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Retail Facility Layout Considering Shopper Path and Door Placement

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RETAIL FACILITY LAYOUT
CONSIDERING SHOPPER PATH AND DOOR PLACEMENT

A thesis submitted in partial fulfillment of the
requirements for the degree of
Master of Science in Industrial and Human Factors Engineering

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Sagarkumar D. Hirpara ENTITLED Retail Facility Layout Considering Shopper Path and Door Placement BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Industrial and Human Factors Engineering.

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ABSTRACT

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The physical design of a retail store is known to influence the attitude and behavior of shoppers, in turn affecting the store's performance. While literature in retail design has alluded to the impact of changes in department placements on impulse revenue, it has not accounted for the changes in the path of a shopper due to such modifications. Shopper path changes can alter a department's visibility to the shoppers as they pass by, and such visibility eventually impacts that department's impulse revenue. To address this gap, we study the retail facility layout problem by accounting for changes in the shopper path and door placement; we refer to it as RFLP-SPDP. We propose an optimization model for RFLP-SPDP that optimally places departments in the store in order to maximize the expected per shopper impulse revenue for the retailer. Because the dependency of shopper path changes on with changing layouts could not be expressed in a closed analytical form, we propose a Simulated Annealing based shortest path heuristic. This is then embedded in a Particle Swarm Optimization based solution approach to solve the overall RFLP-SPDP and implemented using parallel processing. Our experiments indicate that the derived solutions are sensitive to the shopper basket size, the shape of the store, and the number of doors and their location. Our results suggest up to 13.71% increase in impulse revenue for a deeper store over a square-shaped store, while up to 9.65% increase in a one side-door store over other door combinations. We illustrate the use of our proposed approach using the layout of a leading US retailer's store.

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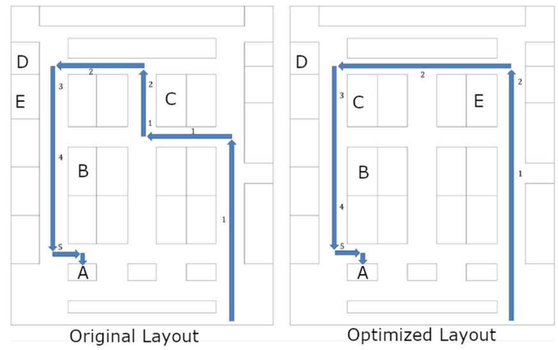
1. INTRODUCTION

While e-commerce has grown significantly in the recent years, brick-and-mortar stores have continued to play a key role in meeting shopper needs. Research shows that, for most Americans, upwards of 65% of online shoppers prefer to buy from physical locations (Pew, 2016); Kohl's recently reported that 90% of their sales still occur within a physical store (Mansell, 2018). In fact, across $5 trillion in total retail sales in 2017, in-store purchasing was nearly 9 times higher than online purchasing (Cordero & Levy, 2018). Shoppers continue to visit brick-and-mortar stores as they (i) can touch and/or see the product, (ii) can try products (especially, apparel), and (iii) often enjoy the experience of going to shop (Leadem, 2017).

For a retailer to be successful, merchandizing, visual communication (e.g., signage), store ambience and lighting, and store planning have been alluded to as key factors (Dunne et. al., 1995). The latter, store planning, refers to the physical store area including layout and space allocation to departments, entrance and exit, checkout counters, aisle and other small stores. Better physical store environment created by the retailer can significantly affect the attitude and behavior of shoppers (Shankar et. al, 2011; Bloomberg, 2018); an increase in the positive mood of a consumer can produce a 12% increase in spending (Babin & Darden, 1996).

For the retailer, revenue comprises planned and impulse purchases. A planned purchase is the list of products the shopper intends to purchase when they visit a store. In contrast, an impulse purchase is a sudden and immediate purchase (with no pre-shopping intention) of either a specific product category or to fulfill a specific buying task (Beatty & Ferrell, 1998). It has been shown that changes in department placements result in a change in shopping path of the shopper given a planned purchase list (Ballester et al., 2014). Figure 1 provides an illustration from Boros et al.

(2016) that indicates that changes in department locations can affect the shopping path, which can affect the number and type of departments that a shopper is exposed to during their path. While the retailer often has limited control over planned purchases, they may possibly impact the degree of impulse purchase by placing high impulse products along the shopper's path, sometimes by over 30% (Sorensen, 2009).



A: Cashier, B: Sweets and Cakes, C: Chocolate Rice, Salt and Cornflakes, D: Bread and Bakery, E: Cakes

Figure 1 Comparison of path and department expose by changing assignment of department [Boros et al., 2016]

To effectively account for such dynamics, and how it impacts the visibility and revenue for the retailer, we propose the retail facility layout problem accounting for shopper path and door placement (RFLP-SPDP). Our study provides the foundation of understanding how shopper path changes can impact impulse revenue. The key contributions of our research are as follows. First, we propose a mixed integer optimization model for the RFLP-SPDP where we explicitly capture the dynamics of changes in shopper path with changes in the layout. Second, because of the difficulty in expressing these shopper path changes in a closed analytical form, we propose a Simulated Annealing based heuristic to generate up to k such paths for a candidate layout (for a given planned purchase list). Third, because the estimation of the visibility of each department depends on these shopper paths, and is consequently difficult to express in a closed analytical form, we propose a procedure to estimate visibility, and in turn impulse revenue. Accordingly, we estimate the proportion of each department visible to a shopper for each of the k shopper paths and then, using department-specific impulse rates, calculate the average expected per shopper impulse revenue. Fourth, considering that two procedures (for shopper path and visibility) are not available in a closed form limiting the use of the state-of-the-art commercial solvers, we propose a Particle Swarm Optimization (PSO) based approach to efficiently derive (near) optimal solutions to realistic problem instances. Finally, using realistic data (some available from a retailer and existing literature, and other collected through personal store visits), we evaluate the sensitivity of the solutions to the basket size, number and location of doors, and aspect ratio of the store. These experiments helped us derive key managerial insights of possible use to a retailer.

Our experiments suggest that expected impulse revenue is highly sensitive to the location of 'high' planned purchase and 'high' impulse purchase departments. Further, the expected impulse revenue from a store that is deep could be up to 13.71% higher compared to a store that is square-shaped. We also observed that a one side-door store could generate up to 9.65% higher impulse revenue compared to other door scenarios. We also illustrate the use of our approach in evaluating and optimizing an existing store layout of a leading US retailer considering two difference scenarios.

In the following sections, we first review the existing literature in Section 2. Our proposed optimization model for RFLP-SPDP and the PSO heuristic to solve that model are discussed in Sections 3 and 4, respectively. We discuss our experimental study in Section 5 and illustration use of our approach for an actual store in Section 6. Finally, in Section 7, we summarize our key findings and discuss avenues for further research.

2. LITERATURE REVIEW

The facility layout problem (FLP) is to find the optimum placement of departments within the facility in order to minimize operating cost (by minimizing distance or material flow) or maximize utilization of space and facilities. Various models and approaches to solve FLP for manufacturing and warehousing facilities have been proposed in the literature (Meller & Gue, 1996; Singh & Sharma, 2006; Yapicioglu et al., 2007; Gue & Meller, 2009; Kundu & Dan, 2012).

Retail facility layout problem (RFLP), in contrast, differs from the traditional FLP in many ways. While the objective of FLP is often to minimize distance, the objective of RFLP is to maximize revenue for the retailer. RFLP involves mainly two levels of decision making: physical store layout and shelf space allocation. Several articles present approaches to address the shelf-space allocation problem (see Corstjens & Doyle, 1981; Botsali & Peters, 2005; Irion et al., 2012; Flamand et al., 2016). Below we focus on the literature related to physical store layout and its impact on retailer's revenue.

Marketing literature suggests that exposure of products is an important metric of retail layout and a sales stimulus (Cairns, 1962; Cairns, 1963; Anderson, 1979; Dreze et al., 1994); "shoppers will only buy what they see" (Suher & Sorensen, 2010; Ebster & Garaus, 2015). Although shoppers only visit one-third of all store areas (Hui & Bradlow, 2012, Hui et al., 2013), store sales increase with increase in the path length (Granbois, 1968; Inman et al., 2009; Anic et al., 2010; Hui et al., 2013) and time spent in the store (Okada & Hoch, 2004; Inman et al., 2009; Anic et al., 2010; Bell et al., 2011). Additionally, shopper traffic density in the store varies (Farley & Ring, 1966; Larson et al., 2005) and that departments in the store vary in their impulse rates (West, 1951; Bellenger et al., 1978).

Based on these observations, a few approaches have been proposed in the IE/OR literature to optimize the store layout. Peters et al. (2004) present models to calculate the expected tour length for three types of retail layouts; aisle, hub-and-spoke, and serpentine. They extend this work by proposing a network-based model for serpentine layout with the objective of maximizing revenue by increasing exposure of impulse purchase items to the shoppers (Botsali and Peters, 2005). Yapicioglu & Smith (2012a) develop a nonlinear optimization model, and solve it using Tabu Search, to optimize retail revenue by assigning high impulse rate departments to high traffic density zones for a racetrack layout. They extend this work by reformulating this problem as a bi-objective model, where the first objective maximized store revenue based on department layout, while the second objective maximized the satisfaction of departmental adjacencies (Yapicioglu & Smith, 2012b). Recently, Ozgormus & Smith (2018) propose a nonlinear bi-objective optimization model to maximize revenue and department adjacencies by considering space requirements and solved it using Tabu Search. They use a case study of a Turkish retail store to illustrate the use of their approach.

Ballester et al. (2014) show that the shopping path, and eventually traffic density within a store, are likely to change with changes in department locations. They analyzed the effect of change in the location of high planned purchase departments on shopper path length and traffic density within a store. Boros et al. (2016) optimize supermarket layout to increase the path length of shoppers. In their approach, they only allowed swapping a department with a restricted set of potential positions and then measure the quality of the new layout using shopper's path length.

