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Yield design upper bound approach to uplift bearing capacity of offshore strip anchors

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Abstract. The offshore oil and gas exploration activities require several subsea infrastructures to be installed over the ocean floor, this equipment being commonly supported by shallow foundations namely anchor plates or mudmats. By the environmental conditions, live loads imposed during the structure lifetime, and the requirement for extraction processes, the design practice for anchor plates strongly relies on the evaluation of the vertical uplift capacity of the foundation system under undrained settings, being subjected to suction forces acting below the anchor interface which compose a rather complex subject in geomechanics, usually not addressed by the classical foundation theories. In this context, this paper aims to evaluate the uplift bearing capacity of a shallow foundation embedded in a purely cohesive material based on the theoretical framework of limit analysis theory and related kinematic approach. The latter allows for the formulation of rigorous upper bound estimates of the uplift bearing capacity by implementation and analysis of appropriate failure mechanisms that involve the anchor, the surrounding soil, and their mechanical interaction along with the interface. It is shown that the optimal uplift estimate bearing capacity amounts to minimizing a non-convex functional with respect to the parameters defining the geometry of considered class of failure mechanisms. The considered minimization procedure is handled by analyzing the stationary condition of the associated variational form, thus leading to the analytical determination of the optimal failure mechanism. For application purposes, the strength capacity of the soil is modeled through a Tresca-like yield condition with a tension cut-off, whereas the soil/anchor interface is modeled by a tensile stress threshold. The predictions obtained from the approach are compared to available finite element solutions derived in a particular situation with a perfectly smooth soil/anchor interface. An extensive parametric study is then carried out to assess the effects of relevant problem parameters on the uplift bearing capacity, such as the threshold stress controlling the breakaway condition at the interface and the strength parameters of the soil.

Keywords: Limit analysis, strip anchors, uplift bearing capacity

1 Introduction

As global oil demand has grown in the past decades, the supply generation increased its dependence on the discovery and development of deepwater reserves, which relies on multi-component seabed facilities to extract the hydrocarbons from deep waters and pipe the production to a nearby platform or production facility, allowing operations in virtually any water depth. This subsea infrastructure demands several instruments to be installed over the seabed such as pipeline end manifolds, terminations, sleepers, and protection systems, usually being supported by shallow foundations namely anchor plates or mudmats. These foundations are typically strip rectangular anchors, with dimensions ranging from 2 to 40 m and embedment depths around 5 to 50% of the width, being subjected to vertical loads from equipment/structure dead weight, and horizontal loads arising from pipeline thermal expansion and ocean currents (Randolph et al. [1], Chandler et al. [2], Shen et al. [3].

Many offshore infrastructure has been in service for several decades and at some point reached or will reach its "life-extension", arising the concern with its decommission operations demanded by several environment protection agencies that request the complete extraction of all seabed equipment present in the facility, an operation also desired when maintenance of the equipment is required (Chandler et al. [2], Agência Nacional do Petróleo [4], Lai [5]). A crucial aspect emerges when the extraction of the anchor plate is taking place, since its recommended design practice (e.g. American Petroleum Institute [6]) usually only concerns with operational loads and does not account for relevant suction forces in the anchor's base during intentional uplift of the foundation system, hindering the decommissioning process and elevating its costs. In this manner, the anchor design process should be optimized regarding both service loads and extraction processes.

The uplift bearing capacity of shallow foundation is a well know subject in foundation engineering theory due to its wide range of applications in geotechnical problems, being object of constant revisions (e. g. Vesić [7], Rowe [8], Rowe and Davis [9], Das and Piccornell [10], Merifield et al. [11], Merifield et al. [12], Merifield et al. [13], Khatri and Kumar [14], Feng et al. [15], Li [16], Aghazadeh Ardebili et al. [17], Shen et al. [3]) due to the uncertainties regarding the mechanisms acting and the interface effects on the anchor's base. In general, the uplift bearing capacity is usually represented in terms of a break-out factor, obtained experimentally or numerically as a function of anchor shape, embedment depth, and soil properties. Inspired from Rowe and Davis [9] work, all the analysis has been provided in two distinct categories: *immediate breakaway*, where the interface soil/anchor is unable to sustain tension and thus there is no suction or adhesion in the anchor base, and *no breakaway*, a case in which the interface remains in contact with the soil at all stages, meaning it has unlimited tension resistance. More recently, the effect of negative pore pressure within the soil mass beneath the anchor has been taken into account by Li [16], yet the interface resistance magnitude remains an open issue.

