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POSITIVE EQUILIBRIUM SOLUTIONS TO GENERAL POPULATION MODEL

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Abstract: In this paper, we investigate conditions for the existence of positive solution to the following general elliptic system with various growth conditions:

$$\begin{cases} \Delta u + u(a + g(u, v)) = 0 \\ \Delta v + v(d + h(u, v)) = 0 \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0. \end{cases} \quad \text{in } \Omega,$$

Our arguments mainly rely on super-sub solutions, maximum principles, spectrum estimates, and some detailed properties for the solution of logistic equations.

AMS Subject Classification: 35A05, 35A07

Key Words: elliptic system with various growth conditions, existence of positive solution, super-sub solutions, maximum principles, spectrum estimates, logistic equations

1. Introduction

One of the prominent subjects of study and analysis in mathematical biology concerns the competition, predator-prey or cooperation of two or more species of animals in the same environment. Especially pertinent areas of investigation include the conditions under which the species can coexist, as well as the con-

ditions under which any one of the species becomes extinct, that is, one of the species is excluded by the others (see [1], [2], [3], [4], [6], [7], [11], [12], [13]). In this paper, we focus on the general population model to better understand the interactions between two species.

2. Preliminaries

Before entering into our primary arguments and results, we must first present a few preliminary items that we later employ throughout the proofs detailed in this paper. The following definition and lemmas are established and accepted throughout the literature on our topic.

Definition 2.1. (*Super and sub solutions*) Consider

$$\begin{cases} \Delta u + f(x, u) = 0 & \text{in } \Omega, \\ u|_{\partial\Omega} = 0, \end{cases} \quad (1)$$

where $f \in C^\alpha(\bar{\Omega} \times \mathbb{R})$ and Ω is a bounded domain in \mathbb{R}^n .

(A) A function $\bar{u} \in C^{2,\alpha}(\bar{\Omega})$ satisfying

$$\begin{cases} \Delta \bar{u} + f(x, \bar{u}) \leq 0 & \text{in } \Omega, \\ \bar{u}|_{\partial\Omega} \geq 0 \end{cases}$$

is called a super solution to (1).

(B) A function $\underline{u} \in C^{2,\alpha}(\bar{\Omega})$ satisfying

$$\begin{cases} \Delta \underline{u} + f(x, \underline{u}) \geq 0 & \text{in } \Omega, \\ \underline{u}|_{\partial\Omega} \leq 0 \end{cases}$$

is called a sub solution to (1).

Lemma 2.1. Let $f(x, \xi) \in C^\alpha(\bar{\Omega} \times \mathbb{R})$ and let $\bar{u}, \underline{u} \in C^{2,\alpha}(\bar{\Omega})$ be, respectively, super and sub solutions to (1) which satisfy $\underline{u}(x) \leq \bar{u}(x), x \in \bar{\Omega}$. Then (1) has a solution $u \in C^{2,\alpha}(\bar{\Omega})$ with $\underline{u}(x) \leq u(x) \leq \bar{u}(x), x \in \bar{\Omega}$.

In our proof, we also employ accepted conclusions concerning the solutions of the following logistic equations.

Lemma 2.2. (*Established in [13]*) Consider

$$\begin{cases} \Delta u + uf(u) = 0 & \text{in } \Omega, \\ u|_{\partial\Omega} = 0, u > 0, \end{cases}$$

where f is a decreasing C^1 function such that there exists $c_0 > 0$ such that $f(u) \leq 0$ for $u \geq c_0$ and Ω is a bounded domain in R^n .

If $f(0) > \lambda_1$, then the above equation has a unique positive solution, where λ_1 is the first eigenvalue of $-\Delta$ with homogeneous boundary conditions whose corresponding eigenfunction is denoted by ϕ_1 . We denote this unique positive solution as θ_f .

The most important property of this positive solution is that θ_f is increasing as f is increasing.

We specifically note that for $a > \lambda_1$, the unique positive solution of

$$\begin{cases} \Delta u + u(a - u) = 0 & \text{in } \Omega, \\ u|_{\partial\Omega} = 0, u > 0, \end{cases}$$

is denoted by $\omega_a \equiv \theta_{a-x}$. Hence, θ_a is increasing as $a > 0$ is increasing.