Related to product exposure in a retail store, Mowrey et al. (2018) introduce the retail rack layout problem (RRLP) to optimize the rack layout within a section of a store to maximize exposure of the products. They consider a shopper's 2D field of vision and allowed racks to be orientated at an angle. Guthrie & Parikh (2019) extend this work and introduce the rack orientation and curvature problem (ROCP) that considers a shopper's 3D field of vision to maximize marginal impulse profit (after discounting for the cost of space). They suggest that higher marginal impulse profits can be achieved by racks that are high-acute and straight-to-medium-curved or high-obtuse and high-curved.

While many of these studies incorporate some aspect of product visibility to estimate impulse revenue, they do not explicitly model the changes in the shopper path and traffic corresponding to changes in the department locations, in turn impacting the visibility of the departments along the shopper path. Our work attempts to fill some of these gaps by explicitly accounting for shopper path changes with layout changes. We also consider how the changes in the number and location of the doors affect these paths. We now present the details of our approach, starting with an optimization model for the RFLP-SPDP.

3. AN OPTIMIZATION MODEL FOR THE RFLP-SPDP

Recall that the objective of the RFLP-SPDP is to determine the optimal location of departments considering shopper path in order to maximize the expected impulse revenue. In developing our model, we make the following assumptions:

- A shopper's planned purchase list may contain more than one item from a department.
- Shoppers will follow one of the k shortest paths to purchase these planned items.
- If a shopper passes by another department on their way to the next planned item, then the shopper may purchase one product from that department based on the department's impulse purchase probability.
- In mapping the departments to nodes, a department may be split across multiple, consecutive nodes. Also, the location of checkout counters and structure of the aisle network is given.

Tables 1 and 2 summarize the list of parameters and decision variables used in the model, respectively.

Table 1 Parameters used in model

Notation	Definition
I	Set of departments; $i \in I$
J	Set of nodes; $j \in J$
S	Set of shoppers; $s \in S$
D	Set of doors; $d \in D$
B_s	List of all departments corresponding to planned purchase list of shopper *s*
R_i	Revenue of department i,
P_i	Impulse rate of department *i* (0< P_i<1)
L_d	Location of door *d*
T_d	Proportion of shoppers that use door *d*
AR	Aspect ratio of store
A_i	Area of department *i*
A_j^N	Area of node *j*
$A_{jj'}$	1, if nodes *j* and *j'* are consecutive; 0, otherwise
M	A big number

Table 2 Decision variable used in model

Notation	Definition
x_{ij}	Proportion of department *i* assigned to node *j*
y_{ij}	1, if department *i* assigned to node *j*; 0, otherwise
z_i	Number of times department *i* is split between consecutive nodes
$\overline{p_s}$	Vector describing the path of shopper *s*
v_{is}	Proportion of department *i* visible to shopper *s*

We propose the following model for the RFLP.

maximize: $\frac{1}{|S|}\sum_{i,s} v_{is}\, R_i P_i$

subject to:

$$v_{is} = f\left(x_{ij}, \overline{p_s}\right);\ i \in I,\ s \in S \tag{1}$$

$$\overline{p_s} = g(x_{ij}, B_s, D, L_d, T_d, AR);\ s \in S \tag{2}$$

$$\sum_i x_{ij} A_i = A_j^N;\ j \in J \tag{3}$$

$$\sum_j x_{ij} A_j^N = A_i;\ i \in I \tag{4}$$

$$y_{ij} \le M\, x_{ij};\ i \in I,\ j \in J \tag{5}$$

$$\sum_j \sum_{j'>j} (y_{ij} y_{ij'} A_{jj'}) \le z_i - 1;\ i \in I, j, j' \in J \tag{6}$$

$$0 \le x_{ij} \le 1, y_{ij} \in \{0,1\};\ i \in I,\ j \in J \tag{7}$$

$$0 \le v_{is} \le 1;\ i \in I, s \in S \tag{8}$$

The objective of the RFLP-SPDP model is to maximize the average expected impulse revenue (per shopper) across all shoppers visiting the store. Visibility of department i by shopper s (v_{is}), which depends on the assignment of departments (x_{ij}) and shopper path ($\overline{p_s}$), is modeled through function f in Constraints (1). The shopper path, in turn, depends on the assignment of departments, shopper's planned purchase list (B_s), number of doors (D), door location (L_d), proportion of traffic entering through door d (T_d), and aspect ratio (AR) of the store, and is modeled through function g in Constraints (2). Constraints (3) ensure that the total area of departments assigned to a node is equal to the prespecified area of that node. Similarly, Constraints (4) ensure that the total area of any department assigned at each node should be equal to the area of that department. Constraints (5) ensure that a department is considered assigned to a node only if some part of (or the entire) department is assigned to that node. Constraints (6) ensure that if a department is split, it is split across consecutive nodes in the mapping sequence. Constraints (7) and (8) indicate bounds on the decision variables.

It was difficult to express the two functions f and g in closed analytical forms making the proposed model extremely difficult to solve via state-of-the-art solution approaches. We, therefore, estimated both these functions via tailored procedures; function g via an enhanced shortest path over a grid considering door locations and function f that uses the shortest paths (generated using function g) and the mapping of departments to estimate visibility. Both these procedures were embedded in a heuristic algorithm based on the Particle Swarm Optimization framework to solve the overall problem. We now detail our proposed solution approach.

4. A HEURISTIC BASED ON PARTICLE SWARM OPTIMIZATION TO SOLVE THE RFLP-SPDP

PSO is a naturally inspired metaheuristic for the optimization of nonlinear functions (Kennedy & Eberhart, 1995). PSO uses the population of particles (where each particle represents one solution) to find an optimal or nearly optimal solution by exploring discrete or continuous search space. Particle exchange information between them using some social behavior inherent in a flock of flying birds or a school of fish to find the best solution. Particle movement in the search space is influenced by the particle's local best position and the best particle's position. This information sharing between particles reduces the chance of the solution being stuck in a local optima (Kennedy & Eberhart, 1995).

Each particle in our proposed PSO represents a solution indicated as a string of k real numbers (i.e., position vectors), where k is the number of departments. For example, for a 4-department store the position vector would be represented as (-0.321, 2.589, -3.687, -0.267), while for a 7-department store it would be represented as (3.258, -1.256, -3.449, -0.679, 2.139, 1.325, 0.123).

Figure 2 shows a flowchart of our proposed PSO-based heuristic. We first perform several data preprocessing steps to extract the data from an existing layout and also help set up the initial particles in the PSO, and then employ 4 subroutines to evaluate each candidate solution (particle). Briefly, the (i) Sequence Subroutine encodes the solution into department assignment sequence, (ii) Mapping Subroutine maps the departments to the node, (iii) Path Subroutine converts a shopper's planned shopping list (which is a list of departments to visit) to the list of nodes and estimates the k shortest path through these nodes for each shopper, and (iv) Impulse Subroutine estimates the resulting per shopper impulse revenue.

To better elucidate our approach, we use an example problem and use it as a reference for the rest of this section. This data corresponding to this problem is shown in Figure 3 (Block I), which includes the existing layout, summary of area requirements for various sections of the store, and an example shopper planned purchased list (mapped to the departments to be visited).

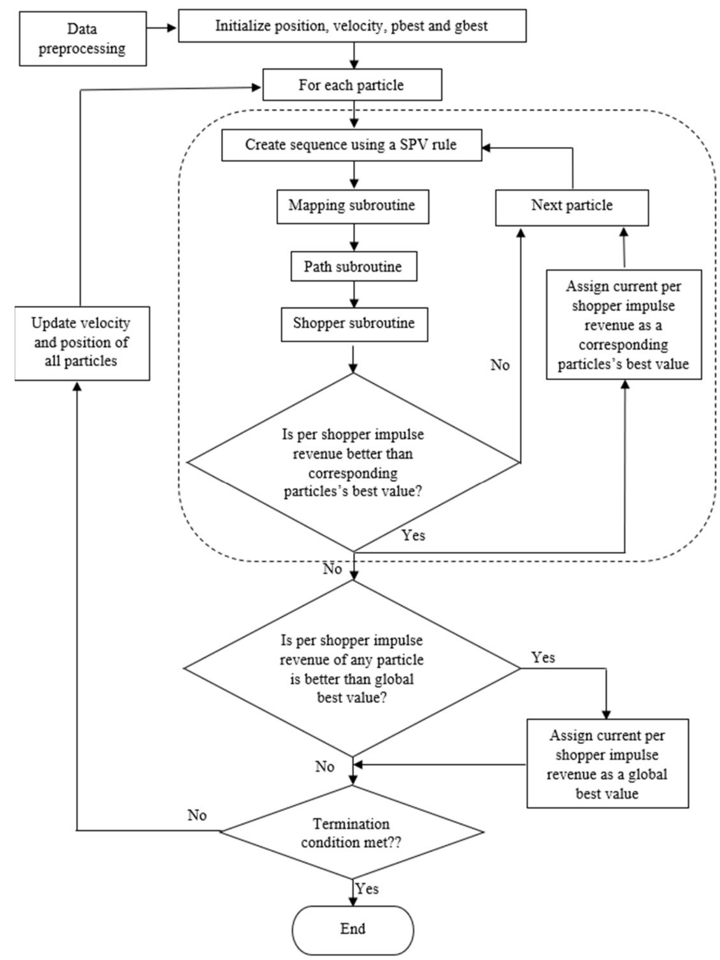


Figure 2 Flowchart of the PSO-based algorithm

4.1 Data Preprocessing

For an existing layout or a 'greenfield' design for which we know the aisle structure, we first represent superimpose a grid that naturally aligns with given aisle structure of the store (Figure 3 - Block II - Grid Representation). We also calculate the left over area after subtracting area of aisles and checkout counter (if any) at each node (Figure 3 - Block II - Nodal Area). With this data, we then generate n particles, each with randomly initialized position vectors, and then invoke the 4 subroutines shown in Figure 2. Although not an input to our proposed heuristic, as a side step to help in comparing the existing layout with the (near) optimal layouts from the PSO, we assign departments to nodes (Figure 3 - Block II - Department to Node Mapping) and calculate impulse revenue per shopper for existing layout.

