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Predicting Efficiency in Threshold Public Good Games: A Learning Direction Theory Approach

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Abstract

In this paper we propose a tractable model of behavior in threshold public good games. The model is based on learning direction theory. We find that individual behavior is consistent with the predictions of the model. Moreover, the model is able to accurately predict the success rate of groups in providing the public good. We apply this to give novel insight on the assurance problem by showing that the problem (of coordinating on the inefficient equilibrium of no contributions) is only likely with a relatively low endowment. In developing the model we compare and contrast best reply learning and impulse balance theory. Our results suggest that best reply learning provides a marginally better fit with the data.

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1 Introduction

A threshold public good is a public good that is provided if and only if total contributions towards its provision reach some critical threshold. Many goods and services can be approximated as threshold public goods (Taylor and Ward, 1982; Andreoni, 1998; van Lange et al., 2013; Mak et al., 2015; Hudik and Chovanculiak, 2017; Iris et al., 2019). A fundamental question is whether such goods are provided at the Pareto efficient level. Experimental evidence suggests they are not. In particular, the observed success rates of providing threshold public goods (when the choice set is continuous) are usually in the range of 30 to 70 percent (e.g. Croson and Marks, 2000; Cadsby et al., 2008; Alberti and Cartwright, 2015).[1] Such inefficiency has been observed in many different settings and is a robust empirical result. What is lacking is a theoretical model that can explain it.

The approach we take in this paper is based on learning direction theory. The theory says that players will have a tendency to change their behavior in a way that is consistent with ex-post rationality (Selten and Stoecker, 1986; Selten, 1998, 2004; see also Cason and Friedman, 1997, 1999). It, thus, encapsulates a general notion of adaptive learning in which players adjust behavior based on the last iteration of play. We apply learning direction theory to threshold public good games and derive two testable hypotheses. These hypotheses detail 'experience conditions' in which we would expect players to increase their contribution, to decrease their contribution, or to leave their contribution unchanged. We also show that predictions are sensitive to whether or not there is a refund. In particular, we analyse individual level data from experiments reported in Alberti and Cartwright (2015) and Cartwright and Stepanova (2015) to test our hypotheses. We find strong support for learning direction theory.

Learning direction theory does not, of itself, allow us to model and predict success rates in providing the public good. To go this extra step we compare and contrast two alternative ways of modelling ex-post rationality - best reply and impulse balance.

[1] It has been shown that various institutions such as sanctions (Andreoni and Gee, 2015), membership fee (Bchir and Willinger, 2013), refund bonus (Cason and Zubrickas, 2015) or requirement for full agreement (Alberti and Cartwright, 2016) can increase efficiency above the 'baseline' level.

Best reply learning posits that players will change their contribution in proportion to the gap between their last contribution and that which was ex-post optimal (Fudenberg and Levine, 1998). For instance, if a player contributes \$20 but can see it would have been optimal to contribute \$30 then she will increase her contribution in proportion to the \$10 gap. By contrast, impulse balance theory posits that players will change their contribution in proportion to ex-post impulse where impulse is measured by foregone profit (Selten, 2004; Ockenfels and Selten, 2005, 2014; Selten and Chmura, 2008; Chmura et al., 2012). For instance, suppose in our previous example that by contributing \$30 the player would have increased her payoff by \$50. Then she will increase her contribution in proportion to the \$50 foregone payoff.

We show that the predictions of best reply learning and impulse balance theory diverge in the case where contributions fall 'just short' of the threshold. Best reply learning posits that a player will have a tendency to increase her contribution a 'little bit' because there was only a small shortfall. By contrast, impulse balance theory posits that she will have a tendency to increase her contribution a lot because there is a large payoff gain from reaching the threshold. This prediction is consistent with the psychology literature on counter-factual thinking (e.g. De Cremer and van Dijk, 2010; Scholl and Sassenberg, 2014). It provides a way to directly compare the two models of learning. We find that both models provide, overall, good predictions of success rates. Best reply learning seems, however, to provide the better fit with both individual and group level data.

Learning direction theory is particularly appealing for our purposes because it can model deviations from Nash equilibrium (Selten and Chmura, 2008). This is crucial because there is strong evidence that play in threshold public good games does not converge to equilibrium. For instance, groups that behave consistent with Nash equilibrium in one round of repeated interaction usually deviate from Nash equilibrium in subsequent rounds (e.g. Cadsby and Maynes, 1999). Theoretical analysis of threshold public good games has to, therefore, take account of non-equilibrium behavior. In Section 5 we show that a model based on learning direction theory can accurately predict success rates at providing the public good. In doing so we revisit the results of Cartwright and Stepanova (2015) looking at whether a refund increases success in providing the public good. We show that a refund is likely to increase efficiency (relative to the setting of no refund) only if the endowment is very low. This has interesting implications, as we discuss in Section 5, on the extent to which inefficiency in threshold public good games is driven by the assurance problem or coordination problem. Our results suggest the coordination problem is foremost.

In relating our work to that on threshold public good games we highlight that our focus is on games with a continuous choice set.[2] Such games are of wide application and are most often used in the experimental literature. There is, however, a distinct lack of theoretical modeling of this type of game. Instead, the theoretical literature has almost exclusively focused on threshold public good games with a binary choice set (e.g. Palfrey and Rosenthal, 1984; Rapoport, 1985, 1987; Au, Chen, and Komorita, 1998; Offerman, Sonnemans, and Schram, 2001; Makris, 2009; Cartwright and Stepanova, 2017; Spiller and Bolle, 2017). Given that very different issues arise with a continuous choice set, as opposed to binary choice set, (Suleiman and Rapoport, 1992; Cadsby and Maynes, 1999) it is crucial to develop theoretical insight specific to the continuous case.

There are two papers we know of that offer theoretical insight on threshold public good games with a continuous choice set. First, Bagnoli and Lipman (1989) show that every perfect Nash equilibrium of a continuous threshold public good game (with refund) results in the public good being provided.[3] This prediction of a 100 percent success rate does little to explain the experimentally observed success rates well below 100 percent. Second, Suleiman and Rapoport (1992) question whether subjects in a threshold public good game behave consistent with expected utility maximization or, what they call, a cooperative model. They found some support for both models, but also found that neither model was able to predict contributions with much accuracy.[4] A new approach, therefore, is needed and in this paper we

[2]Strictly speaking we shall consider games where the choice set is discrete, but large.

[3]A related theoretical literature considers the subscription game, which is a form of threshold public good game (e.g. Admati and Perry, 1991; Laussel and Palfrey, 2003; Barbieri and Malueg, 2008). For the class of game considered in the experimental literature (i.e. simultaneous move games of complete information) the prediction would again be that groups efficiently provide the public good.

[4]More specifically, in their experiments Suleiman and Rapoport (1992) obtained the beliefs of a

provide an approach that appears to work well in capturing both individual and aggregate level behavior.

We proceed as follows: In Section 2 we introduce threshold public good games. In Section 3 we detail the predictions of learning direction theory and the main assumption of impulse balance theory. In Section 4 we test the predictions of learning direction theory with experimental data. In Section 5 we show how impulse balance theory can be used to predict aggregate outcomes. In Section 6 we conclude.

2 Threshold public good games

We shall consider variations of a simultaneous move, symmetric threshold public good game. A game is characterized by four positive integers: the number of players n, the size of endowment E, a threshold T, and a value of the public good V. Each player in set $N = \{1, ..., n\}$ is endowed with E units of a private good. Simultaneously, and independently of each other, players decide how much of their endowment to contribute towards a public good. For each player $i \in N$, let $x_i \in \{0, 1, ..., E\}$ denote the contribution of player i. Let $Y = \sum_{j=1}^{n} x_j$ denote total contributions and let $Y_{-i} = Y - x_i$ denote the total contribution of players other than i.

If total contributions, Y, equal or exceed the threshold T then the public good is provided and each player receives an additional V units of private good. If total contributions are less than the threshold then the public good is not provided. We shall assume that if total contributions are above the threshold there is no rebate of the excess contributions.[5] If total contributions are less than the threshold then we allow two possibilities, either (i) players get a *full refund* (R) of their contribution, or, (ii) they get *no refund* (NR) of their contribution. In the case of a full refund

subject about the likely contributions of others. From this one can ask whether own contribution is consistent with beliefs. They found evidence of consistency with both an expected utility model and cooperative model. This did not, however, translate into an accurate prediction of contributions. Also note, that such predictions rely on knowing the beliefs of subjects. To apply these models in a general setting, therefore, one would require a model of belief formation.

[5]This is standard in the empirical literature, e.g. Cadsby et al. (2008). Exceptions include Coats et al. (2009).

the payoff function of player $i \in N$ can be written

$$\pi_i(x_i, Y_{-i}) = \begin{cases} E - x_i + V & \text{if } x_i + Y_{-i} \geq T \\ E & \text{if } x_i + Y_{-i} < T \end{cases}.$$

In the case of no refund the payoff function of player $i \in N$ can be written

$$\pi_i(x_i, Y_{-i}) = \begin{cases} E - x_i + V & \text{if } x_i + Y_{-i} \geq T \\ E - x_i & \text{if } x_i + Y_{-i} < T \end{cases}.$$

It will be assumed that $nV > T$ meaning that it is socially efficient to provide the public good. It will also be assumed that $nE \geq T$ meaning that it is feasible for players to provide the good.

