5-2020

Understanding Social Factors in Small Group Work in Undergraduate Mathematics Classrooms

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UNDERSTANDING SOCIAL FACTORS IN SMALL GROUP WORK IN UNDERGRADUATE MATHEMATICS CLASSROOMS

By

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B. S. Boston University, 2015

A Thesis

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Teaching

The Graduate School

University of Maine

May 2020

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By Jeremy Bernier

Thesis Advisors: Dr. Janet Fairman and Dr. Natasha Speer

An Abstract of the Thesis
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To address the ongoing labor shortage for jobs in science, technology, engineering, and mathematics (STEM) fields, many different initiatives have been undertaken by practitioners, instructors, and researchers. Two major ones have been efforts to improve undergraduate mathematics instruction and to increase diversity and inclusiveness in STEM fields, including with regards to gender identity and sexual orientation. One major ongoing shift in undergraduate mathematics instruction is a shift to increase active learning, often through tasking students to engage in collaborative problem solving in small groups. It is known that active learning strategies like these improve student outcomes over the use of lecture alone. However, there is much less research considering how the social nature of group work can affect student experience in their undergraduate mathematics classes that use it. Social factors outside of the mathematical content could be expected to play a role when learning through group work, an inherently social activity; moreover, these factors could play a greater role for students who have traditionally been excluded from STEM environments.

To better understand how social factors may influence student participation and experience in small group work in undergraduate mathematics classrooms, a study was conducted that incorporated video-taped in-class observations of students working in small groups along with stimulated recall interviews of students individually. A taxonomy by Chiu

(2000b) was used to interpret, code, and analyze actions taken by the participants in group work, with interviews coded in terms of what ideas students discussed in response to selected interactions. From analysis of the observations and interviews, three main findings are drawn. First, social unfamiliarity among group members can negatively influence a student's experience within a group and the group's overall ability to collaborate. Second, student gender identities and beliefs about how gender and mathematics are related can also play a role, especially when students are unfamiliar with each other, although these data do not suggest exactly when or how this can happen. Third, students may work together ways that are socially productive, but are not mathematically productive. These takeaways broaden our understanding of how groups work in undergraduate mathematics classes while also setting some clear directions for future research on this topic.

ACKNOWLEDGEMENTS

I have to start by acknowledging my committee – Dr. Janet Fairman, Dr. Natasha Speer, and Dr. Timothy Boester. Without Janet, Natasha, and Tim's valuable feedback on my ideas and advisement on methodology at each step of the process, I would not have been able to even begin to conduct a study like I have here. Thank you for all the endless support, patience, and flexibility you've provided.

I also need to acknowledge the faculty and staff beyond my committee who made valuable contributions to my thesis – whether it was through discussion of ideas or letting me use their classroom for my research. In particular, I want to thank Laura Millay, Todd Zoroya, Dr. Justin Dimmel, Dr. Franziska Peterson, and Dr. Michael Wittmann.

I must acknowledge the students who agreed to participate in this study, for allowing me access to their lived classroom experiences.

Finally, I want to acknowledge the academic and social support provided by the community of grad students I have called my colleagues for the past two years. I've had the privilege to work with exceedingly clever folks from the M.S.T., STEM PhD, Physics PhD, and Mathematics M.S. programs in a wide variety of contexts. Between discussions of mathematics, research, and teaching strategies, to sessions of Dungeons and Dragons, you all have contributed to my development as a researcher and a person over this time. I want to give particular shout-outs to Mia, Isaac, Justin, Erin, Michael, Haley, Chrissy, Ryan, and Kristin.

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CHAPTER 1: INTRODUCTION

The Mathematical Association of America (MAA) Instructional Practices guide includes a call for action towards significant reform in collegiate mathematics instruction:

> It is our responsibility to examine the system within which we educate students and find ways to improve that system. It is our responsibility to help our colleagues improve and to collectively succeed at teaching mathematics to all students so that our discipline realizes its full potential as a subject of beauty, of truth, and of empowerment for all. Such a sea change will require transforming how mathematics is taught and facing our own individual and collective roles in a system that does not serve all students well. Societal norms tend toward a belief that only a certain kind of individual can do mathematics and other kinds of people need not even try. We in the profession of teaching mathematics must look inward to determine if we are doing our part to dispel this myth. (Abell, Braddy, Ensley, Ludwig, & Soto, 2018, p. vii)

This call for change reflects broader societal concerns regarding instruction in mathematics and science and recruitment into STEM jobs – specifically, the 21st century has seen a labor shortage in STEM fields (Freeman et al., 2014; Tsui, 2007). While changing instructional practices is one response to this shortage, another has been to consider the diversity or lack thereof of individuals employed in STEM fields (Tsui, 2007). Women, African-Americans, Latino, and Native Americans are underrepresented in STEM fields (Tsui, 2007). Women in particular leave STEM programs at higher rates than men, even when controlling for performance in STEM courses (Ellis, Fosdick, & Rasmussen, 2016; Seymour & Hewitt, 1997). While clear data on the prevalence of individuals who identify as lesbian, gay, bisexual, transgender, queer, or with other non-heteronormative and/or non-cisnormative gender identities and sexual orientations (collectively, LGBTQ)[1] in STEM fields are not yet available, there is evidence of negative effects on LGBTQ employees in STEM jobs due to apparent cultural heteronormativity (Cech & Pham, 2017; Cech & Waidzunas, 2011; Cooper & Brownell, 2016; Yoder & Mattheis, 2016).

While it is well-established (Freeman et al., 2014; Springer, Stanne, & Donovan, 1999) that, at the undergraduate level, active learning strategies improve outcomes for students generally over lecture-based instruction, research on how students actually participate and experience their participation in classrooms that have students work in small groups is limited. In particular, most studies that consider the effects of small groups in classrooms have either used only during-instruction data such as observations and assessment scores (Bianchini, 1997; Chiu, 2000a; Sullivan, Ballen, & Cotner, 2018)or post-instruction data such as interviews and surveys (Cooper & Brownell, 2016; Esmonde, Brodie, Dookie, & Takeuchi, 2009; Theobald, Eddy, Grunspan, Wiggins, & Crowe, 2017).This means that while we might have some sense of what happens in the classroom and what students think or feel about what happens in the classroom, we have very little sense about how these are connected. Moreover, existing literature does not consistently explain how issues of identity, such as gender identity and sexual orientation, influence how students work together and experience small group work in the classrooms (Bianchini, 1997; Chiu, 2000a; Cooper & Brownell, 2016; Esmonde et al., 2009; Sullivan et al., 2018).

The goal of this study was to explore what actually happens when students work together in small groups in undergraduate mathematics classrooms, and how social factors influence how

[1] The initialism LGBTQ will be used throughout this study to refer to this grouping of identities. However, other terms are used for this group and additional identities exist that are included in this grouping, with the exact categorization depending on theoretical perspective. A glossary (Appendix G) that further explicates various terms related to gender identity and sexual orientation is included in the appendix to provide further insight.

students participate in and experience these interactions. To explore this, I first observed students while they worked in small groups in their regularly-scheduled undergraduate mathematics classes, analyzing their work using a taxonomy by Chiu (2000b). This taxonomy was used to interpret and understand each action by each student in the group relative to each other and to the problem. Then, I conducted stimulated recall interviews with the individual students to gain a rich perspective on why things happened the way they did in their groups.

The following chapters outline in more detail the body of work that justifies this study, the methodology employed in conducting it, and findings supported by the data. Ultimately, what I conclude is: 1) social unfamiliarity among group members can negatively influence a student's experience within a group and the group's overall ability to collaborate; 2) student gender identities and beliefs about how gender and mathematics are related can also play a role, especially when students are unfamiliar with each other; and 3) students may work together in ways that are socially productive, but are not mathematically productive. However, limitations on this study including the size and diversity of the sample alongside length of the interviews mean that significantly more work is needed to better understand how groups can get into these situations and what instructors can do about it.

CHAPTER 2: LITERATURE REVIEW

The context of this study is the intersection of two areas of scholarship – the experience of women and LGBTQ individuals in mathematics and science and the use of small group work as an instructional strategy in mathematics and science courses. The following sections provide an overview of relevant scholarship from these two areas and their intersections. In the first section, I describe the theoretical foundations of this study. In the second section, I describe the literature on gender differences in performance and persistence in mathematics, as well as the literature on the experience of LGBTQ individuals in STEM fields. In the final section, I describe research on group work in mathematics and science education. This includes discussion of the effectiveness of group work at the undergraduate level and of what work has been done on how social factors influence experiences and learning in group work.

Theoretical foundations

To better understand how students collaborate in undergraduate mathematics courses and how gender identity and sexual orientation may influence this collaboration, this study has incorporated ideas from two theoretical perspectives. As a contextual background for the ways in which this study considers gender identity and sexual orientation, this study has adopted ideas from intersectionality theory (Crenshaw, 1991; Levya, 2017). While this provides a broader theoretical context within which this study was conducted, the methodology of this study will be informed by role theory (Chiu, 2000b; Tatsis & Koleza, 2006). While intersectionality theory primarily concerns who individuals are with regards to their social identities, role theory concerns how individuals perform their roles in any given situation; in one sense, the roles as understood by role theory and identities as understood by intersectionality theory can each hold influence over each other; thus, these theories complement each other in the design of this study.

Intersectionality theory. Intersectionality theory is a perspective originating from feminist literature and applied across disciplines when studying differences related to identity, with the core tenant being that one cannot isolate the effects of one identity from another and must instead attend to the nuanced interactions between identities (Crenshaw, 1991). Intersectionality theory also treats gender, race, ethnicity, class, sexual orientation, and other identities as social constructs – rather than pre-determined genetic or biological constructs (Crenshaw, 1991; Levya, 2017). In a review of literature on gender in mathematics education, Levya (2017) argues that most such literature has not taken an intersectional perspective.. Past studies have, for example, conflated gender and sex, conducting comparisons between men and women without attending to cisgender and transgender identities or to intersex and non-binary identities. Achievement-focused studies have not always attended to the intersections of gender, sex, and race – reporting results on these variables separately without considering their interactions or leaving their interactions implicit (Levya, 2017). This, Levya (2017) argues, means our understanding of the experiences of diverse individuals in learning mathematics is significantly limited.

Levya (2017) does discuss the work of Esmonde, Brodie, Dokie, and Takeuchi (2009) as an example of how an intersectional analysis can be conducted in math education research. This study – discussed in more detail later in this chapter – was viewed as an intersectional analysis because it explicitly defined gender, race, ethnicity, and sexual orientation as being socially constructed identities, and “offered qualitative, situated accounts of students’ mathematics experiences to glean more nuanced insights of contextual influences at intersections of race, class, and sexuality” (Levya, 2017, p. 424). Research that uses ideas from intersectionality theory can use a very broad set of methodologies, so long as they are attending to the intersections between socially constructed identities.

While this literature review cites many studies that have not adopted intersectional perspectives or conducted intersectional analyses, the fundamental tenets of intersectionality provide the foundation for thinking about gender identity that I have used in designing and implementing this study. The focus of this study is on gender and sexual orientation, but these identities interact with each other and other identities. Most importantly, I take the perspective that these are all socially constructed identities, not biologically constructed or otherwise determined, and that one cannot simply "control" for one of a person's identities to understand their other identities.

Role theory. Within the context of classrooms using small groups, instructors may assign students roles. For example, Cohen and Lotan (2014, p. 121) recommend teachers assign some combination of the following roles, depending on the needs of the assignment: facilitator, checker, materials manager, safety officer, and reporter. . While Cohen and Lotan (2014) ground their framework for group work in sociology, the idea of assigned roles in the classroom varies somewhat from the notion of "role" in role theory. Role theory refers to a broad set of sociological theories that are centered on the idea that "human beings behave in ways that are different and predictable depending on their social identities and the situation" (Biddle, 1986, p. 68). The language of role theory borrows from the language of theatre – so a "role" in role theory is analogous to a role in a play. Role theory is used across different orientations in sociology, but these different orientations view roles quite differently. Functional, structural, and organizational role theorists are three orientations towards role theory that are, broadly speaking, considered with how expectations for roles are established and enacted at a big-picture, system-level (Biddle, 1986). These perspectives are not relevant to this study other than in their influence on role theory as a whole.

The two sociological orientations toward role theory that are most relevant are cognitive and symbolic interactionist role theory. These orientations are focused on the individual actors, rather than social systems. Cognitive role theorists operate in the realm of social psychology, which incorporates both sociological and psychological ideas. In particular, they are concerned with "relationships between role expectations and behavior" (Biddle, 1986, p. 73) Symbolic interactionists are also interested in how individuals develop expectations for these roles, with a focus on how roles evolve depending on not just norms, but individual social interactions; symbolic interactionists also tie ideas in role theory back to the theatre analogy that inspired role theory in the first place (Biddle, 1986).

Role theory has been applied to group work in mathematics education previously in work by Tatsis and Koleza (2006); they situate their work within the symbolic interactionist tradition of role theory, and focus on the idea of role performance – how the collective of actions a student takes fulfills a role Tatsis and Koleza (2006) used this perspective in analysis of video recordings of undergraduate students working in pairs on mathematical problems in three separate sessions for each pair, with each session focusing on a single problem. They used a list of sixteen categories of actions one can take in group problem solving, adapted from Bales (1966), to code transcripts of each pair's work. These categories included, for example, "shows certainty," "shows agreement," "makes suggestion," as well as categories that could be seen as opposites of these – "shows uncertainty," "shows disagreement," and "asks for suggestion" (Bales, 1966; Tatsis & Koleza, 2006, p. 448). Each action was coded in exactly one of these categories. Tatsis and Koleza (2006) then synthesized patterns of actions into four roles that students played in these pairs across groups – the dominant initiator, the collaborative initiator, the collaborative evaluator, and the insecure conciliator (Tatsis & Koleza, 2006, p. 453). They then applied these roles back to each student in each pair, categorizing the role that they played in each session. From these emergent roles, they noted some patterns that occurred across groups in terms of the roles students performed; for example, they found that in groups where the participants' age difference was greater than three years, the older student was more likely to take on a collaborative initiator role.

Ultimately, the focus of this study is to understand how the different aspects of an interaction, including social factors and mathematical content, influence how participants in group work act. In the language of role theory, I am concerned with the role and role performance that students are taking on and what situational factors contribute to their role and role performance. Thus, my analysis follows some of the same patterns of Tatsis and Koleza's (2006). However, I elected not to follow the categories for actions used by Tatsis and Koleza (2006). I made this decision because of my discovery of a taxonomy of group work actions described by Chiu (2000b).

Chiu's (2000b) taxonomy. Chiu's (2000b) taxonomy, rather than placing each action in an individual category like with Tatsis and Koleza (2006), instead places each action in one of three categories in each of three dimensions. The three dimensions of Chiu's (2000b) taxonomy are the evaluation of previous action (EPA) dimension, the knowledge content (KC) dimensions, and the invitational form (IF) dimension. The evaluation of previous action dimension (Chiu, 2000b, pp. 29–30) regards how the individual's action relates to action by the previous individual. In this dimension, an action can be supportive of (+), be critical of (-), or be unresponsive to (0) the previous action. The knowledge content dimension (Chiu, 2000b, p. 31) describes the mathematical content of the action and how it relates to the problem at hand. An individual's action can be characterized in this dimension as a contribution (C), a repetition (R), or a null content action (N). The invitational form dimension (Chiu, 2000b, pp. 31–32) describes the degree to which the action does or does not invite follow-up from other group members. Under this dimension, an action can be characterized as a statement (_), a question (?), or a command (!).[2]

While Chiu (2000b) does not explicitly contextualize his work within role theory, he does refer to how potential actions align with five roles that students may take on in collaborative problem solving – facilitator, proposer, supporter, critic, and recorder. Facilitators perform the widest array of actions – using questions and commands (? or !) to invite the participation of others, phrasing critiques as questions (-?), and alternating between supportive and critical remarks (+ and –). Proposers suggest new ideas for discussion (C), with supporters making contributions that build on prior contributions (+C) while critics identify flaws in and alternatives to ideas being discussed (-C). Finally, recorders summarize existing contributions (+R).

