

Incoherent thermal transport from dirty black holesSašo Grozdanov,^{1,*} Andrew Lucas,^{2,†} and Koenraad Schalm^{1,‡}¹*Instituut-Lorentz for Theoretical Physics, Leiden University, Niels Bohrweg 2, Leiden 2333 CA, The Netherlands*²*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
(Received 3 December 2015; published 21 March 2016)

We study thermal transport in strongly disordered, strongly interacting quantum field theories without quasiparticles using gauge-gravity duality. We analyze linear perturbations of black holes with broken translational symmetry in Einstein-Maxwell-dilaton theories of gravity. Using general geometric arguments in the bulk, we derive bounds on thermal conductivity for the dual disordered field theories in one and two spatial dimensions. In the latter case, the thermal conductivity is always nonzero at finite temperature, so long as the dilaton potential is bounded from below. Hence, generic holographic models make nontrivial predictions about the thermal conductivity in a strongly disordered, strongly coupled metal in two spatial dimensions.

DOI: [10.1103/PhysRevD.93.061901](https://doi.org/10.1103/PhysRevD.93.061901)**I. INTRODUCTION**

One of the most dramatic and unique consequences of quantum mechanics is the localization of electronic wave functions in disordered systems [1], which leads to exponentially suppressed thermal (and electrical) conductivity in noninteracting theories at finite temperature [2]. In recent years, there has been an intense effort to study whether localization extends to interacting theories [3]. Under many circumstances, at least in one spatial dimension [4–6], this has been shown to be the case, and the resulting phase is coined many-body localization. It is of great interest to understand whether this phenomenon persists in strongly interacting quantum systems in higher dimensions.

One of the only analytical techniques to study higher dimensional strongly coupled systems is gauge-gravity duality [7–9]. In particular, finite temperature computations become tractable. In the past few years, numerous holographic models with heuristic “mean field” approximations to disorder have conjectured (perhaps inadvertently) that the simplest holographic models are thermal conductors *at all disorder strengths*, in two or more spatial dimensions [10–13]. Such models correctly approximate weak disorder [14–16], as the theories are in a hydrodynamic regime. Numerical work on holographic thermal transport [17–19] has also been performed by explicitly constructing disordered black holes [20,21]. What remains an open question is whether mean field results are also sensible when disorder is strong.

Recently, the tools to analytically address this problem have been developed. First, the computation of thermal DC conductivity has been reduced to a hydrodynamics problem for an “artificial” fluid on the black hole horizon [22,23]. Second, a general hydrodynamic framework has been developed which can provide nonperturbative bounds on

transport coefficients [24]. Recently, these formalisms have been combined to prove that the DC electrical conductivity of Einstein-Maxwell holographic models in two spatial (boundary) dimensions is strictly finite [25], so long as the black hole horizon is connected. The goal of this paper is to obtain similar results for the thermal conductivity of disordered Einstein-Maxwell-dilaton (EMD) holographic models.

II. RESULTS

We bound the thermal conductivity κ of a relativistic (at high energies) theory dual to a disordered black hole in EMD gravity in $d + 2$ spacetime dimensions, for $d = 1, 2$. These holographic models are dual to field theories at finite density and temperature, deformed by a charge-neutral relevant scalar operator. The dual theory is sourced by arbitrarily strong disorder, so long as in the bulk, the horizon remains connected. Assuming isotropy on long length scales, κ is defined as

$$\kappa \equiv -\left. \frac{Q_x}{\partial_x T} \right|_{J_x=0}, \quad (1)$$

where Q_x is the heat current, J_x is the charge current, and $-\partial_x T$ is an externally imposed uniform temperature gradient. Note that κ is positive by the second law of thermodynamics, has (relativistic) mass dimension $[\kappa] = [M]^{d-1}$, and that it is defined under the boundary condition that no average electric current flows. The boundary theory is at a uniform temperature T .

