Manipulation of a Two-Photon Pump in Superconductor-Semiconductor Heterostructures

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(Received 26 August 2013; published 20 February 2014)

We investigate the photon statistics, entanglement, and squeezing of a *p-n* junction sandwiched between two superconducting leads and show that such an electrically driven photon pump generates correlated and entangled pairs of photons. In particular, we demonstrate that the squeezing of the fluctuations in the quadrature amplitudes of the emitted light can be manipulated by changing the relative phase of the order parameters of the superconductors. This reveals how macroscopic coherence of the superconducting state can be used to tailor the properties of a two-photon state.

DOI: 10.1103/PhysRevLett.112.077003 PACS numbers: 74.45.+c, 74.50.+r, 78.67.-n

The realization of solid-state photon sources that are capable of on-demand generation of entangled photon pairs is highly desired for quantum-information processing and communication [1], as well as for high-precision measurements [2–4]. Such two-photon processes are inherently nonclassical; i.e., they cannot be expressed naturally in terms of simple coherent states. Pairs of entangled photons are routinely generated by parametric down-conversion [5,6]. In this approach, a laser pumps a nonlinear optical crystal, leading to extremely low overall conversion efficiencies from electrons pumped into the laser to photon pairs out of the nonlinear crystal. These obstacles could be overcome by the two-photon counterpart of a light-emitting diode (LED), i.e., a device into which electrons are injected and which emits entangled photon pairs directly, leading to squeezed light. The overall quantum efficiency of such a device could be close to unity. Recently, entanglement of an electrically driven source of photon pairs was demonstrated, based on the recombination of biexcitons [7]. Cooper pairs are an alternative to biexcitons. In both cases, one expects that upon radiative recombination, the entanglement of the electrons is inherited by the two-photon final state. In distinction to biexcitons, Cooper pairs form coherent two-electron states that scatter weakly among each other, are only weakly affected by impurities, and, quite crucially, can be manipulated externally, e.g., using SQUID geometries, Andreev reflection at applied magnetic fields, or electrically tunable Josephson coupling. The proximity effect at superconductor-semiconductor junctions was indeed demonstrated for InAs/InAlAs coupled to niobium [8]. This is in accordance with the theoretical prediction [9,10] and observation [11–14] of an enhanced luminescence rate at such an interface. The exciting physics of a Josephson LED was discussed in the context of quantum-dot [15–18] and solid-state-based devices [19].

In this Letter, we show that a superconductor-LED-superconductor heterostructure is a source of nonclassical light and, most importantly, demonstrate how one can manipulate the two-photon coherence by varying the relative phase between the two superconductors that are coupled to the *p-n* junction. The key physical idea of our theory can be rationalized as follows: the nonequilibrium dynamics of the photon pump can be described in terms of an effective photon Hamiltonian that is similar to the Hamiltonian of a quantum parametric amplifier:

$$H_{\rm PA} = \omega b^\dagger b + (\zeta e^{-ieV_0 t} b^\dagger + i\gamma e^{-i2eV_0 t} b^\dagger b^\dagger + {\rm H.c.}), \tag{1}$$

where $b^\dagger$ is a photon creation operator, ω is the photon frequency (we set $\hbar = 1$), and the coefficients ζ and γ arise from pumping photons electronically via superconducting leads with applied potential difference eV_0. The resulting

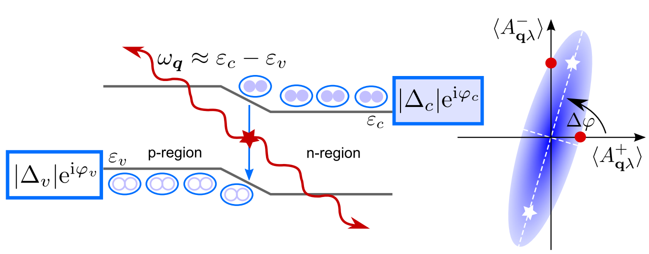

FIG. 1 (color online). The schematic setup of a squeezed light-emitting diode consists of a *p-n* junction with proximity induced superconductivity present in both valence (v) and conduction (c) bands. Electronic coherence of Cooper pairs is transferred to the photons, leading to two-mode squeezing of the quadrature operators $A^{\pm}_{\mathbf{q}\lambda}$ in vacuum controlled by the relative phase of the two superconductors $\Delta\varphi = \varphi_c - \varphi_v$. Entangled photon pairs of frequency of the order of the band gap $\omega_{\mathbf{q}} \approx \varepsilon_c - \varepsilon_v$ are emitted in the active region (red star).

