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# Victory by the Weakest: Effects of Negative Advertising in N>2 Candidate Campaigns

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# *Victory by the Weakest: Effects of Negative Advertising in N>2 Candidate Campaigns*

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**ABSTRACT** The truel, or three way duel, has distinct properties from duels: the weakest contestant often has a very good chance to win. This paper explores application of the logic of truels to election campaigns involving negative advertising. We show that negative campaigning that pits the leading candidates against each other can create circumstances in which the third (or worse) place candidate wins in one or more of the Nash equilibria of the game. We then study whether the simulated existence of an opportunity for Nash equilibrium victory by third place candidates predicts such outcomes in U.S. state-wide elections.

*Keywords: truel, campaign, advertising, game theory*

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## INTRODUCTION

Cree Deeds was trailing in the 2009 Virginia Democratic gubernatorial primary. In third place in the polls. With a smaller budget than his rivals. But then one of his rivals initiated a negative advertising campaign targeting the other. Deeds partly joined in the attack, while also making his positive case to the public. Ultimately, each rival seemingly persuaded the public that the other should not be chosen, and Deeds won. Like Deeds, sometimes candidates polling in third place (or even less than third place) stage surprising election victories. This study explains why, as a matter of logic, some of these victories should be expected. We synthesize insights from the game theoretic study of truels and the political science study of negative campaigning to develop an original model which explains why and when third place candidates sometimes stage surprising come-from-behind victories.

This study applies to three-candidate elections with negative and positive campaigning the logic of three-way duels (called truels) which have principally been studied in game theory and mathematics. Our key theoretical results show that negative advertising campaigns with more than two candidates share a key property with other truels -- the potential for the weakest of the contestants to have a good chance of victory. Writing principally in mathematics and game theory journals, Shubik (1954), Kilgore (1971/72), and Kilgour and Brams (1997) developed analyses showing that the poorest shot sometimes has the best chance to win a three-way gun fight. Bringing that insight into political science, this study shows that electoral campaigns with negative advertising share similar dynamics with gun fight truels even though the campaign context is significantly distinct from the gun battles analyzed in these earlier models – positive advertising for instance might undermine the chances of the weakest to win. For a range of candidate capabilities, the theoretical result of victory by the weakest from the truels literature in game theory extends to the electoral campaign context in politics.

No previous model of negative advertising in multi-candidate electoral competition has developed the logic of victory by the weakest from the truels literature to identify conditions for victory by the third place candidate. In the only previous model of three-candidate negative advertising contests, Skaperdas and Grofman (1995) concluded that in a 2 candidate race the frontrunner would engage in more positive advertising than negative, whereas in a 3 candidate race the third place candidate will only engage in positive advertising, and in a 3 candidate race no candidate will engage in negative campaigning against the weaker of his opponents. Our model upends all three conclusions. Unlike the model of Gandhi et. al. (2016) which focuses on the frequency of negative advertising as a function of the number of candidates and the externalities generated by negative advertising, our analysis focusses on the conditions in which various advertising strategies are in equilibrium in three-candidate races, including equilibria where one or more candidates engage in negative advertising.

For some political scientists and social choice theorists, a model in which an initially weak candidate wins may seem familiar to the point of unoriginality. Yet the synthesis offered here of the truels and negative campaigning literatures is unique. Obviously, the fundamental instability results for models of n>2 candidate spatial competition imply opportunities for weaker candidates to win. And ours is not the first model in which such victories are anticipated. Elections are well known to be uncertain affairs, polling can be flawed, coordination or band-wagoning can create a late surge, and multi-candidate multi-dimensional races are inherently unstable or subject to cycling. Yet while useful, all of these answers suggest that third (or worse) place is generally a disadvantageous place to be, but the weak candidate might get lucky. Unlike previous explanations for the puzzle of victory by third or worse place candidates, we do not attribute their victory to accident, luck, not really being in third place, or (directly at least) to the inherent instability of multi-candidate choice.

We show that victory by the weakest in a negative campaign truel occurs in a Nash equilibrium -- in a context in which in an honest and full-information retrospection, the managers of the losing yet stronger campaigns would have nothing specifically to regret. In the same circumstance and with full information, each losing candidate could not have won through any unilateral alteration of strategy. Indeed, they would have incentives to repeat those strategies if the campaign was to develop in the same way again, even though those strategies produced victory by a third place or weaker candidate with weaker initial public support and less money to spend on advertising.