Techniques based upon limit analysis theory have proven a powerful tool to evaluate the collapse loads of mechanical systems, such as foundations, beams, plates, and earth slopes, playing an important role in the design of different geostructures (Salençon [18], Salençon [19]). In this context, this paper aims to evaluate the uplift resistance of shallow embedded anchor plates based on the theoretical framework of limit analysis and its related kinematic approach, allowing for the formulation of a rigorous upper bound solution based on the adoption of appropriate failure mechanisms of the foundation system. The analysis relies on a plane strain condition, where the soil is modeled as a Tresca material with a tension cut-off criterion whereas the interface soil/anchor base is modeled by a tensile stress threshold which controls the breakaway conditions of the plate, simulating the suction/adhesion effect on the anchor's interface.

Initially, the yield design kinematic approach is briefly presented, followed by the investigation of an optimal mechanism based upon the minimization of a non-convex functional regarding the geometrical parameters of the considered class of failure mechanisms. Later, the predictions obtained are compared with available finite element results for the particular interface condition of the *immediate breakaway* case. Finally, an extensive parametric study is conducted to assess the effect of the relevant parameters upon the uplift bearing capacity, such as anchor width, embedment ratio, and soil parameters.

2 Framework of analysis

The evaluation of the uplift bearing capacity of anchor plates adopted in the present paper is based on the kinematic exterior approach of the limit analysis. This approach states that, if exists a kinematically admissible (KA) virtual velocity field $\underline{\hat{U}}$ in which the virtual rate of work of the external forces P_{ext} exceeds the maximum resisting rate of work P_{mr}, then this load is certainly unsafe, i.e., the system will collapse. The adoption of those virtual velocity fields presents a necessary condition for stability, which enables one to estimate an upper bound value for the ultimate load of the mechanical system (Salençon [19]). In this manner, the kinematic approach condition states that, for a load to admissible, it must obey:

$$\forall \underline{\hat{U}} \text{ KA}, \quad \underline{Q} \cdot \underline{q}\left(\underline{\hat{U}}\right) \leq P_{mr} \tag{1}$$

where $\underline{Q} = \{Q_1, Q_2, ..., Q_n\}$ denotes the set of loads acting of the mechanical system whose components represent the loading parameters, and the vector $\underline{q}\left(\underline{\hat{U}}\right) = \left\{q_1\left(\underline{\hat{U}}\right), q_2\left(\underline{\hat{U}}\right), ..., q_n\left(\underline{\hat{U}}\right)\right\}$ is the generalized virtual velocity of the system. The right-hand side of eq. (1) is evaluated through the adoption of "π *functions*" that represent the density of the maximum work that can be developed according to the material strength restrictions, characterizing P_{mr} along all the domain. In this context, the maximum resisting rate of work can be expressed as a sum of two components, one referring to the rate of work caused by the virtual strain rate $\underline{\underline{\hat{d}}}\left(\underline{x}\right)$ in every material point x belonging to the domain Ω, and another relative to the rate of work caused by the discontinuous velocity fields $\left[\!\left[\underline{\hat{U}}\right]\!\right] = \underline{\hat{U}}_2 - \underline{\hat{U}}_1$ acting on a surface of discontinuity Σ with normal $\underline{n}\left(\underline{x}\right)$. Hence:

$$\underline{Q} \cdot \underline{q}\left(\underline{\hat{U}}\right) \leq \int_{\Omega} \pi\left(\underline{x}, \underline{\underline{\hat{d}}}\right) d\Omega + \int_{\Sigma} \pi\left(\underline{n}\left(\underline{x}\right), \left[\!\left[\underline{\hat{U}}\right]\!\right]\right) d\Sigma \tag{2}$$

The method then consists to construct relevant kinetically admissible virtual velocity fields and evaluate the upper bound for the pullout force. Since they constitute an upper bound estimative, the minimization of this value constructing relevant failure mechanisms will lead to better solutions, allowing us to establish a convex domain of admissible loads from the outside, which contains the actual resistance domain inside. This approach has a very practical appeal for engineering purposes since those velocity fields can be inferred based on the observation of the failure modes (e.g. displacements, shear bands) in real phenomena.