Summary of the theoretical framework. Ultimately, in designing this study, I was concerned with how different aspects of identity could influence behaviors in classroom. I drew from intersectionality theory ideas about identities as social constructs that intersect with each other. I drew from role theory to determine how I would understand the behavior of participants in this study. I drew from Chiu's (2000b) taxonomy to supplement the ideas from role theory. Each of these ideas together gave me a framework to deeply consider how each individual student in my sample was participating in and experiencing group work in their undergraduate mathematics classroom.

[2] See Appendix D for an example of how this coding scheme can be applied.

Experiences of Women and LGBTQ people in STEM

In considering the influence of gender identity and sexual orientation on group work in an undergraduate math classroom, it is important to understand what is known about how gender identity and sexual orientation influence experiences more broadly in mathematics and science. To that end, two strands of literature seem relevant. First, a number of studies have considered differences in achievement, participation, persistence, and interest in mathematics and science between men and women and have rather consistent conclusions, except regarding achievement. Second, a handful of studies have considered the experience of LGBTQ individuals in STEM in broader terms, and while results in these studies are more preliminary, some consistency is found in their conclusions.

Gender differences in mathematics and other STEM disciplines. Broadly, cultural and societal norms, beliefs, and stereotypes imply that math and science are domains where men have greater inherent capabilities than women (Eddy & Brownell, 2016; Levya, 2017). While this 'male superiority myth' is pervasive (Levya, 2017), existing literature paints a more nuanced picture of the relationship between gender and STEM learning (Eddy & Brownell, 2016). There is little evidence to support a belief in inherent ability differences between men and women in mathematics and science. Studies generally find either no or small differences in performance between men and women in STEM (Eddy & Brownell, 2016), while work suggests that beliefs about ability can explain at least some of the observed disparities (Schmader, Johns, & Barquissau, 2004; Spencer, Steele, & Quinn, 1999). More definitive are studies showing that the experience and interests of men and women in STEM are different (Eddy & Brownell, 2016).

Achievement in mathematics and other STEM disciplines. Studies that look at the relationship between gender and achievement in mathematics and other STEM disciplines neither strongly support nor strongly refute the existence of gender achievement gaps in STEM disciplines (Eddy & Brownell, 2016; Hyde & Mertz, 2009). Repeated studies of standardized test data from millions of United States elementary and secondary students, comparing both overall math performance and performance across different domains of mathematics, have generally found no or small gender differences, especially in those conducted in the 21st century (Hyde & Mertz, 2009). Studies using standardized test data at the high school level do seem to show greater rates of gender differences than those at the elementary level (Hyde & Mertz, 2009). In a review of studies of undergraduate classrooms across multiple STEM disciplines, Eddy and Brownell (2016) note that "[they] do not see a consistent gender gap in performance across or within disciplines" (p. 3); studies reviewed by Eddy and Brownell sometimes found that women outperformed men, sometimes found that men outperformed women, and sometimes found no gender difference. However, Eddy and Brownell (2016) do note that when studies controlled for prior academic performance, they were more likely to find evidence that men were outperforming women.

Within undergraduate math, a large study across multiple universities found that courses using inquiry-based learning did not have a gender achievement gap, while lecture-based courses did (Laursen, Hassi, Kogan, & Weston, 2014). On the other hand, a study by different researchers but using similar methodology – collecting data from undergraduate math courses using inquiry-oriented learning across multiple universities – still found a gender gap in achievement in inquiry oriented classes (Keller, Johnson, Keene, Andrews-Larson, & Fortune, 2020).

While studies do not provide strong evidence disproving or proving the existence of a gender achievement gap between men and women, other work suggests that, when gender achievement gaps are found, they can be explained at least in part by beliefs and stereotypes related to women in mathematics and science (Spencer et al., 1999). Spencer, Steele, and Quinn (1999) administered difficult mathematics tests in randomized studies; before each test, participants were either told that there *were* typically gender differences in the test results, that there *were not* typically gender differences in the test results or were not told anything regarding gender and test results. In these studies, while men did perform better in the groups where participants were told there would be gender differences and where participants were not told anything regarding gender differences, outcomes among the participants who were told the results would not show gender differences did not show statistically significant gender differences (Spencer et al., 1999). Ultimately, this work suggests that the awareness of stereotypes and the concern that one may be judged based on stereotypes related to their identity has a significant impact on academic outcomes. The implication, then, is that when gender differences in achievement are observed, their underlying causes may be due to cultural and social factors, rather than any inherent or existing differences in ability..

Later work by Schmader, Johns, and Barquissau (2004) supports this implication of Spencer et al. (1999). In a study following the framework of Spencer et al. (1999), participants (all women who were undergraduate students) were either told that a researcher was primarily looking at test results as reflective of individual ability or as reflective of gender differences; previously, the participants had been surveyed regarding their beliefs about gender and mathematics (Schmader et al., 2004). Moreover, the more participants agreed with statements like "in general, men may be better than women at math," the worse they performed on the assessment (Schmader et al., 2004).

Gender differences in experiences and interest in STEM. Beyond achievement, there is ample research suggesting that students' experiences in classrooms for mathematics and other STEM disciplines differ by gender (Eddy & Brownell, 2016; Levya, 2017). Student participation (i.e., student-teacher or student-student interactions) is one area where studies find consistent gender-based differences in STEM classrooms – namely, that men and boys participate more frequently than women and girls (Eddy & Brownell, 2016). This has result has been found in studies using different environments – K-12 classrooms (Becker, 1981; Greenfield, 1997) and undergraduate classrooms (Crombie, Pyke, Silverthorn, Jones, & Piccinin, 2007; Eddy, Brownell, Thummaphan, Lan, & Wenderoth, 2015; Eddy, Brownell, & Wenderoth, 2014; Fritschner, 2000; Sternglanz & Lyberger-Ficek, 1977) – and using different methodologies – self-reporting by students (Crombie et al., 2007; Eddy et al., 2015), and classroom observations (Becker, 1981; Eddy et al., 2014; Fritschner, 2000; Greenfield, 1997; Sternglanz & Lyberger-Ficek, 1977).

Another clear gender disparity is that women show less interest and persistence in pursuing STEM careers than men, and are underrepresented in STEM jobs (Ellis et al., 2016; Robnett & Leaper, 2013; Sadler, Sonnert, Hazari, & Tai, 2012; Seymour & Hewitt, 1997; Tsui, 2007). Girls report lower rates of interest in STEM careers at the high school level than boys (Robnett & Leaper, 2013; Sadler et al., 2012); Sadler et al. (2012) found in a study of 6,000 students that rates of interest in STEM careers decreased for girls and increased for boys during high school, while Robnett and Leaper (2013) found in a survey of 468 high school students from five North Carolina high schools that gender differences were exacerbated among girls who said they were in friend groups consisting mostly of other girls. Beyond interest, there are also gender differences in terms of what happens when students do choose to pursue STEM. As undergraduates, men are more likely to persist in their study of STEM subjects than women, having lower rates of switching majors and exiting the two semester calculus sequence required by many majors (Ellis et al., 2016; Seymour & Hewitt, 1997). In Ellis et al. (2016), the researchers specifically found that women were 1.5 times more likely to not continue to second semester calculus. Ellis et al. (2016) also found that, even among students with high mathematical ability, women reported significantly lower rates of confidence in their mathematical ability.

LGBTQ experiences in STEM. A handful of studies have looked at the experiences of LGBTQ individuals as STEM workers and students (Cech & Pham, 2017; Cech & Waidzunas, 2011; Cooper & Brownell, 2016; Linley, Renn, & Woodford, 2018; Yoder & Mattheis, 2016). Studies focused on students have used interviews to gain information on student perspectives on the experience of being LGBTQ in STEM (Cech & Waidzunas, 2011; Cooper & Brownell, 2016; Linley et al., 2018). While reported experiences vary greatly among studies, in general, students report some difficulty navigating STEM fields that seem dominated by straight white men and informed by heternormativity (Cech & Waidzunas, 2011; Cooper & Brownell, 2016; Linley et al., 2018). In Linley et al. (2018), a study that pulled students from multiple universities and programs, students did uniformly report acceptance from faculty regarding their identities, which was especially important for transgender participants. However, students also reported difficulty being "out" in math and science classes, particularly when considering their interactions with non-LGBTQ students and in comparison to courses in the humanities and social sciences (Linley et al., 2018). Difficulty interacting with non-LGBTQ STEM students, ranging from casual discrimination to the use of slurs, is relatively consistently reported by LGBTQ STEM majors (Cech & Waidzunas, 2011; Cooper & Brownell, 2016).

Studies that have looked at the experiences of LGBTQ-identifying individuals in STEM workplaces have provided data that are suggestive of differences and discomforts for LGBTQ people in STEM (Cech & Pham, 2017; Yoder & Mattheis, 2016). Cech and Pham (2017) collected data from the Federal Employee Viewpoint Survey for science-focused departments and agencies of the Federal government, while Yoder and Matthies (2016) used a snowball survey strategy – wherein each participant is encouraged to recruit additional participants – to collect responses from 1,427 LGBTQ people in a broad array of STEM fields and jobs. In comparing LGBTQ and non-LGBTQ respondents, Cech and Pham (2017) found that various workplace and personal satisfaction measures were significantly lower for LGBTQ respondents at science-focused departments and agencies of the Federal government. Yoder and Matthies (2016) found that participants generally report feeling safe in their workplace, but also reported they were much less likely to be "out" or open about their identity with their colleagues and students than with their family and friends. While these studies are suggestive, they are primarily about LGBTQ individuals who have completed their education and so are not conclusive about differences in the STEM classroom environment for LGBTQ individuals.

Learning Math and Science Through Small Group Work

This section reviews literature regarding group work in classes in mathematics and other STEM disciplines at the undergraduate and secondary level. It first reviews the results of achievement-focused meta-analyses comparing group work and other active learning strategies to lecture-based strategies (Freeman et al., 2014; Springer et al., 1999). This section also reviews studies which consider different factors that can influence the outcomes of group work, with outcomes being defined broadly to include effects on achievement, student experience, and other outcomes where measured. While the focus of this study is on student experiences and student interactions, to fully contextualize that work, it is important to understand more broadly what has been studied relating to group work in undergraduate math classes.

Moreover, when it comes to *undergraduate math* classes, there are fewer studies than one might think that look at specific implementations of group work. On the one hand, there is an extensive and developing collection of studies on the use of inquiry-based or inquiry-oriented instruction in undergraduate level math courses (Keller et al., 2020; Laursen et al., 2014; Mullins, Serbin, & Johnson, 2020; Rasmussen & Kwon, 2007; Wagner, Speer, & Rossa, 2007). These approaches rely on constructivist principles and use at least some amount of group work and collaborative problem solving in an effort to improve students' development of conceptual understanding of mathematics. However, existing literature does not always focus on the impact that group work has on students. Some studies look at the impact of inquiry-based approaches as a whole, while others focus on the changing role of the instructor in the classroom. So, while elements of this literature contribute to our understanding of how working in small groups impacts students, it is in the interest of this study that research in small group work in other STEM fields and other grade levels be reviewed as well to provide a fuller picture of the context of this study.

Effectiveness at the undergraduate level. Research literature comparing group work and other kinds of active learning instructional techniques to lecture-based instruction in undergraduate math and science has consistently found that active learning is associated with strong positive effects (Freeman et al., 2014; Springer et al., 1999). Springer, Stanne, and Donovan (1999) conducted a meta-analysis of 39 studies of group work in undergraduate classes across STEM disciplines. Across studies, their analysis showed statistically significant positive effects on student achievement, student persistence in STEM courses, and attitudes toward

STEM disciplines. Freeman et al. (2014) conducted a broader meta-analysis on 158 studies across STEM disciplines which evaluated active learning, a category including but not limited to small group work. They found statistically significant effects on student achievement from the use of active learning - average grades were higher and failure rates were lower across studies and STEM disciplines, including mathematics.

Interactions between social factors and group work. While past research establishes the effectiveness of having students work in small groups as an instructional strategy, other research establishes ways in which social factors can influence the experience and outcomes of working in groups for students as individuals and as participants in groups (Bianchini, 1997; Chiu, 2000a; Cooper & Brownell, 2016; Esmonde et al., 2009; Heller & Hollabaugh, 1992; Laursen et al., 2014; Sullivan et al., 2018; Theobald et al., 2017). The findings of these studies are not always consistent – particularly in regards to issues of gender and group work – but there are more consistent trends in the literature around friendship and perceived academic ability. These studies have different academic contexts andtake a variety of approaches, including observing students at work to learn about the group experience (Bianchini, 1997; Chiu, 2000a), interviewing or surveying students after work to learn about the group experience (Cooper & Brownell, 2016; Esmonde et al., 2009; Theobald et al., 2017), or measuring student achievement after working in a group to identify learning effects (Sullivan et al., 2018; Theobald et al., 2017). Each approach elicited distinct but overlapping results, and some studies used multiple approaches. The factors considered by the researchers through all studies can largely be broken down into two categories – ones related to friendship and perceived academic ability, and ones related to gender identity, race, ethnicity, and sexual orientation.

Friendship and perceived academic ability. When considering peer friendship and perceived academic ability, precise definitions thereof vary. In general, researchers studying the effects of these social factors consider friendship in terms of how well-liked a student is reported to be by their peers, and perceived academic ability by how well a student is perceived to be in the course as reported by their peers (Bianchini, 1997; Chiu, 2000a; Esmonde et al., 2009).

In research that observed groups at work, the way in which individuals participated was judged to be influenced by these factors in secondary classrooms (Bianchini, 1997; Chiu, 2000a). Students who were perceived by other students as having higher academic ability and of being friends with the other members in the groups were more likely to act as leaders within their groups, either in terms of having more on-task participation in the middle school science course observed by Bianchini (1997) or in terms of being judged as a leader by peers in a high school math classroom observed by Chiu (2000a).

Student perspectives on how friendship and perceived academic ability influence group work vary greatly by student and context (Esmonde et al., 2009; Theobald et al., 2017). High school math students interviewed by Esmonde et al. (2009) reported difficulties with perceived differences in academic ability. Many students reported a preference to work with peers who they perceived had the same level of academic ability. Among those who reported a preference for heterogenous groups, they still expressed frustration over difficult situations when working with peers of higher or lower perceived ability (Esmonde et al., 2009). These same students reported mixed feelings about working with friends. Students reported that having good peer relationships proved helpful in group work, but close friends could prove to be more of a distraction. However, students also discussed the importance of being comfortable in their group

(Esmonde et al., 2009). Relatedly, Theobald et al. (2017) found that undergraduate biology students reported higher levels of comfort when around friends.

In terms of linking these factors to student achievement, results are less clear (Bianchini, 1997; Chiu, 2000a; Theobald et al., 2017). While Chiu (2000a) could not establish a link between these social factors and student achievement, Bianchini (1997) did. Bianchini (1997) found that students with higher levels of peer friendship and perceived academic ability performed better in the course than their peers. Theobald et al.'s (2017) finding is less directly related, as they found that students in groups where one person dominated discussion during the class session performed worse on an after-class assessment. However, a dominator of discussion within a group and Bianchini's (1997) description of students having more on-task participation could be related. Thus, while the link between these social factors and student experience is strongly suggested, the link between these factors and achievement is weakly suggested.