Consider the system

$$\begin{aligned} \Delta u + f(x, u) &= 0 & \text{in } \Omega, \\ u &= 0 & \text{on } \partial\Omega, \end{aligned} \tag{2}$$

where $u = (u_1, \dots, u_m)$ and $f = (f_1, \dots, f_m)$ is quasimonotone increasing, i.e. $f_i(x, u)$ is increasing in u_j for all $j \neq i$.

Lemma 2.3. ([12]) *Let w_λ be a family of subsolutions ($\alpha \leq \lambda \leq \beta$) to (2), increasing in λ such that*

$$\Delta w_\lambda + f(x, w_\lambda) \geq 0 \text{ in } \Omega, w_\lambda = 0 \text{ on } \partial\Omega.$$

Assume also $u \geq w_\alpha$, w_λ does not satisfy (2) for any λ , and $\frac{\partial w_\lambda}{\partial n}$ changes continuously in λ on $\partial\Omega$. Then $u \geq \sup w_\lambda$.

3. Cooperating Species

In [12], Korman and Leung established a sufficient and necessary condition for the existence of positive solution to the cooperation system

$$\begin{cases} \Delta u(x) + u(x)(a - u(x) + cv(x)) = 0 \\ \Delta v(x) + v(x)(d - v(x) + eu(x)) = 0 \\ u(x)|_{\partial\Omega} = v(x)|_{\partial\Omega} = 0. \end{cases} \text{ in } \Omega, \tag{3}$$

where Ω is a bounded domain in R^n , $u(x)$ and $v(x)$ designate the population densities for the two species.

The following is their result:

Theorem 3.1. *For existence of a positive solution to (3) it is necessary and sufficient that $ce < 1$.*

In this section, we develop their result to more general population model:

$$\begin{cases} \Delta u(x) + u(x)(a + g(u(x), v(x))) = 0 \\ \Delta v(x) + v(x)(d + h(u(x), v(x))) = 0 \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0, \end{cases} \quad \text{in } \Omega, \tag{4}$$

where a and d are positive constants, $g, h \in C^1$ are such that $g_u < 0, g_v > 0, h_u > 0, h_v < 0$ and $g(0, 0) = h(0, 0) = 0$.

The following Theorem proves a necessary condition for the existence of a positive solution to (4).

Theorem 3.2. *If $a > \lambda_1, d > \lambda_1, \inf(h_v) \geq -1$, and $\inf(h_u) > 0$, then the existence of a positive solution to (4) implies*

$$\inf(g_v) \inf(h_u) + \inf(g_u) < 0.$$

Proof. Suppose $\inf(g_v) \inf(h_u) + \inf(g_u) \geq 0$. Consider a family $(u_\lambda, v_\lambda) = (\lambda\phi_1, \lambda \inf(h_u)\phi_1)$ with any $\lambda > 0$.

Then by the assumption and Mean Value Theorem,

$$\begin{aligned} & \Delta u_\lambda + u_\lambda[a + g(u_\lambda, v_\lambda)] \\ &= -\lambda\lambda_1\phi_1 + \lambda\phi_1[a + g(\lambda\phi_1, \lambda \inf(h_u)\phi_1)] \\ &= \lambda\phi_1[-\lambda_1 + a + g(\lambda\phi_1, \lambda \inf(h_u)\phi_1)] \\ &= \lambda\phi_1[a - \lambda_1 + g(\lambda\phi_1, \lambda \inf(h_u)\phi_1) - g(\lambda\phi_1, 0) + g(\lambda\phi_1, 0) - g(0, 0)] \\ &\geq \lambda\phi_1[a - \lambda_1 + \inf(g_v) \inf(h_u)\lambda\phi_1 + \inf(g_u)\lambda\phi_1] \\ &> 0, \end{aligned}$$

and

$$\begin{aligned} & \Delta v_\lambda + v_\lambda[d + h(u_\lambda, v_\lambda)] \\ &= -\lambda \inf(h_u)\lambda_1\phi_1 + \lambda \inf(h_u)\phi_1[d + h(\lambda\phi_1, \lambda \inf(h_u)\phi_1)] \\ &= \lambda \inf(h_u)\phi_1[-\lambda_1 + d + h(\lambda\phi_1, \lambda \inf(h_u)\phi_1)] \\ &= \lambda \inf(h_u)\phi_1[d - \lambda_1 + h(\lambda\phi_1, \lambda \inf(h_u)\phi_1) - h(\lambda\phi_1, 0) + h(\lambda\phi_1, 0) - h(0, 0)] \\ &\geq \lambda \inf(h_u)\phi_1[d - \lambda_1 + \inf(h_v)\lambda \inf(h_u)\phi_1 + \inf(h_u)\lambda\phi_1] \\ &> 0. \end{aligned}$$