2.1 Nash Equilibria

To provide a starting point for the analysis we describe the set of Nash equilibria. Vector of contributions $(x_1^*, ..., x_n^*)$ is a pure strategy Nash equilibrium of the game if and only if $\pi_i(x_i^*, Y_{-i}^*) \geq \pi_i(x_i, Y_{-i}^*)$ for all $x_i \in \{0, 1, ..., E\}$ and $i \in N$. There are typically multiple pure strategy Nash equilibria in simultaneous threshold public good games. These can be partitioned into two broad categories. There is a set of equilibria where the sum of contributions equals the threshold and also a set of equilibria where the sum of contributions is less than the threshold. We describe each in turn.

It is simple to show that there will always exist a set of Nash equilibria with public good provision where

$$x_i^* + Y_{-i}^* = T \text{ and } x_i^* \leq V$$

for all $i \in N$. If $\min\{E, V\} > T/n$ there will be several of such equilibria. All of the equilibria of this type yield a total payoff to players of $nV - T$ but differ in how this payoff is distributed amongst players. A player who contributes relatively less receives a relatively higher payoff.

If $T > \min\{E, V\}$ then there will also exist a set of Nash equilibria with no public good provision. In the case of full refund this set includes any vector of contributions $(x_1^*, ..., x_n^*)$ where

$$x_i^* + Y_{-i}^* < T \text{ and } T - Y_{-i}^* > \min\{E, V\}$$

for all $i \in N$. In interpretation, the public good is not provided and it was not in the interests of any player to contribute enough to satisfy the threshold. In this case every player receives payoff E. In the case of no refund the set of equilibria with no public good provision consists of only the zero vector $(0, ..., 0)$. Given that the public good is not provided and there is no refund, any contribution is costly.

3 Ex-post rationality

In this section we apply learning direction theory. The theory says that players will have a tendency to adjust their behavior in accordance with ex-post rationality (Selten and Stoeker, 1986; Selten, 1998; Selten et al., 2005). This does not mean players will always adjust behavior in accordance with ex-post rationality; rather, there is a tendency to do so that is stronger than would be expected from random behavior. This allows us to make qualitative predictions on the tendency of players to increase or decrease their contribution.

To formalize the notion of ex-post rationality, consider a vector of contributions $(x_1', ...x_n')$. Let ex_i denote the ex-post optimal contribution of player i, where[6]

$$ex_i = \underset{x_i \in \{0,...,E\}}{\arg\max}\, \pi_i\left(x_i, Y_{-i}'\right).$$

We denote by $\pi_i = \pi_i\left(x_i', Y_{-i}'\right)$ the payoff received by player i and by $e\pi_i = \pi_i\left(ex_i, Y_{-i}'\right)$ the payoff player i would have got by contributing the ex-post optimal amount. We refer to $ex_i - x_i'$ as the *ex-post adjustment gap* and $e\pi_i - \pi_i'$ as the *ex-post payoff impulse*. If $ex_i > x_i'$ then learning direction theory predicts a tendency for player i to increase her contribution. Similarly, if $ex_i < x_i'$ it predicts a tendency for player i

[6]For now, we assume there exists a unique ex-post optimum contribution. When detailing specific experience conditions in Section 3.1 we address instances with non-uniqueness.

to decrease her contribution.

To make quantitative predictions on changes in contributions we need to model the strength of tendency to change contribution. We will compare and contrast two approaches - best reply learning and impulse balance theory. Best reply learning says that changes in contribution will be proportional to the adjustment gap. Impulse balance theory says that they will be proportional to the ex-post payoff impulse (Ockenfels and Selten, 2005; Selten and Chmura, 2008). Specific predictions will, thus, depend on the vector of contributions. We, therefore, distinguish the set of possible ex-post outcomes or experience conditions (Selten, 1998).

3.1 Experience conditions

We distinguish seven possible experience conditions. In interpretation, these classify possible outcomes of a game, as given by a vector of contributions $(x_1, ...x_n)$, into different categories. We begin by defining the experience conditions for the case of no refund. The conditions are defined for any player $i \in N$. To help to understand the conditions we highlight that $x_i + Y_{-i}$ is the total contribution and $T - Y_{-i}$ is the amount player i would have needed to contribute to provide the public good. As you work through the conditions it may be useful to refer to Figure 1 which illustrates the conditions for the case where $E > V$ and $n = 5$.[7] On the horizontal axis we measure player i's own contribution and on the vertical axis we measure the sum of contributions by the four other players Y_{-i}.

Lost opportunity (LO). Total contributions were less than the threshold and the player would have done better to contribute the amount needed to just achieve the threshold. Formally, x_i and Y_{-i} satisfy,

$$x_i + Y_{-i} < T \text{ and } T - Y_{-i} \leq \min\{E, V\}. \tag{LO}$$

The first condition says that the public good was not provided. The second condition

[7]These figure correspond to the High NR Treatment in our experiment, to be discussed in Section 4.

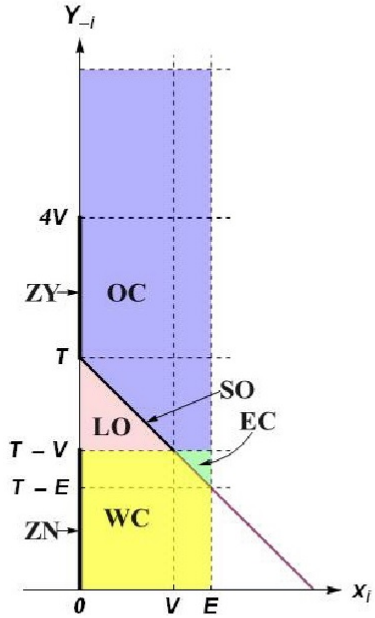

Figure 1: The seven experience conditions when $E > V$ and $n = 5$.

says that it could have been provided if player i had contributed more and that player i's payoff would have been higher by doing so.[8] The optimal ex-post contribution is $T - Y_{-i}$ and the ex-post payoff impulse is $x_i + V - (T - Y_i)$.

Wasted contribution (WC). Total contributions were less than the threshold and the player would have done better to contribute less. Formally, x_i and Y_{-i} satisfy,

$$x_i + Y_{-i} < T \quad \text{and} \quad x_i > 0 \quad \text{and} \quad T - Y_{-i} > \min\{E, V\}. \tag{WC}$$

The first condition says that the public good is not provided. The third condition says that the player either could not have done enough on her own to provide the good or would not have had an incentive to do so. The optimal ex-post contribution

[8] If $T - Y_{-i} = V$ the player is indifferent between contributing 0 or contributing $T - Y_{-i}$. So, this can be seen as either the lost opportunity or wasted contribution experience condition. For simplicity, we shall treat it as the lost opportunity experience condition. This was very rare in the experimental data and so is not significant for our results.

is 0 and the ex-post payoff impulse is x_i.

Spot on contribution (SO). Total contributions are equal to the threshold and the player benefits from the public good. Formally, x_i and Y_{-i} satisfy,

$$x_i + Y_{-i} = T \text{ and } 0 < x_i \leq V. \tag{SO}$$

There is no incentive to increase or decrease the contribution.

Overcontribution (OC). Total contributions exceed the threshold, and the player would have done better to contribute the amount needed to just achieve the threshold. Formally, x_i and Y_{-i} satisfy,

$$x_i + Y_{-i} > T \text{ and } x_i > 0 \text{ and } T - Y_{-i} \leq V. \tag{OC}$$

There is an incentive to reduce the contribution. The optimal ex-post contribution is $T - Y_{-i}$ if $Y_{-i} < T$ or 0 if $Y_{-i} \geq T$. The ex-post payoff impulse is $\min\{x_i, x_i + Y_{-i} - T\}$.

Excessive contribution (EC). Total contributions exceed the threshold, but the player does not, and cannot, benefit from the public good. Formally, x_i and Y_{-i} satisfy,

$$x_i + Y_{-i} \geq T \text{ and } T - Y_{-i} > V. \tag{EC}$$

There is an incentive to reduce the contribution. The optimal ex-post contribution is 0 and the ex-post payoff impulse is $x_i - V$.

Zero contribution (ZY, ZN). If the player contributes 0 and could not have increased her payoff by contributing more then there is no incentive to change her contribution. We distinguish whether the public good is provided or not (zero yes and zero no). Formally, x_i and Y_{-i} satisfy,

$$x_i = 0 \text{ and } Y_{-i} \geq T,\ x_i = 0 \text{ and } T - Y_{-i} > \min\{E, V\}. \tag{1}$$

In the case of a full refund we distinguish the same seven possible ex-post experi-

Table 1: The ex-post optimum, ex_i, the absolute value of adjustment gap, $|ex_i - x_i|$, the direction of predicted change, and the ex-post payoff impulse, $e\pi_i - \pi_i$, by experience condition.

Experience condition	Ex-post optimum			Impulse
	Contribution (ex_i)	Gap ($\|ex_i - x_i\|$)	Dir.	Strength ($e\pi_i - \pi_i$)
LO (NR)	$T - Y_{-i}$	$T - Y_{-i} - x_i$	$\uparrow$	$x_i + V - T + Y_{-i}$
LO (R)	$T - Y_{-i}$	$T - Y_{-i} - x_i$	$\uparrow$	$V - T + Y_{-i}$
WC (NR)	0	x_i	$\downarrow$	x_i
WC (R)	–	–	–	–
SO	x_i	0	0	0
OC	$\max\{0, T - Y_{-i}\}$	$\min\{x_i, x_i + Y_{-i} - T\}$	$\downarrow$	$\min\{x_i, x_i + Y_{-i} - T\}$
EC	0	x_i	$\downarrow$	$x_i - V$
ZY and ZN	0	0	0	0
ZN (R)	–	–	–	–

Notes: The signs of direction are increase, $\uparrow$, decrease, $\downarrow$, ambiguous, $-$, no change, 0.

ence conditions but note changes to the wasted contribution and zero no conditions. The full refund means that there is no clearly defined ex-post optimum in these two conditions. So, there is no ex-post incentive to either increase or decrease the contribution. Intuitively, one might expect an increase in contribution up to a maximum of $\min\{E, V\}$, but this is not a prediction of best reply learning or learning direction theory. We summarize the seven experience conditions for the case of no refund and full refund in Table 1.[9]

3.2 Hypotheses

Having distinguished the seven experience conditions above we now propose a set of hypotheses that can be tested with experimental data. We begin with two hypotheses that state specific testable predictions of learning direction theory. The

[9]Different players in the same group may face different experience conditions. For example, one player may face the lost opportunity condition while another faces the wasted contribution condition. Similarly, one player may face the excessive contribution condition while another faces the overcontribution condition.

first hypothesis is a direct application of learning direction theory in saying that players have a tendency to change contributions consistent with ex-post rationality. It summarises the 'Dir.' column in Table 1.