photon state of such a system is squeezed $|\psi_{\rm photon}(t)\rangle \sim \exp(\gamma t b^\dagger b^\dagger - \gamma^* t b b)|0\rangle$ [20]. We show that the pair production amplitude $\gamma \sim |\Delta_v||\Delta_c|e^{i\Delta\varphi}$ is determined by the gap of the two superconductors (see Fig. 1) and depends on the phase difference $\Delta\varphi$ between them. Thus, by changing the relative phase $\Delta\varphi$ of the Cooper-pair wave functions, e.g., via an external field in a SQUID geometry, one can control the squeezing in a detuned parametric amplifier [21]. This demonstrates how the coherence of the Cooper pair, together with the macroscopic coherence of the superconducting state, can be used to tailor the properties of a two-photon state. In what follows, we substantiate this qualitative picture by explicit many-body calculations, show that the squeezing of the resulting photon state can indeed be manipulated by changing $\Delta\varphi$, and determine the two-photon correlation function that emerges as a result of the superconducting coherence.

The setup of our system is sketched in Fig. 1. We couple a p-n junction on each side to superconducting leads and apply an external voltage. The system is characterized by the Hamiltonian

$$H = H_{\rm ph} + H_c + H_v + \sum_{\mathbf{k},\mathbf{q},\sigma,\lambda} (g b^\dagger_{\mathbf{q}\lambda} v^\dagger_{\mathbf{k}-\mathbf{q}\sigma} c_{\mathbf{k}\sigma} + \text{H.c.}), \quad (2)$$

where $H_{\rm ph} = \sum_{\mathbf{q},\lambda} \omega^0_{\mathbf{q}} b^\dagger_{\mathbf{q}\lambda} b_{\mathbf{q}\lambda}$ is the bare photon Hamiltonian. We assume emission of linearly polarized photons with $\lambda = \pm$, but the case of circular polarization, such as that which occurs, for example, in GaAs due to spin-orbit coupling [16], is a straightforward generalization of our model. The electronic part $H_c + H_v$ describes the leads consisting of a superconducting conduction band on the right, with creation operators $c^\dagger_{\mathbf{k}\sigma}$, band dispersion $\varepsilon_{c\mathbf{k}}$ and proximity induced superconducting gap Δ_c, and a superconducting valence band on the left, with creation operators $v^\dagger_{\mathbf{k}\sigma}$, band dispersion $\varepsilon_{v\mathbf{k}}$, and superconducting gap Δ_v. If we include the respective chemical potentials μ_c and μ_v with $\mu_c - \mu_v = eV_0$, given by the applied voltage V_0, we have ($\alpha = c$ or v)

$$H_\alpha - \mu_\alpha N_\alpha = \sum_{\mathbf{k}\sigma} (\varepsilon_{\alpha\mathbf{k}} - \mu_\alpha) \alpha^\dagger_{\mathbf{k}\sigma} \alpha_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\Delta_\alpha \alpha^\dagger_{\mathbf{k}\uparrow} \alpha^\dagger_{-\mathbf{k}\downarrow} + \text{H.c.}). \quad (3)$$

The coupling between photons and electrons is described by a coupling constant g and leads to emission of a photon for each electron transition from conduction to valence band. We give an estimate for g in the Supplemental Material [22].

We first derive an effective photon Hamiltonian $H_{\rm eff}$ for this heterostructure. Technical details are provided in the Supplemental Material [22]. Its purpose is to elucidate the nature of the photon dynamics and to obtain a tool to investigate the properties of the photon subsystem.

