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Strong lensing time delay constraints on dark energy: a forecast

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Abstract. Measurements of time delays between multiple quasar images produced by strong lensing are reaching a sensitivity that makes them a promising cosmological probe. Future surveys will provide significantly more measurements, reaching unprecedented depth in redshift, making strong lensing time delay (SLTD) observations competitive with other background probes. We forecast constraints on the nature of dark energy from upcoming SLTD surveys, simulating future catalogues with different numbers of lenses distributed up to redshift $z \sim 1$ and focusing on cosmological parameters such as the Hubble constant H_0 and parametrisations of the dark energy equation of state. We also explore the impact of our ability to precisely model the lens mass profile and its environment, on the forecasted constraints. We find that in the most optimistic cases, SLTD will constrain H_0 at the level of $\sim 0.1\%$, while the CPL equation of state parameters, w_0 and w_a , can be determined with errors $\sigma_{w_0} \sim 0.05$ and $\sigma_{w_a} \sim 0.3$, respectively. Furthermore, we investigate the bias introduced when a wrong cosmological model is assumed for the analysis. We find that the value of H_0 could be biased up to 10σ , assuming a perfect knowledge of the lens profile, when a Λ CDM model is used to analyse data that really belong to a w CDM cosmology with $w = -0.9$. Based on these findings, we identify a consistency check of the assumed cosmological model in future SLTD surveys, by splitting the dataset in several redshift bins. Depending on the characteristics of the survey, this could provide a smoking gun for dark energy.

Keywords: gravitational lensing, dark energy experiments

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1 Introduction

The phenomenon of cosmic acceleration, i.e. the late phase of accelerated expansion of the Universe, has posed a major challenge for Cosmology since it was first established in 1998 [1]. The standard cosmological model Λ CDM, with a cosmological constant Λ as the candidate mechanism responsible for cosmic acceleration, has so far been the most successful model in describing both early Universe observations, such as Cosmic Microwave Background (CMB), as well as the late time dynamics of the Universe, probed by observations of Baryon Acoustic Oscillations (BAO), galaxy clustering and weak lensing.

Despite the successes of Λ CDM, recent observations highlighted a discrepancy between the value of the Hubble constant today, H_0 , inferred from CMB observations and the local measurements performed through the distance ladder technique. While the former estimate of H_0 depends on the assumed cosmological model, the latter does not depend strongly on any cosmological assumption, as it relies on the observation of standard candles (type Ia supernovae) whose absolute luminosity is calibrated using Cepheids as an anchor. Recent estimates of H_0 obtained using the latter technique have been provided by the SH0eS team [2], with their latest value achieved exploiting observations of Cepheids in the Large Magellanic Cloud from the Hubble Space Telescope, $H_0 = 74.03 \pm 1.42$ km/s/Mpc [3].

The CMB estimates of H_0 rely instead on constraints of the size of the sound horizon at the last scattering surface (θ_*) a measurement which allows to extrapolate bounds on the current expansion rate. This extrapolation however implies an assumption for the expansion history of the Universe. Assuming a Λ CDM background, measurements of the CMB from the Planck collaboration provide $H_0 = 67.36 \pm 0.54$ km/s/Mpc [4], a value which is in tension with the local measurement of the SH0eS collaboration at 4.4σ .

There is currently no consensus on what is causing the discrepancy in the measure of the Hubble constant between low and high redshift data. One possibility is that the results are biased by neglected systematic effects on observational data (see e.g. [5–9]), while, on the other hand, this tension could indicate that we need to abandon the Λ CDM assumption when extrapolating results to present time. Investigations of the latter possibility have highlighted how early time deviations from standard physics have the potential to ease the tension (see e.g. [10–16]), while other studies have tried to solve this issue allowing for non standard late time evolution, which might be produced by dynamical Dark Energy (DE) models, modified theories of gravity or interactions between DE and dark matter (such as [17–21]).

In order to shed light on this tension, the value of H_0 has been determined also with other kind of observations. For example, the discovery of the first binary neutron stars merging event, GW170817 [22–25] and the detection of an associated electromagnetic counterpart have lead to the measurement $H_0 = 70_{-8}^{+12}$. Even though this constraint is much weaker than those obtained by SNe and CMB observations it is expected to significantly improve with the discovery of new merging events with an associated counterpart [26–28].

Along with standard sirens (as gravitational wave events are called nowadays because of their analogy with standard candles), observations of the time delay between multiply imaged strongly lensed system has become a compelling method to obtain measurements of H_0 together with other cosmological parameters. The observational method of SLTD was first proposed in 1964 and it can now produce precise, although cosmology dependent, estimations of the Hubble constant thanks to accurate measurements of the time delays between multiple images of specific lensed quasar [29]. The analysis of four well-measured systems from the H0LiCOW lensing program [30] has recently provided a bound on the Hubble constant of $H_0 = 72.5 \pm 2.1$ assuming a flat Λ CDM cosmology [31]. While the H0LiCOW program is ambitiously aiming to bring the SLTD estimates of H_0 to the 1% precision (see e.g. [30] and [32] where a 2.4% constraints on H_0 is obtained combining six well-measured lensing systems), observations of lensed system from future surveys, such as the Large Synoptic survey Telescope (LSST) which should start taking data in 2023 [33], are expected to significantly improve the number of well-measured strongly lensed systems [34]. The increase in the number of observed lensed sources will also open the possibility to constrain non standard cosmologies, e.g. extensions in the dark and neutrino sector (see [32] for recent constraints on these extended parameter space from SLTD). SLTD is sensitive to the cosmological model through a combination of distances, but unlike Type Ia SNe, SLTD measurements do not require any anchoring to known absolute distances. Typically however, obtaining cosmological constraints with SLTD systems requires precise measurements and modeling of the mass profile and of the environment of the lens system in order to have systematics reasonably under control. Future surveys, like LSST, are also expected to provide enough well-measured systems to allow sufficient statistics with a selected subset of lenses for which a precise modelling of the lens properties can be obtained. This will certainly limit the impact on the cosmological constraints of the uncertainties in the modeling of lens mass and environment. LSST, for instance, has the advantage of having both the wide field-of-view to detect many quasars, and the frequent time sampling to monitor the lens systems for time delay measurements. Several thousand lensed quasar systems should be detectable with LSST, and, as shown in [34], around 400 of these should yield time delay measurements of high enough quality to obtain constraints on cosmological models [35].

It is timely to investigate the constraints on cosmological parameters that can be obtained from future observations of strongly lensed systems. In this paper we focus on simple

extensions of the Λ CDM expansion history and forecast SLTD constraints on these, as well as on the current expansion rate H_0 . We do so, by creating synthetic mock catalogues of future survey with variable number of lenses up to 1000 and building a Gaussian likelihood to compare data with theory. We also include estimates of the lens galaxy stellar velocity dispersion in our analysis. A pioneering work in this direction [36] explored the complementarity of time delay measurements with other cosmological probes, such as Supernovae and Cosmic Microwave Background, with special focus to well observed systems where the lens properties can be well determined. In this paper we focus instead on the possibility to use SLTD as a stand alone probe, quantifying its constraining power for different number of observed lenses and investigating the possible constraints on DE brought by this probe alone. With respect to [36] we also deal with nuisance effects due to the properties of the lens environment, and include forecasted kinematic data of the lens system in order to break the degeneracies between these and cosmological properties.

The paper is organized as follows. In section 2 we outline the connection between the time delays and the cosmological model, describe the theoretical modeling of the lens velocity dispersion and illustrate how it can improve the time delay constraints on DE parameters. In section 3 we outline our analysis method, describing the likelihood expression used to infer the posterior distributions of cosmological parameters, and explaining the procedure used to generate mock datasets, which are used to forecast the constraints displayed in section 4. In section 4.1 we discuss the constraining power of SLTD on the DE models of interest, assessing at the same time how observational uncertainties on the lens model and on line of sight effects impact the figure of Merit of future surveys. Section 5 contains our investigation of the possible bias brought on the inferred parameters by a wrong assumption of the underlying cosmological model. We also propose a consistency check that could be performed on future SLTD datasets to verify this possibility. Finally, we summarize our conclusions in section 6.

2 Cosmology with time delay measurements

We shall describe the connection between gravitational lensing time delay and the cosmological model, and how we account for the velocity dispersion of the lensing galaxy for our cosmological inference. We also specify the lens and environment mass modeling used for our analysis and include a description of the mass-sheet degeneracy, which provides a transformation of the lens mass profile that has no observable effect other than to rescale the time delays [37].