Therefore, this study consists in evaluating the pullout force which produces the vertical displacement at collapase of the strip anchor embedded in the soil mass under plane strain conditions. The soil is considered as an isotropic homogeneous Tresca material, presenting unit weight γ and a limitation of tensile stress given by a tension cut-off criterion. For the Tresca material with tension cut-off, the π functions present in eq. (3) are written as (Salençon [19]):

$$\pi\left(\underline{x},\underline{\underline{\hat{d}}}\right) = C\left(\sqrt{2\left(|\hat{d}_1|^2+|\hat{d}_2|^2\right)} - \text{tr}\left(\underline{\underline{\hat{d}}}\right)\right) + T\text{tr}\left(\underline{\underline{\hat{d}}}\right) \qquad \text{if tr}\left(\underline{\underline{\hat{d}}}\right)\geq 0 \tag{3}$$

$$\pi\left(\underline{n}\left(\underline{x}\right),\left[\!\left[\underline{\hat{U}}\right]\!\right]\right) = C\left(\left[\!\left[\underline{\hat{U}}\right]\!\right] - \left[\!\left[\underline{\hat{U}}\right]\!\right]\cdot\underline{n}\right) + T\left[\!\left[\underline{\hat{U}}\right]\!\right]\cdot\underline{n} \qquad \text{if } \left[\!\left[\underline{\hat{U}}\right]\!\right]\cdot\underline{n}\geq 0 \tag{4}$$

where C is the material cohesion, T is the tension stress limit, and $\hat{d}_1$ and $\hat{d}_2$ are the rate of deformation eigeinvalues. It can be seen from eq. (3) and eq. (4) that the π functions presented are only valid for tr $\left(\underline{\underline{\hat{d}}}\right)\geq 0$ and $\left[\!\left[\underline{\hat{U}}\right]\!\right]\cdot\underline{n}\geq 0$ as for values outside this range the functions and consequently the pullout force will assume infinite values. Since we are only looking for relevant estimations, we must construct only pertinent velocity fields that lead to finite values of those functions. The interface soil/anchor base is considered perfectly smooth, modeled by a tensile stress threshold t to simulate the suction/adhesion effects acting on the interface for positive virtual velocities normal to the anchor's base $\hat{U}_n$ if the tangent velocity $\hat{U}_t$ is zero (no slipping allowed along soil/anchor interface). Hence, the interface π function is written as:

$$\pi\left(\underline{n},\hat{U}_n\right) = t\hat{U}_n \qquad \text{if } \hat{U}_t = 0 \text{ and } \hat{U}_n \geq 0 \tag{5}$$

2.1 Optimal mechanism development

The uplift mechanism considered is presented on Fig. 1. It is assumed that, in collapse, region 1 will only displace vertically with velocity $\underline{\hat{U}} = U\underline{e}_z$ (rigid body motion) while region 2 remains at rest. Both the vertical velocity and its associated extraction force F are considered to act in the center of a strip anchor of width B, while the discontinuous velocity field is assumed to occur along indeterminate surfaces delimited by the faillure lines AB, given by some unknown function $x = f\left(y\right)$, and $A'B'$ by simmetry, which should be obtained in order to minimize the pullout force. For the proposed mechanism, the virtual rate of external forces is written as:

$$P_{ext} = FU - \gamma UBH - 2\gamma U\int_0^H f\left(y\right)dy \tag{6}$$

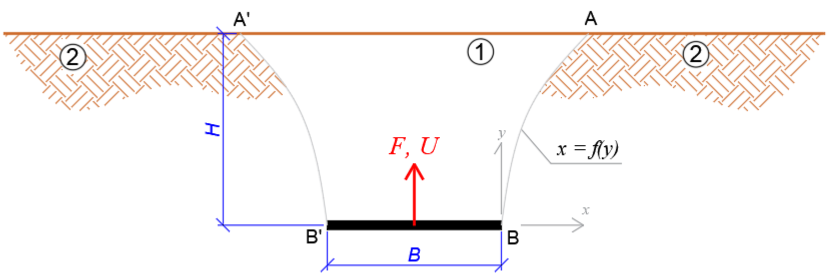

Figure 1. Anchor pullout problem under analysis: the discontinuity surface is given by the unknown function $x = f\left(y\right)$.