Analytically computing bounds on κ , in $d = 1$ we find

$$\kappa \geq 16\pi^3 \left(\frac{1}{1 - \frac{1}{2}V_{\min}} \right) \frac{T}{s} \times \frac{k_B^2}{\hbar} (N^2)^2, \quad (2)$$

where s is the (spatially averaged) entropy density in the boundary theory, N^2 is a large prefactor due to the many

*grozdanov@lorentz.leidenuniv.nl

†lucas@fas.harvard.edu

‡kschalm@lorentz.leidenuniv.nl

degrees of freedom in holographic models, and V_{\min} denotes the global minimum of the dilaton potential; note that $V_{\min} \leq 0$ by construction. In $d = 2$ we find

$$\frac{\kappa}{T} \geq \frac{4\pi^2}{3} \left(\frac{1}{1 - \frac{1}{6}V_{\min}} \right) \times \frac{k_B^2}{\hbar} N^2. \quad (3)$$

Hence, if the dilaton potential is bounded from below, the (relativistically) dimensionless number κ/T is strictly finite. Such sharp exact results for κ have never before been obtained in higher dimensions in a theory where (generic) disorder and interactions are both treated nonperturbatively at finite temperature and density. Henceforth, we work in units where $\hbar = k_B = c = 1$.

Let us briefly compare (3) with our previous result [25] showing that in $d = 2$, σ is strictly finite in Einstein-Maxwell models. The bound on σ derived in [25] relies entirely on curious properties of holographic “horizon fluids” governing transport, but not on details of the near-horizon geometry. Crudely speaking, the horizon fluid is always a local conductor, and so is necessarily a global conductor, as is the dual theory. No such local bound exists for κ ; the bulk Einstein’s equations play an essential role in enforcing the bound (3). For this reason, it is much more subtle. It is also more powerful: while in general EMD models $\sigma(T) \sim T^p$ can be either metallic ($p < 0$) or insulating ($p > 0$) [18,26–28], we show that many EMD models are never better thermal insulators than naive dimensional analysis predicts. Many models with IR hyperscaling-violating exponent $\theta \geq 0$ have $V_{\min} > -\infty$ [29]; for these models (3) is nontrivial.

Our result (3) in $d = 2$ is reminiscent of the proposal for “incoherent” metallic transport presented in [30], whereby a metal with badly broken translational invariance may remain a conductor of thermal (and electric) currents. The incoherent metal was proposed as a conceptual framework to understand how a strongly interacting system, such as the strange metal phase of the high- T_c superconductors [31], remains a conductor even when disorder is strong enough to destroy the Drude peak. In an incoherent metal, transport is diffusion-limited, and remains finite even with strong disorder. This phenomenon is challenging to realize using traditional condensed matter approaches, as interaction strength and disorder strength cannot both be treated nonperturbatively. Mean field holographic approaches have recovered incoherent thermal transport before [13]. Our results demonstrate that generic disordered holographic models can realize incoherent thermal transport.

Evidence for hydrodynamic electronic flow in a strongly coupled metal has emerged in experiments on clean graphene [32,33]. However, this hydrodynamic behavior is limited by disorder in real systems, and the possibility for holography to capture transport physics beyond the hydrodynamic limit of [33] is intriguing. Our qualitative bound $\kappa/T \gtrsim \text{constant}$ in $d = 2$ may therefore be relevant to “messier” materials. It also is interesting to extend

these ideas to transport at finite frequency and magnetic field [34–42].

Returning to our opening question of whether holographic models may realize a many-body localized state, our work rules this out under a broad set of circumstances: κ/T is bounded, regardless of disorder strength, and not vanishing with increasing disorder. Indeed, it appears likely that the only possible holographic model of localization contains black holes with horizons of disconnected topologies. Although some progress has been made in this direction [43–45], this phenomenon has not been shown to be generic, nor are the properties of such holographic models well understood. Hence, further analyses and constructions of such fragmented black holes remain an interesting and important direction for future work.

III. DIRTY BLACK HOLES

We now present the details of our derivation of the bounds (2) and (3) on the thermal conductivity κ . We will consider a family of theories dual to EMD holographic models, for which the Lagrangian is

$$\mathcal{L} = R - 2\Lambda - \frac{1}{2}(\partial\Phi)^2 - \frac{V(\Phi)}{\ell^2} - \frac{Z(\Phi)F^2}{4e^2}, \quad (4)$$

with $\Lambda = -d(d+1)/2\ell^2$ a negative cosmological constant. Here Φ is the scalar dilaton and F is the Maxwell tensor for a $U(1)$ gauge field. The prefactor N^2 defined previously is $N^2 = \ell^2/16\pi G_N$. We assume $Z(\Phi = 0) = 1$ and $V(\Phi = 0) = 0$; V_{\min} as defined previously is $V_{\min} = \min_{\Phi}(V(\Phi))$, which is assumed to be finite, i.e. the dilaton potential is everywhere bounded from below. We have set $16\pi G_N = 1$, and will also set $e = \ell = 1$ henceforth [46].