Hypothesis 1: In the case of no or full refund a player will have a tendency to increase her contribution in the lost opportunity experience condition and decrease her contribution in the overcontribution and excessive contribution condition. In the case of no refund a player will, in addition, have a tendency to decrease her contribution in the wasted contribution condition.

Learning direction theory has previously been applied in settings where there is always an upward or downward impulse (Selten 1998 and Selten et al. 2005). We suggest a natural extension whereby no ex-post payoff impulse implies no tendency to change contribution.

Hypothesis 2: In the case of no or full refund a player will have a tendency to keep her contribution unchanged in the spot on and zero yes experience conditions and to change her contribution in the lost opportunity, overcontribution and excessive contribution conditions. In the case of no refund a player will, in addition, have a tendency to keep her contribution unchanged in the zero no condition and to change her contribution in the wasted contribution condition.

Our next two hypotheses draw on impulse balance theory and best reply learning respectively. In order to state the hypotheses we denote by x_i^r, π_i^r and $e\pi_i^r$ the contribution of player i at time r, her payoff, and the payoff that she would have realized with the ex-post optimal amount.

Hypothesis 3 (Impulse Balance Theory): Changes in contribution tend to be proportional to the strength of ex-post payoff impulse,

$$\left|x_i^{r+1} - x_i^r\right| \propto \left|e\pi_i^r - \pi_i^r\right|. \tag{2}$$

Hypothesis 4 (Best reply Learning): Changes in contribution tend to be proportional to the adjustment gap,

$$\left|x_i^{r+1} - x_i^r\right| \propto \left|ex_i^r - x_i^r\right|. \tag{3}$$

Hypotheses 3 and 4 complement Hypothesis 1 by saying how much players will tend to change their contribution. We can see from Table 1 that impulse balance theory and best reply learning are indistinguishable except for the lost opportunity and excess contribution experience conditions. The lost opportunity condition is particularly interesting because the strength of ex-post payoff impulse is inversely related to the ex-post adjustment gap. Hypothesis 3 predicts that players will increase their contribution by more the closer are contributions to the threshold, because the impulse is stronger. This prediction is consistent with the evidence that counter-factual thinking occurs more often in the case of a 'near miss' rather than 'large miss' (Kahneman and Tversky, 1982; Kahneman and Miller, 1986; De Cremer and van Dijk, 2010).[10] Hypothesis 4, by contrast, predicts that players will increase their contribution the further are contributions from the threshold.

4 Experimental results

To evaluate our hypotheses we shall draw on data from laboratory experiments. We use data reported in Alberti and Cartwright (2015) and Cartwright and Stepanova (2015). We shall only provide a brief overview of the experimental procedure; for full details see Alberti and Cartwright (2015). Subjects played the same threshold public good game over 25 rounds using a fixed matching protocol. At the end of each round feedback was given on own payoff, total contributions to the public good and whether or not the public good was provided. The experiments were programmed using z-tree (Fischbacher, 2007).

Altogether there were 8 different treatments, summarized in Table 2. The Baseline treatment corresponds to the baseline treatment commonly used in the literature (e.g. Croson and Marks, 2000). The High, High 2, Low and Low 2 treatments are

[10]See also Roese (1997) and Parks et al. (2003).

Table 2: Summary of experimental treatments

Treatment	n	E	V	T	Refund	No. of subjects
Baseline	5	55	50	125	Yes	40
Baseline NR	5	55	50	125	No	40
High	5	70	50	125	Yes	30
High NR	5	70	50	125	No	45
High 2	5	55	20	50	Yes	30
Low	5	30	50	125	Yes	30
Low NR	5	30	50	125	No	45
Low 2	5	55	100	250	Yes	30
						290

motivated and discussed in detail by Alberti and Cartwright (2015). The main thing to note is that the high or low refers to a relatively high or low endowment when compared to the threshold. The NR treatments allow insight on the consequences of a refund (Cartwright and Stepanova, 2015). A total of 290 subjects took part in the experiments which were run at the University of Kent.

4.1 Learning direction

We begin the analysis of the experimental data by evaluating Hypotheses 1 and 2. At the end of each round we can work out the experience condition faced by each subject. We also know whether a subject increased, decreased, or kept unchanged their contribution in the subsequent round (excluding round 25). These two things allow us to report the proportion of times subjects increased or decreased their contribution for each experience condition, which we shall denote ρ_{up} and ρ_{dw}. We also report the proportion of times the contribution stayed unchanged σ_{no}.

Table 3 details the number of instances of each experience condition (in the first 24 rounds), as well as ρ_{up}, ρ_{dw} and σ_{no}, aggregated across all treatments. (In the supplementary material we provide the data broken down by treatment, where you can see that the proportions are similar across treatments.) While the formal tests of

Table 3: Change of contribution in the refund and no refund treatments sorted by experience condition

Experience Condition	Refund treatments				NR treatments			
	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}
LO	1009	10.4	**51.1**	38.5	455	16.5	**48.8**	34.7
WC	349	17.2	37.5	45.3	572	**46.9**	24.5	28.7
SO	405	11.1	15.8	**73.1**	279	3.2	5.7	**91.0**
OC	2014	**38.3**	12.1	49.7	1014	**39.1**	19.9	42.0
EC	28	**53.6**	7.1	39.3	26	**15.4**	11.5	73.1
ZY	28	–	17.9	**82.1**	66	–	18.2	**81.8**
ZN	7	–	85.7	14.3	708	–	10.0	**90.0**

Notes: For example, in the refund treatments, there were 1009 instances where a subject faced the lost opportunity experience condition; in 10.4% of these instances we observed a decrease in contribution, in 51.1% an increase, and in 38.5% no change. The proportions in bold correspond to Hypotheses 1 and 2.

Hypothesis 1 and 2 follow immediately below, we highlight that the data summarized in Table 3 is consistent with Hypotheses 1 and 2. For instance, we observe that $\rho_{up} > \rho_{dw}$ in the lost opportunity experience condition. We also observe that $\rho_{up} < \rho_{dw}$ in the overcontribution and excessive contribution conditions. Moreover, $\rho_{up} < \rho_{dw}$ in the wasted contribution condition where there is no refund. Finally, the value of σ_{no} is, as predicted, high in the spot on, zero no and zero yes experience conditions.

To formally evaluate Hypothesis 1 we need to take into account a possible regression effect (Ockenfels and Selten, 2005). The regression effect here is that purely by chance a large contribution is likely to be followed by a smaller contribution and a small contribution is likely to be followed by a larger contribution.[11] We, therefore, test Hypothesis 1 against an alternative *model of random choice*. This alternative model can be explained as follows: (i) In period 1 each player randomly, and in-

[11]This effect is unlikely to explain the data for the wasted contribution experience condition in the case of no refund because in this case the regression effect works in the opposite direction to that predicted by learning direction theory. For the lost opportunity and overcontribution experience conditions the regression effect is a possible concern.

dependently, chooses a contribution between 0 and $\min\{E, V\}$.[12] (ii) In periods 2 to 25 each player independently with probability $s \in [0, 1]$ leaves their contribution unchanged and otherwise randomly choose a contribution between 0 and $\min\{E, V\}$. If $s = 0$ then all choice is random. The higher is s the more persistence in choice.

To explain how we evaluated Hypothesis 1 against the model of random choice consider the lost opportunity condition. In a particular group let LO_{up} and LO_{dw} denote respectively the number of times (during the 25 periods for all 5 players) we observe a subject increasing or decreasing their contribution in response to the lost opportunity experience condition. We then look at the upward ratio $LO_{up}/(LO_{up} + LO_{dw})$. Learning direction theory predicts this ratio would be high. We simulated outcomes in the model with random choice (for different values of s) and compared our observed data to that in the random model. In Table 4 we summarize by treatment the average ratio we observe (Obs), what we would expect with the random model when $s = 0.5$ (Ran) and the probability the observed mean would be so high with the random model (p). Note that this test treats the group as the unit of observation. We performed the same exercise for the overcontribution and wasted contribution conditions with the caveat that we are now looking at whether the observed mean would be so low. Comprehensive details on our methods and a robustness analysis are provided in the Supplementary Material.

You can see in Table 4 that we find strong support for Hypothesis 1. We find a significantly higher tendency to increase contributions in the lost opportunity condition and to decrease contributions in the overcontribution condition than would be expected with random choice. The one caveat is the Low, Low NR and Low 2 treatments. The issue here, however, is that a strong upward and downward tendency is predicted with the random model and so there is little space for observed behaviour to be significantly different. In the wasted contribution condition a highly significant difference is apparent in the Low NR condition.

In evaluating Hypothesis 2 we note that inertia in the random model is determined by parameter s. We cannot, therefore, directly compare with the random model. Instead we compare across experience conditions. Consider, first, the refund setting.

[12] Contributions are still forced to be integers.