Gender, race, ethnicity, and sexual orientation. Research on the effects of identity-related social factors does suggest some influence on group work in STEM classrooms, but no clear consensus on whether or how each factor influences group work has been found (Chiu, 2000a; Cooper & Brownell, 2016; Sullivan et al., 2018; Thompson & Sekaquaptewa, 2002)

Gender. Studies considering the effects of gender on groups do not yet suggest any single conclusion (Chiu, 2000a; Esmonde et al., 2009; Heller & Hollabaugh, 1992). Chiu (2000a) found that gender did not have a statistically significant effect on either perceived leadership roles or the score on group's problem solution in his study using group observations of high school math students. Esmonde et al. (2009), in interviewing high school math students about their group work experiences, found that students – both boys and girls – consistently reported that boys were more likely to take on leadership roles in their groups. In this, it seems possible that a student perspective may identify ways that gender influences group work that an outsider perspective cannot necessarily identify. Similarly, in a study of undergraduate physics courses, Heller and Hollabaugh (1992) found that groups composed of two men and one woman tended to collaborate worse than all other gender combinations, with the two men typically dominating discussions.

The influence of gender on the achievement of students in groups is not well understood either (Keller et al., 2020; Laursen et al., 2014; Sullivan et al., 2018). In inquiry-based learning, we have the previously-discussed contradictory findings of Laursen et al. (2014) and Keller et al. (2020). While inquiry-based learning incorporates small group work, there are other components to implementing it that could be contributing to the existence or nonexistence of a gender gap in these studies.. Working in an undergraduate introductory biology class, Sullivan et al. (2018) assigned students to groups with varying female-to-male gender ratios ranging from 0:4 and 4:0; they found that as the female-to-male ratio increased, the performance of all members in the group (not just the women) increased. Taken together, these studies paint a complicated picture for how a student's gender may influence their learning when working in small groups.

Race and ethnicity. Several of the studies that considered gender also considered the effects of race and ethnicity. Chiu (2000a) did not find a link between race and achievement or the participation of individuals. Participants in Esmonde et al.'s (2009) study reported mixed influences of race and ethnicity on group work. While most students reported a preference for heterogenous groups, there were also frequent reports of difficulties when groups included members of different racial and ethnic backgrounds. Black and Latinx students reported struggles in being treated as equal by white peers in their groups. Moreover, white students were reported as frequently taking leadership roles in mixed-race or mixed-ethnicity groups (Esmonde et al., 2009).

LGBTQ identities. Very limited literature specifically looks at the experience of LGBTQ individuals in classrooms employing small group work. In a qualitative study, Cooper and Brownell (2016) interviewed seven students who identified with various LGBTQ identities (including at least one person identifying in each of the categories of gay, lesbian, bisexual, asexual, queer, and transgender) about their experience in an undergraduate biology class using groupwork and active learning strategies. The study was focused on finding trends in their reflections after the course, with the researchers conducting semi-structured interviews with the participants focused on how their LGBTQ identities interacted with their learning and experiences in the class.

While the students in this study did not report active discrimination against them by their peers, they did generally perceive a level of homophobia from their peers who did not have a LGBTQ identity. They reported that working in teacher-assigned groups could be more discomforting than getting to choose their groups, as they might be working with new people and would have to re-assert their identity with these new peers. On the other hand, they reported that the increased socialization offered by a class using groupwork gave them more opportunities to connect with other LGBTQ peers (Cooper & Brownell, 2016).

Limitations of these studies. From these studies, it seems likely that friendship and perceived academic ability play large roles in small group work at the secondary level, and there's some suggestion that students at least report this at the undergraduate level as well. On the other hand, in terms of issues of identity, the findings of these studies are inconsistent and at times contradictory. Moreover, much of the research cited here was conducted in environments similar to but distinct from undergraduate mathematics classrooms. While there are certainly similarities between these environments and undergraduate math classrooms, the differences inherent in those learning environments limit their applicability.

Additionally, at a theoretical level, most of these studies are inconsistent with the framework I've adopted for this study. Other than Esmonde et al. (2009) and Cooper and Brownell (2016), all studies that considered gender considered it synonymously with sex. Only Cooper and Brownell (2016) considered any gender identities outside of the male-female binary. On the other hand, only Esmonde et al. (2009) conducted an intersectional analysis, by relating the multiple identities each individual student held. Thus, the review of literature in this area serves in part to illustrate how little is established regarding the influence of gender identity and sexual orientation on small group work in undergraduate math classrooms.

CHAPTER 3: RESEARCH METHODS

This study used qualitative research methods to explore the behavior of undergraduate students in small groups in mathematics classrooms. The focus of this analysis was on how gender identity or sexual orientation, among other social factors, might play a role in student behavior and experiences in group work.. This chapter outlines the purpose for the study and research question, describes how data were collected, and identifies how the collected data were analyzed to answer this question.

Purpose of the Study and Research Questions

As previously established, while research is clear that active learning strategies like small group work improve outcomes of undergraduate mathematics classes, literature that considers the experience of students participating in group work and ways in which particular implementations of group work affect students differently is less clear. Most studies that exist have either used only classroom-collected data (observations and/or assessment scores) or only post-classroom data (surveys and interviews) to understand how students experience small group work. Understanding the way that individual students experience and participate in small groups and how that relates to what can be seen by observers in the classroom would be informative to instructors of undergraduate mathematics wishing to transition from lecture-style instruction to more active-learning based classrooms in helping to identify and potentially address groups that may be working in a way to reinforce inequities in STEM. While literature analyzing small group work in elementary and secondary education may provide some insights, undergraduate instruction has significant differences as a learning environment from elementary and secondary schools. Thus, additional study at the undergraduate level is needed to understand this environment in particular. Moreover, findings at the undergraduate level alongside findings at earlier grade levels could allow for the development of a more consistent, universal understanding of group work.

A separate concern is an apparent diversity problem in STEM fields. As discussed in chapter two, women show less interest in STEM and leave STEM majors at higher rates than men, despite the lack of a large gender-based achievement gap. While less data on LGBTQ individuals in STEM exists, what there is suggests some level of discomfort and/or discrimination exists for these individuals when in STEM environments. Since individuals' interest in STEM careers may be influenced by their experience in STEM classes, it is reasonable to evaluate instructional strategies in the context of how individuals of different gender identities and sexual orientations experience them. Understanding how instructional techniques affects individuals of different gender identities and sexual orientations could be used to recommend ways for instructors and administrators to foster STEM classrooms that encourage diversity in STEM recruitment.

With these two goals in mind, my primary research question for this study prior to data collection was: do gender identity and sexual orientation influence how individuals experience and participate in small group work in an undergraduate introductory mathematics class, and if so, how? In literature reviewed in the second chapter of this thesis, 'influence' was defined broadly, and studies reviewed often were focused on achievement or included achievement as one of the factors being studied. For the purposes of this study, the focus is instead on how the process of collaboration may differ within these environments. That is, this study looked at how students' actions and interactions with their group members along with their perceptions of those actions. While there may or may not also be effects on achievement or beliefs about mathematics, those are beyond the scope of this study.

To understand how an individual experiences and participation in small group work could be influenced by these factors, I observed students at work and looked at their behaviors to understand how each student participated in the group and how the group was collaborating. Then, students were interviewed individually in stimulated recall interviews to discuss their group work experience to understand what students' perspectives were on how their experiences in group work may or may not be influenced by gender identity or sexual orientation.

Unfortunately, as detailed in the subsequent findings chapter, I was unable to recruit a sufficient number of LGBTQ participants to make claims about their experiences. One asexual student did participate in this study, but in a group of students that largely did not interact with each other during the scheduled observation session. This student did not agree to a follow-up interview. Yet, when conducting follow-up interviews across students, students offered interesting insights to their group work experience on a number of factors beyond gender identity and sexual orientation, including social unfamiliarity and perceived academic ability. Thus, through the process of data collection and analysis, the research questions for this study changed. The new research questions that developed during data collection are: 1) how do students interact with each other when working in small groups in an undergraduate math class; and 2) what social factors can explain productive and unproductive interactions? Secondarily, this study still explores the original question about gender identity and sexual orientation – but with an understanding that the limited diversity of the study sample limits the claims that can be made, particularly with regards to sexual orientation.

Phase One: Observations

The initial phase of this study consisted of a round of observations of students working in groups in undergraduate precalculus courses. This phase of the study included the distribution of two surveys prior to the observations: an initial interest survey to recruit participants, and a survey of demographic information and mathematical beliefs, distributed on the day of the observation. Three groups were observed, and the data from these groups were coded and analyzed using Chiu's (2000b) taxonomy.

Participant recruitment. Participants were recruited from undergraduate precalculus courses at a public university in the northeast. The courses have approximately 75 students each meeting three times per week in a lecture section and is led by the faculty instructor. Each lecture section has three associated recitation sections of 25 students each; these are led by the teaching assistant and meet twice per week. These classes use small group work as an instructional strategy on a regular, recurring basis across both lecture and recitation sections and with all instructors. I distributed an initial interest form (see Appendix A) in three sections of precalculus early in the semester. Ultimately, 21 students across three lecture sections returned their initial interest forms indicating that they would be willing to participate in the study. Based on the distribution of these students across sections and logistical needs of myself and the instructors, I attended one lecture and two recitation sections to conduct the first phase of data collection.

Data collection. Immediately prior to each small group observation, students were asked to complete a demographic and mathematical beliefs survey (Appendix B). This survey asked students to first rate their agreement with a series of statements from the modified Fennema-Sherman beliefs scales on Confidence in Mathematics and Learning Mathematics with Others (Fennema & Sherman, 1976; Herzig & Kung, 2003) on a six-point Likert scale; once aggregated, scores closer to 1 indicate more negative views of working on math in groups or of their own mathematical ability, while scores closer to 6 indicate more positive views of working on math in groups and of their own mathematical ability. These scales have often been used to understand how student attitudes towards and beliefs about mathematics are influenced by participation in mathematics classes using different instructional techniques (Herzig & Kung, 2003; Murphy, Chang, & Suaray, 2016), often with quantitative tests applied. In this study, the scores on these scales were used at the individual level to contextualize broader patterns observed in the students' collective work. The second half of the survey asked students to answer several demographic questions, including questions beyond the focus of this study (e.g. race and ethnicity, major, year in school). The use of additional demographic questions beyond the focus of this study was to avoid priming students to thinking about gender identity and sexual orientation in regard to their mathematics performance.

Following completion of their surveys, students began to work in small groups on their respective section's regularly assigned materials. This group work was documented using audio and video recording equipment. While set ups were established for multiple groups, the number of students who actually attended each section during which the observations were conducted was insufficient to have multiple groups – one section had only two opted-in attendees, while the other two had four opted-in attendees. As such, three groups were observed in total. These groups were observed for the entire duration of their respective class periods (50 minutes apiece); the actual amount of time students spent on group work on the assigned tasks for the day that was coded and analyzed varied from group to group. One group did not collaborate at all on the assigned tasks, while the other two worked together for about 30 minutes each on the assigned tasks. Using the audio and video recordings, I created transcriptions that included both students' statements and descriptions of relevant non-verbal interactions, such as pointing to indicate the location of something.

Data analysis. While many analytical tools exist to analyze collaboration, social interaction, and small groups, this study used the taxonomy described by Chiu (2000b). Chiu's (2000b) taxonomy is specifically designed for understanding the actions and interactions among students working together in small groups on math problems – thus, it is particularly applicable to the situation described in this study. While it has not historically seen much use outside of Chiu's own work, newer studies of group work in undergraduate mathematics are also using this taxonomy (cf. Quinn, 2020). Chiu's taxonomy categorizes each action along three dimensions, with each dimension having three possible categories. This allows for multiple levels of granularity of analysis – from looking at actions based on only one category of one dimension, to the potential for statistical analysis of different groups of participants based on the number of actions of certain types given a sufficiently large data set.

As discussed earlier, the three dimensions of Chiu's (2000b) taxonomy are the evaluation of previous action (EPA) dimension, the knowledge content (KC) dimensions, and the invitational form (IF) dimension. The evaluation of previous action dimension (Chiu, 2000b, pp. 29–30) regards how the individual's action relates to action by the previous individual. In this dimension, an action can be supportive of (+), be critical of (-), or be unresponsive to (0) the previous action. The knowledge content dimension (Chiu, 2000b, p. 31) describes the mathematical content of the action and how it relates to the problem at hand. An individual's action can be characterized in this dimension as a contribution (C), a repetition (R), or a null content actions (N). The invitational form dimension (Chiu, 2000b, pp. 31–32) describes the degree to which the action does or does not invite follow-up from other group members. Under this dimension, an action can be characterized as a statement (_.), a question (?), or a command

(!). Throughout the findings chapter, transcripts of data will include the relevant codes for each line.

Following the transcription and coding of each group's work, the number of actions assigned in each category in each dimension by each student were tabulated and summarized together to provide a clear picture of how each person in the group participated. Each student's participation in the group was considered for trends both in overall participation and types of actions, as well as for comparisons between how the individual members of the group participated. With these overviews in hand for each group, I was then able to use these to as a guide to return to the transcripts and identify interactions of interest, in preparation for the interview phase of the study. Note that for Group 3, these analysis methods were not used. This is because the group did not collaborate on the day's assigned work and instead remained silent through most of the observation. As such, the content of their follow-up interviews was primarily about this *lack* of interaction, as there was very little interaction between the group's members.

Phase Two: Interviews

For the second phase of the study, follow-up interviews were conducted with seven of the ten students who participated in the first phase of the study. Parts of these interviews were stimulated recall interviews (O'Brien, 1993; Williams, Lopez Torres, & Barton Odro, 2020), where students were shown clips from their own group interactions and asked questions about these interactions. As such, preparing for these interviews required returning to the coded data to select interactions of interest, using Chiu's (2000b) taxonomy and the overview tables for each group as a guide. Then, I invited all participants in the study to interviews which were conducted one-on-one. These interviews were coded for which ideas participants brought up in terms of each interaction, with a focus on the data from the stimulated recall interviews.

Preparing for the interviews. Selecting interactions for the stimulated recall portion of the interviews was a systematic process. Based off of the overview of each group generated through my initial analysis and my impressions of the groups from coding the transcripts, I came to three different categories of interactions that I selected.

The first category of interactions I selected were interactions that reflected some broader trend in one or more student's actions in the overview generated by my initial analysis. For example, when a student made more actions coded as contributions than their peers, then the transcripts were searched for an interaction where that student was making contributions and their peers were not (or were making fewer). I selected these interactions because they were representative of a larger pattern in the data; thus, what students had to say about these interactions would give me insight about the group's collaboration on the date of observation as a whole.

The second category of interactions I selected were ones that reflected poor collaboration. Interactions showing poor collaboration were defined at this stage as interactions in which most actions were coded as unresponsive or negative on the EPA dimension and/or null on the KC dimension. I operationalized poor collaboration this way because interactions with few supportive responses could indicate that the group was not building to any consensus, and interactions with fewer contributions and repetitions may not be focused on the mathematics enough to be effective. These interactions were selected because I wanted to understand how the participating students experienced and felt about interactions where they struggled to build consensus. Moreover, if social factors were a barrier to effective collaboration, this could only come out in discussion of moments of poor collaboration.

Conversely, the third category of interactions that I searched for were interactions that appeared to reflect strong collaboration. This was operationalized by looking for interactions which were mostly supportive or a mix of supportive and critical on the EPA dimension, mostly contributions or repetitions on the KC dimension, and a mix of statements and questions on the IF dimensions. I operationalized good collaboration this way because supportive actions, discussion of mathematical ideas, and invitations to participate are necessary parts for a group to come to a consensus. I wanted to understand what led students to be able to collaborate effectively in these interactions, from their own perspective.

Once I identified these categories, I returned to the transcripts for each group to identify two to three interactions that fell into one or more of these categories for each group. Through this search, specific interactions were selected to be taken from the observational videos, clipped, and shown to students during their interviews to discuss the mathematical and social factors at play in those interactions. Once selected, the actions each participant took were reviewed again to understand what role each participant was playing in that interaction using Chiu's (2000b, p. 35-36) alignment of actions to the roles of facilitator, proposer, supporter, critic, and recorder. Within each interaction, each student typically played only one or two roles.