It is most convenient to consider static background black hole geometries in Gaussian normal coordinates:

$$ds^2 = -f(r, \mathbf{x})^2 dt^2 + dr^2 + G_{ij}(r, \mathbf{x}) dx^i dx^j, \quad (5)$$

with a connected black hole horizon at $r = 0$ [47]. The coordinate r denotes the holographic radial direction (r increases toward the boundary, which we take to be asymptotically AdS), and (t, \mathbf{x}) are boundary theory directions. The (uniform) temperature of the black hole horizon is T , which is also the temperature of the dual field theory. This follows from the zeroth law of black hole thermodynamics and is true for any amount of disorder. Assuming a smooth horizon, we can constrain the near-horizon expansion of f and G (see e.g. [48]):

$$f(r, \mathbf{x}) = 2\pi T r + \frac{F(\mathbf{x})}{6} r^3 + \dots, \quad (6a)$$

$$G_{ij}(r, \mathbf{x}) = \gamma_{ij}(\mathbf{x}) + \frac{h_{ij}(\mathbf{x})}{2} r^2 + \dots \quad (6b)$$

A d -dimensional diffeomorphism symmetry $x_i \rightarrow X_i(x_j)$ is still remaining, and we may use this to our advantage. The dilaton admits the near-horizon expansion

$$\Phi(r, \mathbf{x}) = \phi(\mathbf{x}) + \dots \quad (7)$$

and the Maxwell field

$$A = (\pi T \mathcal{Q} r^2 + \dots) dt. \quad (8)$$

Although our choice of radial coordinate is a bit different than what is used in [23], since this metric obeys all general constraints demanded in their paper, we may still use their results in our new coordinate system. We assume our boundary theory and black hole horizon have topology \mathbb{T}^d , and so we can introduce normalized spatial averaging over the horizon, which we denote by

$$\mathbb{E}[\circ] = \frac{1}{L^d} \int d^d \mathbf{x}_\circ. \quad (9)$$

In the above definition, we have not included a factor of the induced horizon metric. This convention follows [25]. The boundary spatial torus has a length L in each direction, and spatial metric δ_{ij} , for simplicity.

The requirement that the Ricci scalar curvature of the induced horizon metric be nonsingular implies that the extrinsic curvature of the horizon must vanish. As a result, the horizon is the hypersurface (at constant t) with minimal area A_{hor} (see e.g. [49]). In our setup, the black hole horizon is a d -dimensional hypersurface of topology \mathbb{T}^d on a fixed time slice. Let us now consider the hypersurface of points lying along the curve $r = \varepsilon R(\mathbf{x})$, with $R(\mathbf{x}) \geq 0$ and ε an infinitesimally small positive number. This hypersurface thus lies just outside of the horizon for all \mathbf{x} and must as a consequence of our above discussion obey

$$A_{\text{hor}} \leq A_R. \quad (10)$$

When ε is small, the area A_R is

$$A_R \approx \int d^d \mathbf{x} \sqrt{\det \left(\gamma_{ij} + \frac{\varepsilon^2}{2} h_{ij} R^2 + \varepsilon^2 \partial_i R \partial_j R \right)}. \quad (11)$$

Note that A_{hor} is obtained by setting $R = 0$. Expanding (11) to leading order in ε and using (10) gives

$$0 \leq \mathbb{E} \left[\sqrt{\gamma} \left(\gamma^{ij} \partial_i R \partial_j R + \frac{\gamma^{ij} h_{ij}}{2} R^2 \right) \right], \quad (12)$$

which should hold for all possible choices of $R(\mathbf{x}) \geq 0$. The inequality in Eq. (12) allows us to place nontrivial constraints on the possible h_{ij} , and will play a crucial role in finding bounds on κ .

