Primordial Black Holes from Sound Speed Resonance during Inflation

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We report on a novel phenomenon of the resonance effect of primordial density perturbations arisen from a sound speed parameter with an oscillatory behavior, which can generically lead to the formation of primordial black holes in the early Universe. For a general inflaton field, it can seed primordial density fluctuations and their propagation is governed by a parameter of sound speed square. Once if this parameter achieves an oscillatory feature for a while during inflation, a significant non-perturbative resonance effect on the inflaton field fluctuations takes place around a critical length scale, which results in significant peaks in the primordial power spectrum. By virtue of this robust mechanism, primordial black holes with specific mass function can be produced with a sufficient abundance for dark matter in sizable parameter ranges.

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Introduction. – Investigations on primodrial black holes (PBHs) offer an inspiring possibility to probe physics in the early Universe [1–3]. In recent years, the cosmological implications of PBHs have been extensively studied, especially since they could be a potential candidate for dark matter (DM) [4–10]. Moreover, the PBHs can also be responsible for some gravitational wave (GW) events [11–14], for instance, the first direct detection of the GW event announced by the LIGO collaboration [15]. In the literature, many theoretical mechanisms producing PBHs rely on a spectrum of primordial density fluctuations with extra enhancement on certain length scales, which are usually accomplished by a particularly tuned background dynamics of the quantum fields in the early Universe (e.g. see [16–27] for various analyses within inflationary cosmology, see [28, 29] for the investigations within bounce cosmology, and see [30, 31] for comprehensive reviews).

Primordial density fluctuations, that seeded the large-scale structure (LSS) of our Universe, are usually thought to arise from quantum fluctuations during a dramatic phase of expansion at early times, as described by inflationary cosmology, from which a nearly scale-invariant power spectrum with a standard dispersion relation is obtained [32]. This prediction was confirmed by various cosmological measurements such as the cosmic microwave background (CMB) radiation and LSS surveys at extremely high precision. It is interesting to note that, however, as advocated by the theoretical developments of quantum gravity, modifications of the dispersion relation of the primordial density fluctuations are naturally expected [33–36], which could have non-trivial phenomenological consequences, as we will illustrate in this work.

Sound speed resonance. – We begin with a general discussion on the dynamical evolutions of primordial cosmological perturbations in the framework of the standard inflationary paradigm. The causal mechanism of generating primordial power spectrum suggests that, cosmological fluctuations should initially emerge inside a Hubble radius, and then leave it in the primordial epoch, and finally re-enter at late times. One often uses a gauge-invariant variable ζ, the curvature perturbation in comoving gauge, to characterize the primordial inhomogeneities. For the general case with a non-trivial sound speed c_s [37, 38], one can make use of a canonical variable $v \equiv z\zeta$, where $z \equiv \sqrt{2\epsilon}a/c_s$ with $\epsilon \equiv -\dot{H}/H^2$. The perturbation equation for a Fourier mode $v_k(\tau)$ in the context of General Relativity is given by: $v_k'' + \left(c_s^2k^2 - \frac{z''}{z}\right)v_k = 0$, where the prime denotes the derivative w.r.t. the conformal time τ.

To generate PBHs within inflationary cosmology, one needs to consider how to amplify the primordial curvature perturbations for certain ranges of modes. In the literature, most studies focuses on non-conventional behaviors of the inflationary background, such as a sudden change of the slow-roll parameter ϵ caused by an inflection point in the inflaton potential [17]. Soon it was realized that, a fine-tuning of model parameters is inevitable in this type of mechanism to obtain a sufficient large enhancement of ζ to generate PBHs in abundance [39]. In this *Letter*, we explore a novel possibility – a parametric amplification of curvature perturbations caused by resonance with oscillations in the sound speed of their propagation, which, as we will show, provides a much more efficient way to enhance the primordial power spectrum around the astrophysical scales where PBHs could account for DM in the current experimental bounds.