Our analysis has implications for scholarship, showing that despite the incentives (Gandhi et. al., 2016) that discourage negative advertising in multi-candidate elections, negative advertising can play a prominent role in the equilibria of these elections. It also has implications for campaign strategists and practitioners, highlighting the opportunities that negative advertising-focused contests among the leading candidates can create for victory by a candidate with less initial support and resources.

We proceed as follows. Sections 1 and 2 develop our key theoretical results, first illustrating victory by the weakest in the gunfight n>2 duel from the game theory literature in mathematics, and then extending that result to the more complex context of campaigns with positive and negative advertising. Section 3 then applies those results to illustrative cases and an analysis of state-wide U.S. elections. Section 4 concludes.

### 1.1 A Gunfight Truel

Before proceeding into the analysis of campaigns, we begin by offering an example of a three way gun fight truel to sharpen intuitions concerning the way the outcomes of a three way duel studied in the mathematics literature often involve victory by the weakest (Amengual and Toral 2006, Kilgour (1971/72; 1975, 1977), Kilgour and Brams 1997). A fundamental intuition behind the result is that the two most accurate marksmen in a three-way gun fight each pose a greater threat to each other than does the weakest. Consequently, each has an incentive to initially expend resources attacking the other, which increases the odds that the weakest survives. Kilgour (1971/72) summed up that “in many cases, the truel has a unique equilibrium point at which the player who is the poorest marksman has the greatest chance of survival.”

Imagine that three cowboy gunslingers have mutually offended each other and are determined that the only way to settle their disagreements is with a three-way duel – a truel. This truel will have two rounds, and each combatant has one bullet to use in each of the two rounds. In each round each shooter decides whom to target and all three shoot simultaneously. In the second round, the combatants only know who is still alive. It is common knowledge that the three gunslingers vary in the probability with which they will hit a target they aim at, as illustrated in Table 1. In light of the depth their grievances, each would rather be the only one alive at the end of the truel of the law by citizens.

The solution to this game is by Subgame Perfect Nash equilibrium (Osborne 1995). To solve by backward induction we begin with the last round. If all three players survive, then any targeting strategy is an equilibrium. If only two survive, then they target each other. Because each would rather be targeted by a worse shot in round two, each cowboy targets their opponent who is the best shot in the first round. In equilibrium one targets two, two targets one, and three targets one. As shown in the final row of Table 1, the result of these targeting decisions is that there is a very high probability that cowboy three (the worst shot of the three) will be the only one to survive into the second round of the truel. And the cowboy who is the best shot has the worst probability of surviving to the second round. In this instance, as in many other n>2 duels, the outcome is often not victory by the strong or the skilled, but victory by the weakest.

**2.1 A Model of Election Advertising**

This section proves that a third place candidate can win in equilibrium as the result of a negative advertising truel, and that the optimal strategy of the third place candidate may involve either positive, or negative advertising depending upon the context and the relative effectiveness of positive versus negative advertising. We then generalize this result to show that it persists even if candidates can deviate to any mix of positive and negative advertising strategies.

We study a model of campaign competition which is deliberately simplified to include merely candidate support levels (the candidate with the most support will win the election) on the one hand, and candidate advertising budgets (which candidates spend to influence support through positive and negative ads) on the other hand. The underlying psychological mechanisms driving candidate support might include voter perceptions of candidate ideological positions (Jessee 2012, Endersby and Thomason 1994), it might be candidate valence or likes / dislikes (Clarke, et al., 2011, Stokes 1963, 1992), it might be party loyalty (Campbell et. al. 1960), retrospective evaluations of party or presidential performance (Fiorina 1981), or something else.

The assumption that support (S) can be shifted through advertising is a basic precondition for any model of advertising strategy, and seems borne out by the enormous sums candidates often spend on advertising during election campaigns. We assume that positive advertising (P) boosts the support of the candidate using it by an amount proportionate to the budget B allocated to it such that the change in support equals BP or alternately P(B). Conversely negative advertising (N) decreases the support of the targeted candidate by -BN or alternately N(B) (Geer, 2006). The specific mechanisms of advertising influence might vary depending upon which mechanisms (discussed above) shaping candidate support are targeted.