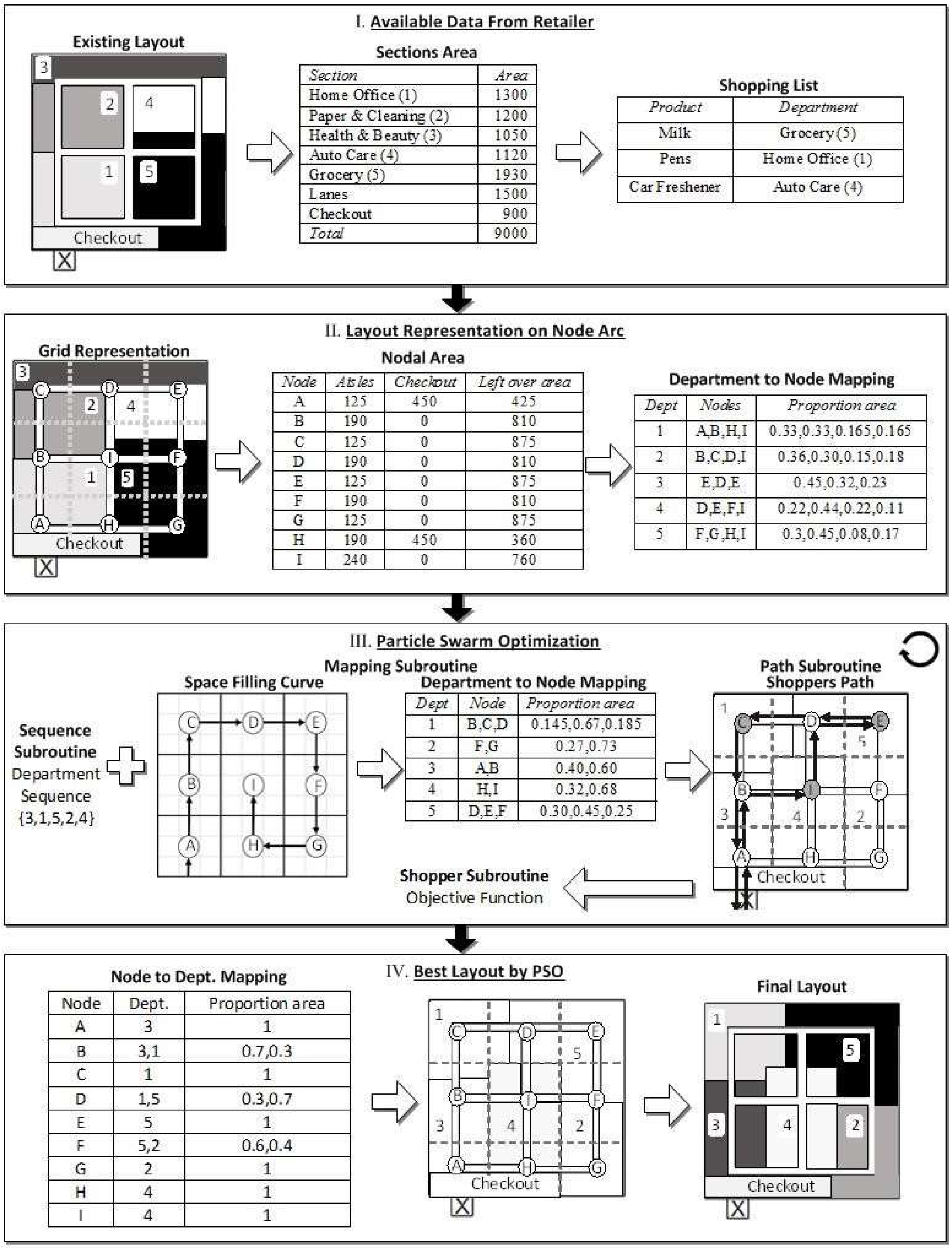


Figure 3 Illustration of our approach

4.2 Sequence Subroutine

To encode a solution (represented via a position vector) into a department sequence, we use the smallest position value (SPV) rule. The SPV rule enables the continuous PSO approach to be applied to all classes of NP-hard discrete sequencing problems (Tasgetiren, 2004). According to this rule, the index of the smallest position value is assigned to be the first in the sequence, the index of the second smallest position value is assigned to be the second in this sequence, and so on. For instance, consider the example in Table 3 which illustrates a particle with 5 departments and their corresponding position vector. In this case, Department 3 has the smallest position value, so it will be assigned to the first position in the sequence, followed by Department 1, and then Department 5, and so on; the final department sequence will be {3, 1, 5, 2, 4}.

Table 3 Generating department sequence based on the positive vector

Dept #	**1**	**2**	**3**	**4**	**5**
Position vector	-2.13	1.24	-4.54	3.78	-0.32
Dept position in the sequence	2^{nd}	4^{th}	1^{st}	5^{th}	3^{rd}

4.3 Mapping Subroutine

This subroutine interprets the department sequence derived from the SPV rule and maps it to the nodes. To do this, we first represent the store layout as an $m \times n$ grid, where m depicts the length and n depicts the depth of the store. The nodes represent the center of the store section and arcs represents aisles (Figure 3 - Block II - Grid Representation). Each node will be associated with a certain area; sum of areas across all nodes will equal to the store area. The arc length between two successive nodes can be prespecified based on the actual distances for corresponding locations in the store.

The door location(s) are also mapped to nodes; checkout counters are assumed to be located close to the door and mapped to ≥1 nodes depending on its size in the real store. For instance, for a store with one door at left-bottom corner (e.g., node A; see Figure 3), the checkout counters are assumed to be located at node A with additional nodes (e.g., H and/or B) as needed. For a store with two doors located on the same side (e.g., nodes A and G), the checkout counters are located at the door (as preassigned) along with other predefined nodes (say, nodes H or nodes B and F).

We used a space filling curve to map the departments to the nodes and to specify which nodes are consecutive. For instant, the following node sequence is used to map departments in the store represented as a 3×3 grid A, B, C, D, E, F, G, H, I (Figure 3 - Block III - Space Filling Curve). For a one side-door store with the door located at node A and checkout counters located at nodes A and H fully occupying the area of those nodes, the first department in the sequence is assigned to node B (the second node in the node sequence) after removing nodes A. If the area required by the checkout counters does not fully consume the area of nodes A and H, then the assignement will start from node A itself. In the given exemple of 3×3 grid, checkout counters occupy nearly half of the area of nodes A nad H; the rest of the area of node A is used to begin the assignment of departments. If the first department requires more area than available at node A, then the left over area of this department is assigned to Node B (the second node in the node sequence); i.e., we allow departments to split across consecutive nodes in node sequence (SFC). Otherwise, in the case of the left over area at node A (after assigning the first department), we assign the second deparment to this node, and so on, until all departments are assigned. If a department is split between nodes, we retain information about the proportional split for use later in the shortest path and visibility estimates. A pseudocode of the mapping subroutine is as follows.

For department sequence (*DS*)

  For assignment sequence (*AS*)

    If $RaD_i > 0$

      If $RaN_j > A_j^N$

        If $RaD_i > RaN_j$

          $RaN_j = RaN_j - RaD_i$

          $RaD_i = 0$

        Else

$RaD_i = RaD_i - RaN_j$

$RaN_j = 0$

End

End

Department sequence (*DS*) is derived from the SPV rule. Assignment sequence (*AS*) is a node sequence for mapping after removing nodes that are fully occupied by the entrance-exit door and checkout counters. For example, in a one side-door store on a 3×3 grid if node A fully occupied by a door and the checkout counters, the corresponding assignment sequence is B, C, D, E, F, G, H, I. RaD_i stands for unassigned area of department i, while RaN_j stands for the unassigned area of node j.

To illustrate this further, let us revisit the example problem with one side-door store shown in Figure 3 with 5 departments mapped to a 3×3 grid. Area of different sections of the store is given in Figure 3 (Block I - Sections Area). The door and checkout counters are assigned to node A and H consuming half area of both node. The department sequence using the SPV rule is 3, 1, 5, 2 and 4 (per the Sequence Subroutine). The mapping would start with the first department in the sequence, Department 3 in this case, beginning from node A. Area remaining at node A after removing area of aisles and checkout is 425 ft^2. Because the area of Department 3 (RaD_3=1050) is more than the area available at node A (RaN_A=425), it is split between node A (425 ft^2) and node B (625 ft^2). Proportional areas of Department 3 assigned to nodes A and B are 40.48% (425/1050) and 59.52% (625/1050) respectively. The left over area at node B is now 185 ft^2 (RaN_B=185). Because the area of Department 1 (RaD_1=1300), the second department in the sequence, is higher than left over area at node B and total area of node C, so we split Department 1 between nodes B, C and D. The next department in the sequence is assigned to node D; this process is continued until all five departments are assigned.

4.4 Path Subroutine

For a given particle (i.e., candidate layout), this subroutine first determines the list of nodes to be visited and then generates up to k shortest paths for each shopper based on their planned purchase list.

For generating the list, we first convert each shopper's planned purchase list (which is a list of departments to visit during a shopping trip) to a list of nodes to visit based on the given department to node mapping (from the Mapping Subroutine). If any department is split between nodes, we choose one of the nodes based on the proportion of department areas assigned to these nodes. For the example we have been considering with the shopper requiring to visit Home Office, Auto care and Grocery departments, assume that in a candidate layout (presented as a particle) these have been mapped by the Mapping Subroutine to nodes C, I, and E respectively. To this list, we then add nodes corresponding to the doors (for entrance and exit). For example, if the list of nodes to be visited is C-I-E and the door is located at node A, then the resulting list becomes A-C-I-E-A. In case of two doors located at A and G, we get four possible combinations based on prespecified door entrance probabilities; A-C-I-E-A, A-C-I-E-G, G-C-I-E-A, and G-C-I-E-G. For instance, if 75% of shoppers typically use door at node A, then we add A to the front and end of the list with a probability of 0.75.

Path generation corresponding to the list of nodes for each shopper is divided into two subsections; identifying the best sequence of nodes and generating shortest path for that sequence. For generating the best node sequence, we propose a Simulated Annealing algorithm (see Appendix). Succinctly, the SA uses initial list of nodes as a current solution and employs the 2-way exchange method in order to reduce the total rectilinear travel distance for the shopper. The first and the last nodes define the shopper's entrance and exit node and so they are not allowed to be swapped.