2.1 Theory of gravitational lensing time delays

In strongly lensed systems, the time that light rays take to travel between the source and the observer depends sensibly both on their path and on the gravitational potential of the lens. For a given i -th light ray, the time delay with respect to its unperturbed path is given by [38, 39]:

$$t(\boldsymbol{\theta}_i, \boldsymbol{\beta}) = (1 + z_l) \frac{D_l D_s}{c D_{ls}} \left[\frac{(\boldsymbol{\theta}_i - \boldsymbol{\beta})^2}{2} - \psi_{\perp}(\boldsymbol{\theta}_i) \right], \quad (2.1)$$

where, as shown in figure 1, $\boldsymbol{\beta}$ and $\boldsymbol{\theta}_i$ stand, respectively, for the source and the image position, z_l is the redshift of the lens and $\psi_{\perp}(\boldsymbol{\theta}_i)$ is the projected gravitational potential calculated on the lens plane. D_l , D_s and D_{ls} are, respectively, the angular diameter distance

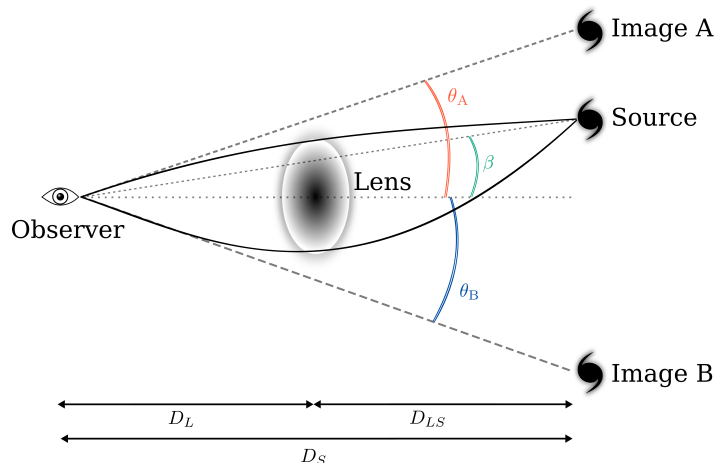


Figure 1. The schematic view of a strongly lensed system. D_l , D_s and D_{l_s} are, respectively, the angular diameter distance from the observer to the lens, from the observer to the source, and from the lens to the source. The solid angles β and θ_i indicate the position of the source and the images, with respect to the lens plane.

from the observer to the lens, from the observer to the source, and from the lens to the source; they satisfy the relation

$$D_{l_s} = \frac{(1 + z_s)D_s - (1 + z_l)D_l}{1 + z_s}, \quad (2.2)$$

where z_s is the redshift of the source. The Fermat principle provides us with a lens equation for the relative angle between the true position of the source and each of the, possibly multiple, images:

$$\boldsymbol{\theta}_i - \boldsymbol{\beta} = \nabla \psi_{\perp}(\boldsymbol{\theta}_i), \quad (2.3)$$

where ∇ is the transverse gradient computed on the plane orthogonal to the direction of propagation of light. It can be shown [38, 39] that the combination $(\boldsymbol{\theta}_i - \boldsymbol{\beta})^2/2 - \psi_{\perp}(\boldsymbol{\theta}_i)$ in eq. (2.1) is only dependent on the geometry and mass distribution of the deflectors; it is usually referred to as the *Fermat potential* $\phi(\boldsymbol{\theta}_i, \boldsymbol{\beta})$.

As eq. (2.1) shows, the background cosmological parameters impact the gravitational lensing time delays through the ratios of angular diameter distances. In a flat Universe the angular diameter distance can be written as

$$D(z) = \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')}, \quad (2.4)$$

where $E(z) = H(z)/H_0$ is the dimensionless Hubble rate, and c is the speed of light.

The relative time delay between two images A and B of a lensed system is given by the difference in the excess time of the two images, which can be rewritten in a simple form using the Fermat potential

$$\Delta t_{AB} = (1 + z_l) \frac{D_l D_s}{c D_{l_s}} [\phi(\boldsymbol{\theta}_A, \boldsymbol{\beta}) - \phi(\boldsymbol{\theta}_B, \boldsymbol{\beta})], \quad (2.5)$$

where we can isolate the factor containing the dependence on cosmological parameters

$$D_{\Delta t} = (1 + z_l) \frac{D_l D_s}{c D_{l_s}}, \quad (2.6)$$

which is referred to as the *time delay distance*.

2.2 Lens mass model and mass-sheet degeneracy

While assuming a cosmological model is enough to define $D_{\Delta t}$ through eq. (2.6), in order to be able to obtain theoretical predictions for Δt_{AB} , the Fermat potential needs to be computed. This requires an accurate modeling of the mass profile of the lens galaxy which is challenging both experimentally and theoretically, and in fact has been proven to be the major source of bias for cosmological inference [40, 41]. The complicating factor arises from the so-called *mass-sheet degeneracy* (for a detailed discussion see e.g. [38, 39]). In fact, a transformation of the lens convergence $\kappa(\boldsymbol{\theta}) = \nabla\psi_{\perp}/2$ of the form:

$$\kappa'(\boldsymbol{\theta}) = \lambda\kappa(\boldsymbol{\theta}) + (1 - \lambda) \quad (2.7)$$

will result in the same dimensionless observables, e.g. image positions and shapes, but will rescale the time delays by a factor λ . This effect, together with the lack of available data on density distributions, implies that there is more than one density profile which can reproduce the observed image positions.

The additional mass term can be due to perturbers that are very massive or close to the lens galaxy (which may need to be included explicitly into the lens mass model and affects stellar kinematics) or to the structures that lie along the LOS (see e.g. [42–45]). Both effects can effectively be summed up into a constant external convergence term, $\kappa_{\text{ext}} = 1 - \lambda$, due to the mass sheet transformation described by eq. (2.7). The overall effect of this degeneracy, as stated above, is a rescaling of the value of the observed time delay distance and thus, the cosmological parameters [46, 47]:

$$D'_{\Delta t} = \frac{D_{\Delta t}}{1 - \kappa_{\text{ext}}} \quad (2.8)$$

Such an effect can be reduced by the combination of lensing data with stellar kinematics measurements, tracing the internal mass distribution of the lens galaxy [48–50].

Considering simple parameterized mass models, it has been shown that the profiles of most discovered lens galaxies are well fitted by a nearly elliptical power-law mass distribution [51]. We will follow the same assumption through this paper (see eq. 2.11), and when describing our lens systems, we will use a projected potential on the lens plane ψ_{\perp} obtained assuming the Softened Power-law Elliptical Potential (SPEP) for the main lens [52]:

$$\psi_{\text{SPEP}}(\boldsymbol{\theta}) = \frac{2A^2}{(3 - \gamma')^2} \left[\frac{\theta_1^2 + \theta_2^2/q^2}{A^2} \right]^{(3-\gamma')/2}, \quad (2.9)$$

where q is the galaxy axis ratio, $A = \theta_E / [\sqrt{q} (3-\gamma')^{1/(1-\gamma')}]$ is an overall normalization factor depending on the Einstein radius θ_E and $\gamma' \approx 2$ being the slope of the mass profile [which we define in eq. (2.11)]. θ_1 and θ_2 are the projections on the lens plane of the two dimensional image position $\boldsymbol{\theta}$.

Additionally, as common in the modeling of the mass profile of quadruple lenses (see e.g. [38, 39]), we include in our modeling of the lens mass profile a constant external shear yielding a potential in polar coordinates of the form:

$$\psi_p(\theta, \phi) \equiv \frac{1}{2}\theta^2\gamma_{\text{ext}} \cos 2(\phi - \phi_{\text{ext}}). \quad (2.10)$$

where γ_{ext} and ϕ_{ext} are the shear strength and angle. It is worth stressing that both ψ_p and ψ_{SPEP} contribute to the projected potential ψ_{\perp} .

We conclude this section noting that a power-law density profile may not be an accurate description of the true profile of a galaxy and that this can induce a large bias in the cosmological bound derived with SLTD (see e.g. [40, 53]). The problem can be partially mitigated including measurements of the stellar kinematic, which allow to reduce the number of models reproducing the data. However, still more than one class of model would be in agreement with observations even since lensing observables can only probe the density profile in a region close to the Einstein ring. As the true density profile of galaxies is not known, one has to rely on a large set of radial density profile with sufficient degree of freedom in order to minimise the systematic error associated with an erroneous description of lens mass profile [50]. In this paper, however, we do not intend to study the bias coming from wrong assumptions in the description of density profile and we stick therefore to the most common assumption of a power law profile for the lensing galaxy.

2.3 Stellar dynamics modeling

In order to model the measurable stellar velocity dispersion σ_v we need to model the 3D gravitational potential of the lens galaxy Φ , in which stars are orbiting. This potential will have contributions from the mass distributions of both the lens and the nearby galaxies physically associated with the lens. To model the stellar velocity dispersion we follow the analysis of [44, 45]. As previously mentioned, we approximate the overall mass density associated to Φ as a spherically symmetric power law profile:

$$\rho_{local} = \rho_0 \left(\frac{r_0}{r} \right)^{\gamma'} \quad (2.11)$$

the overall normalization $\rho_0 r_0^{\gamma'}$ can be determined quite well by lensing measurements, since it is a function of the lens profile characteristic only, and can be written as [44]:

$$\rho_{local}(r) = \pi^{-1/2} (\kappa_{ext} - 1) \Sigma_{cr} R_E^{\gamma'-1} \frac{\Gamma(\gamma'/2)}{\Gamma\left(\frac{\gamma'-3}{2}\right)} r^{-\gamma'}, \quad (2.12)$$

where R_E is the Einstein radius and Σ_{cr} is the critical surface density. As in [44], to calculate the LOS velocity dispersion we follow [54]. The three-dimensional radial velocity dispersion σ_r is then found solving a spherical Jeans equation:

$$\frac{\partial(\rho^* \sigma_r^2)}{\partial r} + \frac{2\beta_{ani}(r)\rho^* \sigma_r^2}{r} + \rho^* \frac{\partial\Phi}{\partial r} = 0, \quad (2.13)$$

where, Φ is the galaxy gravitational potential generated by the density of eq. (2.11). Furthermore, we assume the Osipkov-Merritt model for the stellar anisotropy profile in the lens galaxy $\beta_{ani} = r^2/(r^2 + r_{ani}^2)$. For the modeling of the stellar distribution ρ^* , we have assumed the Hernquist profile [55]

$$\rho^*(r) = \frac{I_0 a}{2\pi r(r+a)^3}, \quad (2.14)$$

with I_0 being a normalization factor, $a = 0.551r_{eff}$ and r_{eff} being the effective radius of the lensing galaxy. The luminosity-weighted velocity dispersion σ_s is then given by:

$$I(R)\sigma_s^2 = 2 \int_R^\infty \left(1 - \beta_{ani} \left(\frac{R}{r} \right)^2 \right) \frac{\rho^* \sigma_r^2 r dr}{\sqrt{r^2 - R^2}}. \quad (2.15)$$