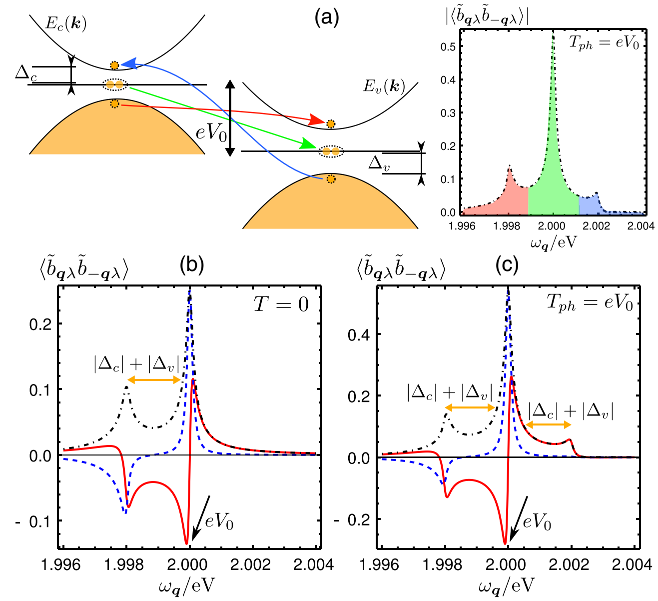

FIG. 2 (color online). (a) Three processes that give rise to the peaks in $|\langle \tilde b_{\mathbf{q}\lambda} \tilde b_{-\mathbf{q}\lambda}\rangle|$ are quasiparticle tunneling from conduction to valence bands and the reverse process, leading to the peaks at $\omega_{\mathbf{q}} = eV_0 \pm (|\Delta_c| + |\Delta_v|)$, as well as tunneling of Cooper pairs, which gives the peak at $\omega_{\mathbf{q}} = eV_0$. (b),(c) Photon coherence $\langle \tilde b_{\mathbf{q}\lambda} \tilde b_{-\mathbf{q}\lambda}\rangle$ at time $t = 0$ and at (b) zero temperature and (c) finite (photon) temperature $T_{\rm ph} = eV_0$. Other parameters are chosen as $eV_0 = 2\text{eV}$, $\Delta_c = 1$ meV, $\Delta_v = 1$ meV, $\Delta\varphi = 0$, $g^2\rho_c = \Delta_c/20$, and $\eta = 0.1$ meV. The plot shows the real part (solid red line), imaginary part (dashed blue line), and absolute value (dash-dotted black line).

The effective Hamiltonian is designed to generate, up to second order in perturbation theory, the same Heisenberg equations of motion for the photonic operators as the full Hamiltonian. We thus determine the equations of motion for the photon operators $b^\dagger_{\mathbf{k}\sigma}(t)$ perturbatively in the photon-electron coupling constant g and deduce $H_{\rm eff}$ from them. We deal with a nonequilibrium problem in steady state. The external bias voltage that drives the system out of equilibrium leads to time-dependent phases in the effective Hamiltonian, and we obtain

$$H_{\rm eff} = \sum_{\mathbf{q},\lambda} \{\omega_{\mathbf{q}} b^\dagger_{\mathbf{q}\lambda} b_{\mathbf{q}\lambda} + [g e^{-ieV_0 t} \zeta_{\mathbf{q}}(t) b^\dagger_{\mathbf{q}\lambda} + g^2 e^{-2ieV_0 t} \gamma_{\mathbf{q}} b^\dagger_{\mathbf{q}\lambda} b^\dagger_{-\mathbf{q}\lambda} + \text{H.c.}]\}. \quad (4)$$

The effective Hamiltonian $H_{\rm eff}$ has the form of the Hamiltonian of the quantum parametric amplifier in Eq. (1) and describes electronic pumping of photons via coupling to superconducting leads. It consists of three parts that correspond to different aspects of the junction.

The first part is the photon energy renormalized by the interaction $\omega_{\mathbf{q}} - \omega^0_{\mathbf{q}} \propto |g|^2$, which is of no fundamental importance to our analysis.