Table 4: Evaluating Hypothesis 1 compared to a model of random choice ($s = 0.5$).

Treatment	LO			OC			WC		
	Obs	Ran	p	Obs	Ran	p	Obs	Ran	p
Baseline	0.82	0.69	0.0001	0.24	0.36	0.0001	-	-	-
Baseline NR	0.78	0.69	0.013	0.34	0.36	0.16	0.48	0.48	0.36
High	0.82	0.69	0.004	0.21	0.36	0.0001	-	-	-
High NR	0.79	0.69	0.01	0.29	0.36	0.0031	0.38	0.48	0.07
Low	0.91	0.61	0.029	0.18	0.10	0.85	-	-	-
Low NR	0.80	0.61	0.12	0	0.10	0.34	0.29	0.48	0.0001
High 2	0.91	0.70	0.0001	0.22	0.36	0.0001	-	-	-
Low 2	0.85	0.55	0.048	0.25	0.06	0.99	-	-	-

In a particular group let G_{no} denote the proportion of times (during the 25 periods for all 5 players) we observe a subject keeping their contribution unchanged when in the lost opportunity or overcontribution experience conditions.[13] Similarly, let B_{no} denote the proportion of times the contribution is unchanged in the spot on and zero yes experience conditions. Hypothesis 2 predicts that inertia difference $B_{no} - G_{no}$ should be relatively high. We compared the inertia difference we observe to that expected with a model of random choice. In the no refund condition we also take account of the wasted contribution and zero no experience conditions.

In Table 5 we detail the average inertia difference by treatment. We also detail the probability the average is this level or higher with the random model (for three different values of s). Full details on the procedure and results are contained in the supplementary material. While the results are somewhat sensitive to the value of s and treatment, you can see that we find broad support for Hypothesis 2. In particular, the inertia difference is consistently well above zero implying that subjects were more likely to keep contributions unchanged in the spot on, zero yes, and where relevant zero no, experience conditions than in the lost opportunity, overcontribution

[13] That is, $G_{no} = (LO_{no} + OC_{no})/(LO_{obs} + OC_{obs})$ where LO_{no} and OC_{no} is the number of time during the 25 periods for all 5 players we observe a subject in the LO and OC experience condition keeping their contribution unchanged and LO_{obs} and OC_{obs} denotes the number of times experience condition LO and OC occurred.

Table 5: Evaluating Hypothesis 2 compared to a model of random choice.

Treatment	$B_{no} - G_{no}$	$p_{s=0}$	$p_{s=0.25}$	$p_{s=0.5}$
Baseline	0.13	0.008	0.106	0.076
Baseline NR	0.25	0.0001	0.061	0.087
High	0.42	0.0001	0.007	0.002
High NR	0.17	0.0001	0.23	0.48
Low	0.16	0.020	0.073	0.064
Low NR	0.32	0.0001	0.002	0.003
High 2	0.53	0.0001	0.0001	0.0001
Low 2	0.34	0.0001	0.0001	0.0001

and, where relevant wasted contribution, experience conditions.

4.2 Strength of impulse

We turn now to Hypotheses 3 and 4. In Table 6 we report the results of mixed effects regressions in which we regress the change in a subject's contribution against the strength of ex-post impulse or adjustment gap. We, thus, compare impulse balance theory (IBT) and best reply learning (BR). In the treatments with a full refund the impulse in the wasted contribution experience condition is undefined. For comparison purposes, we regress against the contribution (which equals the size of impulse there would have been if there was no refund). The results reported in Table 6 reaffirm evidence in support of Hypothesis 1. In particular, we observe an increase in contributions in the lost opportunity experience condition and a decrease in contributions in the overcontribution, excess contribution and, in the no refund treatments, wasted contribution experience conditions. Note also that, with the only exception of the Low treatment with refund, the results are not different across treatments both with a full refund and no refund.

Recall that the ex-post impulse and ex-post adjustment are equivalent in the wasted contribution and overcontribution experience conditions. Moreover, the excess contribution condition is relatively rare. Attention, therefore, in comparing

Table 6: Mixed-effects regression results with the change of contribution as dependent variable. Cluster robust standard errors in parentheses, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Variable	Refund IBT	Refund BR	No refund IBT	No refund BR
LO×Impulse	0.0941***		0.0151	
	(0.00939)		(0.0129)	
LO×Adjustment		0.281***		0.242***
		(0.0189)		(0.0252)
OC×Impulse	-0.216***		-0.271***	
	(0.0153)		(0.0220)	
OC×Adjustment		-0.196***	-0.243***	
		(0.0152)		(0.0210)
EC×Impulse	-0.452***		-0.582***	
	(0.0940)		(0.128)	
EC×Adjustment		-0.102***		-0.162***
		(0.0212)		(0.0326)
WC×Contribution	-0.0170*	-0.0151		
	(0.00988)	(0.00966)		
WC×Impulse			-0.402***	
			(0.0182)	
WC×Adjustment				-0.387***
				(0.0178)
Low	0.740	1.366**	-0.740	-0.0128
	(0.532)	(0.568)	(0.825)	(0.809)
High	0.366	0.383	-0.577	-0.364
	(0.531)	(0.563)	(0.818)	(0.803)
Low 2	-0.246	0.678		
	(0.532)	(0.569)		
High 2	-0.134	0.299		
	(0.533)	(0.567)		
Constant	0.531	-0.0582	2.651***	1.756***
	(0.368)	(0.392)	(0.621)	(0.607)
Observations	3,840	3,840	3,120	3,120
Number of groups	32	32	26	26
AIC	24668.36	24548.13	22542.56	22445.32
BIC	24743.40	24623.17	22603.02	22505.78

Hypotheses 3 and 4, falls on the lost opportunity condition where impulse balance and best reply make very different predictions. Impulse balance theory suggests contributions change in proportion to ex-post impulse while best reply suggests they change in proportion to ex-post adjustment. The results in Table 6 lend support to best reply learning (Hypothesis 4) over impulse balance theory (Hypothesis 3). There are two pieces of evidence to support this view. First, the LO $\times$ Adjustment coefficient is highly significant and similar to the OC $\times$ Adjustment coefficient in both the refund and no refund setting. By contrast, the LO $\times$ Impulse coefficient is smaller and not significant in the setting of no refund. Second, we can look at the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). A lower number indicates better fit and you can see that the AIC and BIC are slightly lower in both the refund and no refund setting for the model based on best reply learning. Our results, thus, support best reply learning over impulse balance theory, even if the difference is relatively marginal.

5 Simulations

In the preceding section we have shown that individual behavior is consistent with the predictions of learning direction theory. In this section we demonstrate that this can be used to predict aggregate outcomes and in particular to predict the success rate in providing the public good. As we highlighted in the introduction, to predict success rates is a crucial objective of our work because there is no model in the existing literature that comes close to predicting empirically observed success rates. To focus the analysis we shall pay attention to the assurance problem of how to avoid 'coordination' on the inefficient Nash equilibrium of zero contributions (Isaac et al., 1989). A refund should alleviate the assurance problem because it directly lowers the cost of being short of the threshold and indirectly increases confidence others will contribute (Isaac et al., 1989; Coats et al., 2009; see also Rapoport, 1987, and Bchir and Willinger, 2013, for an alternative approach). The comparison between a setting with and without a refund is, therefore, of particular interest.

In order to predict outcomes we simulate contributions over time. For completeness we introduce and discuss in turn one model based on best reply learning and one on impulse balance theory. Let x_i^r denote the contribution of player i in round r. The simulation method for best reply learning can be described as follows: (1) Individual contributions in the first period are independently determined for each player according to a normal distribution centered on T/n with standard deviation σ.[14] (2) In subsequent rounds any player with an ex-post adjustment gap of size Δ_{BR} changes their contribution by $\beta\Delta_{BR}$ where $\beta > 0$ is a parameter. Note that Δ_{BR} can be positive (resulting in an increased contribution) or negative (resulting in a decreased contribution). In other words, $x_i^{r+1} = min\{x_i^r + \beta\Delta_{BR}, E\}$ if $\Delta_{BR} > 0$ and $x_i^{r+1} = max\{x_i^r + \beta\Delta_{BR}, 0\}$ if $\Delta_{BR} < 0$. (3) Any player with no impulse leaves their contribution unchanged.

This simulation method encapsulates Hypothesis 4 in a very simplistic way. We would argue, however, that the simplicity of the method is a virtue in that we are not imposing any assumptions on contributions other than that suggested by Hypothesis 4. If this suffices to reliably predict the probability of the public good being provided then we have a method that can easily be applied and extended. The model has two free parameters, σ and β, that we discuss in turn. Values for β can be obtained from the coefficients in Table 6. The results we report here are obtained with $\beta = 0.25$, to approximately fit coefficients in the LO, WC and OC conditions. In the supplementary material we show that our results remain unchanged for alternative values of β.[15]

The value of σ influences the initial contribution profile and so can clearly be critical in determining the subsequent dynamics. Our choice of a distribution centered on T/n has two appealing properties. First, T/n is a focal contribution in threshold public good games (Isaac et al., 1989; Alberti and Cartwright, 2016). Indeed, the

[14] Contributions are rounded to the nearest integer and capped at 0 and E.