Here R is the projected radius and $I(R)$ is the projected Hernquist profile. Finally, the luminosity-weighted LOS velocity dispersion within a measuring device aperture \mathcal{A} is:

$$(\sigma_v)^2 = \frac{\int_{\mathcal{A}} [I(R)\sigma_s^2 * \mathcal{P}] R dR d\theta}{\int_{\mathcal{A}} [I(R) * \mathcal{P}] R dR d\theta}. \quad (2.16)$$

where $*\mathcal{P}$ indicate convolution with the seeing (see also [44, 45]). A prediction of the measurable velocity dispersion σ_v is therefore obtained accounting for the observational characteristics of the survey, i.e. through the convolution, over \mathcal{A} , of the product $I(R)\sigma_s^2$ with the seeing \mathcal{P} . Note that the cosmological dependence of σ_v is contained only in the combination $\Sigma_{cr} R_E^{\gamma'-1}$, therefore separate σ_v as:

$$(\sigma_v)^2 = (1 - \kappa_{\text{ext}}) \frac{D_s}{D_{ls}} \mathcal{F}(\gamma', \theta_E, \beta_{\text{ani}}, r_{\text{eff}}) \quad (2.17)$$

where the terms \mathcal{F} accounts for the computation of the integral in eq. (2.16) without the cosmological terms, θ_E is the angle associated with the Einstein radius, and all the cosmological information is contained in the ratio D_s/D_{ls} . In this work, we follow the spectral rendering approach of [45] to compute the luminosity-weighted LOS velocity dispersion from eq. (2.16).

3 Analysis method and mock datasets

The final goal of this paper is to assess how well future surveys of strongly lensed systems will constrain cosmological parameters, with a particular focus on simple extensions to the Λ CDM model. We do so by comparing the theoretical predictions of different cosmological models with forecasted datasets, based on mock catalogues. In practice we aim at calculating the *posterior distribution* $P(\vec{\pi}|\vec{d})$ for a set of cosmological parameters $\vec{\pi}$ given the set of (forecasted) data \vec{d} . Using the Bayes' theorem, this can be written as

$$P(\vec{\pi}|\vec{d}) \propto P(\vec{d}|\vec{\pi})P(\vec{\pi}), \quad (3.1)$$

where $P(\vec{d}|\vec{\pi})$ is the *likelihood* of \vec{d} given $\vec{\pi}$, and $P(\vec{\pi})$ is the prior distribution.

This expression for the posterior distribution does not include possible nuisance parameters which would account for uncertainties in the modeling of the lensed system, its environment and LOS effects. We will first generalize it to include these parameters and then marginalize over them in order to obtain the final distribution only for the cosmological parameters. We consider the following nuisance parameters $\vec{\pi}_{\text{nuis}} = (r_{\text{ani}}, \kappa_{\text{ext}})$. The final posterior can be obtained as [44]

$$P(\vec{\pi}|\vec{d}) \propto \int dr_{\text{ani}} d\kappa_{\text{ext}} P(\vec{d}|\vec{\pi}, \vec{\pi}_{\text{nuis}}) P(\vec{\pi}) P(r_{\text{ani}}) P(\kappa_{\text{ext}}), \quad (3.2)$$

where $P(r_{\text{ani}})$ and $P(\kappa_{\text{ext}})$ are the prior distributions on each nuisance parameter. Notice that in this work we do not include as nuisance parameters the other terms that enter in the lens model, e.g. the Einstein radius θ_E , the external shear γ_{ext} and the slope of the density profile power law γ' . In this paper we assume these to be perfectly known, and we focus instead mainly on the degeneracy between H_0 and κ_{ext} which may, in turn, also affect the constraints on DE equation of state through the well-known degeneracy between H_0 and DE parameters. It is important to stress that also the lens modeling has an impact on parameter

estimation; in particular, it has been shown how assumptions on the lens density profile significantly affect the measurement of H_0 [53]. We leave however the study of the impact on our results of the inclusion of the whole parameter space of the lens model for a future work.

As discussed in section 2, in order to break the mass-sheet degeneracy SLTD surveys combine measurements of the time delay between different images (Δt) and of the projected velocity dispersion within the lens (σ_v). The latter contains also a dependence on the cosmological parameters. Hence, our data vector will be therefore composed of this pair of measurements for each lensed system included in the dataset, with $\vec{d} = (\vec{\Delta t}, \vec{\sigma}_v)$. In order to constrain our cosmological models, these measurements need to be compared with the theoretical predictions $\vec{\Delta t}^{\text{th}}$ and $\vec{\sigma}^{\text{th}}$. Assuming that a Gaussian likelihood this can be written as

$$P(\vec{d}|\vec{\pi}) = \exp \left[-\frac{1}{2} \left(\sum_{i,j} \frac{(\Delta t_{i,j}^{\text{th}}(\vec{\pi}, \vec{\pi}_{\text{nuis}}) - \Delta t_{i,j})^2}{\sigma_{\Delta t_{i,j}}^2} + \sum_i \frac{(\sigma_i^{\text{th}}(\vec{\pi}, \vec{\pi}_{\text{nuis}}) - \sigma_{v,i})^2}{\sigma_{\sigma_{v,i}}^2} \right) \right], \quad (3.3)$$

where the index i runs over all the lensed systems in the dataset, j runs over the image pairs for each of the systems and we assume there is no correlation between the measurements of different systems. For a given set of cosmological ($\vec{\pi}$) and lens model ($\vec{\pi}_{\text{nuis}}$) parameters, the theoretical predictions $\Delta t_{i,j}^{\text{th}}$ and $\sigma_{v,i}^{\text{th}}$ can be obtained from eq. (2.1) and eq. (2.16) respectively. We compute the angular diameter distances that appear in these equations using EFTCAMB [56, 57], a public patch to CAMB [58, 59].

With these predictions we can then reconstruct the posterior distribution $P(\vec{\pi}|\vec{d})$ sampling the parameter space and computing the likelihood of eq. (3.3) for each sampled point. The parameter space is sampled through the public Monte-Carlo Markov-Chain (MCMC) code CosmoMC [60], with the parameter vector $\vec{\pi}$ including the total matter density Ω_m , the Hubble constant H_0 and w_0 and w_a , which parameterize the DE equation of state via the CPL form [61, 62]

$$w(z) = w_0 + w_a \frac{z}{1+z}. \quad (3.4)$$

Using this parameterization, the dimensionless Hubble rate $E(z)$ appearing in eq. (2.4) can be written as

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_{\text{DE}}(1+z)^{3(1+w_0+w_a)} \exp \left[-w_a \frac{z}{1+z} \right]}. \quad (3.5)$$

In the following, we will explore three different DE models and this will determine whether or not we sample w_0 and w_a . The cases we investigate are:

- Λ CDM, where both parameters are fixed to $w_0 = -1$ and $w_a = 0$, recovering the standard cosmological constant equation of state $w(z) = -1$;
- w CDM, where $w_a = 0$, but we keep w_0 free to vary, obtaining a constant equation of state which might however deviate from -1 ;
- $w_0 w_a$ CDM, where both w_0 and w_a are free to vary and we explore the possibility of a DE with a time dependent equation of state.

We always assume a flat Universe, with the DE density set by the relation $\Omega_{\text{DE}} = 1 - \Omega_m$.

Parameter	Ideal case	Realistic case
Ω_m	[0, 1]	[0, 1]
H_0	[40, 140]	[40, 140]
w_0	[-3, 0]	[-3, 0]
w_a	[-4, 4]	[-4, 4]
κ_{ext}	$\delta_D(-0.003)$	$\mathcal{G}(-0.003, 0.05)$
$r_{\text{ani}} (")$	$\delta_D(3.5)$	[0.665, 6.65]

Table 1. Prior ranges on the cosmological and nuisance parameters sampled in our analysis.

$\theta_E (")$	q	$\theta_q (^\circ)$	γ'	γ_{ext}	$\phi_{\text{ext}} (^\circ)$	κ_{ext}	$r_{\text{eff}} (")$	$r_{\text{ani}} (")$	β_1	β_2
1.18	0.8	-16.8	1.93	0.03	63.7	-0.003	1.33	3.5	0.03	-0.03

Table 2. Fiducial values for the mock lens parameters.

As stated above, sampling only over the parameters $\vec{\pi}$, while keeping the nuisance parameters $\vec{\pi}_{\text{nuis}}$ fixed to their fiducial values, implicitly assumes that the lensed system is perfectly known: we label such cases as *ideal*. We consider also a *realistic* cases, where we also include two of the important nuisance parameters to the free parameter set (κ_{ext} and r_{ani}). In table 1 we show the prior distributions assumed for all the parameters, with the cosmological ones always sampled using a uniform prior $[\pi_{\text{min}}^i, \pi_{\text{max}}^i]$. In the w CDM and w_0w_a CDM cases, we additionally impose an *acceleration prior*, which limits the DE equation of state to $w(z) < -1/3$. In the realistic case, we additionally sample the nuisance parameters using Gaussian prior \mathcal{G} for κ_{ext} and a uniform prior for r_{ani} . As stated by [44], we stress that the uncertainty in r_{eff} has a negligible effect on the velocity dispersion modeling.