The second term describes the effect of the device being a source of single photons. It also occurs in the normal state, where it describes radiation of the usual light-emitting diode and produces single photons at a constant rate. In the presence of superconducting leads and the macroscopic electronic coherence, however, this term also contributes to the emergence of two-photon coherence. The different physical processes that contribute to the coherence are depicted in Fig. 2(a). The "coefficients" $\zeta_{\mathbf{q}}(t)$ contain fermionic creation and annihilation operators. As a consequence of the nonequilibrium state, these operators depend on the initial values of the fermionic operators well in the past, where we assume that the system was decoupled and in local equilibrium. They act as random external (noncommuting) fields, which arise from the coupling of the photons to the fermionic bath, and commute with the photon operators $[b_{\mathbf{q}\lambda}, \zeta^\dagger_{\mathbf{q}}] = 0$ but not with their Hermitian conjugate $[\zeta_{\mathbf{q}}, \zeta^\dagger_{\mathbf{q}}] \neq 0$. At $T = 0$ in the superconducting state, they are defined by correlators such as (for details, consult the Supplemental Material [22])

$$\int_{-\infty}^{t} dt' dt'' \langle \zeta_{\mathbf{q}}(t') \zeta_{-\mathbf{q}}(t'') \rangle e^{i(\omega_{\mathbf{q}} - eV_0 - i0^+)(t'+t'')}$$
$$= -2 \sum_{\mathbf{k}} \frac{u_{c\mathbf{k}} v_{c\mathbf{k}} u^*_{v\mathbf{k}} v^*_{v\mathbf{k}} e^{2i(\omega_{\mathbf{q}} - eV_0)t}}{(\omega_{\mathbf{q}} - eV_0 - i0^+)^2 - (E_{c\mathbf{k}} + E_{v\mathbf{k}})^2}. \quad (5)$$

They contain the superconducting energy dispersions $E_{\alpha\mathbf{k}} = \sqrt{(\varepsilon_{\alpha\mathbf{k}} - \mu_\alpha)^2 + |\Delta_\alpha|^2}$ and the BCS coherence factors $u_{\alpha,\mathbf{k}}$ and $v_{\alpha,\mathbf{k}}$, and we have neglected corrections to the quasiparticle energies by photon momenta $E_{\alpha\mathbf{k}+\mathbf{q}} \approx E_{\alpha\mathbf{k}}$. Most important is the third term, where at $T = 0$ holds

$$\gamma_{\mathbf{q}} = \sum_{\mathbf{k}, s=\pm} \frac{-\frac{1}{2} s u_{c\mathbf{k}} v_{c\mathbf{k}} u^*_{v\mathbf{k}} v^*_{v\mathbf{k}}}{(\omega_{\mathbf{q}} - eV_0 - i0^+) + s(E_{c\mathbf{k}} + E_{v\mathbf{k}})}. \quad (6)$$

This term is responsible for the fact that H_{eff} does have the form of a parametric amplifier, producing entangled photon pairs. Entanglement is meant in the sense that the emitted photon pairs have opposite momentum and the same chirality. If the coefficients $\zeta^\dagger_{\mathbf{q}}$ and $\zeta_{\mathbf{q}}$ were numbers and would not contain fermionic operators of the initial state, we would immediately see that Eq. (4) describes a system that produces squeezed light [20]. The subsequent analysis shows that this is still the case, even with the more complicated form of H_{eff}.

To analyze whether the emitted light is squeezed, we determine the uncertainties of the quadrature amplitudes of the electric field

$$\mathbf{E}(\mathbf{x}, t) = \sum_{\mathbf{q},\lambda} i E_{\omega_{\mathbf{q}}} (\tilde{b}_{\mathbf{q}\lambda} \boldsymbol{\epsilon}_\lambda e^{i(\mathbf{q}\cdot\mathbf{x} - \omega_{\mathbf{q}} t)} - \mathrm{H.c.}), \quad (7)$$

with $\tilde{b}_{\mathbf{q}\lambda}(t) = b_{\mathbf{q}\lambda}(t) e^{i\omega_{\mathbf{q}} t}$, $E_{\omega_{\mathbf{q}}} = \sqrt{\omega_{\mathbf{q}}/2\epsilon_0 \epsilon_r L^3}$, vacuum and relative permittivity ϵ_0 and ϵ_r, volume L^3, and linear polarization vector $\boldsymbol{\epsilon}_\lambda$. We consider the fluctuations of the two-mode quadrature operators

$$A^\pm_{\mathbf{q}\lambda} = \mathcal{N}_\pm [\tilde{b}^\dagger_{\mathbf{q}\lambda} + \tilde{b}^\dagger_{-\mathbf{q}\lambda} \pm (\tilde{b}_{\mathbf{q}\lambda} + \tilde{b}_{-\mathbf{q}\lambda})], \quad (8)$$