[15] There are critical values of β that do significantly affect dynamics. To illustrate, suppose contributions are in excess of the threshold and so all players experience the overcontribution condition. The downward adjustment in this case is proportional to the overcontribution. That means that if $n\beta \leq 1$ contributions will smoothly converge to the threshold. By contrast, if $n\beta > 1$ contributions will overshoot. The coefficients in Table 6 and the prior experimental evidence, however, strongly suggest that overshooting is likely to occur. Within the range $n\beta > 1$ results are insensitive to changes in β.

median and modal choice in the first round was T/n in all of the 8 treatments we are studying here (with the exception of the median being 20 in the Low treatment). A second advantage of using a normal distribution centered on T/n is that it means the probability of providing the public good in the first round must be approximately 50 percent.[16] This would seem a very neutral starting point and one that allows the learning dynamics to determine outcomes. The results we report are obtained with $\sigma = 6$. Again the supplementary material tests the robustness of this assumption. Figure 2 plots the cumulative distribution of choices we observed in the first round (in each treatment) compared to that with the fitted distribution when $\sigma = 6$. You can see that the fit is a reasonable approximation.

The simulation method we propose for impulse balance theory differs to that described for best reply learning in terms of step (2). We assume a potential asymmetry between upward and downward impulse. (2a) Any player with an upward ex-post impulse of size Δ_{IB} increases their contribution by $\alpha\Delta_{IB}$ where $\alpha > 0$ is a parameter. In other words, $x_i^{r+1} = min\{x_i^r + \alpha\Delta_{IB}, E\}$. (2b) Any player with a downward ex-post impulse of size Δ_{IB} decreases their contribution by $\gamma\Delta_{IB}$. So, $x_i^{r+1} = max\{x_i^r - \gamma\Delta_{IB}, 0\}$. A potential asymmetry between upward and downward impulse is considered in prior work (e.g. Selten and Chmura, 2008; Cartwright and Stepanova, 2017). It also seems consistent with the coefficients in Table 6. We report here results obtained with $\alpha = 0.05$ and $\gamma = 0.25$.

To test the model we analyse the relationship between success rates, endowment and refund. Cartwright and Stepanova (2015) argue that the assurance problem is exasperated when the endowment, ceteris paribus, falls below a certain level. To be more specific, define the endowment multiple as $EM = En/T$. A review of the available experimental evidence suggested that a refund makes no difference to success rates in providing the public good if $EM > 2$ but does if $EM < 1.3$ (where success rate is the proportion of times the public good is provided). The gap between 1.3 and 2 exists for the simple reason that no experiments have been run with parameters in this range. To appreciate the issue consider the left hand

[16] It will not be exactly 50%, depending on the treatment, because of the need to constrain contributions within $[0, E]$.

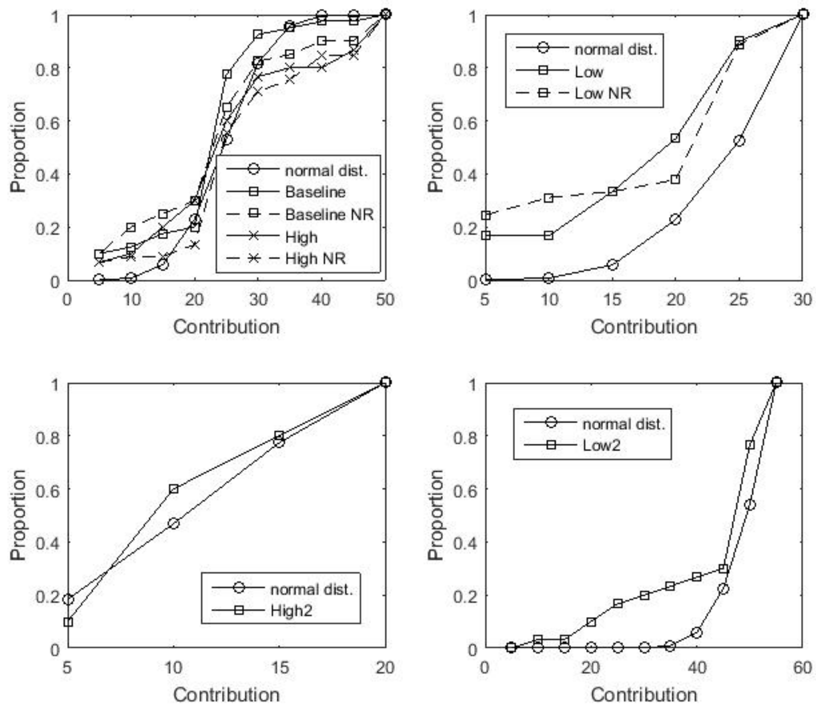

Figure 2: Distribution of choices in the first round compared to a fitted normal.

side of Table 7 in which we report the observed results from the Baseline, High and Low treatments. In the High and Baseline treatments (with an EM of 2.8 and 2.2 respectively) the success rate is essentially the same with and without the refund. But in the Low treatment (with an EM of 1.2) the success rate drops significantly with no refund.

On the right hand side of Table 7 we detail the predicted success rates based on our models for best reply learning and impulse balance theory. These report the average number of times the public good was provided over 25 rounds and so replicate our experimental data. As you can see the predicted success rates are a reasonable fit with the data and, crucially, pick up that the refund only makes a significant difference in the Low treatments. We also see that best reply learning provides a

Table 7: Observed success rates (%) comparing treatments with and without a refund and predicted success rates (setting $\beta = 0.25, \alpha = 0.05, \gamma = 0.25, \sigma = 6$).

Treatment	EM	Observed			Best Reply			Impulse Balance		
		R	NR	Diff	R	NR	Diff	R	NR	Diff
High	2.8	65	61	4	59	59	0	84	70	14
Baseline	2.2	55	50	5	59	59	0	84	70	14
Low	1.2	61	16	45	57	26	31	71	36	35

better fit than impulse balance. In Figure 3 we plot the predicted difference in success rates due to the refund for the full range of E. Our results suggest a refund makes a significant difference if $E \leq 40$ which would equate to $EM \leq 1.6$. Clearly, the number 1.6 should not be treated as definitive, but our analysis demonstrates the potential for learning direction theory to capture observed experimental results and make novel predictions. In this instance, our results would suggest that the assurance problem occurs only if the endowment is very low relative to the threshold.

6 Conclusion

Threshold public good games are of wide practical interest and the subject of a large empirical literature. This literature has shown that groups are inefficient at providing public goods, with success rates typically varying between 30 and 70 percent (when the choice set is continuous). Up to this point, there was no theoretical model that could make sense of these empirical findings. In this paper we apply learning direction theory and show that it is consistent with observed individual behavior. We also show that a model based on best reply learning can be used to reliably predict aggregate success rates in providing the public good. We would argue that this is a fundamentally important step forward in our ability to model and understand threshold public good games.

As mentioned, our approach applies learning direction theory in saying that players will tend to change their contribution in accordance with ex-post rationality (Sel-

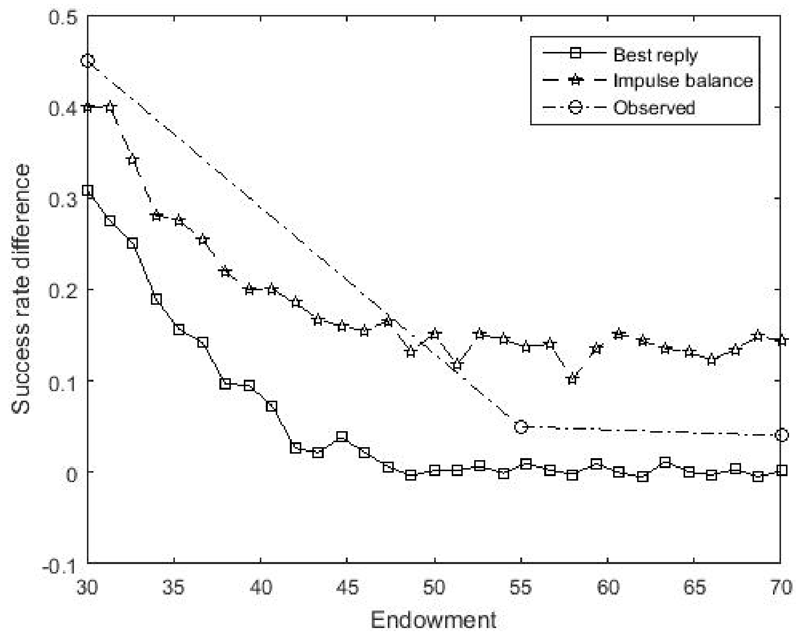

Figure 3: Predicted difference in success rates due to a refund as a function of E. Experimentally observed difference in the Low, Baseline and High treatments.

ten, 2004). We compared two models of learning consistent with learning direction theory - best reply and impulse balance. Our results suggest that best reply learning provides a better fit with both individual level and group level data. This 'win' for best reply learning is, though, relatively marginal and so further work would be desirable to explore this issue. Indeed, it may be that some players react to the adjustment gap (best reply learning) while some may react to foregone profit (impulse balance). We would, thus, see heterogeneity in learning. Exploration of this possibility would require more detailed individual level data in the lost opportunity experience condition.

In this paper we applied our model to study whether a refund enhances efficiency. Our results suggest that a refund only enhances efficiency if the endowment is very low. Thus, a refund is unlikely to be an effective way of increasing success in providing threshold public goods, and other mechanisms or institutions are needed (Cartwright and Stepanova, 2015). There are many other avenues that could be explored with the model. These include the effect that changes in the threshold for public good provision have on success in providing the public good (Isaac et al., 1989). Also, the effect of a rebate on contributions in excess of the threshold (Marks and Croson 1998; Spencer et al., 2009). An additional avenue of research could be changes in the size of strategy set. In particular, to bridge the gap between studies with a binary strategy set (contribute or not) and those with a large, essentially continuous, strategy set (Suleiman and Rapoport, 1992).

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Predicting Efficiency in Threshold Public Good Games: A Learning Direction theory Approach - Supplementary Material

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JEL codes: C72, H41, C92

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1 Change of contribution by treatment

Table 1: Change of contribution in the Baseline and the Baseline NR treatments sorted by experience condition.