The sound speed parameter c_s can deviate from unity during the primordial era, namely, in a general single-field model with a non-canonical kinetic term. This arises when inflation models are embodied in UV-complete theories, such as D-brane dynamics in string theory [40, 41], or, from the effective field theory viewpoint, when heavy modes are integrated out [42, 43]. How variation of the primordial sound speed affects curvature perturbations has already been extensively studied, but mainly in the context of primordial features on CMB scales [44, 45].

In this *Letter*, we put aside the theoretical constructions, and take a phenomenological approach to study the effects of an oscillating sound speed on the power spectrum at much smaller scales. As a starting point, we consider the following parametrization for the sound speed:

$$c_s^2 = 1 - 2\xi\big[1-\cos(2k_*\tau)\big] \ , \qquad \tau > \tau_0 \ , \tag{1}$$

where ξ is the amplitude of the oscillation and k_* is the oscillation frequency. We note that, $\xi < 1/4$ is required such that c_s is positively definite. The oscillation begins at τ_0, where k_* is deep inside the Hubble radius, *i.e.* $|k_*\tau_0| \gg 1$. To simplify the analysis we set $c_s = 1$ before τ_0, and then it transits to oscillation smoothly.

We are interested in the behavior of the perturbation modes on sub-Hubble scales, where some of the terms from z''/z in the perturbation equation becomes negligible. Thus, the perturbation equation can be approximately written as:

$$\frac{d^2v_k}{dx^2} + \big(A_k - 2q\cos 2x\big)v_k = 0 \ , \tag{2}$$

where $x \equiv -k_*\tau$, $A_k = k^2/k_*^2 + 2q - 4\xi$ and $q = 2\xi - (k^2/k_*^2)\xi$. This is the Mathieu equation, which presents a parametric instability for certain ranges of k. This equation has been widely applied in the preheating stage after inflation, where excitations of an additional particle can be resonantly amplified, leading to an efficient energy transfer from the inflaton into other particles (see [46–48] for early studies and see [49, 50] for comprehensive reviews). For the process of preheating, the parametric resonance of fluctuations is driven by oscillations of the inflaton field, leaving the possibility of an amplification of the perturbation modes in the whole infrared regime.

In our case, the parametric resonance is seeded by an oscillatory contribution in the sound speed during inflation. In addition, since ξ is always small and thus $|q| \ll 1$, resonance bands are located in narrow ranges around harmonic frequencies $k \simeq nk_*$ of the oscillating sound speed. Since the first band ($n = 1$) is significantly more enhanced than the subsequent harmonic bands, in the following analysis we focus on the resonance of modes around the frequency k_*. By setting the mode function at the beginning of the resonance to the Bunch-Davies vacuum $v_k(\tau_0) = e^{-ik\tau_0}/\sqrt{2k}$, we get full numerical solutions of $v_k(\tau)$, which is plotted in Fig. 1. We see that, for modes $k \neq k_*$ that are not resonating, $v_k(\tau) \sim const.$ inside the Hubble radius, and $\sim 1/\tau$ after Hubble-exit: they evolve as usual in their Bunch-Davies state. Meanwhile, the k_* mode enters in resonance. On sub-Hubble scales, its exponential growth can be captured by:

$$v_k(\tau) \propto \exp(\xi k_*\tau/2) \ , \tag{3}$$

as shown by the green line in the figure. This amplification stops around the Hubble-exit, since the friction term z''/z becomes important on super-Hubble scales.