3.1 Mock catalogues

The last ingredient that we need in order to compute the likelihood, is the data vector \vec{d} . We generate three mock catalogues containing different numbers of observed systems, i.e. with $N_{\text{lenses}} = 10, 100, 1000$ lenses, uniformly distributed in the redshift range $0 < z \leq 1$. Furthermore, we assume the mass profile of the lenses to be given by eq. (2.9), choosing identical θ_E and γ for all the systems. The fiducial values of the mass model parameters are set to those of the H0LiCOW resolved quadruply lensed system HE0435-1223 [63], listed in table 2. For simplicity, we assume that the redshift difference between the lens and the background source is the same for all the systems, with $\Delta z = 1.239$.

In order to calculate the image positions we assume that the source position $\beta = (\beta_1, \beta_2)$ is known and is the same for all the systems, with the value given in table 2. We then solve eq. (2.3) analytically to find the image positions for each system, assuming that all the systems in our dataset produce quadruple images of the background source.

Notice that our modeling of the mock catalogue implies also that we assume the same external convergence κ_{ext} for all systems. We fix the value of the external convergence on the best fit value of the distribution of κ_{ext} estimated by the analysis of the environment of the lens HE0435-1223 [64]. The assumption of a constant κ_{ext} should be seen as a first approximation, as this parameter encompasses effects due both to the local environment of the lens systems and to the presence of structures along the LOS, and therefore can significantly vary between different systems. In appendix A we investigate the possible impact of this oversimplification

of our mock dataset, finding an overall induced bias on Ω_m of $\approx 2\sigma$. We leave a more extensive study of possible solutions to a dedicated work. The external convergence might also carry cosmological information, in particular if one wants to explore deviations from General Relativity; the different evolution of Large Scale Structures in modified gravity theories might indeed imprint characteristic features in the effect that these structures have on SLTD measurements, which can in principle be exploited to constrain departure from the standard General Relativity description [65].

In addition to the lens parameters, in order to generate our mock datasets we also need to assume a fiducial cosmology. We choose two different fiducials, thus creating two classes of mock data:

- Λ -mock, where the DE equation of state parameter is constant in time and equal to $w(z) = -1$ (thus assuming $w_0 = -1$ and $w_a = 0$), and the cosmological parameters are chosen to be in agreement with the constraints obtained by the Planck collaboration [66], i.e. $\Omega_m = 0.295$, $H_0 = 67.3$ Km/s/Mpc.
- w -mock, which differs from the Λ -mock only in the value of the DE equation of state parameter, which is again constant but set to $w(z) = -0.9$.

In both cases, we assume a flat Universe, with $\Omega_{\text{DE}} = 1 - \Omega_m$.

Once the lens and fiducial cosmological parameters are assumed, the relative time delays and the velocity dispersion can be computed following eq. (2.5) and eq. (2.16). Computing these for each of the N_{lens} lensed system contained in our dataset allows us to create our simulated data points; for each of these we assume that the time delays are observed with an error of $\sigma_{\Delta t} = 0.8$ days,¹ while for the velocity dispersion measurements we assume a constant error $\sigma_{\sigma_v} = 15$ km/s. As an example, we show in figure 2 the Λ -mock obtained for a forecasted survey of $N_{\text{lens}} = 10$ lensed systems.

4 Forecasts for cosmological parameters

In this section we present the forecasted bounds on cosmological parameters obtained following the analysis procedure and the mock datasets described in section 3. We focus on the Λ -mocks, containing $N_{\text{lens}} = 10, 100, 1000$ lensed systems, and we analyse them, both in the ideal and realistic cases, using the three DE models we introduced: Λ CDM, w CDM and w_0w_a CDM.

Λ CDM. In a standard Λ CDM scenario, we find that future strong lensing surveys will be able to constrain H_0 at the same level of Planck [66], $\sigma_{H_0} \sim 1\%$, already with $N_{\text{lens}} = 10$ in the ideal case; this result is consistent with what was found in [68] for a catalogue of 55 lenses. Increasing the number of systems to $N_{\text{lens}} = 100$, improves the bound on H_0 by a factor of ~ 3 , while with our most optimistic dataset ($N_{\text{lens}} = 1000$) we find that H_0 could be constrained with an error of $\sim 0.1\%$. These results are shown in the left panel of figure 3 and in the Λ CDM entries of table 3 shown in appendix B.

In the realistic cases, where the nuisance parameters are let free to vary, the constraints on the Hubble rate are worsened by a factor of ~ 4 for $N_{\text{lens}} = 10$. This worsening is mainly due to the strong degeneracy between H_0 and κ_{ext} described by eq. (2.8), which is clearly visible in the right panel of figure 3. Increasing the number of lenses improves the bounds on

¹Based on our generated time delay values, this estimate fulfills the requirement of 0.2% level time delay accuracy which, as pointed out by [67], is needed for a low biased cosmological inference.

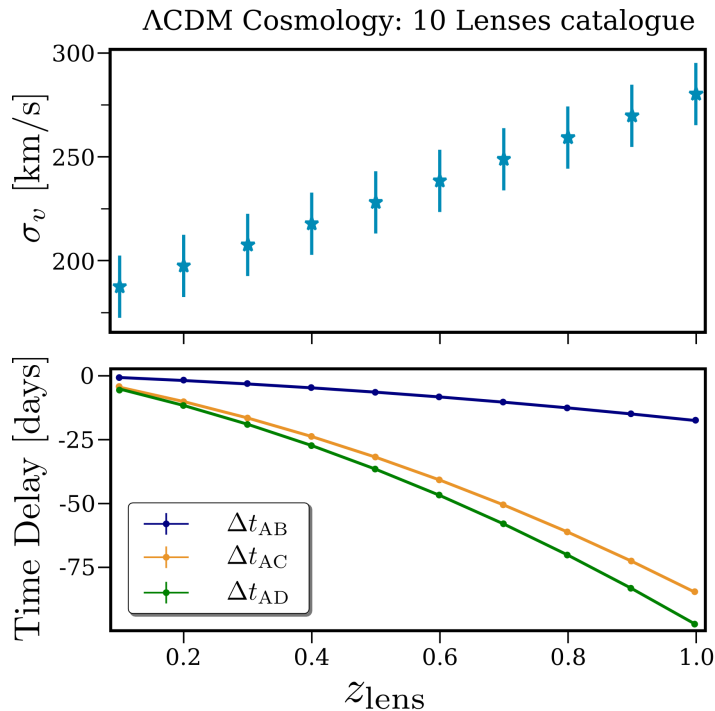


Figure 2. Λ CDM mock for a survey with $N_{\text{lens}} = 10$. The upper panel shows the velocity dispersion of the lens galaxies projected along the line of sight. The bottom panel shows the absolute time delay differences between image *A* and the other three images.

both parameters, and we reach a $\sim 2\%$ constraint on H_0 when $N_{\text{lens}} = 1000$. All the results for the realistic cases are shown in table 4 in appendix B.

w CDM. Using the mock datasets to constrain this simple extended DE model, we find that in the ideal case with $N_{\text{lens}} = 10$, H_0 can now be measured with an error of $\sim 4\%$, which is improved to $\sim 1\%$ and $\sim 0.3\%$ for $N_{\text{lens}} = 100$ and $N_{\text{lens}} = 1000$ respectively. The parameter determining the equation of state for DE, w_0 , is constrained at the level of $\sim 34\%$ for the 10 lenses case, while moving to the optimistic 1000 lenses configuration boosts the constraining power on this parameter up to $\sim 2\%$, thanks to the breaking of the degeneracy between H_0 and w_0 . Such a result highlights how the improvement of SLTD measurements will significantly impact the investigation of DE alternatives to Λ CDM. These results are shown in figure 4, while the constraints on all the sampled parameters are included in appendix B in table 3.

When considering the realistic case (see right panel of figure 4 and table 4), the worsening of the constraints due to the nuisance parameters has a different trend with respect to the Λ CDM model; in the 10 lenses case, the additional degeneracy introduced by κ_{ext} worsen the bounds on H_0 only by a factor ~ 2 (with respect to the factor ~ 4 of the Λ CDM case), due to the already existing degeneracy between H_0 and w_0 , while in the 100 and 1000 lens cases, when this degeneracy is broken, the constraints become looser by a factor ~ 5 and ~ 7 respectively.