with $\mathcal{N}_+ = 2^{-3/2}$ and $\mathcal{N}_- = -i2^{-3/2}$. In vacuum, where $\langle b^\dagger_{\mathbf{q}\lambda} b_{\mathbf{q}\lambda} \rangle = 0$, it follows directly from the bosonic commutation relations that (for details, see the Supplemental Material [22])

$$\langle (\Delta A^\pm_{\mathbf{q}\lambda})^2 \rangle = \frac{1}{4} \Big(1 \pm 2\, \mathrm{Re} \langle \tilde{b}_{\mathbf{q}\lambda} \tilde{b}_{-\mathbf{q}\lambda} \rangle \Big), \quad (9)$$

where $(\Delta A)^2 = (A - \langle A \rangle)^2$. Here, $\langle \tilde{b}_{\mathbf{q}\lambda} \tilde{b}_{-\mathbf{q}\lambda} \rangle$ depends on the superconducting phase difference $\Delta\varphi$ and can easily be determined from our model. The key finding is that the expectation value [see Eq. (12)] that determines the uncertainties of the quadrature amplitudes of $A^\pm_{\mathbf{q}\lambda}$ can be changed if one changes the relative phase of the superconductor. If we picture the squeezing as an uncertainty ellipse (see Fig. 1), changing $\Delta\varphi$ simply rotates it.

Next, we address the problem of squeezing and photon statistics using a more rigorous approach that allows more easily for a generalization to higher order processes and feedback of the photon system onto the superconducting leads. To this end, we use the Schwinger-Keldysh formulation [23] of the Hamiltonian in Eq. (2) and integrate out the fermionic degrees of freedom to arrive at an effective photonic action on the Keldysh time contour given by

$$S^{\mathrm{eff}}_{\mathrm{ph}} = \int_{-\infty}^{\infty} dt dt' \sum_{\mathbf{q},\mathbf{q}',\lambda} \bar{B}_{\mathbf{q}\lambda}(t) D^{-1}_{\mathbf{q}\mathbf{q}';\lambda}(t, t') B_{\mathbf{q}'\lambda}(t'), \quad (10)$$

where $B_{\mathbf{q}\lambda}(t) = (b^{cl}_{\mathbf{q}\lambda}, \bar{b}^{cl}_{-\mathbf{q}\lambda}, b^{q}_{\mathbf{q}\lambda}, \bar{b}^{q}_{-\mathbf{q}\lambda})^T$ carries both the Keldysh $\{cl, q\}$ and the Nambu structures. The photonic propagator

$$D^{-1} = D^{-1}_0 - \Pi_{\mathrm{el}} - \Pi_{\mathrm{bath}} \quad (11)$$

acquires self-energy corrections due to the coupling to the superconducting leads Π_{el} as well as a coupling to an external (Markovian) photon bath Π_{bath}, which leads to a finite photon linewidth η [24,25]. We consider one-loop processes such as those shown in Figs. 3(a) and 3(b). Importantly, the electronic coherence of the Cooper pairs is transferred to the photons via the anomalous elements of the photon self-energy [see Fig. 3(b)].

We first focus on the zero temperature limit in both the leads and the photonic system, and calculate the resulting photonic coherence between modes of opposite momenta by inverting the Dyson equation (11) up to second order in the electron-photon coupling g to find

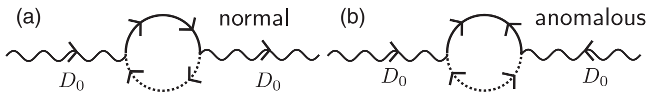

FIG. 3. (a),(b) Normal and anomalous diagrams contributing to the dressed photon propagator $D = D_0 + D_0\Pi_{\rm el}D_0$, where D_0 denotes the free photon propagator and $\Pi_{\rm el}$ the photon self-energy due to coupling to electrons. Solid (dashed) lines denote conduction (valence) electron propagators, and wiggly lines denote photon propagators.

$$\langle\tilde{b}_{\mathbf{q}\lambda}(t)\tilde{b}_{-\mathbf{q}\lambda}(t)\rangle = \sum_{\mathbf{k}}\frac{2g^2 u_{c\mathbf{k}}v_{c\mathbf{k}}u^*_{v\mathbf{k}}v^*_{v\mathbf{k}}e^{2i(\omega_{\mathbf{q}}-eV_0)t}}{(\nu_{\mathbf{q}}-i\eta)(\nu_{\mathbf{q}}+E_{\mathbf{k}}-i\eta)}. \quad (12)$$