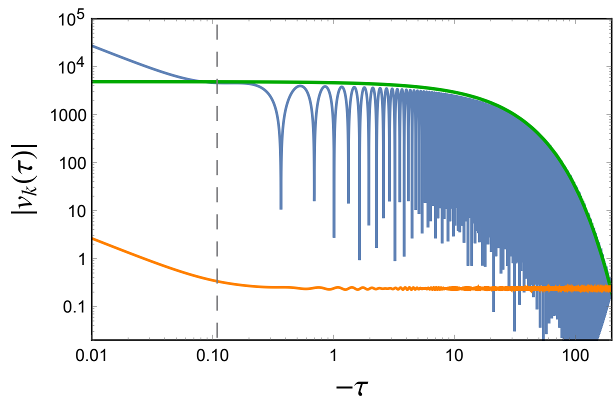

FIG. 1. Parametric amplification of the resonating k_* mode. Here the conformal time evolves from right to left. The full numerical solution is given by the blue line, the green line is the analytical profile, Eq. (3), and the orange line represents modes $k \neq k_*$ that do not enter in resonance. The vertical dashed gray line gives the time of Hubble-crossing for the k_* mode.

In terms of curvature perturbations, before Hubble-crossing, the k_* mode evolves as $\zeta_{k_*}(\tau) \simeq \zeta_{k_*}(\tau_0)e^{\xi k_*(\tau-\tau_0)/2}(\tau/\tau_0)$. It freezes at Hubble-exit, $\tau_* = -1/k_*$, with an enhanced amplitude:

$$\zeta_{k_*} \simeq \zeta_{k_*}(\tau_0)\left(\frac{-1}{k_*\tau_0}\right)e^{-\xi k_*\tau_0/2} \simeq \frac{H}{\sqrt{4\epsilon k_*^3}}e^{-\xi k_*\tau_0/2} \ ,$$

where, in the second equality, we have used $\zeta_{k_*}(\tau_0) = \frac{-H\tau_0}{\sqrt{4k_*\epsilon}}$, as given by the Bunch-Davies vacuum. The resulting primordial power spectrum $P_\zeta \equiv k^3|\zeta_k|^2/(2\pi^2)$ thus presents the following feature: while for modes $k \neq k_*$, we get the standard scale-invariant result, $P_\zeta = \frac{H^2}{8\pi^2\epsilon}$, there is a significant peak from the exponential amplification at the resonance frequency k_*, $P_\zeta = \frac{H^2}{8\pi^2\epsilon}e^{-\xi k_*\tau_0}$, as shown in Fig. 2. The enhancement factor $e^{-\xi k_*\tau_0}$ arises from the interplay of two effects: the oscillation in the sound speed, controlled by its amplitude ξ, and the expansion of the Universe from the beginning of the resonance to Hubble-crossing of the k_*-mode: $-k_*\tau_0 = \tau_0/\tau_* \simeq e^{\Delta N}$, where ΔN is the e-folding number for this period of inflation. By a rough estimate, even for very small oscillation amplitudes $\xi \sim 10^{-4}$, a few e-folds $\Delta N \simeq 12$ is enough to get a peak of order 1 in the power spectrum. Fig. 2 also shows peaks for harmonic frequencies $2k_*$, $3k_*$, $4k_*$, ..., with relatively much lower amplitudes. For simplicity, we keep the discussion only on the k_* mode, and parametrize the power spectrum using a δ-function:

$$P_\zeta(k) \simeq A_s\left(\frac{k}{k_p}\right)^{n_s-1}\left[1+\frac{\xi k_*}{2}e^{-\xi k_* \tau_0}\delta(k-k_*)\right] , \quad (4)$$

where $A_s = \frac{H^2}{8\pi^2\epsilon}$ is the amplitude of the power spectrum as in standard Inflation and n_s is the spectral index at pivot scale $k_p \simeq 0.05\ \mathrm{Mpc}^{-1}$ [51]. The coefficient in front of the δ-function is determined by estimating the area of the peak using a triangle approximation.

