderingen van hun 'temperatuur', toontrappen en functies, gekoppeld aan visualiseringen in de 'harmonische ruimte' en enkele voorbeelden hoe daarvan gebruik kan worden gemaakt bij het schrijven van instrumentale muziek.

Appendix I *Technical details of the implementation of dissonance curves.*

At first, the code for dissonance curves in *DissonanceLib* was based in Sethares, whose formula is expressed in terms of frequencies and amplitudes. By recommendation of Barlow, I adapted the code with Parncutt's mathematical approximation to Plomp and Levelt's weighting curve, which is adjusted to psychoacoustic units. It has the advantage of giving results with a slightly higher resolution as well as being around 30% faster to calculate. Both formulas are selectable, with Parncutt as the default. His approximation of the Plomp and Levelt weighting curve is:

$$P(x) = 4|x|e^{1-4|x|}$$ Exponential curve whose maximum is attained at 1/4.

From this the dissonance measure D for a pair of partials is derived:

$$D(p_1, p_2) = \sqrt{s_1 \cdot s_2} \cdot P(bk_1 - bk_2)$$

Where

$$p_n = (bk_n, s_n)$$

is a partial with a pitch *bk* in *barks* and amplitude *s* in *sones*.

The curve is calculated by measuring the dissonance D for every pair of partials between the fixed spectrum and the transposed one. Depending on its application, usually 6 to 10 partials are required for a spectrum. The sum of the individual measures between all partials gives the total roughness for that transposition, which represents one point on the curve:

$$R_T(t) = \sum_{\substack{j,k \\ 2 \leq j,k \leq n}} D(p_j, t \cdot p_k)$$ Total roughness for transposition t: the sum of D for all pairs of partials p_j against transposed partials $t \cdot p_k$ (j,k between 2 and n)

To obtain the whole curve the total roughness is calculated for each of the transposition intervals within the analysis range, sweeping the transposition in small steps, usually of a hundredth of an octave. In set notation:

$$C = \{R_T | t \in [t_0, t_1]\}$$ The curve is the set of all R_T in the interval $[t_0, t_1]$

A fair amount of calculations are required to obtain a curve: if we use 6 partials inside the range of an octave, D will need to be executed some 6 x 6 x 100 = 3600 times. After this the minima in the curve are detected and the intervals rationalized (comparing with interval tables of around a thousand intervals in an octave and picking the one with highest harmonicity within the tolerance range). From this point the harmonicities of the rationalized interval are calculated and then the intervals are collected as pitch sets which represent each interval as harmonic vectors in harmonic space, also partitioning them into harmonic and timbral subsets.

For real time uses, a fast Fourier analysis (FFT) of the input signal must be performed (usually with windows of 4096 or more samples to have a high frequency resolution). Once the spectrum is obtained (by triggering of some sort, either manually or automatically), the most prominent partials are selected and converted to *barks* and *sones* (with the option of compensating the amplitudes for masking) before the curve is calculated. From the time the curves where implemented up to now computers have become faster, so with *SuperCollider* (version 3.5) and a 3 year old laptop, a curve made out of 8 partials and the range of an octave does not take to calculate and rationalize more than 0.05 seconds.

Appendix II Comparative table of sevenths

ℚ	16/9	9/5	7/4	25/14	225/128	11/6	64/35	98/55	125/72
¢	996	1018	969	1004	977	1049	1045	1000	955
V	<4,-2>	<0,2,-1>	<-2,0,0,1>	<-1,0,2-1>	<-7,2,2>	<-1,-1,0,0,1>	<6,0,-1,-1>	<1,0,-1,2,-1>	<-3,-2,3>
polarity	-	-	+	+	+	+	-	-	+
ℍ	0.107	0.085	0.081	0.042	0.039	0.045	0.044	0.021	0.036
gS	9	9	9	16	20	14	17	28	20
Hd	4.97	3.807	3.332	5.858	10.268	4.19	7.714	8.592	9.105
mag	4.472	2.236	2.236	2.45	7.55	1.732	6.164	2.646	4.69
smooth	3	5	7	7	5	11	7	11	5
hues	2-3	3-5	2-7	2-5-7	2-3-5	2-3-11	2-5-7	2-5-7-11	2-3-5
function	2d	2Dm	S	2Ms	2D2M	dE	ms	m2Se	2d3M
	double subdominant	*submediant of double dominant*	*septimal*	*subseptimal of double mediant*	*double mediant of double dominant*	*eleventh of the subdominant*	*subseptimal of submediant*	*eleventh of double septimal of submediant*	*Triple mediant of double subdominant*
name	*Pythagorean minor 7th*	*Just minor 7th*	*Harmonic 7th*	*Middle minor 7th*	*Augmented 6th*	*Undecimal neutral 7th*	*Septimal neutral 7th*	*Quasi-equal minor 7th*	*Classic Augmented 6th*