Given this sequence, we then use the Dijkstra's algorithm to find shortest path between each pair of successive nodes; we use the *igraph* package in *R* that implements this algorithm (see Appendix). We combine these paths together to generate up to *k* shortest paths corresponding to the given shopping list and mapping of the departments. For any shopper, if generated shortest paths are less than *k*, then we use all those paths; if they are >*k*, then we randomly select *k* among those. For our example with one side-door located at node A, the shortest node sequences to visit are A-I-E-C-A and A-C-E-I-A, both with a path length of 8 units. We randomly select one of these shortest sequences to generate a path corresponding to the node sequence. Then, for the shortest node sequence (A-I-E-C-A), there are four shortest paths (A-B-I-D-E-D-C-B-A, A-H-I-F-E-D-C-B-A, A-H-I-D-E-D-C-B-A, A-B-I-F-E-D-C-B-A) with same path length (Figure 3 - Block III - Shopper Path). If *k*=3, then will randomly select three path among these paths and use it for subsequent estimation in the next subroutine.

4.5 Shopper Subroutine

This subroutine estimates the expected impulse revenue for each path across each shopper and then average them to estimate the per shopper impulse revenue across all shoppers. Remember that the Path Subroutine provides *k* shortest paths, each of the shortest path represents a sequence of nodes to be visited (some for planned purchases and others on route). A department is considered to be entirely visible if it is completely assigned to a node or split between nodes all of which are on the shopper's path. But if a department is split across nodes and only one node is along the path, then the %-area of that department assigned to this node is considered as a surrogate for the visibility of this department. For each of the *k* paths for a shopper, we use department-specific visibility probability to determine if that department will be visible to the shopper. Consider the path shown in Figure 3 (Block III - Shopper Path; A-B-I-D-E-D-C-B-A) which includes nodes A, B, I, D, E and C. Using the department mapping shown in Figure 3 (Block III - Department to Node Mapping), visibility of Departments 1 and 3 will be 100% because nodes A, B, C, and D are along the shopper's path. Department 5 was split between nodes D, E, and F; among them nodes D and

E are on the shopper's path, which means that the visibility of Department 5 will be 75% (30% Department 5 assigned at node D + 45% at node E). Similarly, for Department 4 the visibility will be 68% because only node I is on shopper's path. While Department 2 will not be visible to the shopper because nodes F and G are not on shopper's path.

Given the visibility probability for each department on a path, we then use that department's impulse probability to determine if the shopper would make an impulse purchase from that department. We do this for all departments that the shopper comes across in each path and record the estimated impulse revenue; we repeat this for each of the *k* paths for this shopper. Finally, the impulse revenue for each shopper is obtained by aggregating these values assuming each of the *k* paths are equally likely (i.e., probability of 1/*k* for a shopper). Averaging across all shoppers provides an estimate of per shopper impulse revenue for a candidate layout.

4.6 Solution Updating

To update the candidate layout (i.e., update the particle's position vector), we us fact that each particle retains memory of its personal (particle) best solution and neighborhood (global) best solution. The personal best is an incumbent best solution for each particle, while the neighborhood best is the best solution among all particles. Each particle's velocity and position are updated after each iteration (i.e., after the evaluation of each particle). Velocity updating, a significant part in PSO, accounts for three things; (i) velocity of the particle in its previous iteration, (ii) cognitive or selfish influence (which drives the particle toward the corresponding best position of that particle in search space), and (iii) global influence (which drives the particle towards best incumbent solution among all particles); this influence is known as a social influence which helps PSO to converge (Shi & Eberhart, 1998; Ozturkoglu et al., 2014; Mowrey et al., 2018).

Let V_{ij}^{t} represent the velocity of the j^{th} index of particle *i* at current iteration, X_{ij}^{t} represent the position of j^{th} index of particle *i* at current iteration, $Pbest_{ij}^{t-1}$ represent the position of j^{th} index of best particle *i* until last iteration among the all particles of *i* ,$Gbest_{j}^{t-1}$ represent the position of

j^{th} index of best particle among all particles until last iteration, C_1 and C_2 represent acceleration constants, and K represents the constriction coefficient. Then, a particle's new position in search space is given by:

$$V_{ij}^{t} = K\,(V_{ij}^{t-1} + C_1 r_1\left(Pbest_{ij}^{t-1} - X_{ij}^{t-1}\right) + C_2 r_2(Gbest_{j}^{t-1} - X_{ij}^{t-1}))) \quad (9)$$

$$X_{ij}^{t} = X_{ij}^{t-1} + V_{ij}^{t} \quad (10)$$

The movement towards personal best or local best is determined by r_1 and r_2, which follow Uniform (0,1). To prevent this randomness causing the particle velocity to approach infinity, Eberhart and Shi (2000) propose the use of a liberal velocity limit, V_{max}. Preliminary experiments with our PSO suggested setting V_{max}=100 helped convergence. We set acceleration constants $C_1 = C_2 = 2.05$, constriction coefficient $K = 0.7298$ and $X_{max} = V_{max}$ as suggested by Clerc and Kennedy (2002). Termination criteria included maximum iterations of 1000 and maximum of 150 iterations with solutions not improving by more than 0.1%.

4.7 PSO Performance

To evaluate the performance of the proposed PSO-based heuristic, we compared the solution quality and solution time with those obtained via total enumeration. Recall that for an N-department store, we represented the department location in the store as a string of size N; in that case, there are N! distinct layouts that would need to be evaluated during total enumeration. For this comparison, we let N=8 departments, which resulted in a total of 8! = 40,320 possible solutions. We let all 8 departments be of identical size and generated random input data such as impulse rate, department area, and dollar purchase amount. We used two basket sizes (planned purchase list) of 4 and 8 items, three aspect ratios of 0.5, 1.0, 2, and one side-door store. Both approaches (PSO and total enumeration) were implemented on a personal computer with an Intel Core™ i7-3770 8-processor system, each processor of 3.4 gigahertz and a total of 16 GB RAM. PSO was coded in *R* programming language and used 16 particles with parallel implementation.

Table 4 shows a comparison of the two approaches, which suggested that the PSO was able to find high quality solutions in a fraction of time compared to the total enumeration approach. The objective value was within 1.25%, while the run time reduced by over 90%.

Table 4 Comparison of objective and solution time of PSO and total enumeration for different scenarios

		Objective ($)			**Solution Time (hr)**		
	Basket size	***Total Enumeration***	***PSO***	***% Diff***	***Total Enumeration***	***PSO***	***% Diff***
AR=0.5	4	$9.13	$9.02	1.23%	23.31	1.74	92.54%
	8	$10.96	$10.83	1.17%	28.48	2.25	92.10%
AR=1	4	$8.96	$8.88	0.85%	26.43	2.48	90.62%
	8	$10.37	$10.32	0.50%	29.53	1.98	93.29%
AR=2	4	$10.19	$10.07	1.18%	22.95	2.28	90.07%
	8	$11.44	$11.35	0.80%	29.36	2.53	91.38%

To ensure that the particles converged, we conducted additional tests using the baseline layout (i.e., 20 departments, basket size=8, AR=1.0, one side-door). We used 16 particles, initialized them randomly, and used the stopping criteria of maximum 1,000 iterations and less than 0.1% non-improving 150 iterations. We ran this instance 5 times and summarized several statistics; see Table 5.The within-particle variation for a given run ranged from 0.44% to 1.18%, while the mean objective value across the 5 runs was $13.8; the resulting standard deviation across the runs was $0.094 and MAPD (mean absolute percentage deviation) between the 5 runs was 0.63%.

Table 5 Summary of five runs with the baseline layout

Run	**Objective ($)**	**Iterations**	**Time (hours)**	**Per iteration time (s)**	**Within particle variation in this run**
1	$13.94	328	3.52	38.634	0.54%
2	$13.88	677	7.39	39.297	0.50%
3	$13.70	271	2.93	38.923	0.44%
4	$13.77	302	3.29	39.219	0.65%
5	$13.71	227	2.47	39.172	1.18%

In short, the results from Tables 4 and 5 collectively indicated that our proposed heuristic based on the PSO framework was able to achieve high quality solutions in a short amount of time, and that the solutions were fairly robust. We, therefore, use this approach for further experimentation to generate managerial insights.

5. EXPERIMENTAL STUDY

We used the model presented in Section 3 to evaluate the sensitivity of PSO solutions to system parameters. We considered a 40,000 ft^2, 20-department store in all our analysis. The individual department areas are presented in Table 6.

Table 6 Sections and their areas in the store

	Section	Area in ft^2		Section	Area in ft^2
1	Snacks, Cookies & Chips	2700	9	Autocar	722
	Breakfast and Cereal		10	Sporting Goods	575
	Beverages		11	Toys	1375
	Beer, Wine & Spirits		12	Cards & Party supplies	934
2	Bakery & bread	3331	13	Clothing	5173
	Dairy, Eggs & Cheese		14	Baby	663
	Deli		15	Pharmacy	303
	Meat, Seafood & Poultry		16	Health & Beauty	2255
3	Produce	3574	17	Household Essentials	5143
4	Frozen Food	3346			
	Other Grocery		18	Cosmetics	382
5	Paper & cleaning	729	19	Jewelry	369
6	Pet Care	775	20	Fabric and Crafts	342
7	Home office	342	*21*	*Path/lane area*	*4000*
8	Electronics	1367	*22*	*Checkout counters*	*1600*

We considered three different basket sizes to mimic various shopping conditions; e.g., small basket size representative of a convenience store, medium for grocery or departmental store, and large for hypermarkets or wholesale clubs.

We also considered up to two doors for the store, similar to most retail stores. In a one-door store, the door is typically located at either one of the sides or the center of the store. In a two-door store, both doors are often on the same side, equally spaced, but may have different proportion of shoppers entering and exiting the store from these doors. For the 2-door store, we used equal (50%) and unequal (75% and 25%) as the probability of shopper entry/exit.