Here, $\nu_{\mathbf{q}} = \omega_{\mathbf{q}} - eV_0$ is the photon frequency measured relative to the applied potential difference and the location of one of the resonances is determined by the Bogoliubov dispersion $E_{\mathbf{k}} = E_{c\mathbf{k}} + E_{v\mathbf{k}}$. The product of BCS coherence factors $u_{c\mathbf{k}}v_{c\mathbf{k}}u^*_{v\mathbf{k}}v^*_{v\mathbf{k}} = |\Delta_c||\Delta_v|e^{i\Delta\varphi}/4E_{c\mathbf{k}}E_{v\mathbf{k}}$ depends on the relative phase $\Delta\varphi = \varphi_c - \varphi_v$ between the two superconductors. Using the Keldysh approach, we can thus confirm the above results for the phase-dependent quadrature amplitudes and our ability to manipulate light squeezing by coupling to the superconducting phase difference $\Delta\varphi$. Note that in the rotating frame of the photon fields, the squeezing ellipse rotates with the detuning off the central (Cooper-pair) peak like $\langle\tilde{b}_{\mathbf{q}\lambda}(t)\tilde{b}_{-\mathbf{q}\lambda}(t)\rangle \sim \exp[2i(\omega_{\mathbf{q}} - eV_0)t]$.

While we can perform the momentum sum numerically for the most general parameters, here we make a simplifying assumption of parabolic band dispersions $\varepsilon_{c\mathbf{k}} = \mathbf{k}^2/2m^*_c + D/2$ and $\varepsilon_{v\mathbf{k}} = -\mathbf{k}^2/2m^*_v - D/2$ with effective masses m^*_c, m^*_v, and band gap D. For a symmetric choice $|\Delta_c| = |\Delta_v| \equiv |\Delta|$, $m^*_c = m^*_v$, $\mu_c = D/2 + \delta$, and $\mu_v = -\mu_c$, where δ defines the (quasi-)Fermi energies in the two bands related to the applied voltage by $eV_0 = D + 2\delta$, we can evaluate the correlator analytically. The summation over momenta in Eq. (12) in this case yields

$$\begin{aligned}\langle\tilde{b}_{\mathbf{q}\lambda}(t)\tilde{b}_{-\mathbf{q}\lambda}(t)\rangle &= g^2\rho_c|\Delta|^2 e^{i\Delta\varphi}e^{2i(\omega_{\mathbf{q}}-eV_0)t}\\ &\times\frac{2\arcsin(\tilde{\nu}_{\mathbf{q}}-i\tilde{\eta}) + \pi\left[-1+\sqrt{1-(\tilde{\nu}_{\mathbf{q}}-i\tilde{\eta})^2}\right]}{(\nu_{\mathbf{q}}-i\eta)^2\sqrt{4|\Delta|^2-(\nu_{\mathbf{q}}-i\eta)^2}},\end{aligned} \quad (13)$$

where ρ_c denotes the fermionic density of states at the Fermi surface, and $\tilde{\nu}_{\mathbf{q}} = \nu_{\mathbf{q}}/2|\Delta|$ and $\tilde{\eta} = \eta/2|\Delta|$ are dimensionless frequencies and decay rates.

In Fig. 2(b), we present the zero temperature expectation value $\langle\tilde{b}_{\mathbf{q}\lambda}(t)\tilde{b}_{-\mathbf{q}\lambda}(t)\rangle$ of Eq. (13) for a particular choice of parameters. The function exhibits two peaks, one at frequency $\omega_{\mathbf{q}} = eV_0 - |\Delta_c| - |\Delta_v|$ and one at $\omega_{\mathbf{q}} = eV_0$. The lower frequency peak corresponds to quasiparticle tunneling from the conduction to the valence band, while the peak at $\omega_{\mathbf{q}} = eV_0$ is due to tunneling of Cooper pairs. The sum over momenta only broadens the quasiparticle peak. Both processes involve the emission of photons and are thus possible at $T = 0$ in the absence of a finite photon density.