Data collection. After interactions were selected, follow-up interviews using the interview protocol in Appendix C were scheduled with seven of the participating 10 students; the three remaining students did not respond to invitations to schedule an interview. These interviews were scheduled for 30 minutes each, though the actual length of each interview varied from a low of 8 minutes to a high of 29 minutes. Part of the variation in interview length is explained by the number of clips. For Group 1, two clips were used; for Group 2, three clips were used; for Group 3, no clips were used, as that was the group that did not interact with each other. A summary of who was interviewed, how long they were interviewed for, and when the interviews took place relative to the initial observation is presented in Table 3.1. Pseudonyms are used for all participants in this study.

Table 3.1: Summary of Interviews

Group	Participant	Interview?	Time Between Observation & Interview	Length of Interview
1	Carlton	N	N/A	N/A
1	Flora	Y	7.5 weeks	16 minutes
1	Leo	Y	8.5 weeks	15 minutes
1	Jim	Y	7 weeks	17 minutes
2	Krista	Y	5 weeks	30 minutes
2	Steven	Y	6 weeks	29 minutes
3	Josh	Y	7.5 weeks	8 minutes
3	Mary	N	N/A	N/A
3	Leticia	N	N/A	N/A
3	Eva	Y	7.5 weeks	11 minutes

The goal of these interviews was to gather more data on the student's perspective of the selected interactions, to better understand how they viewed their role in the groups, and to also address whether or not gender identity and sexual orientation had ever influenced their experiences with group work. During the follow-up interviews, students were first asked to reflect generally on their thoughts and feelings regarding group work in their current math classroom. Then, in the format of a stimulated recall interview, they were presented with clips of interactions from their own group's work, selected as described in the earlier sections. Students were asked to describe the events shown in terms of how they occurred from their own perspective. Students were then asked whether they felt social factors or mathematical content were the greater influence on the way the episode resolved; when students asked for clarification, I elaborated by explaining social factors to be anything related to how they were interacting with their peers, while mathematical content was referring to the question at hand and the mathematical concepts involved. Follow-up questions were asked when students answers were short or unclear to gather a more complete understanding of their perspective. After reviewing several episodes, students were then asked a series of more general questions regarding their participation in small groups in the class and whether they could identify any ways in which gender identity and sexual orientation influenced group work. Note that for students in the group that did not interact with each other, no video clips were used; they were simply reminded by the interviewer that they had not interacted and asked to discuss the mathematical and social factors that influenced that.

Data analysis. These follow up interviews were coded using grounded theory-style (Willig, 2013) open coding to identify themes in the social and mathematical factors that they discussed in response to questions about each specific interaction that they were shown. Responses to these questions were compared with the individual's actions and roles assumed during the interaction and to responses of other students about that interaction to paint the fullest possible picture of *what* happened for each student in that interaction and *why* it played out the way it did. Responses to questions about gender identity and sexual orientation in small group work were coded into one of a handful of categories based on whether the student indicated that they thought it was *possible* for gender identity and sexual orientation to influence group work in math class and whether or not they had *specific* moments that they felt that had occurred.

Summary of Research Methods

Ultimately, the questions I explored in this study were 1) how do students interact with each other when working in small groups in an undergraduate math class?; 2) what social factors can explain productive and unproductive interactions?; and 3) do gender identity and sexual orientation influence how individuals experience and participate in small group work, and if so, how? To explore these questions, I conducted a study in two parts. The first part was to observe students working in small groups in their undergraduate precalculus courses. The second part was to conduct follow-up interviews including structured recall portions to understand each student's perspective on their work and the work of their peers from the date observation. I combined my analysis of their group work with their interview responses to explore my research questions and how they might be answered for each group. The next chapter explains the findings that were produced from this collection and analysis.

CHAPTER 4: FINDINGS

In this chapter, I present the analysis of the collected data in two parts. The first section describes the results from the group observations and from the stimulated recall portions of the interviews. This entire section addresses my first research question, exploring how students interact with each other when working in small groups in an undergraduate math class. The discussion of each selected interaction for each group addresses my second research question, exploring what social factors can explain productive and unproductive interactions. In Group 2's third selected interaction, I address the third research question, exploring how gender identity and sexual orientation may influence how individuals experience and participate in small group work. The second section describes student responses to more general questions from the interviews about gender identity and sexual orientation in math class. This section primarily addresses the third research question. This chapter concludes with a brief summary of the key findings that are discussed more in the final chapter.

Group Observations & Stimulated Recall Interviews

In reporting the findings from the group observations and stimulated recall portions of the interviews, I focus on one group of observed students at a time. For each group, I first describe the students and give a broad overview of the actions that they took within the observation, as coded with Chiu's (2000b) taxonomy. These summaries are followed by selected excerpts from the individual interview transcripts with students' thoughts about the group interactions.

Group 1: Carlton, Flora, Leo, and Jim. The first group observed consisted of four students – Leo, Jim, and Carlton, all men, and Flora, a woman. None of these students identified as LGBTQ. These students were observed during their regularly-schedule precalculus lecture section, where time was divided between lecture, small group work, and whole class discussion.

The section met in the afternoon and was attended by about four dozen additional students. Flora and Leo had known each other prior to the observation and each did not know Jim or Carlton; Carlton and Jim did not know any of the other participants. Three additional students had opted-in to the study but were not present on the date of observation. Throughout the observed session, students worked on problems on function composition; the complete problem set can be found in Appendix E. Table 4.1 shows an overview of the group's actions using Chiu's taxonomy, along with their aggregate scores on the Learning with Others and Mathematical Confidence scales.

Table 4.1: Summary of Group 1 Actions & Fennema-Sherman Scale Scores

		Carlton		Flora		Leo		Jim	
		No. of Actions	% of Total	No. of Actions	% of Total	No. of Actions	% of Total	No. of Actions	% of Total
EPA Evaluation of Previous Action	Supportive (+)	9	64.29%	20	57.14%	26	46.43%	28	48.28%
	Critical (-)	2	14.29%	8	22.86%	8	14.29%	9	15.52%
	Unresponsive (0)	3	21.43%	7	20.00%	22	39.29%	21	36.21%
KC Knowledge Content	Contribution (C)	2	14.29%	15	42.86%	20	35.71%	24	41.38%
	Repetition (R)	5	35.71%	9	25.71%	14	25.00%	13	22.41%
	Null (N)	7	50.00%	11	31.43%	22	39.29%	21	36.21%
IF Invitational Form	Statement (_)	11	78.57%	30	85.71%	41	74.55%	52	89.66%
	Question (?)	3	21.43%	5	14.29%	14	25.45%	6	10.34%
	Command (!)	0	0.00%	0	0.00%	0	0.00%	0	0.00%
	TOTAL	14		35		55		58	
Modified F-S Scales	Learning w/ Others	4.25		3.42		4.58		3.75	
	Confidence	5.00		4.42		4.92		3.67	

Overall, we can see that Leo and Jim participated more than Flora, and the three of them all participated far more than Carlton. When taking their seats at the group's table, Carlton sat further away from the other three students and often did not acknowledge their conversations. Carlton was the only one of the four students in this group who did not agree to participate in a follow-up interview. For Flora, the reason for why she took fewer actions is less apparent in these data. One possible reason is indicated by her middling score on the Learning with Others scale – perhaps she simply has a lesser propensity towards working in groups. However, this contrasted with Carlton's higher score on this same scale. Along the Invitational Form dimension, Leo asked many more questions than the other participants; Leo was often the student initiating a discussion by asking a clarifying question about the problem or task. Differences along the Knowledge Content dimension seem minor. Along the Evaluation of Previous Action dimension, Flora made far fewer unresponsive actions than Leo or Jim.

Flora, Jim, and Leo each participated in an individual interview approximately a month and a half following their observation. During these interviews, two interactions of interest were clipped from the original video and shown to each participant. These were presented in the interviews in chronological order, with each participant discussing the events from the interaction after watching the respective clip.

Group 1, interaction 1. This first interaction occurred early in the observation. The students had been working independently for a few minutes on the first problem in Appendix E, when Leo asked a question to verify his answer on part d of that problem, "define a function h that inputs the square's side length x (in inches) and outputs the square's area A (in square inches)" (Carlson, Oehrtman, & Moore, 2018). Initially, Flora and Jim respond with confusion over which problem Leo is referring to and ask to cycle back to part c ("How does the square's side length change as the perimeter changes from 6 inches to 20 inches") (Carlson et al., 2018). Mathematically, on part c, students are expected to recognize that an increase of 16 inches to the perimeter corresponds to an increase of 4 inches in the side length; on part d, students are expected to recognize that if x is the side length, then $h(x) = x^2$ gives the area. Jim gave a detailed explanation of his process, and Leo and Flora both expressed some agreement with his solution. Carlton did not participate in this interaction, though he appeared to be working in his workbook.

The transcribed dialogue around this interaction along with the appropriate codes under Chiu's taxonomy are presented in Table 4.2; the video clip was about thirty seconds in length.

Table 4.2: Group 1, Interaction 1, Transcript and Codes

Student	Action	EPA	KC	IF
Leo	So is it three over four squared yeah?	0	C	?
Flora	For the function notation?	-	N	?
Leo	For d	+	N	_
Jim	Uhh	-	N	_
Flora	Well I think it's like, you're solving g as in like six over four	-	C	_
Leo	I thought you were solving h and then [mumbling]	-	C	_
Jim	Wait what problem are you on?	0	N	?
Leo	d	+	N	_
Flora	Well what did you get for the function notation in c?	0	N	?
Jim	I got g of fourteen equals three and a half	+	C	_
Leo	How is that [overlapping] I just wrote down g of [overlapping]	-	N	?
Jim	It's the same as g of fourteen is three and a half inches it is the same as g of twenty minus g of six which is the difference of the sides how much the sides changed	0	C	_
Leo	And then we just plug that into h of x	+	C	?
Jim	No um h of x is like a completely different function	-	C	_
Leo	Yeah I mean like h of x and then we put that in	-	R	_
Jim	Yeah then we put all those in in part [inaudible]	+	R	_
Leo	So it would be equal [inaudible]	0	N	_
Flora	oh yeah because you subtracted the both perimeters from each other and that's how you got the fourteen	+	R	_
Jim	yeah that's like the	+	N	_
Flora	and then you just divided that by four	+	R	_
Jim	right	+	N	_

Key	EPA:		KC		IF	
	+	Supportive	C	Contribution	_	Statement
	-	Critical	R	Repetition	?	Question
	0	Unresponsive	N	Null	!	Command

This interaction was selected for falling in two of the categories – being representative of broader trends within the group, and for appearing to show productive group work. In this interaction, Leo and Jim participated the most, while Flora made a few comments and Carlton did not participate, reflecting their cumulative rates of participation throughout the session. Moreover, the interaction has a number of lines coded as contributions and repetitions, a good mix of supportive and critical remarks, and a mix of statements and questions – meeting the criteria for potentially productive group work. Considering the alignment of actions to roles made in Chiu's taxonomy (2000b), we can see that Leo takes on the role of the proposer in this interaction by making the initial contributions to start the discussion. Flora acts as a facilitator by using a mixture of supportive and critical actions and by phrasing her critical actions as questions. Jim acts more as a critic, with more critical contributions presented as alternatives to Leo's ideas.

Flora, Leo, and Jim were each asked whether this interaction was best explained by social factors or the mathematical content. Flora and Jim both spoke about how the social factor of having the ability to collaborate and have this interaction at all made their work process different than if they had been working alone on the same problem. For example, Flora said, "sitting in a social situation like this, with other classmates around who are doing the same exact thing, it was easiest to just reach out to them and say, 'Hey, what'd you get? Does this look correct?'" while Jim said, "just comparing to if we hadn't been working together, like we would have maybe like been working on the problem and wouldn't have understood it and that would have been it." On the other hand, Leo stated that the interaction was centered on and best understood through thinking about the mathematics involved, without going into additional detail

Group 1, interaction 2. While the first interaction ended with the participating students all in agreement, this second interaction was somewhat more contentious. In the final segment of small group work during the observed class, students were asked to discuss problem 5 parts a and b from Appendix E. This problem refers back to two functions from problem 4: *f*, which takes expected attendance as an input and outputs expected revenue, and *g*, which takes temperature in degrees Fahrenheit as input and gives expected attendance as its output. Problem

5a asks students to explain the meaning of $f(g(x))$ while 5b asks students to explain why $g(f(x))$ does not have a valid meaning. The mathematically correct interpretation of $f(g(x))$ is that it gives the expected revenue given the temperature in degrees Fahrenheit. On 5b, students are meant to recognize that there is no valid meaning of $g(f(x))$ because g takes temperature as an input, but f does not output temperature.

Jim and Leo mostly discuss the first part of the question amongst themselves, with Flora and Carlton not chiming in until the second half – where Jim and Flora both hold tight to their own understanding of the problem, while Carlton and Leo express a few statements of agreement or disagreement without getting into the mathematical details of the discussion. Their group discussion was ended by a whole-class discussion without the group reaching a resolution. The dialogue around this interaction along with the appropriate codes under Chiu's taxonomy are presented in Table 4.3; the video clip for this interaction was about two minutes in length.

Table 4.3: Group 1, Interaction 2, Transcript and Codes

Student	Action	EPA	KC	IF
Leo	All right, we're on five now.	0	N	_
Jim	I guess we just have to talk about this we don't have to find the answer.	0	N	_
Leo	All right, so.	0	N	_
Jim	So f of g of seventy means find g of seventy and whatever the value of g of seventy is we can use to find the f of whatever that is	0	C	_
Leo	Yeah the output of g seventy is the input of f	+	C	_
Jim	Yeah of f of g seventy	+	R	_
Flora	Yeah	+	N	_
Leo	Is that it?	0	N	?
Flora	Is uh that's the opposite	0	N	_
Jim	We need to do uh exercise four that's the fair carnival - so in the context of exercise four explain why...	0	N	_
Leo	So the temperature is affects the revenue	0	N	_
Jim	Yeah temperature is f oh so the temperature is 70	+	C	_
Leo	So the attendance - so they're giving us the temperature which gives us the attendance	0	C	_
Jim	The attendance of five hundred - I'm not gonna try to figure out the little...	+	C	_
Leo	Yeah and so that attendance shows the revenue	+	R	_

Table 4.3, Continued

Jim	Revenue would then be like thirteen hundred	+	C	_
Jim	I don't know why it says in b explain why g of f of seventy does not represent a real world quantity	0	N	_
Flora	Because it's like you have to find it's telling you like how much you're going to make and the fair is gonna tell you how hot the day is	-	C	_
Leo	Oh yeah you can't go backwards. can we go backwards?	+	R	?
Jim	Why not? If we figure out how much we make at the fair.	-	C	?
Flora	Well you can like technically go backwards but like in the real world however much you make at the fair isn't gonna tell how hot the day is	-	C	_
Carlton	Yeah that's true yeah	+	N	_
Jim	But however much you made at the fair tells you how many bookings	-	C	_
Flora	Right but that's still not gonna affect the heat of the day	-	R	_
Jim	You could make a reasonable guess that like if it's fifteen degrees why are they even having the carnival if it is fifteen degrees out.	-	C	_
Flora	That's why you only get like seven people there.	+	C	_
Carlton	Maybe it's in Canada.	0	N	_
Leo	Alaska somewhere.	+	N	_

Key	EPA:		KC		IF	
	+	Supportive	C	Contribution	_	Statement
	-	Critical	R	Repetition	?	Question
	0	Unresponsive	N	Null	!	Command

This interaction was selected because it fell into the category of appearing to show productive group work. Like the earlier interaction, the group members here engage in a diverse mix of supportive and critical, contributions and repetitions, and statements and questions. Based on that definition, and not considering the mathematics of the assigned problem, this interaction appeared to be productive. Considering Chiu's (2000b) alignment of roles with actions again, we see that Jim has the role of bringing forth the original proposal. Flora takes on the role of critic towards Jim's later ideas, with Jim also taking on the role of critic towards Flora's alternative explanation. Carlton and Leo play a supporter role throughout this interaction, though whose ideas they are supporting vary.