Experience Condition	Baseline				Baseline NR			
	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}
LO	385	12.5	**50.4**	37.1	201	17.4	**49.8**	32.8
WC	49	30.6	40.8	28.6	172	**45.3**	29.1	25.6
SO	105	13.3	15.2	**72.4**	21	14.3	38.1	**47.6**
OC	401	**48.1**	15.5	36.4	432	**36.1**	20.6	43.3
EC	7	**42.9**	0.0	57.1	4	25.0	0.0	75.0
ZY	12	–	33.3	**66.7**	23	–	26.1	**73.9**
ZN	1	–	100.0	0.0	107	–	8.4	**91.6**

Table 2: Change of contribution in the High, High NR and High 2 treatments sorted by experience condition.

Experience Condition	High				High NR				High 2			
	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}
LO	220	12.3	**45.9**	41.8	225	16.0	**48.9**	35.1	188	5.3	**41.1**	43.6
WC	34	20.6	47.1	32.4	57	**61.4**	24.6	14.0	12	33.3	58.3	8.3
SO	30	6.7	10.0	**83.3**	98	6.1	8.2	**85.7**	80	16.3	16.3	**67.5**
OC	403	**33.7**	10.2	56.1	567	**39.5**	19.9	40.6	435	**41.1**	14.7	44.1
EC	21	**57.1**	9.5	33.3	22	**13.6**	13.6	72.7	0	–	–	–
ZY	11	–	9.1	**90.9**	43	–	14.0	**86.0**	5	–	0.0	**100.0**
ZN	1	–	100.0	0.0	68	–	8.8	**91.2**	0	–	–	–

Table 3: Change of contribution in the Low, Low NR and Low 2 treatments sorted by experience condition.

Experience Condition	Low				Low NR				Low 2			
	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}	no.	ρ_{dw}	ρ_{up}	σ_{no}
LO	123	8.9	**61.8**	29.3	29	13.8	**41.4**	44.8	93	9.7	**52.7**	37.6
WC	159	14.5	40.3	45.3	343	**45.2**	22.2	32.7	85	11.6	25.3	63.2
SO	35	2.9	20.0	**77.1**	160	0.0	0.0	100	155	9.7	16.1	**74.2**
OC	400	**36.8**	8.5	54.8	15	**40.0**	0.0	60.0	375	**30.9**	11.2	57.9
ZY	0	–	–	–	0	–	–	–	0	–	–	–
ZN	3	–	66.7	33.3	533	–	10.5	89.5	2	–	100.0	0.0

2 Testing Hypothesis 1

2.1 Lost opportunity experience condition

In Figure 1 we plot the cumulative distribution of upward ratio, $LO_{up}/(LO_{up}+LO_{dw})$, obtained with a model of random choice in the Baseline and High treatments. Given that $min(E,V)$ is the same in all four treatments the model of random choice gives identical predictions. We plot the distribution of the random model for $s = 0$ and 0.5 to provide an upper and lower 'bound'. As you can see, the results are not sensitive to changes in s. You can also see that the upward ratio is predicted to be around 0.6 to 0.8 illustrating the regression effect. In particular, purely by chance we would expect an upward trend in the lost opportunity condition.

In Figure 1 we also plot the observed outcomes in the groups in the experiment data together with a 95% confidence interval (for $s = 0$). You can see that several groups have an upward tendency that is unlikely with random choice. To explain the statistical test we report in the paper consider the Baseline treatment. Here we have 8 groups and the mean upward ratio in those 8 groups is 0.82. We bootstrap the probability that with 8 groups the mean upward ratio would be 0.82 or above. For both $s = 0, 0.5$ the probability is $p < 0.0001$. Hence it is statistically unlikely we would observe such a high upward ratio with random choice. We perform a similar exercise of the other 7 treatments.

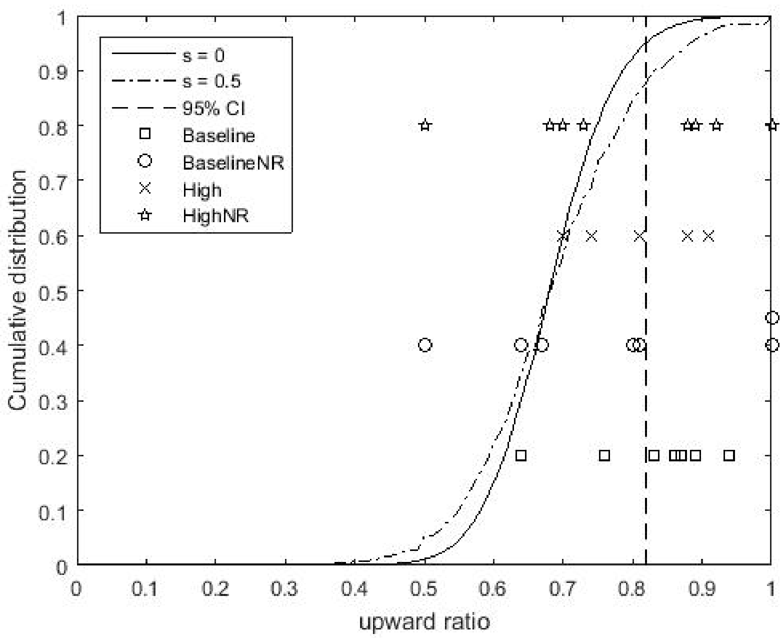


Figure 1: Cumulative distribution of upward ratio in the Baseline, Baseline NR, High, and High NR treatments with a random choice model and lost opportunity experience condition. The observed outcomes in experimental groups.

For completeness, in Figure 2 we provide the corresponding plot for the Low, Low NR, Low 2 and High 2 treatments. In the Low treatments distinguishing a high upward ratio is difficult because the lost opportunity condition is rare. Hence, the random model predicts a ratio of either 0 or 1 is common because there was only one observation of the condition. In the High 2 treatment, by contrast, the lost opportunity condition is more common and so it is much easier to pick out a relatively high upward ratio.

2.2 Overcontribution experience condition

In Figure 3 we plot the cumulative distribution upward ratio, $OC_{up}/(OC_{up}+OC_{dw})$, obtained with a model of random choice in the Baseline and High treatments. Here you can see the upward ratio is relatively low indicating a tendency to decrease contribution in the overcontribution condition even with the random model. That said, with the exception of the Baseline NR treatment the observed upward ratio is statistically lower than would be expected with the random choice model (see the main paper). In Figure 4 we plot the corresponding distribution for the other four treatments. You can see that in the Low, Low NR and Low 2 treatments an upward ratio of 0 is highly likely with random choice. This means it is not possible to statistically distinguish observed group outcomes from the random model.

2.3 Wasted contribution experience condition

In Figure 5 we plot the cumulative distribution upward ratio, $WC_{up}/(WC_{up}+WC_{dw})$, obtained with a model of random choice in the Baseline and High treatments. In Figure 6 we plot the corresponding distribution for the Low treatments. Note that wasted contribution only has a clearly defined ex-post payoff impulse in the no refund treatments and so the Baseline NR, High NR and Low NR treatments are the natural focus. You can see that in the Low NR treatment the upward ratio is relatively low. In the Baseline NR and High NR treatments we observe no clear distinction between observed behaviour and the random model.

3 Testing Hypothesis 2

In Figure 7 we plot the cumulative distribution of inertia difference predicted by the random choice model in the baseline and high treatments. In Figure 8 we plot the distribution for the low treatments and in Figure 9 we compare between the Low 2 and High 2 treatments. You can see that the inertia difference in groups is nearly always positive, consistent with Hypothesis 2. Statistical significance drops when

Table 4: Predictions of the best reply model as a function of σ and β. In the main paper we report the results for $\sigma = 6$ and $\beta = 0.25$.

σ	6	6	6	6	6
β	0.15	0.2	0.25	0.3	0.35
Baseline NR	59	60	59	59	52
Baseline	59	60	59	59	52
Baseline diff.	0	0	0	0	0
Low NR	37	29	26	13	9
Low	40	44	57	55	53
Low diff.	3	15	31	41	44

Table 5: Predictions of the best reply model as a function of σ and β. In the main paper we report the results for $\sigma = 6$ and $\beta = 0.25$.

σ	3	6	10
β	0.25	0.25	0.25
Baseline NR	60	59	55
Baseline	60	59	58
Baseline diff.	0	0	3
Low NR	30	26	11
Low	58	57	42
Low diff.	27	31	31

$s = 0.5$ only because this adds an element of noise to the random model and so makes a positive inertia difference more likely.

4 Robustness of simulation results

Our simulation results for best reply learning are based on $\sigma = 6$ and $\beta = 0.25$. In Tables 4 and 5 we provide a sensitivity analysis to changes in σ and β. As you can see our results are robust. Only if β drops as low as 0.15 we see a notable change in the efficiency consequences of a refund. We remind, however, that our analysis in the main paper suggests that β is well above 0.15.

Our simulation results for impulse balance theory are based on $\alpha = 0.05, \gamma = 0.25$

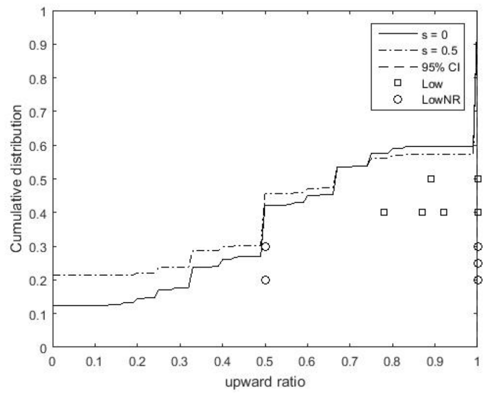
and $\sigma = 6$. In Tables 6 and 7 we provide a sensitivity analysis to changes in the parameters. Here you can see more variation in predictions than in the best reply model. Moreover, the predictions fit less well with observed success rates because they consistently overestimate success. Even so, we do observe relative consistency in predicting the efficiency consequences of a refund.