We also considered the impact of aspect ratio of a store (AR=length/width) on the location of departments and shopping path. While we acknowledge that it is difficult to generalize, stores in a shopping mall often tend to be deeper, while stores in a strip mall are often wider. In our experiments, AR=1 represents a square-shaped store; AR>1 refers to a deeper store, while AR<1 refers to a wider store.

Table 7 Parameter values used in experimentation

Parameter	Level	Values
Basket size (B_s)	3	4, **8**, 12
Door type (D, L_d)	4	**One side-door,** one center-door, two unequal-door, two equal-door
Aspect ratio (AR)	3	0.5, **1.0**, 2

Table 7 summarizes the levels and values of each parameter used in our experimental study. Bold entries in the last column indicate the baseline store (B_s=8, one side-door, and AR=1.0). For each of these 36 experiments, we used our proposed PSO discussed in Section 4.

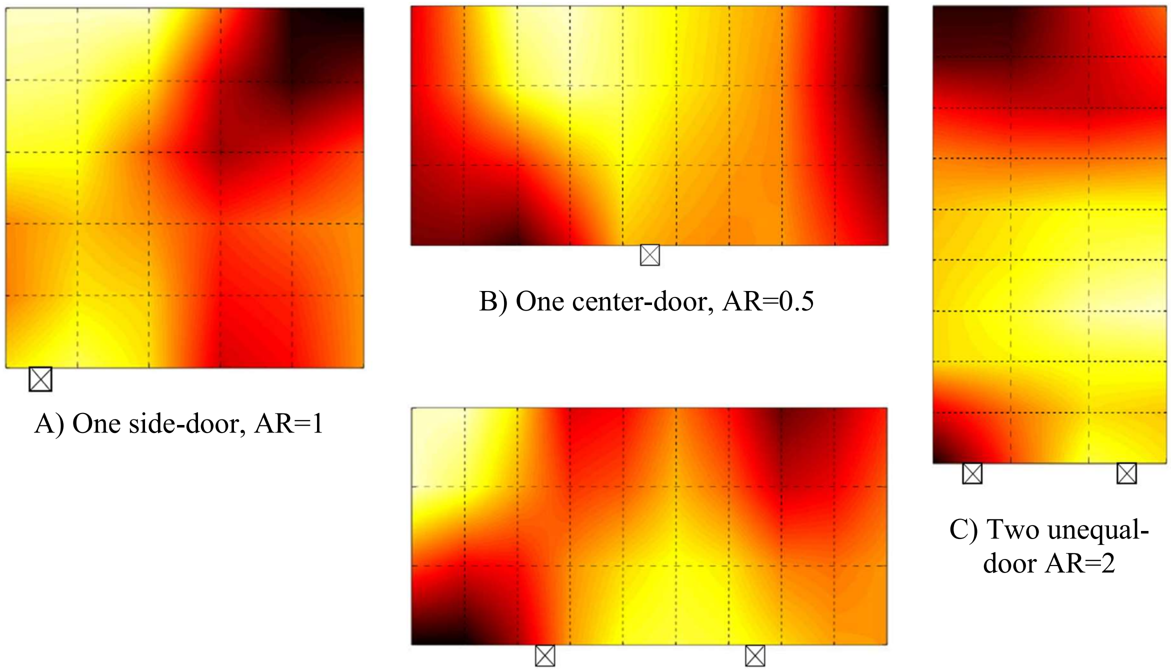
In our analysis, we used 20 of the most common departments observed across all nearby major retail stores. Impulse rate data of departments was obtained from literature (West, 1951; Bellenger et al., 1978; Flamand et al., 2016), while purchase data was collected from the literature (Flamand et al., 2016) and by visiting nearby stores. We use a Triangular distribution to estimate expected impulse revenue; i.e., if x_d represents a random variable for purchase amount in Department d, then x_d~Triangular(a,b,c), where a, b, and c are the minimum, most likely, and maximum purchase amounts in \$. We used transaction data of a major retailer to estimate planned purchase probability of each department. Area of each department was estimated by visiting nearby stores.

To help with the analysis, we grouped the departments into 'high,' 'medium,' and 'low' in terms of the maximum impulse revenue that can potentially be generated from them if they were fully visible to a shopper. Note that the amount of impulse revenue from any department depends on the impulse rate and mean amount of purchases from that department. For example, Department X with impulse rate of 0.25 and mean purchase amount of $10 would generate high expected impulse revenue ($2.5) compared to Department Y with impulse rate of 0.75 and mean purchase amount of $3 ($2.25). In our data, out of the 20 departments, 7 departments contributed to over 60% of this impulse revenue and so were grouped as 'high,' while 7 departments generated less than 15% and so were grouped as 'low.' The remaining were grouped as 'medium' impulse departments. In case of planned purchase, 6 dominating departments that accounted for 70% of planned purchase were grouped as 'high' planned purchase departments, while 8 departments that covered about 10% of planned purchase were grouped as 'low' planned purchase departments; the rest were grouped as 'medium' planned purchase departments.

Given this data and the experimental set up in Table 7, we observed the following.

Observation 1. Jointly placing 'high' planned departments away from the doors and 'high' impulse departments along the high traffic paths can increase impulse revenue substantially, with minor increase in path length.

We observed that 'high' planned purchase departments were grouped and assigned away from the door(s) or placed on the periphery of the store in all 36 layouts. In the optimized baseline layout (one side-door, basket size = 8, AR=1), the average aisle distance from the door to the nodes where 'high' planned purchase departments were assigned was 187 ft, followed by 'medium' with 141 ft and 'low' with 114 ft. This is fairly intuitive as placing such departments away from the door would cause the shoppers to pass through several other departments leading to higher visibility of those departments with no planned purchases. Figures 4(a), 4(b), 4(c) and 4(d) illustrate this, where dark regions represent 'high' planned purchase departments and box(s) represent door location.



A) One side-door, AR=1

B) One center-door, AR=0.5

C) Two unequal-door AR=2

D) Two equal-door, AR=0.5

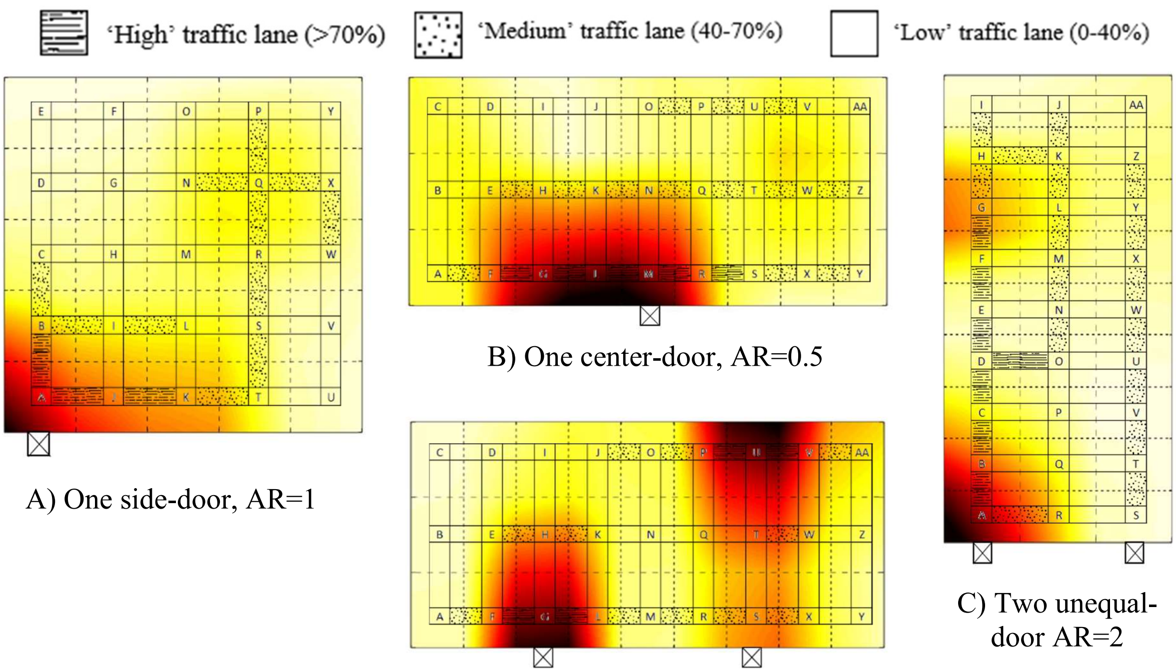
Figure 4 Heat map of planned purchase department for different layout

To understand the placement of the ‘high’ impulse departments, we first had to understand shopper movement in the store. For this, we estimated ‘traffic density per lane’ as a metric, measured as $\frac{\text{expected number of shoppers traveling that lane}}{\text{total number of shoppers}}$. That is, if each shopper were to travel through a lane at least once, then % traffic density for that lane is 100%. To calculate the numerator, we assumed that a shopper is considered to have navigated through a lane if they did so for at least once during their shortest path. We then averaged the shopper movement across *k* paths, and then across all shoppers, to estimate the expected number of shoppers traveling that lane. We considered traffic density per lane of more than 70% as ‘high’ density, 40%-70% as ‘medium’ density, and less than 30% as ‘low’ density lanes.

Across all 36 configurations we considered, we noticed that the ‘high’ impulse departments were grouped and assigned around ‘high’ traffic density lanes, irrespective of the type of layout.

This resulted in increased visibility of those departments and, subsequently, increased expected impulse revenue. Figures 5(a), 5(b), 5(c) and 5(d) illustrate this, where dark regions represent 'high' impulse departments, lanes with three different pattern represent % range of traffic density in each lanes and box(s) represent door location.

Such strategic placement of 'high' planned and 'high' impulse purchase departments alters shopper movement in the store, which increases expected impulse revenue by an average of 21.81% in best PSO- generated layouts compared to initial layouts.



A) One side-door, AR=1

B) One center-door, AR=0.5

C) Two unequal-door AR=2

D) Two equal-door, AR=0.5

Figure 5 Heat map of impulse department and traffic density for different layout

So would that increase the travel distance of a shopper? To understand this, in a post-hoc analysis, we estimated average travel distance of shopper. In the optimized layout, we noticed an average of 2.13% increase in the path length compared to the average path length across initial layouts (best of 16 particles in terms of impulse revenue). That is, without a substantial increase in the shopper path (and potential frustration), a retailer can increase its revenue considerably by optimizing the placement of a selected few departments.