Following eq. (4) and eq. (5), the maximum resisting rate of work along the surface discontinuity and the interface discontinuity are given respectively by:

$$P_{mr}^{A'B' \cup AB} = 2 \int_{AB} \left[C \left(U - \frac{U f'(y)}{\sqrt{1 + f'(y)^2}} \right) + T \frac{U f'(y)}{\sqrt{1 + f'(y)^2}} \right] d\Sigma \tag{7}$$

$$P_{mr}^{B'B} = \int_{B'B} \min(t, T)\, U d\Sigma = UB \min(t, T) \tag{8}$$

As can be seen, the term relative to the anchor breakout in eq. (8) can assume two values depending on t and T magnitude, meaning that if $t < T$ the breakaway will occur in the interface, as if $T < t$ the discontinuity occurs in the soil immediately below the anchor base due to the lack of tension resistance in the soil mass. Since $d\Sigma^2 = dx^2 + dy^2$ and $dx/dy = f'(y)$, the upper bound theorem expressed in eq. (1) can be written for the proposed analysis as:

$$F \leq 2 \int_0^H \left[C\sqrt{1 + f'(y)^2} + (T - C) f'(y) + \gamma f(y) \right] dy + B\left(\min(t, T) + \gamma H\right) \tag{9}$$

where the function describing the discontinuity surface $x = f(y)$ must be found in order to minimize the functional form for F subjected to the pertinence condition $\left[\!\left[\underline{\hat{U}} \right]\!\right]_{12} \cdot \underline{n}_{12} \geq 0$ and therefore $f'(y) \geq 0$. Following classical procedures of the calculus of variations to find extremum values for functionals with constraints described by Petrov [20], it is seen that the stationary condition for $f(y)$ will lead to two solutions in the geometric domain $y \in [0, H]$. The first one is given by the constraint condition $f'(y) = 0$ and consenquently $f(y) = 0$, to cope with the boundary condition $f(y = 0) = 0$, occurring in one segment of the vertical axis $0 \leq y \leq H_0$ with $H_0 \in [0, H]$ being an intermediate point of the domain, providing a straight line discontinuity surface until the height H_0. On the other hand, the second solution is given by the stationary condition of the functional unconstrained, which will occur in the segment $H_0 \leq y \leq H$. Thus, applying the Euler-Lagrange equation to the functional presented in eq. (9), the following nonlinear second-order differential equation is obtained:

$$\frac{f''(y)}{\left(1 + f'(y)^2\right)^{3/2}} = \frac{\gamma}{C} \tag{10}$$

Solving the differential equation above and complying with the boundary conditions, it is seen that the prevalence of the first or second solutions mentioned above will depend on the magnitude of the material parameters and the height H. In order to maintain the pertinence conditions, the following conclusion can be stated:

1. If $C \geq \gamma H + T$ the optimal mechanism will be given only by the solution to eq. (10), i.e., $H_0 = 0$, hence:

$$\begin{aligned} F \leq \tfrac{C^2}{\gamma} \left[\arcsin\left(\tfrac{\sqrt{\alpha}(C-T) + \sqrt{\beta}(\gamma H - C + T)}{C^2} \right) \right] + \tfrac{\sqrt{\alpha}}{\gamma}(\gamma H - C + T) + \\ \tfrac{\sqrt{\beta}}{\gamma}(C - T) + B\left(\gamma H + \min(t, T)\right) \end{aligned} \tag{11}$$

   where $\alpha = (\gamma H + T)(2C - \gamma H - T)$ and $\beta = T(2C - T)$. The function describing the discontinuity surface will then be given by:

$$x = f(y) = \frac{1}{\gamma}\left(\sqrt{\alpha} - \sqrt{(\gamma H - \gamma y + T)(2C + \gamma y - T - \gamma H)} \right) \qquad y \in [0, H] \tag{12}$$

   for the faillure line AB as $A'B'$ is obtained by simmetry.

2. If $T \leq C \leq \gamma H + T$ the surface will be given by the two segments of the stationary conditions, with $H_0 \in [0, H] = (\gamma H - C + T)/\gamma$. The upper bound for the pullout force is then given by:

$$F \leq 2C\left(\frac{\gamma H - C + T}{\gamma}\right) + \frac{C^2}{\gamma}\arcsin\left(\frac{C - T}{C}\right) + \frac{\sqrt{T(2C - T)}}{\gamma}(C - T) + B\left(\gamma H + \min(t, T)\right) \tag{13}$$

   and the function describing the discontinuity:

$$x = f(y) = \begin{cases} 0 & \text{if } y \leq \frac{\gamma H - C + T}{\gamma} \\ \frac{1}{\gamma}\left(C - \sqrt{(\gamma H - \gamma y + T)(2C + \gamma y - T - \gamma H)}\right) & \text{if } y \geq \frac{\gamma H - C + T}{\gamma} \end{cases} \tag{14}$$

for the faillure line AB as $A'B'$ is obtained by simmetry.