Therefore, $(u_\lambda, v_\lambda) = (\lambda\phi_1, \lambda \inf(h_u)\phi_1)$ with any $\lambda > 0$ is a family of subsolutions to (4).

Furthermore, if (u, v) is a positive solution to (4), then $u > \lambda_0\phi_1$ and $v > \lambda_0 \inf(h_u)\phi_1$ for sufficiently small $\lambda_0 > 0$, and so by the lemma 2.3, we conclude that $u \geq \lambda\phi_1$ and $v \geq \lambda \inf(h_u)\phi_1$ for any $\lambda \geq \lambda_0$.

Hence, there is no positive solution to (4).

For a sufficient condition for the existence of a positive solution to (4), we need the following Lemma.

Lemma 3.3. *If $bf > ce$, then we can choose arbitrary large $M, N > 0$ such that*

$$\begin{aligned} a - bM + cN &< 0, \\ d + eu - fN &< 0. \end{aligned}$$

We now establish a sufficient condition for the existence of a positive solution to (4).

Theorem 3.4. *If $a > \lambda_1, d > \lambda_1$ and $\sup(g_v)\sup(h_u) < \sup(g_u)\sup(h_v)$, then (4) has a positive solution.*

Proof. Let $\underline{u} = \alpha\phi_1, \underline{v} = \beta\phi_1$, where $\alpha, \beta > 0$.

Then since $a > \lambda_1$ and $d > \lambda_1$, by the Mean Value Theorem, for small enough $\alpha, \beta > 0$,

$$\begin{aligned} &\Delta \underline{u} + \underline{u}[a + g(\underline{u}, \underline{v})] \\ &= -\alpha\lambda_1\phi_1 + \alpha\phi_1[a + g(\alpha\phi_1, \beta\phi_1)] \\ &= \alpha\phi_1[-\lambda_1 + a + g(\alpha\phi_1, \beta\phi_1)] \\ &= \alpha\phi_1[-\lambda_1 + a + g(\alpha\phi_1, \beta\phi_1) - g(0, \beta\phi_1) + g(0, \beta\phi_1) - g(0, 0)] \\ &\geq \alpha\phi_1[-\lambda_1 + a + \inf(g_u)\alpha\phi_1 + \inf(g_v)\beta\phi_1] \\ &\geq 0, \end{aligned}$$

and

$$\begin{aligned} &\Delta \underline{v} + \underline{v}[d + h(\underline{u}, \underline{v})] \\ &= -\beta\lambda_1\phi_1 + \beta\phi_1[d + h(\alpha\phi_1, \beta\phi_1)] \\ &= \beta\phi_1[-\lambda_1 + d + h(\alpha\phi_1, \beta\phi_1)] \\ &= \beta\phi_1[-\lambda_1 + d + h(\alpha\phi_1, \beta\phi_1) - h(0, \beta\phi_1) + h(0, \beta\phi_1) - h(0, 0)] \\ &\geq \beta\phi_1[-\lambda_1 + d + \inf(h_u)\alpha\phi_1 + \inf(h_v)\beta\phi_1] \\ &\geq 0, \end{aligned}$$

and so, $(\underline{u}, \underline{v}) = (\alpha\phi_1, \beta\phi_1)$ is a subsolution to (4) for sufficiently small $\alpha, \beta > 0$.