Candidates are indexed c = 1, … n, where 1 is the initially strongest candidate in support and budget, and n is the weakest candidate. The specific context in which the campaign is fought is characterized by two parameters that set the relative strength of each candidate. Each candidate has a starting support and a starting budget. B1, B2 and B3 indicate the starting budget for each candidate with B1 > B2 > B3. S1, S2 and S3 indicate the starting level of support for each candidate with S1> S2>S3. As a result of each candidate's strategy decisions, the budget and support levels change during the game, and the player with the highest ending support wins the election. Candidate payoffs are assumed to be based upon whether they win or not with U(win) > U(loss).

Each candidate has 4 options. The first option is positive advertising: P indicates that a candidate has chosen a positive advertising campaign and B1P is the impact on support for candidate 1 from spending budget B on positive advertising. The next two options are negative advertising against each one of the opponents: N2 indicates an attack on candidate 2 and N3 indicates an attack on candidate 3. Thus, B1N2 is the change in candidate 2's support brought about by candidate 1 spending budget B1 on negative advertising that attacks candidate 2. Finally, we allow candidates to decide not to spend (option O for out): B1O indicates that candidate 1 has made the choice to not spend which leaves all candidate support levels unchanged. Since candidate support is always higher under positive advertising, this option is weakly dominated by positive advertising (P) so it is of little importance in most of our analyses. We also assume that negative advertising potentially carries a cost "L" for the support of the advertiser as studies have found evidence that some types of negative advertising can reduce public approval of the candidate doing the attack (Brooks & Geer, 2007). We generalize the model below to allow for mixing between options.

Our model does not assume a specific order of play and the interaction modeled occurs within a single round in which all candidates move simultaneously. It is thus solved using Nash Equilibrium. Since each of the three candidates has four available strategies, there are 64 possible strategy combinations, each associated with a distinct set of final candidate support values. The support levels resulting from a selection of the strategy combinations are described in Table 2. Each cell contains the support for candidate 1, candidate 2, and then candidate 3. For instance, in the top left cell of the table we see the support of each candidate resulting from a choice by candidate 1 to attack candidate 2, and choices by candidate 2 and candidate 3 to attack candidate 1. Here the utility of candidate one reflects his or her starting support (S1) combined with the reduced support imposed by the attack of candidate 2 (-B2N1), the reduced support imposed by the attack of candidate 3 (-B3N1) and the loss to support resulting from running a negative campaign against candidate 2 (-L).

To find equilibria, we identify best responses by each candidate to actions by the other candidates, based on the assumption that each candidate cares only about victory – about having a higher support than any other candidate. On the basis of these best responses, Nash equilibria in pure strategies can be identified in which no campaign had an incentive to change strategy. There are several pure strategy combinations that can produce victory for the weakest candidate in equilibrium including (N2, N1, N1), (N2, N1, P), and (P, N1, P). Each strategy profile can be a Nash equilibria of the game for some parameter values.

Claim 1. If all six inequalities (1 through 6) listed below are satisfied there a Nash equilibrium (N2, N1, P) in which the third place candidate wins the election by adopting a positive advertising strategy.

| **Conditions for Victory by Third Place Candidate Running Positive Campaign While Opponents Attack Each Other:** | | |
|---|---|---|
| $S_3 + B_3P >$ | $S_1 - B_2N_1 - L$ | (1) |
| $S_3 + B_3P >$ | $S_2 - B_1N_2 - L$ | (2) |
| $S_3 + B_3P >$ | $S_1 - B_2N_1 + B_1P$ | (3) |
| $S_3 + B_3P >$ | $S_2 - B_1N_2 + B_2P$ | (4) |
| $S_2 >$ | $S_1 - B_2N_1$ | (5) |
| $S_1 >$ | $S_2 - B_1N_2$ | (6) |

Proof: if first two inequalities (equations 1 and 2) are satisfied, then C3 will win the election if no player deviates from the strategy profile. If the second two inequalities are satisfied (equations 3 and 4), then neither of the other players can benefit from deviating from this strategy profile by running a positive campaign because each will still suffer a loss. If the last two inequalities are satisfied (equations 5 and 6) then neither of the other candidates (1 and 2) can benefit from deviating from this strategy profile by attacking candidate 3 instead because this will lead to the other of these two candidates winning. For instance, if candidate 1 attacks C3 instead of C2, then C2 wins. Equations 5 and 6 constitute a closeness condition: the first and second place candidates must be close enough to each other than each cannot win if the other is permitted to run unanswered negative attacks.