IV. ONE DIMENSION

We begin by considering boundary theories with one spatial dimension, where it has recently been shown [23] that given any EMD theory (4): [50]

$$\kappa = \frac{16\pi^2 T}{\mathbb{E}[\gamma^{-1/2}(\partial_x \phi)^2 + \gamma^{1/2} Z(\phi) \mathcal{Q}^2]}. \quad (13)$$

In this case, the Gaussian normal coordinates used in (5), (6) and (7) prove to be particularly convenient. By using the leading order coefficients of the tt component of Einstein's equations near the horizon, we find the following:

$$\mathbb{E} \left[\frac{(\partial_x \phi)^2}{\sqrt{\gamma}} + \sqrt{\gamma} Z(\phi) \mathcal{Q}^2 \right] = \mathbb{E} \left[4\sqrt{\gamma} - \frac{2h_{xx}}{\sqrt{\gamma}} - 2\sqrt{\gamma} V(\phi) \right]. \quad (14)$$

By substituting the ansatz $R = 1$ into the bound (12), we find

$$\mathbb{E} \left[\frac{h_{xx}}{\sqrt{\gamma}} \right] \geq 0. \quad (15)$$

Using $s = 4\pi \mathbb{E}[\sqrt{\gamma}]$ and $V \geq V_{\text{min}}$ directly implies (2).

V. TWO DIMENSIONS

Let us now consider the thermal conductivity of a disordered black hole in EMD gravity when $d = 2$. We assume the black hole geometry is isotropic (in the boundary spatial directions) on thermodynamically large length scales. Following [24,25], it is straightforward to derive a variational bound on the thermal conductivity. Reference [23] showed that the DC transport coefficients may be computed as follows: solve the following equations

$$\nabla_i \mathcal{I}^i = \nabla_i \mathcal{J}^i = 0, \quad (16a)$$

$$\mathcal{I}_i = Z(\phi)(E_i - \nabla_i \mu) + (4\pi T)^{-1} Z(\phi) \mathcal{Q} \mathcal{J}_i, \quad (16b)$$

$$\begin{aligned} & Z(\phi) \mathcal{Q} (\nabla_i \mu - E_i) + 4\pi (\nabla_i \Theta - T \zeta_i) \\ &= (4\pi T)^{-1} [2\nabla^j \nabla_{(j} \mathcal{J}_{i)} - \mathcal{J}^j \nabla_j \phi \nabla_i \phi], \end{aligned} \quad (16c)$$

where ϕ and \mathcal{Q} are the horizon data defined previously, covariant derivatives are taken with respect to the induced horizon metric γ_{ij} , μ and Θ are scalar functions and E_i and ζ_i are an applied electric field and thermal ‘‘drive’’ (ζ_i is analogous to $-\partial_i \log T$) in the boundary theory. The spatially averaged boundary charge and heat currents are

$$J_{\text{charge}}^i = \mathbb{E}[\sqrt{\gamma} \mathcal{I}^i], \quad J_{\text{heat}}^i = \mathbb{E}[\sqrt{\gamma} \mathcal{J}^i]. \quad (17)$$

The Joule heating in the boundary theory is given by

$$\begin{aligned}
\mathcal{P} &= E_i J_{\text{charge}}^i + \zeta_i J_{\text{heat}}^i \\
&= \frac{1}{16\pi^2 T^2} \mathbb{E} \left[2\sqrt{\gamma} \nabla^{(i} \mathcal{J}^{j)} \nabla_{(i} \mathcal{J}_{j)} + \sqrt{\gamma} (\mathcal{J}^i \nabla_i \phi)^2 \right. \\
&\quad \left. + \sqrt{\gamma} Z(\phi) \left(\mathcal{Q} \mathcal{J}_i - \frac{4\pi T}{Z(\phi)} \mathcal{I}_i \right) \left(\mathcal{Q} \mathcal{J}^i - \frac{4\pi T}{Z(\phi)} \mathcal{I}^i \right) \right], \tag{18}
\end{aligned}$$

as is simple to check by integrating each term in the above expression by parts, and employing (16) and (17).