In discussing the social and mathematical factors at play in this interaction, Jim again felt that just being able to discuss the problem played an important role: “Probably if I was alone I probably would have continued to try to rationalize if I could reverse the function. So yeah, I mean, it's just helpful to have other people's point of view with things a lot of the time, I think.” Leo discussed how, being near the end of the class, the students were more familiar with each other and so were able to “talk more and more brave, I don't know, to talk with each other.” Flora felt that her own behavior in the interaction was best explained by her own understanding of the mathematics: “once I understood what was happening and I understood that they didn't understand what was happening, I was like, they don't know what's going on, I'll try and explain it.”

While this interaction had a greater number of negative responses to previous actions, these responses from Jim, Leo, and Flora indicate that they did not view this as a bad or unproductive interaction. Carlton’s participation, however brief, is also indicative that one might consider this interaction productive. The mathematical story here is different, particularly in relation to their discussion of the composition $g(f(x))$. The problem with this composition is that g takes temperature as an input, but f does not give temperature as an output; thus, the composition is not conceptually valid. The students’ discussion of this composition does not reach this point, with the group instead talking about going “backwards” on the functions and using the y-axis as an input and the x-axis as an output – interpreting $g(f(x))$ as though it were $g^{-1}(f^{-1}(x))$. While Flora objects to that interpretation, on the basis of a lack of a causal link from sales to temperature: “like in the real world however much you make at the fair isn't gonna tell how hot the day is.” Thus, we see that this interaction might have been socially productive but was not mathematically productive. My initial interpretation of the interaction using Chiu's (2000b) taxonomy was unable to detect this difference.

The follow-up interviews, which were conducted a month later, did not reveal any apparent change in understanding. While the participants were not directly asked about their mathematical understanding of the problem, none indicated that they had a different understanding than what seemed apparent in the clip. Flora's response in particular would suggest that she stands by her understanding of the prompt as initially posed. Moreover, each of the interviewed participants from this group indicated that this discussion was particularly memorable for them. Therefore, despite this discussion being socially productive and the participants feeling comfortable to express differing opinions, mathematically, it did not benefit their developing understanding of function composition.

Group 2: Krista and Steven. The second group observed consisted of two students – Krista, a woman, and Steven, a man. These students were observed during their regularly-scheduled precalculus recitation, which was held in the morning and was attended by around a dozen other students. An additional student had opted-in to the study but was not present on the date of observation. Krista and Steven had not previously worked together prior to this observation, and according to their responses during the post-observation interviews, they did not work together during any additional class sessions between their observation and interviews. They worked on selected problems on function composition, with the full problem set given as Appendix F. Table 4.4 shows an overview of Krista and Steven's actions using Chiu's (2000b) taxonomy.

Table 4.4: Summary of Group 2 Actions & Fennema-Sherman Scale Scores

		Krista		Steven	
		No. of Actions	% of Total	No. of Actions	% of Total
EPA Evaluation of Previous Action	Supportive	36	36.36%	46	59.74%
	Critical	6	6.06%	7	9.09%
	Unresponsive	57	57.58%	24	31.17%
KC Knowledge Content	Contribution	41	41.41%	18	23.38%
	Repetition	14	14.14%	10	12.99%
	Null	44	44.44%	49	63.64%
IF Invitational Form	Statement	83	83.84%	73	93.51%
	Question	15	15.15%	4	6.49%
	Command	1	1.01%	0	0.00%
	TOTAL	99		77	
Modified F-S Scales	Learning w/ Others	4.00		4.00	
	Confidence	4.33		5.25	

Along the Evaluation of Previous Action dimension, far more of Krista's actions were coded as unresponsive than for Steven. A partial explanation for this is that Krista often initiated discussions about problems, while Steven's actions more often were direct responses to Krista's. Along the Knowledge Content dimension, a majority of Steven's actions were coded as null. Many of Steven's actions were coded as such because they were simple statements of agreement or disagreement with Krista's actions, and so did not specifically refer to any mathematical idea. Finally, along the Invitational Form category, Krista asked far more questions than Steven. Again, this is partially explained by the fact that Krista was often initiating discussion, and this occasionally occurred with a question rather than a statement.

Both Krista and Steven participated an in interview approximately a month following their observation. During these interviews, three interactions of interest were clipped from the original video and shown to each of them. These were presented in the interviews in chronological order, with Krista and Steven each discussing the events from the interaction after watching the respective clip.

Group 2, interaction 1. This interaction occurred early on during the second group's observation. Krista and Steven had worked independently on the early problems of the assignment and had moved on to comparing their answers to verify them. When they came to a disagreement in their solutions on the final part of problem 55a, Krista questioned her answer and stated that she was "lost;" this was followed by Steven explaining his solution process step-by-step, physically referring to parts of Krista's written work from her paper. Mathematically, solving the problems in 55a simply involves correctly reading the graphs to compute the result of the function composition. A transcript of the dialogue making up this interaction along with the appropriate codes under Chiu's taxonomy are presented in Table 4.5; the clip for this interaction was about a minute and a half long.

Table 4.5: Group 2, Interaction 1, Transcript and Codes

Student	Action	EPA	KC	IF
Krista	Okay so did you get seven six?	0	C	?
Steven	Yup	+	N	_
Krista	Um, one three.	0	C	_
Steven	I got one one.	-	C	_
Krista	What did I do wrong? So for four you got um you got one?	+	R	?
Steven	Yeah I got one.	+	R	_
Krista	Okay so	0	N	_
Steven	Negative [mumbling]	0	N	_
Krista	[overlapping] So you go to three first right?	0	C	?
Steven	Yeah negative three in the f.	+	C	_
Krista	Oh	+	N	_
Steven	That's negative one and then you scooch it over to the g x equals negative one.	0	C	_
Krista	Okay, why am I getting so lost?	-	N	?
Krista	Wait wait wait	0	N	!
Krista	Okay negative three oh so you start there...	0	R	_
Steven	[pointing to Krista's paper] Negative three in that one [points again] negative three in the x, then you scooch it down to negative one, then you get a negative one over here, then you get a one.	0	R	_
Krista	Oh pssht okay, I was trying to do it like these ones. [points to page] I have no idea why. Okay anyway [laughter]	+	N	_

Table 4.5 Continued

	EPA		KC		IF	
Key	+	Supportive	C	Contribution	_	Statement
	-	Critical	R	Repetition	?	Question
	0	Unresponsive	N	Null	!	Command

This interaction was selected because it seemed to show productive group work. Again, like the interactions for Group 1, we see that actions are a mix of supportive and critical, contributions and repetitions, and questions and statements. In terms of the roles assumed by the participants, we see Krista takes in the role of the proposer by bringing forth her own answers to initiate the conversation. Steven acts as a critic to one of Krista's initial proposals, and then as a proposer in offering an alternate solution. Krista then acts as a facilitator for the rest of the interaction by asking Steven clarifying questions and making supportive and critical responses until she arrives at a point of agreement with Steven.

When asked in their corresponding interviews about whether mathematical or social factors explained this interaction, Krista and Steven both gave nuanced answers. While Steven initially said that social factors were the primary explainer, he elaborated by saying "…it was just a wrong answer on the math problem. And we figured that... We didn't really discuss the math problem... Or, well, we did solve it. And it was a simple solution to a simple mistake." His response emphasized the mathematical solution mismatch as being easily resolved and did not specifically mention any social factors that inhibited or supported the mismatch being resolved. He followed by saying "it would be a mix" of social and mathematical factors. Asked how the interaction could play out differently, he referred to the role that perceived differences in mathematical ability could play in the group, saying "if someone is better than someone else on a topic, and the person that doesn't understand the topic, trying to get help from the person that knows it, it would be difficult if the person that already knows it would be rude or thinking it as easy."

Krista's reflection on this episode was focused on how her unfamiliarity with Steven and her confusion with the mathematics both had roles in how this interaction played out: "I feel like that specific moment I was really just worried about the math, but I think generally working together as a whole…I had no idea who he was and I still have no idea who he was. So, I feel like the social factors going into it where it was being like possibly 'this guy's better at math than I am.'" Moreover, she explicitly links her unfamiliarity with Steven to how she perceives his mathematical ability, which she later, in discussing another interaction, linked to his gender.

Group 2, interaction 2. This interaction occurred near the middle of the observation. Krista and Steven were discussing how to start with problem 57, which tasked students with coming up with a decomposition for each given function. On 57a, the intended route is to decompose the function $h(x) = 3(\mathrm{x} - 1) + 5$ into functions $f(x) = 3\mathrm{x} + 5$ and $g(x) = \mathrm{x} - 1$, where $h(x) = f(g(\mathrm{x}))$. In discussing how to start solving the problem, Krista made many contributions of mathematical ideas, while Steven mostly responded positively but without making many of his own contributions. A transcript of the dialogue making up this interaction along with the appropriate codes under Chiu's taxonomy are presented in Table 4.6; the clip for the interaction is approximately one and a half minutes long.

Table 4.6: Group 2, Interaction 2, Transcript and Codes

Student	Action	EPA	KC	IF
Krista	For each function defined below redefine the function in terms of two new functions. What? Using function composition and function arithmetic...okay I don't know what that means...oh yeah no. This is basically just like this problem [points at another page]	0	C	_
Steven	Yeah it's the other page yeah [overlap]	+	N	_
Krista	It's just like, why did it word it like this?	0	N	?
Steven	It's just a complicated way of saying y.	+	N	_

Table 4.6, Continued

Krista	It's like let's just throw more English at you and hope that you can understand.	+	N	_
Krista	Okay so a wait no number one wait no it’s like going backwards is it	0	C	?
Steven	These look ugly	0	N	_
Krista	Yeah, wait, I have no idea what it is asking actually	+	N	_
Steven	For each of the functions below define that function in terms of two new functions f and g	0	R	_
Krista	Oh yeah, I see now I think	0	N	_
Steven	Oh yeah.	0	N	_
Krista	So I think it's like f of x would equal I don't know which one would be f and which one would be g but I guess it doesn't matter...be like three x plus five because you're taking out that like that like x minus one	0	C	_
Steven	Yeah	+	N	_
Krista	Then it would be like g of x is x minus one	0	C	_
Steven	That makes sense	+	N	_
Krista	[overlapping] I think… Then why did they throw the h in there?	0	N	?
Steven	[shakes his head and shrugs]	+	N	_
Krista	Like in the example one I don't know why they throw the h in...I don't know whatever.	0	R	_
Steven	That example just doesn't...	+	N	_
Krista	They said to use f of x and g of x so, oh well we're gonna do that I guess	0	R	_

Key	EPA		KC		IF	
	+	Supportive	C	Contribution	_	Statement
	-	Critical	R	Repetition	?	Question
	0	Unresponsive	N	Null	!	Command

This interaction was selected because it reflected the overall trends with this group. Krista made many more contributions and unresponsive actions than Steven did, while most of Steven’s actions were supportive and contained no mathematical idea; we see this clearly in this interaction. In terms of the roles assumed here, Krista’s actions place her squarely in the roles of proposer (as the person who suggests the initial ideas) and as facilitator (by asking questions to invite Steven’s participation). Most of Steven’s actions are positive responses to Krista’s actions, placing him squarely in the role of a supporter.

Asked about this interaction, Krista and Steven each said that mathematical factors played a greater role than social factors. Krista contrasted this interaction to earlier interactions:

Maybe in the beginning, before I got to know him, I was thinking more of, ‘oh, this is a guy. This is ... he could be really good at math.’ But I think once I got to know him, once we got started working on the math, I was more focused on the math.

So in this case for Krista, she felt that at this stage in their time working together, she was comfortable enough with Steven and thus able to collaborate effectively with him. Steven’s answer relied on the idea that he and Krista were on the same page in this interaction: “we were both reading it, we were reading the same words and were processing it the same. We were just both confused on where they got all these functions from when it was telling us to do something else.” While their explanations of the interaction are not identical, this is the interaction for which Krista and Steven viewed things most similarly.

Group 2, interaction 3. This interaction was over the final problem from the assigned tasks for the day, problem 65: “Let $160 = 2t + 4m$ and let $p = \frac{1}{4}tm^2$. Write a function k that gives p in terms of t” (Carlson et al., 2018). To solve this problem, I would start by solving the first equation for m in terms of t, and then combine that result with the second equation, to get the function $k(t) = \frac{1}{4}t(40 - t/2)^2$. Krista and Steven both expressed confusion over what the problem was actually asking them to do and how to arrive at the answer. Each suggested different strategies, and both were critical of each other’s suggestions. A transcript of the dialogue making up this interaction along with the appropriate codes under Chiu’s taxonomy are presented in Table 4.7; the clip is about three and a half minutes long.

Table 4.7: Group 2, Interaction 3, Transcript and Codes

Student	Action	EPA	KC	IF
Krista	So I think it means like k of p of t right...I think that's what it means	0	C	_
Steven	I think that makes sense. Yeah, I would assume so	+	N	_
Krista	So k of p of t so	0	R	_
Steven	alright [mumbling] write a function	0	N	_
Krista	so [pausing, both looking at their books]	0	N	_
Steven	I don't like this	0	N	_

Table 4.7, Continued

Krista	I know, I am trying to figure it out	+	N	_
Krista	So I think if you like made t into x then you'd wanna replace whatever the x is so you wanna replace whatever the t is right?	0	C	_
Steven	Would you try to solve for t ?	-	C	?
Krista	I don't think it wants us to solve, I think it just wants a function but its weird cause it does give one sixty equals	-	R	_
Steven	Yeah	+	N	_
Krista	But since it wants a function I'm wondering if you subtract that one sixty to make it two t plus four um minus	0	C	_
Steven	One sixty and then you've got - it's so weird.	-	R	_
Krista	I think what you do is you yeah i'm not sure if that's correct	0	N	_
Steven	I would try to solve for t	0	R	_
Krista	Four m minus one sixty and then k and p and t wait okay [mumbling] because there's an m in there too	0	C	_
Steven	I think that's just a random variable just they're always kind of...	-	N	_
Krista	But what I was trying to do is I was like trying to put in whatever that is into t but then who is to say you don't do that for m as well. Like, you don't know which would be your x value in your typical so that's not the answer.	-	C	_
Steven	I tried moving the two t to the other side and the 160 to the other and then dividing by two so that we get something with t and then you plug that in to p which gives you k, I think. That gives p in terms of t. Actually would you solve for m?	0	C	?

Key	EPA		KC		IF	
	+	Supportive	C	Contribution	_	Statement
	-	Critical	R	Repetition	?	Question
	0	Unresponsive	N	Null	!	Command

This interaction was selected because it appeared to show an example of poor collaboration. As per my operational definition, this interaction contains a lot of critical and unresponsive remarks and very few supportive ones, indicating that Krista and Steven are not coming to a consensus about how to solve this problem. Asked about this interaction, both participants stated that they did not collaborate as effectively as they could have, citing multiple mathematical elements and social factors that explained this difficulty. Both referred to their own tendency to work independently as a barrier; Steven stated that he doesn't "really talk what [he's] thinking during math," while Krista said that she was "trying to independently think about it first." They also both referred to their unfamiliarity with each other as being a barrier, with both saying that they would have likely interacted differently if they were friends with each other.