Since inequalities 1, 2 and 4 will be satisfied if 3 is satisfied (following from the relatively weaker support and budget of the second place candidate), we solve for the critical starting support and budget levels of the third place candidate using inequality 3. The critical support level for the third place candidate to be able to win in equilibrium with a positive campaign if the closeness condition is satisfied is:

S3 > S1 – B2N1 + B1P - B3P

And the critical budget level for victory by the third place candidate is:

B3 > (S1 – B2N1 + B1P - S3)/P

Claim 2. An alternate set of closeness conditions obtain when the first place candidate adopts a positive advertising strategy in the Nash equilibrium (P, N1, P) with candidates 1 and 3 running positive campaigns, and candidate 2 running a negative campaign. Equations 1 through 4 must still be satisfied. The alternative conditions to Equations 5 and 6 are 8 and 9 below:

| **Conditions for Victory by Third Place Candidate Running Positive Campaign While Opponents Attack Each Other:** | | |
|---|---|---|
| $S_3 + B_3P >$ | $S_1 - B_2N_1 - L$ | (1) |
| $S_3 + B_3P >$ | $S_2 - B_1N_2 - L$ | (2) |
| $S_3 + B_3P >$ | $S_1 - B_2N_1 + B_1P$ | (3) |
| $S_3 + B_3P >$ | $S_2 - B_1N_2 + B_2P$ | (4) |
| $S_2 >$ | $S_1 - B_2N_1$ | (5) |
| $S_1 >$ | $S_2 - B_1N_2$ | (6) |

If the conditions in equations 3 and 7 are satisfied, then candidate 3 will win. If the condition in equation 8 is satisfied, then player 1 cannot win by shifting to an attack on player 3 (and obviously shifting to an attack on player 2 will not lead to victory if equation 3 holds). If the condition in equation 9 is satisfied, then player 2 cannot win by shifting to a positive campaign (and obviously shifting to an attack on player 3 will not lead to victory if equation 9 is true).

Claim 3. For the third place candidate to win in equilibrium through a negative advertising strategy of attacking the first place candidate (N2, N1, N1), the following conditions must hold:

| Conditions for Victory by Third Place Candidate Running Negative Campaign: | | |
|---|---|---|
| $S_3 >$ | $S_1 - B_2N_1 - B_3N_1$ | (10) |
| $S_3 >$ | $S_2 - B_1N_2$ | (11) |
| $S_3 >$ | $S_1 - B_2N_1 - B_3N_1 + B_1P + L$ | (12) |
| $S_3 >$ | $S_2 - B_1N_2 + B_2P + L$ | (13) |
| $S_2 >$ | $S_1 - B_2N_1 - B_3N_1$ | (14) |
| $S_1 >$ | $S_2 - B_1N_2 + B_3N_1$ | (15) |

Proof: if the conditions outlined in equations 10 and 11 prevail, then candidate 3 will win because the third candidate will have the highest level of support. If the conditions outlined in equations 12 and 13 prevail then neither of the other candidates can win by deviating from the posited equilibrium strategy individually to run a positive campaign, and if the closeness conditions in 14 and 15 obtain, then neither candidate can deviate to an attack on candidate 3 without losing to the other candidate: if candidate 1 attacks 3, then candidate 2 wins. Condition 15 always obtains by assumption because S1>S2 and B1N2 > B3N1.