As κ_{ext} affects w_0 only through its degeneracy with H_0 , moving from the ideal to the realistic case does not have an extreme impact on the DE parameter, with the constraints getting worse by a factor of ~ 2 for $N_{\text{lens}} = 10, 100, 1000$

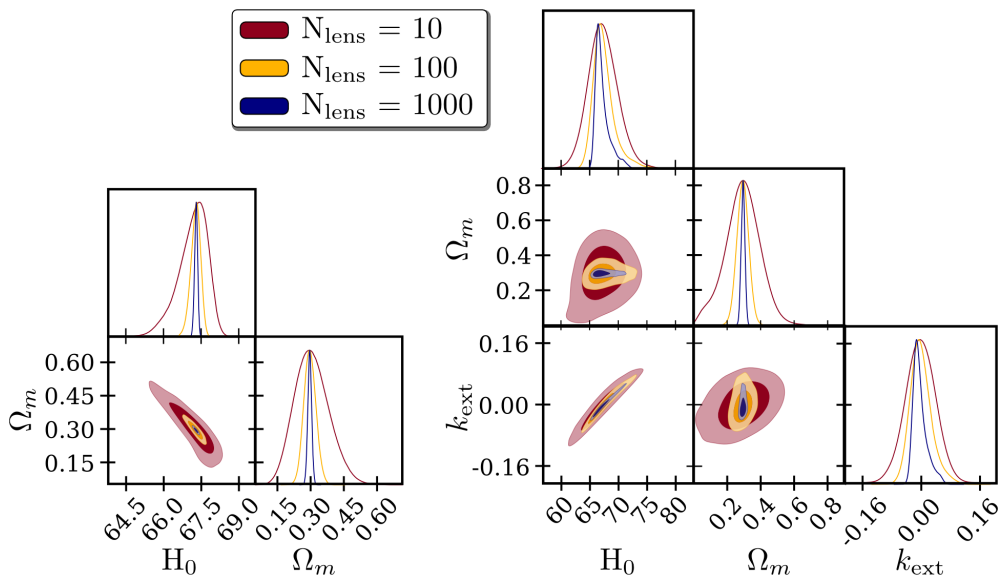


Figure 3. Constraints on the Λ CDM cosmological model obtained using the Λ -mock datasets with $N_{\text{lens}} = 10$ (red contours), 100 (yellow contours) and 1000 (blue contours). The left panel shows the results for the ideal case, with nuisance parameters fixed to their fiducial values, while the right panel refers to the realistic case where these parameters are free. We show here only κ_{ext} as this is the only nuisance parameter with a significant degeneracy with the cosmological ones.

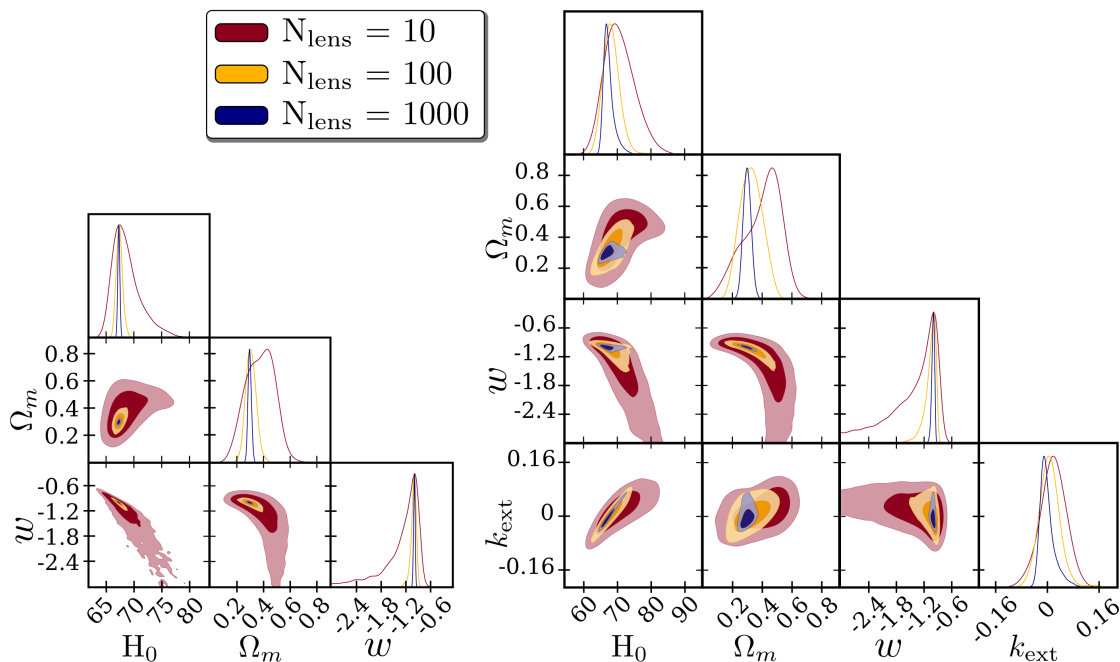


Figure 4. Constraints on the w CDM cosmological model obtained using the Λ -mock datasets with $N_{\text{lens}} = 10$ (red contours), 100 (yellow contours) and 1000 (blue contours). The left panel shows the results for the ideal case, with nuisance parameters fixed to their fiducial values, while the right panel refers to the realistic case where these parameters are free. We show here only κ_{ext} as this is the only nuisance parameter with a significant degeneracy with the cosmological ones.

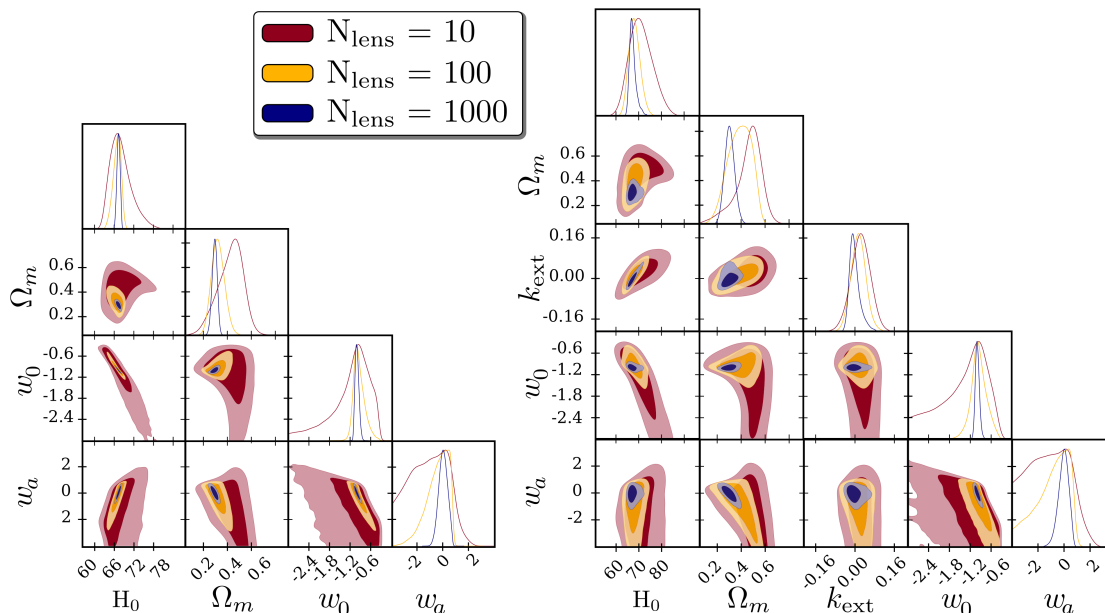


Figure 5. Constraints on the w_0w_a CDM cosmological model obtained using the Λ -mock datasets with $N_{\text{lens}} = 10$ (red contours), 100 (yellow contours) and 1000 (blue contours). The left panel shows the results for the ideal case, with nuisance parameters fixed to their fiducial values, while the right panel refers to the realistic case where these parameters are free. We show here only κ_{ext} as this is the only nuisance parameter with a significant degeneracy with the cosmological ones.

w_0w_a CDM. In this case, we find that due to the degeneracies between H_0 and the DE parameters w_0 and w_a , the constraints on H_0 are significantly worsened. We see that a strong lensing survey could reach a $\sim 1\%$ level bound on the Hubble parameter only with the most optimistic configuration of this paper, i.e. $N_{\text{lens}} = 1000$ in the ideal case. Due to their degeneracy, w_0 and w_a are not efficiently constrained solely with SLTD data; the best constraint is of the order of $\sim 5\%$ on w_0 and $\sigma_{w_a} \sim 0.3$ on w_a , in the most optimistic case. Of course, possible synergies of future SLTD surveys with other background probes, such as SNIa or BAO, would significantly improve this situation, breaking the degeneracy between the DE parameters and allowing to obtain again a bound on H_0 competitive with respect to CMB or local measurements.

The effect of nuisance parameters when considering the realistic case is similar to what is found for the w CDM case, with the additional parameters affecting mainly the bounds on H_0 , whose error reaches now $\sim 2\%$ for $N_{\text{lens}} = 1000$, while not showing significant impact on the DE parameters.

The results for the w_0w_a case are shown in figure 5, while numerical constraints are reported in tables 3 and 4 shown in appendix B.