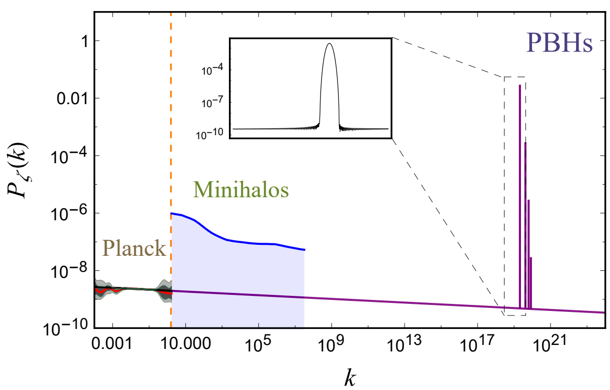

FIG. 2. The power spectrum of primordial curvature perturbations with sharp peaks caused by sound speed resonance, and the comparison with various observational windows [52]. The first peak around the resonating mode k_* (here given by the Schwarzschild radius of PBHs with one solar mass) is the most significant one, while others at subsequent harmonics $2k_*$, $3k_*$, $4k_*$ ... are sub-dominant by at least two orders of magnitude.

The PBHs formation. – We now study the formation of PBHs due to the enhancement in the primordial power spectrum. As we see, the width of the peak in the power spectrum being very narrow ($\sim \xi k_*$), only modes very close to the resonance frequency k_* may have sufficiently large amplitude to collapse into black holes. After Hubble-exit, if the density perturbations produced by these modes are larger than a critical value δ_c, then, after re-entering the Hubble radius, they could collapse into black holes due to gravitational attraction. The Schwarzschild radius of PBHs with mass M is related to the physical wavelength of the mode k_M at Hubble re-entry, $k_{M,\mathrm{ph}} = k_M/a_M \simeq R_S^{-1} = \left(\frac{M}{4\pi M_p^2}\right)^{-1}$. Accordingly, the PBH mass can be expressed as a function of k_M via:

$$M \simeq \gamma \frac{4\pi M_p^2}{H(t_{\mathrm{exit}}(k_M))} e^{\Delta N(k_M)} , \quad (5)$$

where $\Delta N(k_M) = \ln[a(t_{\text{re-entry}}(k_M))/a(t_{\mathrm{exit}}(k_M))]$ is the the e-folding number from the Hubble-exit time of the mode k_M to its re-entry. The correction factor γ represents the fraction of the horizon mass responsible for PBH formation, which can be simply taken as $\gamma \simeq 0.2$ [53]. Given the sharpness of the peak in the power spectrum, the PBHs formed in this context are likely to possess a rather narrow range of masses, as we will discuss now.

To estimate the abundance of PBHs with mass M, one usually defines $\beta(M)$ as the mass fraction of PBHs against the total energy density at the formation, which can be expressed as an integration of the Gaussian distribution of the perturbations:

$$\beta(M) \equiv \frac{\rho_{\mathrm{PBH}}(M)}{\rho_{\mathrm{tot}}} = \frac{\gamma}{2}\mathrm{Erfc}[\frac{\delta_c}{\sqrt{2}\sigma_M}] , \quad (6)$$

where Erfc denotes the complementary error function. Here σ_M is the standard deviation of the density perturbations at the scale associated to the PBH mass M, which can be expressed as $\sigma_M^2 = \int_0^\infty \frac{dk}{k} W(k/k_M)^2 \frac{16}{81}\left(\frac{k}{k_M}\right)^4 P_\zeta(k)$, where $W(x) = \exp(-x^2/2)$ is a Gaussian window function. Since the scale-invariant part of the power spectrum is smaller than the critical density, no black holes will form except at scales around the resonance peak. Considering that we are working in the perturbative regime, the height of the peak in $\sqrt{P_\zeta(k)}$ should be no more than 1, corresponding to a maximal variance of $\sigma_M^2 \lesssim \frac{8}{81}\xi\left(\frac{k_*}{k_p}\right)^{n_s-1}\left(\frac{k_*}{k_M}\right)^4 e^{-(k_*/k_M)^2}$, within which our analysis is restricted [1].