Note that a critical condition for this equilibrium is that attack by the first-place candidate on the second place candidate must be sufficiently powerful that the second place candidate has a lower support than the third place candidate (equation 11). Thus, the second-place candidate must be weaker than was required in claim 1, so there are conditions in which a positive campaign by the third-place candidate would bring victory but a negative campaign would not. On the other hand, when B3P < B3N1 – L, the condition in equation 12 will be met more readily than the condition in equation 3, indicating the existence of circumstances in which the unique winning strategy in equilibrium is for the third place candidate to attack if negative advertising has a sufficiently larger impact than positive advertising.

Figure 1 illustrates that when positive and negative campaigning have equal effectiveness, the weakest candidate is generally best off running a positive campaign. For the selected parameter values, a positive campaign by the weakest candidate can result in victory in every circumstance in which a negative campaign would also lead to victory, but there are also a range of initial budget and support levels for the weakest candidate for which only a positive campaign can generate a Nash equilibrium in which the weakest wins. The area within the triangle bounded by dotted lines indicates the range of parameter values for which the third-place candidate is weakest yet can win in equilibrium by running a positive campaign. The quadrilateral bounded by dashed lines indicates the range of parameter values for which the third-place candidate is weakest in respect to both support and budget, yet can win in equilibrium by running a negative campaign against the first place candidate.

### 2.2 Extension to Mixed Advertising Campaigns

So far, we have maintained the simplifying assumption that candidates must devote the entirety of their resources to a single advertising strategy. The purpose of this section is to show that equilibria involving victory by the third-place candidate (3) still occur when candidates can devote their budget to a mix of advertising strategies.

We now allow candidates to select any combination of strategies. Let b1P + b1N2 + b1N3+ b1O=B1 represent the portions of candidate 1's budget being devoted to each of the available strategies. Thus, if b1P = b1N2 = B1/2, candidate 1 is devoting half of his or her budget to positive campaigning, and half to negative attacks on candidate two. For simplicity we drop the assumption (L) that negative campaigning hurts the candidate engaging in it below.

Claim 4. For candidate three to win in equilibrium under mixed advertising strategies the following conditions must hold:
Proof: If equation 16 is true, then the support of candidate 3 exceeds that of candidate 1, and if equation 17 is true, then the support of candidate 3 exceeds that of candidate 2. Hence, candidate 3 will have the highest support, and will win.

For this to be an equilibrium, all candidates must be choosing budget shares that best respond to all other candidates with the above inequalities satisfied. Therefore, it must be the case that neither candidate 1 nor

**Table 1. A Gun-Fight Truel**

| | Probability (hits target) | Equilibrium probability of survival into second round |
|---|---|---|
| Cowboy 1 | 0.9 | 6 percent |
| Cowboy 2 | 0.8 | 10 percent |
| Cowboy 3 | 0.7 | 100 percent |