Observation 2: Deeper stores could generate more impulse revenue.

Compared to a square-shaped store (AR=1), we observed an average of 8.84% increase in the expected impulse revenue in a deeper store (AR=2) and 3.29% higher in a wider store (AR=0.5). We noticed the same trend in average traffic density along the lanes; 20.08% higher in a deeper store, and 8.07% higher in a wider store, compared to a square-shaped store. Clearly, a higher average traffic density means more number of shoppers per lane, which increases visibility of departments and subsequently increases the expected impulse revenue of the store.

To better understand the impact of the shape on traffic density, we conducted post-hoc analysis where we calculated interdepartmental travel distance between 'high' planned purchase departments to quantify the relative placement of 'high' planned purchase departments with respect to each other and to the door location. Clearly, the higher the value of this metric, the more dispersed the placement of such 'high' planned purchase departments. Due to the shape of the store, across all door(s) location, interdepartmental travel distance for 'high' planned purchase departments was on average 11.55% higher for deeper store and 5.86% higher in a wider store compared to a square-shaped store. Such higher distances mean that shoppers would have to pass by several lanes to reach the required 'high' planned purchase departments corresponding to their shopping list, which subsequently increases the traffic density on the lanes connecting those departments, further increasing the visibility of the 'high' impulse departments along those lanes and subsequently the expected impulse revenue.

Observation 3: A store with one side-door can potentially generate high expected impulse revenue.

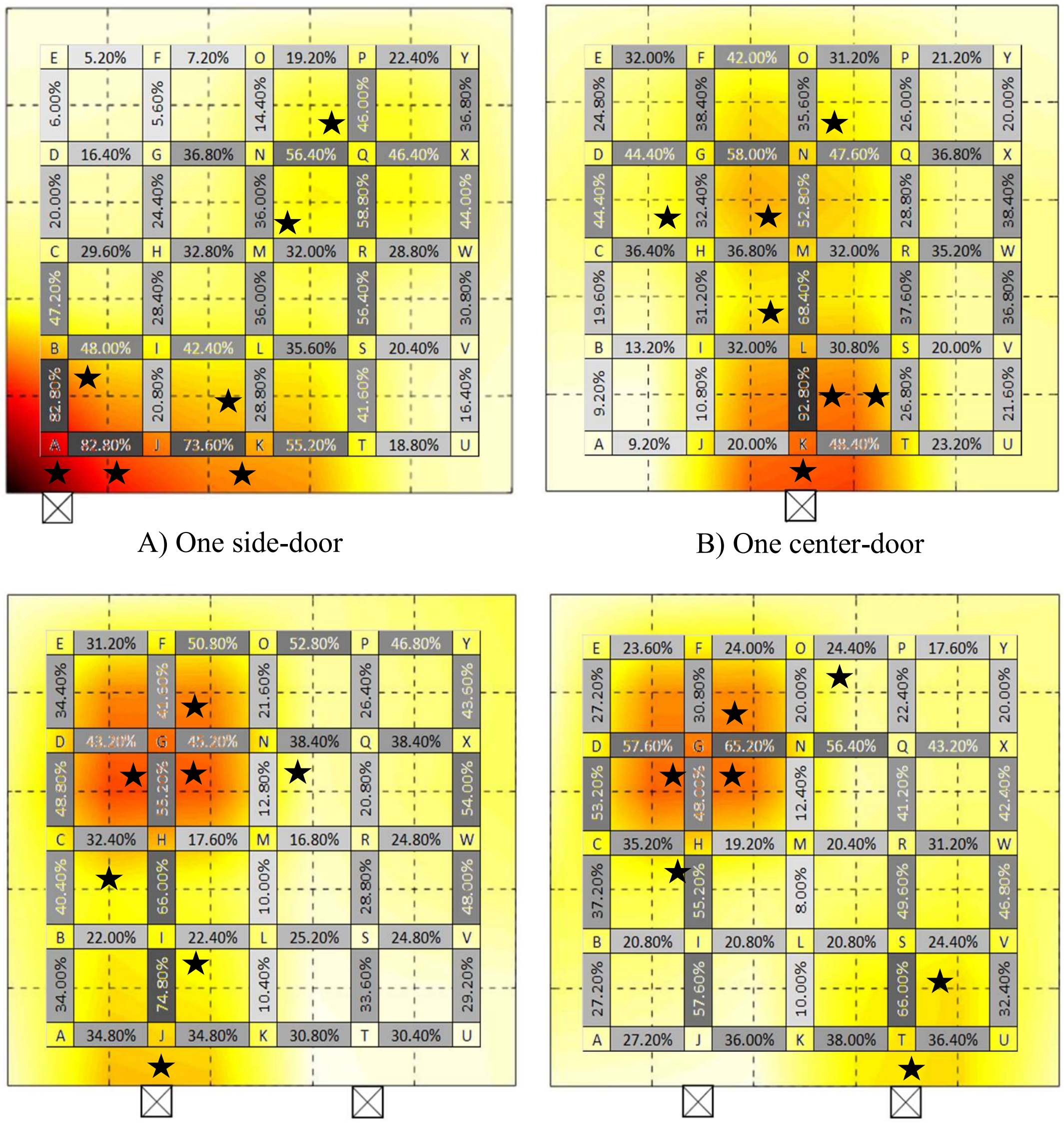
We noticed that for any given basket size and AR of 1 and 0.5, the expected impulse revenue from one side-door store was the highest. Table 8 summarizes the impulse revenue and ranking of different door scenarios for each of the AR and basket size combinations, where 1S=one side-door, 1C=one center-door, 2U=two unequal-door, and 2E=two equal-door. The %-decreases are summarized keeping 1S as the reference; e.g., in case of AR=0.5 and basket size=4, the choice of 1C over 1S would reduce the expected per shopper impulse revenue by 2.56%. Interestingly, for AR=2, while 1S still generated the highest revenue, 2U was the second best compared to 1C.

Table 8 Door ranking for a given AR and basket size

AR	Basket size	Door ranking	Impulse revenue	% Decrease compared to 1S (reference)
0.5	4	1S-1C-2U-2E	$12.78, $12.46, $12.17, $11.96	ref, -2.56%, -4.83%, -6.47%
	8	1S-1C-2U-2E	$14.24, $13.79, $13.66, $12.87	ref, -3.14%, -4.07%, -9.65%
	12	1S-1C-2U-2E	$14.96, $14.64, $14.37, $14.25	ref, -2.18%, -3.99%, -4.75%
1	4	1S-1C-2U-2E	$12.42, $12.22, $11.99, $11.40	ref, -1.67%, -3.50%, -8.21%
	8	1S-1C-2U-2E	$13.71, $13.40, $13.35, $12.68	ref, -2.26%, -2.60%, -7.48%
	12	1S-1C-2U-2E	$14.19, $14.10, $13.87, $13.58	ref, -0.62%, -2.24%, -4.24%
2	4	1S-2U-1C-2E	$13.76, $13.63, $12.86, $12.77	ref, -0.89%, -6.49%, -7.19%
	8	1S-2U-1C-2E	$14.89, $14.41, $14.10, $13.89	ref, -3.24%, -5.30%, -6.73%
	12	1S-2U-1C-2E	$15.52, $15.31, $14.92, $14.61	ref, -1.38%, -3.89%, -5.88%

To better understand such differences in impulse revenue across the four door scenarios, in a post-hoc analysis we observed that ‘high’ impulse departments were often assigned along top 3 ‘high’ traffic density lanes (in term of percentage of shoppers). We noticed that for AR of 0.5 and 1, the average traffic density across the top 3 lanes were higher in 1S (84.45%) followed by 1C (79.84%), 2U (74.87%), and 2E (68.98%). Higher traffic density means increased visibility of the ‘high’ impulse departments assigned around those lanes to the shoppers, in turn increasing the expected per shopper impulse revenue.

So why do the average traffic densities differ with changes in the door position? To understand this, we compared traffic density distributions across all 4 door placements for basket size=8 and AR=1. In case of one side-door (1S), shoppers either travel straight (along the width) or travel across (along the length) creating dominating lanes near the door; the traffic density in this scenario ranged from 73.60-82.80% (see lanes AJ, AB, and JK in Figure 6(a)). But the position of the door at the center (1C) allows for 3 potential directions for the shoppers to enter the store, reducing these traffic densities, which now range from 58.00-92.80% (see lanes KL, LM and NG in Figure 6(b)).



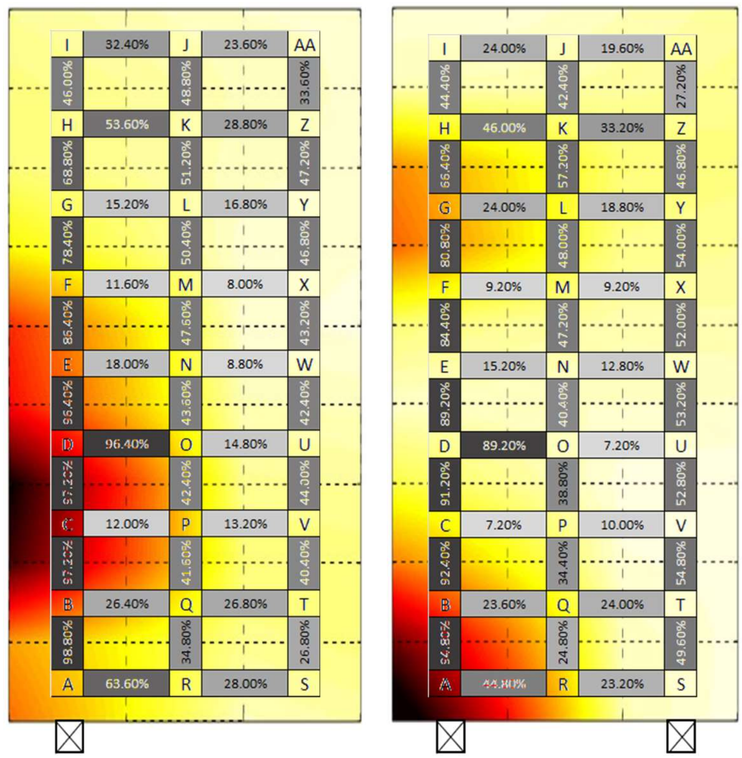
A) One side-door B) One center-door

C) Two unequal-door D) Two equal-door

Figure 6 Traffic density per lane for different door scenarios for basket size = 8 and AR=1; stars indicates the 7 ’high’ impulse purchase departments

The situation appears to be a bit complex in case of two-door stores (i.e., 2U and 2E). In case of 2U (with 75%/25%), naturally, one door is utilized more by the shoppers for entry/exit than the other door. This creates dominating lanes (lanes JI, IH and HG in Figure 6(c)) with traffic densities ranging from 55.20-74.80%. But in the case of 2Ethe shoppers are equally likely to select one of the two doors for entry/exit, which resulted in a relatively lower traffic density range along the top 3 lanes (57.20-66.00%). As discussed earlier, placing 'high' impulse departments along 'high' traffic lanes would increase visibility, and in turn impulse revenue (see Figure 6 where 'stars' indicate the placement of these 'high' impulse departments).