We now think of \mathcal{P} as a functional of arbitrary currents \mathcal{I}^i and \mathcal{J}^i which are conserved. If $\bar{\mathcal{I}}$ and $\bar{\mathcal{J}}$ are the currents that solve (16) for fixed J_{charge} and J_{heat} , then writing an arbitrary current as $\mathcal{I} = \bar{\mathcal{I}} + \tilde{\mathcal{I}}$ and $\mathcal{J} = \bar{\mathcal{J}} + \tilde{\mathcal{J}}$ we find that [using that $\bar{\mathcal{I}}$ and $\bar{\mathcal{J}}$ obey (16)]

$$\mathcal{P}[\mathcal{I}, \mathcal{J}] = \mathcal{P}[\bar{\mathcal{I}}, \bar{\mathcal{J}}] + \mathcal{P}[\tilde{\mathcal{I}}, \tilde{\mathcal{J}}] + \mathbb{E}[2\sqrt{\gamma}(E_i \tilde{\mathcal{I}}^i + \zeta_i \tilde{\mathcal{J}}^i)]. \tag{19}$$

The last term vanishes if we fix normalization (17). As (18) is positive definite, we conclude that $\mathcal{P} \geq \mathcal{P}[\bar{\mathcal{I}}, \bar{\mathcal{J}}]$. Furthermore, the power dissipated in the absence of electric current is $J_{\text{heat}}^2/T\kappa$. Hence, if \mathcal{I} and \mathcal{J} are trial currents which are conserved and normalized to $J_{\text{heat}} = 1$ and $J_{\text{charge}} = 0$, we may employ (18) to find an upper bound on $1/\kappa$. A simple guess for \mathcal{I} and \mathcal{J} is

$$\mathcal{I}^i = 0, \quad \mathcal{J}^i = \frac{1}{\sqrt{\gamma}} \delta_x^i. \tag{20}$$

It is helpful to use the residual diffeomorphism invariance to fix

$$\gamma_{ij} = \delta_{ij} e^{\omega(\mathbf{x})}. \tag{21}$$

Because of the emergent isotropy of our model, this diffeomorphism should not change the length of the spatial torus in either direction in the thermodynamic limit (we may add a constant to ω to achieve this). Combining our variational ansatz (20) with the induced horizon metric choice (21), we obtain the bound

$$\begin{aligned}
\frac{T}{\kappa} &\leq \frac{1}{16\pi^2} \mathbb{E}[e^{-\omega}((\partial_x \omega)^2 + (\partial_y \omega)^2 + (\partial_x \phi)^2) + Z(\phi) Q^2] \\
&= \frac{1}{16\pi^2} \mathbb{E}[-R_2 + e^{-\omega}(\partial_x \phi)^2 + Z(\phi) Q^2], \tag{22}
\end{aligned}$$

where R_2 is the induced Ricci scalar on the horizon:

$$R_2 = -e^{-\omega}(\partial_x^2 \omega + \partial_y^2 \omega). \tag{23}$$

We now derive an upper bound on $\mathbb{E}[R_2]$. The four dimensional Ricci scalar R_4 in the bulk is given by

$$R_4 = -12 + \frac{1}{2}(\partial\Phi)^2 + 2V(\Phi), \tag{24}$$

for any solution of the Einstein's equations that follow from the Lagrangian (4). General arguments [48] about geometries near black hole horizons, independent of the equations of motion, then allow us to relate R_4 to R_2 . At the horizon, i.e. at $r = 0$,

$$R_4 = R_2 - \frac{F}{\pi T} - 2e^{-\omega}(h_{xx} + h_{yy}). \tag{25}$$

Einstein's equations at leading order in the near-horizon expansion then force

$$\frac{F}{\pi T} = 6 + \frac{Z(\phi) Q^2}{2} - e^{-\omega}(h_{xx} + h_{yy}) - V(\phi). \tag{26}$$

Combining (24), (25) and (26) at $r = 0$, we obtain

$$\begin{aligned}
R_2 &= -6 + e^{-\omega}(h_{xx} + h_{yy}) \\
&\quad + \frac{Z(\phi) Q^2}{2} + \frac{(\partial\phi)^2}{2} + V(\phi). \tag{27}
\end{aligned}$$