3. If $C \leq T$ then the surface will be a constant straight line on all the domain once $H_0 = H$ and thus $x = f(y) = 0$. The pullout force is then estimated as:

$$F \leq 2CH + B\left(\gamma H + \min\left(t, T\right)\right) \tag{15}$$

Figure 2 presents a visualization of the solution obtained.

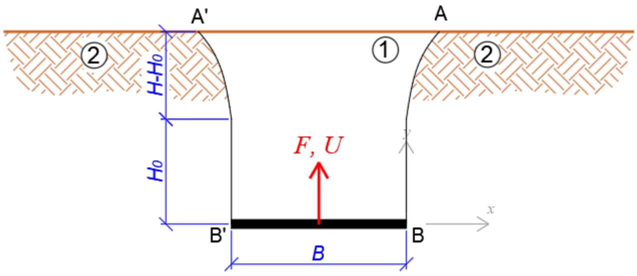

Figure 2. Optimal mechanism developed

2.2 Model Prediciton vs. available results

The predictions obtained for the anchor pullout capacity through the mechanism proposed are compared with available finite element analysis results presented by Merifield et al. [11] for the *immediate breakaway* case, where the interface soil/anchor is unable to sustain tension and thus $t = 0$, and the Tresca criterion is unlimited in tension $(T = \infty)$. This comparison can be seen in Fig. 3 where the vertical axis presents the pullout force F normalized by the anchor width B and the soil cohesion C as a function of the soil's unit weight γ, anchor depth H, and C.

As can be seen, for an embedment ratio H/B equals to 1, the results obtained are very similar to the ones presented by Merifield et al. [11], exceeding them in about 1%. As expected, when the embedment ratio increases this difference also increases due to changes in the considered failure mechanism, once the velocity field in the collapse will no longer be purely vertical, changing from global to a local failure mechanism. Even though it may be a large outcome for deeper anchors, the mudmats commonly adopted in offshore enginering presents embdement ratios up to 0.5, but more tipically 0.05-0.2 (Randolph et al. [1]).

3 Parametric Analysis

In order to evaluate how the different parameters involved in the present problem affects the anchor pullout capacity, a parametrical analysis concerning the normalized quantities $\gamma H/C$ and H/B was conducted to evaluate its effects on the normalized pullout force $F/\left(BC\right)$.

Figure 4(a) presents the parametrical analysis concerning the *immediate breakaway* $(t = 0)$ case for the cases where the soil tension stress limit is null $(T = 0)$ or equal to the cohesion $(T = C)$ at three different embedment ratios. As can be seen, the normalized pullout strength increases with the ratio $\gamma H/C$ due to increases in anchor depth, presenting a linear evolution for the case where $T = C$ as the pullout force is given by the straight-line mechanism presented by eq. (15) and non-linear increasing for the $T = 0$ case, a condition given by the discontinuities predicted by eq. (11) and eq. (13), with $0 \leq H_0 < H$. It is also interesting to note that, as $\gamma H/C$ increases, both cases tend to same values, once that H_0 will tend to H and consequently the mechanism will be majorly depicted by straight lines.

The results for the case where the interface resistance is equal to the cohesion $(t = C)$ are presented by Fig. 4(b). Once again, the same trend discussed for the immediate breakaway case is observed, however, there is a gap between the trends as $\gamma H/C$ increases as a result of the difference in T magnitude. When $T = 0$, the breakaway occurs in the foundation immediately below the anchor base due to the lack of soil tension resistance,