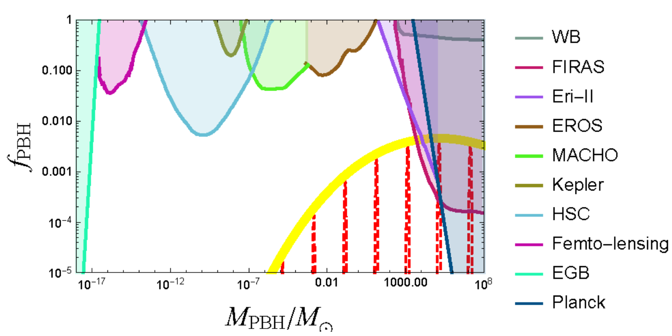

FIG. 3. Estimations for the fraction of PBHs against the total DM density, f_{PBH}, Eq. (7), produced by sound speed resonance, for different values of k_*. Constraints from a number of astronomical experiments are also shown (see main text for refs.): their observational sensitivities are given by colored shadow areas. We choose the oscillation amplitude $\xi = 0.15$ and take a group of typical values for the other parameters: $\gamma = 0.2$, $g_{\mathrm{form}} \simeq 100$, $\delta_c = 0.3$, $n_s = 0.968$.

PBHs formed by sound speed resonance can account for dark matter in wide parameter ranges and easily sat-

[1] The non-perturbative regime can be easily reached in the sound speed resonance (see Fig. 4), which is also interesting for PBHs formation, but is beyond the scope of the current paper.

isfy experimental bounds. To see this, we consider the fraction of PBHs against the total dark matter component at present [31]:

$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}} \qquad (7)$$
$$= 2.7\times 10^8 \left(\frac{\gamma}{0.2}\right)^{1/2}\left(\frac{g_{*,\rm form}}{10.75}\right)^{-1/4}\left(\frac{M}{M_\odot}\right)^{-1/2}\beta(M)\ ,$$

where $g_{*,\rm form}$ is the total relativistic degrees of freedom at the PBH formation time.

We plot estimations of $f_{\rm PBH}$ in Fig. 3, for $\gamma = 0.2$, $g_{\rm form} \simeq 100$ [54] and $\delta_c = 0.3$ [31], representative of the physics of a typical PBHs formation, as well as adopting the Planck result for $n_s = 0.968$ [51], and choose the oscillation amplitude $\xi = 0.15$. We also show current bounds of various astronomical experiments including EGB (extragalactic γ-ray background), microlensing of Kepler, HSC (Hyper Suprime-Cam), MACHO (massive astrophysical compact halo object), EROS (Exprience pour la Recherche d'Objets Sombres), FIRAS (The Far Infrared Absolute Spectrophotometer) and Planck [5]. In this figure, the red dashed curves correspond to the predictions of $f_{\rm PBH}$ with different choices for the resonance frequency k_*. The PBH mass distribution is given by a narrow peak around k_*: this is a distinctive feature of PBHs formed by sound speed resonance from PBHs formed by other processes, for which the mass distribution is usually more spread out. By varying the value of k_*, the peaks form a one-parameter family enveloped by a yellow solid curve that mainly depends on the amplitude parameter ξ. One can see from Fig. 3 that, for the specific case we chose to plot, resonance frequencies $k_* \gtrsim 10^{16}{\rm Mpc}^{-1}$ corresponding to PBH masses $M \gtrsim 10^3 M_\odot$, are excluded by observations.

Because the PBHs formed by sound speed resonance possess a very narrow mass distribution, no particular tuning of the background is needed to generate PBHs in abundance, consistently with current experimental bounds. From previous discussion, we know that the resonance frequency k_* provides the median of the PHBs mass distribution M, while $f_{\rm PHB}$ is mainly determined by the oscillation amplitude ξ, and the e-folding numbers ΔN from τ_0 to the horizon-exit time of k_* mode. Through $f_{\rm PBH}$ in Eq. (7), these model parameters can be bounded by various astronomical constraints. In Fig. 4, we plot contours for different ΔN, above which the parameter space is excluded by various astronomical constraints. One can see that, even within the scope of the perturbative treatment we followed, the sound speed resonance has a large parameter space, left to be probed by future observations.