4.1 Figure of merit for strong lensing time delay

We would like to quantify the constraining power of SLTD surveys, and its improvement with the number of observed systems, in a general way that allows to directly compare the performance of different surveys. For this purpose, we rely on the commonly used *Figure of Merit* (FoM) [69]. For two parameters α and β the FoM is

$$\text{FoM}_{\alpha\beta} = \sqrt{\det \tilde{F}_{\alpha\beta}}, \quad (4.1)$$

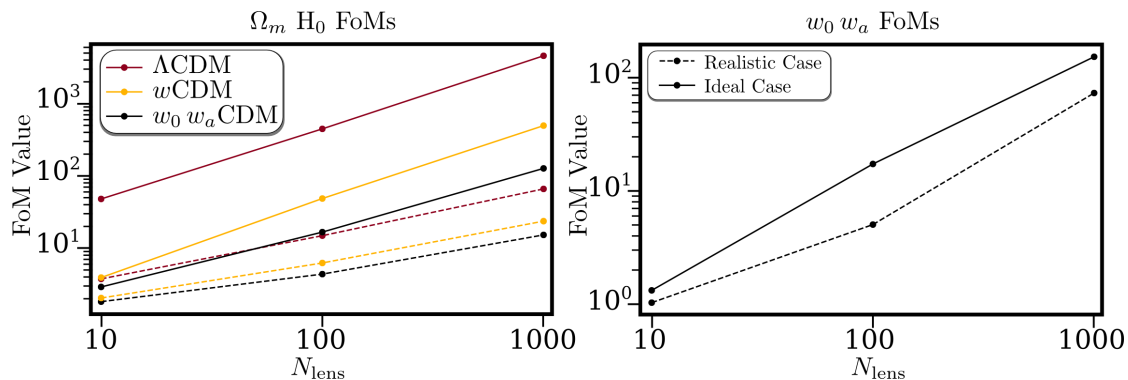


Figure 6. FoM for the Ω_m, H_0 (left panel) and w_0, w_a (right panel) as a function of N_{lens} when analysing Λ CDM (red lines), w CDM (yellow lines) and $w_0 w_a$ CDM (black lines). The solid lines refer to the ideal case, while the dashed lines account for free nuisance parameters (realistic case).

where \mathbf{F} is the *Fisher information matrix* for a generic number of parameters and $\tilde{F}_{\alpha\beta}$ is the Fisher matrix marginalized over all the parameters except for α and β . Given its definition, the FoM gives an estimate of the area of the confidence contours for two parameters, thus quantifying the constraining power of an experiment on them, taking also into account their correlation. It is important to remember that such a definition implies approximating the posterior distribution $P(\vec{\pi}|\vec{d})$ to a Gaussian.

From our MCMC analysis we derived a covariance matrix $\mathbf{C} = \mathbf{F}^{-1}$, which contains all the sampled parameters. Let us focus on two cosmological parameters of interest that are common to all the models investigated in this work: Ω_m and H_0 . We shall marginalize the covariance matrices over all the other parameters and then compare the constraining power of our mock datasets in each of the cases analysed, using the FoM for Ω_m and H_0 :

$$\text{FoM}_{\Omega_m H_0} = \sqrt{\det \tilde{C}_{\Omega_m H_0}^{-1}}. \quad (4.2)$$

The posterior for Ω_m and H_0 is very close to a Gaussian one when the parameters are tightly constrained, e.g. in the Λ CDM case with $N_{\text{lens}} = 1000$; however, the gaussian approximation becomes less and less efficient as the number of lenses in the dataset decreases. Hence, the FoM values for the less constraining cases might be overestimated.

In the left panel of figure 6 we show the trend of the FoM for the ideal (solid lines) and realistic (dashed lines) cases as a function of N_{lens} . Comparing these two cases, we can notice how the improvement in constraining power brought by the number of lenses is less significant when the nuisance parameters are let free to vary. We also notice that the FoM for the ideal and realistic cases become more similar to each other as we go from Λ CDM to the more general DE parametrized by CPL. This is consistent with the trends that we have discussed in section 4.

In the right panel of figure 6 we also show the FoM for the w_0 and w_a parameters, $\text{FoM}_{w_0 w_a}$. Such quantity is commonly used when quantifying the expected sensitivity of future experiment to the DE sector. We find that SLTD surveys can reach values of $\text{FoM}_{w_0 w_a} \approx 100$ for $N_{\text{lens}} = 1000$ (in the ideal case), which is comparable with other future surveys, such as Euclid, expected to reach $\text{FoM}_{w_0 w_a} \approx 400$ with its primary probes, [70] or the $\text{FoM}_{w_0 w_a} \approx 100$ reached by the combination of Weak Lensing measurements from SKA1 and DES, together with Planck observations [71].

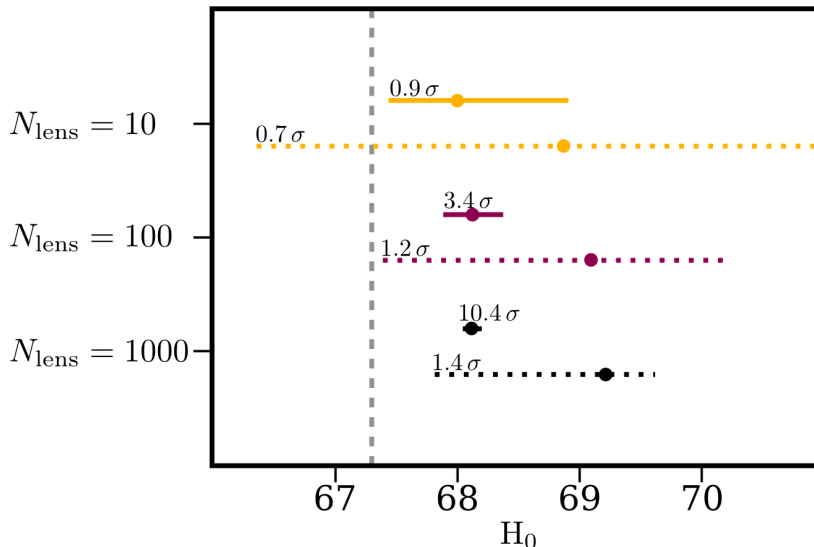


Figure 7. Marginalized means and error estimates on the value of the Hubble constant using the w -mocks ($w_0 \neq -1$) analysed keeping the w fixed to the Λ CDM value. The solid lines represent the ideal cases, while the dashed lines show the results of the realistic cases. The numbers above each line correspond to $S(H_0)$ computed following eq. (5.1).

5 A smoking gun for dark energy?

In section 4 we used the Λ -mock and constrained three DE models which contained the assumed fiducial cosmology as a limiting case. However, when real data will be available, we will have no a priori knowledge of the underlying cosmological model, and assumptions about the latter might affect the results. In this section we test the impact of wrong assumptions about the underlying cosmology on constraints from future surveys. To this extent, we consider the w -mocks, generated with a fiducial $w_0 = -0.9$, and fit the data assuming instead a Λ CDM cosmology. Given that the latter does not contain the true fiducial as a limiting case, we can quantify the sensitivity of future surveys on this assumption by computing the shift of the mean values obtained for cosmological parameters. In particular, for H_0 we have

$$S(H_0) = \frac{|H_0 - H_0^{\text{fid}}|}{\sigma_{H_0}}, \quad (5.1)$$

where the fiducial value is the one used to generate the mock data, i.e. $H_0^{\text{fid}} = 67.3$ km/s/Mpc, and we assume that the H_0 distributions obtained through our analysis can be approximated by a Gaussian of width σ_{H_0} .

In figure 7 we show the bounds on H_0 and the values of $S(H_0)$ changing the sample size, both for the ideal and realistic cases, when fitting the w -mocks with a Λ CDM cosmology. In the realistic case, the shift on this parameter is never statistically significant and reaches the maximum of $S(H_0) = 1.4$ for the 1000 lenses mock. However, in the ideal case the shift can be as high as 10σ . This implies that, if mass modelling of lenses reaches extreme accuracy with future surveys, the assumption of wrong cosmology could lead to significant tensions on H_0 value between SLTD observations and other independent cosmological measurements (e.g. from SH0eS [3]).

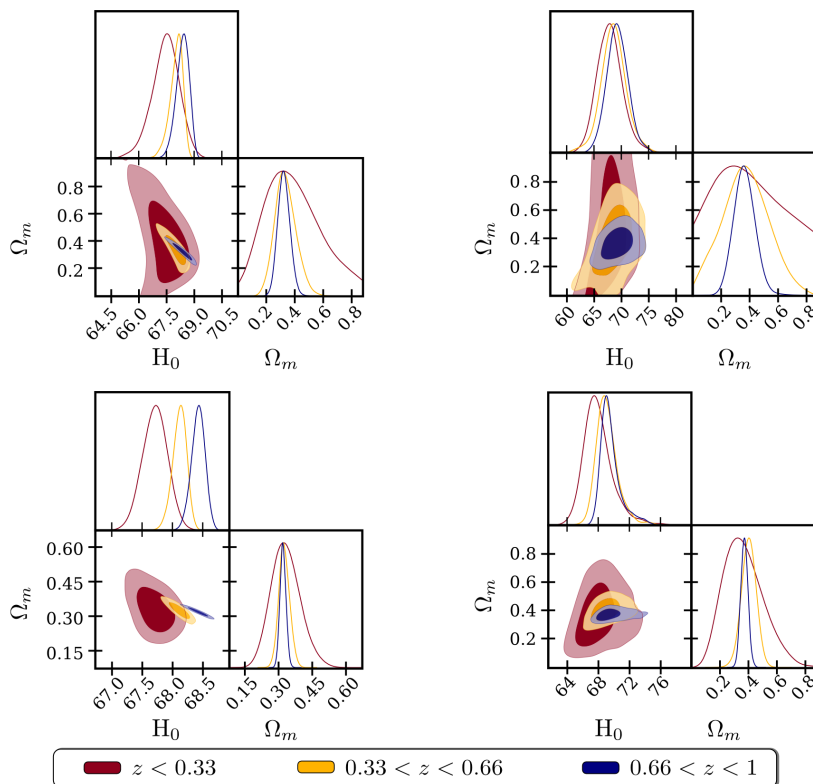


Figure 8. Comparison of the constraints on H_0 and Ω_m from the analysis of the three datasets obtained splitting the original mock data in three redshift bins. Top panels refer to the $N_{\text{lens}} = 100$ dataset, while the bottom panels to the $N_{\text{lens}} = 1000$. Left panels do not include nuisance parameters (ideal case), while the right panels refer to the realistic case.