We have also obtained results for the photon coherence in the presence of a finite temperature $T_{\rm ph} > 0$ thermal background of photons. As shown in Fig. 2(c), a third peak at frequency $\omega_{\mathbf{q}} = eV_0 + |\Delta_c| + |\Delta_v|$ appears at photon temperature $T_{\rm ph} > 0$, which corresponds to the absorption of photons from the thermal background and transfer of quasiparticles from the valence to the conduction band. The three processes are schematically depicted in Fig. 2(a).

In addition, we analyze the photon-photon correlation functions by coupling the photons $B_{\mathbf{q}}(t)$ to external counting fields. As expected, the density correlations between the photons of the same momentum $\langle\langle n_{\mathbf{q}\lambda}n_{\mathbf{q}\lambda}\rangle\rangle \equiv \langle n^2_{\mathbf{q}\lambda}\rangle - \langle n_{\mathbf{q}\lambda}\rangle^2$ obey the thermal relation $\langle\langle n_{\mathbf{q}\lambda}n_{\mathbf{q}\lambda}\rangle\rangle = \langle n_{\mathbf{q}\lambda}\rangle[\langle n_{\mathbf{q}\lambda}\rangle + 1]$. These correlations simply reflect the tendency of bosonic particles to bunch. Two photons with the same momentum $\mathbf{q}$ must have been emitted in uncorrelated events since Cooper pairs have total momentum zero.

In the presence of electronic coherence, however, there also appear density correlations between photons of opposite momenta

$$\begin{aligned}\langle\langle n_{\mathbf{q}\lambda}n_{-\mathbf{q}\lambda}\rangle\rangle = |g|^4\sum_{\mathbf{k},\mathbf{k}'}&\frac{|\Delta_c|^2|\Delta_v|^2}{16E_{c\mathbf{k}}E_{c\mathbf{k}'}E_{v\mathbf{k}}E_{v\mathbf{k}'}}\\ &\times\frac{1}{(\nu^2_{\mathbf{q}}+\eta^2)(\nu_{\mathbf{q}}+E_{\mathbf{k}}+i\eta)(\nu_{\mathbf{q}}+E_{\mathbf{k}'}-i\eta)},\end{aligned} \quad (14)$$

where $\nu_{\mathbf{q}} = \omega_{\mathbf{q}} - eV_0$ and $E_{\mathbf{k}} = E_{c\mathbf{k}} + E_{v\mathbf{k}}$. These correlations are inherited from the coherence of the Cooper pairs within the BCS many-body state. As before, we observe the asymmetric peak structure at zero temperatures with two peaks occurring at $\omega_{\mathbf{q}} = eV_0 - |\Delta_c| - |\Delta_v|$ and $\omega_{\mathbf{q}} = eV_0$ corresponding to tunneling of quasiparticle and Cooper pairs from the conduction to the valence band. Again, at finite (photon) temperatures, a third peak emerges at $\omega_{\mathbf{q}} = eV_0 + |\Delta_c| + |\Delta_v|$ due to the correlated absorption of photons from the thermal background imprinting density correlations between $n_{\mathbf{q}\lambda}$ and $n_{-\mathbf{q}\lambda}$.

In summary, we have shown that a *p*-*n* junction in proximity to two BCS superconductors can be operated to emit squeezed light, produces entangled photon pairs, and affects the photon density correlations. Squeezing occurs between modes of opposite momenta and results from a transfer of the electronic coherence of the Cooper pairs to the photons. The squeezing angle is controlled by the phase difference between the two superconductors. This squeezed light-emitting diode enables us to use the macroscopic coherence of superconductors to manipulate the photon coherence in a two-photon pump.

We acknowledge useful discussions with M. Bechu, M. Hetterich, P. Hlobil, G. Khitrova, D. Rülke, M. Schütt, J. Wang, and M. Wegener. The Young Investigator Group of P. P. O. received financial support from the "Concept for the Future" of the KIT within the framework of the German Excellence Initiative. P. B. and I. V. acknowledge travel support under NSF EPSCoR Cooperative Agreement No. EPS-1003897 with additional support from the Louisiana Board of Regents. I. V. was supported in part by NSF Grant No. DMR-1105339 and ICAM Branches Cost Sharing Fund.

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