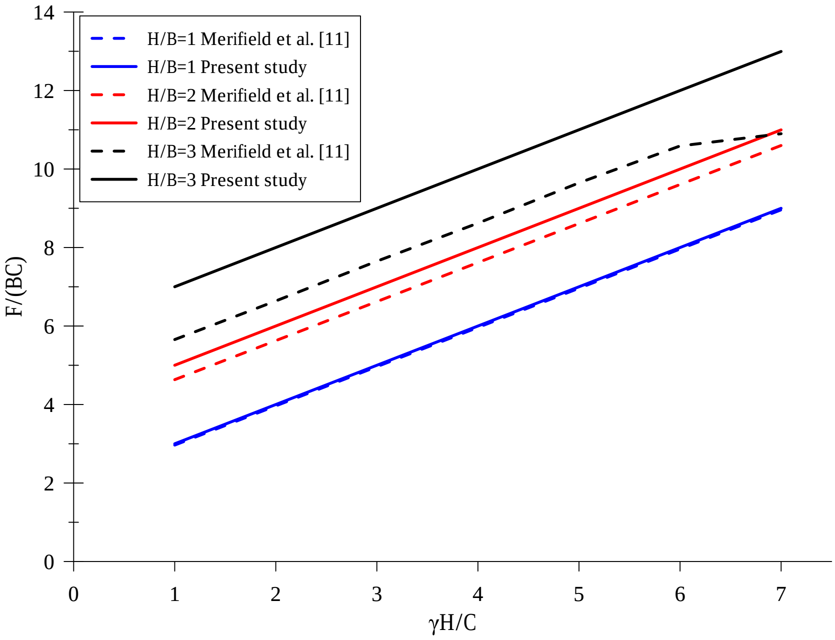

Figure 3. Comparison between the results obtained in the present study vs. available finite element results for the *immediate breakaway* case.

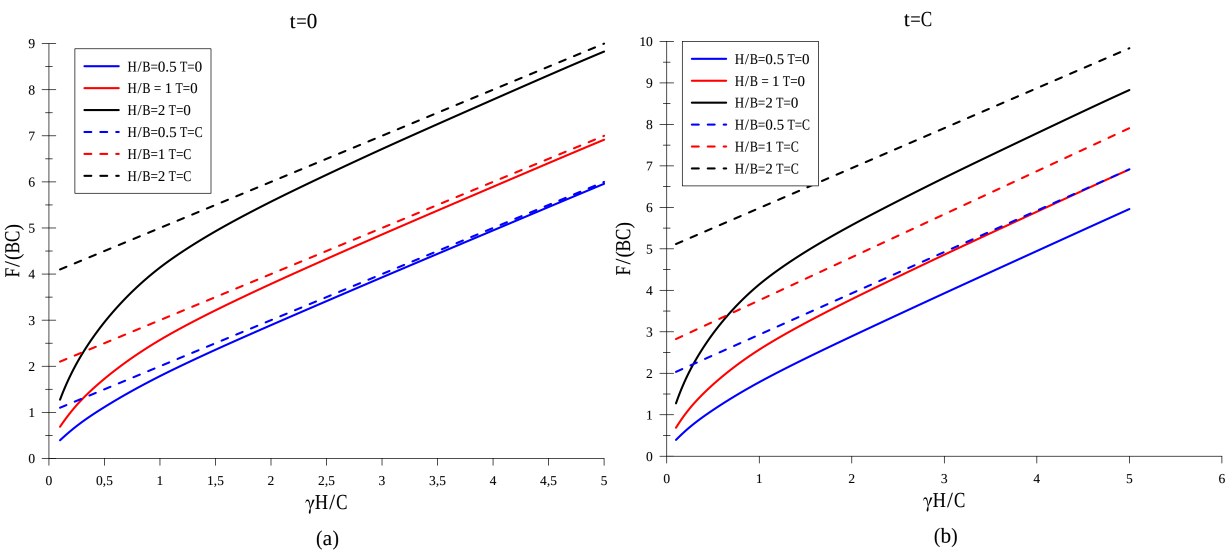

Figure 4. Parametric analysis results for (a) immediate breakaway case and (b) interface resistance magnitude equal to the material cohesion.

even though the interface can sustain some tension. On the other hand, when $t = T = C$, the discontinuity may occur indefinitely on the soil or in the interface, presenting a resistance equal to the cohesion nevertheless.

4 Conclusions

The present paper describes the application of the limit analysis kinematic approach theorem in the estimation of the pullout resistance of anchor plates, providing upper bound reference values of the pullout bearing capacity of strip anchors under plane strain conditions considering its interface resistance. The solution was obtained through the minimization of a non-convex functional with respect to the parameters defining the geometry of considered class of failure mechanisms.

The solution was compared with available finite element analysis results available in the literature for the immediate breakaway case, where can be seen that the present study shows good agreement with numerical results for values of embedment ratios $H/B = 1$, increasing the discrepancy with H/B increases. These results seem satisfying once mudmats adopted in practice usually does not exceed embedment ratios greater than 0.5.

A parametrical analysis was conducted varying the main parameters of the mechanical problem, demonstrating that the pullout normalized strength is mainly affected by the soil resistance, embedment ratio, and interface resistance, changing the governing failure mechanism according to those parameters' magnitude. It is concluded that the upper bound estimative presented shows itself a good tool to estimate the pullout capacity of shallow embedded strip anchors, as for deeper anchors a more suitable mechanism should be investigated.

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