Krista also again brought up the role she felt gender and perceived mathematical ability were playing in her own behavior: “I maybe thought he was better at math than I am. And that, he just is a male in general and, I don't know.” Earlier, she had said that she thought this was maybe only a factor early in their collaboration, but this interaction was over the final problem from the day’s assigned tasks. This suggests that Krista’s own beliefs about gender may have played a greater role in her interactions with Steven than she initially thought. Asked to consider how things might have occurred differently, Krista again talked about ways she feels gender and mathematical ability could influence group work:

> I feel males do tend to be a little bit more assertive and obviously not – that is a big generalization… if it were someone I was working with that I was obviously getting more right answers than they were, I would be more inclined to just not listen to them… I don't know about his math background, I'm not comparing mine to that, but it did feel like we were pretty much on the same page about some things and with slight differences.

It’s not clear whether she felt that Steven was being “more assertive” in this case, but she does seem to indicate that upon reflection, she doesn’t believe there was a significant difference in her and Steven’s mathematical abilities – contrasting with how she says she might have felt in the moment.

Group 3: Josh, Mary, Leticia, and Eva. The third and final group to be observed consisted of four students – Mary, Leticia, and Eva, all women, and Josh, a man. Leticia identified as asexual, and none of the other students in this group identified as LGBTQ. These students were observed during their regularly-scheduled precalculus recitation, which was held in the morning and was attended by around a dozen other students. Three other students from the section had opted-in to the study but did not attend on the day of data collection. None of the students had worked with each other before the date of observation. They worked on the same problems as Group 2 (Appendix F). Table 4.8 shows each student's aggregate score on the two modified Fennema-Sherman scales used.

Table 4.8: Group 3 Fennema-Sherman Scale Scores

		Josh	Mary	Leticia	Eva
Modified F-S Scales	Learning w/ Others	3.00	2.75	5.42	4.42
	Confidence	3.42	4.00	3.08	5.33

Group 3 was unique in that the students did not collaborate with each other for most of the working time on the assigned problems. Of the 50-minute session, approximately 30 minutes of the class time was spent with the members of the group working independently in silence; Mary had headphones on for much of the class time. As students completed their work on the day's tasks, which were assigned out of a textbook, they would transition to working on online homework by taking out their laptops. During the latter part of the class, when most of the students had transitioned to the homework, there were a few conversations that occurred between pairs of students. However, because these conversations were not on the day's assigned material and were not among all of the students, these conversations did not represent the type of interactions that this study was designed to understand.

The participant's scores on the modified Fennema-Sherman scales provide some insight into possible reasons why the students did not collaborate much at all during their observed session. Mary and Josh's scores indicate a moderately negative view of group work – so perhaps they were simply not inclined to collaborate with their peers. Leticia's score on this scale, however, indicates a much more favorable view of group work – but her score on mathematical confidence indicates that she does not feel particularly strong in math. Unfortunately, for Mary and Leticia, this is all the data available to provide clues about their behavior. Only Eva and Josh agreed to participate in a follow-up interview.

When asked about why their group members didn't interact, Eva's and Josh's ideas differed. Eva believed it came down to comfort and familiarity:

> I think it was definitely because we didn't know each other and we did not feel comfortable…it's definitely really hard to collaborate with people you don't know because you feel uncomfortable. You feel like people are going to judge you, and really they're not going to judge you.

She also noted that the recitation section in which the group worked was typically "pretty quiet…I know I sit next to someone, and I actually barely talk to her." Josh indicated that he felt some confusion regarding the instructions: "I think the interpretation that we all sort of came to on our own was that we should only talk if there was an issue." However, Josh also said that students in that recitation tended to work independently: "Normally, we just work on our own especially me."

Summary of these data. To summarize the findings of this section, we saw various social factors, especially unfamiliarity and gender, lead to difficulties in effectively collaborating for Groups 2 & 3. Group 3 did not collaborate at all, likely due to their unfamiliarity and a lack of collaboration in the classroom, while Group 2 did not collaborate as effectively as they could have due to the unfamiliarity between Krista and Steven and Krista's beliefs about the relationship between gender and mathematics. For Group 1, while their interactions seemed productive to the participating students and in analysis with Chiu (2000b), they failed to arrive at a mathematically correct result in one of their interactions.

Gender and Sexual Orientation – Across Interviews

All interview participants were asked near the end of their interview about whether they had ever experienced or witnessed any interactions in their math class that they thought were affected by gender identity or sexual orientation. Like earlier in the interview, Krista indicated that she had experienced instances where she felt her gender caused peers to view her actions differently: “[in the lecture] I sit beside some boys and they typically ... they admit that they're not good at math and they don't really like math and all, that kind of stuff…these boys, even though they say and they admit that I'm probably stronger in pre-calc than they are, they still tend to try to correct me sometimes if I ... when I usually end up being right.” While Krista indicated that she identified as straight and had not experienced any times when her own sexual orientation affected group work, she did discuss how a “homosexual boy” worked effectively with a group of girls, tying it back to her experiences and beliefs regarding gender. Flora also felt that her interactions with male students played out differently than interactions with other students, particularly in her recitation section: “I do know I sit on a side where there's mostly guys, but they do seem to have a different air about them and a different level of interest in asking for help from me than another guy in the classroom.” Leo indicated that while he had never witnessed or experienced any such events in his precalculus class, he did refer to a vague recollection to an instance in his high school math class where someone’s gender identity or sexual orientation impacted a group’s work.

All of the other students indicated that they had never witnessed gender identity or sexual orientation have any effects in small group work in their precalculus class. Several students, such as Josh, Jim, and Steven, referred to the idea that math is a ‘gender-neutral’ subject; for example, Jim stated “I feel like the whole, the sexual orientation, gender identity kind of like isn't really a part of what we're even thinking about. And plus with math, it's just all about numbers anyway." Eva was the only woman to be interviewed to indicate that she had never felt her gender identity had influenced her small group work experience: "not ever in a classroom setting. I've always felt very equal, and the teachers make people feel very equal."

Summary of Findings

The ways in which students collaborated varied greatly across groups. In Group 1, students' behavior on the date of observation and responses to interview questions indicate, except perhaps for Carlton, that the group members felt they collaborated effectively and that being able to collaborate was beneficial to their mathematical work that day. However, their collaboration led to a mathematically unproductive understanding of one of the key topics from the day's material – thus, their interaction was socially productive but mathematically unproductive. In Group 2, the participants struggled to collaborate with each other effectively. While each member of this group cited unfamiliarity with each other as a factor in this, Krista also believed that being a woman working with a man had an impact here as well. In Group 3, students did not work together, and both students who agreed to be interviewed indicated this was at least partially due to their unfamiliarity with each other. So, we saw the social factor of unfamiliarity have a strong impact in two groups, while the mathematical content drove the interactions of the other group. Finally, across all interviews, two out of three female students reported that they had experienced or witnessed cases where their or another student's gender identity or sexual orientation influenced small group work in their math class, while the male students in the study indicated that they had never witnessed or experienced this.

CHAPTER 5: DISCUSSION

From the findings reported in the previous chapter, I believe there are three key takeaways related to the overall research question for this study: 1) social unfamiliarity with one another can negatively influence a student's experience within a group and the group's overall ability to collaborate; 2) student gender identities and beliefs about how gender and mathematics are related can also play a role, especially when students are unfamiliar with each other; and 3) students may work together in ways that seems socially productive, but are not mathematically productive. This chapter explains each of these takeaways in more detail, with a discussion of the limitations of these findings, how future researchers and instructors may use these findings in their work, and a conclusion highlighting why these findings are significant.

Familiarity, Friendship, and Group Work

At the secondary level, work by Bianchini (1997) in science and Chiu (2000a) and Esmonde et al. (2009) in mathematics suggested that students may work more effectively when in groups of friends and when group members are perceived to be at the same or similar ability levels, both of which are related to how familiar group members are with one another. In trying to extend these ideas about the importance of friendship and familiarity in working with peers at the undergraduate level, one cannot assume that they automatically apply in the same way. At the secondary level, students often work with the same peers (or a subset thereof) for most, if not all, of each school day. By contrast, undergraduate students may only see their group once or twice a week for an hour each. While Theobald et al. (2017) did find that undergraduate students reported being more comfortable when working with friends and did better on an achievement assessment tied to that survey when they reported being more comfortable in their group, the post-hoc nature of their study did not provide an in-classroom context for how the students' groups worked.

This study's findings provide evidence that familiarity and friendship remain important in productive group work in undergraduate mathematics classrooms. Based on the observations and interviews, it is clear that unfamiliarity between the students was a social factor in Krista and Steven's mathematically unproductive interactions. Each of them cited their unfamiliarity with the other as preventing them from collaborating effectively, and we see that play out in their occasional inability to fully consider and take up each other's ideas. Meanwhile, Group 3's complete lack of collaboration can also be partially attributed to the fact that they had not ever worked together before. Group 1, which did not encounter these same types of difficulties, had two members who were familiar with each other before the observation (Flora and Leo), which may have limited the role that unfamiliarity played within the group. The way these groups worked (or did not work) together shows that social unfamiliarity among students can sometimes contribute to poor collaboration and negative experiences for the participating students. Theobald et al. (2017) had previously found that students reported higher levels of comfort when working with friends in groups in an undergraduate biology class, and my finding here is consistent with that. But, my finding also provides additional context by showing specific, in-classroom ways in which unfamiliarity can directly affect the behavior of small groups of students; whereas Theobald et al. (2017) only collected data on the aftermath of the group work experience.

Gender and Beliefs About Gender and Mathematics

Research at the secondary and undergraduate levels on group work in STEM courses indicate a mixed picture of how gender could potentially play a role in group work in undergraduate mathematics classrooms (Chiu, 2000a; Esmonde et al., 2009; Heller &

Hollabaugh, 1992; Laursen et al., 2014; Mullins et al., 2020; Sullivan et al., 2018). While this study only adds another example to an area of contradictory literature, I think it is clear that within this study, gender had an impact – particularly with Group 2.

As already discussed, Group 2 had difficulty working together productively which was recognized by both participants in the group. Krista and Steven each believed that their unfamiliarity with each other was a factor in this. However, it seems that their unfamiliarity intersected with the differing gender identities and beliefs about gender's relationship to mathematics in limiting Krista and Steven's ability to collaborate effectively. While Krista stated she believed herself to be a strong math student, she also felt that she had some biases about how working with Steven would be on the basis of their genders, and that those beliefs made it difficult for her to express herself the way she would in a group of all women. On the other hand, Steven expressed an ignorance to the idea that gender might influence group work at all, and did not consider how this might affect Krista's experience.

With Groups 1 and 3, evidence of any influence of gender is less clear. Flora from Group 1 indicated that being a woman has affected how others work with her in math group work, but she did not refer to any specific examples. It is of note that she was the only woman in her group, a group structure that Heller and Hollabaugh (1992) and Sullivan et al. (2018) suggest is most dysfunctional. From a social perspective, it is not entirely clear that this was the case with Group 1, but Flora did participate second-least out of anyone in the group. Perhaps her status as the only woman in the group did influence the group's behavior in a way that Flora was not conscious of. With Group 3, it seems that unfamiliarity was an overriding factor in their lack of collaboration; neither Josh nor Eva indicated any sense of gender influence their group or any other group in math class. While influences from gender are still possible, there's not enough evidence in the data to say.

Ultimately, what this study presents is very clear data that one group was dysfunctional due to its members' genders and beliefs about gender and mathematics, and less clear data that suggests another group may have been influenced by gender. These specific examples contextualize the broader claims other studies are trying to make by demonstrably showing not just that there was an impact, but how there was an impact of gender on these groups.

Mathematically Unproductive, but Socially Productive

While the failure of Groups 2 and 3 to collaborate effectively in solving their assigned problems can be tied in part to the unfamiliarity between those group members, the problems faced by Group 1 are decidedly different. In the second selected interaction, while the students in the group all participated in a way that they viewed as effective, their discussion did not develop towards a mathematically accurate solution for the problem. Despite further work and instruction on the topic in class between the observation and interview, none of the participants suggested that their understanding had significantly changed since that discussion. Yet, when looking at the interactions via Chiu's (2000b) coding scheme and by the interview responses of the students, this appeared to be a socially productive interaction – with a variety of students contributing new ideas and discussing the positive and negative aspects of each.

What this tells us is that group work may be socially productive without being mathematically productive. This is particularly concerning when you consider the role an instructor plays in the classroom where students are doing group work. Since the instructor cannot physically be with more than one group at a time, they must pick and choose which groups to spend time with at each given moment. An instructor may see a group of students engaged in vigorous debate about a discussion question like the one posed to Group 1 and mistake social, on-task engagement for effective mathematical progress. They then might decide to go work with a group that is showing more obvious signs of dysfunction (like Groups 2 and 3), and fail to course-correct the former group.

Thus, it is clear that while social factors are important in understanding how small groups work in undergraduate mathematics, we cannot explain all instances of unproductive work solely in terms of social factors . While it is likely that the relative unfamiliarity between some of the group members or the gender distribution of the group influenced the way the discussion occurred, the fundamental issue with the discussion was a misinterpretation of what $g(f(x))$ could mean in the problem. It is hard for me to imagine a group of students having this misinterpretation and having a mathematically productive discussion, even if one proposed this question to a group of students who identify as friends or with a more balanced gender distribution. I am inclined to consider whether, in this case, the problem could have been differently designed or posed to the students to avoid their initial misinterpretation. Answering this question is beyond the scope of the data I have collected in this study.

Limitations

While the findings of this study provide valuable information about ways that groups can work unproductively, it is clear to me that there are numerous limitations to the application of these findings. This study included a total of ten students split across three groups. Moreover, these groups were not their normal working groups in their courses. Based on the small sample size of the study alone, it is clear that we cannot assume that all groups behave like these groups, or even that these groups were representative of their classrooms.

Moreover, the behavior of the participants of this study presents its own set of limitations. Since Group 3 did not interact with each other over the assigned group work, I was not able to analyze their interactions with each other in the same way that I had hoped. While this phenomenon was still interesting, it means that only two groups provided valuable data about group interactions, with one group providing data about the lack of interaction. Additionally, since one student from Group 1 and two students from Group 3 did not participate in follow-up interviews, any claims about those groups are being made while missing the perspective of the students who declined to be interviewed. When considering the original of this study, the biggest limitations are the difficulty I faced in recruiting LGBTQ students, and the relatively short length of most of the interviews.

Difficulties in recruiting LGBTQ students. Most past studies that have sought to understand the experiences of LGBTQ students have specifically recruited for only LGBTQ students (Cech & Waidzunas, 2011; Cooper & Brownell, 2016; Linley et al., 2018). The challenges this study faced are reflective of one reason why this might be the case: namely, that with an opt-in sample that has not specifically targeted LGBTQ participants, it is a matter of luck as to whether you recruit any. This study did manage to recruit one participant, Leticia, who did identify as asexual (and most would consider asexual individuals as falling under the LGBTQ umbrella), but she participated in a group that did not interact enough for the interactions to be analyzed and did not agree to a follow-up interview.

This study could have taken a different approach more similar to Cooper and Brownell's (2016), recruiting specifically for LGBTQ participants and inquiring about their experiences in small groups in a precalculus course. However, this would have made the classroom observation portion of this study less valuable in a few ways. First, if LGBTQ participants were being observed with the knowledge that the observer was looking into how their LGBTQ status was affecting their participation and experience in group work, it is almost certain that their LGBTQ status *would have* affected their participation and experience in group work – possibly through stereotype threat (Spencer et al., 1999). That is, knowledge of that focus of the study during the group observation phase would have made the behaviors noted during that phase less accurate. Moreover, to fully understand a group, the study design called for analyzing the actions of and interviewing each student from the group, if possible. Non-LGBTQ students might have been unwilling to provide consent for being observed in a study if they were aware of the focus on LGBTQ students and might not have answered interview questions as honestly as they would have otherwise.