ℚ = Ratio; ¢ = distance in cents; V = coordinate in the lattice; polarity = +/-; ℍ = harmonic intensity (absolute value of *harmonicity); gS* = *gradus suavitatis*; *Hd* = harmonic distance; mag = Euclidean distance to 1/1; smooth = highest prime limit; hues = prime mixtures; function and name.

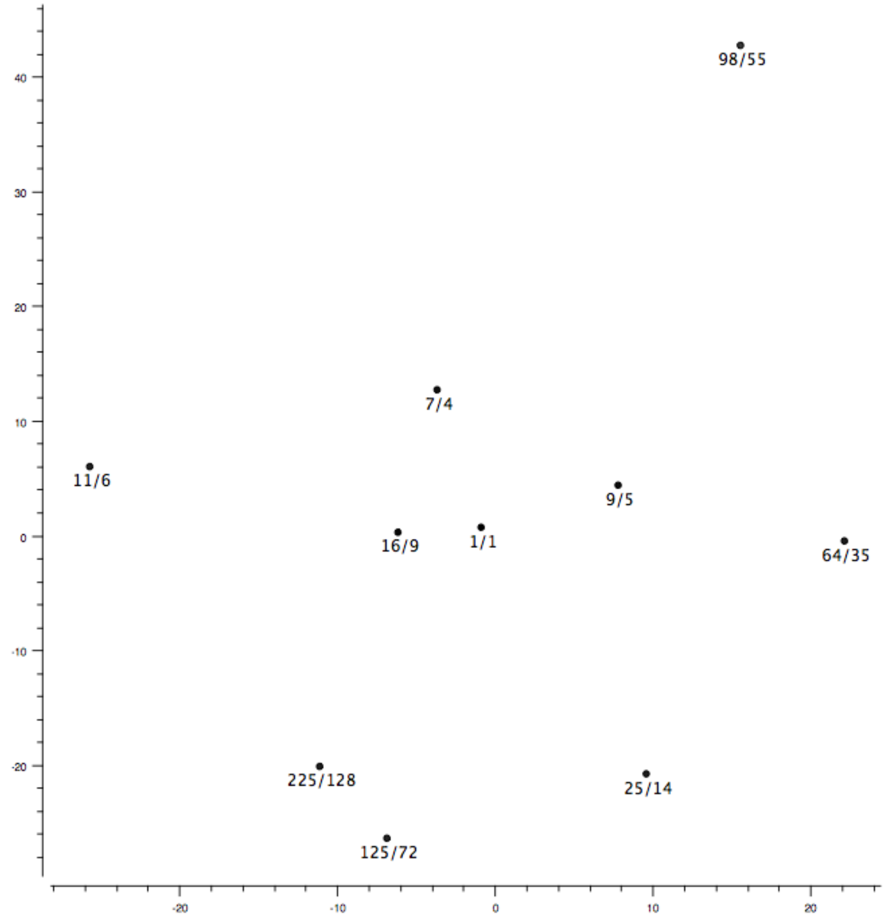
Partch argues for replacing the dissonant 9/5 for the consonant 7/4. He thinks one can postulate 7 by decree, in the same way that 5 was postulated to replace Pythagorean thirds, going even further to declare 11 as a veritable interval (going beyond '9'). ('*The problem of 7*' section in Partch, *op. cit.* p. 119). On the other hand, he acknowledges that 'the importance of an identity in tonality decreases as its number increases' (p. 126) requiring more exact intonation.

Barlow (*Bus Journey To Parametron*, p. 20) argues that 7/4 has a different function that a dominant seventh, because this would imply that the seventh over the dominant degree (in the tonality of C, the note F in the chord G-B-D-F) would be a 21/16 instead of a 4/3, also citing Johannes Fritsch, who in "*The Tonality of Harry Partch*", says "[t]hat the number seven really belongs to 'speculative harmony' and not to our triadic-based musical practice can be readily seen in our inability to sing 7-based intervals, even though they are to be found in every string instrument and in several wind instruments, as well as in important spectral formants in speech and in instrumental timbre".