But, for all (u, v) , by the Mean Value Theorem again,

$$\begin{aligned} g(u, v) &= g(u, v) - g(u, 0) + g(u, 0) - g(0, 0) \\ &\leq \sup(g_v)v + \sup(g_u)u, \end{aligned}$$

and

$$\begin{aligned} h(u, v) &= h(u, v) - h(u, 0) + h(u, 0) - h(0, 0) \\ &\leq \sup(h_v)v + \sup(h_u)u, \end{aligned}$$

so by the condition and the Lemma 3.3, there are constants $M, N > 0$ with $\alpha\phi_1 < M, \beta\phi_1 < N$ such that

$$\begin{aligned} \Delta M + M[a + g(M, N)] &\leq M[a + \sup(g_v)N + \sup(g_u)M] < 0, \\ \Delta N + N[d + h(M, N)] &\leq N[d + \sup(h_v)N + \sup(h_u)M] < 0, \end{aligned}$$

in other words, (M, N) is a supersolution to (4).

We conclude by the Lemma 2.1 that there is a positive solution (u, v) to (4) with $\alpha\phi_1 \leq u \leq M, \beta\phi_1 \leq v \leq N$.

4. Competing Species

In [4], Cosner and Lazer established a sufficient and necessary condition for the existence of positive solution to the competing system

$$\begin{cases} \Delta u(x) + u(x)(a - bu(x) - cv(x)) = 0 \\ \Delta v(x) + v(x)(a - fv(x) - eu(x)) = 0 \\ u(x)|_{\partial\Omega} = v(x)|_{\partial\Omega} = 0, \end{cases} \quad \text{in } \Omega, \tag{5}$$

where Ω is a bounded domain in R^n , $a, b, c, e, f > 0$ are constants, $u(x)$ and $v(x)$ designate the population densities for the two species.

The following is their result:

Theorem 4.1. *In order that there exist positive smooth functions u and v in Ω satisfying (5), it is necessary and sufficient that one of the following three sets of conditions holds.*

- (i) $a > \lambda_1, b > e, c < f$
- (ii) $a > \lambda_1, b = e, c = f$
- (iii) $a > \lambda_1, b < e, c > f$

In this section, we develop their result to more general population model:

$$\begin{cases} \Delta u(x) + u(x)[a + g(u(x), v(x))] = 0 \\ \Delta v(x) + v(x)[a + h(u(x), v(x))] = 0 \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0, \end{cases} \quad \text{in } \Omega, \tag{6}$$

where a is a positive constant, $g, h \in C^1$ are such that $g_u < 0, g_v < 0, h_u < 0, h_v < 0$, there exist constants $c_0 > 0, c_1 > 0$ such that $a + g(u, 0) \leq 0$ for $u \geq c_0$ and $a + h(0, v) \leq 0$ for $v \geq c_1$, and $g(0, 0) = h(0, 0) = 0$.

The following theorem provides a sufficient condition for the existence of a positive smooth solution to (6).

Theorem 4.2. *Suppose one of the following three sets of conditions holds.*

- (1) $a > \lambda_1, \inf(g_u) < \inf(h_u), \inf(g_v) > \inf(h_v)$
- (2) $a > \lambda_1, \inf(g_u) = \inf(h_u), \inf(g_v) = \inf(h_v)$
- (3) $a > \lambda_1, \inf(g_u) > \inf(h_u), \inf(g_v) < \inf(h_v)$

Then (6) has a positive smooth solution.

Proof. By the Theorem 4.1, if one of the above three sets of conditions holds, then there is a positive smooth solution (u, v) to

$$\begin{aligned} \Delta u + u[a - (-\inf(g_u))u - (-\inf(g_v))v] &= 0 \\ \Delta v + v[a - (-\inf(h_u))u - (-\inf(h_v))v] &= 0 \end{aligned} \quad \text{in } \Omega, u|_{\partial\Omega} = v|_{\partial\Omega} = 0.$$

But, by the Mean Value Theorem,

$$\begin{aligned} &\Delta u + u[a + g(u, v)] \\ &= \Delta u + u[a + g(u, v) - g(0, v) + g(0, v) - g(0, 0)] \\ &\geq \Delta u + u[a + \inf(g_u)u + \inf(g_v)v] \\ &= \Delta u + u[a - (-\inf(g_u))u - (-\inf(g_v))v] \\ &= 0, \end{aligned}$$

and

$$\begin{aligned} &\Delta v + v[a + h(u, v)] \\ &= \Delta v + v[a + h(u, v) - h(0, v) + h(0, v) - h(0, 0)] \\ &\geq \Delta v + v[a + \inf(h_u)u + \inf(h_v)v] \\ &= \Delta v + v[a - (-\inf(h_u))u - (-\inf(h_v))v] \\ &= 0. \end{aligned}$$

Hence, (u, v) is a subsolution to (6).