In case of AR=2, average traffic density across the top 3 lanes were higher in 1S (95.64%), followed by 2U (90.93%), 1C (89.38%), and 2E (86.44%). To understand this, note that the AR=2 store has a shorter width (141 ft in given case) compared to AR=0.5 and 1 (279 and 200 ft, respectively). This shorter width means that both doors located would appear to be located closer to the left and right edges of the store (Figure 7(b)). So the case of 2U with 75-25% (where the shoppers are more likely to use one door than the other) would appear much similar 1S (in which one door located at either extreme left or right side). Consequently, we would expect the behavior of the 2U store in terms of the objective function and solution to be closer to 1S.



A) One side-door (1S) B) Two unequal-door (2U)

Figure 7 Traffic density per lane for different door scenarios for basket size = 8 and AR=2

6. CASE STUDY

To illustrate the use of our approach, we modeled a store of a leading US retailer in our region and attempted to identify better layouts using our approach. We obtained data from the retailer's website, published literature, and through personal visits to that store.

Table 9 Existing area allocation to various store sections

	Section	Area in ft^2		Section	Area in ft^2
1	Snacks/Cookies/Chips(G)	2068	14	Baby	1134
2	Bakery/bread (G)	3308	15	Pharmacy	591
3	Produce (G)	1241	16	Health & Beauty	642
4	Frozen Food (G)	1654	17	Household Essentials	3177
5	Paper & cleaning	704	18	Cosmetics	513
6	Pet Care	642	19	Jewelry	309
7	Home office	352	20	Seasonal	429
8	Electronics	1907	21	Paint & hardware	1467
9	Auto care	557	22	Fabrics crafts	792
10	Sporting Goods	1378	23	Shoes	1176
11	Toys	1113	24	Lawn and garden	683
12	Cards & Party supplies	528	*25*	*Lanes (aisles)*	*6543*
13	Clothing	4550	*26*	*Checkout counter*	*2542*

This 2-door store is around 40,000 ft^2 with 21 departments. We modeled the 3 vertical and 2 horizontal dominating aisles, and aggregated short-cuts through the departments into 2 additional vertical and 2 additional horizontal paths

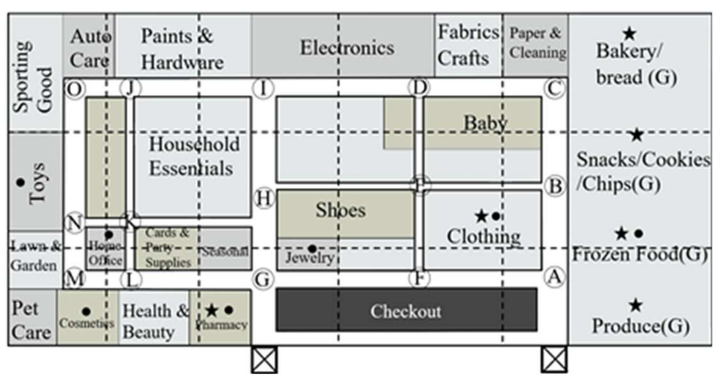


Figure 8 Existing design superimposed with a 5×3 grid; departments assigned to the closest node (stars indicate 'high' planned purchase departments and dots indicate 'high' impulse departments)

(i.e., arcs HK, KN, LK, and KJ). The resulting 5×3 grid representation of this store is shown in Figure 8, while Table 9 summarizes the area allocations.

Based on the original layout, the two doors were assigned to nodes A and G, while checkout counters were assigned to A, F, and G. This resulted in 89%, 54%, and 82% of the remaining areas of nodes A, F, and G, respectively. We further reduced the area of each node by the amount of lane area they contain; this can be calculated based on the floor layout of the store. The remaining area of each node was then used to assign departments. We used Poisson distribution with a mean of 11 items (Numerator, 2019) to generate planned purchase lists for each of the sample 250 shoppers in our analysis. We assumed equal probability of entrance and exit through these 2 doors.

We considered 3 departments as 'high' planned purchase department, which captured over 70% of all products on the planned purchase list of shoppers; Grocery=56% (Snacks/Cookies/Chips=13.6%, Bakery/Bread=20.2%, Produce=8.8%, Frozen Food=13.4%), Clothing=14.4%, and Pharmacy=6.3%; see stars in Figures 8, 9, and 10. Seven departments that generated more than 50% of impulse revenue in the data were considered as 'high' impulse department (marked as dot). We fixed the location of the Lawn and Garden department at the side of the store adjacent to the Lawn and Garden section (similar to the real store).

Acknowledging that the location of the Grocery department (with its 4 sub-sections; Snacks, Cookies and Chips, Bakery and Bread, and Produce and Frozen Food) on one side of the store may just be one specific configuration, and need not be the case at other stores, we considered 2 scenarios in our experiments: Scenario (i) - Grocery department as a whole and Scenario (ii) - all 4 subsections as individual departments.

First, we calculated the expected per shopper impulse revenue for the existing layout using four subroutines. We then used our PSO algorithm to optimize Scenarios (i) and (ii); see Figure 9 and 10. Table 10 summarizes key metrics, while Figure 11 shows the distribution of traffic density per lane.

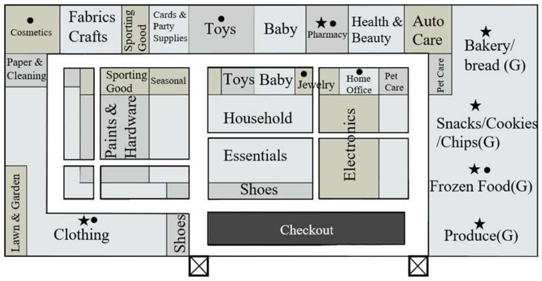


Figure 9 Layout 1 corresponding to Scenario (i)

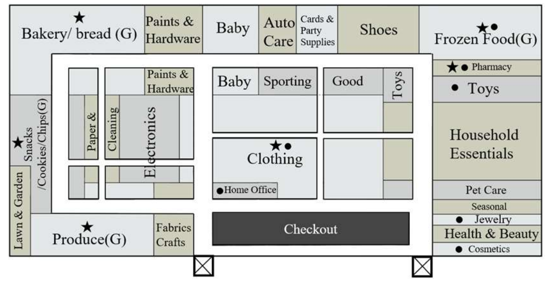


Figure 10 Layout 2 corresponding to Scenario (ii)

Table 10 Summary of finding

	Objective (expected impulse revenue per shopper)	**Avg. distance of 'high' planned purchase dept from door**	**Shopper path length**
Existing layout	$14.04	93.71 ft	481.35 ft
Scenario (i)	$17.47 (24.42%)	109.47 ft (16.82%)	493.21 ft (2.46%)
Scenario (ii)	$18.35 (30.68%)	138.06 ft (47.32%)	528.60 ft (9.81%)

The estimated per shopper impulse revenue for the existing layout was $14.04, around the $15-$20 range indicated in the retail literature and trade magazines (Hui et al., 2013; Stilley et al.,2010). Observe that most of the 'high' planned purchase departments were located towards the right side of the store (average distance from the door was 93.71 ft). We also observed only 3 'high' traffic density lanes, but 8 'low' density lanes; only Clothing, Pharmacy, and Frozen Food (out of 7 'high' impulse departments) were on these 3 lanes.

The best layout obtained via the PSO for Scenario (i) resulted in per shopper impulse revenue of $17.47, a 24.43% increase compared to the existing layout. In line with our observations in Section 5, the 'high' planned purchase departments were placed on the periphery of the store, with an average distance of these departments from the doors as 109.47 ft (16.82% higher compare to existing layout). Six out of 7 'high' impulse departments were placed around 'high' traffic density lanes. There was a 4.47% increase in average traffic density per lane and a slight increase in the shopper path length (2.46%) in this scenario.

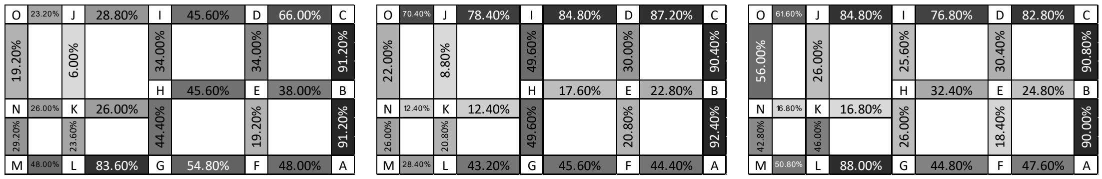


Figure 11 Comparison of traffic density in different lane (doors at nodes A and G)

Interestingly, in Scenario (ii), the 4 sub-sections of the Grocery department were distributed throughout the store. The average distance of 'high' planned purchase departments was 138.06 ft from the door, a 47.32% increase. Among the 22 lanes, 6 were 'high' traffic density lanes, while 8 were 'medium,' with the lowest traffic density of 16.80% compared to 6% in existing layout. The average traffic density per lane was observed to be 50.72%, which was 19.99% higher compared to the existing layout. This layout resulted in an expected per shopper impulse revenue of $18.35 (a 30.68% increase). This seems to allude that, unless critical, avoiding the need to keep these sub-sections together in the Grocery department may be beneficial. However, the corresponding increase by 9.81% in the shopper path length must be weighed accordingly.