Johnston also corroborates, 4:5:6:7 is not a dominant chord, it is stable, thus another kind of harmonic object than 4:5:6 plus either a 16/9 or a 9/5. Comparatively, 7/4 and 9/5 are not distinguished by *gS* and regarded inversely by *Hd* – for which 7/4 is slightly more harmonic than 9/5 – and *H* – whose reverse difference is even smaller. This despite the very different connotations of both intervals and their 35 ¢ difference in distance.

Regarding the other intervals in the table it is clear that most of them belong to other functions: augmented sixths, neutral sevenths (11/6 and 64/35 being quite close in terms of *H*, for which the complexity of function almost matches the complexity of the number 11, both also being very close in distance). They are followed by a middle minor 7th, 25/14 and two very distant and functionally complex 98/55 (the one regarded lowest by the *gS* and *H*) and 125/72. For *Hd*, the most harmonically complex correspond to augmented sixths. These last intervals are included to show how far they are from each other and from the functions usually associated with that region of pitch distance space (the semitone spanning from 950 to 1050).

These considerations can be seen in the multi dimensional scaling analysis (made using *harmonicity* as a metric). Of the three main minor sevenths discussed, it is clear that 9/5 and 16/9 are of inverse polarity (lying symmetrically around 1/1), the latter being the closest. 7/4 has an altogether different character and position. All the other candidates lie quite far, the distinction between neutral sevenths (11/6 and 64/35) being also their symmetric inversion.



Appendix III   Research into a septimal pitch set

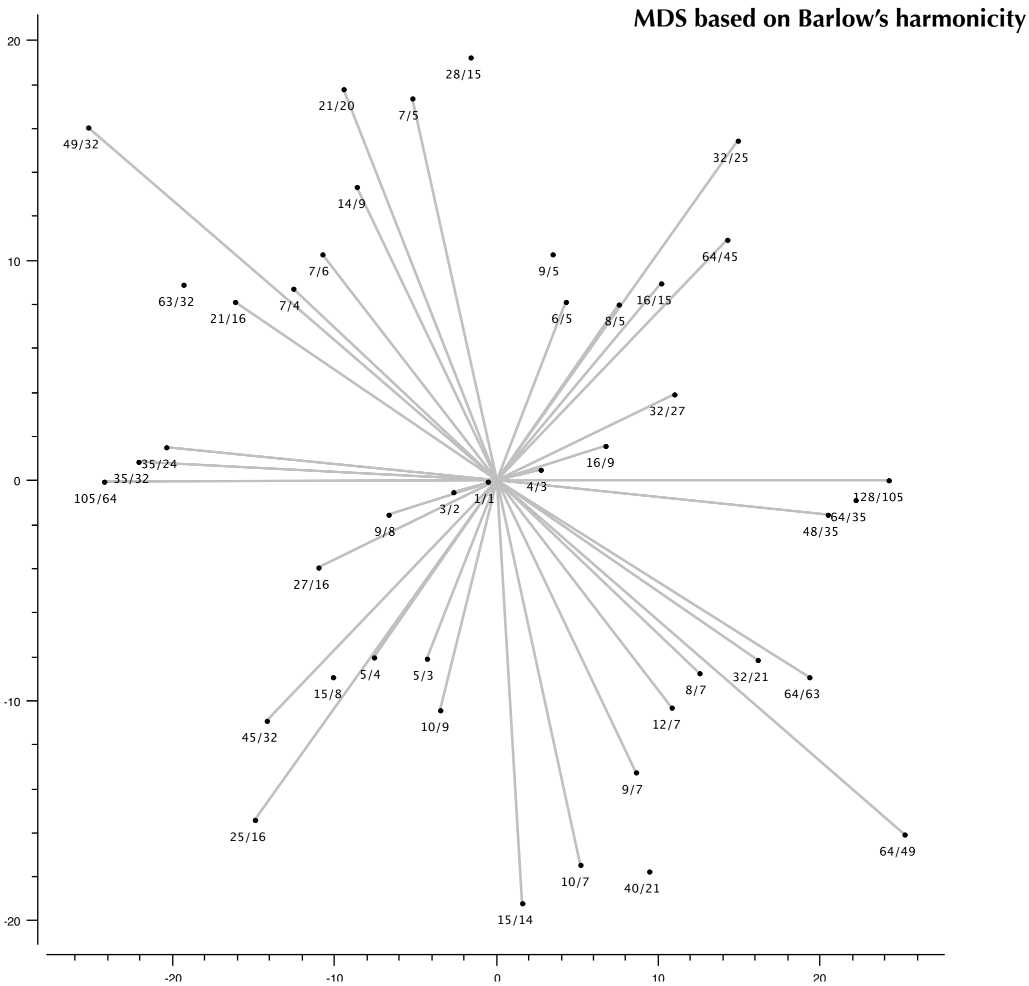
$\alpha_1 = 3$, $\alpha_2 = 5$, $\alpha_3 = 7$; exponents of the axes in harmonic space, ***h*** = the maximum value a coordinate can take. The number of intervals arising from combinations of each ***h*** is: $h_0 = 1$, $h_1 = 6$, $h_2 = 18$, $h_3 = 38$. Total number = 63. ***N*** = fundamental number; ***R*** = Ratio (octave reduced); ***¢*** = cents; ***k*** = degree number in 53 ET