But by the conditions of g, h , any large positive constant M satisfying $u < M, v < M$ in Ω is a supersolution to (6).

Therefore, by the Lemma 2.1, (6) has a positive smooth solution.

The next theorem establishes a necessary condition for the existence of a positive smooth solution to (6).

Theorem 4.3. *If (6) has a positive smooth solution, then $a > \lambda_1$ and one*

of the following six sets of conditions holds.

- (1) $g_u \equiv h_u$ are constants, $\inf(h_v) \leq \sup(g_v), \sup(h_v) \geq \inf(g_v)$
- (2) $\inf(h_u) = \sup(g_u), \sup(h_u) > \inf(g_u), \inf(h_v) \leq \sup(g_v)$
- (3) $\inf(h_u) > \sup(g_u), \sup(h_u) > \inf(g_u), \inf(h_v) < \sup(g_v)$
- (4) $\inf(h_u) < \sup(g_u), \sup(h_u) = \inf(g_u), \sup(h_v) \geq \inf(g_v)$
- (5) $\inf(h_u) < \sup(g_u), \sup(h_u) < \inf(g_u), \sup(h_v) > \inf(g_v)$
- (6) $\inf(h_u) < \sup(g_u), \sup(h_u) > \inf(g_u)$

Proof. Suppose (u, v) is a positive smooth solution to (6).

By the Mean Value Theorem, there are \tilde{u}, \tilde{v} with $0 \leq \tilde{u} \leq u, 0 \leq \tilde{v} \leq v$ such that

$$\begin{aligned} g(u, 0) - g(0, 0) &= g_u(\tilde{u}, 0)u, \\ g(u, v) - g(u, 0) &= g_v(u, \tilde{v})v. \end{aligned}$$

Hence, by the Green's Identity,

$$\begin{aligned} &\int_{\Omega} u\phi_1[\lambda_1 - a - g_u(\tilde{u}, 0)u - g_v(u, \tilde{v})v]dx \\ &= \int_{\Omega} u\phi_1[\lambda_1 - a + g(0, 0) - g(u, 0) + g(u, 0) - g(u, v)]dx \\ &= \int_{\Omega} u\phi_1[\lambda_1 - a - g(u, v)]dx \\ &= \int_{\Omega} \phi_1[-au - ug(u, v)] + u\lambda_1\phi_1 dx \\ &= \int_{\Omega} \phi_1\Delta u - u\Delta\phi_1 dx \\ &= 0. \end{aligned}$$

But, since $-g_u(\tilde{u}, 0)u - g_v(u, \tilde{v})v > 0$ in Ω , $a > \lambda_1$.

By the Mean Value Theorem again, there are u_1, u_2, v_1, v_2 with $0 \leq u_1, u_2 \leq u, 0 \leq v_1, v_2 \leq v$ such that

$$\begin{aligned} g(u, v) - g(0, v) &= g_u(u_1, v)u, \\ h(u, v) - h(0, v) &= h_u(u_2, v)u, \\ g(0, v) - g(0, 0) &= g_v(0, v_1)v, \\ h(0, v) - h(0, 0) &= h_v(0, v_2)v. \end{aligned}$$

Therefore, by the Green's Identity again,

$$\begin{aligned} &\int_{\Omega} uv([h_u(u_2, v) - g_u(u_1, v)]u + [h_v(0, v_2) - g_v(0, v_1)]v)dx \\ &= \int_{\Omega} uv[h_u(u_2, v)u + h_v(0, v_2)v - g_u(u_1, v)u - g_v(0, v_1)v]dx \\ &= \int_{\Omega} uv[h(u, v) - h(0, v) + h(0, v) - h(0, 0) + g(0, v) - g(u, v) - g(0, v) \\ &\quad + g(0, 0)]dx \\ &= \int_{\Omega} uv[h(u, v) - g(u, v)]dx \\ &= \int_{\Omega} v\Delta u - u\Delta v dx \\ &= 0, \end{aligned}$$