**Table 2. Candidate Support as a Result of Strategic Choices**

| | C3=$N_1$ | | | |
|---|---|---|---|---|
| | C2 = $N_1$ | C2 = $N_3$ | C2 = P | C2 = O |
| C1 = $N_2$ | $S_1 - B_2N_1 - B_3N_1 - L, S_2 - B_1N_2 - L, S_3 - L$ | $S_1 - B_3N_1 - L, S_2 - B_1N_2 - L, S_3 - B_2N_3 - L$ | $S_1 - B_3N_1 - L, S_2 + B_2P - B_1N_2, S_3 - L$ | $S_1 - B_3N_1 - L, S_2 - B_1N_2, S_3 - L$ |
| C1 = $N_3$ | $S_1 - B_2N_1 - B_3N_1 - L, S_2 - L, S_3 - B_1N_3 - L$ | $S_1 - B_3N_1 - L, S_2 - L, S_3 - B_1N_3 - B_2N_3 - L$ | $S_1 - B_3N_1 - L, S_2 + B_2P, S_3 - B_1N_3 - L$ | $S_1 - B_3N_1 - L, S_2, S_3 - B_1N_3 - L$ |
| C1 = P | $S_1 + B_1P - B_2N_1 - B_3N_1, S_2 - L, S_3 - L$ | $S_1 + B_1P - B_3N_1, S_2 - L, S_3 - B_2N_3 - L$ | $S_1 + B_1P - B_3N_1, S_2 + B_2P, S_3 - L$ | $S_1 + B_1P - B_3N_1, S_2, S_3 - L$ |
| C1 = O | $S_1 - B_2N_1 - B_3N_1, S_2 - L, S_3 - L$ | $S_1 - B_3N_1, S_2 - L, S_3 - B_2N_3 - L$ | $S_1 - B_3N_1, S_2 + B_2P, S_3 - L$ | $S_1 - B_3N_1, S_2, S_3 - L$ |
| | C3=$N_2$ | | | |
| C1 = $N_2$ | $S_1 - B_2N_1 - L, S_2 - B_1N_2 - B_3N_2 - L, S_3 - L$ | $S_1 - L, S_2 - B_1N_2 - B_3N_2 - L, S_3 - B_2N_3 - L$ | $S_1 - L, S_2 + B_2P - B_1N_2 - B_3N_2, S_3 - L$ | $S_1 - L, S_2 - B_1N_2 - B_3N_2, S_3 - L$ |
| C1 = $N_3$ | $S_1 - B_2N_1 - L, S_2 - B_3N_2 - L, S_3 - B_1N_3 - L$ | $S_1 - L, S_2 - B_3N_2 - L, S_3 - B_1N_3 - B_2N_3 - L$ | $S_1 - L, S_2 + B_2P - B_3N_2, S_3 - B_1N_3 - L$ | $S_1 - L, S_2 - B_3N_2, S_3 - B_1N_3 - L$ |
| C1 = P | $S_1 + B_1P - B_2N_1, S_2 - B_3N_2 - L, S_3 - L$ | $S_1 + B_1P, S_2 - B_3N_2 - L, S_3 - B_2N_3 - L$ | $S_1 + B_1P, S_2 + B_2P - B_3N_2, S_3 - L$ | $S_1 + B_1P, S_2 - B_3N_2, S_3 - L$ |
| C1 = O | $S_1 - B_2N_1, S_2 - B_3N_2 - L, S_3 - L$ | $S_1, S_2 - B_3N_2 - L, S_3 - B_2N_3 - L$ | $S_1, S_2 + B_2P - B_3N_2, S_3 - L$ | $S_1, S_2 - B_3N_2, S_3 - L$ |
| | C3 = P | | | |
| C1 = $N_2$ | $S_1 - B_2N_1 - L, S_2 - B_1N_2 - L, S_3 + B_3P$ | $S_1 - L, S_2 - B_1N_2 - L, S_3 - B_2N_3 + B_3P$ | $S_1 - L, S_2 + B_2P - B_1N_2, S_3 + B_3P$ | $S_1 - L, S_2 - B_1N_2, S_3 + B_3P$ |
| C1 = $N_3$ | $S_1 - B_2N_1 - L, S_2 - L, S_3 - B_1N_3 + B_3P$ | $S_1 - L, S_2 - L, S_3 - B_1N_3 - B_2N_3 + B_3P$ | $S_1 - L, S_2 + B_2P, S_3 - B_1N_3 + B_3P$ | $S_1 - L, S_2, S_3 - B_1N_3 + B_3P$ |
| C1 = P | $S_1 + B_1P - B_2N_1, S_2 - L, S_3 + B_3P$ | $S_1 + B_1P, S_2 - L, S_3 - B_2N_3 + B_3P$ | $S_1 + B_1P, S_2 + B_2P, S_3 + B_3P$ | $S_1 + B_1P, S_2, S_3 + B_3P$ |
| C1 = O | $S_1 - B_2N_1, S_2 - L, S_3 + B_3P$ | $S_1, S_2 - L, S_3 - B_2N_3 + B_3P$ | $S_1, S_2 + B_2P, S_3 + B_3P$ | $S_1, S_2, S_3 + B_3P$ |
| | C3 = O | | | |
| C1 = $N_2$ | $S_1 - B_2N_1 - L, S_2 - B_1N_2 - L, S_3$ | $S_1 - L, S_2 - B_1N_2 - L, S_3 - B_2N_3$ | $S_1 - L, S_2 + B_2P - B_1N_2, S_3$ | $S_1 - L, S_2 - B_1N_2, S_3$ |
| C1 = $N_3$ | $S_1 - B_2N_1 - L, S_2 - L, S_3 - B_1N_3$ | $S_1 - L, S_2 - L, S_3 - B_1N_3 - B_2N_3$ | $S_1 - L, S_2 + B_2P, S_3 - B_1N_3$ | $S_1 - L, S_2, S_3 - B_1N_3$ |
| C1 = P | $S_1 + B_1P - B_2N_1, S_2 - L, S_3$ | $S_1 + B_1P, S_2 - L, S_3 - B_2N_3$ | $S_1 + B_1P, S_2 + B_2P, S_3$ | $S_1 + B_1P, S_2, S_3$ |
| C1 = O | $S_1 - B_2N_1, S_2 - L, S_3$ | $S_1, S_2 - L, S_3 - B_2N_3$ | $S_1, S_2 + B_2P, S_3$ | $S_1, S_2, S_3$ |