Next, we plug the ansatz $R(\mathbf{x}) = e^{-\omega(\mathbf{x})/2}$ into (12), which gives us the inequality

$$0 \leq \mathbb{E} \left[e^{-\omega} \left(\frac{1}{2}(\partial_x \omega)^2 + \frac{1}{2}(\partial_y \omega)^2 + h_{xx} + h_{yy} \right) \right]. \tag{28}$$

Recognizing R_2 from Eq. (23) after partial integration inside (28), we find

$$\mathbb{E}[R_2] \leq 2\mathbb{E}[e^{-\omega}(h_{xx} + h_{yy})]. \tag{29}$$

Combining (27) and (29), along with $V \geq V_{\min}$, we obtain

$$\begin{aligned}
&\mathbb{E}[e^{-\omega}(\partial_x \phi)^2 + Z(\phi) Q^2 - R_2] \\
&\leq \mathbb{E}[(\partial\phi)^2 + Z(\phi) Q^2 - R_2] \leq 12 - 2V_{\min}, \tag{30}
\end{aligned}$$

which can be used in (22) to establish (3). In both $d = 1$ and $d = 2$, we may replace V_{\min} by $\mathbb{E}[V(\phi)]$, if we are able to compute this quantity. In particular, even if $V(\Phi \rightarrow \pm\infty)$ is unbounded, κ is finite so long as ϕ is finite everywhere at finite T . There can be no analogue of many-body localization if $V(\phi)$ is finite on the horizon.

A simple extension of the model we have studied so far in this paper includes $n > 1$ scalar fields Φ^a in $d = 2$,

$$\mathcal{L} = R - 2\Lambda - \sum_{a=1}^n \frac{(\partial\Phi^a)^2}{2} - V(\Phi^a) - \frac{Z(\Phi^a) F^2}{4}. \tag{31}$$

The bounds for the theory in (31) are still (2) and (3).

VI. MEAN FIELD MODELS

Let us now compare our exact results with the predictions of “mean field” models of disorder in holography. A simple model in $d = 2$ consists of $n = 2$ copies of the same scalar field, with Lagrangian (31), but with $V = 0$ and $Z = 1$ [51,52]. Choosing the scalar fields to be $\Phi^i = mx^i$, one finds [11]

$$\kappa = \frac{4\pi sT}{m^2 + \mu^2}, \quad (32)$$

where s is the entropy density and μ is the chemical potential. In order for the temperature of the dual theory to be non-negative,

$$2m^2 + \mu^2 \leq \frac{3}{\pi}s. \quad (33)$$

As it must, our bound (3) is obeyed, as

$$\kappa \geq \frac{4\pi^2 T}{3} \frac{2m^2 + \mu^2}{m^2 + \mu^2} \geq \frac{4\pi^2 T}{3}. \quad (34)$$

The fact that mean field models capture the correct behavior of κ in explicitly disordered models is a pleasing surprise to us.

ACKNOWLEDGMENTS

We thank Jerome Gauntlett, Blaise Goutéraux, Sean Hartnoll, Moshe Rozali and Subir Sachdev for helpful discussions. A. L. is supported by the NSF under Grant No. DMR-1360789 and MURI Grant No. W911NF-14-1-0003 from ARO. S. G. and K. S. are supported by a VICI grant of the Netherlands Organization for Scientific Research (NWO), by the Netherlands Organization for Scientific Research/Ministry of Science and Education (NWO/OCW) and by the Foundation for Research into Fundamental Matter (FOM).