7. CONCLUSION AND FUTURE RESEARCH

A retail store's layout has a direct impact on the path followed by the shopper to purchase items per their planned purchase list. Strategically placing departments in the store can effect the density of shoppers traveling along the shopping lanes and impact the visibility of departments (and the products therein). Higher visibility of products often leads to higher impulse revenue generation for the retailer. With this basis, and realizing a gap in the retail facility layout literature, this paper proposed an optimization model to optimize the placement of departmetns in the store to maximize expected impulse revenue for the retailer. The key contribution of our work was to dynamically account for the changes in the shopper path with changes in the department placements, in conjunction with consideration for door placements. We also propose an approach to derive these shopper paths and embed it in a particle swarm optimization based heuristic to efficiently solve real-world instances. We also illustrated how our approach could be used for a real store, and suggested potential improvements for that store.

The key findings from our study are as follows:

- The expected per shopper impulse revenue of the store was largely driven by the placement of 'high' planned purchase departments away from the door or on periphery of the store, and the subsequent placement of 'high' impulse purchase departments along the resulting high traffic lanes. This resulted in increased visibility of 'high' impulse purchase departments, in turn increasing the expected impulse revenue.
- The shape of the store can play a key role; a deeper store demonstrated higher impulse revenue compared to a wider store or a square-shape store.
- A one side-door store was able to generate high impulse revenue than other door placements.

These findings clearly point towards the need to incorporate changes in shopper paths, and resulting traffic density along major shopping lanes, when designing the layout of a store. In so doing, a retailer would be able to increase the visibility of the departments, and in turn, impulse revenue.

Future research in this area could extend this work by making door placement a decision by itself. Additionally, this model can be extended to account for the relayout cost given an existing layout. Placement of aisles can also be incorporated as a decision in the model to make it even more realistic, but this will make the problem even more complex and difficult to solve. Considering shopper objectives such as reduction in path length or emotions, along with the retailer's revenue maximizing objective, in a multiobjective setting could be a viable future endeavor.

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APPENDIX: SIMULATED ANNEALING ALGORITHM FOR DERIVING THE SHORTEST PATH FOR A GIVEN SHOPPING LIST

We designed a Simulated Annealing (SA) algorithm to find the shortest node visit sequence to be followed by the shopper based on a given planned purchase list. As we mention in the Path Subroutine, we first map the departments to be visited to nodes and then determine the node sequence that results in the minimum travel distance. Figure A.1 shows nodes for the 5×5 nodal structure. The distance between two successive nodes is can be easily calculated for a prespecified aspect ratio of the store.

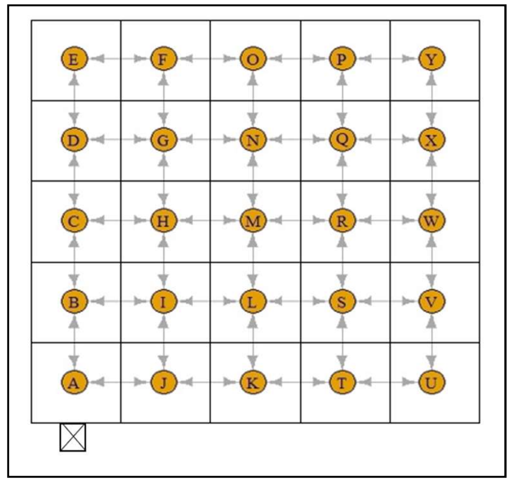

Figure A.1 5×5 nodal structure

We used the 2-way exchange method to evaluate the neighborhood of a candidate solution. In 2-way exchange, we randomly choose 2 nodes (except for the first and last nodes) from the list and swap their positions. First and last nodes define the entrance and exit of the store, and so they are not allowed to be swapped. The quality of a candidate solution is measured by the corresponding travel distance, which is a sum of the distance between all 2 adjacent nodes in the list. We use rectilinear distances between nodes. For instance, in a one side-door store (door located at node A), for a shopping list that requires a shopper to visit nodes A, K, F, M, D, and A, the resulting travel distance is 16 units.

After every iteration, if the distance associated with a candidate solution is lower than the incumbent solution, then this candidate solution is accepted as the new incumbent solution. If it is worse, we use an acceptance probability to determine whether or not to accept this worse solution;

$$a = e^{\frac{d_{best} - d_{new}}{T}} \tag{1}$$

where, a= acceptance probability,

d_{new}= distance corresponding a new (candidate) solution,

d_{best}= distance corresponding to the best(incumbent) solution, and

T= current temperature of an algorithm

To avoid getting stuck in a local minima, we also incorporated a 3-way exchange in situation where the best (incumbent) solution has not improved for 50 iterations. When the current temperature becomes less than or equal to the minimum temperature or the incumbent solution does not improve after 50 iterations after a 3-way exchange, we terminate the algorithm. A pseudocode of our SA implementation is presented below.

Initial conditions:

Step 1: Set T_0 = 10.0, T_F = 0.0000001, α = 0.9, I_{iter}=1000;

Step 2: Take a list of nodes to visit for purchase along with enter and exit node as an initial sequence of a node to visit (path) X;

Step 3: Calculate the total distance of this initial sequence of a node, d(X);

Step 4: Let $T = T_0$, I = 1, N = 0, kick=0; append X to list $[S]_{best}$ of best solution; append d(X) to list $[D]_{best}$ of best solution's distance

Iterative process:

Step 5: Generate a solution Y in the neighborhood of X using a two-way exchange;

Step 6: Calculate the total distance of a new sequence of node d(Y);

Step 7: If d(Y) < d(X) {

d(X)= d(Y);

X=Y;

$[S]_{best}$=Y;

$[D]_{best}$= d(Y);

N=I;

}

Else If $a = e^{\left\{\frac{d(Y)-d(X)}{T}\right\}} > rand$ (0,1) {

d(X)= d(Y);

X=Y;

N=I;

}

Step 8: T = T×α;

I = I + 1;

Step 9: If I-N>50 {

If kick=0 {

Generate a solution *Y* in the neighborhood of *X* using three-way exchange;

Calculate the total distance of new sequence of node *d*(*Y*);

d(X)= d(Y);

X=Y;

N=I;

Kick = 1;

} else {

Terminates the SA procedure

}

}

Step10:If T ≤ T_F {

Terminate the SA procedure

Best solution= $[S]_{best}$

}

Else {Go to Step 5;}

We convert the best solution (node list) obtained from the SA to a path using a shortest path function (*igraph* package in *R*). This shortest path function uses well known Dijkstra's algorithm to find all shortest paths between two nodes. We find shortest paths between all succeeding nodes and combine them together to form the overall shortest path(s) for a shopper given a shopping list and a department mapping.

For the example discussed earlier, an optimal shopping sequence is A, D, F, M, K and A (for one side-door). One of the shortest paths for this sequence is A-B-C-D-E-F-G-H-M-L-K-J-A and the corresponding travel distance is 12 units. This path is a combination of A-B-C-D, D-E-F, F-G-H-M, M-L-K and K-J-A, which are the shortest paths between two nodes A and D, D and F, F and M, M and K, K and A respectively.

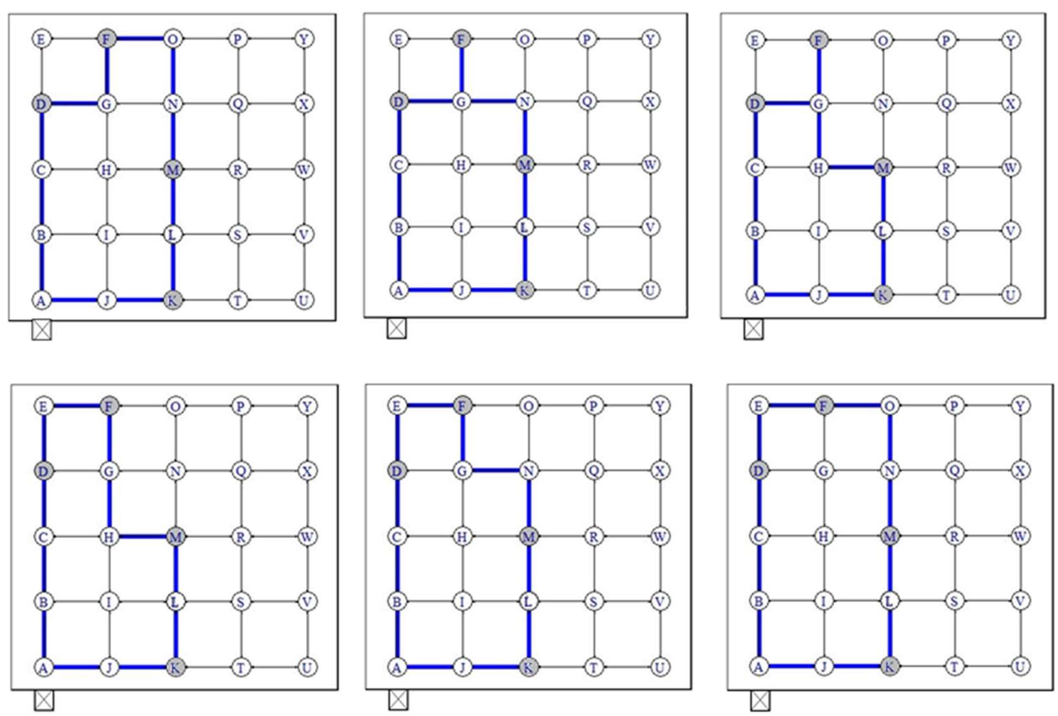

Figure A.2 Illustration of the 6 alternative shortest paths for the example node sequence; all have the same travel distance of 12 units

For this example, there are a total of 6 alternative paths with the same travel distance. These paths are highlighted in Figure A.2. As mentioned earlier, if generated shortest paths are less than k, then we use all those paths; if they are $>k$, then we randomly select k among those.