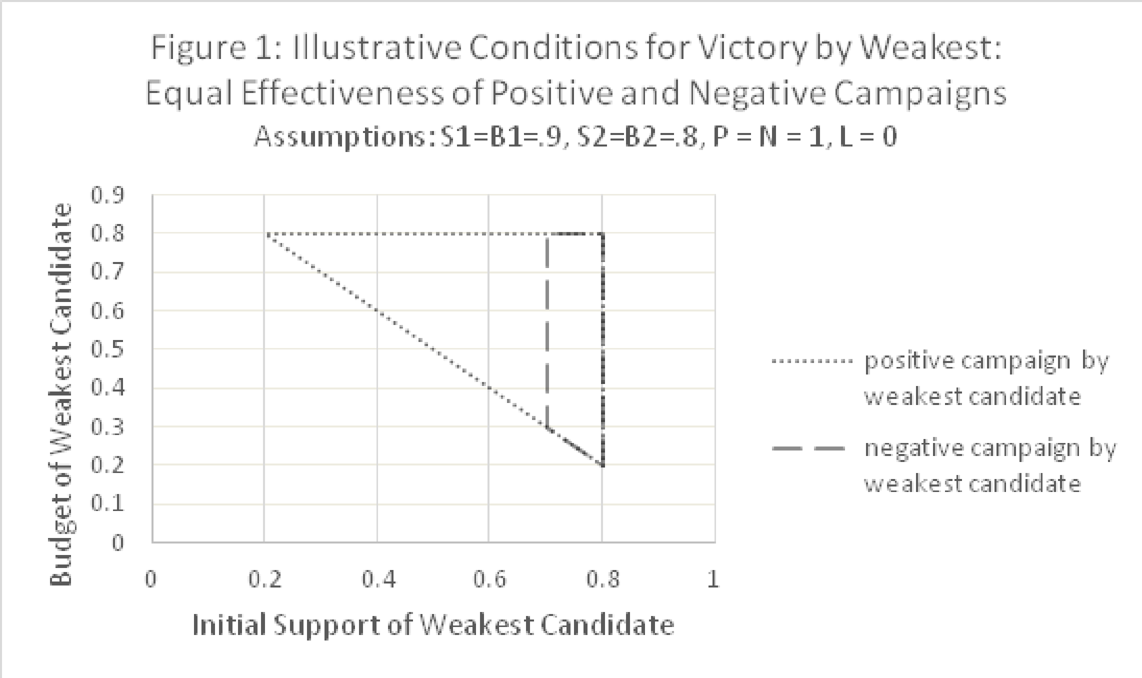
Figure 1: Illustrative Conditions for Victory by Weakest:
Equal Effectiveness of Positive and Negative Campaigns
Assumptions: S1=B1=.9, S2=B2=.8, P = N = 1, L = 0
Budget of Weakest Candidate
Initial Support of Weakest Candidate
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
0 0.2 0.4 0.6 0.8 1
positive campaign by weakest candidate
negative campaign by weakest candidate