Table 6: Predictions of the impulse balance mode as a function of α, γ and σ. In the main paper we report the results for $\alpha = 0.05, \gamma = 0.25$ and $\sigma = 6$.

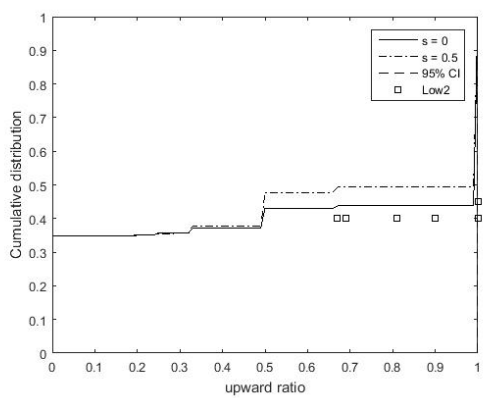
σ	6	6	6	6	6
α	0.02	0.05	0.1	0.05	0.05
γ	0.25	0.25	0.25	0.15	0.35
Baseline NR	73	70	60	90	56
Baseline	66	84	85	90	74
Baseline diff.	-7	14	25	0	18
Low NR	29	36	32	50	8
Low	63	71	75	75	58
Low diff.	34	35	43	25	50

Table 7: Predictions of the impulse balance model as a function of α, γ and σ. In the main paper we report the results for $\alpha = 0.05, \gamma = 0.25$ and $\sigma = 6$.

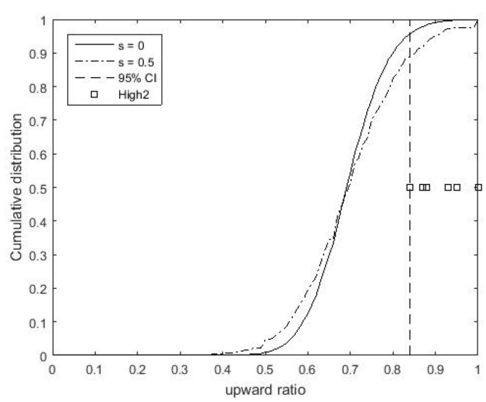
σ	3	6	10
α	0.05	0.05	0.05
γ	0.25	0.25	0.25
Baseline NR	74	70	60
Baseline	78	84	79
Baseline diff.	5	14	19
Low NR	54	36	19
Low	78	71	61
Low diff.	24	35	41



(a) Low, Low NR



(b) Low 2



(c) High 2

Figure 2: Cumulative distribution of upward ratio in the Low, Low NR, Low 2 and High 2 treatments with a random choice model and lost opportunity experience condition. The observed outcomes in experimental groups.

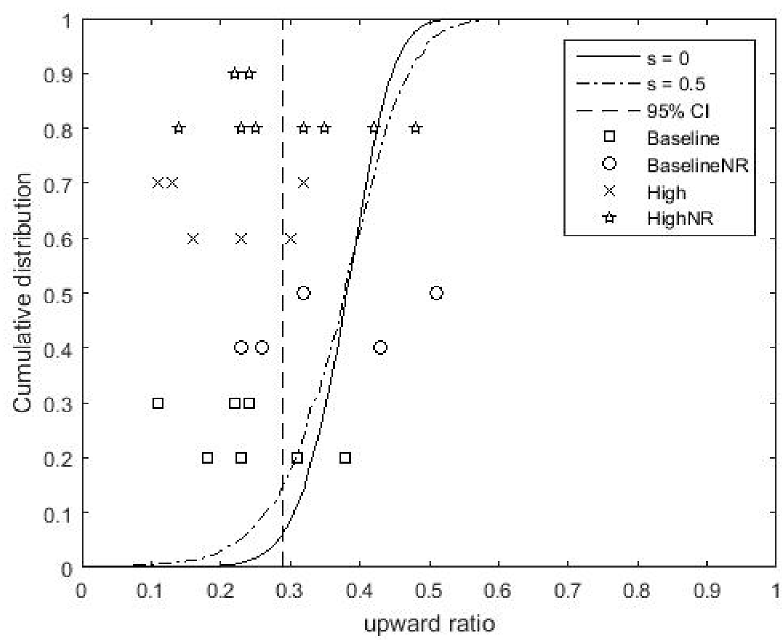
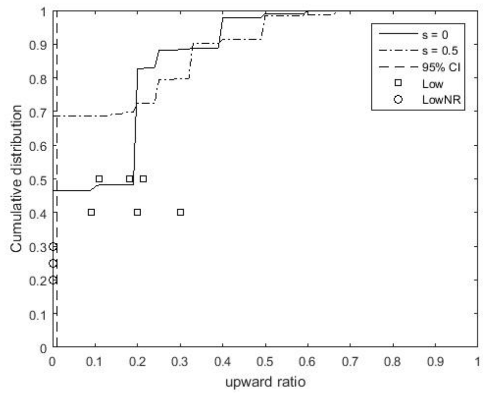
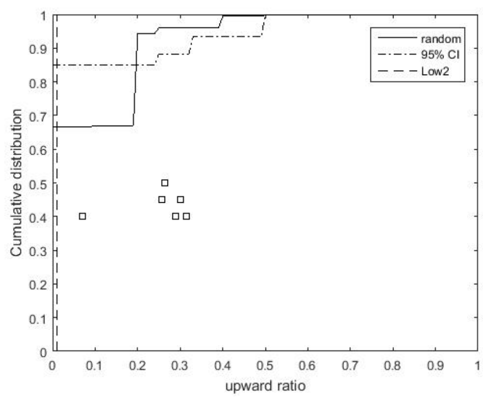


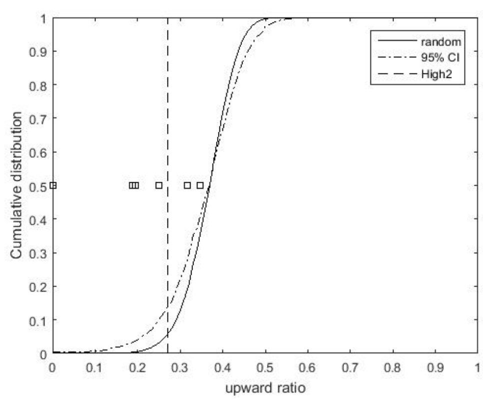
Figure 3: Cumulative distribution of upward ratio in the Baseline, Baseline NR, High, and High NR treatments with a random choice model and overcontribution condition. The observed outcomes in experimental groups.



(a) Low, Low NR



(b) Low 2



(c) High 2

Figure 4: Cumulative distribution of upward ratio in the Low, Low NR, Low 2 and High 2 treatments with a random choice model and overcontribution experience condition. The observed outcomes in experimental groups.

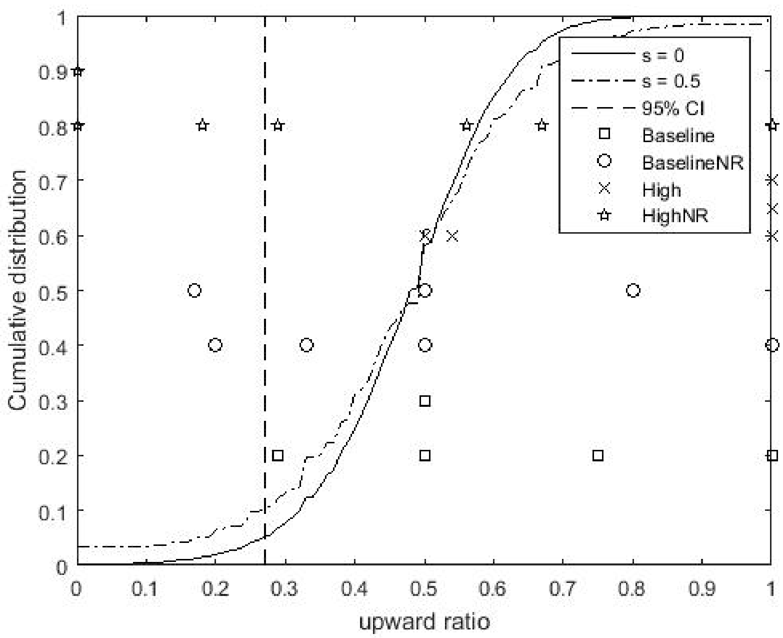


Figure 5: Cumulative distribution of upward ratio in the Baseline, Baseline NR, High, and High NR treatments with a random choice model and wasted contribution condition. The observed outcomes in experimental groups.