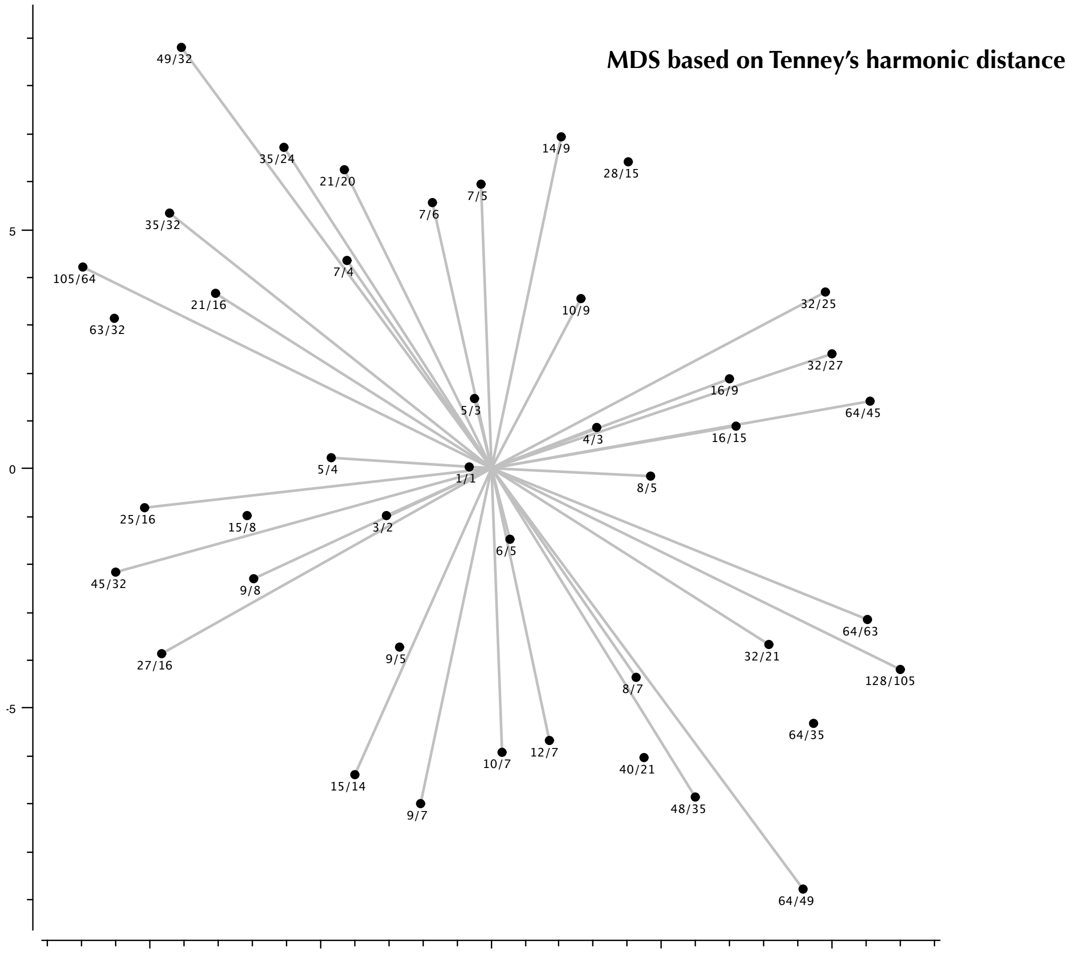
f = function: D/d, dominant, M/m, mediant, S/s, septimal; ***g*** = diatonic degree; ***acc*** = accidental (in Helmholtz-Ellis notation) ***H*** = *harmonicity*; ***Hd*** = harmonic distance; ***tmp*** = an alternative interval conflated by 53ET (black) or an interval of interest which lies very close and could replace the main one (blue)