Conclusions.– In this Letter we proposed a novel mechanism generating PBHs from resonating primordial density perturbations in inflationary cosmology with an oscillatory feature in the sound speed of their propagation.

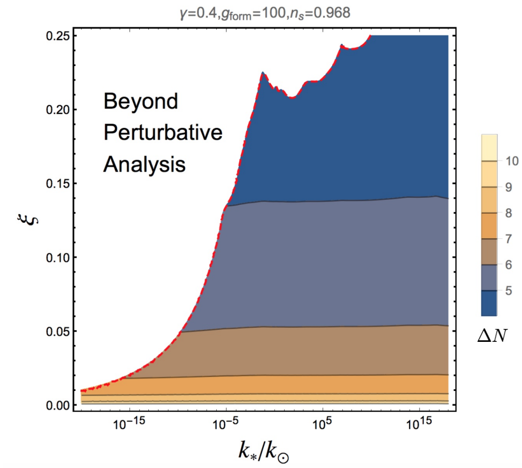

FIG. 4. Constraint contours on the parametric resonance parameters space by the various astronomical experiments shown in Fig. 3. The white regime is beyond our consideration since the enhancement there yields $\zeta(k_*) > 1$ which invalidates the perturbative treatment in this paper.

This scenario may be realized in the context of the effective field theory of inflation or by non-canonical models inspired by string theory. Using a parametrization of the oscillating part of the sound speed, our analysis demonstrates that primordial curvature perturbations could be resonantly enhanced in a narrow band of comoving wavenumbers around the oscillation frequency of the sound speed. As a result, the power spectrum of primordial density fluctuations presents a sharp peak around this resonance frequency, while remains nearly scale-invariant on large scales as predicted by standard inflationary cosmology. Accordingly, a considerable amount of PBHs could eventually form when these amplified modes re-enter the Hubble radius, that may be testable in various forthcoming astronomical observations. Note that, with this mechanism, enhancement of primordial density fluctuations on specific small scales can be extremely efficient in comparison with other existing mechanisms for PBHs formation. Besides, PBHs generated by sound speed resonance can easily account for DM in current experimental bounds, especially since their mass distribution is very narrow.

We end by highlighting the implications of the proposed mechanism that could initiate future studies from several perspectives. First of all, in this work we mainly study the first peak in the power spectrum, but as discussed, the parametric resonance effect also gives rise to discrete peaks on smaller scales. Although they are not as significant as the first one, it is still possible to have PBHs formation on these higher harmonic scales, therefore our model may yield a distinct feature for PBHs mass distribution. Phenomenologically, an important lesson from our study is that, as we started with small oscillations in the sound speed of the propagation of primordial curvature fluctuations in Inflation, and ended up with a dramatic production of PBHs, the observational windows on the early universe are no longer limited within the CMB and LSS surveys, but also include other astronomical instruments probing at much smaller scales. On one hand, this motivates theoretical investigations on the possible inflation models from fundamental theories or effective field theories, which could yield oscillating behaviors in the sound speed. Moreover, it is important to further explore how a general time-varying sound speed may affect the evolutions of primordial density fluctuations nonlinearly. On the other hand, in the era of multi-messenger astronomy, PBHs are becoming more and more testable, making for a more and more serious DM candidate, which may inspire designs for future experiments. In particular, detection of GWs produced in black holes merger events could provide great insights on the black holes distribution and their masses.

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This Letter is dedicated to the memory of the giant Prof. Stephen Hawking, who inspired numerous young people to pursue the dream about the Universe, and beyond.

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