Interestingly, it might be possible to exploit this shift effect, to build a consistency check of the assumed cosmological model. Using a dataset of N_{lens} observed systems, we can split it in N_{bin} redshift bins and use the resulting datasets separately to constrain the parameters of a given cosmological model, e.g. Λ CDM. Should this model differ from the “true” cosmology (or the fiducial one in the case of forecasts), the results obtained analysing separately the three datasets will be in tension with each other. As a test case, we split the w -mock, both for $N_{\text{lens}} = 100$ and $N_{\text{lens}} = 1000$, in $N_{\text{bin}} = 3$ redshift bins and we fit these with a Λ CDM cosmology. In figure 8 we show the constraints on H_0 and Ω_m obtained through this analysis for $N_{\text{lens}} = 100$ (top panels) and $N_{\text{lens}} = 1000$ (bottom panels), with the left (right) panels showing the results in the ideal (realistic) case. While for 100 overall lenses both the ideal and realistic case show no tensions on the cosmological parameters, in the ideal case with $N_{\text{lens}} = 1000$ a tension between the results on H_0 appears, with a $\sim 2\sigma$ significance between the first and the third bin. Such a result highlights how, with a sufficient number of observed systems, the assumption of a Λ CDM cosmology could be checked internally using only this observable; a statistically significant tension on the measured parameters in different redshift bins would then provide a smoking gun for the breakdown of Λ CDM, after internal systematics effect are excluded.

6 Conclusions

In this paper we explored constraints on the nature of Dark Energy (DE) from future Strong Lensing Time Delay (SLTD) measurements. We simulated SLTD datasets starting from a fiducial cosmological model and a description of the lens profile. For the latter, we assumed a common lens profile for all the systems. We distributed the lenses uniformly in the redshift range $0 < z_{\text{lens}} < 1$, and we simulated the time delay that these would generate among different images of a background source, always placed at a $\Delta z = 1.239$ from the lens, assuming different cosmologies.

In the ideal case, in which the lens profile and external environment parameters are perfectly known, SLTD measurements can provide constraints that are competitive with other upcoming cosmological observations; H_0 can be constrained with an error as small as $\sim 0.1\%$ assuming a Λ CDM model and an optimistic dataset of $N_{\text{lens}} = 1000$ observed systems, while this error increases up to $\sim 1\%$ when the DE equation of state is allowed to vary.

We also evaluated the figure of Merit (FoM) for the w_0 and w_a in the Chevallier-Polarski-Linder parametrization of DE. We found that in our most optimistic case, the FoM can reach a value of ~ 100 , which is competitive with what is expected from upcoming Large Scale Structure surveys. When considering a more realistic case, with κ_{ext} included as a nuisance parameter, that encodes the external convergence brought by additional structures along the line of sight between the observer and the lens. When we allow κ_{ext} to vary (according to a prior), we find that H_0 can be constrained only up to $\sim 2\%$ both in the Λ CDM and $w_0 w_a$ CDM cases. In the latter case, the FoM on w_0, w_a can reach only ~ 60 .

Furthermore we quantified the bias on cosmological parameters arising from a wrong assumption on the cosmological model in the analysis of future data. We analysed the mock dataset generated assuming $w = -0.9$ with a Λ CDM cosmology, i.e. with a fixed $w = -1$, and computed the shift $S(H_0)$ on the Hubble constant with respect to the fiducial value used to obtain the mock data. Interestingly, we found that in the ideal case this shift can reach 10σ , highlighting how comparing the results obtained from SLTD observations with other measurements of H_0 could produce significant tensions on this parameter. Such a shift is however almost completely washed out in the realistic case, where $S(H_0)$ never exceeds $\sim 1.5\sigma$.

The study of the shift in H_0 , suggested an interesting, and potentially powerful, consistency check of the cosmological model, entirely based on SLTD data. We split our mock datasets constructed with a w CDM cosmology, with $w = -0.9$, and analysed the three resulting datasets separately, (wrongly) assuming Λ CDM cosmology. In the ideal case with $N_{\text{lens}} = 1000$, we found that the measurements of H_0 in the different bins would be in tension with each other up to $\sim 2\sigma$. This result shows how, with an accurate modeling of the observed lenses, future SLTD datasets can be used to internally test the assumptions on the cosmological model.

The future of SLTD looks bright; measurements are reaching the same accuracy of other, more traditional probes of background cosmology. As we have shown with our analysis, in the upcoming years, SLTD will provide competitive and complementary constraints on dark energy. It would be of great interest to not only further explore SLTD in the context of extended theories of gravity [72–74], but also in combination with other cosmological probes. This work represents a first step in all these directions. In particular, the likelihood pipeline that we have built in CosmoMC will be of great use to explore complementarity of SLTD with other cosmological probes.

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A Impact of κ_{ext} on cosmological parameters

While producing our mock datasets, we made several assumptions which allowed us to simplify the procedure of generating simulated data. As we are interested in cosmological parameter estimation, the assumption of having the same perfectly known lens system with the same density and kinematic properties, repeated N_{lens} times over our redshift range, coincides with the selection of well observed lenses only, out of a future SLTD survey.

There is one lens property however that escapes this logic, namely the external convergence κ_{ext} . Our assumption of it taking the same value for each lens system is clearly an oversimplification. As κ_{ext} traces the mass distribution along the LOS, each lens has its own LOS mass distribution and in a realistic sample the value of κ_{ext} will then differ from lens to lens. For isolated lenses² like the ones assumed in our mock, external convergence will be dominated by the perturbations to the gravitational potential (i.e. perturbations of the matter density) along the LOS. In a homogeneous and isotropic Universe, density fluctuations have zero mean but they do possess a non-zero variance, which will reflect in the distribution of κ_{ext} [44, 75]. Estimations show this variance to be $\mathcal{O}(10^{-2})$, see e.g. [31, 76–78].

Fixing the value of the external convergence, can therefore have a significant impact on parameter extraction due to κ_{ext} being degenerate with the cosmological parameters as it can be seen from eqs. (2.8)–(2.17).

In order to estimate the impact of this assumption we generate again our 100 lenses Λ CDM fiducial mock data, assuming this time a different value of κ_{ext} for each lens system. We sample these values from a Gaussian distribution with mean zero and standard deviation 0.05, i.e. now our mock dataset accounts for a 5% spread in the value of $1 - \kappa_{\text{ext}}$.

We analyze this dataset using the same pipeline described in section 3, in particular a single, common nuisance parameter κ_{ext} . As it can be seen in figure 9, this yields a posterior distribution on Ω_m shifted towards values higher than the fiducial cosmology, which is now excluded at $\approx 2\sigma$. In order to remove this problem and to still account for the external convergence as a nuisance parameter, one would need to account for a different κ_{ext} for each lens system. This approach would be unfeasible to include in our MCMC analysis, given the large number of extra parameters introduced.

One other possible approach is to include the dispersion of the κ_{ext} values of our dataset as an extra systematic error. As showed in [79], the lack of systematic error may in fact lead to biased results in MCMC analysis, even when the data are fitted with the right cosmological model. For this reason, we add to the observational errors σ_X discussed in section 3.1, with

²Here with isolated we mean galaxies whose potential is not significantly distorted by neighbouring galaxies.

X being our observable Δt and σ_v , the systematic error contribution $\sigma_{\text{sys}} = 0.05$, set to coincide with the standard deviation of the κ_{ext} distribution. We obtain therefore a total error $\sigma_{X,\text{tot}}$

$$\sigma_{X,\text{tot}}^2 = \sigma_X^2 + \sigma_{\text{sys}}^2. \quad (\text{A.1})$$

Figure 9 shows the results obtained using this updated dataset, highlighting how the bias with respect to the fiducial cosmology is now removed, at the price of a reduced precision, with the errors on Ω_m and H_0 increasing by a factor 2 and 1.5, respectively.

It is interesting to note that in this analysis the parameter suffering the largest shift is Ω_m , rather than H_0 as one would expect from the interplay between the latter and κ_{ext} (see e.g. eq. 2.8).

Below we take a closer look into these results.

As discussed in section 2, the effect of κ_{ext} on $D_{\Delta t}$ (containing all the cosmological information) is to rescale its value at each point in redshift by a constant amount which may be different from a lens to another. Since $D_{\Delta t} \propto H_0^{-1}$, the Hubble constant is also rescaled by κ_{ext} , therefore one would expect the bounds on H_0 to be the most affected by the underlying κ_{ext} distribution.

However it must be noted that in our analysis we assume the distribution of κ_{ext} to have zero mean: therefore, on average, the overall value of the time delay distance is not affected by the inclusion of a Gaussian-distributed κ_{ext} and so is the average (i.e. the best-fit) value of H_0 for our mock. Our assumption about κ_{ext} is merely adding an additional 5% variation to the value of the time delay for each lens in the catalog. However the precision with which we constrain H_0 is strongly sensitive to κ_{ext} since the two parameters are highly degenerate as expected (see right panels of e.g. figure (3–5) and figure 9). The explanation for a biased Ω_m is also rather simple. The sensitivity of $D_{\Delta t}$ to a variation of Ω_m grows with redshift (see figure 10) where also the effect of κ_{ext} becomes more relevant. These two trends combine together and lead to a biased inference of the posterior of Ω_m .