h	α_1	α_2	α_3		*N*	*R*	*¢*	*k*	*f*	*g*	*acc*	*H*	*Hd*	*tmp*	**name**
0	0	0	0		1	**1/1**	0	0	T	I	♮	∞	0	-	unison
1	-1	0	0		1/3	**4/3**	498	22	d	IV	♮	-0.214	3.585	343/256	perfect four
	0	-1	0		1/5	**8/5**	814	36	m	VI	♭↑	-0.106	5.322	-	minor sixth
	0	0	-1		1/7	**8/7**	231	10	s	II	⌐	-0.075	5.807	-	major second
	0	0	1		7	**7/4**	969	43	S	VIIm	L♭	0.081	4.807	225/128 256/147	harmonic seventh
	0	1	0		5	**5/4**	386	17	M	III	♮↓	0.119	4.322	-	major third
	1	0	0		3	**3/2**	702	31	D	V	♮	0.273	2.585	512/343	perfect fifth
2	-2	0	0		1/9	**16/9**	996	44	2d	VII	♭	-0.107	7.17	25/14	minor seventh
	-1	-1	0		1/15	**16/15**	112	5	dm	II	♮↑♭	-0.077	7.907	15/14	minor second
	-1	0	-1		1/21	**32/21**	729	32	ds	V	L	-0.055	9.392	75/49	wide fifth (septimal)
	-1	0	1		7/3	**7/6**	266	12	dS	III	L♭	0.072	5.392	75/64	septimal minor third
	-1	1	0		5/3	**5/3**	884	39	dM	VI	♮↓	0.11	3.907	-	major sixth
	0	-2	0		1/25	**32/25**	427	19	2m	IV	𝄫↑↑	-0.056	9.644	9/7	diminished fourth
	0	-1	-1		1/35	**64/35**	1044	46	ms	VIIn	⌐♮↑	-0.044	11.13	11/6	neutral seventh
	0	-1	1		7/5	**7/5**	583	26	mS	V	L♭↑	0.06	5.129	45/32	septimal tritone (diminished fifth)
	0	0	-2		1/49	**64/49**	462	20	2s	III	⌐⌐	-0.038	11.61	13/10	septatonic major third
	0	0	2		49	**49/32**	737	33	2S	VI	LL	0.039	10.61	20/13	septimal minor sixth
	0	1	-1		5/7	**10/7**	618	27	Ms	IV+	⌐♯↓	-0.057	6.129	64/45	septimal tritone (augmented fourth)
	0	1	1		35	**35/32**	155	7	MS	IIn	L♮↓	0.046	10.13	12/11	septimal neutral second
	0	2	0		25	**25/16**	773	34	2M	V+	♯↓↓	0.06	8.644	14/9 11/7	augmented fifth
	1	-1	0		3/5	**6/5**	316	14	Dm	III	♭↑	-0.099	4.91	-	minor third
	1	0	-1		3/7	**12/7**	933	41	Ds	VI	⌐	-0.067	6.392	128/75	septimal major sixth
	1	0	1		21	**21/16**	471	21	DS	IV	L	0.059	8.392	64/49	septimal narrow fourth
	1	1	0		15	**15/8**	1088	48	DM	VII	♮↓	0.083	6.907	28/15	major seventh
	2	0	0		9	**9/8**	204	9	2D	II	♮	0.12	6.17	28/25	major whole tone