-
- [1] P. W. Anderson, Absence of diffusion in certain random lattices, *Phys. Rev.* **109**, 1492 (1958).
 - [2] V. Ambegaokar, B. I. Halperin, and J. S. Langer, Hopping conductivity in disordered systems, *Phys. Rev. B* **4**, 2612 (1971).
 - [3] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Metal-insulator transition in a weakly interacting many-electron system with localized single-particle states, *Ann. Phys. (Amsterdam)* **321**, 1126 (2006).
 - [4] V. Oganesyan and D. A. Huse, Localization of interacting fermions at high temperature, *Phys. Rev. B* **75**, 155111 (2007).
 - [5] A. Pal and D. A. Huse, Many-body localization phase transition, *Phys. Rev. B* **82**, 174411 (2010).
 - [6] R. Vosk, D. A. Huse, and E. Altman, Theory of the Many-Body Localization Transition in one Dimensional Systems, *Phys. Rev. X* **5**, 031032 (2015).
 - [7] S. A. Hartnoll, Lectures on holographic methods for condensed matter physics, *Classical Quantum Gravity* **26**, 224002 (2009).
 - [8] J. McGreevy, Holographic duality with a view towards many-body physics, *Adv. High Energy Phys.* **2010**, 723105 (2010).
 - [9] S. Sachdev, What can gauge-gravity duality teach us about condensed matter physics?, *Annu. Rev. Condens. Matter Phys.* **3**, 9 (2012).
 - [10] A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, and D. Musso, Thermo-electric transport in gauge/gravity models with momentum dissipation, *J. High Energy Phys.* **09** (2014) 160.
 - [11] A. Donos and J. P. Gauntlett, Thermoelectric DC conductivities from black hole horizons, *J. High Energy Phys.* **11** (2014) 081.
 - [12] A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli, and D. Musso, Analytic DC thermo-electric conductivities in holography with massive gravitons, *Phys. Rev. D* **91**, 025002 (2015).
 - [13] R. A. Davison and B. Goutéraux, Momentum dissipation and effective theories of coherent and incoherent transport, *J. High Energy Phys.* **01** (2015) 039.
 - [14] M. Blake, D. Tong, and D. Vegh, Holographic Lattices Give the Graviton an Effective Mass, *Phys. Rev. Lett.* **112**, 071602 (2014).
 - [15] A. Lucas, S. Sachdev, and K. Schalm, Scale-invariant hyperscaling-violating holographic theories and the resistivity of strange metals with random-field disorder, *Phys. Rev. D* **89**, 066018 (2014).
 - [16] A. Lucas, Conductivity of a strange metal: from holography to memory functions, *J. High Energy Phys.* **03** (2015) 071.
 - [17] A. Donos and J. P. Gauntlett, The thermoelectric properties of inhomogeneous holographic lattices, *J. High Energy Phys.* **01** (2015) 035.
 - [18] M. Rangamani, M. Rozali, and D. Smyth, Spatial modulation and conductivities in effective holographic theories, *J. High Energy Phys.* **07** (2015) 024.
 - [19] S. A. Hartnoll, D. M. Ramirez, and J. E. Santos, [arXiv: 1508.04435](https://arxiv.org/abs/1508.04435).
 - [20] S. A. Hartnoll and J. E. Santos, Disordered Horizons: Holography of Randomly Disordered Fixed Points, *Phys. Rev. Lett.* **112**, 231601 (2014).
 - [21] S. A. Hartnoll, D. M. Ramirez, and J. E. Santos, Emergent scale invariance of disordered horizons, *J. High Energy Phys.* **09** (2015) 160.
 - [22] A. Donos and J. P. Gauntlett, Navier-Stokes on black hole horizons and DC thermoelectric conductivity, *Phys. Rev. D* **92**, 121901 (2015).