So, I do not believe that the methodological decision to not specifically recruit LGBTQ students was a mistake. Instead, I believe the methodological decision to conduct this study as an opt-in study was the mistake. Time and logistical constraints likely would have prevented me from conducting this study as an opt-out study. However, if it had been conducted in that way, potentially all or most students in each class could have been observed in groups. With a larger net as it were, it would be more likely (though still not guaranteed) to naturally encounter LGBTQ participants working with non-LGBTQ participants in a sample. This change would balance the desire not to *cause* LGBTQ participants to change their behavior with the desire to actually be able to analyze and discuss their behavior and experience.

Length of interviews. It was intended for each interview to take approximately 30 minutes to complete; yet, only two interviews (Steven's and Krista's) approached that time. This indicates that I did not get as much of an insight into each student's experience as I might have liked. Some of this can be chalked up to the fact that I only selected two interactions to ask

Group 1 about, while I asked Group 2 about three. For Group 3, I did not have any interactions to ask them about.

I would also attribute some of this simply to how open the students were during the interview process. Steven and Krista gave long, detailed answers to each of my questions on the interview protocol and the follow-up questions I asked them as well. On the opposite end of the spectrum, Josh gave initial responses to most questions of only a few words, and required multiple follow-up questions just to fully answer the initial questions.

I still believe that I have sufficient information between the group observations and the interviews to have a degree of understanding of each interviewed student's experience. However, the length of the interviews leaves open the possibility that my understanding of these students is incomplete or partially incorrect.

Implications for Research and Practice

While the claims made here are limited, there are still valuable findings from this study that future researchers and instructors may build on to help understand and improve group work in undergraduate mathematics classrooms. The claims and limitations thereupon for this study also provide some guidance on how future work may better address goals of equity in STEM instruction. In this section, I discuss these implications in three parts; first, I reflect on the theoretical framework adopted by this study. Second, I discuss how future studies might build on the methodology of this study to build on its findings. Finally, I discuss what instructors might take away from these findings at this time.

Reflections on the theoretical framework. In developing my theoretical framework for this study, I incorporated ideas from intersectionality theory (Crenshaw, 1991; Levya, 2017) and role theory (Biddle, 1986; Tatsis & Koleza, 2006) to conceptualize and analyze group work in exploring my research questions. I believe that this theoretical framework was valuable – although I am not certain I used these theories as effectively as I could have.

Intersectionality theory. In terms of intersectionality theory, it was difficult to be fully intersectional in my analysis, as my sample had less diversity than I would have hoped. Having said that, I believe that my efforts to deeply consider each individual student's perspective on group work, and giving all students the opportunity to discuss experiences based on their own gender and sexual orientation, I followed the spirit of intersectionality. My analysis of Group 2's difficulty in collaborating also is fairly strong in terms of being intersectional. A less intersectional analysis might use unfamiliarity or gender as the only important factor, discarding the other. In fact, it was the intersection of those two factors, as well as Steven and Krista's beliefs about mathematics and gender, that influenced the interaction. I don't believe you could isolate the impact any one factor had on the interaction, as they each affected one another.

However, I could have attuned myself more to the other identities of the participants in interviewing them about their experiences in math class. While my goal was to focus on gender and sexual orientation, I did collect additional demographic data from each participant, but I elected not to use it. I could have incorporated additional discussion in the interviews about their racial and ethnic identities, and their identities as students and learners, to build a more complete picture of why each student participated in each group as they did. However, this is not a limitation on the theory, so much as a limitation on my implementation of the theory.

Role theory & Chiu's (2000b) taxonomy. The elements of cognitive and symbolic interactionist role theory and Chiu's (2000b) taxonomy that I used proved to be a valuable way to understand group work. I broke down each group's work into individual actions, used Chiu's taxonomy to understand those actions, used those actions to understand what role that student played in the group in certain interactions and overall, and then used my interviews with the students to better understand why they behaved the way they did and how they viewed the functioning of the group. This focus on individual actions and roles as a way to understand groups was productive towards addressing my research questions.

However, I do think that while the role theoretic perspective I took offered great insights, these insights were primarily about the social aspects of group work. The social interactions were my focus for this study, so that wasn't necessarily hugely problematic. However, it meant that my analysis only had anything to say about the mathematical learning aspect of the group work when those implications were obvious to me (i.e. the second interaction with Group 1). Even Chiu's (2000b) taxonomy, which is situated more within mathematics education than role theory, was limited in that regard. The only distinction between mathematical ideas that the taxonomy alone makes is between a new mathematical idea and a repeated one. Therefore, while role theory was great for understanding the social interactions involved with the group work, I would need to have used ideas from learning theories to say more about the mathematical learning outcomes.

Building on methodology. Overall, the methodology used in this study produced some great insight into the observed groups. However, there are several changes that any study that would build on this one should make in design and implementation. To begin with, I would suggest that any future study building upon this study be conducted as an opt-out study. Under an opt-out model, one could conduct observations of 5 – 10 small groups simultaneously in the same classroom. Beyond increasing the likelihood of collecting data from LGBTQ-identifying participants, this would also allow for greater generalizability of claims from such a data set. With a large number of groups all working on the same mathematical tasks, one could draw stronger between-group comparisons of groups that are being effective and groups that are not at the same task. This would allow for stronger claims about what factors are related to these unproductive moments.

Moreover, a larger study following this same template could potentially look at additional aspects of identity – including race, socioeconomic status, and ethnicity, among others. Methodologically, a larger study could also potentially use statistical tests to make comparisons between groups of students based on identity. This could allow for more generalizable claims to be made about how gender, sexual orientation, and other aspects of identity might influence how students behave in small groups.

However, any future study using this study as a model should include revisions to the survey and interview protocol instruments. Questions from the modified Fennema-Sherman beliefs scale were posed on the survey but ultimately proved only minimally useful to this study, particularly given the small sample size. While a more quantitatively-oriented study might find data from those questions more useful, a future study that is still focusing on qualitative analysis would likely not need to ask such questions on the survey. The interview protocol ought to have several revisions to questions for the stimulated recall portion. Instead of asking whether social or mathematical factors were most important in understanding the interactions, a revised protocol could ask students to identify specific social factors *and* mathematical ideas influencing the interactions. With the original phrasing, students often (though not exclusively) discussed only social factors or only mathematical ideas. To gain a more complete understanding of how students experienced the interaction, asking them to identify specific factors of both kinds would likely produce more productive responses. In addition, the meaning of the phrase “mathematical factors” is somewhat unclear given that ‘factor’ has multiple mathematical meanings in addition to its common English meaning; the phrase “mathematical ideas” would likely be clearer to students.

Additionally, studies not following the methodology used here could still build upon the work done in this study. While Chiu’s (2000b) taxonomy is very appropriate for mathematical problem-solving, studying group work through discourse analysis or other qualitative methods could provide more information about how students experience small group work. Chiu’s (2000b) taxonomy groups a wide array of different kinds of actions under each of its categories – for example, “I disagree” and “that was the worst suggestion I’ve ever heard” are both coded the same on the evaluation of previous action dimension. Similarly, a new mathematical idea is coded as a contribution, regardless of if the idea is correct or incorrect. So, tools for analysis of group work that pay more attention either to tone or to the mathematical aspects of the group problem solving process, for example, may reveal more about how small groups work together.

As well, to better understand issues of identity and equity in group work, some studies that specifically recruit for students of underserved and underrepresented genders, races, ethnicities, sexual orientations, and socioeconomic statuses would be appropriate. Though such studies may be limited in terms of how they might use in-classroom data, it is unlikely that even large opt-out studies can guarantee enough diversity in their samples to be able to fully capture the experience of these students.

Finally, as future studies develop our understanding of how students interact when working in small groups in undergraduate mathematics classes, they should also keep an eye on implications for instructors. This study doesn’t offer any solutions for instructors wishing to detect, prevent, or address such problems. As our collective understanding of the student perspective on group work grows, it is important that we then connect this understanding back to the practice of teaching, so that math instructors can equitably implement small group work as an instructional strategy in their undergraduate classrooms.

Implications for instruction. Because this study is focused on student interactions with each other without considering the role of the instructor explicitly, the implications for this study are limited but important. For one, instructors might look to the examples of unproductive group work presented here to consider how they might more accurately identify unproductive group work in their own classrooms. As discussed previously, it can be difficult for an instructor to choose which groups to spend time with and on which problems, and instructors can't ever know exactly what happens in groups they aren't working with. By presenting scenarios to instructors in which students fail to collaborate effectively that they can review outside of the classroom environment, alongside many of the participating students' reflections on why things occurred the way they did, an instructor might be able to better identify groups in need of their assistance in the classroom. However, making specific recommendations for what instructors should do if they see a group working together in an unproductive way is beyond the scope of this study.

Significance of the Study

When this study began, I set out to better understand how students work together in small groups in undergraduate mathematics classrooms, and how issues of gender identity and sexual orientation influence those interactions. Ultimately, this study contributes to our understanding of these issues by giving several specific examples of how students may interact during group work and providing additional context using stimulated recall interviews to capture a more complete perspective of each interaction. The instances of unproductive collaboration and how unfamiliarity and gender did or did not influence those instances in this study provide clear evidence of how these factors can have direct impacts on group work, and also, when we need to look beyond social factors to understand why a group is not working productively. While I had less to say about sexual orientation than I had hoped, I believe that the methodologies for observing students, interviewing them, and analyzing those data are productive for answering that type of question. On this basis, I have made recommendations for future researchers and instructors on how to use my findings in their work. Though the sample size of this study is small, by focusing on individual groups and the experiences of each student within those groups, I have provided in-depth data to accompany and contextualize the larger but broader studies conducted by other researchers on these topics.

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APPENDIX A – INITIAL INTEREST FORM

Your name: ____

Your UMAINE email address: _____________________________________@maine.edu

Are you willing to participate in the group observation stage of the study? This will take place during your normal class time and with normal course work.

Yes No

Please indicate the following information regarding your course:

Lecture instructor: ______________________ Lecture days and time: ________________

Recitation instructor: ____________________ Recitation days and time: ______________

Are you willing to be contacted for a follow-up interview after the group observation phase of the study? You will be eligible for up to thirty minutes of tutoring on a math topic of your choice if you fully participate in an interview.

Yes No

Paper copies of this form will be destroyed after this data is digitized and stored; this will occur no later than November 30, 2019.

APPENDIX B – SURVEY

To better understand how social and demographic factors may influence group work in undergraduate mathematics, we need to know a little bit more about you as a person and as a learner of mathematics.

For each of the following statements regarding mathematics and your learning thereof, please rate the extent to which you agree or disagree on a scale of 1 to 6, with 1 indicating that you strongly disagree and 6 indicating that you strongly agree.

Statement	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
Studying math with others helps me see different ways to solve problems.	1	2	3	4	5	6
Generally I feel secure about attempting to learn mathematics.	1	2	3	4	5	6
I am sure I could do advanced work in mathematics.	1	2	3	4	5	6
Math has been my worst subject.	1	2	3	4	5	6
Talking with other students about math problems helps me understand better.	1	2	3	4	5	6
I am sure that I sure that I can learn mathematics.	1	2	3	4	5	6
I can get good grades in mathematics.	1	2	3	4	5	6
When I become confused about something I'm studying in math, I go back and try to figure it out myself.	1	2	3	4	5	6
Math is a solitary activity, done by individuals in isolation.	1	2	3	4	5	6
I don't think I could do advanced mathematics.	1	2	3	4	5	6
I have a lot of self-confidence when it comes to math.	1	2	3	4	5	6

Statement	Strongly Disagree	Disagree	Somewhat Disagree	Somewhat Agree	Agree	Strongly Agree
I learn math best when I study by myself.	1	2	3	4	5	6
For some reason even though I study, math seems unusually hard for me.	1	2	3	4	5	6
I prefer to work with other students when doing math assignments or studying for tests.	1	2	3	4	5	6
When I can't understand material in precalculus class, I like to ask another student in class for help.	1	2	3	4	5	6
Math is more interesting when I work with other people.	1	2	3	4	5	6
When I work on math with other students, I usually end up doing more than my share of the work.	1	2	3	4	5	6
I think I could handle more difficult mathematics.	1	2	3	4	5	6
I work harder when I work in a group with other students.	1	2	3	4	5	6
When I study math with other students, we don't get much done.	1	2	3	4	5	6
I'm not the type to do well in math.	1	2	3	4	5	6
It's hard to work with other students on math because some students work faster or slower than others.	1	2	3	4	5	6
Most subjects I can handle OK, but I have a knack for messing up in math.	1	2	3	4	5	6
I'm no good in math.	1	2	3	4	5	6

Please answer each of the following demographic questions to the best of your ability, as complete data will be most helpful for the study. If you don't understand a question, please ask the researcher for clarification. If you find any question uncomfortable or are otherwise unwilling to answer, you may skip that question.

Did you graduate from high school in Maine? Circle one. (Multiple Choice)

Yes No

What year are you in university? Circle one. (Multiple choice)

Freshman Sophomore Junior Senior Other

What is your major? (Open Response)

What is the gender with which you identify?. (Open Response)

What is your racial and/or ethnic identity? (Open Response)

What is your sexual orientation? (Open Response)

To associate the data from this survey with your group observation, please provide your University of Maine email address:

____@maine.edu

APPENDIX C – INTERVIEW PROTOCOL

1. Throughout this interview, I am going to ask you to think about your participation and the participation of others in small groups in your math class. If at any point any of the questions makes you uncomfortable, you can choose not to answer. If you need to end the interview for any reason, you are welcome to do so.
2. First, I'd like to start with some general questions about your experience with group work in math classrooms.
3. **Q:** Do you think that working in a small group on math problem improves, harms, or makes no difference to your understanding of the math concepts? Why?
4. **Q:** When working in small groups, do you find that your ideas and contributions are valued by your peers? Why or why not?
5. **Q:** When working in small groups, do you find that you value all of your peer's ideas and contributions equally? Why or why not?
6. Now, I'm going to show you a few moments from when I observed your group on (date). After each clip, I will ask you a few questions regarding it. I'm trying to understand what this experience was like from your point of view.
7. [Show clip]
8. **Q:** Can you tell me what was happening from your perspective during this clip?
   a. Ask for elaboration as needed to get a complete narrative of the student perspective, as well as whether the student recalls or does not recall the episode.
9. **Q:** Based on your recollection and your viewing of these moments, do you think that these moments happened the way they did more because of the mathematical content being discussed, or because of social factors? Why?
   a. Ask for elaboration as needed. Potential follow-up: How do you think this moment could have gone differently? Why do you think this moment worked differently than an earlier moment? Other questions based on student responses.
10. [Repeat 7 – 10 for several different moments / episodes]
11. **Q:** Were the clips shown typical representations of what happened in class?
12. **Q:** What does the phrase "gender identity" mean to you? What does the phrase "sexual orientation" mean to you?
   a. To clarify student responses to the following questions
13. **Q:** In working in small groups in your (calculus / precalculus) class, have you ever felt that your gender identity or sexual orientation influenced how your peers responded to something you said or did? If so, describe that.
14. **Q:** In working in small groups in your class, have you felt that someone else's gender identity or sexual orientation influenced how you or your peers responded to something they said or did? If so, describe that.

APPENDIX D – SAMPLE CODING SHEET

Participant	Timecode	Action/Statement	EPA	KC	IF
Person A	[00:00:04]	I think we should try using the Pythagorean theorem.	0	C	_
Person B	[00:00:09]	Okay that's what I was thinking too.	+	N	_
Person C	[00:00:12]	Why would you want to use the Pythagorean theorem instead of the area formula?	-	C	?
Person B	[00:00:25]	Because we don't need the area, we need to use the Pythagorean theorem.	-	R	_
Person A	[00:00:32]	So I got 8 when I did that.	0	C	_
Person B	[00:00:35]	[Person C], do the Pythagorean theorem and tell us what you got so we can compare.	0	R	!