h	α1	α2	α3	N	R	¢	k	f	g	acc	H	Hd	tmp	name
3	-3	0	0	1/27	**32/27**	294	13	3d	IIIm		-0.077	9.755	13/11	Pythagorean minor third
	-2	-1	0	1/45	**64/45**	610	27	2dm	V-		-0.056	11.492	10/7	2nd tritone (just diminished fifth)
	-2	0	-1	1/63	**64/63**	27	1	2ds	'		-0.046	11.977	49/48 81/80	Arquitas' comma (7-3 comma)
	-2	0	1	7/9	**14/9**	765	34	2dS	VIm		0.06	6.977	25/16 11/7	septimal minor sixth
	-2	1	0	5/9	**10/9**	182	8	2dM	II		0.079	6.492	-	minor whole tone
	-1	-2	0	1/75	**128/75**	925	41	d2m	VII-		-0.045	13.229	12/7	diminished seventh
	-1	-1	-1	1/105	**128/105**	343	15	dms	IIIn		-0.038	13.714	11/9	septimal neutral third
	-1	-1	1	7/15	**28/15**	1081	48	dmS	VII		0.047	8.714	15/8	grave major seventh
	-1	1	-2	1/147	**256/147**	960	42	d2s	VI		-0.032	15.2	7/4 225/128	[doubly septimal large sixth]
	-1	0	2	49/3	**49/48**	36	2	d2S	IIm		0.037	11.2	128/125	slendro diesis, 1/6-tone (7^{2}-3^{-1} diesis), {subminor second}
	-1	1	-1	5/21	**40/21**	1116	49	dMs	VII		-0.045	9.714	243/128 21/11	acute major seventh
	-1	1	1	35/3	**35/24**	653	29	dMS	Vsd		0.045	9.714	256/175 16/11	septimal semi-diminished fifth
	-1	2	0	25/3	**25/24**	71	3	d2M	I+		0.054	9.229	256/245	classic chromatic semitone, minor chroma {augmented unison}
	0	-3	0	1/125	**128/125**	41	2	3m	diesis		-0.038	13.966	49/48	minor diesis, diesis {unison}
	0	-2	-1	1/175	**256/175**	659	29	2ms	V-		-0.032	15.451	35/24 16/11	{septimal diminished fifth}
	0	-2	1	7/25	**28/25**	196	9	2mS	III-		-0.04	9.451	9/8	middle second {septimal diminished third}
	0	-1	-2	1/245	**256/245**	76	3	m2s	limma		-0.029	15.937	25/24	{7^{-2}-5^{-1} limma (a syntonic + 2 septimal commas) }
	0	-1	2	49/5	**49/40**	351	16	m2S	IV-		0.033	10.937	11/9 21/17	larger approximation to neutral third {double septimal diminished fourth}
	0	0	-3	1/343	**512/343**	694	31	3s	IV+		-0.025	17.422	3/2	3 septatones or septatonic fifth [large triple septatonic augmented fourth} **
	0	0	3	343	**343/256**	506	22	3S	IV		0.026	16.422	4/3	{triple septatonic diminished fifth}
	0	1	-2	5/49	**80/49**	849	37	M2s	VIn		-0.032	11.937	105/64 18/11	(double septimal) smaller approximation to neutral sixth
	0	1	2	245	**245/128**	1124	50	M2S	VIII-		0.029	14.937	48/25	{septimal diminished octave}
	0	2	-1	25/7	**25/14**	1004	44	2Ms	VI+		0.042	8.451	16/9	middle minor seventh {septimal augmented sixth}
	0	2	1	175	**175/128**	541	24	2MS	IV+		0.033	14.451	48/35 11/8	{septimal semi augmented fourth}
	0	3	0	125	**125/64**	1159	51	3M	VII+		0.04	12.966	96/49 35/18 49/25	classic augmented seventh, octave - minor diesis {triple tertial octave}
	1	-2	0	3/25	**48/25**	1129	50	D2m	VIII-		-0.051	10.229	245/128	classic diminished octave
	1	-1	-1	3/35	**48/35**	547	24	Dms	IV+		-0.043	10.714	175/128 11/8	septimal semi-augmented fourth
	1	-1	1	21/5	**21/20**	84	4	DmS	IIm		0.047	8.714	22/21	(septimal) minor semitone
	1	0	-2	3/49	**96/49**	1164	51	D2s	VII+		-0.035	12.2	125/64	{double septatonic large seventh}
	1	0	2	147	**147/128**	240	11	D2S	III-		0.033	14.2	8/7	{double septatonic diminished third}
	1	1	-1	15/7	**15/14**	119	5	DMs	I+		-0.049	7.714	16/15	[augmented unison] major diatonic semitone
	1	1	1	105	**105/64**	857	38	DMS	VIn		0.039	12.714	80/49 18/11	septimal neutral sixth
	1	2	0	75	**75/64**	275	12	D2M	II+		0.047	12.229	7/6	classic augmented second
	2	-1	0	9/5	**9/5**	1018	45	2Dm	VIIm		-0.085	5.492	-	just minor seventh
	2	0	-1	9/7	**9/7**	435	19	2Ds	III		-0.064	5.977	32/25	septimal major third
	2	0	1	63	**63/32**	1173	52	2DS	VIII		0.048	10.977	160/81	octave minus septimal comma
	2	1	0	45	**45/32**	590	26	2DM	IV+		0.06	10.492	7/5	diatonic tritone
	3	0	0	27	**27/16**	905	40	3D	VI		0.083	8.755	22/13	Pythagorean major sixth