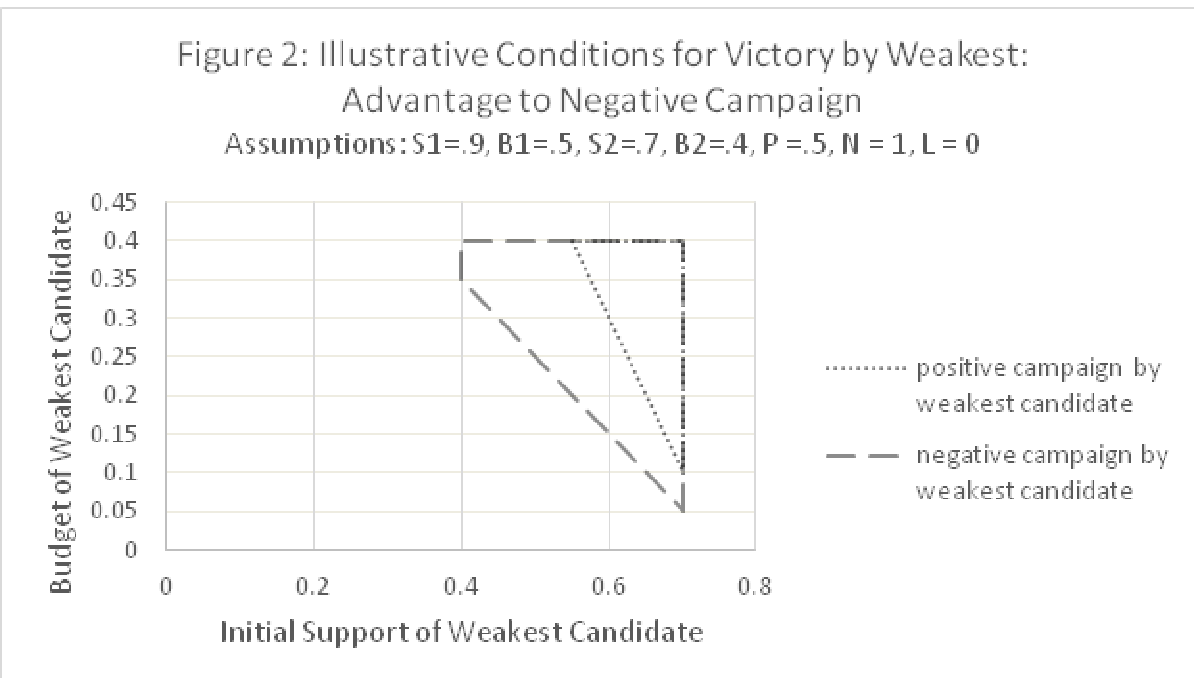
Figure 2: Illustrative Conditions for Victory by Weakest:
Advantage to Negative Campaign
Assumptions: S1=.9, B1=.5, S2=.7, B2=.4, P =.5, N = 1, L = 0
Budget of Weakest Candidate
Initial Support of Weakest Candidate
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0 0.2 0.4 0.6 0.8
positive campaign by weakest candidate
negative campaign by weakest candidate

candidate 2 has an individual incentive to deviate from their strategy. In other words, holding the other candidate's strategies constant, there must be no deviation that allows a losing candidate to gain a larger level of support than the strongest of the other candidates. In the absence of such a strategy, each candidate will be indifferent between their alternatives, which is the necessary condition for maintaining the mixed strategy. If equation 18 holds, then candidate 1 has no feasible combination of positive and negative advertising that allows the candidate to win (though as above in pure strategies, the candidate might be able to choose which of his or her opponents will win). Similarly, if equation 19 holds, then candidate 2 has no feasible combination of positive and negative advertising strategies that allows the candidate to win.

This analysis shows that there continue to be conditions in which a negative advertising truel produces victory for the weakest candidate, even if the other candidates can play a mix of strategies. Figure 3 illustrates the range of budget and support values under which a Nash equilibrium exists that leads to victory by the weakest candidate by adding lines delineating the boundaries of the area in which an equilibrium exists in which the winning candidate is initially the weakest as a function of the initial budget and support of the candidates. To facilitate comparison, the parameters for the effectiveness of positive versus negative campaigning, and the support and budget values of the leading candidates are kept the same as in Figure 2. In addition, we assume that the third place candidate only plays one strategy – either b3P = B3 or b3N1 = B3. For this set of parameters, the region in which positive campaigning by the weakest candidate can produce victory shrinks, but the area in which negative campaigning by the weakest candidate can produce victory increases once all candidates are permitted to select mixed strategies that potentially involve both positive advertising and attacks one or both opponents.

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