and so,

$$\int_{\Omega} uv([\inf(h_u) - \sup(g_u)]u + [\inf(h_v) - \sup(g_v)]v)dx \leq 0,$$

$$\int_{\Omega} uv([\sup(h_u) - \inf(g_u)]u + [\sup(h_v) - \inf(g_v)]v)dx \geq 0,$$

which derives

$$(A) \inf(h_u) = \sup(g_u), \inf(h_v) \leq \sup(g_v),$$

$$(B) \inf(h_u) > \sup(g_u), \inf(h_v) < \sup(g_v),$$

$$(C) \inf(h_u) < \sup(g_u),$$

and

$$(A') \sup(h_u) = \inf(g_u), \sup(h_v) \geq \inf(g_v),$$

$$(B') \sup(h_u) < \inf(g_u), \sup(h_v) > \inf(g_v),$$

$$(C') \sup(h_u) > \inf(g_u).$$

Combining (A), (B), (C) and (A'), (B'), (C') together, we may have

$$(A'') g_u \equiv h_u \text{ are constants, } \inf(h_v) \leq \sup(g_v), \sup(h_v) \geq \inf(g_v),$$

$$(B'') \inf(h_u) = \sup(g_u), \sup(h_u) < \inf(g_u), \inf(h_v) \leq \sup(g_v), \sup(h_v) > \inf(g_v),$$

$$(C'') \inf(h_u) = \sup(g_u), \sup(h_u) > \inf(g_u), \inf(h_v) \leq \sup(g_v),$$

$$(D'') \inf(h_u) > \sup(g_u), \sup(h_u) = \inf(g_u), \inf(h_v) < \sup(g_v), \sup(h_v) \geq \inf(g_v),$$

$$(E'') \inf(h_u) > \sup(g_u), \sup(h_u) < \inf(g_u), \inf(h_v) < \sup(g_v), \sup(h_v) > \inf(g_v),$$

$$(F'') \inf(h_u) > \sup(g_u), \sup(h_u) > \inf(g_u), \inf(h_v) < \sup(g_v),$$

$$(G'') \inf(h_u) < \sup(g_u), \sup(h_u) = \inf(g_u), \sup(h_v) \geq \inf(g_v),$$

$$(H'') \inf(h_u) < \sup(g_u), \sup(h_u) < \inf(g_u), \sup(h_v) > \inf(g_v),$$

$$(I'') \inf(h_u) < \sup(g_u), \sup(h_u) > \inf(g_u).$$

However, it is clear that (B''), (D''), (E'') are not possible, so we establish the result of the Theorem.

Finally, we prove a nonexistence result.

Theorem 4.4. *If $a > \frac{\mu}{d}$, $-1 \leq g_u < 0$, and $h_v \leq -1$, where $\mu = \min[-\sup(h_u), 1]$ and $\nu = \max[-\inf(g_v), 1]$, then there is no positive solution to (6).*

Proof. Suppose there is a positive solution (u, v) to (6).

Then by the Mean Value Theorem, the Green's Identity and the inequality

conditions,

$$\begin{aligned}
 & \int_{\Omega} (a - d + [-\sup(h_u) - 1]u + [1 + \inf(g_v)]v)uv dx \\
 \leq & \int_{\Omega} (a - d + [\inf(g_u) - \sup(h_u)]u + [\inf(g_v) - \sup(h_v)]v)uv dx \\
 \leq & \int_{\Omega} [a - d + g(u, 0) - g(0, 0) - h(u, v) + h(0, v) + g(u, v) - g(u, 0) - h(0, v) \\
 & + h(0, 0)]uv dx \\
 = & \int_{\Omega} [a - d + g(u, v) - h(u, v)]uv dx \\
 = & \int_{\Omega} (v\Delta u - u\Delta v) dx \\
 = & 0.
 \end{aligned} \tag{7}$$

But, if $a > \frac{\nu}{\mu}d$, then since $a \geq u$ and $d \geq v$,

$$a - d + [-\sup(h_u) - 1]u + [1 + \inf(g_v)]v \geq \mu a - \nu d > 0,$$

which contradicts to (7).

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