Multidimensional scaling of the septimal pitch set



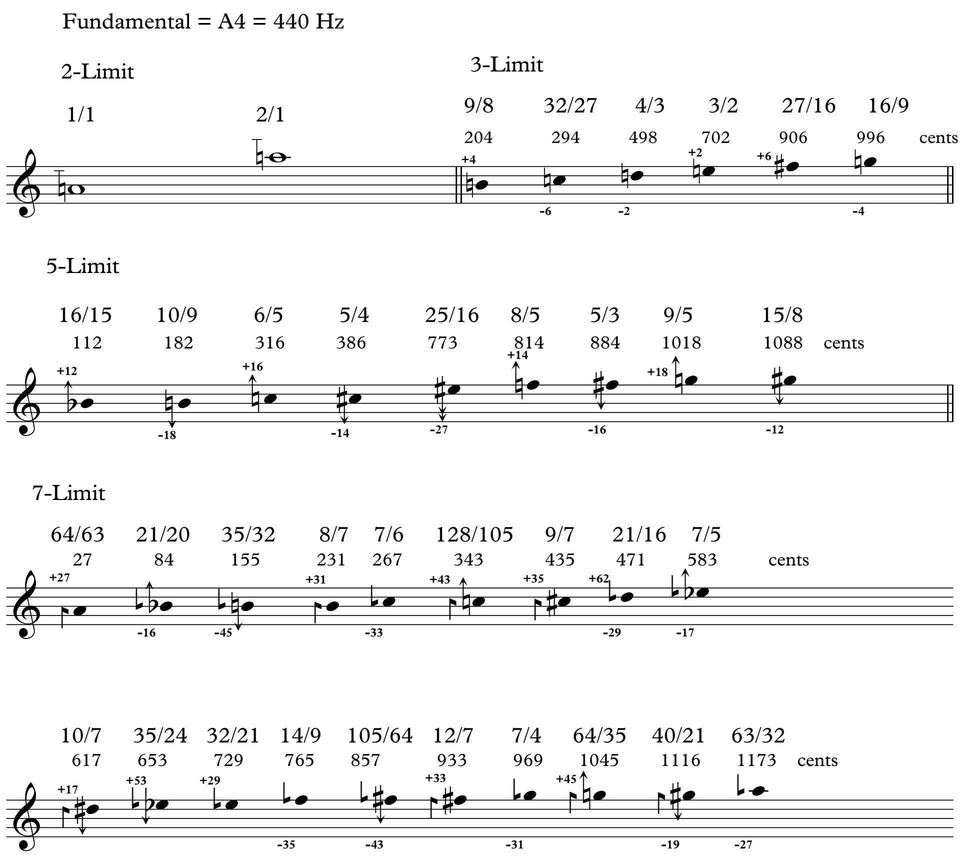


The above table lists the properties of every interval of a septimal pitch set generated from coordinates in harmonic space. The intervals are listed according to their maximum coordinate in each of the main intervallic axes (the ***h***), ordered by their combination of coordinates in harmonic space (the α's). ***N*** is the fundamental number(s) behind the ratio (the mixtures of prime intervals, disregarding octaves), ***R*** (ratio) is the octave reduced proportion. ***¢*** is the interval's pitch distance in cents, ***k*** gives the approximation of the interval in 53 equal temperament, ***f*** is its harmonic function (T = tonic, D/d = dominant/subdominant, M/m = mediant/submediant, and S/s = septimal/subseptimal); ***g*** is the interval's diatonic degree, ***acc*** is the interval's Helmholtz-Ellis accidental corresponding to that degree. ***H*** and ***Hd*** harmonicity and harmonic distance. ***tmp*** shows other interesting interval ratios lying close to that interval; when the ratio is in blue, it means that a higher prime ratio (of 11 or 13) of interest could also be used; when the ratio is black, it means that another ratio from the pitch set is conflated into the same 53ET degree as the current one. Finally the last column shows the interval's name, first according to the Huygens-Fokker nomenclature, and, in cases where the name does not appear in that list or differs from its proper scale degree and function, the name it should have, based on its harmonic properties, is shown in curly brackets.