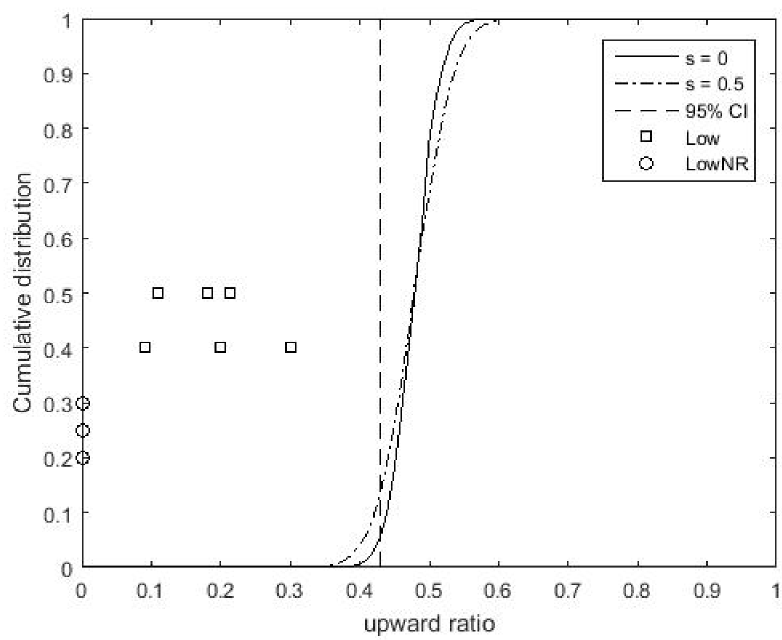
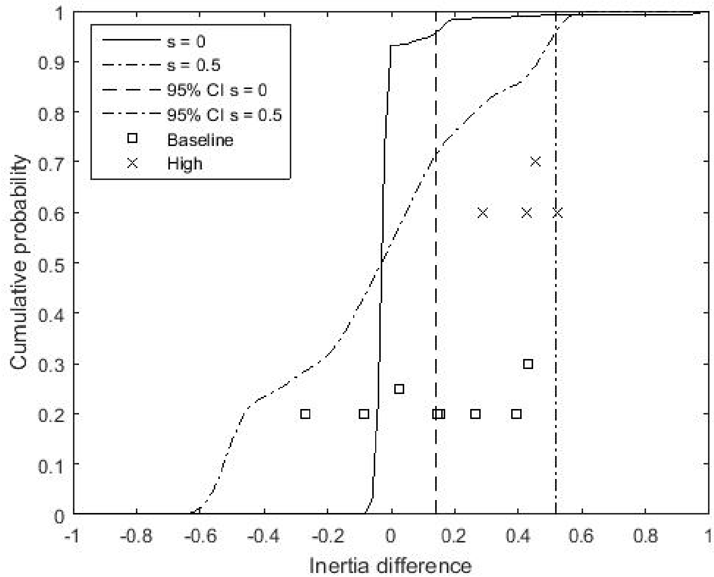
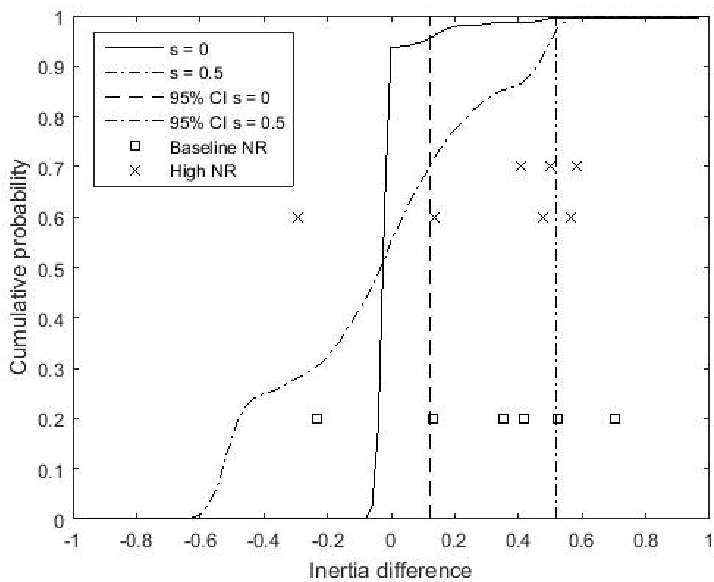


Figure 6: Cumulative distribution of upward ratio in the Low and Low NR treatments with a random choice model and wasted contribution experience condition. The observed outcomes in experimental groups.

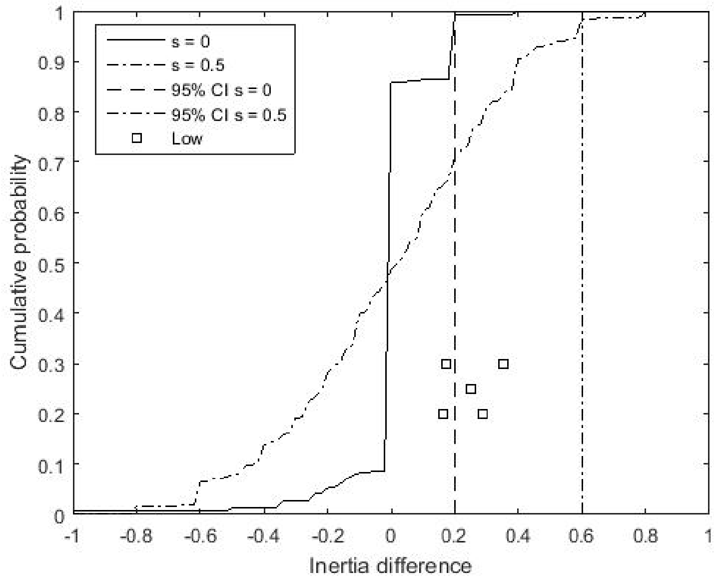


(a) Baseline, High

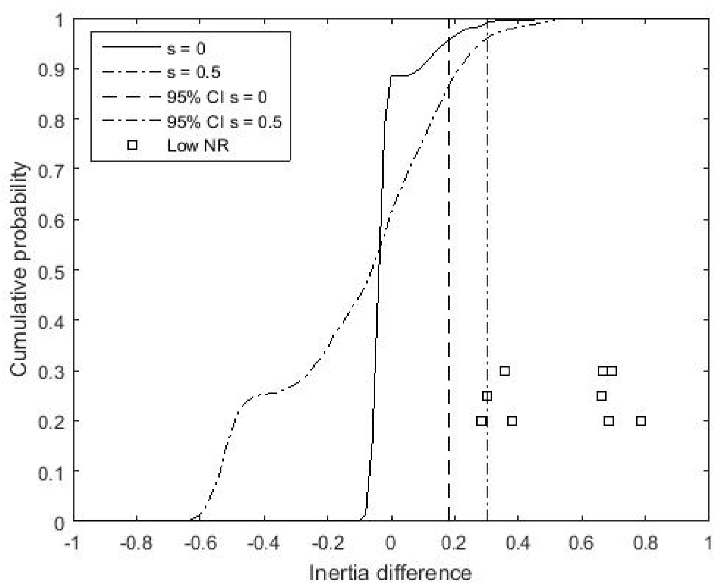


(b) Baseline NR, High NR

Figure 7: Cumulative distribution of inertia difference in the Baseline and High and Baseline NR and High NR treatments with a random choice model. The observed outcomes in experimental groups.

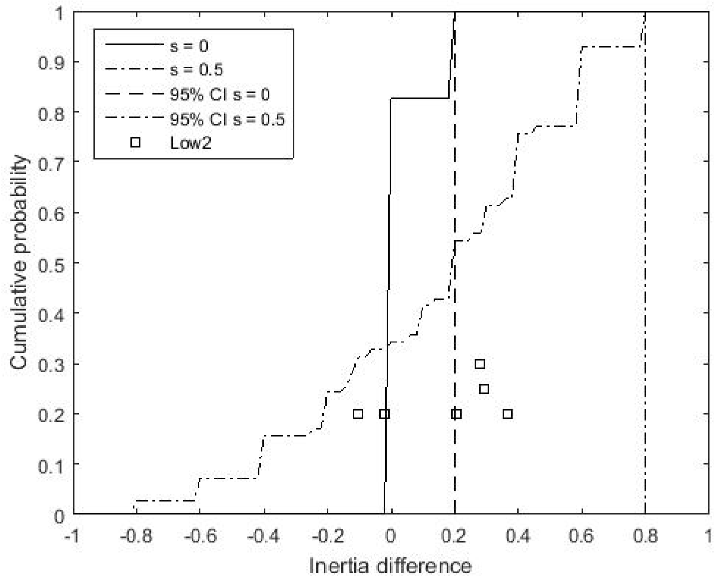


(a) Low

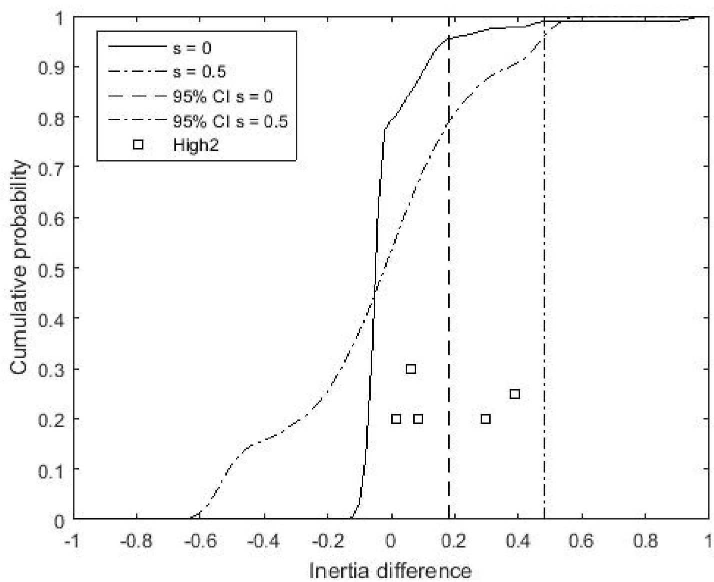


(b) Low NR

Figure 8: Cumulative distribution of inertia difference in the Low and Low NR treatments with a random choice model. The observed outcomes in experimental groups.



(a) Low 2



(b) High 2

Figure 9: Cumulative distribution of inertia difference in the Low 2 and High 2 treatments with a random model of choice. The observed outcomes in experimental groups.