APPENDIX E – PROBLEMS FOR GROUP 1

The problems in Appendices E and F were selected by the respective instructors for the observed sections. As a researcher, I did not request any input and had no input on what problems were selected. These problems come from the textbook used by all precalculus courses at the university where the study was conducted from Carlson, Oehrtman, and Moore (2018). Only pages with problems explicitly referenced in the text of this thesis are included here. Materials are copyrighted by Carlson, Oehrtman, and Moore, and their reproduction here qualifies under fair use under section 107 of the Copyright Act of 1976 due to the scholarly purpose of this thesis and the limited selection of pages reproduced.

There are two key takeaways from Exercise #1.

(i) The expressions we write should be meaningful and related to our understanding of the quantities. For example, note the meaning of each of the following expressions and how new meanings develop as we modify them.

- t represents the number of minutes Nikki has been running
- $720t$ represents the distance Nikki runs (in feet) in t minutes
- $\frac{720t}{5{,}280}$ represents the distance Nikki runs (in miles) in t minutes
- $100\left(\frac{720t}{5{,}280}\right)$ represents the number of calories Nikki burns in t minutes

(ii) Using the results of one calculation or set of calculations as the basis for another calculation is one of the foundations for understanding ***function composition*** – the process of chaining together multiple function processes.

*2. a. Draw a diagram of a square and label the side lengths x (measured in inches). Visualize the square growing and shrinking as the value of x changes and think about how the square's perimeter length (in inches) compares to the side length.

b. Define a function g that inputs the square's perimeter P and outputs the square's side length x (both measured in inches).

c. How does the square's side length change as the perimeter changes from 6 inches to 20 inches? Calculate this value **and** represent it using function notation.

d. Define a function h that inputs the square's side length x (in inches) and outputs the square's area A (in square inches).

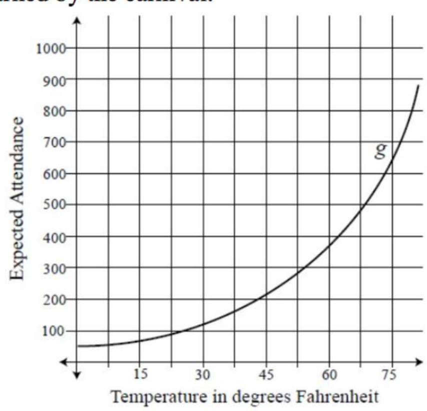
e. Using the functions in parts (b) and (d), determine the area of squares that have each of the following perimeters: 24 inches, 60 inches, 14 inches, and P inches.

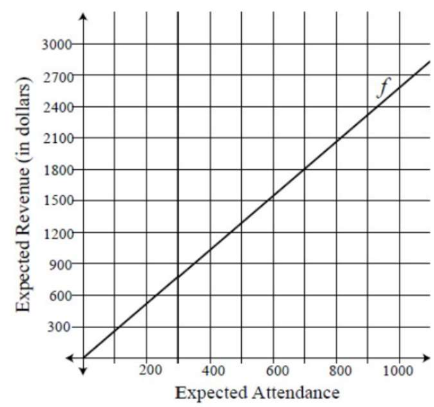
3. Using the context in Exercise #2, imagine that the square's perimeter begins at 0 inches and increases at a rate of 3 inches per second.
   a. What is the square's area 8 seconds after the perimeter started increasing? 10 seconds? 40 seconds?

   b. Define a function that inputs the time elapsed (in seconds) since the perimeter started increasing and outputs the square's area (in square inches).

   c. Discuss with a group (or as a class) how the process of using a time elapsed to determine the square's area involves thinking about function composition.

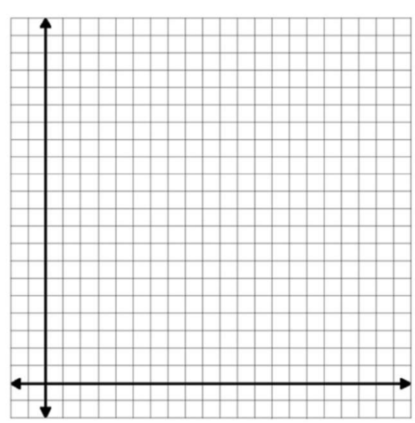
*4. The following graphs show two functions, *f* and *g*. Function *g* takes as its input the high temperature in degrees Fahrenheit and outputs the expected attendance at a neighborhood carnival. Function *f* takes as its input a number of people attending the carnival and outputs the total expected revenue earned by the carnival.





   a. If the forecast predicts clear skies and a high temperature of 45ºF, what is the expected revenue from the carnival today? What if the forecast predicts a high temperature of 75ºF?

   b. Function *h* inputs the high temperature in degrees Fahrenheit and outputs the expected revenue (in dollars). On the given axes plot at least 5 coordinate points representing input/output pairs for *h*.



In Exercise #4 we used outputs of g as inputs to f in order to determine the expected revenue given a forecasted high temperature. The notation for representing expected revenue given the forecasted high temperature should seem very logical.

expected revenue	
$f(\text{expected attendance})$	• f inputs expected attendance and outputs expected revenue
$f(g(\text{high temp °F}))$	• g(temp °F) represents the expected attendance given the high temp °F
$f(g(x))$	• Let x represent the high temperature in °F

So $f(g(x))$ represents the expected revenue (in dollars) given x, the high temperature in degrees Fahrenheit. Evaluating an expression like $f(g(15))$ is similar to following the order of operations. We begin with the inside function. *We used the graph to estimate each value.*

$f(g(15))$	
$f(75)$	• the expected attendance is 75 when the high temperature is 15°F
200	• the expected revenue is \$200 when the expected attendance is 75 people

Note that $f(g(x))$ represents the function's outputs, but it is not the function's *name* (just like $f(x)$ is the output of f, not the function name). We use the notation $f \circ g$ to name the composite function.

Function Composition Notation

If f and g are functions, then $f \circ g$ is the name of the composite function formed by chaining together the two processes where g is the "inside" function, meaning the process involves:

1. Inputting a value into g and producing an output value.
2. Using that output value as an input to f to get another output value. This is the output value for the composite function.

If $f \circ g$ is the name of the composite function, then $f(g(x))$ represents the function's output values. *Note that we sometimes condense the notation by just giving this new composite function a name like h. So we could define a function h to be the composite function $f \circ g$ by saying $h(x) = f(g(x))$.*

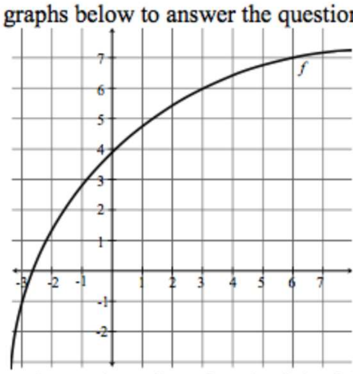
*5. a. In the context from Exercise #4 explain what $f(g(70))$ represents.

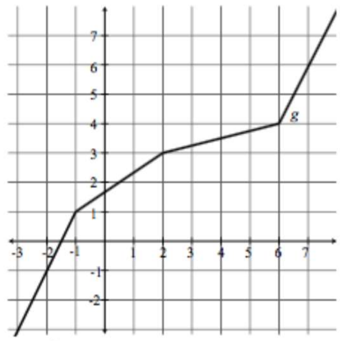
b. In the context from Exercise #4 explain why $g(f(70))$ does not represent a real-world quantity.

c. Let's define a new function k that is the composition of f and g, that is, $k(x) = f(g(x))$. Explain what the equation $1800 = k(x)$ represents, then explain how you can find the value of x that satisfies the equation.

APPENDIX F – PROBLEMS FOR GROUPS 2 AND 3

55. Use the graphs below to answer the questions that follow





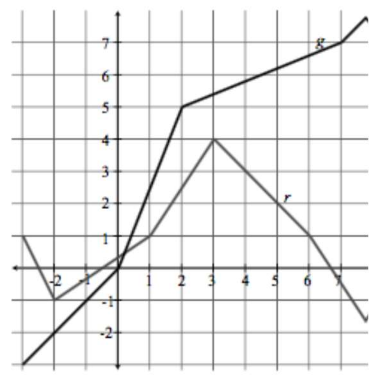
a. Approximate the value of each of the following expressions.
   i. $f(f(3))$ ii. $g(f(6))$ iii. $g(g(-2))$ iv. $g(f(-3))$

b. Find the value of *x* that satisfies each of the following equations.
   i. $f(g(x))=6$ ii. $g(f(x))=4$

56. Functions *g* and *r* are defined by their graphs to the right.

a. Determine the values of each of the following expressions:
   i. $r(g(2))$ ii. $g(r(1))$ iii. $r(r(6))$ iv. $g(r(-2))$

b. How does the output $g(r(x))$ vary as *x* varies from 1 to 3?



57. For each of the functions defined below, redefine that function in terms of two new functions, *f* and *g*, using function composition and function arithmetic. *For example,* $f(x)=(2x)^3$ *can be defined as* $f(x)=h(g(x))$ *if* $g(x)=2x$ *and* $h(x)=x^3$.

a. $h(x) = 3(x-1)+5$
b. $m(x) = (x+4)^2$
c. $k(x) = (x+2)^2 + 3(x+2) + 1$
d. $j(x) = \sqrt{x-1}$
e. $p(x) = \frac{500}{100-x^2}$

V: Extra Practice with Function Composition (TEXT: S5)

58. The standard formula for determining temperature in degrees Fahrenheit when given the temperature in degrees Celsius is $F = \frac{9}{5}C + 32$. We can write this formula using function notation by letting $F = g(C)$ and writing $g(C) = \frac{9}{5}C + 32$. The function *g* defines a process for converting a temperature measure in degrees Celsius to degrees Fahrenheit.

a. State the meaning of $g(100)$ and evaluate $g(100)$.
b. Solve the equation $g(C) = 212$ and explain how you arrived at your answer.
c. Define a function *h* that converts temperature measures in degrees Fahrenheit to the corresponding measure in degrees Celsius. (*Hint: Generalize the steps you described in part (b) that reversed the process of g. You can also solve* $F = \frac{9}{5}C + 32$ *for C.*)
d. State the meaning of $h(212)$ and evaluate $h(212)$.
e. Determine the values of $g(h(212))$ and $h(g(100))$ without performing any calculations. What do you notice about the relationship between *g* and *h*?

Exercise continues on the next page.

62. You are planning a trip to Japan where the currency is the yen. On your way to Japan you will stop in Italy where the currency is the euro. You know that you will need to convert your US dollars to euros before your trip and know that the number of euros is 0.78 times as large the number of dollars. You also know the number of yen is 103 times as large as number of dollars.
    a. Write a formula that expresses the number of yen, y, you will have if you begin with d dollars.
    b. Write a formula that expresses the number of euro, e, you will have if you begin with d dollars
    c. You know that you will not be converting from dollars to yen because you will first stop in Italy. So it would be more helpful to know the conversion rate between euros and yen. Write a function g that expresses the number of yen you will have, y, in terms of the number of euros you convert.

63. The length of a steel bar changes as the temperature changes. Consider a 10-meter steel bar that is placed outside in the morning when the temperature is 20 degrees Celsius. Let l represent the length of a steel bar in meters. Let t be the temperature of the steel bar in degrees Celsius. And let n be the number of minutes since you placed the bar outside. You are given the following relationships: $l = 0.00013(t-20) + 10$ and $n = 12t - 240$. Write a function f that gives the length of the bar, l, (in meters) in terms of the number of minutes elapsed, n, since the bar was placed outside.

64. Let $n = 240s + 123$ and let $t = 0.14s^2$. Write a function h that gives n in terms of t.

65. Let $160 = 2t + 4m$ and let $p = \frac{1}{4}tm^2$. Write a function k that gives p in terms of t.

VI. Inverse Functions: Reversing the Process (Text: S6)

In Exercises #66-68, do the following.

a. Describe the function process (for example, "the input is increased by 2 and then tripled").
b. Describe the process that undoes the function (for example, if f is a process that increases its input by 7, the process that undoes f will decrease its input by 7).
c. Algebraically define a process that undoes the process of the given function – that is, define the inverse function for each of the given functions.

66. $f(x) = \frac{2}{5}x$ 67. $g(x) = 10 + x$ 68. $h(x) = \frac{x}{12}$

69. a. Define algebraically a process f that multiplies its input by 3, and then decreases this result by 1.
    b. Describe the process that undoes the process of f described in part (a).
    c. Algebraically define the process f^{-1} that undoes the process of f that is described in part (a).

70. Given the function f defined by $f(x) = 3x + 2$,
    a. Define f^{-1}, the function that undoes the process of f.
    b. Show that $f\left(f^{-1}(x)\right) = x$ for the function f that is given above.
    c. Show that $f^{-1}\left(f(x)\right) = x$ for the function f that is given above.
    d. What do you conclude about the relationship between f and f^{-1}

71. Given $g(x) = 14x - 9$, what are the value(s) of the input x when $g(x) = 109$?

72. Given $h(x) = \frac{1}{6}x^2 + 7$, what are the value(s) of the input x when $h(x) = 31$?

APPENDIX G – GLOSSARY

This glossary is adapted from definitions offered in multiple sources, including Cooper and Brownell, 2016; LGBTQIA Resource Center, n.d.; Yoder and Mattheis, 2016.	
Asexual	A sexual orientation describing someone who does not experience sexual attraction.
Bisexual	A sexual orientation describing someone who experiences sexual attraction towards both men and women, OR, a sexual orientation describing someone who experiences sexual attraction to people of at least two gender identities.
Cisgender	A term to describe a person whose gender identity matches their assigned gender, e.g. an individual who identifies as male and who was assigned male at birth.
Gay	A sexual orientation describing someone who primarily experiences sexual attraction towards individuals of the same gender.
Gender	A socially construct which classifies individuals as men, women, or some other identity and is distinct from biological sex.
Gender identity	An individual's self-identified gender, which may or may not differ from the gender they were assigned at birth and the gender which other individuals read them as.
Heteronormativity	Norms and practices that assume binary alignment of biological sex, gender identity, and gender roles. Under heteronormativity, it is expected that individuals be cisgender and be attracted to the opposite gender.
Intersex	Describes an individual whose chromosomes, hormones, and primary and secondary sex characteristics differ from expected patterns of male and female.
Lesbian	A term used to describe a woman who experiences sexual attraction primarily to other women.
LGBTQ	The initialism used in this study for individuals whose gender identity or sexual orientation is in some way not heteronormative. Other studies and analyses use different terms, including: LGBT, LGBT+, LGBTQIA, queer, and others.
Non-binary	A gender identity to describe a person who does not strictly identify as a man or a woman. Individuals may identify simply as non-binary or may have other gender identities that could be grouped as non-binary, including genderfluid and genderqueer.
Pansexual	A sexual orientation describing someone who experiences sexual attraction regardless of gender identity.
Queer	Sometimes an umbrella term to refer to LGBTQ individuals. Also used as a descriptor for an individual's sexual orientation or gender identity.
Sex	A biologically or medically constructed categorization, usually assigned based on genitalia or chromosomes depending on context.
Transgender	A term used to describe a person whose gender identity does not match their assigned gender.

BIOGRAPHY OF THE AUTHOR

Jeremy Bernier spent time growing up in Portland, Gray, Skowhegan, and Auburn, Maine, graduating from Edward Little High School in 2011. He then attended Boston University, where he earned a Bachelor of Science in Mathematics Education, graduating in 2015. Jeremy spent a year as an intern at Walt Disney World, before teaching mathematics for two years at Maine Connections Academy, an online charter school serving students across Maine. In 2018, Jeremy enrolled in the Master of Science in Teaching program at the University of Maine, where he concentrated in Mathematics. He will be enrolling in Arizona State University's PhD program in Learning, Literacies, and Technologies in the Fall of 2020. He is a candidate for the Master of Science in Teaching degree from the University of Maine in May 2020.