Finally, let us notice that the analysis performed in this appendix informs us on the bias resulting from assuming the same κ_{ext} when analyzing a (mock) data set where instead κ_{ext} is different for each lens. This is still not the most realistic scenario, since it implies knowing the distribution of κ_{ext} for the lenses considered. While the average of κ_{ext} over the whole sky must be zero by definition, observed lenses seem to be found preferentially in overdense regions (see e.g. the discussion in [80] about the six H0LiCOW lenses). Therefore, the observed κ_{ext} distribution is not a Gaussian but is positive skewed (see e.g. [48, 75, 81]). Kinematic measurements of the lens can be used to reduce the number of possible mass models for a specific system (as we discussed in section 2), but they carry almost no information about lens environment: eq. (2.12) is in fact obtained integrating the mass profile up to the Einstein radius R_E [44], typically of the order of the galactic radius (few kpc), while the group of galaxies interacting with the lens can be extended up to a scale an order of magnitude greater than R_E . Therefore, one has to rely on other methods for the estimation of κ_{ext} such as number counts [81] or weak lensing measurements [82] to trace the mass distribution of the structures around a lens.

Furthermore, the distribution of κ_{ext} is also related to the assumed lens mass profile i.e. κ_{ext} distributions estimated from different mass profiles may slightly differ from one another. To avoid biased cosmological constraints one has then to average over several mass profiles in order to obtain an effective distribution for κ_{ext} (see e.g. [76]).

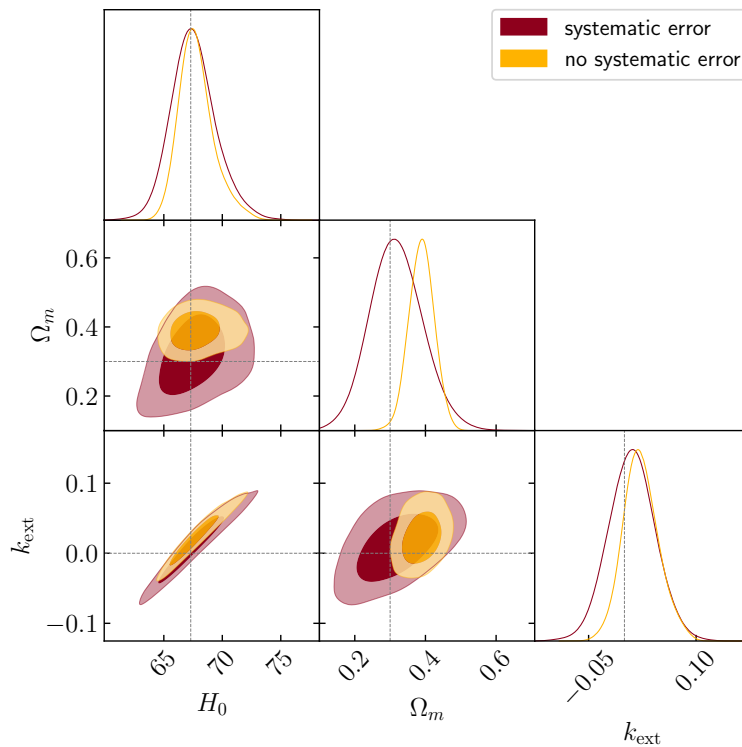


Figure 9. Comparison of the constraints on H_0 , Ω_m and κ_{ext} obtained fitting the $N_{\text{lens}} = 100$ dataset where each lens system has a different value of κ_{ext} drawn from a distribution $\mathcal{G}(0, 0.05)$. Yellow contours are obtained using the same errors on time delay and velocity dispersion as in the original analysis. Red contours use an error where the spread of the κ_{ext} distribution is included in the observations error as a systematic contribution. The gray dashed lines correspond to the fiducial values of the parameters considered in this plot.

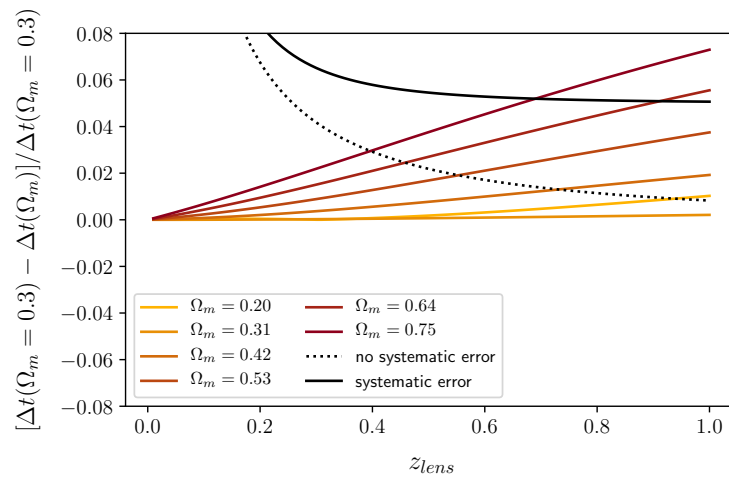


Figure 10. Percentage differences in SLTD for different choices of the matter density Ω_m with respect to our fiducial cosmology. The black lines show the percentage error for the image with the smallest error with (solid line) and without (dotted line) the inclusion of κ_{ext} systematic error. As fiducial here we use the mock dataset of our main analysis i.e. the one where all lenses have the same value of the external convergence.

B Constraints on cosmological parameters

In this appendix we show the constraints obtained on all free cosmological parameters when the Λ -mock is analysed. In table 3 we show the results obtained in the ideal case, when the nuisance parameters are assumed to be perfectly known, while table 4 shows the constraints in the realistic case, where also the κ_{ext} , r_{ani} and γ' are free. In these tables, for each parameter, we show the results obtained assuming different DE models, i.e. Λ CDM, w CDM and w_0w_a CDM.

Parameter	DE model	10 lenses	100 lenses	1000 lenses
Ω_m	Λ CDM	$0.304^{+0.069}_{-0.085}$	0.296 ± 0.027	0.2949 ± 0.0086
	w CDM	$0.37^{+0.12}_{-0.10}$	$0.305^{+0.037}_{-0.042}$	0.296 ± 0.012
	w_0w_a CDM	$0.425^{+0.12}_{-0.083}$	0.322 ± 0.053	$0.294^{+0.022}_{-0.020}$
H_0	Λ CDM	$67.16^{+0.70}_{-0.41}$	$67.28^{+0.21}_{-0.17}$	$67.299^{+0.063}_{-0.057}$
	w CDM	$68.1^{+2.1}_{-4.0}$	$67.42^{+0.51}_{-0.62}$	67.31 ± 0.18
	w_0w_a CDM	$68.3^{+3.0}_{-3.8}$	$66.9^{+1.4}_{-1.2}$	$67.33^{+0.52}_{-0.47}$
w_0	Λ CDM	–	–	–
	w CDM	$-1.30^{+0.47}_{-0.10}$	$-1.021^{+0.073}_{-0.046}$	$-1.002^{+0.020}_{-0.018}$
	w_0w_a CDM	$-1.19^{+0.74}_{-0.21}$	$-0.94^{+0.15}_{-0.19}$	$-1.001^{+0.045}_{-0.063}$
w_a	Λ CDM	–	–	–
	w CDM	–	–	–
	w_0w_a CDM	$-1.1^{+1.8}_{-1.3}$	$-0.43^{+1.0}_{-0.79}$	0.02 ± 0.34

Table 3. Mean marginalized values and their 68% confidence level bounds for the three DE model considered. We show here the results for the ideal case for 10, 100 and 1000 lenses.

Parameter	DE model	10 lenses	100 lenses	1000 lenses
Ω_m	Λ CDM	$0.292^{+0.11}_{-0.096}$	0.293 ± 0.035	0.295 ± 0.011
	w CDM	$0.401^{+0.15}_{-0.091}$	0.327 ± 0.075	0.299 ± 0.027
	w_0w_a CDM	$0.460^{+0.13}_{-0.065}$	$0.388^{+0.11}_{-0.084}$	0.304 ± 0.042
H_0	Λ CDM	$67.3^{+2.3}_{-2.7}$	$67.5^{+1.1}_{-2.1}$	$67.26^{+0.43}_{-1.5}$
	w CDM	$70.8^{+3.6}_{-5.4}$	$68.3^{+2.1}_{-2.8}$	$67.51^{+0.77}_{-1.9}$
	w_0w_a CDM	$71.1^{+4.4}_{-5.8}$	$68.2^{+2.2}_{-2.8}$	$67.48^{+0.82}_{-1.8}$
w_0	Λ CDM	–	–	–
	w CDM	$-1.44^{+0.62}_{-0.20}$	$-1.07^{+0.15}_{-0.050}$	$-1.007^{+0.036}_{-0.024}$
	w_0w_a CDM	$-1.41^{+0.89}_{-0.37}$	-0.96 ± 0.23	$-0.997^{+0.050}_{-0.064}$
w_a	Λ CDM	–	–	–
	w CDM	–	–	–
	w_0w_a CDM	$-1.2^{+1.9}_{-1.6}$	$-0.93^{+1.7}_{-0.64}$	$-0.07^{+0.46}_{-0.33}$
κ_{ext}	Λ CDM	-0.001 ± 0.041	$-0.001^{+0.023}_{-0.032}$	$-0.0040^{+0.0092}_{-0.023}$
	w CDM	0.017 ± 0.041	0.006 ± 0.034	$-0.001^{+0.011}_{-0.027}$
	w_0w_a CDM	0.023 ± 0.040	0.011 ± 0.030	$-0.001^{+0.013}_{-0.025}$
r_{ani}	Λ CDM	> 3.06	> 3.45	4.1 ± 1.5
	w CDM	> 3.46	> 3.33	4.2 ± 1.4
	w_0w_a CDM	> 3.57	> 3.55	$4.2^{+1.9}_{-1.3}$

Table 4. Mean marginalized values and their 68% confidence level bounds for the three DE model considered. We show here the results for the realistic case for 10, 100 and 1000 lenses.

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