Below the table, two figures show multidimensional scaling analyses based on *harmonicity* and *harmonic distance*, giving a visual glimpse of the positions of the intervals in harmonic space.

The research has been the departure point for the final piece written for Ensamble Modelo 62. The pitch set has been adapted based on the microtonal possibilities of the ensemble, which involves scordaturas and special positions for the string instruments and guitar, or special fingerings for the flute. Piano and clarinet play for the most part 3-limit intervals which do not require retuning. For these idiomatic reasons as well as the fact that many intervals lie very closely together, the pitch set was reduced to a more practical and musically useful one. It is shown below, ordering the intervals in pitch distance space, separated into subsets of prime partitions (2,3,5 and 7-limit).

Also shown in the page that follows below is an example of the scordatura for the strings. It corresponds to the fourth string of the violin, violoncello and contrabass, showing the retuning and positions of nodes positions required to obtain many of the sought intervals, especially 7- and 5-based ones (1/7 and 1/5 to be precise). The harmonics and pressed positions correspond to multiples and relatives of the open string tuning with respect to the global fundamental of A 440 Hz. This is shown for the purpose of exeplifying how the intervals were deployed in a practical composition. The explanation for the musicians is also shown.



Each staff corresponds to a string and its natural harmonics, grouped according to their overtone number (2=octaves, 3=fifths, 5=major thirds, 7=harmonic sevenths). In addition to the nodes for playing these natural harmonics, it also indicates notes which are played at the position of the nodes but which are to be pressed in order to produce real notes corresponding to harmonic ratios.

At the left of the staff is the string number and its scordatura. One of each of the instruments' strings is detuned to correspond to some harmonic or subharmonic of A440, which is the fundamental for the whole piece. The strings which do not need scordatura are labelled with a small cent deviation (-4 cents for G, −2 for D, +2 for E, 0 cents for A). They do not need to be exactly retuned (although it is recommended), for the deviations are too small to be noticeable. However, the strings to be detuned by more than 4 cents are shown with a box around their label. The double bass' 4th string (E) is tuned up 114 cents to a F +14¢ which corresponds to the fifth subharmonic; the cello's fourth is moved down 112 cents to a B -12¢ which is the seventh subharmonic, and the violin's third string is moved down 114 cents to a C# -14¢ which is the fifth harmonic.

Each staff has an upper ossia that shows the resulting sound as well as the harmonic proportion with respect to A440. The accidentals are based on the Helmholtz Ellis notation (adagio.calarts.edu/~msabat/ms/pdfs/notation.pdf) which has special symbols for syntonic commas (small arrows for 5-Limit intervals) and septimal commas (7-limit with a symbol similar to a 7 and an inverted 7). Above or below each note is a small number indicating the cent deviation from the closest equal tempered note.

The lower staff shows how the notes are to be played, starting with the open string and including both the nodes for natural harmonics as well as the notes that sound when these nodes are fully pressed to produce notes at that position (this is labelled with 'pressed node'). These notes correspond to a whole number division of the string and therefore to harmonic ratios over the string's fundamental. There is a small number which labels the node number, a small cipher next to the diamond notes in the divisions of 5 and 7. Together with the fact that most of these strings are themselves harmonic ratios of the A440 fundamental, it allows a way of tuning many far away ratios of A440 with high accuracy and a relatively low difficulty in finding their positions in the fingerboard.

This is a table for composing as well as for rehearsing how to play these notes. In two cases (the Vlc and CB 4th string), harmonics go up to the 9th. An electronic tuner is recommended for testing the factibility of these notes.

The last page contains the pitch set for the whole piece. The intervals are in the middle octave (between 1/1 and 2/1) and arranged, first according to their prime intervals (2, 3, 5 and 7) and within each grupo according to their pitch distance. This pitch set is for reference showing the intervals for the whole piece as well as their distance in cents with respect with A440.