- [23] E. Banks, A. Donos, and J. P. Gauntlett, Thermoelectric DC conductivities and Stokes flows on black hole horizons, *J. High Energy Phys.* **10** (2015) 103.
- [24] A. Lucas, Hydrodynamic transport in strongly coupled disordered quantum field theories, *New J. Phys.* **17**, 113007 (2015).
- [25] S. Grozdanov, A. Lucas, S. Sachdev, and K. Schalm, Absence of Disorder-driven Metal-insulator Transitions in Simple Holographic Models, *Phys. Rev. Lett.* **115**, 221601 (2015).
- [26] A. Donos and J. P. Gauntlett, Novel metals and insulators from holography, *J. High Energy Phys.* **06** (2014) 007.
- [27] B. Gout eraux, Charge transport in holography with momentum dissipation, *J. High Energy Phys.* **04** (2014) 181.
- [28] A. Donos, B. Gout eraux, and E. Kiritsis, Holographic metals and insulators with helical symmetry, *J. High Energy Phys.* **09** (2014) 038.
- [29] A. Lucas and S. Sachdev, Conductivity of weakly disordered strange metals: From conformal to hyperscaling-violating regimes, *Nucl. Phys.* **B892**, 239 (2015).
- [30] S. A. Hartnoll, Theory of universal incoherent metallic transport, *Nat. Phys.* **11**, 54 (2015).
- [31] L. Taillefer, Scattering and pairing in cuprate superconductors, *Annu. Rev. Condens. Matter Phys.* **1**, 51 (2010).
- [32] J. Crossno *et al.*, [arXiv:1509.04713](https://arxiv.org/abs/1509.04713).
- [33] A. Lucas, J. Crossno, K. C. Fong, P. Kim, and S. Sachdev, Transport in inhomogeneous quantum critical fluids and in the Dirac fluid in graphene, *Phys. Rev. B* **93**, 075426 (2016).
- [34] S. A. Hartnoll, P. K. Kovtun, M. M uller, and S. Sachdev, Theory of the Nernst effect near quantum phase transitions in condensed matter, and in dyonic black holes, *Phys. Rev. B* **76**, 144502 (2007).
- [35] A. Lucas and S. Sachdev, Memory matrix theory of magnetotransport in strange metals, *Phys. Rev. B* **91**, 195122 (2015).
- [36] A. Amoretti and D. Musso, Magneto-transport from momentum dissipating holography, *J. High Energy Phys.* **09** (2015) 094.
- [37] M. Blake, A. Donos, and N. Lohitsiri, Magnetothermoelectric response from holography, *J. High Energy Phys.* **08** (2015) 124.
- [38] K.-Y. Kim, K. K. Kim, Y. Seo, and S.-J. Sin, Thermoelectric conductivities at finite magnetic field and the Nernst effect, *J. High Energy Phys.* **07** (2015) 027.
- [39] R. A. Davison and B. Gout eraux, Dissecting holographic conductivities, *J. High Energy Phys.* **09** (2015) 090.
- [40] M. Blake, Momentum relaxation from the fluid/gravity correspondence, *J. High Energy Phys.* **09** (2015) 010.
- [41] M. Blake, Magnetotransport from the fluid/gravity correspondence, *J. High Energy Phys.* **10** (2015) 078.
- [42] A. Donos, J. P. Gauntlett, T. Griffin, and L. Melgar, DC conductivity of magnetised holographic matter, *J. High Energy Phys.* **01** (2016) 113.
- [43] D. Anninos, T. Anous, F. Denef, and L. Peeters, Holographic vitrification, *J. High Energy Phys.* **04** (2015) 027.
- [44] G. T. Horowitz, N. Iqbal, J. E. Santos, and B. Way, Hovering black holes from charged defects, *Classical Quantum Gravity* **32**, 105001 (2015).
- [45] Y.-Z. Zou, X.-L. Qi, and C. Xu, [arXiv:1508.03635](https://arxiv.org/abs/1508.03635).
- [46] Note that $\ell^2/16\pi G_N = N^2$, but since N^2 only enters the computation as an overall prefactor in κ , it is not necessary to keep track of it explicitly.
- [47] To obtain the near-horizon Gaussian normal coordinates from the coordinates $(t, \tilde{r}, \tilde{\mathbf{x}})$ that were used in [22,23,25], we write $\tilde{r} = \Gamma_0(\mathbf{x})r^2 + \Gamma_1(\mathbf{x})r^3 + \mathcal{O}(r^4)$, $\tilde{x}^i = x^i + \Omega_0^i(\mathbf{x})r^2 + \mathcal{O}(r^3)$ and appropriately adjust the functions $\Gamma_n(\mathbf{x})$ and $\Omega_n^i(\mathbf{x})$. Importantly, this process does not alter γ_{ij} or ϕ , nor the leading order behavior in A , up to a simple rescaling.
- [48] A. J. M. Medved, D. Martin, and M. Visser, Dirty black holes: Spacetime geometry and near-horizon symmetries, *Classical Quantum Gravity* **21**, 3111 (2004).
- [49] S. N. Solodukhin, Entanglement entropy of black holes, *Living Rev. Relativity* **14**, 8 (2011).
- [50] In this formula, and in (16), we have already transformed the expression to our coordinate system.
- [51] T. Andrade and B. Withers, A simple holographic model of momentum relaxation, *J. High Energy Phys.* **05** (2014) 101.
- [52] A. Donos and J. P. Gauntlett, Holographic Q-lattices, *J. High Energy Phys.* **04** (2014) 040.